Time Series Forecasting with Neural Networks

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Motivation - 1/2

- Accurate time series forecasting is crucial in both economic and financial applications
- Standard econometric approaches rely almost exclusively on *linear models* (AR, MA, ARIMA...) as they usually perform well on standard tasks.
- Developments in deep learning have made available highly flexible models that can infer any complex nonlinearity from the data (**Universal Approximation Theorem**)

Motivation - 2/2

- Existing literature is lacking a unified perspective on the performance of neural networks on economic and financial forecasting tasks.
- Only a few types of neural networks (typically two) are usually tested against each other. Here, we compare *five different neural network models*.
- The difference between *univariate* and *multivariate* performance is not always assessed
- Economic and financial series are never contrasted with more 'standard' (well-behaved) series

Linear Models

The **linear models** estimated in this project are:

- The **Random Walk (RW)**, a simple *persistence model*:

  $$y_t = y_{t-1} + \epsilon_t$$

  where ϵ_t is a *white noise process* (zero mean and constant variance).

- The **Autoregressive Integrated Moving Average (ARIMA)**, taking the form:

  $$\Delta y_t = \alpha + \phi_1 \Delta y_{t-1} + \ldots + \phi_p \Delta y_{t-p} + \theta_1 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q} + \epsilon_t$$

  where:

  - **p** is the number of *autoregressive components*
  - **d** is the *order of integration* (Δ)
  - **q** is the number of *moving average components*

Feedforward Neural Networks - 1/2

A **feedforward neural network** (FFNN) is a sequence of **dense layers** taking the form:

$$f(X) = \beta_0 + \sum_{k=1}^{K} \beta_k h_k(X)$$

where K is the number of **neurons**, each triggered by an **activation** h_k:

$$h_k(X) = g\left(w_{k0} + \sum_{j=1}^{p} w_{kj} X_j\right)$$

where $g(\cdot)$ is a **nonlinear activation function** (e.g. ReLU, tanh, sigmoid) and p are the predictors.

Feedforward Neural Networks - 2/2

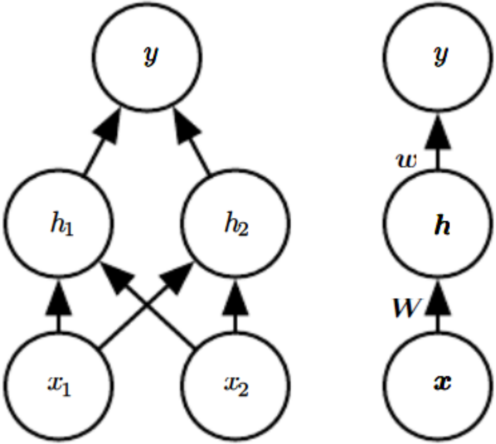


Figure: Single-layer Feedforward Network with two neurons (Extended (left) and schematic (right) representations)

Convolutional Neural Networks - 1/4

Convolutional Neural Networks (CNNs) make use of *two specific layers*:

- **Convolution Layer**
- **Max Pooling Layer**

Convolutional Neural Networks - 2/4

The **convolution layer** uses a **kernel (k)** and *convolves* it over the **input data (X)**, producing a **feature map (s)**:

$$X = [0.4, 0.1, 0.5, 0.3, 0.9] \qquad k = [1, 0, 1]$$

$$\Downarrow$$

$$s = [0.4 + 0 + 0.5, 0.1 + 0 + 0.3, 0.5 + 0 + 0.9] = [0.9, 0.4, 1.4]$$

Convolutional Neural Networks - 3/4

The **max pooling layer** performs *dimension reduction* selecting the highest value within a given **pool size**:

$$s = [0.9, 0.4, 1.4] \qquad \text{pool size} = 2$$

$$\Downarrow$$

$$s = [\max(0.9, 0.4), \max(0.4, 1.4)] = [0.9, 1.4]$$

Convolutional Neural Networks - 4/4

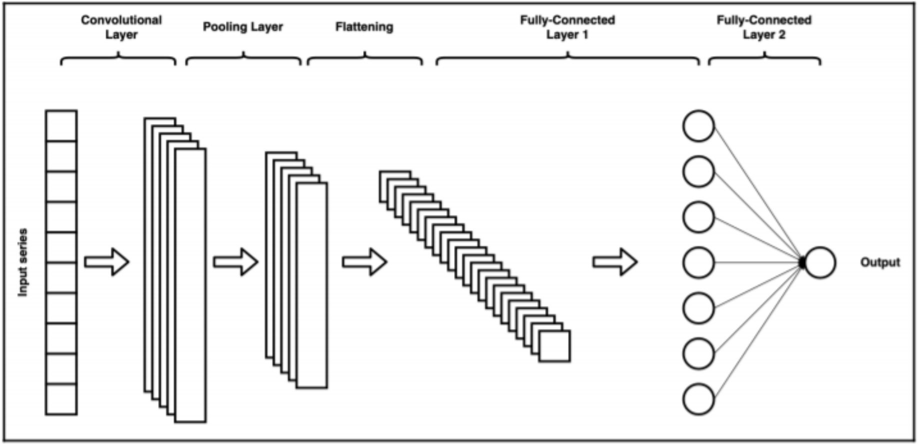


Figure: Convolutional Neural Network for 1-d Time Series Forecasting

Recurrent Neural Networks - 1/5

- Recurrent neural networks (RNNs) are designed to model *sequences of data*.
- They take in a sequence $X = \{X_1, X_2, \ldots, X_T\}$ and pass it through a sequence of **hidden states** $\{h_t\}_1^T = \{h_1, h_2, \ldots, h_T\}$:

$$h_{tk} = g\left(w_{k0} + \sum_{j=1}^{p} u_{kj} X_{tj} + \sum_{s=1}^{K} w_{ks} h_{t-1,s}\right)$$

  for K neurons in each hidden state.
- Note that the value of the present state is **dependent on the value of the previous state**.

Recurrent Neural Networks - 2/5

- The **output** of the RNN at each time step is a weighted sum of the hidden states' neurons

$$O_t = \beta_0 + \sum_{k=1}^{K} \beta_k h_{tk}$$

- The weights in each layer are **not time-dependent**

Recurrent Neural Networks - 3/5

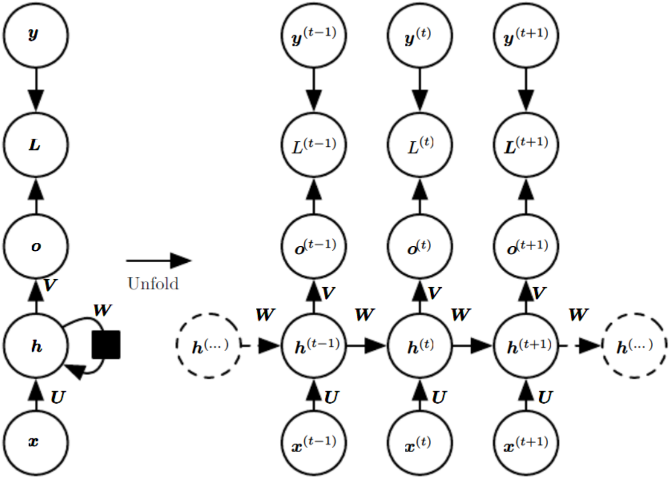
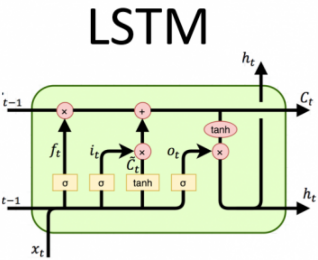


Figure: Single-layer Recurrent Network
(Schematic (left) and extended (right) representations)

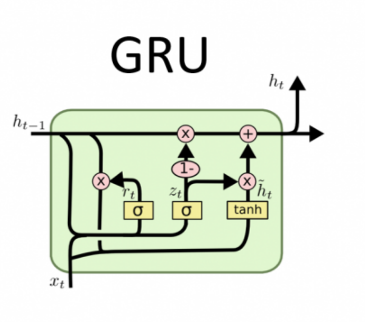
The **Long Short-Term Memory (LSTM)** network is a type of RNN where each cell is made up of:

- **Forget Gate** $\rightarrow$ regulates how much past information is discarded and how much is kept
- **Input Gate** $\rightarrow$ updates the cell state
- **Output Gate** $\rightarrow$ defines the value of the next hidden state



The **Gated Recurrent Unit (GRU)** network is a type of RNN where each cell consists of:

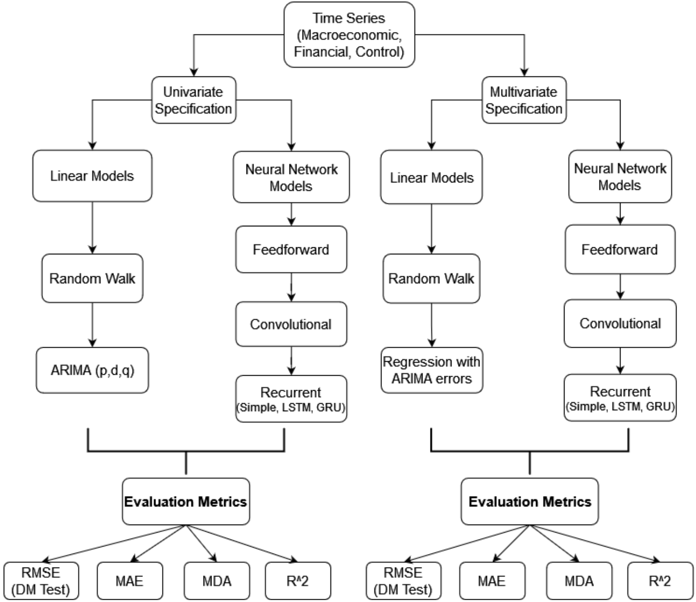
- **Update Gate** $\rightarrow$ decides how much new information to add
- **Reset Gate** $\rightarrow$ decides how much old information to discard



Methodology - 1/4

Three time series were used in this forecasting exercise:

- $\boxed{\textbf{Macroeconomic Time Series}}$: *YoY % change in unemployment* in the United States between July 1955 and December 2019 (*monthly* observations)
  Covariates: *Inflation rate*, *YoY % change in Federal funds rate* and *industrial production*
- $\boxed{\textbf{Financial Time Series}}$: *S&P 500 Trading Volume* between January 4th, 2010 to December 30th, 2020 (*daily* observations)
  Covariates: Daily *FTSE 100 and Nasdaq trading volumes*, *CBOE Volatility Index*
- $\boxed{\textbf{Control Time Series}}$: Daily *visits to a statistical forecasting website* from September 14, 2014 to August 19, 2020
  Covariates: *Unique Visits*, *First Time Visits* and *Returning Visits* to the same website


Time Series (Macroeconomic, Financial, Control)
Univariate Specification
Multivariate Specification
Linear Models
Neural Network Models
Linear Models
Neural Network Models
Random Walk
Feedforward
Random Walk
Feedforward
ARIMA (p,d,q)
Convolutional
Regression with ARIMA errors
Convolutional
Recurrent (Simple, LSTM, GRU)
Recurrent (Simple, LSTM, GRU)
Evaluation Metrics
Evaluation Metrics
RMSE (DM Test)
MAE
MDA
R^2
RMSE (DM Test)
MAE
MDA
R^2

Methodology - 3/4

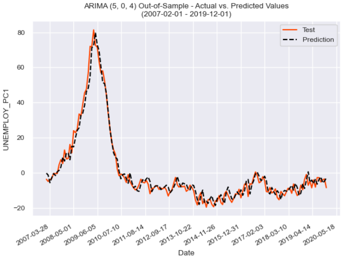
- Each series was split into *training* and *test* sets.
- Fitting of the ARIMA models was carried out using the **auto_arima** function from Python's *pmdarima*
- Neural Networks' **hyperparameters** were selected using a **grid search algorithm** and the models were optimized using **TensorFlow**

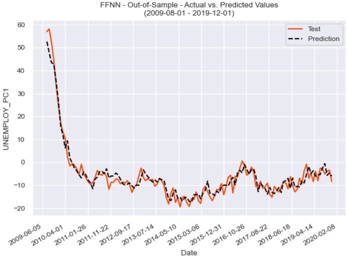
Methodology - 4/4

- Each model produces **one-step ahead forecasts** on both the training (in-sample) and test (out-of-sample) sets
- **Walk-forward validation** was used with linear models, while neural networks performed **sliding-window regression**
- To avoid overfitting, the number of hidden layers is **built conditionally on training set size** (larger dataset = deeper network)

Univariate

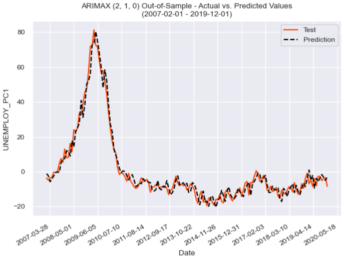
- The ARIMA model has statistically significant lower RMSE **in-sample**
- There is no statistically significant difference between ARIMA and neural networks RMSE **out-of-sample**
- CNN yields a **31% increase in MDA with respect to the ARIMA model** and an **average 34% increase in MDA with respect to other neural networks** (MDA = 0.59)

