CGA5_SampleFile

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パッケージを読み込むフォルダをSetDirectoryで指定しておく.

```
SetDirectory[FileNameJoin[
   {$HomeDirectory, "Dropbox/Ymken2014/2014_M1seminar/20140424_Versor"}]];
<< CGA5Package`;
```

第一章 基本関数

Basis

$\mathbb{R}^{4,1}$ の基底をB={0, 1, 2, 3, ∞} とする.
$\mathbb{G}^{4,1}$ の基底をGB={w[S] | S⊂ B}とする.
GBの元は (2^5 =) 32 個ある.
$\mathbb{R}^3 \subset \mathbb{R}^{4,1} \subset \mathbb{G}^{4,1}$ この包含写像を Inc:$\mathbb{R}^3 \to \mathbb{G}^{4,1}$ とする.
点p∈$\mathbb{R}^3$に対して,Inc(p)=Pnt[p]である.
三点p,q,r∈$\mathbb{R}^3$を通る$\mathbb{R}^3$内の円をCとするとき,Inc(C)=Cir[p,q,r]である.

例

```
Pnt[{1, 2, 3}]
```

w[{0}] + w[{1}] + 2 w[{2}] + 3 w[{3}] + 7 w[{∞}]

```
Cir[{1, 2, 3}, {2, 8, 3}, {2, 8, 4}]
```

$$w[\{0,1,3\}] + \frac{7}{2} w[\{0,1,\infty\}] + 6\, w[\{0,2,3\}] + 21\, w[\{0,2,\infty\}] - \frac{63}{2} w[\{0,3,\infty\}] + 4\, w[\{1,2,3\}] + 14\, w[\{1,2,\infty\}] - 35\, w[\{1,3,\infty\}] - 84\, w[\{2,3,\infty\}]$$

和,差,スカラー倍

$\mathbb{G}^{4,1}$ の和と差とスカラー倍は,線形空間$\mathbb{R}^{32}$と同様で記号は+と-を用いる.

```
Pnt[{1, 2, 3}] + Pnt[{6, 5, 4}]
```

$$2\, w[\{0\}] + 7\, w[\{1\}] + 7\, w[\{2\}] + 7\, w[\{3\}] + \frac{91}{2} w[\{\infty\}]$$

```
Pnt[{1, 2, 3}] - Pnt[{1, 0, 3}]
```
2 w[{2}] + 2 w[{∞}]

```
Pnt[{1, 2, 3}]
```
w[{0}] + w[{1}] + 2 w[{2}] + 3 w[{3}] + 7 w[{∞}]

```
3 Pnt[{1, 2, 3}] // Expand
```
3 w[{0}] + 3 w[{1}] + 6 w[{2}] + 9 w[{3}] + 21 w[{∞}]

CGAProduct : $\mathbb{G}^{4,1}\times\mathbb{G}^{4,1}\to\mathbb{G}^{4,1}$, $(\mathbb{G}^{4,1})^{n}\to\mathbb{G}^{4,1}(n\geq0)$

CGAの基本的な演算.
幾何積(Geometric Product,ジオメトリック積,CGA積:*)

CGAProduct は,幾何積を表す関数である.
引数には,CGAの元の組を用いる.

計算例

```
CGAProduct[ w[{∞}], w[{0}]]
```
-w[{}] - w[{0, ∞}]

```
CGAProduct[w[{0}], w[{∞}]]
```
-w[{}] + w[{0, ∞}]

```
CGAProduct[w[{0, ∞}], w[{0, ∞}]]
```
w[{}]

```
CGAProduct[w[{3, ∞}], w[{3, ∞}]]
```
0

```
CGAProduct[w[{0, ∞}], w[{0}]]
```
-w[{0}]

```
CGAProduct[w[{0, ∞}], w[{1, 2, 3}]]
```
-w[{0, 1, 2, 3, ∞}]

```
CGAProduct[w[{∞}], w[{0, ∞}]]
```
-w[{∞}]

```
CGAProduct[w[{2, ∞}], w[{2}]]
```
-w[{∞}]

```
CGAProduct[w[{2, 3}], 2 w[{2, 3}]]
```
-2 w[{}]

```
CGAProduct[w[{0, 2}], w[{1, 2, 3}]]
```

```
-w[{0, 1, 3}]
```

```
CGAProduct[w[{0, 1, 2, 3, ∞}], w[{0, 1, 2, 3, ∞}]]
```

```
-w[{}]
```

```
CGAProduct[w[{0, 1, 2, ∞}], w[{0, 1, 2, ∞}]]
```

```
-w[{}]
```

```
w[{}] + CGAProduct[w[{∞}], w[{0}]]
```

```
-w[{0, ∞}]
```

```
CGAProduct[(w[{}] + CGAProduct[w[{∞}], w[{0}]]),
  (w[{}] + CGAProduct[w[{∞}], w[{0}]])]
```

```
w[{}]
```

線形性をもつ.

```
CGAProduct[ w[{0}] + 3 w[{2}],  5 w[{3}] - 7 w[{∞}]]
```

```
7 w[{}] + 5 w[{0, 3}] - 7 w[{0, ∞}] + 15 w[{2, 3}] - 21 w[{2, ∞}]
```

```
CGAProduct[ w[{0}],  5 w[{3}] - 7 w[{∞}]] +
   3 CGAProduct[ w[{2}],  5 w[{3}] - 7 w[{∞}]] // Expand
```

```
7 w[{}] + 5 w[{0, 3}] - 7 w[{0, ∞}] + 15 w[{2, 3}] - 21 w[{2, ∞}]
```

```
CGAProduct[ w[{0, ∞}], 2 w[{0}] + 3 w[{1}]]
```

```
-2 w[{0}] - 3 w[{0, 1, ∞}]
```

```
2 CGAProduct[ w[{0, ∞}],  w[{0}]] + 3 CGAProduct[ w[{0, ∞}], w[{1}]]
```

```
-2 w[{0}] - 3 w[{0, 1, ∞}]
```

スカラー倍は w[{}] (またはSca)を用いて表現可能.

```
CGAProduct[w[{2, 3}], 3 w[{}] ]
```

```
3 w[{2, 3}]
```

```
CGAProduct[3 w[{}], w[{2, 3}] ]
```

```
3 w[{2, 3}]
```

```
CGAProduct[w[{2, 3}], Sca[3] ]
```

```
3 w[{2, 3}]
```

引数が三つ以上の場合

{}でくくって,複数の元を引数に

```
CGAProduct[{ w[{2, 3}], w[{0}], w[{0}] }]
```

0

```
CGAProduct[{ w[{0, 3}], w[{0, ∞}], w[{0}] + w[{1}]}]
```

-w[{0, 1, 3}]

```
CGAProduct[{ w[{0, 3}], w[{0, ∞}], w[{0}] + w[{1}],  5 w[{3}] - 7 w[{∞}] }]
```

-5 w[{0, 1}] - 7 w[{1, 3}] + 7 w[{0, 1, 3, ∞}]

```
CGAProduct[{7 w[{0, 1, 3, ∞}], w[{0}] }]
```

-7 w[{0, 1, 3}]

```
CGAProduct[{ w[{0, 3}], w[{0, ∞}], w[{0}] + w[{1}], 5 w[{3}] - 7 w[{∞}], w[{0}] }]
```

-14 w[{0, 1, 3}]

交換法則は成り立たない！

```
CGAProduct[{ w[{0, 3}], w[{0, ∞}], w[{0}] + w[{1}], w[{0}], 5 w[{3}] - 7 w[{∞}] }]
```

0

引数0個は単位元.

```
CGAProduct[ {}]
```

w[{}]

$\mathbb{G}^{4,1}$の幾何積の演算表(32×32).

```
GB = Map[w, Subsets[{0, 1, 2, 3, ∞} (*基底*)]]; (*全ての部分集合*)
Array[CGAProduct[GB[[#1]], GB[[#2]]] &, {Length[GB], Length[GB]}] // MatrixForm
```

w[{}]	w[{0}]	w[{1}]	w[{2}]
w[{0}]	0	w[{0, 1}]	w[{0, 2
w[{1}]	-w[{0, 1}]	w[{}]	w[{1, 2
w[{2}]	-w[{0, 2}]	-w[{1, 2}]	w[{}]
w[{3}]	-w[{0, 3}]	-w[{1, 3}]	-w[{2, 3
w[{∞}]	-w[{}] - w[{0, ∞}]	-w[{1, ∞}]	-w[{2, ∞
w[{0, 1}]	0	w[{0}]	w[{0, 1,
w[{0, 2}]	0	-w[{0, 1, 2}]	w[{0}]
w[{0, 3}]	0	-w[{0, 1, 3}]	-w[{0, 2,
w[{0, ∞}]	-w[{0}]	-w[{0, 1, ∞}]	-w[{0, 2,
w[{1, 2}]	w[{0, 1, 2}]	-w[{2}]	w[{1}]
w[{1, 3}]	w[{0, 1, 3}]	-w[{3}]	-w[{1, 2,
w[{1, ∞}]	-w[{1}] + w[{0, 1, ∞}]	-w[{∞}]	-w[{1, 2,
w[{2, 3}]	w[{0, 2, 3}]	w[{1, 2, 3}]	-w[{3}
w[{2, ∞}]	-w[{2}] + w[{0, 2, ∞}]	w[{1, 2, ∞}]	-w[{∞}
w[{3, ∞}]	-w[{3}] + w[{0, 3, ∞}]	w[{1, 3, ∞}]	w[{2, 3,
w[{0, 1, 2}]	0	-w[{0, 2}]	w[{0, 1
w[{0, 1, 3}]	0	-w[{0, 3}]	-w[{0, 1, 2
w[{0, 1, ∞}]	-w[{0, 1}]	-w[{0, ∞}]	-w[{0, 1, 2
w[{0, 2, 3}]	0	w[{0, 1, 2, 3}]	-w[{0, 3
w[{0, 2, ∞}]	-w[{0, 2}]	w[{0, 1, 2, ∞}]	-w[{0, ∞
w[{0, 3, ∞}]	-w[{0, 3}]	w[{0, 1, 3, ∞}]	w[{0, 2, 3,
w[{1, 2, 3}]	-w[{0, 1, 2, 3}]	w[{2, 3}]	-w[{1, 3
w[{1, 2, ∞}]	-w[{1, 2}] - w[{0, 1, 2, ∞}]	w[{2, ∞}]	-w[{1, ∞
w[{1, 3, ∞}]	-w[{1, 3}] - w[{0, 1, 3, ∞}]	w[{3, ∞}]	w[{1, 2, 3,
w[{2, 3, ∞}]	-w[{2, 3}] - w[{0, 2, 3, ∞}]	-w[{1, 2, 3, ∞}]	w[{3, ∞
w[{0, 1, 2, 3}]	0	w[{0, 2, 3}]	-w[{0, 1,
w[{0, 1, 2, ∞}]	-w[{0, 1, 2}]	w[{0, 2, ∞}]	-w[{0, 1,
w[{0, 1, 3, ∞}]	-w[{0, 1, 3}]	w[{0, 3, ∞}]	w[{0, 1, 2,
w[{0, 2, 3, ∞}]	-w[{0, 2, 3}]	-w[{0, 1, 2, 3, ∞}]	w[{0, 3,
w[{1, 2, 3, ∞}]	-w[{1, 2, 3}] + w[{0, 1, 2, 3, ∞}]	-w[{2, 3, ∞}]	w[{1, 3,
w[{0, 1, 2, 3, ∞}]	-w[{0, 1, 2, 3}]	-w[{0, 2, 3, ∞}]	w[{0, 1, 3,

OuterProduct : $\mathbb{G}^{4,1}\times\mathbb{G}^{4,1}\rightarrow\mathbb{G}^{4,1}$, $(\mathbb{G}^{4,1})^{n}\rightarrow\mathbb{G}^{4,1}$(n≥0)

CGAの外積(ウェッジ積,楔積:^)
Blade同士の積の最大のGradeが外積となっている.

OuterProduct は,外積を表す関数である.
引数には,CGAの元の組を用いる.

