Non-monotonic Reasoning in Description Logics Through Typicality Models

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Description Logics

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- Limited to $\{0, 1, 2\}$–ary predicates ...
- and constants (individual names)
- Describe **concepts** and their relationships.

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Examples of **concept-oriented knowledge**

- A parent is a human being that has at least one child.
- A trojan is a human being natural from Troy.
- A god is an immortal being.

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Examples of **individual-oriented knowledge**

- Hector is a trojan prince.
- Athena is the daughter of Zeus.

Description Logics – Concepts

Concept Constructors

$$C ::= \top \mid \bot \mid A \mid C \sqcap D \mid \exists r.C \mid \forall r.C \mid \neg C \mid C \sqcup D$$

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- Athena is the daughter of Zeus.
  - $\mathrm{daughterOf}(\mathrm{Athena}, \mathrm{Zeus})$

Semantics

FOL-like interpretations

$\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$

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Concept	**Interpretation**
$\top^{\mathcal{I}}$	$\Delta^{\mathcal{I}}$
$\bot^{\mathcal{I}}$	$\emptyset$
$(C \sqcap D)^{\mathcal{I}}$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
$(\exists r.C)^{\mathcal{I}}$	$\{x \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}} \text{ s.t. } (x, y) \in r^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\}$
$(\forall r.C)^{\mathcal{I}}$	$\{x \in \Delta^{\mathcal{I}} \mid \forall y \in \Delta^{\mathcal{I}} \text{ if } (x, y) \in r^{\mathcal{I}},\ y \in C^{\mathcal{I}}\}$

Knowledge Bases

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$\mathsf{Bird}(\mathit{tweety})$

$\mathsf{Bird} \sqsubseteq \mathsf{Animal} \sqcap \exists \mathsf{has}.\mathsf{Wings}$

$\mathsf{Penguin} \sqsubseteq \mathsf{Bird}$

$\mathsf{Sparrow} \sqsubseteq \mathsf{Bird}$

Subsumption Checking

- $\mathcal{K} \models \text{Penguin} \sqsubseteq \textit{Animal}$?
- $\mathcal{K} \models \exists\text{has.Wings} \sqsubseteq \text{Birds}$?

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Instance Checking

- $\mathcal{K} \models \text{Animal}(\textit{tweety})$?

Typicality & Defeasible Reasoning in DLs

Typicality

Prototype theory rivals **classical theory** of concepts.

- Philosophical roots in **family resemblance** (WITTGENSTEIN, 1953).
- Experimental/cognitive roots in **prototype theory of conceptualization** (ROSCH, 1978).

Typicality

According to the **classical theory**, concepts are...

- characterized by a list of **necessary and sufficient conditions**,
- have **all or nothing membership**, and
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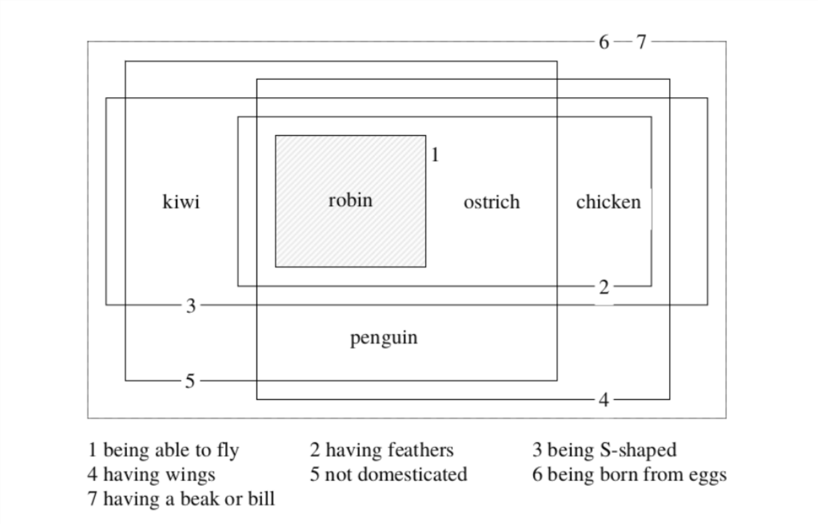
According to the **prototype theory**, concepts are...