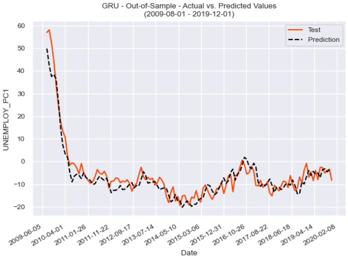




Multivariate

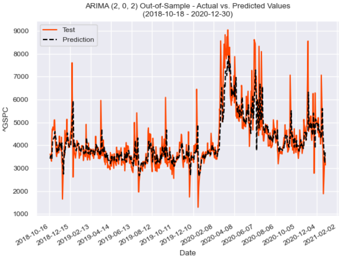
- Neural network models have statistically significant lower RMSE **in-sample**
- Again, there is no statistically significant difference between ARIMA and neural networks RMSE **out-of-sample**
- CNN outperforms all other models on MDA

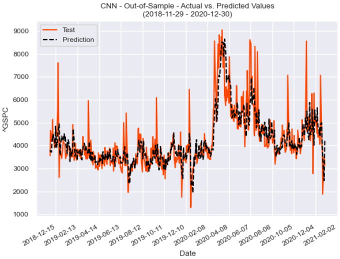




Univariate

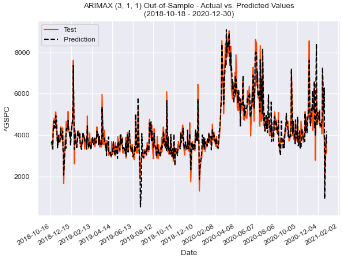
- ARIMA has a significantly lower in-sample RMSE
- Neural networks perform **at least as good as** ARIMA out-of-sample
- GRU yields a **low but statistically significant reduction in RMSE with respect to ARIMA**
- CNN yields a **17% increase in MDA with respect to the ARIMA model**

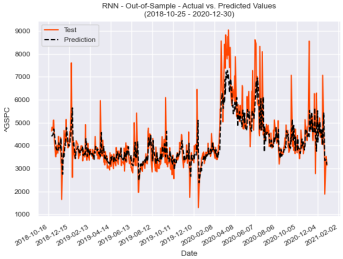




Multivariate

- Neural network models perform **significantly worse** than ARIMAX on all available metrics

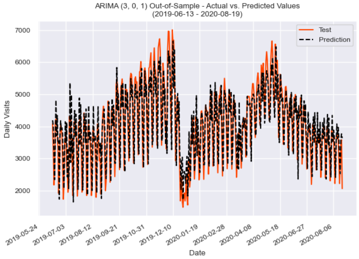


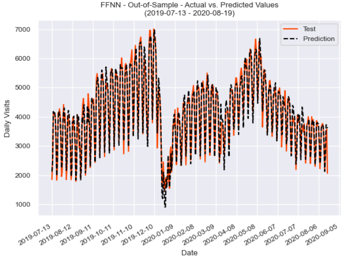


Results - Control Time Series - 1/2

Univariate

- **Neural networks outperform ARIMA** in every metric
- **Lowest** reduction in RMSE is **53**%, **highest** is **55**% (FFNN)
- **Lowest** increase in MDA is **30**% (CNN), **highest** is **34**% (LSTM)

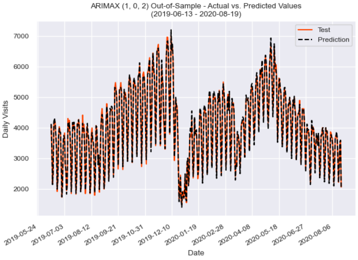


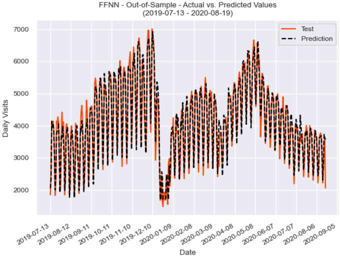


Results - Control Time Series - 2/2

Multivariate

- ARIMAX performs significantly better than neural networks on **all in-sample and out-of-sample metrics** ($R^2 = .99$)
- Neural network out-of-sample performance is *slightly worse* than in the univariate specification





Future Work

Possible improvements on the present approach include:

- Check performance on **different forecasting horizons**
- Operate some **transformation of the input data**
- Widen the grid-search or change **hyperparameter tuning algorithm**
- Check performance of **hybrid models** (e.g. ARIMA-ANN)

Conclusions

- When properly trained, neural networks are able to reach the accuracy of linear models (and sometimes even outperform them) in the **univariate setting**.
- Evidence in the **multivariate setting** is more in favour of linear models.
- CNNs achieve **high accuracy on the MDA measure** in both macroeconomic and financial series.
- **RNNs are not as accurate as expected** in time series forecasting tasks.