ADSEE II Summary of the summary

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Contents

1 Aircraft aerodynamic analysis - fundamentals

1.1 Basic Definitions

In designing a wing, two views are of importance:

- The planform design: i.e. stuff like span, AR, surface area etc. Everything you see when looking top-down.
- Airfoil design: the wing's cross-sectional shape.

DEFINITION

The **lift** is the aerodynamic force normal to the freestream.
The **drag** is the aerodynamic force parallel to the freestream.
The **pitching moment** is the aerodynamic moment around a reference point, usually the aerodynamic center.
The **aerodynamic center** is the point on the airfoil at which the value of the pitching moment does not change as the angle-of-attack.
Aerodynamic coefficients:

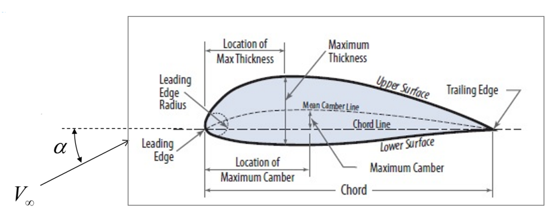
$$C_L = \frac{L}{qS_{ref}} \quad C_D = \frac{D}{qS_{ref}} \quad C_M = \frac{M}{qS_{ref}\bar{c}} \tag{1.1}$$

Remember that the aerodynamic coefficients are largely dependend of the angle of attack, Reynolds number (viscosity effect) and Mach number (compressibility effect).

1.2 Airfoil Aerodynamics

Look at figure 1.1. Most things are pretty obvious, except perhaps for the camber. This is the line exactly in the middle between the upper and lower surface. The location of maximum camber is the location where the mean camber line is farthest away from the chordline.

Figure 1.1: Airfoil geometry definitions



An airflow causes a pressure distribution as depcited in the left figure of figure 1.2.
Now, if you don't remember what c_p was:

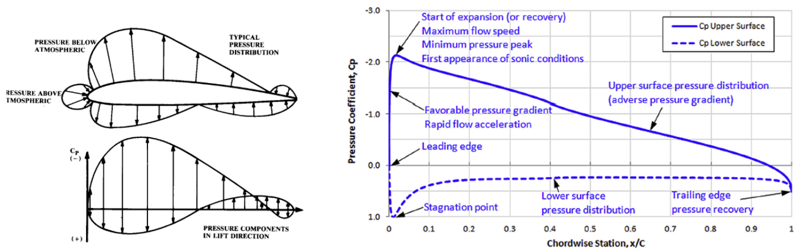
FORMULAS

$$c_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} \tag{1.2}$$

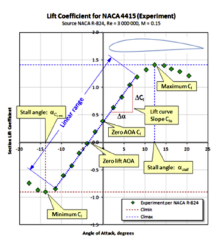
With p the local pressure, p_∞ the freestream pressure, ρ_∞ the freestream density and V_∞ the freestream velocity.

Please note that we use C_l (and similarly C_d) to denote the lift coefficient of the *airfoil*. For a 3D, finite-length wing, we will be using C_L, where we take into account the effect of finite size.

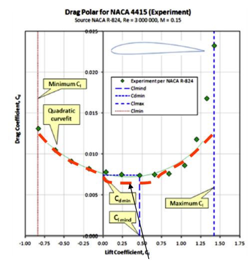
Figure 1.2: Airfoil pressure field



1.2.1 *Airfoil lift, drag and pitching moment*



(a) Lift curve slope.



(b) Drag polar.

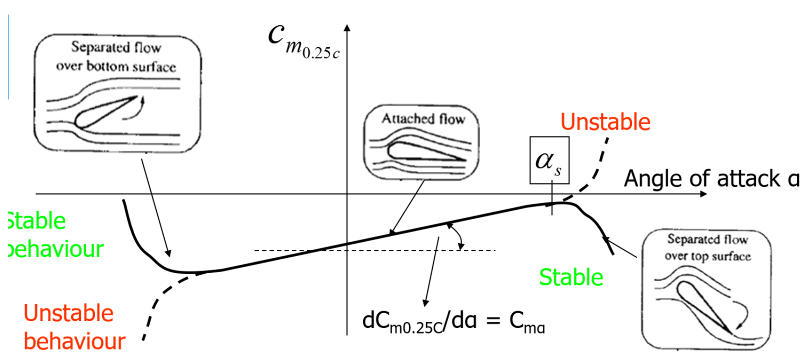
Figure 1.3: Lift curve slope and drag polar.

In figure 1.3a, we clearly see that C_l increases linearly with the angle of attack (AoA) in the linear range. Close to the **stall angle** α_s, it's no longer linear. For thin airfoils, the lift curve slope is 2π per radian. The lift curve *slope* is affected by the Mach number, while the stall angle of attack and the maximum C_l are affected by Re.

Figure 1.3b should look familiar as well.

- At low C_l values, friction drag is predominant. The graph is quadratic here.
- At high C_l values, pressure drag (due to flow separation) is the main factor. A quadratic drag polar is not vlaid anymore.
- Some airfoils have a drag bucket: at low $C_l s$, the drag does not change with lift.

Figure 1.4: Airfoil pitching moment.



Remember Jacco's lectures from intro I: the pitching moment is defined to be positive if it results in a pitch up (nose up). Airfoils (not entire aircraft) normally have a natural negative pitching moment, causing the nose to go down, with increasing pitching moment as the AoA increases. However, to make the airfoil stable, C_{m_α} needs to become negative when you go beyond α_S: you want to create a more nose-down attitude, so C_m needs to become more negative. Similarly, at negative angles of attack, C_{m_α} needs to become negative as well (so that if the AoA decreases further, it becomes less negative), so that it creates a more nose-up attitude.

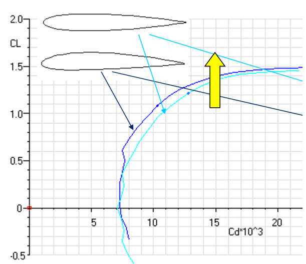
Please note that generally, the C_m is fairly constant, as it is given around the quarter-chord location. This is very close to the aerodynamic center usually, and the aerodynamic center is defined as the place where the pitching moment does not change with the AoA, and hence C_m does not change that much, either.

1.2.2 *Effects of airfoil design parameters*

Theorem: effects of camber

- Increasing camber moves the lift slope curve upwards, meaning that at 0 degrees AoA, lift is still generated.
- Increasing camber does not affect C_{l_α}
- Increasing camber causes less drag for the same amount of lift (see figure 1.5).
- Increasing camber increases pitching moment of the airfoil, meaning a larger tail is required.
- Moving the camber forward increases $C_{l_{max}}$, but causes a sudden stall drop.
- Moving the camber aft decreases $C_{l_{max}}$, but causes a more gradual stall transition.

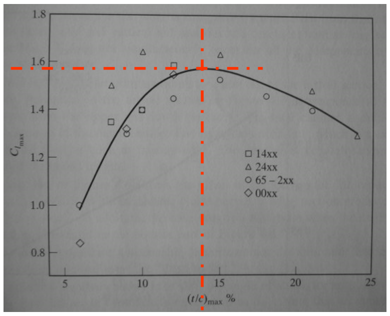
Figure 1.5: Effect of airfoil camber on drag polar.



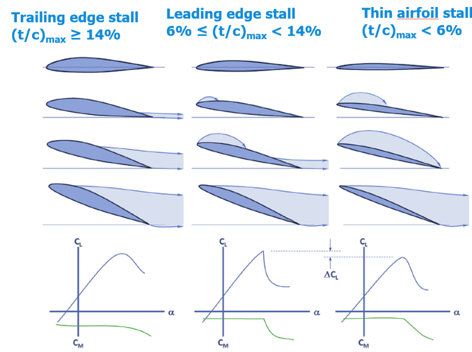
Theorem: effects of thickness ratio

- $C_{l_{max}}$ depends heavily on $(t/c)_{\text{max}}$, as depicted in figure 1.6a.
- Moving the $(t/c)_{max}$ aft moves the maximum pressure location aft as well. This causes:
  - Lower minimum drag.
  - Higher Mach critical (the maximum pressure peak becomes lower as well).
  - Lower $C_{l_{max}}$.
  - Higher drag increase.
- The stall characteristics depend heavily on the the maximum $(t/c)_{\text{max}}$. We see by inspecting figure 1.6b:
  - If the airfoil is very thick (>14%), flow separation starts from the trailing edge, which is something you'd expect. This means that the effect of separation is more gradual, and the stall is thus gradual.
  - If the airfoil is moderately thick (between 6% and 14%), there will actually be flow separation at the leading edge. This flow separation bubble remains constant in size for a while, until at one point it more or less 'burst', causing a very sudden stall.
  - For a thin airfoil (<6%), the flow seperation also occurs at the leading edge, but it grows gradually, causing less sudden C_L. Nevertheless, $C_{l_{max}}$ is smaller, as per figure **??**.
- A thicker airfoil of course has incresed friction drag, because the flow will stay in contact with airfoil over a longer distance.
- Not mentioned in the slides, but a thicker airfoil will actually be structurally lighter. The reason for this is rather simple: remember from your wing box that $\sigma = \frac{M \cdot y}{I}$. Now, y is obviously $\frac{t}{2}$, but if you'd assume a rectangular wing, you see that the moment of inertia is related to the cube of the thickness: thus, increasing the thickness of the wing means the stress on the skin will be smaller, meaning the wing skin itself can be *less* thick. This obviously decreases the weight. A

thicker airfoil can also make sure there's room for a spar to decrease the stresses on the skin.
- Thicker airfoil allows for more fuel capacity.



(a) Effect of $(t/c)_{\max}$ on $C_{l_{max}}$.



(b) Effect of location of $(t/c)_{\max}$ on stall characteristics.

Figure 1.6: Effects of $(t/c)_{max}$.

Theorem: Effects of Reynolds number (viscosity)

Remember that $\text{Re} = \frac{\rho V x}{\mu}$, μ being the viscosity.
- Larger Re increases the $C_{l_{max}}$[a].
- Larger Re decreases friction and form drag.

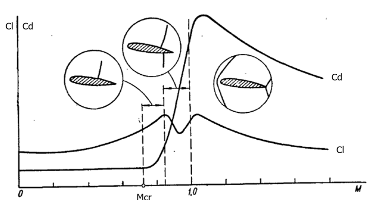
[a]Lift curve slope is unchanged.

Theorem: Effects of Mach number (compressibility)

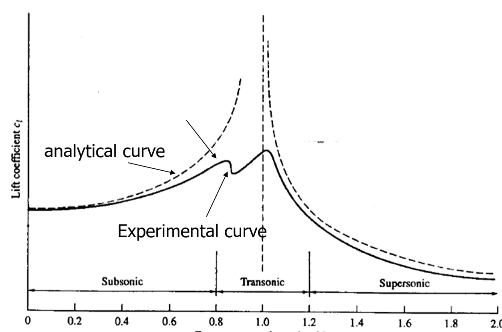
Remember that $M = \frac{V_\infty}{a_\infty}$.
- The transition from subsonic to supsonic speeds drastically increases (wave) drag due to shock waves. The Mach number at which this starts to happen is the M_{cr}, the critical Mach number (see below).
- For subsonic speeds, the $C_{l_{max}}$ increases with Mach number, per the **Prandtl-Glauert compressibility corrections**:

$$C_{l_M} = \frac{C_{l_{M=0}}}{\sqrt{1 - M_\infty^2}} \tag{1.3}$$

A more precise variation of C_l and C_d with M can be seen in figures 1.7a and 1.7b.



(a) Variation of C_l and C_d with M.



(b) Variation of C_l with M.

Figure 1.7: Variations of C_l and C_d with M.

DEFINITION

> The airfoil **critical Mach number** is the freestream Mach number at which the flow velocity on the airfoil reaches the speed of sound

If the freestream velocity is further increased, shock waves are generated. Recompression after the shock wave induces boundary layer thickening and then major flow separation. This can lead to the occurence of buffeting and then to high speed stall.

How can we find out our M_{cr}?

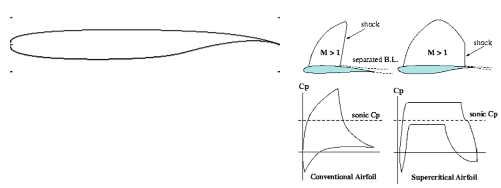
- Look at statistics, historical trends and reference aircraft.
- Use experimental data for different airfoil shapes.
- Calculate it, which, if you remember, you actually partly did during intro I.

