

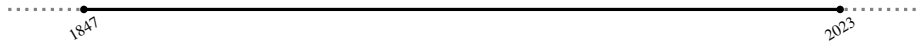
Thinking with **Circuits**

From **Logic** to **Probabilistic** and Back

Renato Lui Geh



Circuits?



Circuits?



Circuits?

George Boole



1847

2023

Boolean Algebra

$$x, y \in \{0, 1\}$$

$$x \wedge y \Leftrightarrow$$

x	y	$x \wedge y$
0	0	0
0	1	0
1	0	0
1	1	1

$$x \vee y \Leftrightarrow$$

x	y	$x \vee y$
0	0	0
0	1	1
1	0	1
1	1	1

$$\neg x \Leftrightarrow$$

x	$\neg x$
0	1
1	0

Propositional logic?

Satisfiability and Model Counting

$$\phi(X, Y, Z) = [X \wedge (\neg Y \vee Z)] \vee [(\neg Z \vee Y) \wedge \neg X]$$

Satisfiability and Model Counting

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Are there $X = x$, $Y = y$, $Z = z$ s.t.

$$\phi(X = x, Y = y, Z = z) = 1?$$

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x	y	z	$\phi(x, y, z)$	x	y	z	$\phi(x, y, z)$
0	0	0	1	1	0	0	1
0	0	1	0	1	0	1	1
0	1	0	1	1	1	0	0
0	1	1	1	1	1	1	1

Satisfiability and Model Counting

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x	y	z	$\phi(x, y, z)$	x	y	z	$\phi(x, y, z)$
0	0	0	1 ✓	1	0	0	1 ✓
0	0	1	0 ✗	1	0	1	1 ✓
0	1	0	1 ✓	1	1	0	0 ✗
0	1	1	1 ✓	1	1	1	1 ✓

SAT = ✓

Satisfiability and Model Counting

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0	0	1	0	1	0	1	1
0	1	0	1	1	1	0	0
0	1	1	1	1	1	1	1

Satisfiability and Model Counting

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$$\#\text{SAT} = 6$$

Satisfiability and Model Counting

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$$\#SAT = 6$$

Normal Forms

CNF

Conjunctive Normal Form

$$(X_1 \vee \neg X_2) \wedge (\neg X_3 \vee X_4)$$

Normal Forms

CNF

Conjunctive Normal Form

$$(X_1 \vee \neg X_2) \wedge (\neg X_3 \vee X_4)$$

DNF

Disjunctive Normal Form

$$(\neg Y_1 \wedge \neg Y_2) \vee (Y_3 \wedge Y_4)$$

Normal Forms

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NNF

Negation Normal Form

$$\neg Z_1 \wedge (Z_2 \vee \neg Z_3) \vee Z_4$$

Normal Forms

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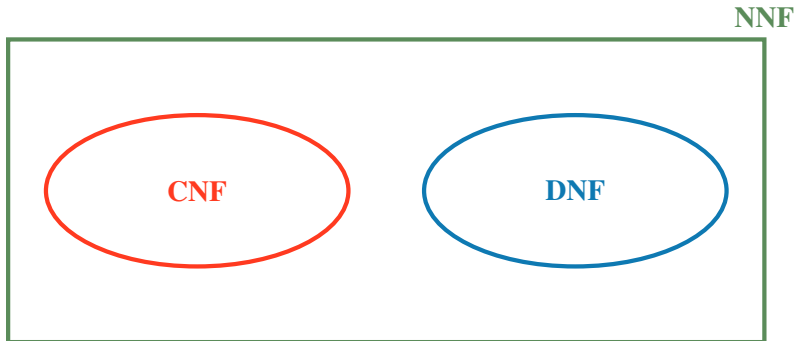
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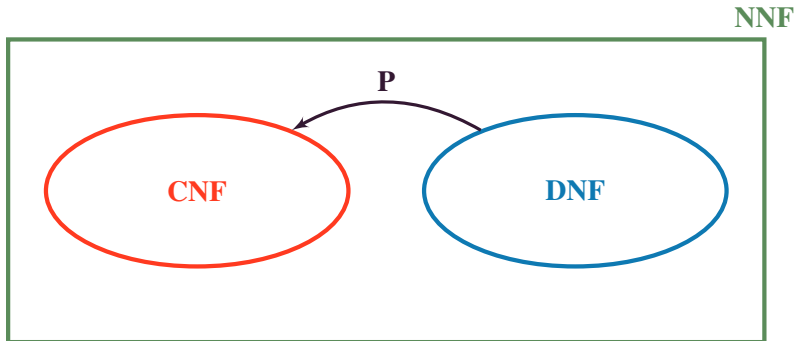
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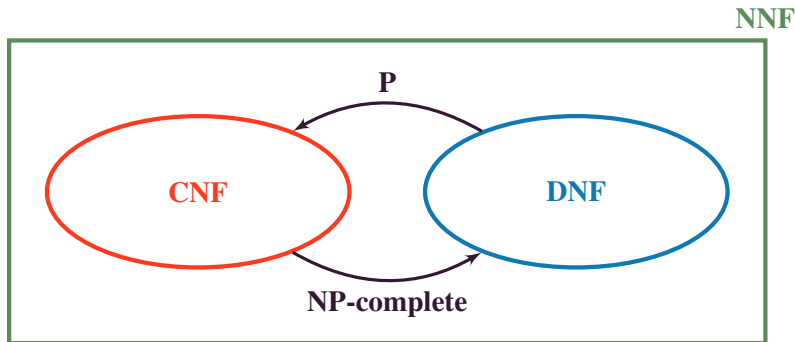
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Circuits?