計算例

```
OuterProduct[ w[{1}], w[{0}] ]
```

-w[{0, 1}]

```
OuterProduct[ w[{2, ∞}], w[{0, 3}]]
```
-w[{0, 2, 3, ∞}]

```
OuterProduct[w[{1, 3}], w[{0}]] + OuterProduct[w[{2}], w[{0}]]
```
-w[{0, 2}] + w[{0, 1, 3}]

```
OuterProduct[ 2 w[{}], 3 w[{}] ]
```
6 w[{}]

```
OuterProduct[w[{0, 1, 3}], 0]
```
0

同じ元同士の外積は0

```
OuterProduct[w[{1, 3}], w[{1, 3}]]
```
0

反対称性

```
OuterProduct[ w[{∞}], w[{0}]]
```
-w[{0, ∞}]

```
OuterProduct[ w[{0}], w[{∞}]]
```
w[{0, ∞}]

線形性(分配則)

```
OuterProduct[w[{1, 3}] + w[{2}], w[{0}]]
```
-w[{0, 2}] + w[{0, 1, 3}]

```
OuterProduct[w[{1, 3}], w[{0}]] + OuterProduct[w[{2}], w[{0}]]
```
-w[{0, 2}] + w[{0, 1, 3}]

結合則

```
OuterProduct[ OuterProduct[w[{0}], w[{∞}]] , w[{3}] ]
```
-w[{0, 3, ∞}]

```
OuterProduct[ w[{0}] , OuterProduct[w[{∞}], w[{3}]] ]
```
-w[{0, 3, ∞}]

引数が三つ以上の場合

{}でくくって,複数の元を引数に

```
OuterProduct[{ w[{2, 3}], w[{0}] + w[{∞}], w[{1}] }]
```
w[{0, 1, 2, 3}] - w[{1, 2, 3, ∞}]

```
OuterProduct[{ w[{0, 3}] + w[{3}],
  w[{0, ∞}] + 5 w[{2}] + 3 w[{0}], w[{1}] + w[{2}], 5 w[{3}] - 7 w[{∞}] }]
```

```
-21 w[{0, 1, 3, ∞}] - 21 w[{0, 2, 3, ∞}] + 35 w[{1, 2, 3, ∞}] + 35 w[{0, 1, 2, 3, ∞}]
```

```
OuterProduct[{w[{0, 3}], w[{0, ∞}], w[{0}] + w[{1}], w[{0}], 5 w[{3}] - 7 w[{∞}] }]
```

```
0
```

InnerProduct : $\mathbb{G}^{4,1}\times\mathbb{G}^{4,1}\to\mathbb{G}^{4,1}$, $(\mathbb{G}^{4,1})^{n}\to\mathbb{G}^{4,1}(n\geq 0)$

CGAの内積(ドット積:・)
Blade同士の積の最小のGradeが内積となっている.

InnerProduct は,内積を表す関数である.
引数には,CGAの元の組を用いる.

計算例

```
InnerProduct[ w[{1}], w[{1}]]
```

```
w[{}]
```

```
InnerProduct[ w[{1}], w[{2}]]
```

```
0
```

```
InnerProduct[ w[{∞}], w[{0}]]
```

```
-w[{}]
```

```
InnerProduct[ w[{0, ∞}], w[{∞}]]
```

```
w[{∞}]
```

```
InnerProduct[ w[{∞}], w[{0, ∞}]]
```

```
-w[{∞}]
```

```
InnerProduct[ w[{0, ∞}], w[{0}]]
```

```
-w[{0}]
```

```
InnerProduct[ w[{0, ∞}], w[{0, ∞}]]
```

```
w[{}]
```

```
InnerProduct[ w[{∞}], w[{∞}]]
```

```
0
```

```
InnerProduct[ 2 w[{}], 3 w[{}] ]
```

```
6 w[{}]
```

```
InnerProduct[w[{0, 1, 3}], 0]
```

```
0
```

```
InnerProduct[w[{1, 3}], w[{1, 3}]]
```

-w[{}]

```
InnerProduct[ 5 w[{1, 3}] + 2 w[{0, 1}]  , 7 w[{1}]]
```

14 w[{0}] - 35 w[{3}]

```
InnerProduct[ 7 w[{1}] ,  5 w[{1, 3}] + 2 w[{0, 1}] ]
```

-14 w[{0}] + 35 w[{3}]

線形性

```
InnerProduct[ 5 w[{1, 3}] + 2 w[{0, 1}] , 7 w[{1}]]
```

14 w[{0}] - 35 w[{3}]

```
InnerProduct[5 w[{1, 3}], 7 w[{1}]] + InnerProduct[2 w[{0, 1}], 7 w[{1}]]
```

14 w[{0}] - 35 w[{3}]

引数が三つ以上の場合

{}でくくって,複数の元を引数に

```
InnerProduct[{ w[{2}],  w[{1, 2}] + w[{1, ∞}],  w[{1, 2, 3}] }]
```

-w[{2, 3}]

Reversion : $\mathbb{G}^{4,1} \rightarrow \mathbb{G}^{4,1}$

CGAの元の逆順

Reversion は,逆順を表す関数である.
引数には,CGAの元を用いる.

計算例

```
Reversion[w[{1}]]
```

w[{1}]

```
Reversion[w[{0, 1}]]
```

-w[{0, 1}]

```
Reversion[w[{1, 2, 3}]]
```

-w[{1, 2, 3}]

```
Reversion[w[{0, 1, 2, 3}]]
```

w[{0, 1, 2, 3}]

```
Reversion[w[{0, 1, ∞}] + w[{0, 1, 2, 3}]]
```

```
-w[{0, 1, ∞}] + w[{0, 1, 2, 3}]
```

Dual : $\mathbb{G}^{4,1} \to \mathbb{G}^{4,1}$

CGAの元の双対

Dual は,双対を表す関数である.
引数には,CGAの元を用いる.

計算例

```
Dual[w[{∞}]]
```

```
w[{1, 2, 3, ∞}]
```

```
Dual[w[{1}]]
```

```
w[{0, 2, 3, ∞}]
```

```
Dual[w[{2}]]
```

```
-w[{0, 1, 3, ∞}]
```

```
Dual[w[{0, 1}]]
```

```
w[{0, 2, 3}]
```

```
Dual[w[{0, ∞}]]
```

```
w[{1, 2, 3}]
```

```
Dual[w[{1, 2, 3}]]
```

```
-w[{0, ∞}]
```

```
Dual[w[{0, 1, 2, 3}]]
```

```
-w[{0}]
```

```
Dual[w[{0, 1, ∞}] + w[{0, 1, 2, 3}]]
```

```
-w[{0}] - w[{2, 3}]
```

BASIC ELEMENTS

ELEMENTS

CGAでの,ベクトル関係の元.

Sca : $\mathbb{R} \to \mathbb{G}^{4,1}$

Sca は,スカラー(Scalar)を表す関数である.
引数には,実数を用いる.

```
Sca[8]
```

```
8 w[{}]
```

```
Sca[.2]
```

```
0.2 w[{}]
```

Vec : $\mathbb{R}^3 \to \mathbb{G}^{4,1}$

Vec は,ベクトル(Vector)を表す関数である.
引数には,実数の組を用いる.

```
Vec[{x, y, z}]
```

```
x w[{1}] + y w[{2}] + z w[{3}]
```

Biv : $\mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{G}^{4,1}$

Biv は,二重ベクトル(Bivector)を表す関数である.
引数には,実数の組を二つ用いる.

```
Biv[{0, 1, 0}, {1, 0, 0}]
```

```
-w[{1, 2}]
```

Tri : $\mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{G}^{4,1}$

Tri は,三重ベクトル(Trivector)を表す関数である.
引数には,実数の組を三つ用いる.

```
Tri[{0, 1, 2}, {1, 0, 2}, {1, 2, 3}]
```

```
3 w[{1, 2, 3}]
```

Round

CGAでの,球関係の元.

Pnt : $\mathbb{R}^3 \to \mathbb{G}^{4,1}$

Pnt は,点(Point)を表す関数である.
引数には,実数の組を用いる.

```
Pnt[{2, 4, 8}]
```

```
w[{0}] + 2 w[{1}] + 4 w[{2}] + 8 w[{3}] + 42 w[{∞}]
```

NPnt : Pnt[$\mathbb{R}^3$] → Pnt[$\mathbb{R}^3$]

NPnt は,点を正規化する関数である.
引数には,CGAの点を用いる.

```
D0 = Dilator[Pnt[{1, 2, 0}], 3]
```

$$\frac{w[\{0\}]}{e^3} + w[\{1\}] + 2\,w[\{2\}] + \frac{5}{2}\,e^3\,w[\{\infty\}]$$

```
NPnt[Dilator[Pnt[{1, 2, 0}], 3]]
```

$$w[\{0\}] + e^3\,w[\{1\}] + 2\,e^3\,w[\{2\}] + \frac{5}{2}\,e^6\,w[\{\infty\}]$$

Par : $\mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{G}^{4,1}$

Par は,点対(PointPair)を表す関数である.
引数には,実数の組を二つ用いる.

```
Par[{6, 1, 4}, {2, 1, -2}]
```

$$-4\,w[\{0, 1\}] - 6\,w[\{0, 3\}] - 22\,w[\{0, \infty\}] + 4\,w[\{1, 2\}] - 20\,w[\{1, 3\}] - 26\,w[\{1, \infty\}] - 6\,w[\{2, 3\}] - 22\,w[\{2, \infty\}] + 71\,w[\{3, \infty\}]$$

Cir : $\mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{G}^{4,1}$

Cir は,円(Circle)を表す関数である.
引数には,実数の組を三つ用いる.

```
Cir[{3, 0, 0}, {0, 3, 0}, {-3, 0, 0}]
```

$$18\,w[\{0, 1, 2\}] + 81\,w[\{1, 2, \infty\}]$$

Sph : $\mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{G}^{4,1}$

Cir は,球(Sphere)を表す関数である.
引数には,実数の組を三つ用いる.

```
Sph[{3, 0, 0}, {0, 3, 0}, {0, 0, 3}, {-3, 0, 0}]
```

$$-54\,w[\{0, 1, 2, 3\}] + 243\,w[\{1, 2, 3, \infty\}]$$

Flat

CGAでの,平面関係の元.

Lin : $\mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{G}^{4,1}$

Lin は,直線(Line)を表す関数である.
引数には,実数の組を二つ用いる.

```
Lin[{0, 0, 0}, {2, 5, 0}]
```

$$2\,w[\{0, 1, \infty\}] + 5\,w[\{0, 2, \infty\}]$$

Dll : $\mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{G}^{4,1}$

Dllは,双対直線(DualLine)を表す関数である.
引数には,実数の組を二つと実数の組一つを用いる.

```
Dll[{0, 1, 2}, {1, 0, 2}, {1, 2, 3}]
```

```
-w[{1, 2}] - 2 w[{1, 3}] + w[{1, ∞}] + 2 w[{2, 3}] + 2 w[{2, ∞}] + 3 w[{3, ∞}]
```

Pln : $\mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{G}^{4,1}$

Plnは,平面(Plane)を表す関数である.
引数には,実数の組を三つ用いる.

```
Pln[{3, 0, 1}, {0, 3, 1}, {0, 2, 1}]
```

```
3 w[{0, 1, 2, ∞}] + 3 w[{1, 2, 3, ∞}]
```

Dlp : $\mathbb{R}^3 \times \mathbb{R} \to \mathbb{G}^{4,1}$

Dlpは,双対平面(DualPlane)を表す関数である.
引数には,実数の組とスカラーを用いる.

```
Dlp[{3, 8, 1}, 9]
```

```
3 w[{1}] + 8 w[{2}] + w[{3}] + 9 w[{∞}]
```

Flp : $\mathbb{R}^3 \to \mathbb{G}^{4,1}$

Flp は,水平点(FlatPoint)を表す関数である.
引数には,実数の組を用いる.

Flp[p] = Pnt[p] ^ w[{∞}]] (p∈$\mathbb{R}^3$)
ポイントペアの一点を無限大にとばしたもの.

```
Flp[{2, 3, 7}]
```

```
w[{0, ∞}] + 2 w[{1, ∞}] + 3 w[{2, ∞}] + 7 w[{3, ∞}]
```

Versors

CGAでの運動,作用.

Rotor : $\mathrm{Pnt}[\mathbb{R}^3] \times \mathrm{Biv}[\mathbb{R}^3 \times \mathbb{R}^3] \times \mathbb{R} \to \mathbb{G}^{4,1}$

Rotorは,回転を表す関数である.
引数には,CGAの元と二重ベクトルとスカラーを用いる.

```
Rotor[Pnt[{1, 0, 0}], Biv[{1, 0, 0}, {0, 1, 0}], θ]
```

$$w[\{0\}] + \mathrm{Cos}[\theta]\, w[\{1\}] + \mathrm{Sin}[\theta]\, w[\{2\}] + \frac{1}{2} w[\{\infty\}]$$

Translator : Pnt$[\mathbb{R}^3]$ × Vec$[\mathbb{R}^3]$ → $\mathbb{G}^{4,1}$

Translatorは,平行移動を表す関数である.
引数には,CGAの元とベクトルを用いる.

```
Translator[Pnt[{x, 0, 0}], Vec[{t, 0, 0}]]
```

$$w[\{0\}] + (t + x)\, w[\{1\}] + \frac{1}{2} (t + x)^2\, w[\{\infty\}]$$

Dilator : Pnt$[\mathbb{R}^3]$ × $\mathbb{R}$ → $\mathbb{G}^{4,1}$

Dilatorは,拡大縮小を表す関数である.
引数には,CGAの元とスカラーを用いる.

```
Dilator[Pnt[{1, 2, 0}], 3]
```

$$\frac{w[\{0\}]}{e^3} + w[\{1\}] + 2\, w[\{2\}] + \frac{5}{2} e^3\, w[\{\infty\}]$$

Direction

CGAでの,方向ベクトル関係の元.