To deal with M_{dd} (Mach divergence), two things can be done:

- The sweep angle can be increased, so that the velocity perpendicular to the wing (which is the velocity that is important) decreases which postpones the M_{dd}.
- Supercritical airfoils can be used.

A supercritical airfoil is depicted in figure 1.8. Compared with a conventional airfoil, a supercritical airfoil has a smaller camber, a larger leading edge radius, a small surface curvature on the suction side (upper surface) and a concavity in the rear part of the pressure side (lower surface). Why is this necessary? The main problem from shockwaves isn't the shockwaves, the problem occurs when shockwaves leave us, as the return to subsonic speeds cause flow separation. Now, remember that C_p was directly related to the local flow velocity, since it was an indication of the dynamic pressure. By 'flattening' the upper surface, you also flatten out the graph of C_p on the upper surface, meaning that if you go above the sonic C_p, you also *stay* above C_p for a very long time. So, you no longer have the nuisance of one point of the airfoil fucking up your entire airfoil because the flow velocity is too high there.

Figure 1.8: Supercritical airfoils.



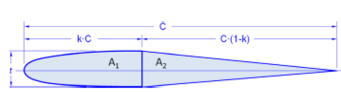
In addition to above, supercritical airfoils need the concavity in the rear as to counteract the increased pitching moment of the airfoil. Furthermore, supercritical airfoils are generally thicker (their location of $(t/c)_{max}$ is also further behind), which means they can be structurally lighter.

1.2.3 Approximation of an airfoil cross-sectional area

Looking at figure 1.9, we can simplify the geometry so that we can calculate the area of the airfoil:

$$\begin{aligned} A_{total} &= A_1 + A_2 = \frac{2kCt}{3} + \frac{C\,(1-k)\,t}{2} = \frac{(k+3)\,Ct}{6} \\ A_{airfoil} &= \frac{k+3}{6}\left(\frac{t}{c}\right)C^2 \end{aligned}$$

Figure 1.9: Approximation of an airfoil cross-sectional area.



1.3 Wing aerodynamics

Now, we can go back to ADSEE I to find a shitload of equation form wing planform designing:

FORMULAS

Wing area: $$S = b\left(\frac{C_r + C_t}{2}\right) \tag{1.4}$$

Aspect ratio - general: $$AR = \frac{b^2}{S} \tag{1.5}$$

Aspect ratio - constant-chord: $$AR = \frac{b}{C_{avg}} \tag{1.6}$$

Taper ratio: $$\lambda = \frac{C_t}{C_r} \tag{1.7}$$

Average chord: $$C_{avg} = \frac{C_r + C_t}{2} = \frac{C_r}{2}(1+\lambda) \tag{1.8}$$

Mean geometric chord: $$C_{MGC} = \left(\frac{2}{3}\right) C_r \left(\frac{1+\lambda+\lambda^2}{1+\lambda}\right) \tag{1.9}$$

Mean aerodynamic chord: $$C_{MAC} = C_{MGC} \tag{1.10}$$

Y-location of MGC_{LE}: $$y_{MGC} = \left(\frac{b}{6}\right)\left(\frac{1+2\lambda}{\lambda}\right) \tag{1.11}$$

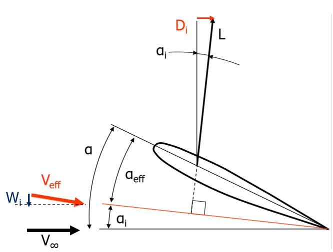
X-location of MGC_{LE}: $$x_{mgc} = y_{mgc} \tan\Lambda_{LE} \tag{1.12}$$

Angle of quarter-chord line: $$\tan\Lambda_{c/4} = \tan\Lambda_{LE} + \frac{C_r}{2b}(\lambda - 1) \tag{1.13}$$

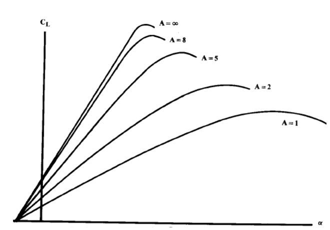
Angle of an arbitrary chord line: $$\tan\Lambda_n = \tan\Lambda_m - \frac{4}{AR}\left[(n-m)\frac{1-\lambda}{1+\lambda}\right] \tag{1.14}$$

Please do remember that C_r is the root chord of the theoretical wing, i.e., the wing that you see when you'd delete the fuselage. The wing that you see in reality (where the fuselage is in the middle) is called the exposed wing. Also, b may not exceed 80 m or otherwise it wouldn't fit at your airport.

You may remember from intro I that vortices exist: since there is lower pressure at the upper surface of the wing, air tries to go from the lower side to the upper side around the tips. This causes a **downwash** velocity component w_i, which affects the aerodynamic flow around the wing.



(a) Downwash.



(b) Effect of aspect ratio.

Figure 1.10: Downwasha and aspect ratio.

We see what happens in figure 1.10a. The downwash velocity causes a slightly downward effective velocity, decreasing the geometric angle of attack α by an induced angle of attack α_i to end up at α_{eff}. This decreases the amount of lift generated. Furthermore, the lift is now tilted slightly backwards, meaning the drag is increased by an amount of induced drag D_i as well.

Theorem: effects of aspect ratio on lift and drag.

- A higher aspect ratio decreases the induced drag.
- A lower aspect ratio decreases the wing lift curve slope.
- A lower aspect ratio increases the stall angle.

Note that a small AR is beneficial for the tailplane: this causes the tailplane to stall only after the main wing stalls, increasing your chance of saving yourself out of stalls. For canards, the opposite is the case, where the AR is larger to ensure it stalls before the main wing.

Theorem: pros and cons of *low* aspect ratio.

The following things become more pronounced when the AR decreases. Obviously, when the AR is very high, the opposite occurs. Pros are:

- High stall AoA.
- High flutter speed[a].
- Low roll damping.
- Low structural weight.
- Great gust penetration capability (because of low C_{L_α}[b].
- Low adverse yaw.

Cons are:

- Inefficient because of high induced drag.
- Shallow C_{L_α} requires large changes in AoA with airspeed.
- Low $C_{L_{max}}$ (high stall speed).
- Low L/D_{max} (bad glide characteristics).

[a]Fluttering is when the wings start wildly bouncing about.

[b]Gusts cause sudden decrease/increase in angle of attack. If C_{L_α} is large, this would mean that it affects performance heavily, which is something you don't want.

The wing taper ratio doesn't do too much, other than controlling the lift distribution. At $\lambda = 0.4$, the lift distribution is almost elliptical.

Theorem: effects of wing sweep angle.

- Higher sweep angle increases the wing critical Mach number[a]:

$$M_{cr_{swept}} = \frac{M_{cr_{unswept}}}{\cos \Lambda} \tag{1.15}$$

- Higher sweep angle decreases the maximum lift coefficient.
- Higher sweep angle causes higher structural weight: a higher sweep increases the pitching moment (because the center of lift is being moved aft), which increases tthe torsional stress and thus requires a heavier structure.

[a]Please note that this the slide says it reduces rather than increases. This is absolutely incorrect: local supersonic speeds will occur at a *higher* Mach number, so M_{cr} is increased. There's also like a graph next to it which shows that the critical Mach number is increased.

Theorem: effects of wing twist.

- Higher wingtwist prevents wingtip region from stalling before wing root. By making the airfoil tip point down relative to the airfoil root, the wing tip will stall later. This is beneficial, because the tip is almost always more aft than the root, so it causes a nose down attitude.
- A higher wingtwist modifies the spanwise lift distribution to help achieve minimum drag at mission condition.

Geometric twist is the angle measured between the airfoil's chords (typically between the root and tip airfoils).
Aerodynamic twist is the angle between the **zero-lift angle** of an airfoil and another.
Generally, twist angles values less than 5 degrees are used. -3 degrees is a typical twist angle value to avoid wing tip stall.

DEFINITION

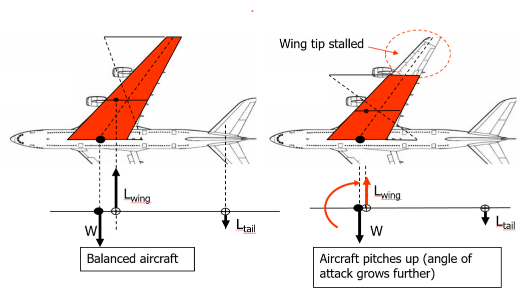
The **wing incidence angle** is the pitch angle of the wing w.r.t. the fuselage, typically measured between the wing root chord[a] and a fuselage longitudinal reference plane, e.g. the fuselage floor.

[a]In case of twisted wings, the incidence angle should be measured w.r.t. the zero lift angle of the wing, rather than the root chord.

Angles of wing incidence in the order of 1-2 degrees are typical. It should be chosen such that when it flies at the incidence angle required by $C_{L_{design}}$ (α_{trim}, the fuselage is not an angle. Otherwise, the flight attendants would have a terrible time pushing the food carts around.

Now, wing stall is never a nice things. However, to survive it, the stall pattern on the wing should always begin at the root and progress toward the tip as AoA increases. The reason for this is obvious in figure 1.11.

Figure 1.11: Wing stall.



Now, tapered wings naturally have a problem with this: since their wing tip is shorter than their wing root, the Reynolds number near the tip is smaller, decreasing $C_{L_{max}}$ and thus stall wil start at the tip. Furthermore, it may not necessarily be that the right wing tip stalls at the same moment as the left wing tip: this causes a roll. This means that it's a bad idea to place ailerons near the tips, as then you wouldn't be able to correct the roll.

THEOREM: COMBATTING TIP STALL.

- Applying wing twist.
- Using different airfoils near the tip and root, where the root airfoil stalls earlier.
- Apply the following formula:

$$AR \leq 17.7\,(2-\lambda)\,e^{-0.043\Lambda_{c/4}} \tag{1.16}$$

1.4 *Airfoil/planform design guidelines*

THEOREM: AIRFOIL SELECTION.

1. Determine design lift coefficient of airfoil:
   - The design lift is the aircraft lift at the flight condition at which the aircraft is supposed most of the time (usually crusie). Then $L_{tot} = W_{\text{fuel intensive mission leg}}$, and for conventional aircraft we can assume $L_{wing} \approx L_{tot}$ where the 10% is to compensate for the negative lift contribution generated by the tail to trim the aircraft.
   - Then the requird wing lift coefficeint can be calculated as follows:

$$\begin{aligned} L &= 1.1W = qSC_L \\ C_L &= 1.1\frac{1}{q}\left(\frac{W}{S}\right) \end{aligned}$$

   But weight is not constant, hence

$$C_{L_{des}} = 1.1\frac{1}{q}\left\{\frac{1}{2}\left[\left(\frac{W}{S}\right)_{\text{start of cruise}} + \left(\frac{W}{S}\right)_{\text{end of cruise}}\right]\right\}$$

- To take into account sweep:

$$C_{l_{des}} = \frac{C_{L_{Des}}}{\cos^2 \Lambda}$$

The cosine squared because the velocity is squared in the dynamic pressure.

Table 1.1: Effects of design parameters on several variables. N.E. is no effect, a $\downarrow$ indicates moderate decrease, $\Downarrow$ significant decrease, $\uparrow$ moderate increase, $\Uparrow$ significant decrease.

Increase in	C_{D_0} (subs.)	C_{D_0} (supers.)	$K = \frac{1}{\pi A e}$	C_{L_α}	$C_{L_{max}}$	Wing Wt	Wing Vol
Aspect ratio	N.E.	$\Uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\Uparrow$	$\Uparrow$
Wing sweep	N.E.	$\downarrow$	$\Uparrow$	$\Downarrow$	$\Downarrow$ (aft) / N.E. (forw)	$\Uparrow$	N.E.
Taper ratio	N.E.	N.E.	$\downarrow\Uparrow$	$\uparrow\Downarrow$	N.E.	$\Uparrow$	$\Downarrow$
Airfoil thickness ratio	N.E.	$\Uparrow$	N.E.	N.E.	$\uparrow$	$\downarrow$	$\uparrow$
Leading edge radius	N.E.	$\Uparrow$	$\downarrow$	N.E.	$\uparrow$	N.E.	$\uparrow$
Camber	$\Uparrow$	$\Uparrow$	$\downarrow$	N.E.	$\uparrow$	N.E.	N.E.