- characterized by shared common properties and similarity to prototypical entities,
- have **graded membership**, and
- are not necessarily **compositional**.

Example



BECKER, L. `https://laurabecker.gitlab.io`

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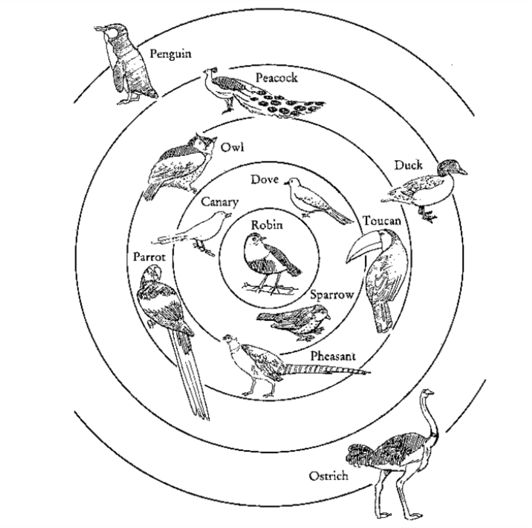

AITCHISON, J. Words in the Mind: An Introduction to the Mental Lexicon. Blackwell: 1987

Defeasibility

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  - Counter-example: situs inversus.

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Seeing concepts through the lens of prototype theory enables defeasible reasoning.

- P1: Birds typically fly.
- P2: **tweety** is a bird.
- C: **tweety** flies.

Typicality and Defeasible DLs

Seeing concepts through the lens of prototype theory enables defeasible reasoning.

- P1: Birds typically fly.
- P2: **tweety** is a bird.
- P3: **tweety** is a penguin.
- C: **tweety** does not fly.

Defeasible Knowledge Bases

$$\mathcal{K} = (\mathcal{A}, \mathcal{T}, \mathcal{D})$$

$\mathrm{Bird}(\mathit{tweety})$

$\mathrm{Bird} \sqsubseteq \mathrm{Animal} \sqcap \exists \mathrm{has}.\mathrm{Wings}$

$\mathrm{Penguin} \sqsubseteq \mathrm{Bird}$

$\mathrm{Sparrow} \sqsubseteq \mathrm{Bird}$

$\mathrm{Bird} \mathrel{\sqsubset\!\!\!\sim} \mathrm{Flies}$

$\mathrm{Bird} \mathrel{\sqsubset\!\!\!\sim} \mathrm{Feathered}$

$\mathrm{Penguin} \mathrel{\sqsubset\!\!\!\sim} \neg \mathrm{Flies}$

Reasoning Task: Defeasible Subsumption Checking

- $\mathcal{K} \models \text{Penguin} \sqsubset\!\!\!\sim \text{Flies}$?
- $\mathcal{K} \models \text{Sparrow} \sqsubset\!\!\!\sim \text{Flies}$?
- $\mathcal{K} \models \text{Penguin} \sqsubset\!\!\!\sim \text{Feathered}$?

Materialisation-based reasoning

Materialisation

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$$S = \{C_1 \precsim D_1, \ldots, C_n \precsim D_n\}$$

$$\overline{S} := \sqcap\{\neg C_i \sqcup D_i \mid C_i \precsim D_i \in S\}$$

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Materialisation-based Reasoning

$\mathcal{K} \models_{\mathsf{mat}} C \precsim D$ **iff** $\mathcal{K} \models C \sqcap \overline{S} \sqsubseteq D$
$S \subseteq \mathcal{D}$

Materialization-based reasoning – Examples

$\mathcal{K} \models_{\mathsf{mat}} C \sqsubsetapprox D$ **iff** $\mathcal{K} \models C \sqcap \overline{S} \sqsubseteq D$