Drv : $\mathbb{R}^3$ → $\mathbb{G}^{4,1}$

Drv は,方向ベクトル(DirectionVector)を表す関数である.
引数には,実数の組を用いる.

```
Drv[{4, 1, 3}]
```

$$4\, w[\{1, \infty\}] + w[\{2, \infty\}] + 3\, w[\{3, \infty\}]$$

```
Drv[{1, 2, 0}]
```

$$w[\{1, \infty\}] + 2\, w[\{2, \infty\}]$$

Drb : $\mathbb{R}^3$ × $\mathbb{R}^3$ → $\mathbb{G}^{4,1}$

Drb は,二重方向ベクトル(DirectionBivector)を表す関数である.
引数には,実数の組を二つ用いる.

```
Drb[{0, 1, 0}, {1, 0, 0}]
```

$$-w[\{1, 2, \infty\}]$$

```
Drb[{1, 4, 2}, {1, 2, 3}]
```

$-2\,w[\{1, 2, \infty\}] + w[\{1, 3, \infty\}] + 8\,w[\{2, 3, \infty\}]$

Drt : $\mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{G}^{4,1}$

Drt は,三重方向ベクトル(DirectionTrivector)を表す関数である.
引数には,実数の組を三つ用いる.

```
Drt[{0, 1, 0}, {1, 0, 0}, {0, 0, 1}]
```

$-w[\{1, 2, 3, \infty\}]$

```
Drt[{0, 1, 2}, {1, 4, 2}, {1, 2, 3}]
```

$-5\,w[\{1, 2, 3, \infty\}]$

Tangent

CGAでの,接線ベクトル関係の元.

Tnv : $\mathbb{R}^3 \to \mathbb{G}^{4,1}$

Tnv は,接線ベクトル(TangentVector)を表す関数である.
引数には,実数の組を用いる.

```
Tnv[{0, 1, 0}]
```

$w[\{0, 2\}]$

```
Tnv[{1, 2, 0}]
```

$w[\{0, 1\}] + 2\,w[\{0, 2\}]$

Tnb : $\mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{G}^{4,1}$

Tnb は,二重接線ベクトル(TangentBivector)を表す関数である.
引数には,実数の組を二つ用いる.

```
Tnb[{0, 1, 0}, {1, 0, 0}]
```

$-w[\{0, 1, 2\}]$

```
Tnb[{1, 4, 2}, {1, 2, 3}]
```

$-2\,w[\{0, 1, 2\}] + w[\{0, 1, 3\}] + 8\,w[\{0, 2, 3\}]$

Tnt : $\mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{G}^{4,1}$

Tnt は,三重接線ベクトル(TangentTrivector)を表す関数である.
引数には,実数の組を三つ用いる.

```
Tnt[{0, 1, 0}, {1, 0, 0}, {0, 0, 1}]
```

```
-w[{0, 1, 2, 3}]
```

```
Tnt[{0, 1, 2}, {1, 4, 2}, {1, 2, 3}]
```

```
-5 w[{0, 1, 2, 3}]
```

Abstract

CGAでの,定数.
引数なし.

$\mathrm{Mnk} \in \mathbb{G}^{4,1}$

Mnk は,ミンコフスキー平面(MinkowskiPlane)を表す関数である.
引数はなし,定数.

```
Mnk
```

```
w[{0, ∞}]
```

$\mathrm{Pss} \in \mathbb{G}^{4,1}$

Pss は,擬スカラー(Pseudoscalar)を表す関数である.
引数はなし,定数.

```
Pss
```

```
-w[{0, 1, 2, 3, ∞}]
```

$\mathrm{Inf} \in \mathbb{G}^{4,1}$

Inf は,無限遠点(Infinity)を表す関数である.
引数はなし,定数.

```
Inf
```

```
w[{∞}]
```

第二章 図形表示

Point

計算

点P = $\{x, y, z\} \in \mathbb{R}^3$ のCGA上での元は,
$w_0 + P + \frac{1}{2} P^2 w_\infty$ で表される.

```
(*CGA上の点に変換する関数Pntを定義した*)
Pnt[{x, y, z}]
```

$$w[\{0\}] + x\,w[\{1\}] + y\,w[\{2\}] + z\,w[\{3\}] + \frac{1}{2}\left(x^2 + y^2 + z^2\right) w[\{\infty\}]$$

Pointの二乗 == 0 (null – vector)

```
CGAProduct[Pnt[{x, y, z}], Pnt[{x, y, z}]]
```

0

原点w_0 と無限遠点w_∞も,もちろん null – vectorとなっている.

```
CGAProduct[w[{0}], w[{0}]]
```

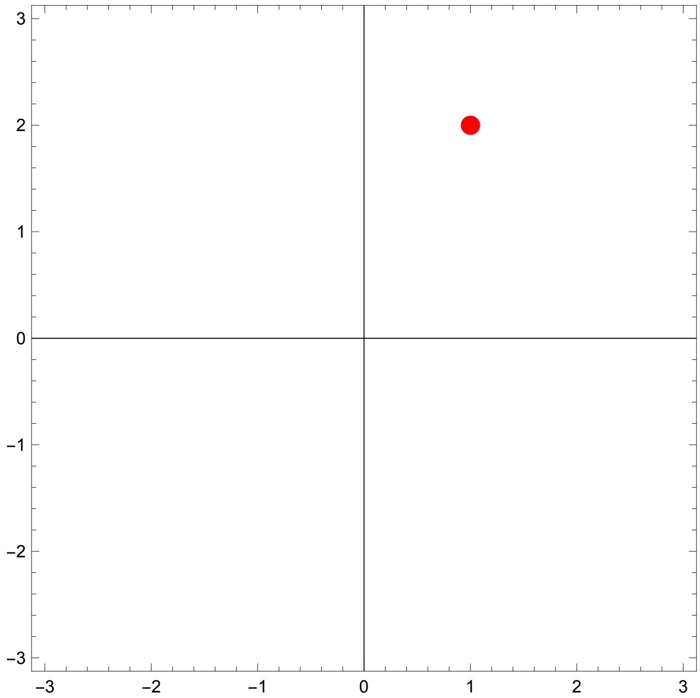
0

```
CGAProduct[w[{∞}], w[{∞}]]
```

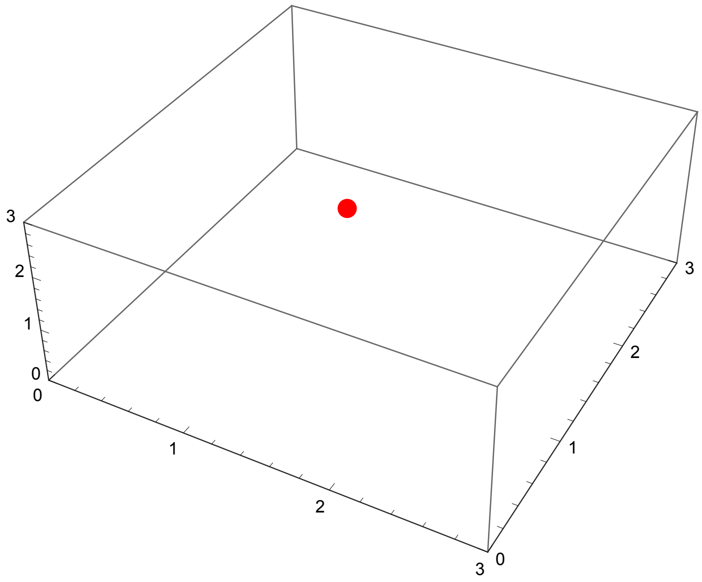
0

図形表示

```
Range0 = {{-3, 3}, {-3, 3}};
CGAOutput2D[Pnt[{1, 2}] , Range0,
 Axes → True, PlotStyle → {AbsolutePointSize[10], Red}]
```



```
CGAOutput3D[Pnt[{1, 2, 1}], {{0, 3}, {0, 3}, {0, 3}},
 PlotStyle → {AbsolutePointSize[10], Red}]
```

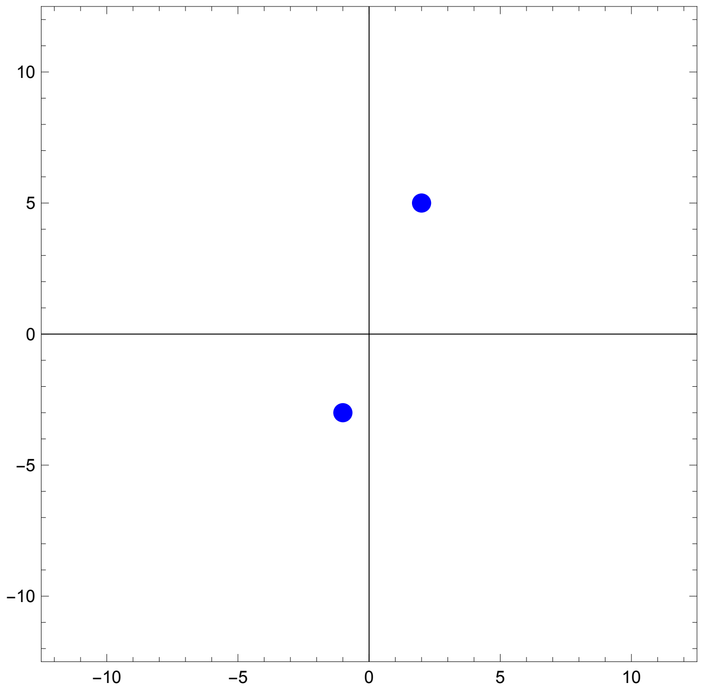


PointPair

図形表示

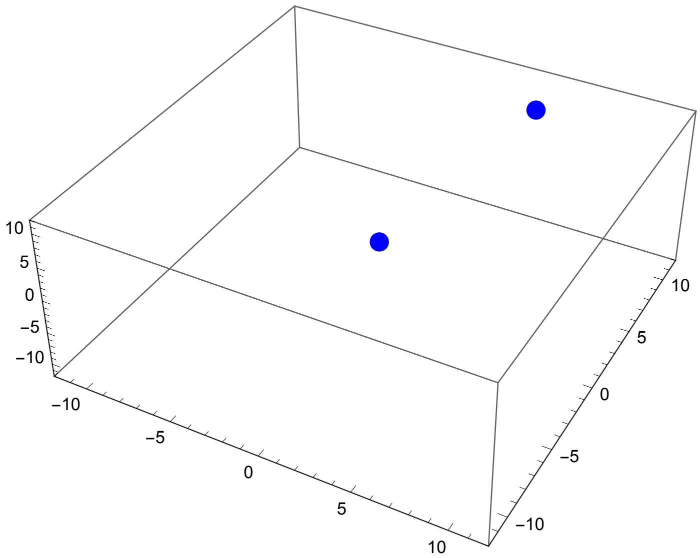
二点 {-1, -3}, {2, 5} ∈ $\mathbb{R}^2$

```
TestPP = Par[{-1, -3}, {2, 5}];
Range0 = {{-12, 12}, {-12, 12}};
CGAOutput2D[TestPP, Range0, Axes → True,
 PlotStyle → {AbsolutePointSize[10], Blue}]
```



二点 {0, 0, 0}, {4, 10, 9} ∈ $\mathbb{R}^3$

```
TestPP2 = Par[{0, 0, 0}, {4, 10, 9}];
Range0 = {{-12, 12}, {-12, 12}, {-12, 12}};
CGAOutput3D[TestPP2, Range0, Axes → True,
 PlotStyle → {AbsolutePointSize[10], Blue}]
```



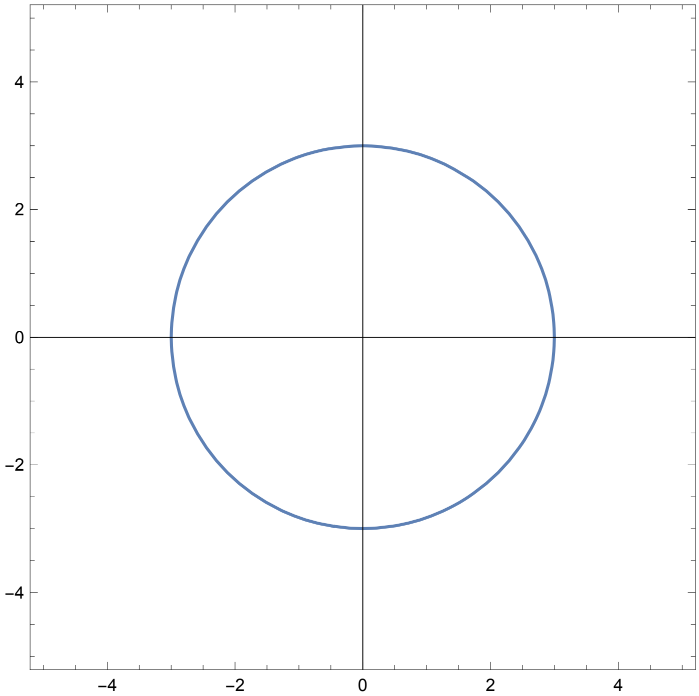
Circle

図形表示

円は三点の外積で表される

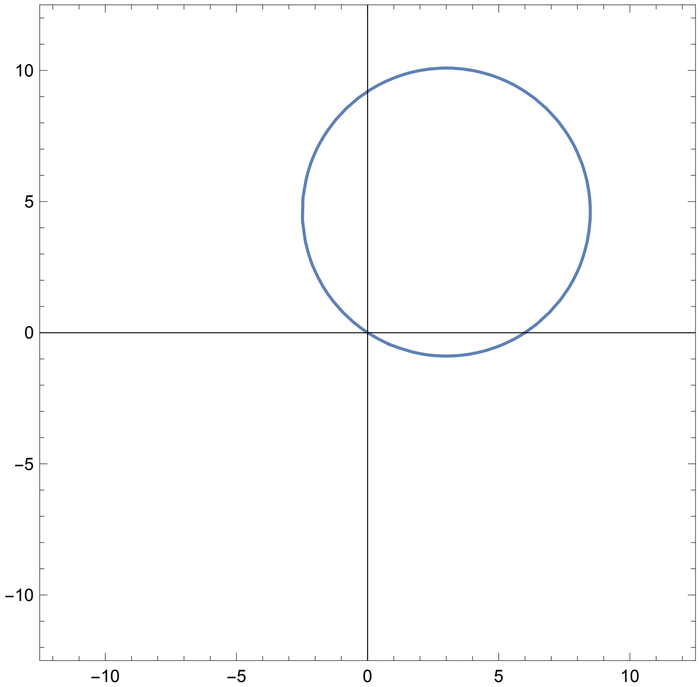
三点 {3, 0}, {0, 3}, {-3, 0} ∈ $\mathbb{R}^2$を通る円

```
(*円の定義*)TestCircle = Cir[{3, 0}, {0, 3}, {-3, 0}];
(*外積の係数を抜き出す*)C1 = CoefPickUp[OuterProduct[TestCircle, Pnt[{x, y}]]];
(*等高線*)ContourPlot[C1 == 0, {x, -5, 5}, {y, -5, 5}, Axes → True]
```