2 Aircraft aerodynamic analysis - lift

2.1 C_{L_α}

We saw before that downwash reduces the effective wing AoA. The lift-curve rotates clockwise, around the zero lift angle α_{0L}[1]. Now, the question is, how much does C_{L_α} differ from C_{l_α}? It depends on three things: compressibility (Mach number), aspect ratio and sweep angle:

FORMULAS

$$\frac{dC_L}{d\alpha} = C_{L_\alpha} = \frac{2\pi A}{2 + \sqrt{4 + \left(\frac{A\beta}{\eta}\right)^2 \left(1 + \frac{\tan^2 \Lambda_{0.5c}}{\beta^2}\right)}} \tag{2.1}$$

With:

$\beta = \sqrt{1 - M_\infty^2}$	Prandtl-Glauert compressibility correction factor
$\Lambda_{0.5C}$	Sweep angle measured at half chord length
$\eta \approx 0.95$	Airfoil efficiency factor

Note that the units is rad^{-1}. Furthermore, when using winglets, you can increase A by 20%. Finally, this equation is based on the DATCOM method, which is basically a method that describes a shitload of aerodynamic stability equations based on both emperical and theoretical stuff (the variables used in the equations are mainly based on theory, but the exact relationships are decided by experimental data, mostly). This equation is very accurate until very high Mach numbers (above M_{dd}).

Note that we can apply two formulas to find C_L:

FORMULAS

$$C_L = C_{L_{\alpha=0}} + C_{L_\alpha} \cdot \alpha \tag{2.2}$$
$$C_L = C_{L_\alpha} \cdot (\alpha - \alpha_{L=0}) \tag{2.3}$$

Where $C_{L_{\alpha=0}}$ is the lift at zero AoA, C_{L_α} the lift-curve slope, α the (geometric) angle of attack and $\alpha_{L=0}$ the AoA at which the lift equals 0.

α_{trim} is the AoA at which the wing has to fly in order to deliver the design lift coefficient $C_{L_{des}}$. With $C_{L_{des}} = C_{L_\alpha}(\alpha_{trim} - \alpha_{L=0})$, we get that

FORMULAS

$$\alpha_{trim} = \frac{C_{L_{des}}}{C_{L_\alpha}} + \alpha_{L=0} \tag{2.4}$$

So, to decrease the α_{trim}, we can choose a higher AR (to increase C_{L_α} or use cambered airfoils (negative values of $\alpha_{L=0}$.

2.2 High lift devices

You can't take off and land at $C_{L_{max,clean}}$, hence you need HLDs. If you designed your airplane that it was able to take off and land at $C_{L_{max,clean}}$, it means you've overdesigned your wing.

HLDs are nice because they fulfill both high and low speed requirements:

- Cruise requirements (speed and low drag):

[1] Why is the zero lift angle still the same? At zero lift, there is no 'net' pressure difference between the upper and lower surface, hence no vortices are generated then, so the zero lift angle isn't altered in any way.

  - Small wing surface (hence low drag)
  - Small camber airfoils (less loaded tail and less drag)
  - Lift generated with low lift coefficient value (to reduce induced drag)
- Take off and landing requirements ($C_{L_{max}}$):
  - High lift coefficient and wing area required because of the low speed
  - The lower the stall speed the better
  - Small drag at take off, but high drag at landing
  - Pilot visibility issues during approach (you need lift at low angle of attack)

HLDs are complex however, and above all, weight.

We distinguish two main types of HLDs:
- Passive lift enhancing devices:
  - Trailing edge devices to increase camber (and wing surface);
  - Leading edge devices to prevent flow separation and stall;
- Active lift enhancing devices (for STOL or VTOL requirements):
- Use air streams from jet propeller to energize the boundary layer and prevent separation.

Examples of active lift enhancement is vectored thrust[2], upper surface blowing[3] and externally blown flaps[4].

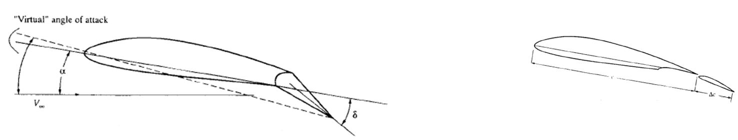
2.2.1 *Trailing edge HLDs*

The working principle of TE HLDs is very simple when looking it figures 2.1a and 2.1b:

Theorem: working principle of TE HLDs.

- The total wing camber is increased;
- The virtual angle of attack is slighty increased;
- The surface of the wing is increased (in case of Fowler momevement);
- Boundary layer control[a] (in case of properly designed slots).

[a]Airflow through slots provides a new boundary layer (it does not energize the already separated flow!).



(a) Plain flap deflected through angle δ.

(b) Extended trailing-edge flap.

Figure 2.1: Working principle of flaps.

The effect of TE HLDs is also fairly straightforward when looking at figure 2.2:

Theorem: effects of TE HLDs.

- Non-extending plain flaps move the lift-curve upward, increasing $C_{L_{max}}$.
- Non-extending slotted flaps do even more, by extending the lift curve to increase $C_{L_{max}}$ even further (due to delay of flow separation).
- Extending flaps both shift the lift-curve upwards, and increase the slope. This is due to the added surface area: C_L is still defined for the original surface area (so without the extention), but naturally, the extra surface area also produces lift. Applying $C_L = \frac{L}{q \cdot S}$ thus yields a higher value than it actually should when you'd use the increased surface area.

[2]Pointing your thrust vector upwards mean you create more lift, namely also the vertical component of the thrust.

[3]Mounting your engines above the wing so that their exhaust blows over the wing, reducing the pressure above the wing and increasing lift ($C_{L_{max}}$ can reach values beyond 4).

[4]Mounting your flaps behind the engine exhaust so they are further blown by the exhaust flow. Similar working to a blown diffuser for a F1-car, but with the exact opposite goal of course. $C_{L_{max}}$ values beyond 3.5-4 can be achieved.

Do note that HLDs actually typically *decrease* the clean wing stall angle.

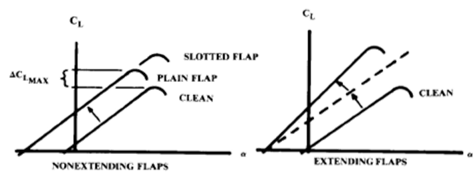


Figure 2.2: Effect of flaps on lift curve.

2.2.2 *LE HLDs*

From figure 2.3, we can easily deduce the working principle of LE HLDs.

Theorem: working principle of LE HLDs.

- Moving the nose of the wing towards the flow (hence bringing up the stagnation point)
- Lowering the angle of attack
- Energizing the boundary layer on the upper wing surface.

In general, TE devices are used to increase $C_{L_{max}}$, whereas LE devices mainly provide a margin against stall.

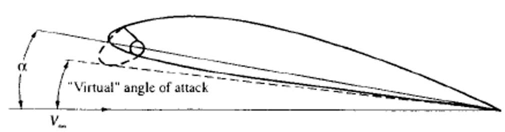


Figure 2.3: Deflected leading edge flap.

From figure 2.4, we can also deduce the effects of LE HLDs.

Theorem: effects of LE HLDs.

- Extend linear part of curve towards higher vlaue of $C_{L_{max}}$ and AoA.
- SLightly shift the whole curve towards the right (their main effect is to decrease the effective AoA).
- In case of extensible elements (slats): increase the lift curve slope.
- *Not* really help improving lift at takeoff and landing, because they are effective only at high angle of attack. However, they work well with TE HLDs, delaying separation.

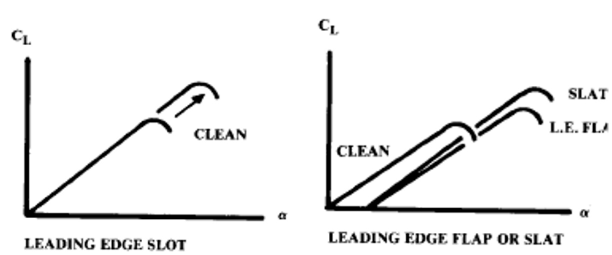


Figure 2.4: Effects of LE HLDs.

Please note that leading edges are not flapped in proximity of the fuselage. This is to make sure the root sections stall before the wing tips. Furthermore, note that if you look a typical commercial aircraft from above, you see that near the root, the trailing edge has a kink, a **yehudi**. These serve to accomodate landing gears and to increase flaps effectiveness, by lowering the wing sweep locally.

2.3 *Conceptual sizing of HLDs and generation of flapped wing lift-curve*

DEFINITION

The **reference wing flapped surface** (S_{wf}) is the spanwise portion of the reference wing area (S) affected by the presence of a certain type of HLD.

See also figure 2.5.

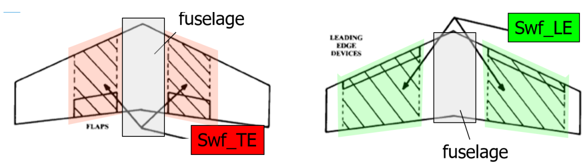


Figure 2.5: S_{wf}.

Please note the following:

- There is a different reference wing flapped surface defined for each type of HLD that is installed on the wing.
- The flapped area S_{wf} is *not* the actual area of flaps. The actual flap area must of course be calculated by using the chord of the flap itself, c_f, which is of course smaller than the wing chord c.
- It is *not* possible to have $S_{wf} \geq S$. Part of S is reserved:
  - The fuselage takes up some of S, and obviously, we can't install HLDs there.
  - The LE near the fuselage is usually left free, as said before.
  - Space is usually reserved near the engines (engine gates).
  - $S_{wf_{LE}}$ is generally larger than $S_{wf_{TE}}$.
- If, e.g., part of the TE edge is covered by HLDs of type X and part by HLDs of type Y, then two reference wing flapped areas must be defined: S_{wf_x} and S_{wf_y}.

THEOREM: SIZING OF HLDS.

1. Evaluate the target $\Delta C_{L_{max}}$.
2. Estimate available wing area to place HLDs.
3. Use two simple equations:

$$\Delta C_{L_{max}} = 0.9\Delta C_{l_{max}} \frac{S_{wf}}{S} \cos \Lambda_{hingeline}$$

$$\Delta \alpha_{L=0} = (\Delta \alpha_{L=0})_{airfoil} \frac{S_{wf}}{S} \cos \Lambda_{hingeline}$$

plus a set of reference vlaues ($\Delta C_{l_{max}}$,δ_f,...) for each type of HLD to find S_{wf}/S.

Now, if you've done this, you know how much the $C_{L_{max}}$ has shifted upwards in the lift curve, and how much $\alpha_{L=0}$ has shifted to the left in the lift curve. Reference values will of course be given to you at the exam.

3 *Aircraft aerodynamic analysis - drag*

3.1 *Fundamentals*

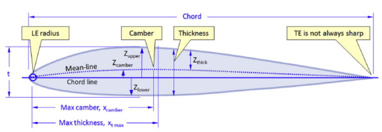
We have two different types of boundary layers:

- Laminar boundary layer. This layer is thinner, has lower friction, but is more prone to separation.
- Turbulent boundary layer. This layer is thicker, has higher friction, but due to its higher energy, it has higher resistance against separation.

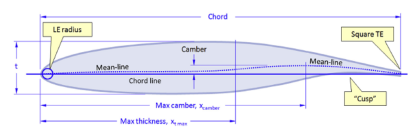
Now, several parameters affect which type we have:

- Effect of roughness: smoother surface can sustain laminar BL more than a rough surface.
- Effect of geometry: position of maximum thickness controls the pressure gradient on an airfoil. At the thickest point of the airfoil, the air above the upper surface has to move the fastest, hence pressure is lowest there. However, at the trailing edge, the pressure should slightly higher than p_∞. Hence, if you move the maximum thickness too far aft, you run the risk that there is too little distance to recover to the correct pressure, causing flow seperation (the adverse pressure gradient is too steep). So, although moving the position of the maximum thickness backwards postpones transition, it also increases the probability of separation.

NLF airfoils (Natural Laminar Flow) are airfoils that can sustain laminar flow as far as 50% or even more of the chord naturally, with the maximum thickness being very far aft. For a conventional airfoil, see figure 3.1a, for an NLF airfoil, see figure 3.1b.