$\mathcal{K} \models \mathsf{Sparrow} \sqcap \overline{\{\mathsf{Bird} \sqsubsetapprox \mathsf{Flies}, \mathsf{Penguin} \sqsubsetapprox \neg\mathsf{Flies}\}} \sqsubseteq \mathsf{Flies}$

$\mathcal{K} \models \mathsf{Penguin} \sqcap \overline{\{\mathsf{Penguin} \sqsubsetapprox \neg\mathsf{Flies}\}} \sqsubseteq \neg\mathsf{Flies}$

Rational reasoning

$C \mathrel{\sqsubset\!\!\!\sim} D$ is **exceptional** w.r.t. $\mathcal{T}$ and $\mathcal{U} \subseteq \mathcal{D}$ iff
$\mathcal{T} \models C \sqcap \overline{\mathcal{U}} \sqsubseteq \bot$.

Rational reasoning

$C \mathrel{\sqsubset\!\!\!\sim} D$ is **exceptional** w.r.t. $\mathcal{T}$ and $\mathcal{U} \subseteq \mathcal{D}$ iff
$\mathcal{T} \models C \sqcap \overline{\mathcal{U}} \sqsubseteq \bot$.

$\mathcal{T} \models \text{Penguin} \sqcap \overline{\mathcal{D}} \sqsubseteq \bot$

Penguin $\mathrel{\sqsubset\!\!\!\sim}$ Flies is exceptional w.r.t. $\mathcal{T}$ and $\mathcal{D}$

Exceptionality Chain

Chain: $\mathcal{E}_0 \supset \mathcal{E}_1 \supset \cdots \supset \mathcal{E}_n$

$\mathcal{E}_0 = \mathcal{D}$

$\mathcal{E}_{i+1} = \{C \precsim D \in \mathcal{E}_i \mid C \precsim D \text{ is exceptional w.r.t. } \mathcal{T} \text{ and } \mathcal{E}_i\}$

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Example

$\mathcal{E}_0 = \{Bird \precsim Flies, Bird \precsim Feathered, Penguin \precsim \neg Flies\}$

$\mathcal{E}_1 = \{Penguin \precsim \neg Flies\}$

$\mathcal{E}_2 = \emptyset$

Rational Reasoning

$\mathcal{K} \models_{\mathsf{mat,rat}} C \sqsubsetapprox D$ iff $\mathcal{T} \models C \sqcap \overline{\mathcal{E}_i} \sqsubseteq D$

For the smallest i s.t. $C \sqsubsetapprox D$ is not exceptional w.r.t. $\mathcal{T}$ and $\mathcal{E}_i$.

Examples

$\mathcal{T}$

Penguin $\sqsubseteq$ Bird

Sparrow $\sqsubseteq$ Bird

$\mathcal{D} = \mathcal{E}_0$

$\mathcal{E}_1$

Penguin $\mathrel{\sqsubset\!\!\!\sim}$ $\neg$Flies

Bird $\mathrel{\sqsubset\!\!\!\sim}$ Flies

Bird $\mathrel{\sqsubset\!\!\!\sim}$ Feathered

Example 1: Sparrows Fly

$\mathcal{T}$

Penguin $\sqsubseteq$ Bird

Sparrow $\sqsubseteq$ Bird

$\mathcal{D} = \mathcal{E}_0$

Penguin $\sqsubsim \neg$Flies

$\mathcal{E}_1$

Bird $\sqsubsim$ Flies

Bird $\sqsubsim$ Feathered

$$\mathcal{T} \models \text{Sparrow} \sqcap \overline{\mathcal{E}_0} \sqsubseteq \text{Flies}$$