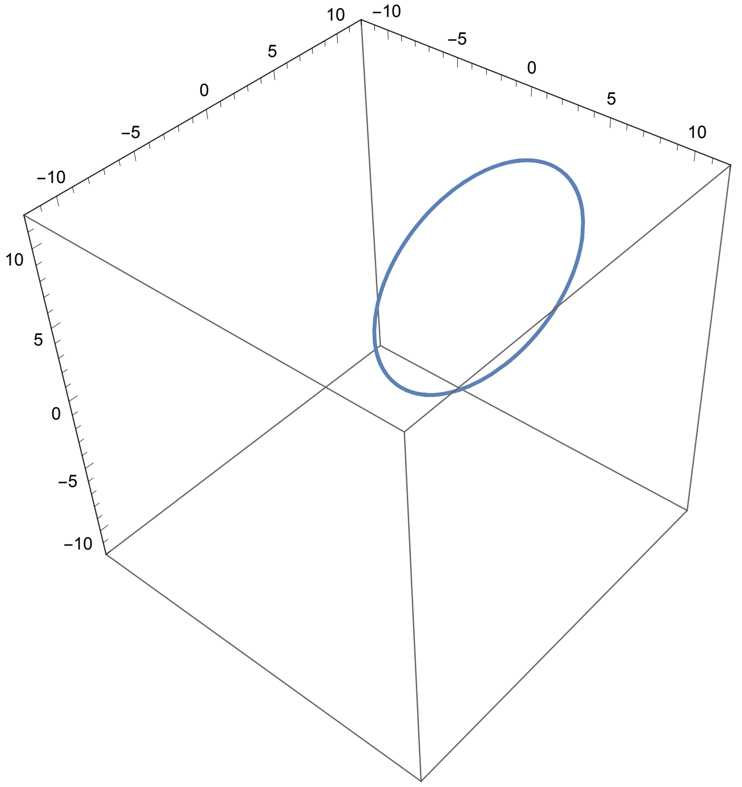
三点 {0, 0}, {2, 10}, {6, 0} ∈ ℝ²を通る円

```
TestCircle2 = Cir[{0, 0}, {2, 10}, {6, 0}];
Range0 = {{-12, 12}, {-12, 12}};
CGAOutput2D[TestCircle2 , Range0, Axes → True]
```



三点 {0, 0, 0}, {4, 10, 9}, {6, 0, 3} ∈ ℝ³を通る円

```
TestCircle3 = Cir[{0, 0, 0}, {4, 10, 9}, {6, 0, 3}];
Range0 = {{-12, 12}, {-12, 12}, {-12, 12}};
CGAOutput3D[TestCircle3 , Range0]
```



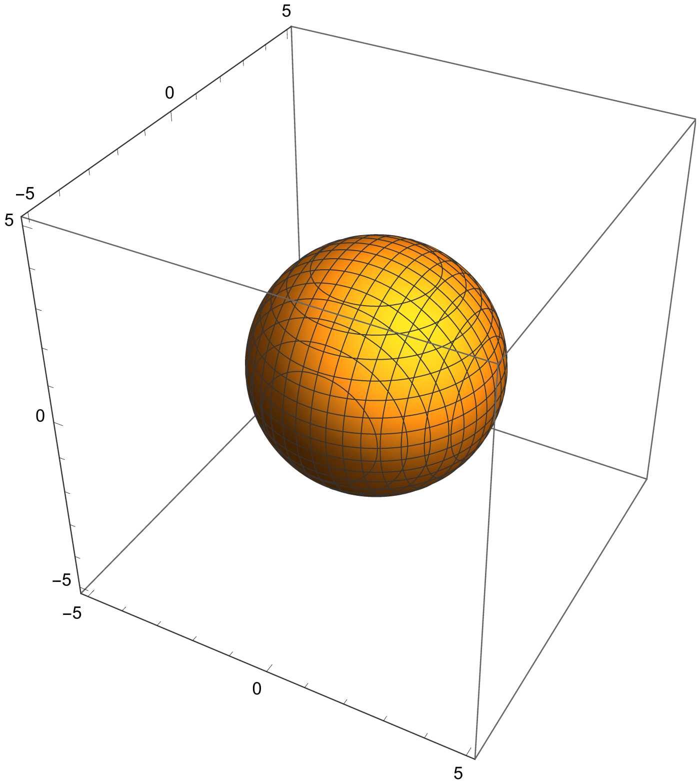
Sphere

図形表示

球は四点の外積で表される

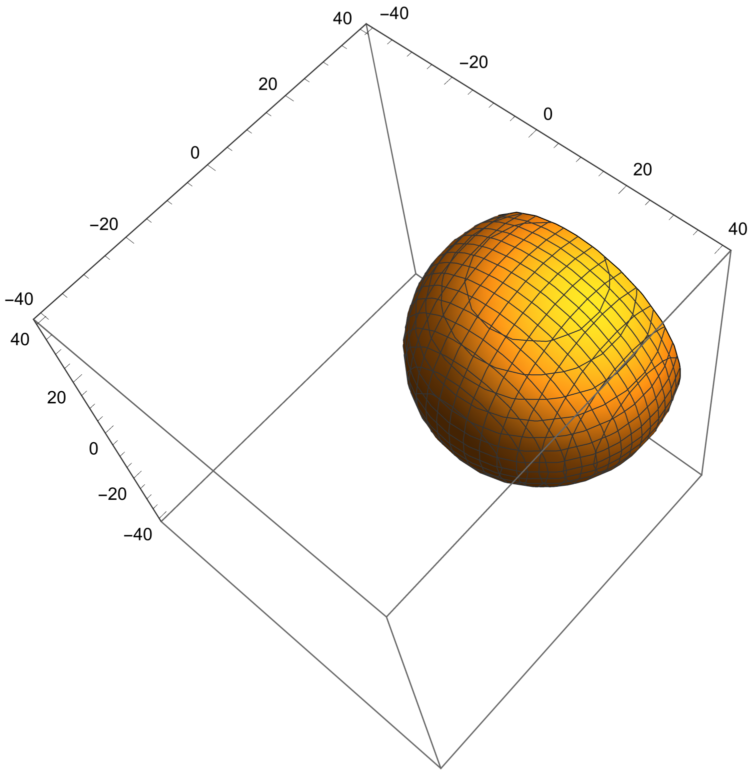
四点 {3, 0, 0}, {0, 3, 0}, {0, 0, 3}, {-3, 0, 0} ∈ $\mathbb{R}^3$を通る球

```
TestSphere = Sph[{3, 0, 0}, {0, 3, 0}, {0, 0, 3}, {-3, 0, 0}];
S1 = CoefPickUp[OuterProduct[TestSphere, Pnt[{x, y, z}]]];
ContourPlot3D[S1 == 0, {x, -5, 5}, {y, -5, 5}, {z, -5, 5}]
```



四点 {5, 2, 1}, {3, 3, 3}, {9, 2, 3}, {-3, 3, 3} ∈ $\mathbb{R}^3$を通る球

```
TestSphere2 = Sph[{5, 2, 1}, {3, 3, 3}, {9, 2, 3}, {-3, 11, 12}];
Range0 = {{-40, 40}, {-40, 40}, {-40, 40}};
CGAOutput3D[TestSphere2 , Range0]
```



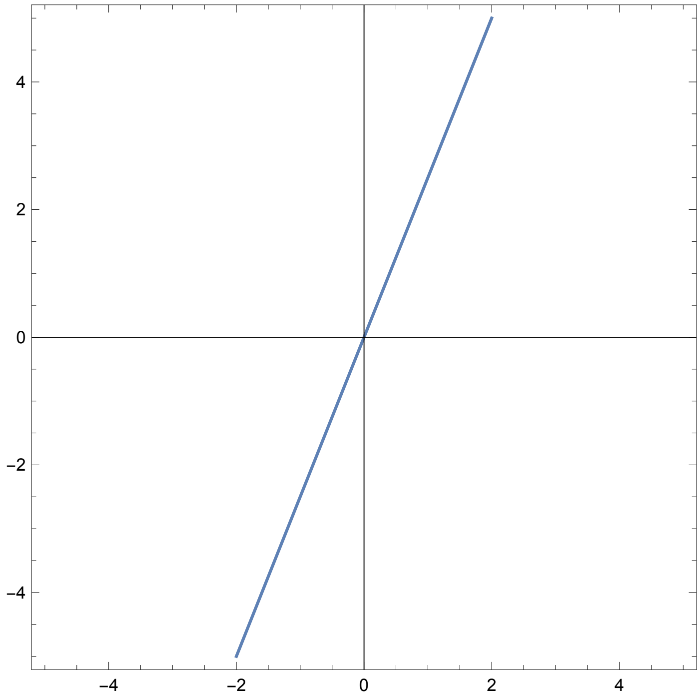
Line

図形表示

直線は二点と無限遠点の外積で表される. つまり三点の内の一点が無限遠にある円

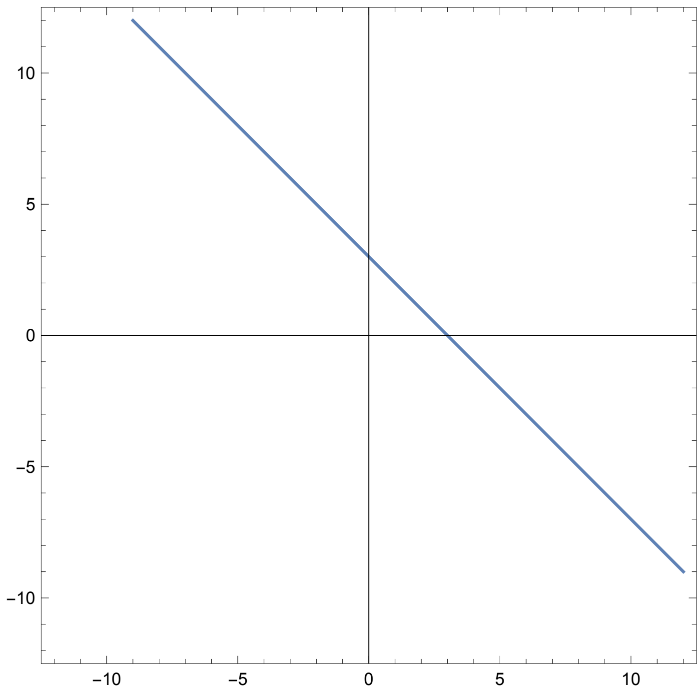
二点 {0, 0}, {2, 5} ∈ $\mathbb{R}^2$を通る直線

```
TestLine = Lin[{0, 0}, {2, 5}];
L1 = CoefPickUp[OuterProduct[TestLine, Pnt[{x, y}]]];
ContourPlot[L1 == 0, {x, -5, 5}, {y, -5, 5}, Axes → True]
```



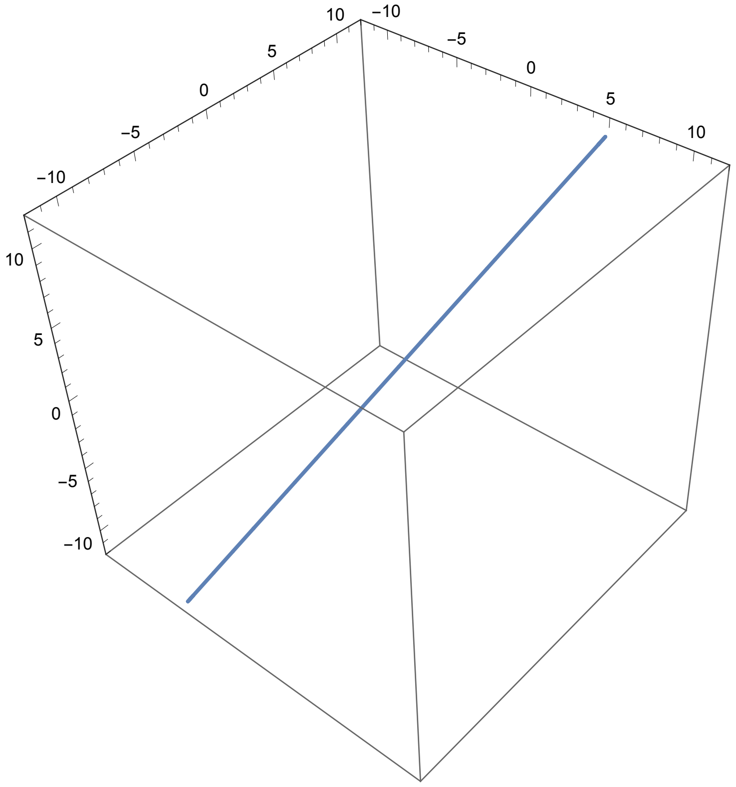
二点 **{3, 0}, {0, 3}** ∈ $\mathbb{R}^2$を通る直線

```
TestLine2 = Lin[{3, 0}, {0, 3}];
Range0 = {{-12, 12}, {-12, 12}};
CGAOutput2D[TestLine2 , Range0, Axes → True]
```