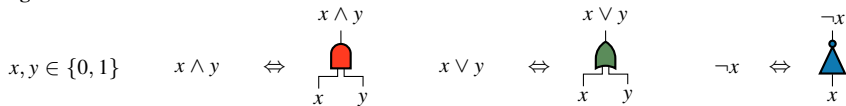
Circuits?



Circuits?



Logic Circuits

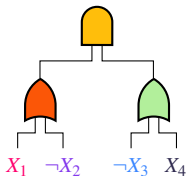


Normal Forms in Logic Circuits

CNF

Conjunctive Normal Form

$$(X_1 \vee \neg X_2) \wedge (\neg X_3 \vee X_4)$$

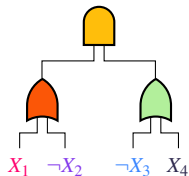


Normal Forms in Logic Circuits

CNF

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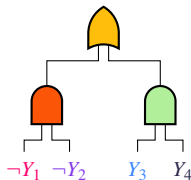
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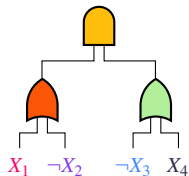


Normal Forms in Logic Circuits

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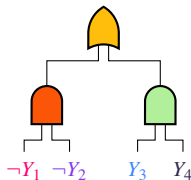
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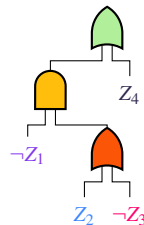
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Satisfiability and Model Counting in Logic Circuits

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Satisfiability and Model Counting in Logic Circuits

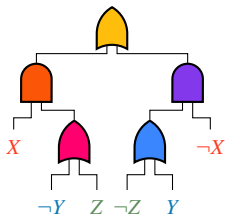
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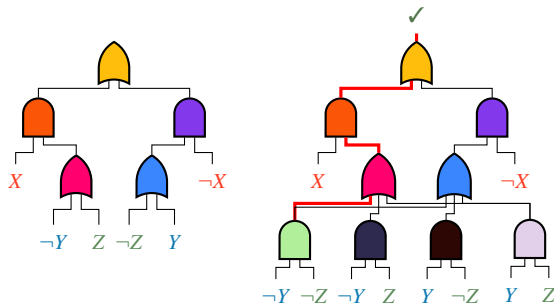
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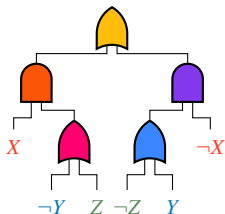


SAT = ✓

Satisfiability and Model Counting in Logic Circuits

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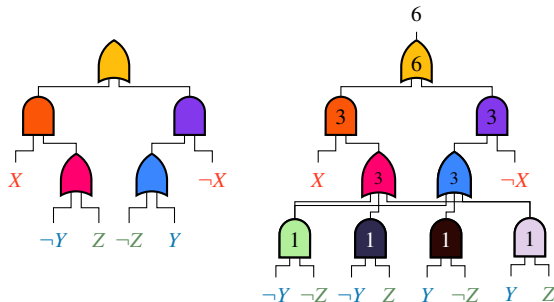
How *many* $X = x, Y = y, Z = z$ s.t. $\phi(X = x, Y = y, Z = z) = 1$?



Satisfiability and Model Counting in Logic Circuits

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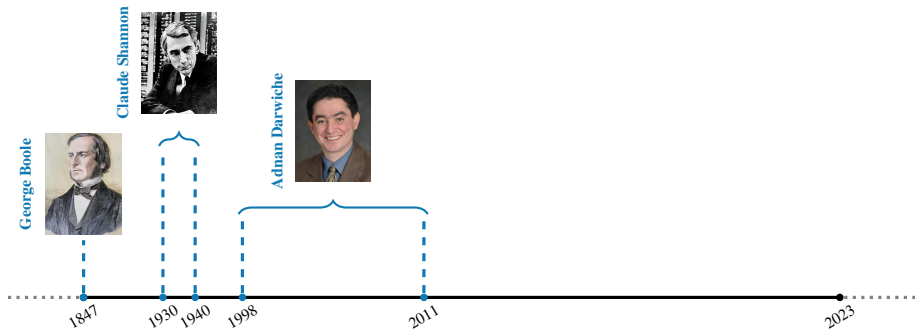


#SAT = 6

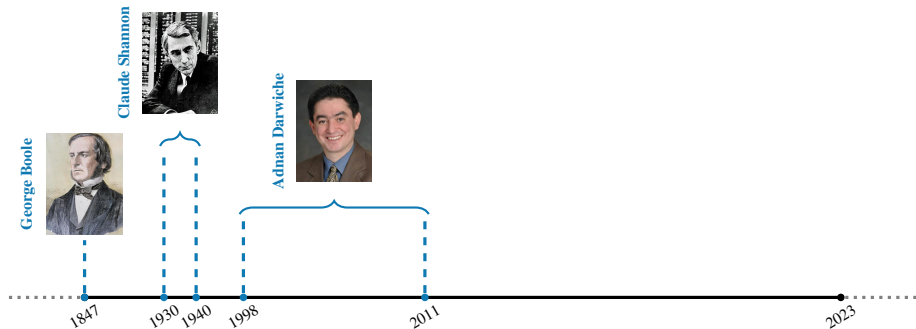
Circuits?