Example 2: Inheritance Blocking

$\mathcal{T}$	$\mathcal{D} = \mathcal{E}_0$	
$\mathsf{Penguin} \sqsubseteq \mathsf{Bird}$	$\mathsf{Penguin} \underset{\sim}{\sqsubset} \neg\mathsf{Flies}$	$\mathcal{E}_1$
$\mathsf{Sparrow} \sqsubseteq \mathsf{Bird}$	$\mathsf{Bird} \underset{\sim}{\sqsubset} \mathsf{Flies}$	
	$\mathsf{Bird} \underset{\sim}{\sqsubset} \mathsf{Feathered}$	

$$\mathcal{T} \not\models \mathsf{Penguin} \sqcap \overline{\mathcal{E}_1} \sqsubseteq \mathsf{Feathered}$$

Example 3: Quantification Neglect

$\mathcal{T}$

$\text{Penguin} \sqsubseteq \text{Bird}$

$\text{Sparrow} \sqsubseteq \text{Bird}$

$\text{Cat} \sqsubseteq \exists \text{eats.Bird}$

$\mathcal{D} = \mathcal{E}_0$

$\text{Penguin} \lesssim \neg\text{Flies}$

$\mathcal{E}_1$

$\text{Bird} \sqsubseteq \text{Flies}$

$\text{Bird} \sqsubseteq \text{Feathered}$

$$\mathcal{T} \not\models \text{Cat} \sqcap \overline{\mathcal{E}_0} \sqsubseteq \exists \text{eats.Flies}$$

Typicality Models

An overview – What are typicality models?

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- The semantics is parametrized with two variables: **strength** X **coverage**.
  - **Strength**: related to the materialisation-based reasoning procedure.
  - **Coverage**: to which depth the semantics propagate defeasible information (**propositional** is shallow, while **nested** is deep).

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  - **Strength**: related to the materialisation-based reasoning procedure.
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- A typicality upgrade procedure takes the semantics from propositional to nested.

A Brief Introduction to Canonical Models for the $\mathcal{EL}$ family

- Elements of the domain represent concepts ($\mathcal{EL}_\bot$) or sets of concepts ($\mathcal{ELI}_\bot$).

A Brief Introduction to Canonical Models for the $\mathcal{EL}$ family

- Elements of the domain represent concepts ($\mathcal{EL}_\perp$) or sets of concepts ($\mathcal{ELI}_\perp$).
- We can check subsumption by looking at membership for concept representatives.
  - $C \in D^{\mathcal{I}^{\mathcal{K}}}$ iff $\mathcal{K} \models C \sqsubseteq D$
  - $M \in A^{\mathcal{I}^{\mathcal{K}}}$ iff $\mathcal{K} \models \sqcap_{C \in M} C \sqsubseteq A$

Typicality Domains

$C_\mathcal{U}$ represents $C \sqcap \overline{\mathcal{U}}$

$\mathcal{U} \subseteq \mathcal{D}$

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Defeasible Satisfiability

$\mathcal{I} \models C \mathrel{\sqsubset\!\!\!\sim} D$ iff $C_{\mathcal{E}_i} \in D^\mathcal{I}$ for every maximal $\mathcal{E}_i$ w.r.t. C in the domain.

Rational Domain for $\mathcal{EL}_{\bot}$

	Bird	Cat	Penguin
$\emptyset$	●	●	●
$\mathcal{E}_1$	●	●	●
$\mathcal{E}_0$	●	●	

Minimal Typicality Model for $\mathcal{EL}_\bot$

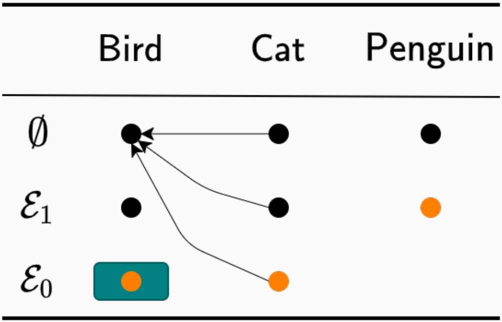
$$C_{\mathcal{U}} \in D^{\mathcal{I}^{\mathcal{K}}_{\textbf{min,rat}}} \text{ iff } \mathcal{T} \models C \sqcap \overline{\mathcal{U}} \sqsubseteq D$$

$$(C_{\mathcal{U}}, D_{\emptyset}) \in r^{\mathcal{I}^{\mathcal{K}}_{\textbf{min,rat}}} \text{ iff } \mathcal{T} \models C \sqcap \overline{\mathcal{U}} \sqsubseteq \exists r.D.$$