二点 **{0, 0, 0}, {4, 10, 9}** ∈ $\mathbb{R}^3$を通る直線

```
TestLine3 = Lin[{0, 0, 0}, {4, 10, 9}];
Range0 = {{-12, 12}, {-12, 12}, {-12, 12}};
CGAOutput3D[TestLine3 , Range0]
```



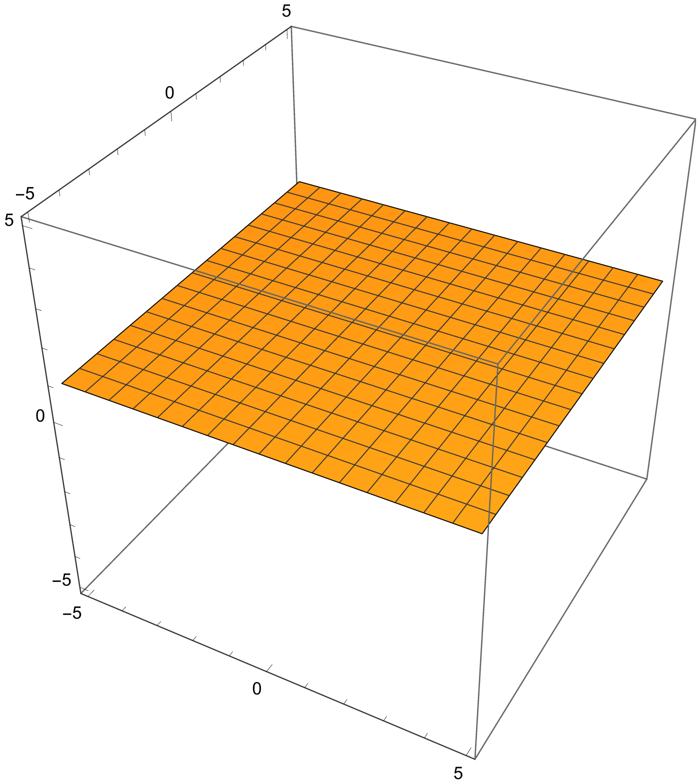
Plane

図形表示

平面は三点と無限遠点の外積で表される. つまり四点の内の一点が無限遠にある球

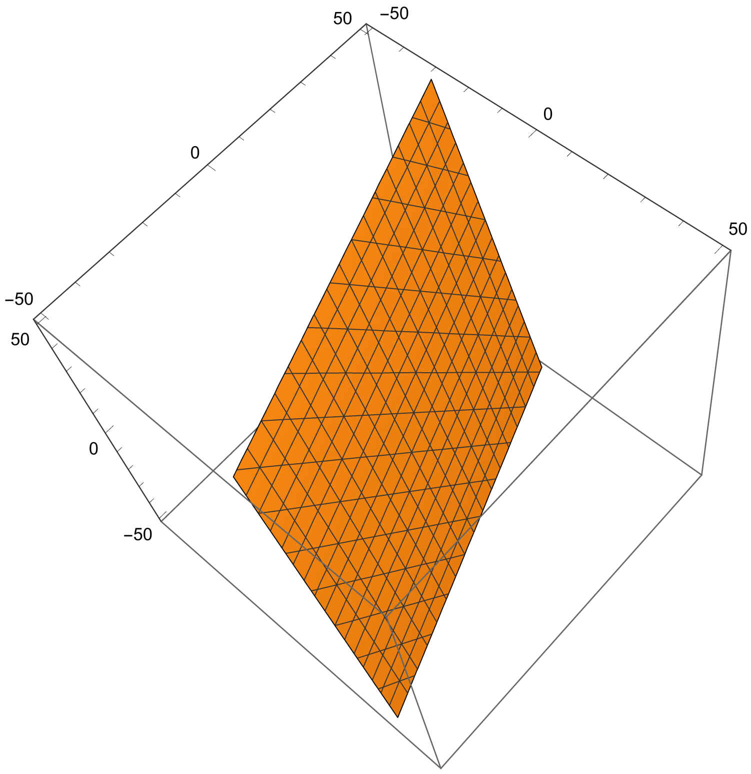
四点 {3, 0, 1}, {0, 3, 1}, {0, 2, 1} ∈ $\mathbb{R}^3$を通る平面

```
TestPlane = Pln[{3, 0, 1}, {0, 3, 1}, {0, 2, 1}];
P1 = CoefPickUp[OuterProduct[TestPlane, Pnt[{x, y, z}]]];
ContourPlot3D[P1 == 0, {x, -5, 5}, {y, -5, 5}, {z, -5, 5}]
```



三点 **{3, 0, 0}, {0, 5, 4}, {0, 2, 8}** ∈ $\mathbb{R}^3$を通る平面

```
TestPlane2 = Pln[{3, 0, 0}, {0, 5, 4}, {0, 2, 8}];
Range0 = {{-50, 50}, {-50, 50}, {-50, 50}};
CGAOutput3D[TestPlane2 , Range0]
```



Translator

計算

点P = {x, y, z} ∈ $\mathbb{R}^3$ のCGA上での元は,
$w_0 + P + \frac{1}{2} P^2 w_\infty$ で表される.

点 {x, 0} ∈ $\mathbb{R}^2$をベクトル {t, 0} ∈ $\mathbb{R}^2$で平行移動

```
Pnt[{x, 0}]
```

$$w[\{0\}] + x\, w[\{1\}] + \frac{1}{2} x^2\, w[\{\infty\}]$$

```
T0 = Translator[Pnt[{x, 0}], Vec[{t, 0}]]
```

$$w[\{0\}] + (t + x)\, w[\{1\}] + \frac{1}{2} (t + x)^2\, w[\{\infty\}]$$

点 {1, 2, 3} ∈ $\mathbb{R}^3$を ベクトル {t_1, t_2, t_3} ∈ $\mathbb{R}^2$で平行移動

```
Pnt[{1, 2, 3}]
```

$$w[\{0\}] + w[\{1\}] + 2\, w[\{2\}] + 3\, w[\{3\}] + 7\, w[\{\infty\}]$$

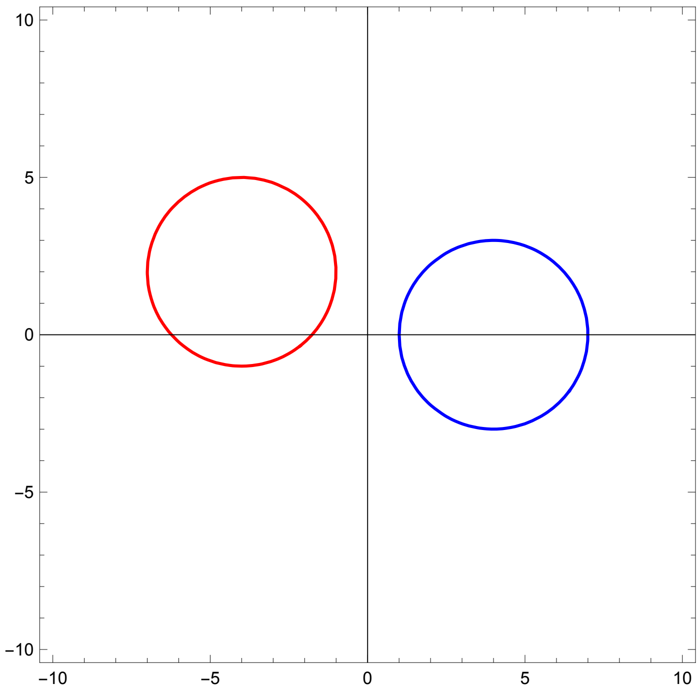
```
Translator[Pnt[{1, 2, 3}], Vec[{t1, t2, t3}]]
```

$w[\{0\}] + (1 + t_1)\, w[\{1\}] + (2 + t_2)\, w[\{2\}] + (3 + t_3)\, w[\{3\}] + \frac{1}{2}\left(14 + 2 t_1 + t_1^2 + 4 t_2 + t_2^2 + 6 t_3 + t_3^2\right) w[\{\infty\}]$

図形表示

三点 {1, 0}, {7, 0}, {4, 3} ∈ $\mathbb{R}^2$ を通る円をベクトル {-8, 2} ∈ $\mathbb{R}^2$ で平行移動

```
TestCircle = Cir[{1, 0}, {7, 0}, {4, 3}];(*元の円:青色*)
TestTranslator = Translator[TestCircle, Vec[{-8, 2}]];(*変換後の円:赤色*)
Range0 = {{-10, 10}, {-10, 10}};
Show[
 CGAOutput2D[TestCircle, Range0, Axes → True, ContourStyle → Blue],
 CGAOutput2D[TestTranslator, Range0, Axes → True, ContourStyle → Red]
]
```



Rotor

計算

点 {1, 0} ∈ $\mathbb{R}^2$ をxy平面上で θ 回転

```
Pnt[{1, 0}]
```

$w[\{0\}] + w[\{1\}] + \frac{1}{2}\, w[\{\infty\}]$

```
R0 = Rotor[Pnt[{1, 0}], Biv[{1, 0}, {0, 1}], θ]
```

$$w[\{0\}] + \text{Cos}[\theta]\, w[\{1\}] + \text{Sin}[\theta]\, w[\{2\}] + \frac{1}{2}\, w[\{\infty\}]$$

点 {1, 2, 3} ∈ $\mathbb{R}^3$を xy平面上でθ回転

```
Pnt[{1, 2, 3}]
```

$$w[\{0\}] + w[\{1\}] + 2\, w[\{2\}] + 3\, w[\{3\}] + 7\, w[\{\infty\}]$$

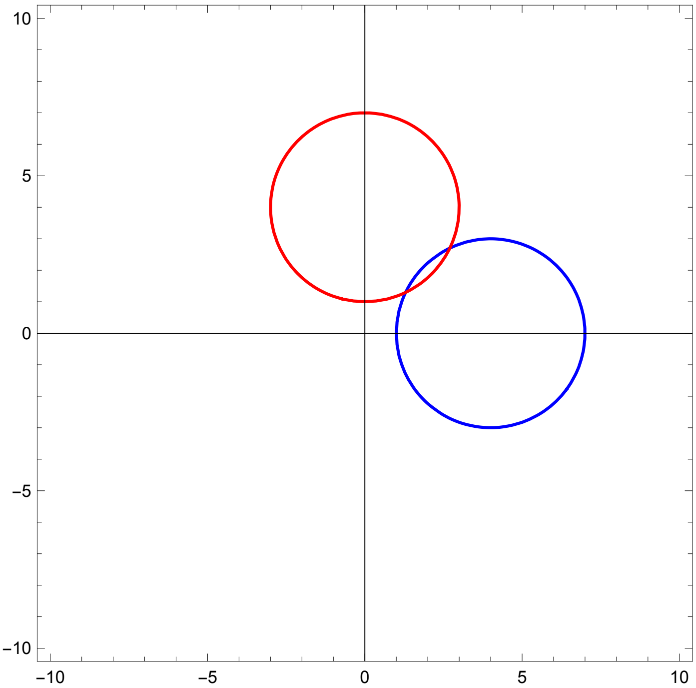
```
R0 = Rotor[Pnt[{1, 2, 3}], Biv[{1, 0}, {0, 1}], θ]
```

$$w[\{0\}] + (\text{Cos}[\theta] - 2\,\text{Sin}[\theta])\, w[\{1\}] + (2\,\text{Cos}[\theta] + \text{Sin}[\theta])\, w[\{2\}] + 3\, w[\{3\}] + 7\, w[\{\infty\}]$$

図形表示

三点 {1, 0}, {7, 0}, {4, 3} ∈ $\mathbb{R}^2$ を通る円をxy平面上で $\frac{\pi}{2}$回転

```
TestCircle = Cir[{1, 0}, {7, 0}, {4, 3}];(*元の円:青色*)
TestRotor = Rotor[TestCircle, Biv[{1, 0}, {0, 1}], π / 2];(*変換後の円:赤色*)
Range0 = {{-10, 10}, {-10, 10}};
Show[
 CGAOutput2D[TestCircle, Range0, Axes → True, ContourStyle → Blue],
 CGAOutput2D[TestRotor, Range0, Axes → True, ContourStyle → Red]
]
```



Dilator

計算

点 {1, 2} ∈ $\mathbb{R}^2$を e^3 倍に拡大

```
Pnt[{1, 2}]
```

$$w[\{0\}] + w[\{1\}] + 2\,w[\{2\}] + \frac{5}{2}\,w[\{\infty\}]$$

```
D0 = Dilator[Pnt[{1, 2}], 3]
```

$$\frac{w[\{0\}]}{e^3} + w[\{1\}] + 2\,w[\{2\}] + \frac{5}{2}\,e^3\,w[\{\infty\}]$$