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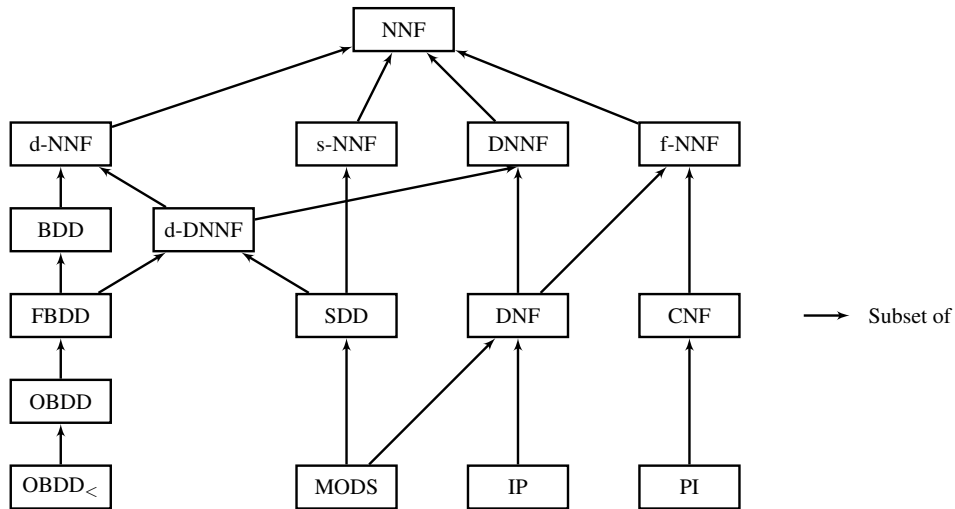
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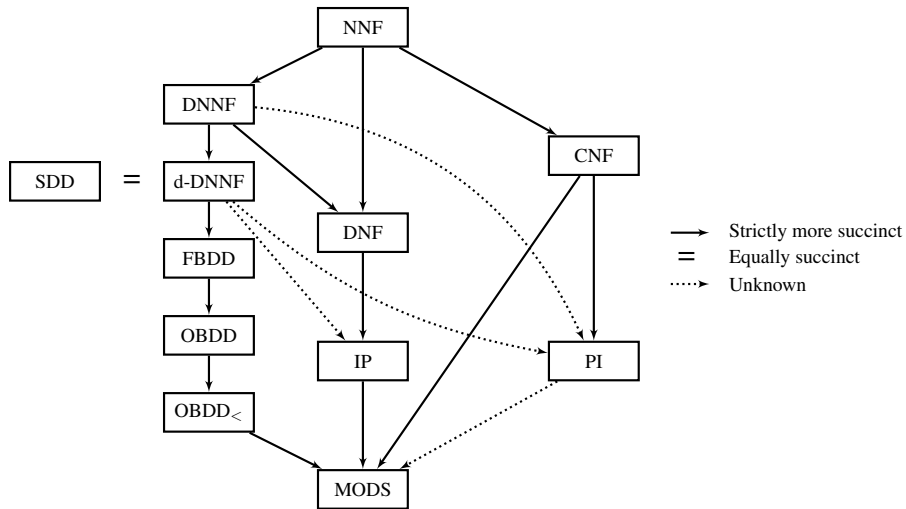
Knowledge Compilation

How much/many ... $\left\{ \begin{array}{l} \dots \text{succinct (i.e. compact/efficient)} \text{ are circuits for } \dots \\ \dots \text{queries in polytime are supported by } \dots \\ \dots \text{transformations in polytime are available for } \dots \end{array} \right\} \dots \text{different classes of logic circuits?}$

The World of Logic Circuits



Succinctness in Logic Circuits



Querying in Logic Circuits

L	CO	VA	CE	IM	EQ	SE	CT	ME
NNF	○	○	○	○	○	○	○	○
DNNF	✓	○	✓	○	○	○	○	✓
d-NNF	○	○	○	○	○	○	○	○
s-NNF	○	○	○	○	○	○	○	○
f-NNF	○	○	○	○	○	○	○	○
d-DNNF	✓	✓	✓	✓	?	○	✓	✓
SDD	✓	✓	✓	✓	?	○	✓	✓
BDD	○	○	○	○	○	○	○	○
FBDD	✓	✓	✓	✓	?	○	✓	✓
OBDD	✓	✓	✓	✓	✓	○	✓	✓
OBDD _{<}	✓	✓	✓	✓	✓	✓	✓	✓
DNF	✓	○	✓	○	○	○	○	✓
CNF	○	✓	○	✓	○	○	○	○
PI	✓	✓	✓	✓	✓	✓	○	✓
IP	✓	✓	✓	✓	✓	✓	○	✓
MODS	✓	✓	✓	✓	✓	✓	✓	✓

Notation	Query
CO	Consistency check
VA	Validity check
CE	Clausal entailment check
IM	Implicant check
EQ	Equivalence check
SE	Sentential entailment check
CT	Model counting
ME	Model enumeration

Notation	Description
✓	In P
✗	Not in P
○	Not in P unless P = NP
?	Unknown

[Darwiche and Marquis, 2002]