(a) Ordinary airfoil.



(b) NFL airfoil.

Figure 3.1: Ordinary vs. NFL airfoil.

Theorem: drag components.

- **Skin friction drag**: the drag on a body resulting from viscous shearing stresses over its wetted surface.
- **Pressure drag (or form drag)**: the drag on a body resulting from the integrated effectr of the static pressure acting normal to its surface resolved in the drag direction.
- **Profile drag (or parasite drag)**: usually defined as the sum of the skin friction drag and the pressure drag for a two-dimensional airfoil.
- **Inviscid drag-due-to-lift**: the drag that results from the influence of a trailing vortex (downstream of a lifting surface of finite aspect ratio) on the wing aerodynamic center (note: it is present with or without viscosity).
- **Interference drag**: drag resulting from placing your fuselage and wing together, for example.
- **Wave drag**: only exists with supersonic flow; this is pressure drag resulting from noncancelling static pressure components on either side of a shockwave acting on the surface of the body from which the wave is emanating.

Drag is usually spoken of as so many counts, with each drag count being 0.0001 drag coefficient (e.g., 38 drag counts mean a drag coefficient of 0.0038). Drag force over the dynamic pressure (D/q) is a commonly used parameter and has a unit of m^2 or ft^2. D/q is called the drag area. If it is multiplied by the dynamic pressure it yields the drag force, if it is divided by the reference area, it yields the drag coefficient.

3.2 Aircraft drag modelling

Remember the drag polar. For uncambered wings, we have

FORMULAS

$$C_D = C_{D_0} + KC_L^2 \tag{3.1}$$

For cambered wings we have

FORMULAS

$$C_D = C_{D_0} + K\left(C_L - C_{L_{minD}}\right)^2 \tag{3.2}$$

Where $C_{L_{minD}}$ is the C_L at minimum C_D. For wings with moderate camber, this is so small that the first equation can be used. Remember that

FORMULAS

$$K = \frac{1}{\pi Ae} \tag{3.3}$$

With A the aspect ratio and e the Oswald efficiency factor.

Hence, for drag estimation, we must estimate the zero lift drag and the Oswald efficiency factor.

3.3 Zero lift drag estimation methods

3.3.1 Very fast estimation of zero lift drag

The fastest method for C_{D_0} estimation of clean aircraft configuration is by applying

$$C_{D_0} = C_{f_e}\frac{S_{wet}}{S_{ref}} \tag{3.4}$$

Where C_{f_e} can be looked up for several types of aircraft (e.g. bomber, light aicraft single engine, etc.), usually between 0.0020 and 0.0070, and $\frac{S_{wet}}{S_{ref}}$ is the ratio between the wetted surface area and the area wing reference area of the aircraft. Several reference aircraft are given in figure 3.2.

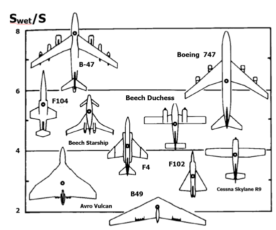


Figure 3.2: S_{wet}/S.

3.3.2 Fast estimation of zero lift drag

A fast method for C_{D_0} estimation of clean aircraft configuration is as follows:

$$C_{D_0} = \frac{1}{S_{ref}}\sum_c C_{D_c}A_c + C_{D_{misc}} \tag{3.5}$$

Where C_{D_c} is the drag coefficient of a component, A_c the area of that component and $C_{D_{misc}}$ an extra margin to take into account interference, roughness and excrescence. Appropriate data will be given if necessary, of course. Similarly, appropriate formulas will be given to you for stuff like wing wetted area, horizontal tail wetted area, etc. You don't need to know them by heart, just remember that this is one method of estimating the zero lift drag.

3.4 Drag due to lift

Remember

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

Where we need to estimate e beforehand. We have estimation formulas for this:

For straight wings $$e = 1.78\left(1 - 0.045AR^{0.68}\right) - 0.64$$

For swept wings $$e = 4.61\left(1 - 0.045AR^{0.68}\right)\left(\cos\Lambda_{LE}\right)^{0.15} - 3.1$$

Furthermore, we have the effect of flap deflection: increasing e by Δe:

Fuselage mounted engines $$\Delta e = 0.0026\delta_f$$

Wing mounted engines $$\Delta e = 0.0046\delta_f$$

Why do wing mounted engines increase e more? Because there exhaust flow positively affects the flaps behind it (is my best guess, the slides don't say anything about it).

3.4.1 Effect of wing twist

Wing twist generally decreases the induced drag. Why? Suppose we have zero net lift on the wing tip. Then there will be no wing tip vortices, but due to the twist, the sections near the root will already producing lift due to the twist. Besides this, the wing twist also changes the lift distribution which alters the induced drag. We can estimate the effect by

$$\Delta C_{D_i} = 0.00004\left(\phi_{tip} - \phi_{MGC}\right)$$

Where ϕ_{tip} is the twist angle at the tip in degree and ϕ_{MGC} the twist angle at the mean geometric chord in degree. Do note that if ΔC_{D_i} becomes negative (so less drag) if $\phi_{tip} < \phi_{MGC}$, which is pretty much always the case.

3.4.2 Effect of wing tip

The shape of the wing tip also effects the induced drag, via the effective aspect ratio:

$$AR_{eff} = AR + \Delta AR \tag{3.6}$$

Where the effect of the wing tip device can be seen in ΔAR. Please note that AR_{eff} must be used in *all* equations for lift and drag estimation. The various values of ΔAR can be seen in figures 3.3a and 3.3b. Furthermore, whether raked wing tips (which Boeiing believes in) or winglets are used also affects ΔAR, as depicted in figure 3.4.

(a) Effect of wing tip shape.

(b) Effect of wing tip shape.

Figure 3.3: Effects of wing tips.

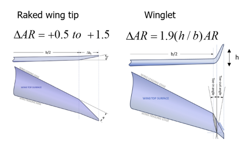


Figure 3.4: Raked wing tip vs. winglet.

3.4.3 Groud effect

Finally, we have that $K = \frac{1}{\pi A e}$ is affected by how close the aircraft is to the ground: if it is close to the ground (say less than half the span away), the ground gets in the way of the downwash, effectively preventing it from fully developing. This effect can be counted by multiplying K by a factor

$$K_{groundeffect} = \Phi K \text{ with } \Phi = \frac{33\left(h/b\right)^{1.5}}{1 + 33\left(h/b\right)^{1.5}} \tag{3.7}$$

Where b is the wingspan and h is the height of the chordline of the airfoil at MGC above the ground.

4 Aileron design

4.1 Aircraft roll control

Ailerons work rather straightforwardly: you extend ailerons upwards on one wing to decrease lift there, and downwards on the other wing to increase the lift there, creating a roll moment. Ailerons can be combined with flaps (flaperons) and elevators (elevons). Depending on the size, weight and maneuvrability of the airplane, a different roll rate is required. For example, a class III aircraft (a large, heavy, low-to-medium maneuverability) is required roll 30 degrees in 1.5 seconds.

4.2 Aileron aerodynamics

By deriving, we have that

$$P = -\frac{C_{l_{\delta_a}}}{C_{l_p}}\delta_a\left(\frac{2V}{b}\right)$$

with $C_{l_{\delta_a}}$ being the slope of the wing rolling moment coefficient (similar to the slope of the lift coefficient, but then for ailerons and their effectiveness as aileron), C_{l_p} being the roll damping coefficient, δ_a the deflection angle of the aileron, V the velocity and b the wingspan.

Now, there are two more problems with ailerons you need to take into account:

- Suppose you're flying right behind an airplane. The plane wants to turn left, so it deflects its left aileron upward and right aileron downward. Now, the left wing will create *less* drag since C_L is smaller and hence C_{D_i} is smaller, whereas the right wing will create *more* drag as C_L is larger and hence C_{D_i} is larger as well. This causes a moment around the Yaw-axis, causing the plane to turn right instead of left, what you intended to do by creating the roll (see also figure 4.1).

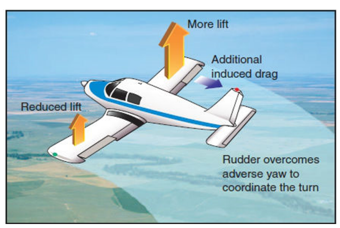


Figure 4.1: Adverse yaw.

  So, to counteract this, we must also use our rudder, to counteract this phenomena known as **adverse yaw**. In addition, **differential ailerons** are used: the aileron going down is deflected less than the aileron going up. Typically, this ratio equals 0.75: if the up-going wing is deflected 10 degrees upward, then the down-going wing is deflected 7.5 degrees downward. For the deflection angle in the formulas, use

$$\delta_a = \frac{1}{2}\left(\delta_{a_{up}} + \delta_{a_{down}}\right)$$

- When the aileron goes down, the rear part of the wing creates extra lift, causing extra torsion, as depicted in figure 4.2. This reduces the angle of attack which reduces the total lift of the wing. If the twist angle is large enough, it might even decrease the wing lift instead of increasing it. This phenomenon is known as **aileron reversal**.

Needless to say, when you design your flap(s), you should keep enough space for the ailerons, and similar to flaps, ailerons are placed after the wing rear span, determining the maximum chord ratio of the aileron.
Furthermore, large passenger aircraft can have two ailerons: inboard and outboard:

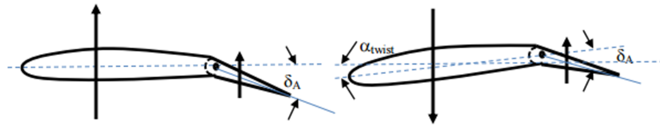


Figure 4.2: Aileron reversal.

- Outboard ailerons are more efefctive as they have a larger arm. However, at high speeds it may suffer the aileron reversal phenonema since the wing rigidity is lower at the wing tip.
- The inboard aileron can be used at high speeds instead. It can also be used as a flaperon.

In some cases, spoilers are used for roll control at high speeds to avoid aileron reversal.

5 Aerodynamic analysis: software & tools

Please note that this chapter is AFAIK new this year. So, don't look through the older exams and be like oh shit I don't need to know all this shit (though you don't actually need to know much except for the basic idea behind how these programs work).

5.1 Introduction

So far during the Aerodynamics course, you've seem some very nice equations. These equations (such as the Navier-Stokes equations) are near impossible to solve for humans, hence computers are asked to do the work. Nevertheless, even computers take a very long time for solving such equations (literally days or weeks)[1]. Now, we don't always have the time for this, and early in the design we aren't dead certain on our design anyway, so we first 'ease' the problems, by assuming inviscid, subsonic, imcompressible & irrotational flow.

Now, if you ever need aerodynamic software, here are some noteworthy programs (please don't learn these by heart of course, but as a further reference, for example for the assignment):

- CFD solvers: Star-CCM+, FLUENT, OpenFoam
- Panel method solvers:
  - 3D: PANAIR, VSAERO, PMARC
  - VLM: AVL, TORNADO, XFLR5
  - AIRFOIL: XFOIL, JAVAFOIL, DESIGNFOIL

And, just as a friendly remember, if your computer generates something strange, it probably *is* strange and a mistake.

In this chapter, we will focus on some panel method solvers.

5.2 Background on the Panel Method theory

Now, what are panel methods? Panel methods solve the so called La-Place equation to the so called Potential flow. We assume steady, subsonic, incompressible, uniform, inviscid and irrotational flow. Now, what we'll learn during aerodynamics is that the flow field can be described by a scalar function Φ, and more importantly,the velocity can be calculated through the gradient of the potential Φ[2]:

$$V = \boldsymbol{\nabla}\Phi \tag{5.1}$$

Now, as we have incompressible flow, we have that the divergence of $\mathbf{V}$ is zero ($\boldsymbol{\nabla}\cdot\mathbf{V} = 0$), hence we also get

$$\nabla^2\Phi = \left(\frac{\partial^2}{\partial x^2}\Phi + \frac{\partial^2}{\partial y^2}\Phi + \frac{\partial^2}{\partial z^2}\Phi\right) = 0 \tag{5.2}$$

Now, for some basic types of flow, we have standard solutions:

Uniform, free stream flow along x-coordinate:	$\Phi = Ax \rightarrow$	$V = A$
Source/sink:	$\Phi = \dfrac{(-2)\,\sigma}{2\pi}\ln r \rightarrow$	$V = V_r = \dfrac{(-)\,\sigma}{2\pi}\dfrac{1}{r}$
Irrotational vortex:	$\Phi = \dfrac{m}{2\pi}\theta \rightarrow$	$V = V_\theta = \dfrac{m}{2\pi}\dfrac{1}{r}$

[1] In addition, it requires human work to input all data. For example, an aircraft model in CFD takes about 20 to 25 working hours, whereas in a simple program, this only takes 1 to 5 hours.