Minimal Typicality Model for $\mathcal{EL}_\perp$

$$C_{\mathcal{U}} \in D^{\mathcal{I}^{\mathcal{K}}_{\textsf{min,rat}}} \text{ iff } \mathcal{T} \models C \sqcap \overline{\mathcal{U}} \sqsubseteq D$$

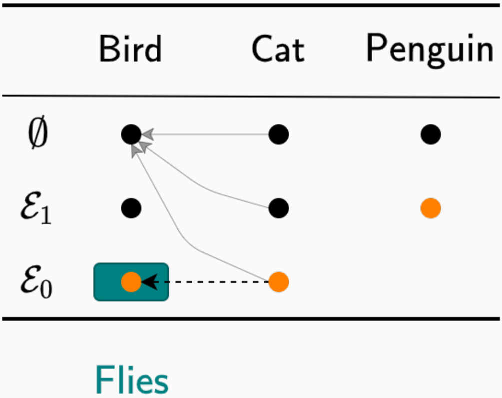
$$(C_{\mathcal{U}}, D_{\emptyset}) \in r^{\mathcal{I}^{\mathcal{K}}_{\textsf{min,rat}}} \text{ iff } \mathcal{T} \models C \sqcap \overline{\mathcal{U}} \sqsubseteq \exists r.D.$$



Flies

Upgrading Edges From the Minimal Typicality Model for $\mathcal{EL}_\perp$

$$\mathsf{Cat}_{\mathcal{E}_0} \in (\exists \mathsf{eats}.\mathsf{Flies})^{\mathcal{I}} \Leftrightarrow \mathcal{I} \models \mathsf{Cat} \mathrel{\underset{\sim}{\sqsubseteq}} \exists \mathsf{eats}.\mathsf{Flies}$$



Upgrade algorithm

1. update an edge,
2. recover the model property.

- Iterating this procedure until there are no more edges to improve creates a set of **preferred models – saturated typicality models**.

Nested Reasoning

Upgrade algorithm

1. update an edge,
2. recover the model property.

- Iterating this procedure until there are no more edges to improve creates a set of **preferred models – saturated typicality models**.
- This defines **reasoning of nested coverage**, which **solves quantification neglect**.

Introducing Inverse Roles – $\mathcal{ELI}_\bot$

Inverse roles: the inverse relation of any given role.

- $r^{-\mathcal{I}} = \{(d, e) \in \Delta^\mathcal{I} \times \Delta^\mathcal{I} \mid (e, d) \in r^\mathcal{I}\}$.
- $\mathsf{parentOf}^- = \mathsf{childrenOf}$.
- Expressing value restrictions: $\exists r^-.C \sqsubseteq D \equiv C \sqsubseteq \forall r.D$.
- Complexity blowup: from linear to ExpTime.

Three Main Obstacles for $\mathcal{ELI}_\bot$ TMs

(1.) Adjusting the domain;

(2.) Identifying elements in the edges;

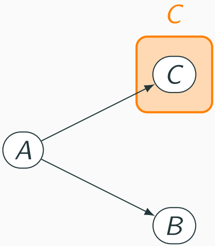
(3.) Recovering the model property.