スカラー倍は同じ元とみなせるので, w_0の係数を1にして簡約化

```
D0 / Coefficient[D0, w[{0}]] // Expand
```

$$w[\{0\}] + e^3\,w[\{1\}] + 2\,e^3\,w[\{2\}] + \frac{5}{2}\,e^6\,w[\{\infty\}]$$

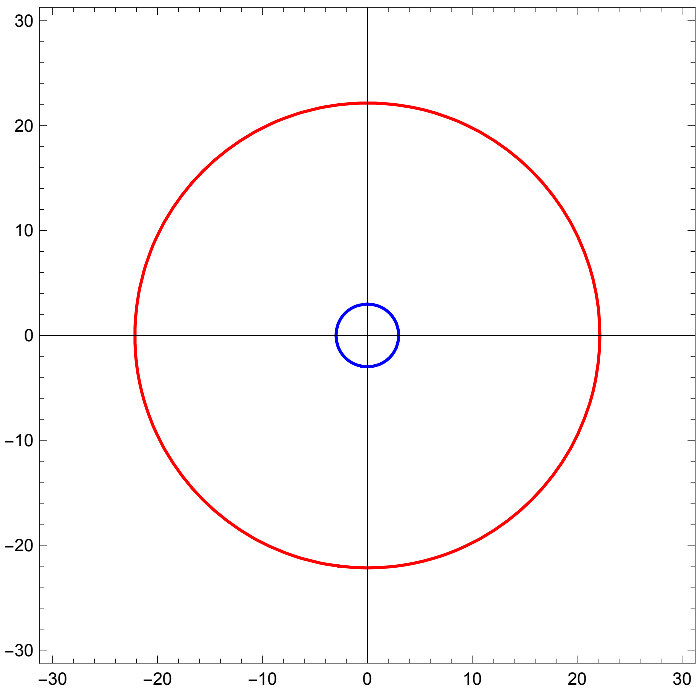
```
NPnt[D0]
```

$$w[\{0\}] + e^3\,w[\{1\}] + 2\,e^3\,w[\{2\}] + \frac{5}{2}\,e^6\,w[\{\infty\}]$$

図形表示

三点 {3, 0}, {0, 3}, {-3, 0} ∈ ℝ²を通る円をe^2倍に拡大

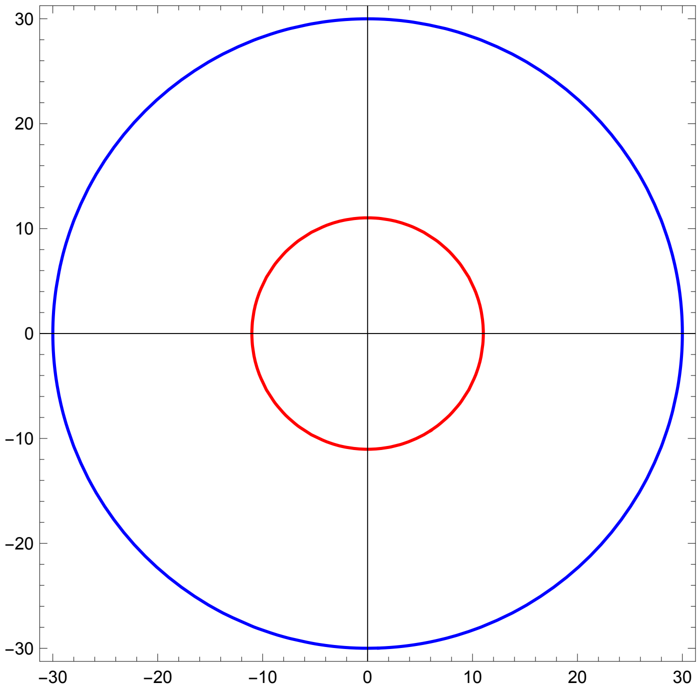
```
TestCircle = Cir[{3, 0}, {0, 3}, {-3, 0}];(*元の円:青色*)
TestDilator = Dilator[TestCircle, 2];(*変換後の円:赤色*)
Range0 = {{-30, 30}, {-30, 30}};
Show[
 CGAOutput2D[TestCircle, Range0, Axes → True, ContourStyle → Blue],
 CGAOutput2D[TestDilator, Range0, Axes → True, ContourStyle → Red]
]
```



```
TestCircle = Cir[{3, 0}, {0, 3}, {-3, 0}];(*元の円:青色*)
TestDilator = Dilator[TestCircle, 2];(*変換後の円:赤色*)
Range0 = {{-30, 30}, {-30, 30}};
Show[
 CGAOutput2D[TestCircle, Range0, Axes → True, ContourStyle → Blue],
 CGAOutput2D[TestDilator, Range0, Axes → True, ContourStyle → Red]
]
```

三点 {30, 0}, {0, 30}, {-30, 0} ∈ $\mathbb{R}^2$を通る円をe^{-1}倍に縮小

```
TestCircle2 = Cir[{30, 0}, {0, 30}, {-30, 0}];(*元の円:青色*)
TestDilator2 = Dilator[TestCircle2, -1];(*変換後の円:赤色*)
Range0 = {{-30, 30}, {-30, 30}};
Show[
 CGAOutput2D[TestCircle2, Range0, Axes → True, ContourStyle → Blue],
 CGAOutput2D[TestDilator2, Range0, Axes → True, ContourStyle → Red]
]
```



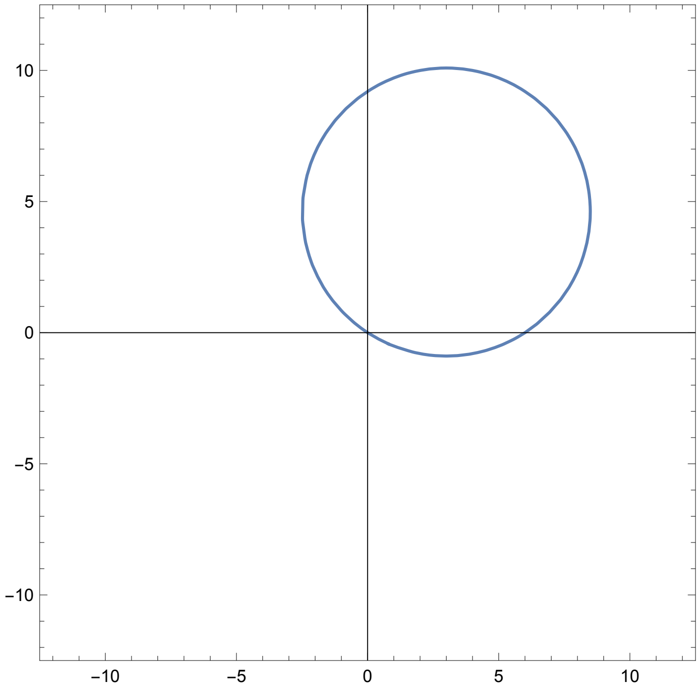
CGAOutput

CGAの元をユークリッド空間上に表示する.

CGAOutput2D

CGAOutput2D は,二次元空間にプロットする関数である.
引数には,CGAの元と表示する範囲を用いる.

```
TestCircle2 = Cir[{0, 0}, {2, 10}, {6, 0}];
Range0 = {{-12, 12}, {-12, 12}};
CGAOutput2D[TestCircle2 , Range0, Axes → True]
```



CGAOutput3D

CGAOutput3D は,三次元空間にプロットする関数である.
引数には,CGAの元と表示する範囲を用いる.

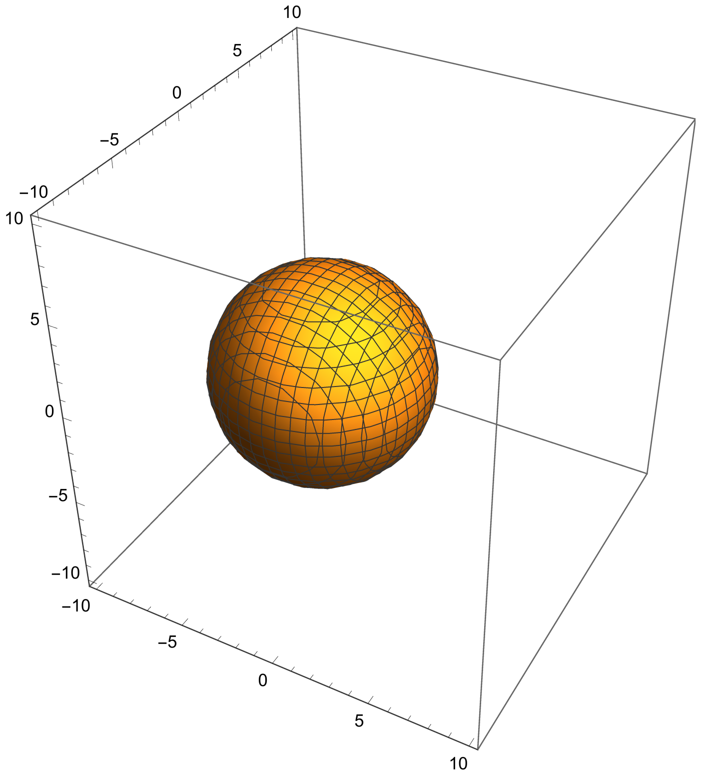
連立方程式１つ

3次元上の物体,面,球 → ContourPlot3Dで等高面を表示

[例] $x^2 + y^2 + z^2 - 1 = 0$

```
TestSphere = Sph[{3, 0, 0}, {0, 3, 1}, {0, 2, 3}, {-3, 3, 2}];
OuterProduct[TestSphere, Pnt[{x, y, z}]]
CGAOutput3D[TestSphere, {{-10, 10}, {-10, 10}, {-10, 10}}]
```

$\left(-37\,x-3\,(-64+11\,y+z)-9\left(x^2+y^2+z^2\right)\right)\,w[\{0,1,2,3,\infty\}]$



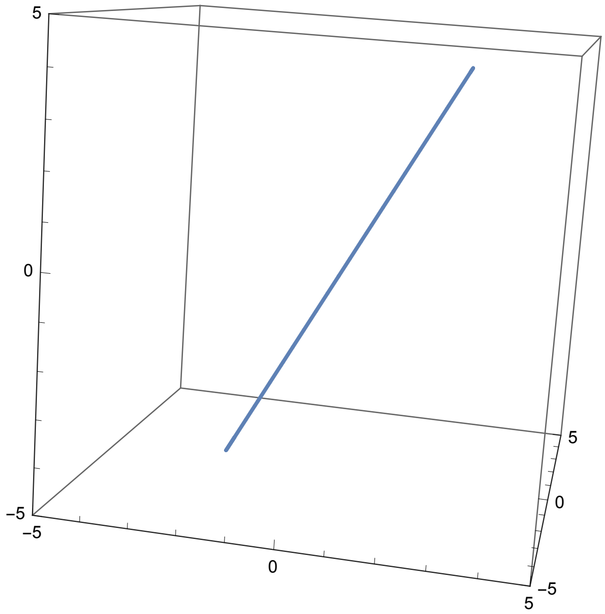
連立方程式２つ

2次元の物体,直線,円 →ParametricPlot3Dで{x,y,z}の内一つをパラメータとして表示

[例] $x-z-1=0,\ y+z-3=0$
$\rightarrow \{x,y,z\}=\{z+1,-z+3,z\}$

```
TestLine = Lin[{0, 3, 0}, {2, 5, 4}];
OuterProduct[TestLine, Pnt[{x, y, z}]]
CGAOutput3D[TestLine , {{-5, 5}, {-5, 5}, {-5, 5}}]
```

```
2 (3 + x - y) w[{0, 1, 2, ∞}] + (4 x - 2 z) w[{0, 1, 3, ∞}] +
 (4 y - 2 (6 + z)) w[{0, 2, 3, ∞}] + 6 (-2 x + z) w[{1, 2, 3, ∞}]
```