[2] You can basically compare this with a 2D-map of the velocity. If you know that at point A a particle has a certain velocity (e.g. 20**i**), and in B it has a certain velocity (e.g. 30**i**), then obviously you can tell something about the acceleration. In fact, if you know the velocity at each infinitesimal point in a flow field, you can nicely calculate the acceleration based on the variation, the gradient of $\mathbf{V}$. Naturally, we can apply something similar to the calculation of $\mathbf{V}$, though this time, the gradient of the potential is a little bit more theoretical. Nevertheless, the idea is the same.

What's so great about this? We can 'add' these simple cases of flow to arrive at more complex cases, and hence can simplify these complex cases by building them from simple elements. For example, we see from figure 5.1 how a uniform flow, source and sink combine (the source is placed where they diverge, the sink is placed where they come together again). In fact, we can place an airfoil in the void in the middle.

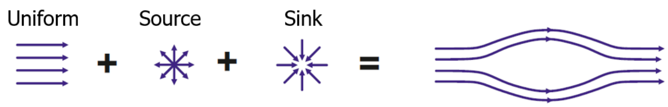


Figure 5.1: Linear composition of potential flow.

So, here's the basic idea: we try to describe an airfoil shape (that is known part of the problem) through the sum of elementary flows. The location and strength of the different elementary contributions can be set by ourselves. So, what the computer does is (see also figure 5.2):

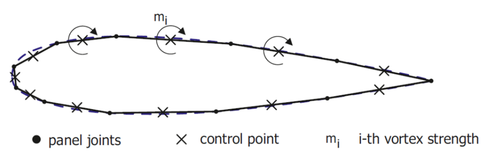


Figure 5.2: Basic formulation of panel method.

1. Discretize the shape with stragith panels.
2. For each panel, place a singularity (e.g. vortices, sink, etc.) at fixed control points.
3. Find the unknown singularities strength in order to solve the problem determined by the following boundary conditions:
   - Far away from the profile the velocity equals the free-stream conditions
   - The streamlines are tangent to the profile shape ($\mathbf{V} \cdot \mathbf{n} = 0$)
   - Specific conditions at the trailing edge.

So, in short:

- The panel method is based on a simple aerodynamic theory: be careful when assumptions are valid.
- The method uses superimposition of elementary solutions of the La-Place equation.
- The unknowns are the strengths of these elementary contributions whose location is set. The number and location of the unknowns depend on the panel distribution over the body.
- Once the problem is numerically solved, the velocity field and pressure coefficients can be calcualted.
- The accuracy notably depends on the discretization of the body: the more discrete, the more accurate it becomes, similar as how for numerical integration smaller time steps also increase accuracy.
- The model can be easily extended for the 3D case: the procedure remains conceptually the same.

5.3 *Panel methods for airfoil analysis*

JAVAFOIL is a nice program that uses the Panel Method. It can be used to evaluate:

- The lift at a certain AoA;
- The ptich moment at a certain AoA;
- The velocity distribution;
- The pressure distribution (very closely related to the velocity distribution per $c_p = 1 - \left(\frac{V}{V_\infty}\right)^2$.

Now, what it can't calculate is anything that is severely affected by the assumptions made: the polar drag can't be calculated as friction drag is neglected since it is assumed to be inviscid; similarly, separation and bubbles can't be calculated because you need a boundary layer for that, which you don't have with inviscid flow; since you don't have any information about separation, you can't tell the maximum C_l and the stall angle; and since you've assumed incompressibility, it's not a very smart thing to do calculations close to the critical Mach number, even less so calculate the actual critical Mach number.

XFOIL is another nice program, but it lacks a user-interface.

5.4 Vortex lattice methods

The VLM is a particular sub-case of the panels method. The VLM is widely used to evaluate the 3D aerodynamics of a finite wing (or for certain number of lifting surfaces located in the space) at a conceptual level.

The main assumption is about the modelling criteria of the lifting surface: a lifting surface is reduced to a flat plate which a grid of discrete (straight) vortices is defined on. The boundary conditions for the resolution of the potential flow are applied on this surface. Thickness (but not camber) is totally ignored. Now, what can the VLM tell us?

- The global lift coefficient;
- The pressure and lift distribution on the surface(s);
- The induced drag;
- The stability conditions & some useful information about aerodynamic derivatives[3]

The AVL is a nice piece of software using VLM. This one as well lacks a user interface. It is reliable until $M = 0.6$ regarding compressible effects. You can read the slides for an explanation of the program, which, apparently, will learn us:

- Although VLM is very simple, it is capable to determine several aerodynamic parameters;
- You should always keep the limitations of the method in mind: inviscid, subsonic and only small perturbations;
- You should pay attention to three different aspects: the geometry modelling, the grid and setting the analysis case;
- You should always be aware from the results: try to compare them with other data;
- You deal with numbers, so don't use inches when you're supposed to use centimeters because mistakes like those come back to haunt you.

[3] Think about the deflection of ailerons: $c_l = c_{l_{\delta_a}} \cdot \delta_a$. VLM can give you $c_{l_{\delta_a}}$.

6 Weight estimation and iterations in a/c design

6.1 Introduction

Weight plays a rather important role in the design of your aircraft. Now, sucky thing is that when you do a first estimation of the weight, the weight generally starts growing already during the design process[1]. It also keeps growing during the operational life, e.g. due to trapped fluids, debris, repairs, etc.

Nowadays, the aircraft ends up weighing 2% more than originally planned. Now, you might think you're a smart kid and say but why don't we automatically add 3-6% to this weight so that we don't go over it? The reason is that you then also build your engine etc. based on this extra contingency factor, meaning that your engine ends up heavier than strictly necessary. This warrants the question: is my airplane genuinely heavier, or is it heavier because I assumed it'd be a few percent heavier than planned (a self-fulfilling prophecy).

Now, there are four types of weight estimation, depending on the accuracy of the method:

- Class I: remember the very first two ADSEE aircraft tutorials last year? This is class I estimation: using the fuel fractions methods and statistical data, the MTOW, OEW and fuel weight could be estimated. It happens during conceptual design.
- Class II: this is what we'll be focussing on this chapter. We will now focus more on individual components (made up of subcomponents of course) and their mass (e.g. the mass of the wing group, based on the surface area, sweep angle,e tc.). This will give the weights of the a/c main components and systems and the OEW. It happens during conceptual design.
- Class III will be more detailed still, using material data, some preliminary CAD models of ouer surfaces and internal structure layout, etc. Don't need to know this for this corse though (not even ADSEE III). It happens during preliminary design.
- Class IV is when you start invoking the CATIA-gods. You do this during the detail design phase.
- In the end, you obviously place your aircraft on a scale. This is done when you've actually built your aircraft.

6.2 Class II weight estimation methods

You may remember figure 6.1 from ADSEE last year. It gives tons of definitions of weights. The most important ones are of course the OEW and the TOW. The items to the left of the payload are part of the OEW, everything to the right of the payload are part of the fuel group, and the payload is of course the payload group. Now, let us consider the parts of the basic empty weight, as that is what we, as engineers, have most influence on:

- Airframe structure: these are the wing group, tail group, body group (fuselage etc.), landing gear group, surface controls group (cockpit control, autopilot, system controls), and engine section or nacelle group (bot not the engine itself! That's part of the propulsion group. It includes the engine mounts, nacelles, pylons, engine cowlings, etc.);
- Propulsion group: engine installation, air induction system, exhaust system, powerplant controls, starting and ignition system, fuel system, propeller installation, thrust reverser.
- Airframe services and equipment: APU (Auxilary Power Unit); instrumentation; hydraulig, pneumatic & electrical systems; fuel systems (the part collocated in the wing, tail and fusealge and not directly attached to the engine); air conditioning, pressurization and anti-/de-icing systems; furnishing; flight deck accomodations; passenger cabin accomondations; cargo accomondations; standard emergency equipment, etc. All these together can be in the order of 50% of the fuselage airframe weight.

[1] Note the snowball efect. For example, if W increases and T/W is to remain the same, then a higher engine thrust is required, so more fuil is required, the fuel tank must increase and the wing must be enlarged, leading to a higher OEW. If W/S is to remain the same the wing mst be enlarged and strengthened, the undercarraige must be beefed-up, leading to a higher OEW.

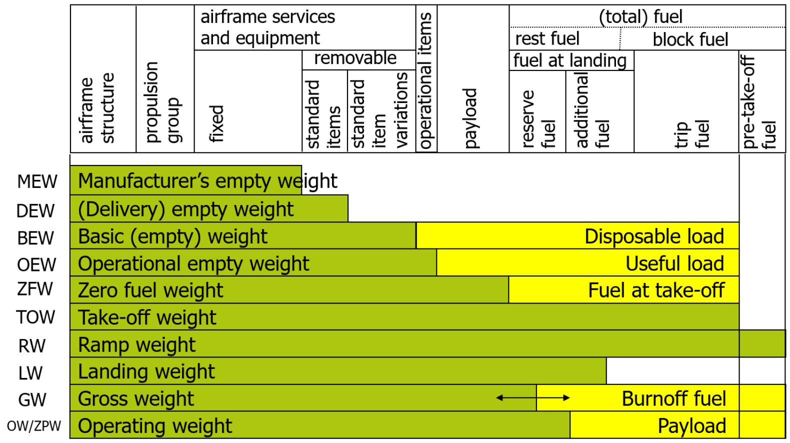


Figure 6.1: Definitions of weights.

- Operational items: crew, passenger cabin supplies, potable water, toilet chemicals and lavatory supplies, safety equipment (life jackets), oil in engines & unusable fuel (trapped fuel) and carge handling equipment.

Please note that you obviously don't need to know these all by heart, just as an indication what belongs to what.

Class II cannot predict the weight of single structural details (e.g. ribs, spars, frames, etc.), but can calculate the weight of component groups (e.g. wing, fuselage, etc.), and allows the user to evaluate the sensitivity of the weight to a number of relevant design parameters, such as sweep angle, taper ratio, wing span, etc.

Now, onto an actual example of a class II weight estimation. Let's consider, for example, the weight estimation of a wing:

$$W_{wing} = 0.0051 \left(W_{dg} + N_z\right)^{0.557} S_w^{0.649} A^{0.5} \left(t/c\right)_{root}^{-0.4} \left(1+\lambda\right)^{0.1} \cdot \left(\cos\Lambda\right)^{-1.0} S_{csw}^{0.1}$$

With W_{dg} the design gross weight, N_z the maximum load factor, S_W the wing surface, A the aspect ratio $(t/c)_{root}$ the thickness ratio at the root, λ the taper ratio, Λ the wing sweep angle, and S_{csw} the control surface area.

Just to be absolutely clear: you really don't have to learn this formula by heart, not even when you want to join SAWE. What you *do* have to understand is how and why each of the variables relate to the wing weight the way they do:

- Having a higher W_{dg} obviously increases the load on the structure, requiring it to be stronger and thus heavier. Multiplying by the maximum load factor to end up at the maximum load makes sense. Hence, the positive correlation between this term and the wing weight makes sense.
- Increased S_W obviously increases the wing weight.
- Similarly, incresed S_{CSW}[2] increases the wing weight. It makes sense, however, that the relationship is much weaker (compare the exponents): if you double the control surface area, then obviously the effect on the weight must be smaller than when you double the wing surface area, simply because the control surface area is much smaller in an absolute sense.
- Increase in aspect ratio should yield an increase in wing weight: increasing the aspect ratio (with constant wing surface) 'thins' out the wing, making it have a higher wing span. This increases the bending load, thus increasing the required structural integrity and hence increasing the weight.
- As said a few times before, thicker airfoils lead to a *decrease* in wing weight.
- A higher taper ratio should increase the wing weight, as this means the wing tip will have increased chord: this increases the loading near the wing tip, increasing the bending load, and hence increasing the required structural strength and weight.