Transformations in Logic Circuits

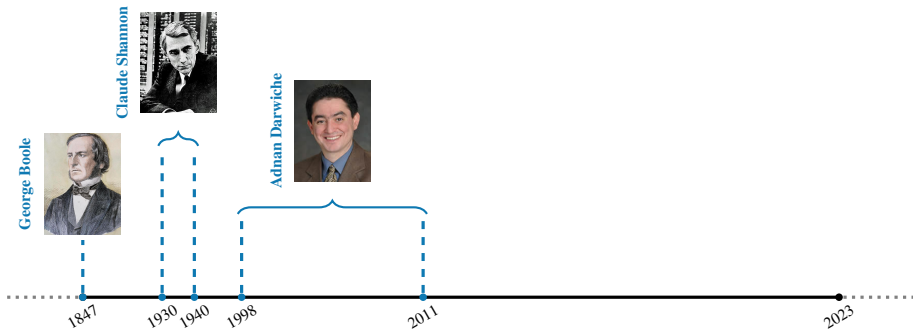
L	CD	FO	SFO	$\wedge C$	$\wedge BC$	$\vee C$	$\vee BC$	$\neg C$
NNF	✓	○	✓	✓	✓	✓	✓	✓
DNNF	✓	✓	✓	○	○	✓	✓	○
d-NNF	✓	○	✓	✓	✓	✓	✓	✓
s-NNF	✓	○	✓	✓	✓	✓	✓	✓
f-NNF	✓	○	✓	✗	✗	✗	✗	✓
d-DNNF	✓	○	○	○	○	○	○	✓
SDD	✓	○	○	○	○	○	○	✓
BDD	✓	○	✓	✓	✓	✓	✓	✓
FBDD	✓	✗	○	✗	○	✗	○	✓
OBDD	✓	✗	✓	✗	○	✗	○	✓
OBDD _{<}	✓	✗	✓	✗	✓	✗	✓	✓
DNF	✓	✓	✓	✗	✓	✓	✓	✗
CNF	✓	○	✓	✓	✓	✗	✓	✗
PI	✓	✓	✓	✗	✗	✗	✓	✗
IP	✓	✗	✗	✗	✓	✗	✗	✗
MODS	✓	✓	✓	✗	✓	✗	✗	✗

Notation	Query
CD	Conditioning
FO	Forgetting
SFO	Singleton forgetting
$\wedge C$	Conjunction
$\wedge BC$	Bounded conjunction
$\vee C$	Disjunction
$\vee BC$	Bounded disjunction
$\neg C$	Negation

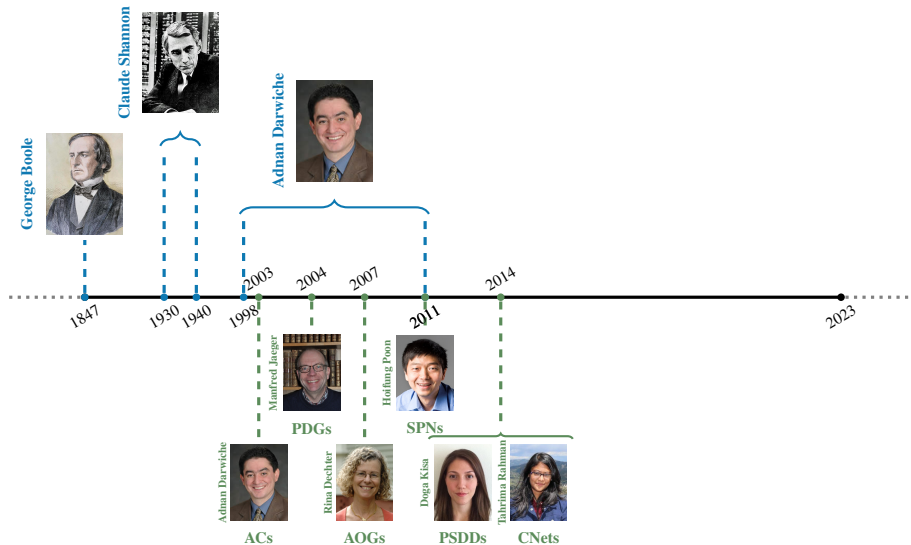
Notation	Description
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✗	Not in P
○	Not in P unless P = NP
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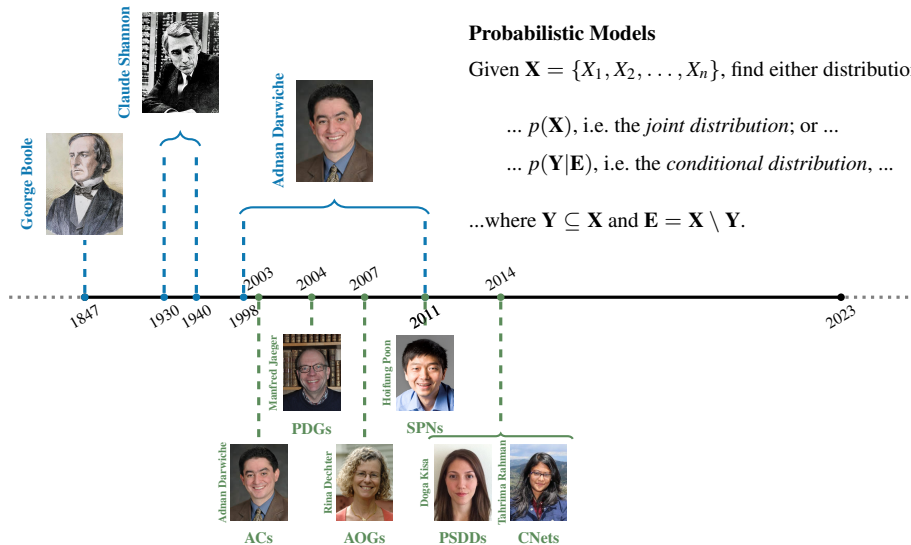
Circuits?