Adjusting the Domain

$$\mathcal{EL}_\bot$$

$$C \sqsubseteq \exists r.D \Rightarrow (C, D) \in r^{\mathcal{I}^{\mathcal{K}}_{\mathsf{min,rat}}}$$

$A \sqsubseteq \exists r.B + A \sqsubseteq \forall r.C$



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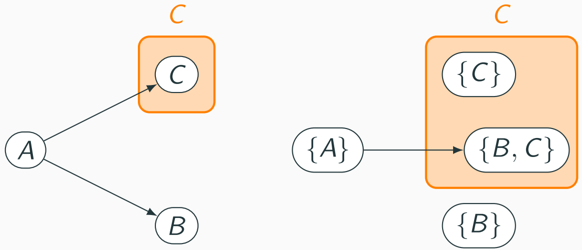


$C \sqsubseteq \exists r.M$ **and** M is maximal w.r.t. $\mathcal{K}, C, r \Rightarrow (C, M) \in r^{\mathcal{I}^{\mathcal{K}}_{\mathsf{min,rat}}}$

Overview

(1.) Adjusting the domain;

(2.) Identifying elements in the edges;

(3.) Recovering the model property.

Causal Direction of the Edges

$$\lceil M \rceil \sqcap \overline{\mathcal{E}_j} \sqsubseteq \exists r.\lceil N \rceil$$

$$M_{\mathcal{E}_i} \longrightarrow N_{\mathcal{E}_k}$$

Initiator labeling – each edge is labeled with *label* $\subseteq \{s, p\}$.

Causal Direction of the Edges

$$\lceil M \rceil \sqcap \overline{\mathcal{E}_j} \sqsubseteq \exists r.\lceil N \rceil$$

$$\begin{tikzcd} M_{\mathcal{E}_i} \arrow[r] & N_{\mathcal{E}_k} \end{tikzcd}$$

$$l < k$$

$$\lceil M \rceil \sqcap \overline{\mathcal{E}_j} \sqsubseteq \exists r.\lceil N \rceil$$

$$\begin{tikzcd} & N_{\mathcal{E}_l} \\ M_{\mathcal{E}_i} \arrow[r] \arrow[ur, dashed] & N_{\mathcal{E}_k} \end{tikzcd}$$

Initiator labeling – each edge is labeled with *label* $\subseteq \{s, p\}$.

Causal Direction of the Edges

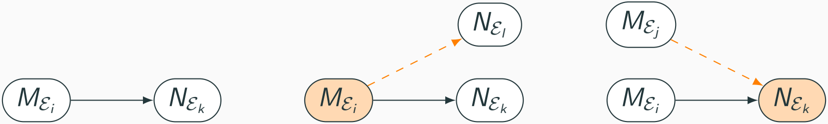
$\lceil M \rceil \sqcap \overline{\mathcal{E}_j} \sqsubseteq \exists r.\lceil N \rceil$

$l < k$

$\lceil M \rceil \sqcap \overline{\mathcal{E}_j} \sqsubseteq \exists r.\lceil N \rceil$

$i < j$

$\lceil N \rceil \sqcap \overline{\mathcal{E}_k} \sqsubseteq \exists r^{-}.\lceil M \rceil$



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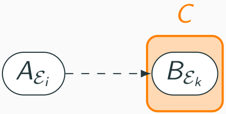

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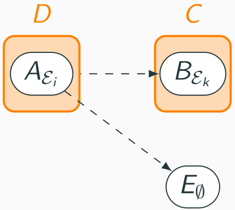

Add $A_{\mathcal{E}_i}$ to D

Add edge landing on $E_\emptyset$

Causal Direction of the Edges

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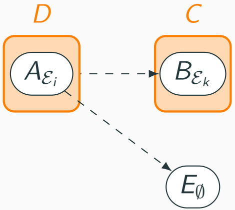
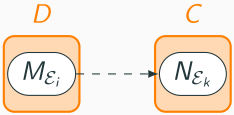
$$\exists r.C \sqsubseteq D$$
$$D \sqsubseteq \forall r.E$$





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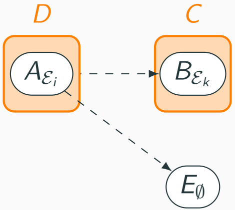

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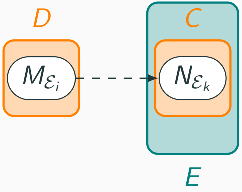
$\exists r.C \sqsubseteq D$