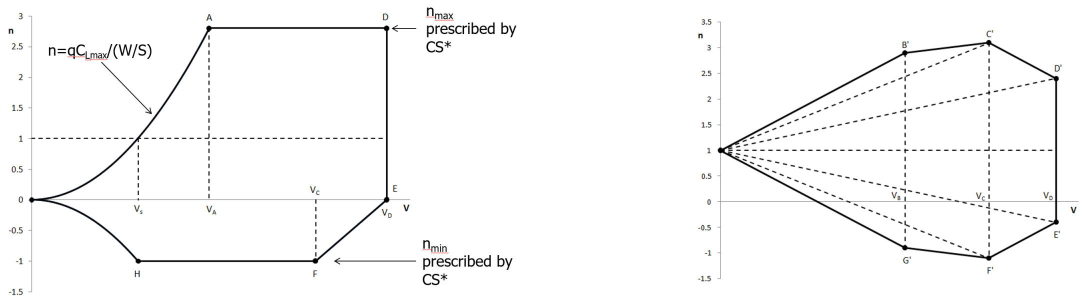
[2] The control surfaces are the ailerons, elevators, etc.

- An increase in sweep angle should increase the wing weight[3] as the torsional load is increased, hence requiring a stronger structure and thus an increase in wing weight.

Such formulas exist for more components, and if you manage to sneak into the SAWE, you may very well be able to get your hands on some of those supersecret formulas.

6.3 *Load factors*

Now, I mentioned the maximum load factor above. You may remember the two diagrams depicted in figure 6.2a and 6.2b from intro I, the part given by Mark Voskuijl. From these graphs, the maximum load factor can be deduced.



(a) Maneuver load diagram.

(b) Gust load diagram.

Figure 6.2: $V - n$ diagrams.

6.4 *Some notes on Class II Weight estimation accuracy*

- Different methods may vary in the way components and systems are grouped. E.g., one method might include the weight of the wing center section in the fuselage weight estimation, another in the wing weight estimation of the wing. Therefore, never combine weight estimates from different methods, unless there is no alternative.
- Large errors may be observed for predicted component weights, but much better correlation is found for their summation. This is something that will also be discussed during probability and statistics.
- For structural components, there are many load cases, but only one is chosen for the computational model, so it may be that your computations base your structure on the wrong critical load[4].
- Suppose you know that the ratio W_{exp}/W_{calc} for the wing group is usually 1.1, i.e., the weight of the wing group (found by experiments) W_{exp} is 10% larger than the afore calculated estimated weight W_{calc}, then it makes sense to calibrate further estimations with this factor (called the calibration or fudge factor):

$$W_{est_{des}} = [W_{calc}]_{des} \left[\frac{W_{exp}}{W_{calc}}\right]_{case}$$

  So, you multiply the weight you've estimated by a ratio you've found in literature from experimental data, which is supposed to increase the accuracy of your estimate.

Now, it may be that the OEW you've calculated using class II differs significantly from the one you've calculated using class I (from methods discussed in last year's ADSEE). If this is the case, reiterate your calculations, until the percentage difference becomes less than 1%.

[3]Note that the formula does ensure this: a higher Λ decreases $\cos\Lambda$, but by applying $^{-1}$, the wing weight is actually increased.

[4]For example, the formula may be calibrated such that the torsional load is critical, but perhaps for your wing, bending load will be critical, which obviously requires a different kind of reinforcements, thus altering the weight of your actual wing and meaning your original estimate was inaccurate.

Obviously, if you want to know the c.g. of the aircraft, you need to know the location of the c.g.s of the components, for which you can use literature to look them up. Next year, we'll be looking at the variation of the c.g. in presence of payload and fuel (and during burning of fuel, when you push people off the plane, etc.).

6.5 N^2 diagrams

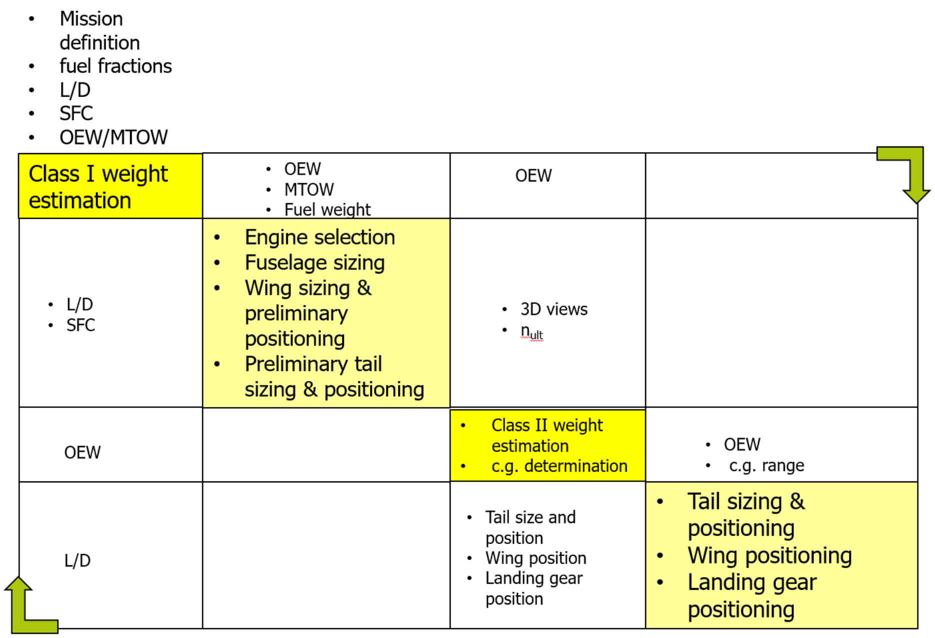


Figure 6.3: Example of N^2 diagram.

An example of a N^2 diagram is shown in figure 6.3. Basically, everything in the horizontal rows is output of that process (the yellow boxes), and everything in the vertical columns is input for that process. For example, we see that for the class I weight estimation, the required input are mission definition (range, endurance etc.), fuel fractions, L/D, SFC (specific fuel consumption, c_j) and OEW/MTOW (the ratio between these, at least). It then generates as output the OEW, the MTOW and the fuel weight. These are then used in the process of engine selection, fuselage sizing etc., which will generate for you a new estimate of the L/D and the SFC (which you can use later in an iteration of the class I weight estimation), and 3D views and n_{ult}, which you'll need as input for the class II weight estimation and the c.g. determination. Class II weight estimation produces the OEW and c.g. range as output, which you can then later use for some positioning, which gives you input for a reiteration of the class II weight estimation, and also an even more accurate L/D so that you can even do class I again. These kind of N^2 diagrams can by the way also be used in spacecraft, where you show the interaction between all of the aircraft subsystems. For example, the ADCS requires from the EPS the required pointing for the solar arrays as input to function properly, and the TT&C requires proper antenna pointing as input from the ADCS.

Intermezzo: coding system and main characteristics of NACA airfoils

Pretty sure you ought to know this stuff too, as there was an exam question that literally asks you to recommend a specific NACA airfoil for some given requirements. It'll probably also be of use for the assignment.

NACA 4 (4-digit series)

E.g. the NACA4415:

- NACA4415: maximum camber is 0.04c.
- NACA4415: position of maximum camber is 0.4c.
- NACA4415: thickness/chord ratio (t/c) is 0.15c.

NACA 4 digits have rather gradual stall behaviour. They feature gradual increase in drag and pitching moment for increasing AoA. They are also good in (low speed) trainers, which have to operate in many different conditions (not one fixed $C_{l_{des}}$.

NACA 5 (5-digit series)

E.g. the NACA23012:

- NACA23012: maximum camber is at 0.02c *and* the $C_{l_{des}} = 0.15 \cdot \underline{2}$, which allows for direct selection of the airfoil based on the required design lift coefficient, simply by multiplying the first digit by 0.15.
- NACA23012: position of maximum camber is $0.5 \cdot \underline{30}c = 0.15c$.
- NACA23012 Thickness/chord ratio (t/c) equals 0.12c.

NACA 5 digits are generally appropriate for personal/utility or regional commuter aircraft ($M_{cruise} < 0.4$) with $(t/c)_{max} = 14\%$. The position of the max camber is closer to the nose w.r.t. the 4 digit series. They feautre high values of $C_{l_{max}}$ (highest of all NACA families) but a rather abrupt stall behaviour. Due to this poor stall behaviour, NACA 5 airfoils are quite often used together with NACA 4 airfoils installed at the wing tip.

NACA 6 (laminar airfoil family)

E.g. NACA64_3-212:

- NACA64_3-212: indicate series.
- NACA64_3-212: location of maximum pressure is $0.\underline{4}c$.
- NACA64_3-212: extent of drag bucket: $C_{l_{des}} \pm 0.\underline{3}$.
- NACA64_3-212: design lift coefficient is 0.2.
- NACA64_3-212: thickness/chord ratio $(t/c) = 0.\underline{12}c$.

Associated graphs with some clarification are depicted in figure 6.4. NACA 6 digits are designed to reach extended zones of laminar flow and typically feature a gradual stall behaviour. The drag bucket, however, is very sensitive to the cleanness and smoothness of the wing surface. Dirt, water drops and even insects might annihilate this advantage (and the bucket gets filled). However, the smooth pressure gradient at the nose turned out to be very positive for increasing the critical Mach number. Hence, these airfoils became soon very popular for high speed aircraft in general, even for supersonic military aircraft. Generally, NACA 6 airfoils reach lower $C_{l_{max}}$ values than NACA 4 and 5 airfoils. A final comparison between the characteristics of these families is given in figure 6.5.

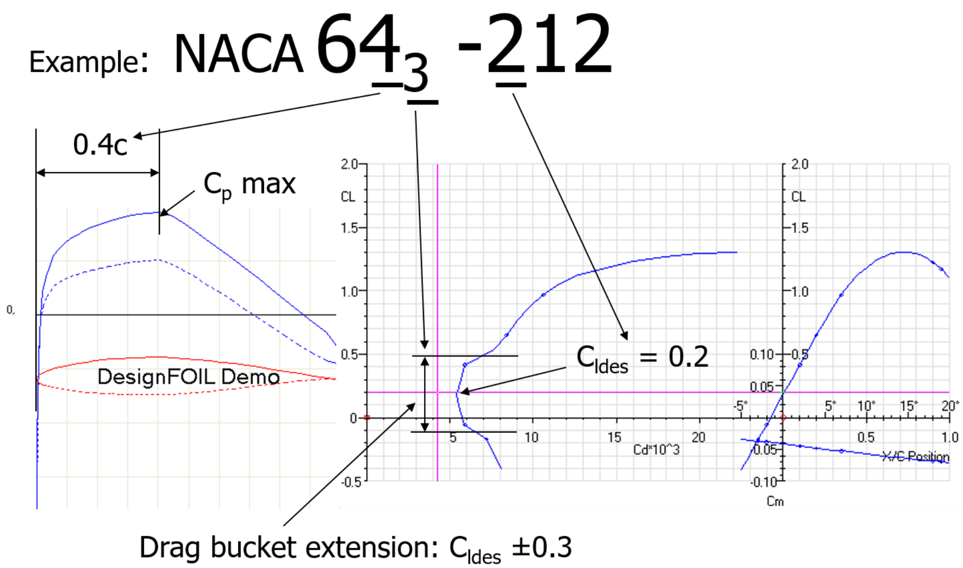


Figure 6.4: Characteristics of NACA 6 series.

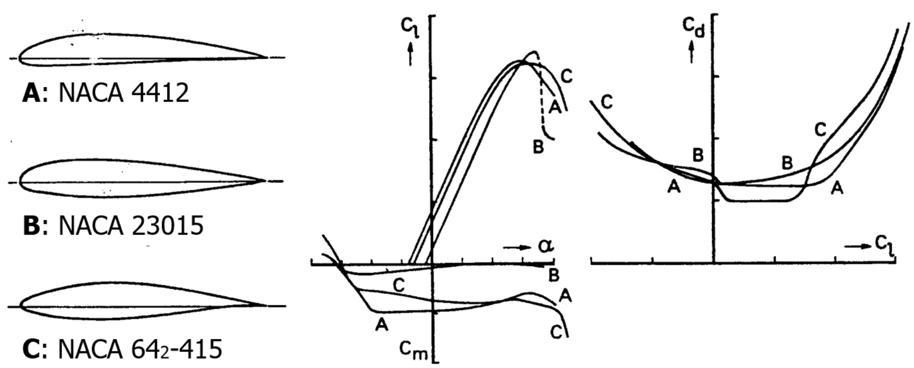


Figure 6.5: Comparison of different NACA families.