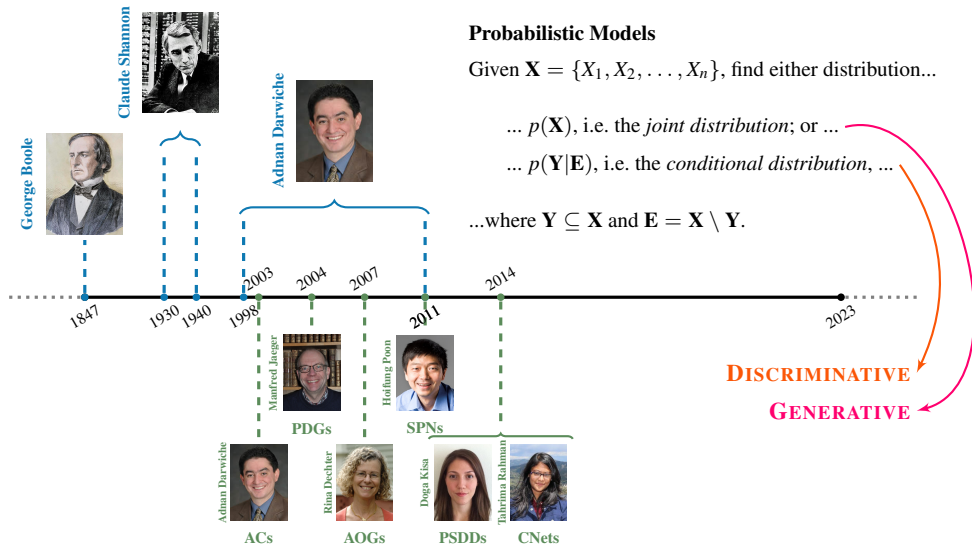
Circuits?



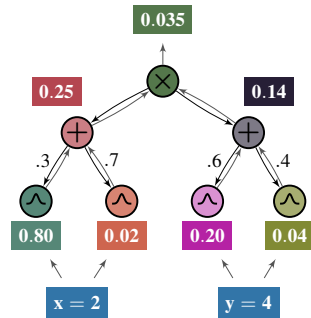
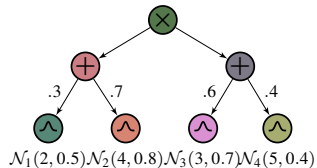
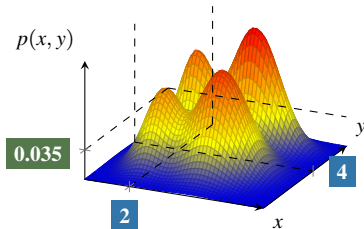
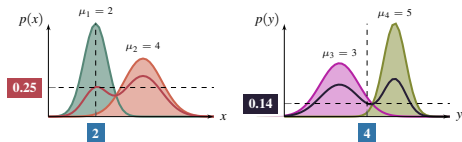
Circuits?



Circuits?



Probabilistic Circuits



Querying in Probabilistic Circuits

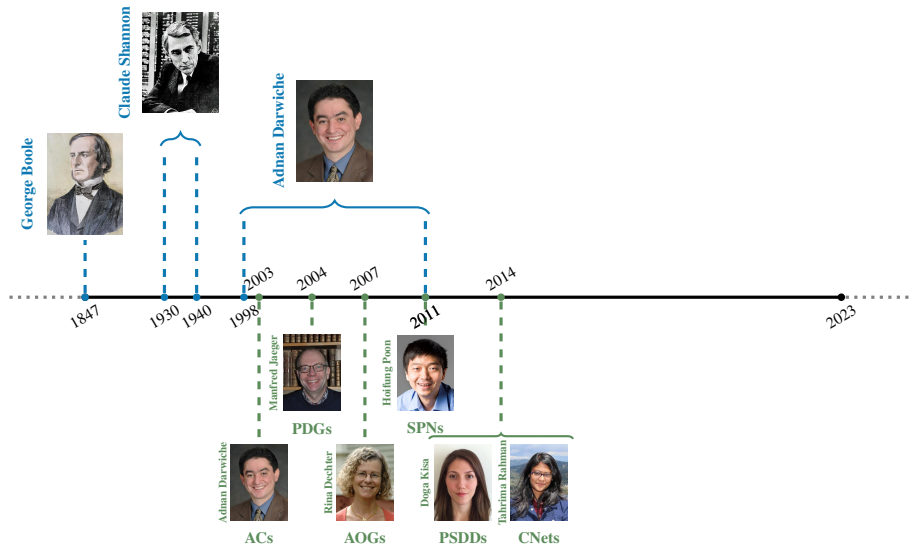
Query	+Sm?	+Dec?	+Det?	+Str Dec?
Evidence	✓	✓	✓	✓
Marginals	○	✓	✓	✓
Conditionals	○	✓	✓	✓
MPE	○	○	✓	✓
Shannon Entropy*	○	○	✓	✓
Rényi Entropy*	○	○	✓	✓
Cross Entropy*	○	○	○	✓
Kullback-Leibler Div*	○	○	○	✓
Rényi's Alpha Div*	○	○	○	✓
Cauchy-Schwarz Div*	○	○	○	✓
Logical Events	○	○	○	✓
Mutual Information*	○	○	○	✓