$D \sqsubseteq \forall r.E$



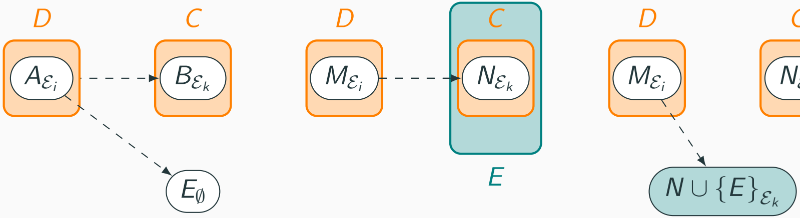
Add r successors to E

Causal Direction of the Edges

$\exists r.C \sqsubseteq D$
$\exists r.C \sqsubseteq \exists s.E$

$\exists r.C \sqsubseteq D$
$D \sqsubseteq \forall r.E$

Move r edges to include E



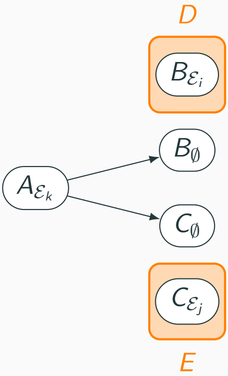
Add $A_{\mathcal{E}_i}$ to D
Add edge landing on $E_\emptyset$

Add r successors to E

Nested Reasoning

Nested Reasoning

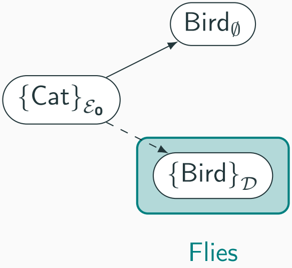
$\exists r.D \sqcap \exists r.E \sqsubseteq \bot$



Skeptical Reasoning:

$\mathcal{K} \models_{\mathsf{nest,rat}} A \sqsubseteq\!\!\!\!\sim B$ *iff* $\{A\}_{\mathcal{E}_{\min}} \in B^{\mathcal{I}}$
In every maximally upgraded $\mathcal{I}$.

$$
\begin{aligned}
\mathcal{T} = \{ & \text{Penguin} \sqsubseteq \text{Bird}, \\
& \text{Cat} \sqsubseteq \exists \text{eats.Bird} \} \\
\mathcal{D} = \{ & \text{Penguin} \underset{\sim}{\sqsubset} \neg \text{Flies} \\
& \text{Bird} \underset{\sim}{\sqsubset} \text{Flies} \\
& \text{Bird} \underset{\sim}{\sqsubset} \text{Feathered} \}
\end{aligned}
$$



$\{\text{Cat}_{\mathcal{E}_0}\} \in \exists \text{eats.Flies}^{\mathcal{I}}$

$\mathcal{K} \models_{\text{nest,rat}} \text{Cat} \underset{\sim}{\sqsubset} \exists \text{eats.Flies}$

Summary

- Defeasible reasoning is a robust way to approach typicality within DLs.
- Semantics based on typicality models are general enough to model distinct materialisation-based reasoning procedures.
- The typicality upgrade procedure defines reasoning of nested coverage, which solves the problem of information spread through roles.
- Increase in expressivity come with costs and the procedure depends fundamentally on the simplicity of the canonical structures.

Related work not covered here

- Typicality semantics accomodate ABoxes and instance checkings with some adjustments.
- There are canonical domains for other materialisation-based reasoning procedures, such as relevant and lexicographic closure.
- A full comparison between all the strengths $\times$ coverages for both $\mathcal{EL}_\bot$ and $\mathcal{ELI}_\bot$.

Avenues for future research

- Extending the typicality framework for Horn-DLs.
- Including more complex reasoning tasks, such as conjunctive query answering.
- Explore the complexity of the calculus for $\mathcal{ELI}_\bot$.
- Implementing typicality-based reasoners.