7 Spacecraft Attitude Determination & Control System

7.1 Introduction

First, a short recap of last year, in case you don't recall:

Definition

Attitude is the orientation with respect to a certain reference frame.
Attitude determination is the process of measuring and computing the attitude.
Attitude control is the process of orienting the spacecraft in a specified, predetermined direction based on the determined attitude.

ADCS is all about that angular momentum, and the fact that angular momentum is constant in an isolated system. The most notable examples of attitude control are spin, 3-axis, gravity gradient and magnetic stabilization. Some typical sensors are Sun/Earth/Star sensors, magnetometers and rate gyroscopes. Some actuators are reaction wheels, magnetic torquers, thrusters.

Now, we have three types of requirements if you rememeber from ADSEE I:

- Functional requirements: these describe the function of the ADCS w.r.t. another subsystem (e.g., orientation of the solar array).
- Non-functional requirements: these describe the performance of the ADCS, *without* relating it directly to another subsystem:
  - Accuracy: how precise is the ADCS in setting a certain satellite attitude? For example, the attitude pointing accuracy shall be better than 0.1° 3σ (so in 99.7% of the cases, the accuracy is better or equal to 0.1°;
  - Stability or jitter (allowed angle deviation in a fixed time interval): how good is the ADCS in maintaining a certain attitude? For example, the Cryosat shall have a pointing stability better than 0.0005° for 0.5 s.
  - Agility or maneuvering rate (angular rate to change between two different attitudes): how fast is the ADCS in modifying the satellite attitude? For example, the German SAR-Lupe satellites shall have a maneuvering rate of 1.33 °/s.
- Mission requirements and constraints:
  - Mission requirements:
    * Cost;
    * Schedule;
    * Reliability;
    * Compatibility.

    For example, the ADCS shall have a reliability of 95% over its first 10 years of life.
  - Constraints:
    * Mass/power/volume/thermal budgets;
    * Required orbit;
    * Allowed number of single points of failure (how many single component failures shall be manageable?).

    For example, the ADCS shall have no single points of failure (how many single component failures shall be manageable?).

  For example, the ADCS shall have no single points of failure (the failure of any single component shall be manageable without losing the system).

7.2 *Fundamental principles*

Definition

A **body-fixed system** is a system anchored with the body, with the yaw axis (w) pointing to the Earth, the roll axis (u) pointing to the velocity vector, and the pitch axis (v) pointing to the "right wing".
A **reference system** is a system independent on the body attitude.

Examples of a reference system are:

Definition

The **inertial geocentric system** is a system with origin at the Earth's center, with:

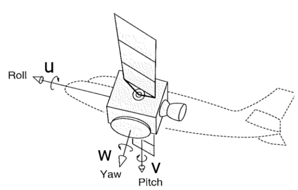
- x-axis pointing to the vernal equinox (first point of Aries)[a];
- z-axis pointing to the Earth's North Pole;
- y-axis forming a right-handed system with x- and z-axes.

The **orbit system** is a system with origin at the satellite poistion in orbit, with:

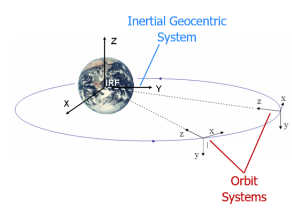
- z-axis pointing to Earth's center (Nadir);
- y-axis pointing to the negative orbit normal;
- x-axis forming a right-handed system with x- and z-axes.

[a] This can be obtained by the intersection of the elliptical orbit plane and the equitorial plane of the Earth.

See also figures 7.1a and 7.1b. All coordinate systmes have the characteristics that it is 3D, Cartesian, right-handed and that the origin is not relevant (only rotations are important, not translations).



(a) Body-fixed systems.



(b) Reference systems.

Figure 7.1: Coordinate systems.

Now, one friendly reminder, the attitude is something else than the orbital position. The attitude is part of the ADCS, the orbital position is part of the GNC (Guidance, Navigation & Control). However, for most of the time, you need to know both the orbital position and the attitude to do something useful (see figure 7.2).

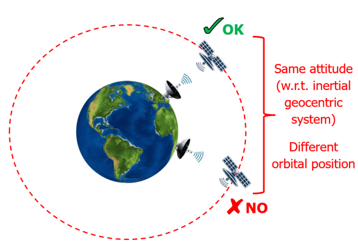


Figure 7.2: Difference between attitude and position.

Now, onto some mathematics, including some nice vectors and matrices. Let's look at some generic vector $\mathbf{a}_B = < a_u, a_v, a_w >$ in a *body-fixed* system. This may for example be a vector that is parallel to the pitch

axis (and thus is given by $< 0, 1, 0 >$), a vector that connects your spacecraft with the Sun[1]. Now, the length of the vector does not matter at all: you may make your vector connnecting the Sun to your s/c a whole AU long, but you're also allowed to normalize it if you want to. Direction is the only important thing for this generic vector.
Now, this vector pointing from the S/C to the Sun may be given by $< 5, 3, 1 >$. Now we want to know what how that vector would like in a reference sytem, so the vector $\mathbf{a}_R =< a_x, a_y, a_z >$ (it may, for example, be that in this reference system, the vector that in the body-fixed system pointed in the direction of $< 5, 3, 1 >$, now points in the direction $< 2, -1, 0 >$ when using this reference system). So, how do we go from one system to the other? We can relate these as follows:

$$\mathbf{a}_B = \begin{bmatrix} a_u \\ a_v \\ a_w \end{bmatrix} = A_B^R \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = A_R^B \mathbf{a}_R \tag{7.1}$$

So, by multiplying the vector $\mathbf{a}_R$ with matrix A_B^R, we end up at the vector $\mathbf{a}_B$. A_B^R performs a rotation upon $\mathbf{a}_R$ and is called the transformation matrix. We can naturally also find $\mathbf{a}_R$:

$$\mathbf{a}_B = A_B^R \mathbf{a}_R$$
$$\mathbf{a}_R = \left(A_B^R\right)^{-1} \mathbf{a}_B$$

However, since A_B^R is purely rotational (it does not alter the length of the original vector), we have that $\left(A_B^R\right)^{-1} = \left(A_B^R\right)^T$, so

$$\mathbf{a}_R = \left(A_B^R\right)^T \mathbf{a}_B \tag{7.2}$$

Now, we have two ways of finding A_B^R: we can use a direction-cosine matrix, and Euler angles. First, the direction-cosine matrix: this is a rather simple method in principle, but rather difficult in execution. What you do is making the following matrix:

$$A_B^R = \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{bmatrix}$$

Where each element of the matrix is the cosine of the angle between one axis of the body-fixed system and one axis of the reference system. For example, u_x is the cosine of the angle between the u-axis and the x-axis of the reference system; v_z is the cosine of the angle between the v-axis (pitch axis) and the z-axis. So, you have to draw the relative orientation of the coordinate systems[2] and measure all possible angles and plug them in this matrix. However, this requires some solid 3D-thinking, which can be rather difficult. So, there's another way to do it, namely using the Euler angles.

The approach by Euler angles uses the fact that the reference system is always obtained from the body-fixed one by a set of (at most) three successive rotations around one of the body-fixed system axes. Each of those individual rotations are rather easy to construct in a matrix, as we have:

FORMULAS

Rotation around u-axis with angle ϕ (roll):
$$A_u = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \tag{7.3}$$

Rotation around v-axis with angle θ (pitch):
$$A_v = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \tag{7.4}$$

Rotation around w-axis with angle ψ (yaw):
$$A_w = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7.5}$$

[1] Since the distance is so huge, the direction of the vector is pretty much constant, regardless of where you let the vector start on your spacecraft.

[2] Questions about constructing this matrix give you this relative orientation by telling you how the reference frame can be constructed by rotating the body-fixed system in certain ways (or the other way around).

And we have the combined vector rotation be

$$\mathbf{a}_R = A_R^B \mathbf{a}_B = A_u(\phi)\, A_v(\theta)\, A_w(\psi)\, \mathbf{a}_B \tag{7.6}$$

Now, you may be wondering, this is nice and all, but what do I actually need to use it for? We can use above equations to find the transformation matrix ourselves, which we can use to find the angles of rotation of the transformation, hence yielding us the attitude of the spacecraft. Now, how do we compute the transformation matrix?

Theorem: computing the transformation matrix

1. Measure two non-parallel unit vectors $\mathbf{a}_{1b}$ and $\mathbf{a}_{2b}$ in the body-fixed system. For example, measure one vector from a Sun sensor and another from a Star sensor. This (probably) gives you two different vectors.
2. Compute the same vectors $\mathbf{a}_{1r}$ and $\mathbf{a}_{2r}$ in the reference system, by means of a model that tells us where the identified star and Sun are, at that certain moment, in the reference system.
3. Build two sets of three mutually perpendicular vectors (three because 3D). Let the first vector be[a]

$$\mathbf{q}_B = \mathbf{a}_{1B}$$

Then, the second vector will be the cross product of $\mathbf{q}_B$ and $\mathbf{a}_{2B}$, as this vector will be perpendicular to $\mathbf{q}_B$:

$$\mathbf{r}_B = \mathbf{q}_B \times \mathbf{a}_{2B}$$

Finally, let the third vector be the cross-product of $\mathbf{q}_B$ and $\mathbf{r}_B$, so that this one is perpendicular to these two as well:

$$\mathbf{s}_B = \mathbf{q}_B \times \mathbf{r}_B$$

And do exactly the same for the two vectors from the reference system.
4. Calculate the transformation matrix from the two sets of vectors:

$$\begin{bmatrix}\mathbf{q}_B & \mathbf{r}_B & \mathbf{s}_B\end{bmatrix} = A_B^R \begin{bmatrix}\mathbf{q}_R & \mathbf{r}_R & \mathbf{s}_R\end{bmatrix} \rightarrow A_B^R = \begin{bmatrix}\mathbf{q}_B & \mathbf{r}_B & \mathbf{s}_B\end{bmatrix} \cdot \begin{bmatrix}\mathbf{q}_R & \mathbf{r}_R & \mathbf{s}_R\end{bmatrix}^T \tag{7.7}$$

[a] Yes it's literally exactly the same vector, but just another letter.

Please note that any errors on the measured or computed vectors have a big impact on the computed attitude. To mitigate the problem, more than two vectors are (redundantly) measured and a statistical method is used to determine the attitude. Another way to decrease the number of operations (and hence the number of error propagation sources) is to take two non-parallel unit vectors that are perpendicular to each other: this way, you have one less cross-product to calculate.

7.2.1 Attitude sensor errors

Ideally, you want value measured by your sensors to equal to 'true' value. However, this is often not the case, because there are usually two errors in play:

- Systematic erros: these are errors caused by e.g. manufacturing offset (mechanical tolerances), imperfect calibration or secondary effects (e.g. temperature offset). Systematic errors come into two flavours: bias and scale. Bias is when for each measurement, you need to add or substract a fixed amount (so the 'mistake' is constant, regardless of the measured value). Scale is when you need to multiply each measurement with a fixed amount (so how large the 'mistake' is, depends on the measured value). If you know how large the bias or scale is, they are easily be compensated[3].
- Random errors, or noise: you just have to deal with this shit, but their effects can be reduced by averaging or filtering techniques.

See also figure 7.3.

[3] For example, if we look at American election polls, a daily tracking poll conducted by the LA Times (everyday a poll is published) consistently gives an advantage of about 6 points to Trump. So, when you look at a LA Times poll showing Trump 6 up on HRC (which, at the time of writing, he is), then you just have to think that in reality, it's actually a tie.

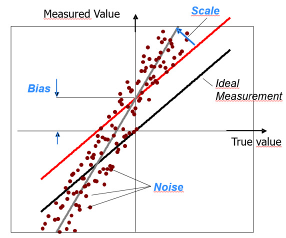


Figure 7.3: Attitude sensor errors.