Transformations in Probabilistic Circuits

	Query	+Sm?	+Dec?	+Det?	+Str Dec?
Sum	$w_1 \cdot p + w_2 \cdot q$	✓	✓	○	○
Product	$p \cdot q$	○	○	○	✓
Power	$p^n, n \in \mathbb{N}$	○	○	○	✓
	$p^\alpha, \alpha \in \mathbb{R}$	○	○	✓	✓
Quotient	$\frac{p}{q}$	○	○	✓	✓
Log	$\log(p)$	○	○	✓	✓
Exp	$\exp(p)$	○	○	✓	✓

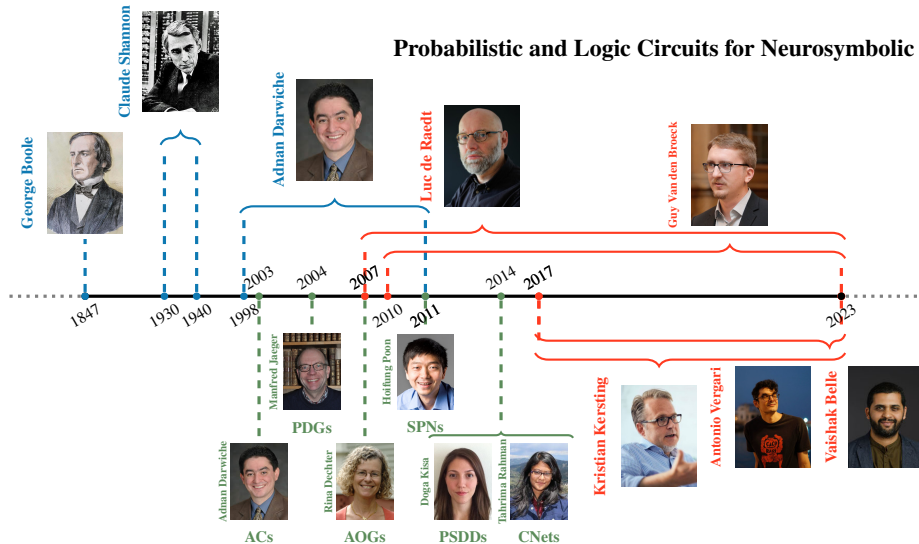
[Vergari et al., 2021]

Circuits?



Circuits?

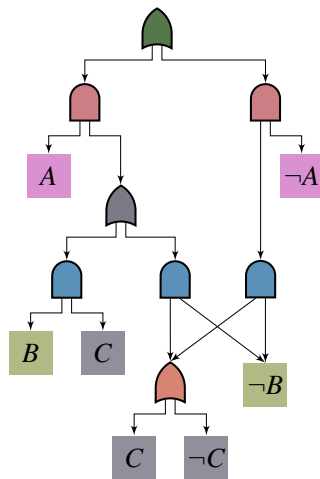
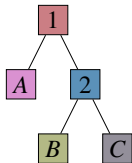
Probabilistic and Logic Circuits for Neurosymbolic Reasoning



Logic Circuits \subset Probabilistic Circuits

A	B	C	$\phi(\mathbf{x})$
0	0	0	1
1	0	0	1
0	1	0	0
1	1	0	0
0	0	1	1
1	0	1	1
0	1	1	0
1	1	1	1

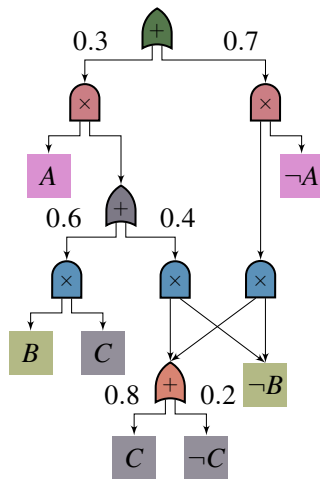
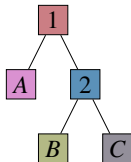
$$\phi(A, B, C) = (A \vee \neg B) \wedge (\neg B \vee C)$$



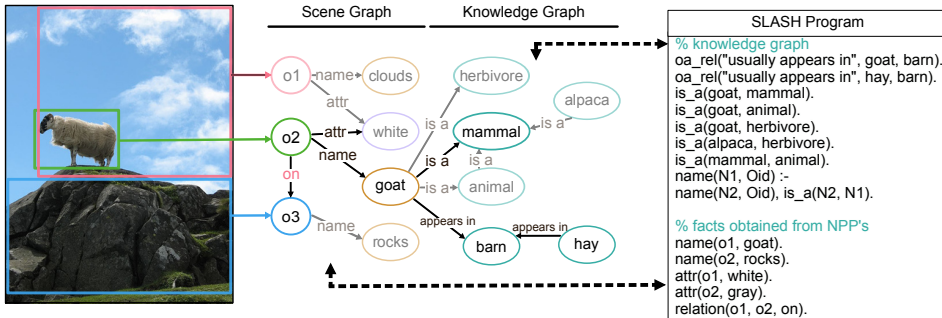
Logic Circuits \subset Probabilistic Circuits