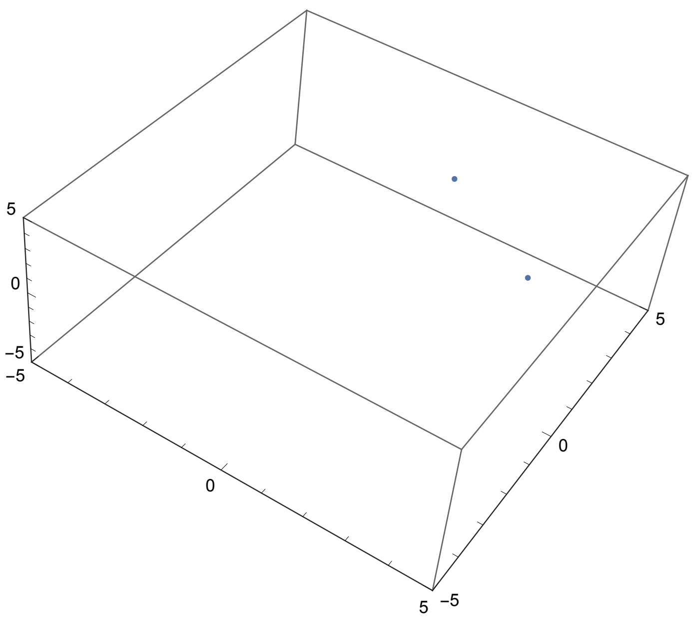
連立方程式３つ

1次元の物体,点 → ListPointPlot3Dで点を表示

[例] $x^2 - 1 = 0,\ y - 3 = 0\ ,\ z + 4 = 0$
$\rightarrow \{x, y, z\} = \{1, 3, -4\},\ \{-1, 3, -4\}$

```
TestPointPair = OuterProduct[Pnt[{3, 2, 0}], Pnt[{0, 4, 0}]];
OuterProduct[TestPointPair, Pnt[{x, y, z}]]
CGAOutput3D[TestPointPair, {{-5, 5}, {-5, 5}, {-5, 5}}]
```

$$(12 - 2\,x - 3\,y)\;w[\{0,1,2\}] - 3\,z\,w[\{0,1,3\}] - \frac{3}{2}\left(-16 + x + x^2 + y^2 + z^2\right) w[\{0,1,\infty\}] + 2\,z\,w[\{0,2,3\}] + \left(-10 + x^2 - \frac{3\,y}{2} + y^2 + z^2\right) w[\{0,2,\infty\}] - \frac{3}{2}\,z\,w[\{0,3,\infty\}] + 12\,z\,w[\{1,2,3\}] + \left(-10\,x + 6\,x^2 + 6\left(-4\,y + y^2 + z^2\right)\right) w[\{1,2,\infty\}] - 24\,z\,w[\{1,3,\infty\}] + 10\,z\,w[\{2,3,\infty\}]$$



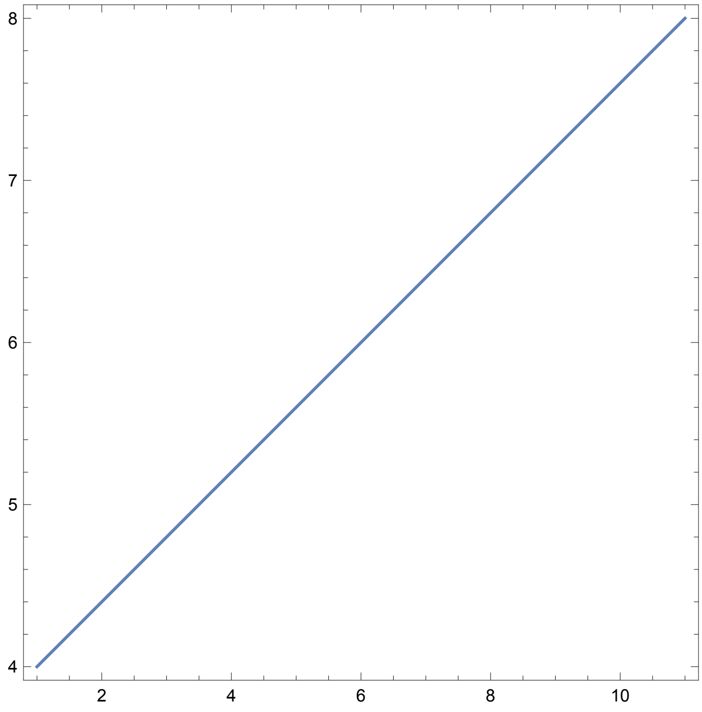
SegOutput

線分をユークリッド空間上に表示する.

SegOutput2D

SegOutput2D は,二次元空間にプロットする関数である.
引数には,CGAの点を二つ用いる.

```
SegOutput2D[Pnt[{1, 4}], Pnt[{11, 8}]]
```

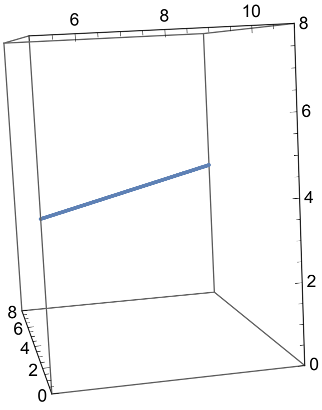


SegOutput3D

SegOutput3D は,三次元空間にプロットする関数である.
引数には,CGAの点を二つ用いる.

入力された二点間の線分を表示.

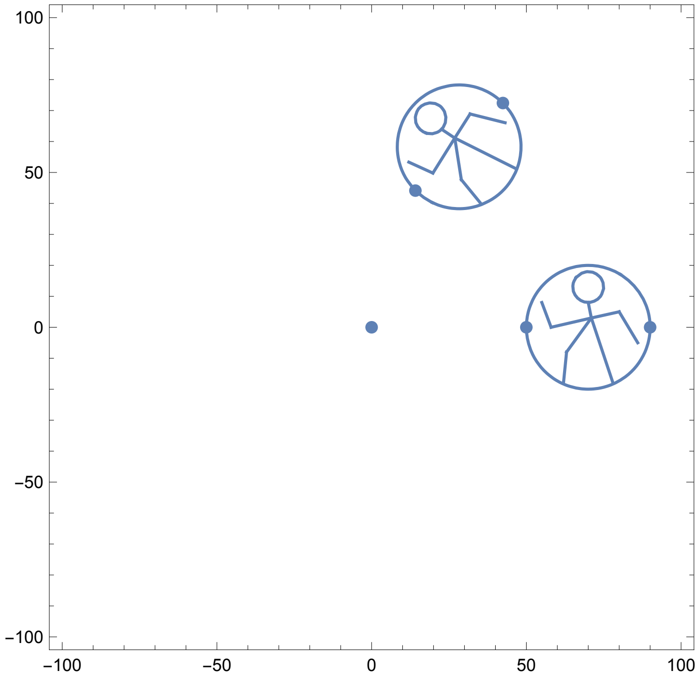
```
SegOutput3D[Pnt[{5, 0, 4}], Pnt[{11, 8, 4}]]
```



面白い例

```
i = 50;
TestPoint1 = Pnt[{50, 0}]; TestPoint2 = Pnt[{90, 0}];
TestCircle1 = Cir[{50, 0}, {90, 0}, {70, 20}];
TestCircle2 = Cir[{65, 13}, {75, 13}, {70, 18}];

SPoint1 = Pnt[{70, 8}];
SPoint2 = Pnt[{71, 3}];
```

```
SPoint3 = Pnt[{58, 0}];
SPoint4 = Pnt[{55, 8}];
SPoint5 = Pnt[{80, 5}];
SPoint6 = Pnt[{86, -5}];
SPoint7 = Pnt[{78, -18}];
SPoint8 = Pnt[{63, -8}];
SPoint9 = Pnt[{62, -18}];

FVec = Vec[{-30, 30}]; FBiv = Biv[{1, 0}, {0, 1}];

TestFP1 = F[TestPoint1, FVec, FBiv, π / 2, i / 100];
TestFP2 = F[TestPoint2, FVec, FBiv, π / 2, i / 100];
TestFC1 = F[TestCircle1, FVec, FBiv, π / 2, i / 100];
TestFC2 = F[TestCircle2, FVec, FBiv, π / 2, i / 100];

TestFSP1 = F[SPoint1, FVec, FBiv, π / 2, i / 100];
TestFSP2 = F[SPoint2, FVec, FBiv, π / 2, i / 100];
TestFSP3 = F[SPoint3, FVec, FBiv, π / 2, i / 100];
TestFSP4 = F[SPoint4, FVec, FBiv, π / 2, i / 100];
TestFSP5 = F[SPoint5, FVec, FBiv, π / 2, i / 100];
TestFSP6 = F[SPoint6, FVec, FBiv, π / 2, i / 100];
TestFSP7 = F[SPoint7, FVec, FBiv, π / 2, i / 100];
TestFSP8 = F[SPoint8, FVec, FBiv, π / 2, i / 100];
TestFSP9 = F[SPoint9, FVec, FBiv, π / 2, i / 100];

Range1 = {{-100, 100}, {-100, 100}};
Show[CGAOutput2D[Pnt[{0, 0}], Range1 ],

 SegOutput2D[SPoint1, SPoint2],
 SegOutput2D[SPoint2, SPoint3],
 SegOutput2D[SPoint3, SPoint4],
 SegOutput2D[SPoint2, SPoint5],
 SegOutput2D[SPoint5, SPoint6],
 SegOutput2D[SPoint2, SPoint7],
 SegOutput2D[SPoint2, SPoint8],
 SegOutput2D[SPoint8, SPoint9],

 SegOutput2D[TestFSP1, TestFSP2],
 SegOutput2D[TestFSP2, TestFSP3],
 SegOutput2D[TestFSP3, TestFSP4],
 SegOutput2D[TestFSP2, TestFSP5],
 SegOutput2D[TestFSP5, TestFSP6],
 SegOutput2D[TestFSP2, TestFSP7],
 SegOutput2D[TestFSP2, TestFSP8],
 SegOutput2D[TestFSP8, TestFSP9],

 CGAOutput2D[TestPoint1, Range1 ],
 CGAOutput2D[TestPoint2, Range1 ],
 CGAOutput2D[TestCircle1, Range1] ,
 CGAOutput2D[TestCircle2, Range1] ,

 CGAOutput2D[TestFP1, Range1],
 CGAOutput2D[TestFP2, Range1] ,
 CGAOutput2D[TestFC1, Range1] ,
 CGAOutput2D[TestFC2, Range1]
]
```



第三章 応用例

交点-CGAIntersection-

CGAの元の交点を調べる.

CGAIntersection は,交点を調べる関数である.
引数には,CGAの元を二つ用いる.

計算

```
TestLine = Lin[{3, 4, 0}, {3, 4, 1}];
TestPlane = Pln[{3, 0, 1}, {0, 3, 1}, {0, 2, 1}];
CGAIntersection[TestLine, TestPlane]
```

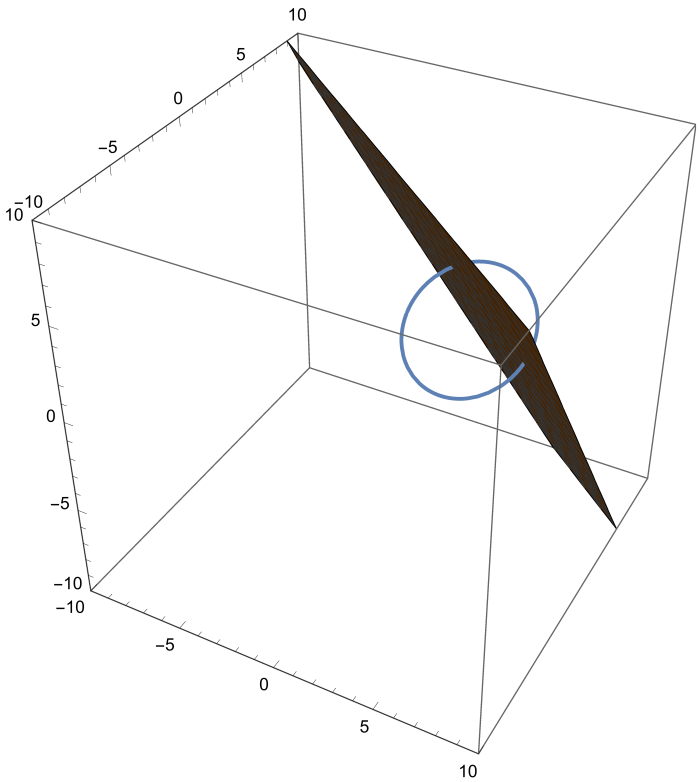
```
{{3, 4, 1}}
```

```
TestPointPair = Par[{4, 2, 3}, {2, 3, 4}];
TestPlane = Pln[{4, 2, 3}, {2, 3, 4}, {0, 2, 8}];
CGAIntersection[TestPointPair, TestPlane]
```

```
{{2, 3, 4}, {4, 2, 3}}
```

```
TestCircle = Cir[{1, 0, 2}, {7, 0, 3}, {4, 3, 5}];
TestPlane = Pln[{4, 2, 3}, {2, 3, 4}, {0, 2, 8}];
CGAIntersection[TestCircle, TestPlane]
Show[CGAOutput3D[TestCircle, {{-10, 10}, {-10, 10}, {-10, 10}}],
 CGAOutput3D[TestPlane, {{-10, 10}, {-10, 10}, {-10, 10}}]]
```

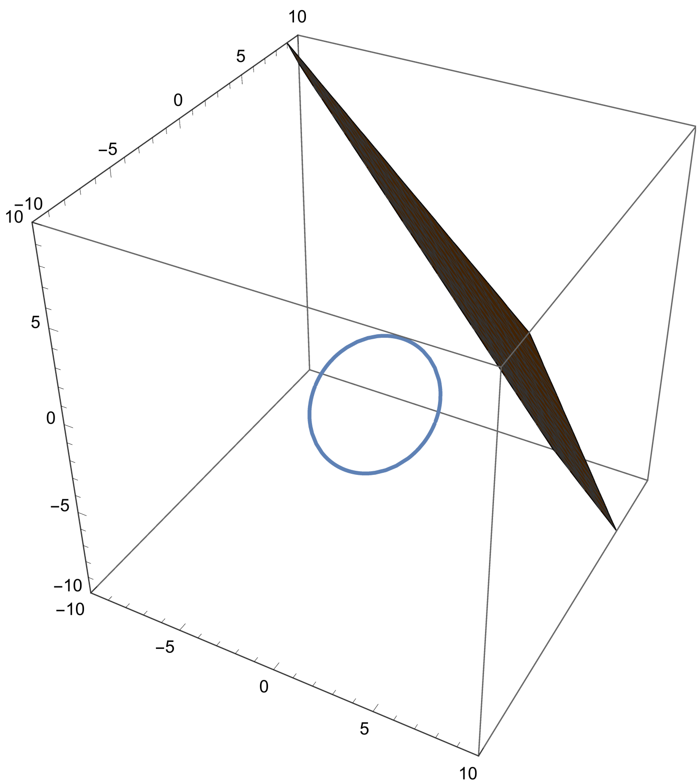
$$\left\{\left\{\frac{2\left(10447+14\sqrt{163681}\right)}{4653}, \frac{5594-17\sqrt{163681}}{4653}, \frac{33349-19\sqrt{163681}}{9306}\right\}, \left\{\frac{2\left(10447-14\sqrt{163681}\right)}{4653}, \frac{5594+17\sqrt{163681}}{4653}, \frac{33349+19\sqrt{163681}}{9306}\right\}\right\}$$