A	B	C	$\phi(\mathbf{x})$	$p(\mathbf{x})$
0	0	0	1	0.140
1	0	0	1	0.024
0	1	0	0	0.000
1	1	0	0	0.000
0	0	1	1	0.560
1	0	1	1	0.096
0	1	1	0	0.000
1	1	1	1	0.180

$$\phi(A, B, C) = (A \vee \neg B) \wedge (\neg B \vee C)$$

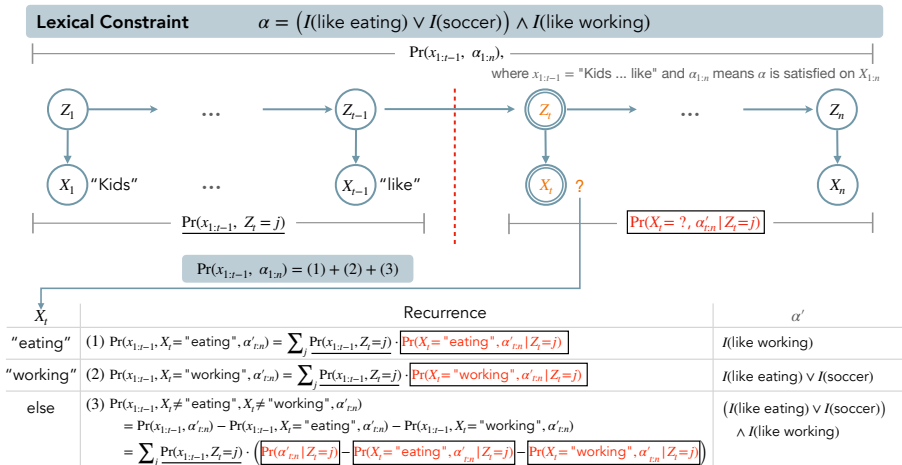


Circuits in Visual Query Answering

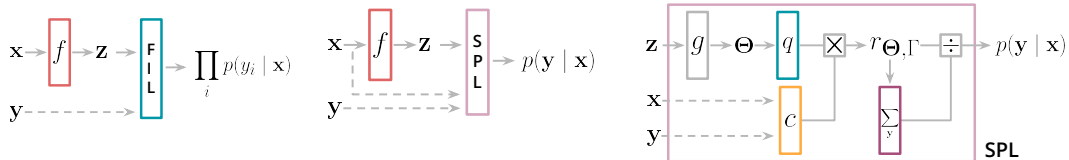


	Natural language question	Programmatic queries	Answer
difficulty	"Identify the goat "	target(O):-name(O, goat).	target(o2) 
	"Identify the white mammal "	target(O):-name(O, mammal), attr(O, white).	
	"Identify the mammal on the rocks , that usually appears where hay appears "	target(O):- name(O, mammal), name(O2, rocks), relation(on, O, O2), oa_rel("usually appears in", O, P), oa_rel("usually appears in", hay, P).	

Circuits in Natural Language Processing



Circuits in Neural Networks



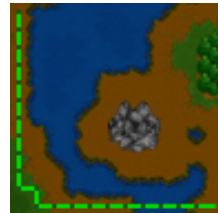
GROUND TRUTH



FIL

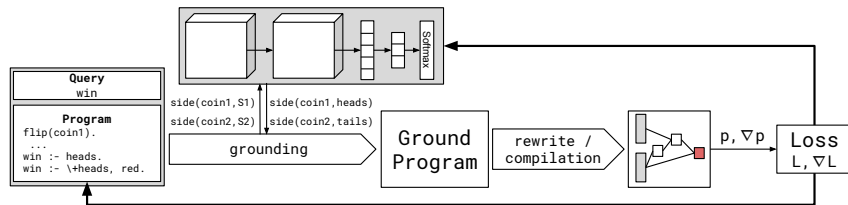


Semantic Loss



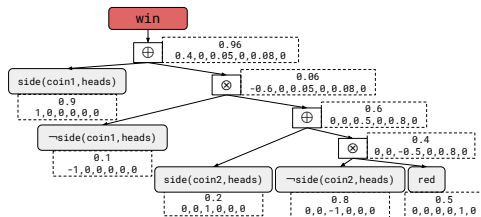
Semantic Probabilistic Layer

Circuits in Deep Probabilistic Logic Programming

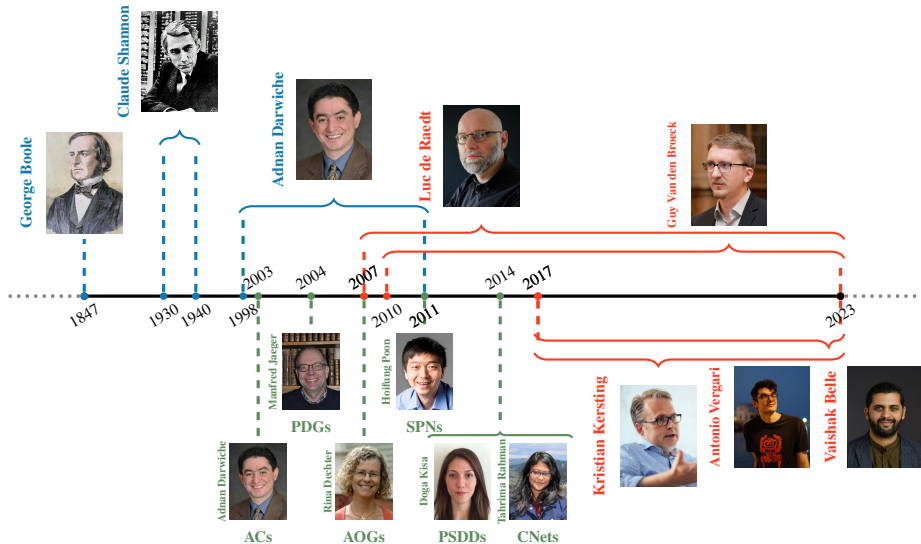


```
flip(coin1). flip(coin2).
nn(side(C,[heads,tails]))::side(C,heads);
                                side(C,tails).

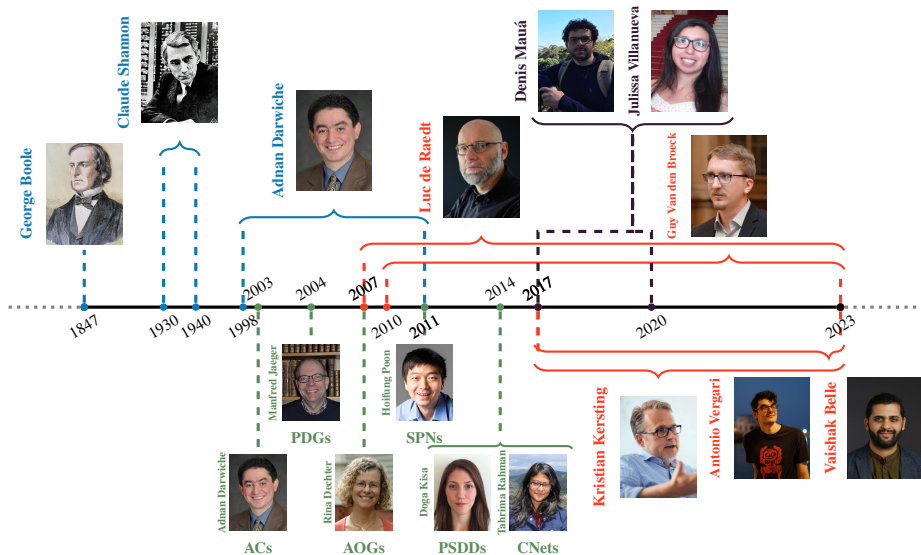
t(0.5)::red; t(0.5)::blue.
heads :- flip(X), side(X,heads).
win :- heads.
win :- \+heads, red.
query(win).
```



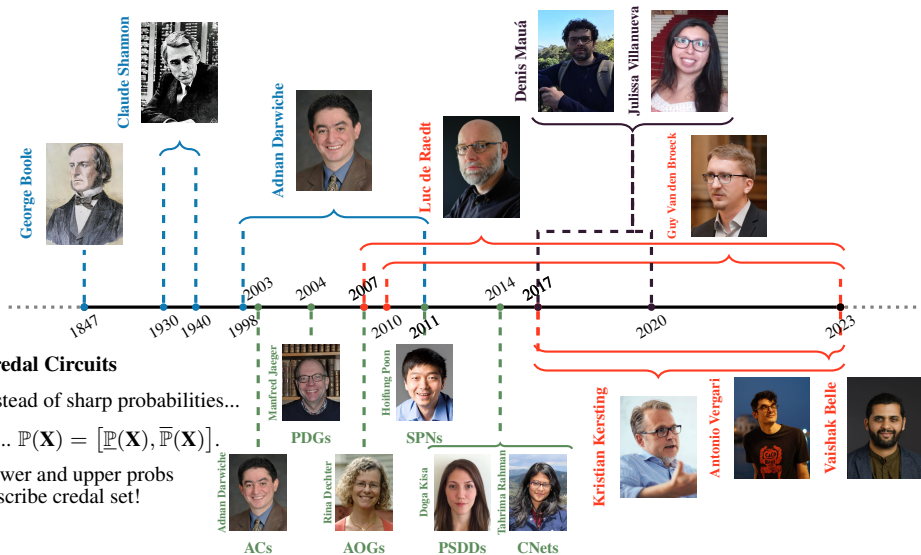
Circuits?



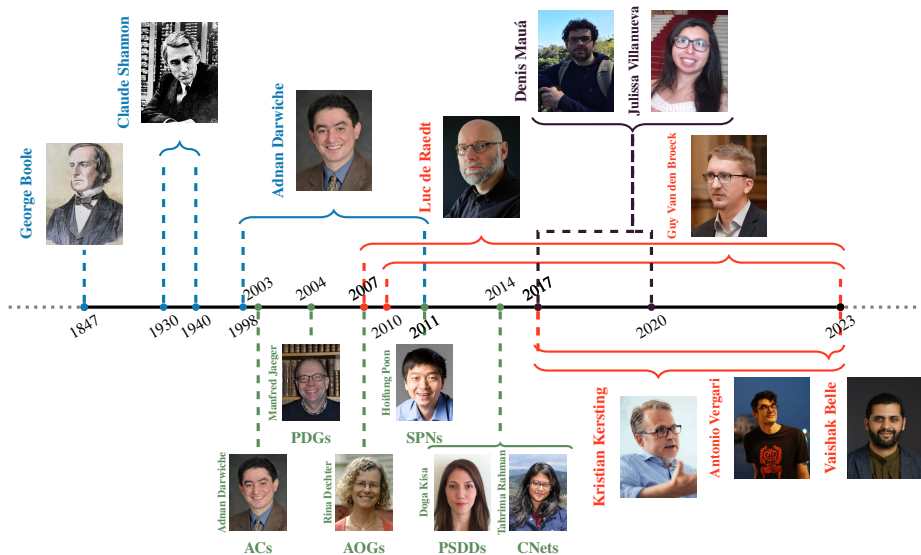
Circuits?



Circuits?



Circuits?



Thinking with **Circuits**

From **Logic** to **Probabilistic** and Back

Renato Lui Geh