交点が無いときは虚数がでる

```
TestCircle = Cir[{1, 0, 2}, {7, 0, 3}, {4, 3, 5}];
TestCircle = Translator[TestCircle, Vec[{-3, -3, -3}]];
TestPlane = Pln[{4, 2, 3}, {2, 3, 4}, {0, 2, 8}];
CGAIntersection[TestCircle, TestPlane]
Show[CGAOutput3D[TestCircle, {{-10, 10}, {-10, 10}, {-10, 10}}],
 CGAOutput3D[TestPlane, {{-10, 10}, {-10, 10}, {-10, 10}}]]
```

$$\left\{\left\{\frac{2\left(10195+14\,i\sqrt{536879}\right)}{4653},\frac{5900-17\,i\sqrt{536879}}{4653},\frac{33691-19\,i\sqrt{536879}}{9306}\right\},\left\{\frac{2\left(10195-14\,i\sqrt{536879}\right)}{4653},\frac{5900+17\,i\sqrt{536879}}{4653},\frac{33691+19\,i\sqrt{536879}}{9306}\right\}\right\}$$



Dual

片方Dualの内積は交点となる

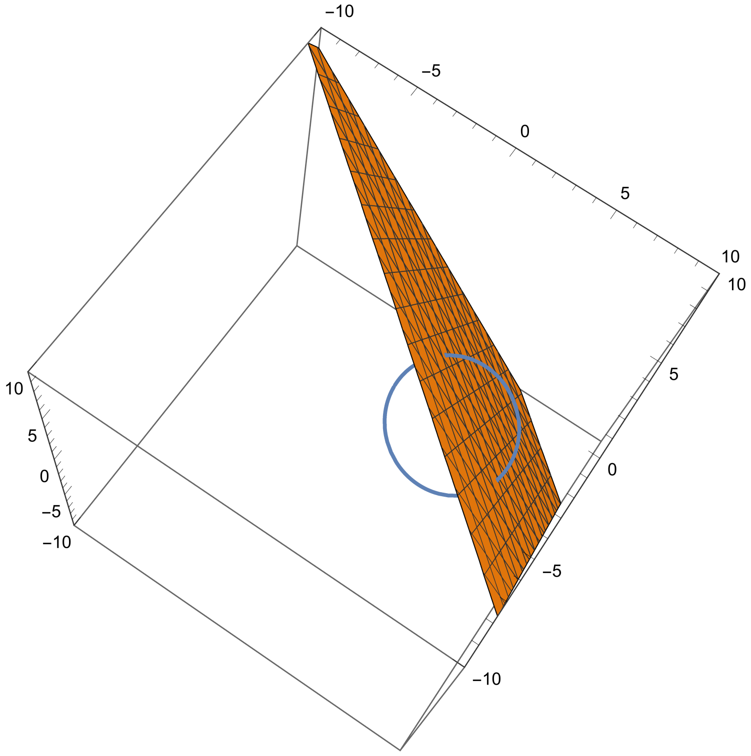
```
TestCircle = Cir[{1, 0, 2}, {7, 0, 3}, {4, 3, 5}];
TestPlane = Pln[{4, 2, 3}, {2, 3, 4}, {0, 2, 8}];
I0 = CGAIntersection[TestCircle, TestPlane]
```

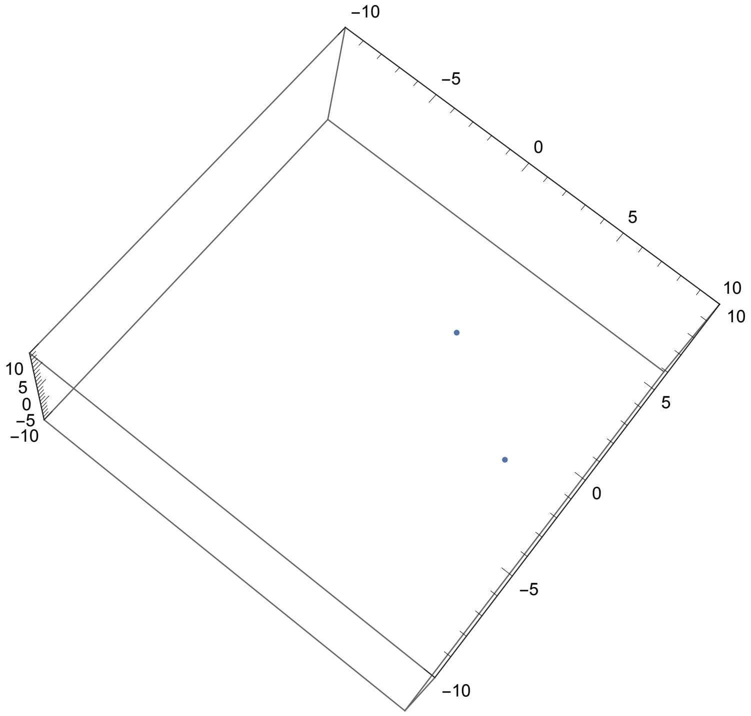
$$\left\{\left\{\frac{2\left(10447+14\sqrt{163681}\right)}{4653},\frac{5594-17\sqrt{163681}}{4653},\frac{33349-19\sqrt{163681}}{9306}\right\},\left\{\frac{2\left(10447-14\sqrt{163681}\right)}{4653},\frac{5594+17\sqrt{163681}}{4653},\frac{33349+19\sqrt{163681}}{9306}\right\}\right\}$$

```
dual0 = Dual[TestCircle];
C0 = InnerProduct[TestPlane, dual0];
CGAIntersection[TestCircle, C0]
```

$$\left\{\left\{\frac{2\left(10447+14\sqrt{163681}\right)}{4653},\frac{5594-17\sqrt{163681}}{4653},\frac{33349-19\sqrt{163681}}{9306}\right\},\right.$$
$$\left.\left\{\frac{2\left(10447-14\sqrt{163681}\right)}{4653},\frac{5594+17\sqrt{163681}}{4653},\frac{33349+19\sqrt{163681}}{9306}\right\}\right\}$$

```
Range0 = {{-10, 10}, {-10, 10}, {-10, 10}};
Show[CGAOutput3D[TestCircle, Range0],
 CGAOutput3D[TestPlane, Range0],
 CGAOutput3D[C0, Range0]
]
CGAOutput3D[C0, Range0]
```





```
TestCircle = Cir[{1, 0, 2}, {7, 0, 3}, {4, 3, 5}];
TestPlane = Pln[{4, 2, 3}, {2, 3, 4}, {0, 2, 8}];
I0 = CGAIntersection[TestCircle, TestPlane];
P0 = Par[I0[[1]], I0[[2]]]

dual0 = Dual[TestCircle];
C0 = InnerProduct[TestPlane, dual0]

CGAEquationCheck[P0, C0]
```

$$-\frac{56\sqrt{163681}\ w[\{0,1\}]}{4653}+\frac{34\sqrt{163681}\ w[\{0,2\}]}{4653}+\frac{19\sqrt{163681}\ w[\{0,3\}]}{4653}-\frac{95\sqrt{163681}\ w[\{0,\infty\}]}{3102}+\frac{20}{423}\sqrt{163681}\ w[\{1,2\}]+\frac{26}{423}\sqrt{163681}\ w[\{1,3\}]+\frac{571\sqrt{163681}\ w[\{1,\infty\}]}{4653}-\frac{1}{47}\sqrt{163681}\ w[\{2,3\}]-\frac{1813\sqrt{163681}\ w[\{2,\infty\}]}{9306}-\frac{1843\sqrt{163681}\ w[\{3,\infty\}]}{9306}$$

$$-168\,w[\{0,1\}]+102\,w[\{0,2\}]+57\,w[\{0,3\}]-\frac{855}{2}w[\{0,\infty\}]+660\,w[\{1,2\}]+858\,w[\{1,3\}]+1713\,w[\{1,\infty\}]-297\,w[\{2,3\}]-\frac{5439}{2}w[\{2,\infty\}]-\frac{5529}{2}w[\{3,\infty\}]$$

```
True
```

CGAEquationCheck

CGAの元がユークリッド空間上で一致するか調べる.

CGAEquationCheck は,同値判定を表す関数である.
引数には,CGAの元の組を用いる.

計算例

```
TestCircle = Cir[{1, 0, 2}, {7, 0, 3}, {4, 3, 5}];
CGAEquationCheck[TestCircle, CGAProduct[TestCircle, Sca[2]]]
True

TestSphere = Sph[{3, 0, 0}, {0, 3, 0}, {0, 0, 3}, {-3, 0, 0}];
CGAEquationCheck[TestSphere, CGAProduct[TestCircle, Sca[2]]]
False
```

```
A1 = 2 w[{0, 1, 2}] + 4 w[{0, 1, 3}] + 11 w[{0, 1, ∞}] +
   2 w[{0, 2, 3}] + 22 w[{0, 2, ∞}] + 33 w[{0, 3, ∞}] + 14 w[{1, 2, ∞}] +
   28 w[{1, 3, ∞}] + 14 w[{2, 3, ∞}] + 6 w[{0, 1, 2, 3}] + 44 w[{0, 1, 2, ∞}] +
   55 w[{0, 1, 3, ∞}] + 66 w[{0, 2, 3, ∞}] + 46 w[{1, 2, 3, ∞}];
```

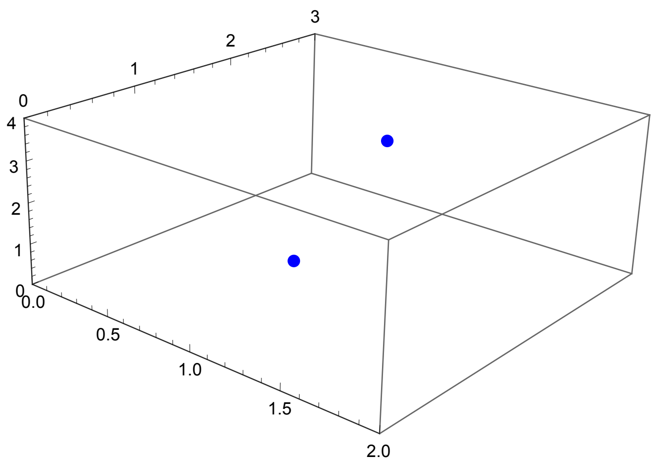
$$A2 = w[\{0, 2\}] + 2\,w[\{0, 3\}] + \frac{11}{2}\,w[\{0, \infty\}] + w[\{1, 2\}] + 2\,w[\{1, 3\}] + \frac{11}{2}\,w[\{1, \infty\}] + w[\{2, 3\}] + 4\,w[\{2, \infty\}] + \frac{5}{2}\,w[\{3, \infty\}];$$

```
CGAEquationCheck[
 A1,
 A2]
```

True

図

```
CGAOutput3D[2 w[{0, 1, 2}] + 4 w[{0, 1, 3}] + 11 w[{0, 1, ∞}] +
  2 w[{0, 2, 3}] + 22 w[{0, 2, ∞}] + 33 w[{0, 3, ∞}] + 14 w[{1, 2, ∞}] +
  28 w[{1, 3, ∞}] + 14 w[{2, 3, ∞}] + 6 w[{0, 1, 2, 3}] + 44 w[{0, 1, 2, ∞}] +
  55 w[{0, 1, 3, ∞}] + 66 w[{0, 2, 3, ∞}] + 46 w[{1, 2, 3, ∞}],
 {{0, 2}, {0, 3}, {0, 4}}, PlotStyle → {PointSize[0.02], Blue}]
```




```
Par[{1, 1, 1}, {1, 2, 3}]
```

$$w[\{0, 2\}] + 2\,w[\{0, 3\}] + \frac{11}{2}\,w[\{0, \infty\}] + w[\{1, 2\}] + 2\,w[\{1, 3\}] + \frac{11}{2}\,w[\{1, \infty\}] + w[\{2, 3\}] + 4\,w[\{2, \infty\}] + \frac{5}{2}\,w[\{3, \infty\}]$$

```
CGAOutput3D[w[{0, 2}] + 2 w[{0, 3}] + 11/2 w[{0, ∞}] + w[{1, 2}] +
  2 w[{1, 3}] + 11/2 w[{1, ∞}] + w[{2, 3}] + 4 w[{2, ∞}] + 5/2 w[{3, ∞}],
  {{0, 2}, {0, 3}, {0, 4}}, PlotStyle → {PointSize[0.02], Red}]
```