Now, we also have absolute vs. relative attitude:

DEFINITION

The **absolute attitude** is based on the direction of two non-parallel unit vectors in the two coordinate systems (body-fixed and reference). This is what we've done before.
The **relative attitude** is the attitude relative to the attitude at an earlier point in time. Periodic alignment with absolute attitude measurements is needed to avoid drifts.

Relative attitude is helpful because it can be measured much more often than the absolute attitude (which can be necessary because you want a really, really stable camera).

7.2.2 *Attitude kinematics and dynamics*

First, an equation to find the time derivative of the rotation matrix, so that you can integrate it later to obtain the transformation matrix as a function of time (I'm pretty sure most of this is beyond the scope of this course for now, though):

$$\frac{d}{dt}A_B^R = \Omega \cdot A_B^R = \begin{bmatrix} 0 & \omega_w & -\omega_v \\ -\omega_w & 0 & \omega_u \\ \omega_v & -\omega_u & 0 \end{bmatrix} \cdot \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{bmatrix}$$

Now, onto the dynamics. For an inertial system, we have that the sum of the moments equals the time rate of change of angular momentum:

$$\frac{d}{dt}\mathbf{H} = \sum \mathbf{M}$$

However, for a body-fixed system, which is already rotating itself, we need to add another term[4] to arrive at the **Euler equations of motion**:

$$\frac{d}{dt}\mathbf{H} + \begin{bmatrix} \omega_u \\ \omega_v \\ \omega_w \end{bmatrix} \times \mathbf{H} = \sum \mathbf{M} \tag{7.8}$$

These can be worked out to

FORMULAS

Along u axis: $$I_{xx}\frac{d}{dt}\omega_u + \omega_v\omega_w\left(I_{zz} - I_{yy}\right) = M_u \tag{7.9}$$

Along v axis: $$I_{yy}\frac{d}{dt}\omega_v + \omega_u\omega_w\left(I_{xx} - I_{zz}\right) = M_v \tag{7.10}$$

Along w axis: $$I_{zz}\frac{d}{dt}\omega_w + \omega_u\omega_v\left(I_{yy} - I_{xx}\right) = M_u \tag{7.11}$$

[4] I don't exactly know where it comes from, just believe me.

8 ADCS sensors and actuators

8.1 ADCS sensors

In this section, six sensors will be discussed: magnetometers, Earth sensors, Star sensors, Sun sensors, GPS-based systems and gyroscopes.

Definition

A **magnetometer** measures the direction and magnitude of Earth's[a] magnetic field in the body-fixed system.

[a]Of course, other planets can also be used, as long as you're close to them of course.

This gives you one vector of the two necessary. Note that this vector is compared to the vector in the reference system (which you know from a model), so you must know the orbital position too. The sensing part of a magnetometer is typically deployed away from the satellite by means of a boom, to avoid influence on the measurement from the residual magnetic field of the satellite, and to avoid magnetization of the sensing part during integration, test and operations on ground.

Definition

An **Earth sensor** detects Earth's horizon in the thermal infrared at a wavelength of about 15 µm.

It provides the angle between the nominal sensor axis and the Nadir direction. It can be affected by radiation distribution not being spatially uniform (or uniform throughout time), oblateness of the Earth, and thermal variations on the sensor. We have two types:

- Static sensors: from the distribution of the radiation, the angle between the sensor axis and the Nadir direction can be measured. So, only one vector is generated, of the two necessary. The typical field of view is 20° by 15° when used in GEO[1].
- Scanning Earth sensor: this sensor field of view sweeps around and hits the Earth twice. By knowing the period of rotation and the time interval between the two Earth hits, and the scan angle and angular size of the Earth, it is possible to calculate the direction between the Nadir vector and the scan spin axis. These type of sensors are more accurate than static Earth sensors, and can be used at any altitude by adjusting the scan angle.

Definition

A **star sensor** identifies star patterns (4-stars or more) in the body-fixed coordinate system.

This is then compared to a model of the position of the same star patterns in a reference system. Each star generates one measured vector. One sensor describes the location of a star as $x - y$ coordinates. So, to arrive at three coordinates, another non-parallel sensor is needed. A star sensor typically has a narrow FoV ($20° \times 20°$). It provides sufficient accuracy only at low spacecraft rotation rates. Star sensors are, however, presently the dominant technology for space attitude determination.

Definition

A **Sun sensor** measures the direction of the Sun rays in a body-fixed frame.

Only one vector is measured. A thin entry slit allows Sun rays to hit a set of photodetectors. Depending on the angle between the Sun rays and the entry slit, only specific photodetectors are hit, from which the direction can be derived. However, two non-parallel sensors must again be used. Two categories of Sun sensors are available, with coarse being simple but not accurate, and fine Sun sensors the opposite.

[1]Due to this large field of view, it is necessary to use this sensor at high altitudes to make sure the Earth's horizon is inside its field of view. If you're too close to the Earth, you don't see the horizon on your sensor, making it useless.

DEFINITION

A **GPS sensor** receives the same GPS signal at tree different antennas. By comparing the phase differences between the received signal, the attitude can be determined.

DEFINITION

A **gyroscope** is a device that measures the satellite rotation rates w.r.t. a given reference system.

This gives the relative attitude, i.e. compared to an earlier point in time. However, the measured attitude slowly drifts from the true one due to error sources (scale, bias, noise). Three types of gyroscope are typically used: mechanical gyro (very common, but its use is decreasing), fiber optic gyro (FOG) (rare, but its use is increasing) and ring laser gyro (RLG) (very common). We'll focus on the mechanical gyro for now.

You may remember from the previous chapter that

$$I\frac{d}{dt}\boldsymbol{\omega} + \boldsymbol{\omega} \times I\boldsymbol{\omega} = \sum \mathbf{M}$$

Now, let's look at figure 8.1.

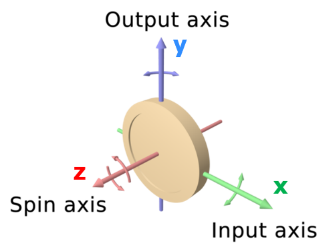


Figure 8.1: Spinning gyroscope.

Now, what happens exactly in this figure? The spacecraft rotates with angular velocity Ω around the input axis. This is the angular velocity we eventually want to know. Now, the gyroscope wheel itself rotates with a known angular velocity ω around the spin axis. The term $\omega \times I\omega$ will create a component on the output axis, proportional to both Ω and ω. Since we measure this component and know ω, Ω can rather easily be computed. Now, from the scalar equations listed in the previous chapter, we have that the scalar equation along the y-axis is given by[2][3]:

$$I_{yy}\frac{d}{dt}\omega_{output} + \omega\Omega\left(I_{xx} - I_{zz}\right) = -M_y \tag{8.1}$$

Example So if, for example, it is given that $I_x = I_y = 10^{-5}\,\mathrm{kgm}^2$, $I_z = 2 \times 10^{-5}\,\mathrm{kgm}^2$ is spinning with a velocity $\omega_Z = 2000\,\mathrm{rpm}$. At a certain moment, to balance the gyro around the output axis, the actuator needs to provide a torque $M_y = 6 \times 10^{-3}\,\mathrm{Nm}$. What is the angular velocity of the spacecraft (in deg/s) around the input axis x.

So, assuming a constant output axis angular velocity, we have that:

$$\begin{aligned}
\omega\Omega\left(I_x - I_z\right) &= -M_y \\
\Omega \cdot \frac{2000 \cdot 2\pi}{60} \cdot \left(10^{-5} - 2 \cdot 10^{-5}\right) &= -6 \cdot 10^{-3} \\
\Omega &= 2.865\,\mathrm{rad/s} = 164.1\,^\circ/\mathrm{s}
\end{aligned}$$

[2]In the three scalar equations, replace u everywhere with x, v with y and w with z.

[3]Note the minus sign: the scalar equations gave M as an *external* torque. However, as we produce this torque here ourselves, it should be negative. Another way to see it is that the angular momentum is constant as no external torque is applied. Hence, if the angular velocity is increased clockwise, there must a torque pointing counterclockwise (similar to how one of those cars you played with as a kid that you could pull back and then sort of 'loaded' the car. First, you apply an external force on the car by pulling it back. You feel that the thing inside the car already tries to produce a reaction force for this. This reaction force points forward. However, once you stop applying your external force (and neglecting friction), the car will accelerate forward, but the force inside the car accelerating it will decrease, so it'll point more backwards. This ensures that $\mathbf{F} + m\mathbf{a} = \mathbf{0}$, as there is no external force working on it anymore (except gravity)). The same principal is at work for this gyroscope.

Finally, an overview of the the pros and cons of each type of sensor given in table 8.1.

Table 8.1: Overview of pros and cons of each type of sensor.

Type	Strenghts	Weaknesses
Magnetometer	• Cheap • Low power • Reliable • Lightweight	• Offsets from temperature effects and residual magnetic field of satellite • Limited accuracy due to inaccurate models of Earth magnetic field
Star sensor	• Orbit independent • 3-axis attitude with 2 sensors • High accuracy	• Heavy, complex and expensive • High power consumption
Sun sensor	• Bright, easy-to-find target • Low power • Reliable	• Sun not visible at all times (eclipses) • Limited accuracy due to Sun's diameter
Earth sensor	• Bright, easy-to-find target • Target always available • Robust, easy analysis	• Low accuracy due to fuzzy Earth • Complexity (for scanning sensors)
Gyroscope	• No external inputs • Orbit independent • High short-term accuracy	• Measures attitude changes only • Poor long-term accuracy • Errors from drift

8.2 *ADCS actuators*

8.2.1 *Passive attitude control strategies*

Passive control does not require any actuators. It makes use of the spacecraft properties (rotation) or fields in which it is immersed (gravitational, magnetic).

8.2.2 *Active attitude control*

Active attitude control allows for 3-axis spacecraft stabilization through the use of actuators. In this subsection, we will take a closer look at:

- Reaction wheel
- Momentum wheel
- Control momentum gyro
- Thrusters
- Magnetic torquers

The first three require internal torques. This mean that they tend to become saturated after a certain time (reach maximum allowed rotating speed). They must therefore always be used in combination with thrusters or magnetic torquers, in order to de-saturate them (also called momentum dumping).

Table 8.2: Passive attitude control strategies

Type	Typical accuracy	Comments
Pure spin stabilization	$\pm 0.1^{\circ} - \pm 1^{\circ}$ (2 axis). Proportional to spin rate.	• Maintains fixed orientation in inertial space. • Requires passive nutation dampers or thruster control. • Spinning around axis with largest inertia ($I_z > I_x = I_y$). • Simple
Dual spin	Same as pure spin	• Payload section de-spun to facilitate pointing. • Complex and expensive.
Gravity gradient	$\pm 5^{\circ}$ (2 axis)	• Stable orientation relative to the local vertical of central body. • Significant asymmetry in inertia needed: $I_z << I_x, I_y$. • Displays oscillatory motion. • Limited to LEO; requires elongated mass distribution.
Passive magnetic	$\pm 5^{\circ}$ (2 axis)	• Satellite is magnetically stabilized. • Only applicable to LEO.

Internal torque actuators

As aforementioned, these three types of actuators are reaction are reaction wheel, momentum wheel and control moment gyro. The latter two are simply extensions of the basic principle behind the reaction wheel. All of them rely on the Euler equation in the body-fixed system[4]:

$$I\frac{d}{dt}\boldsymbol{\omega}+\boldsymbol{\omega}\times I\boldsymbol{\omega}=\sum\mathbf{M} \tag{8.2}$$

Let's look at the reaction wheel first, as depicted in figure 8.2. From the scalar component of the Euler

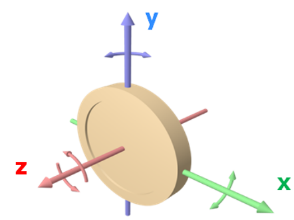


Figure 8.2: Reaction wheel.

equation, we have that if ω is the velocity around the z-axis, and Ω_x the angular velocity of the *spacecraft* around x and Ω_y around y[5]:

$$I\frac{d}{dt}\begin{bmatrix}\Omega_x\\ \Omega_y\\ \omega\end{bmatrix}+\begin{bmatrix}\Omega_x\\ \Omega_y\\ \omega\end{bmatrix}\times\begin{bmatrix}I_x & 0 & 0\\ 0 & I_y & 0\\ 0 & 0 & I_z\end{bmatrix}\cdot\begin{bmatrix}\Omega_x\\ \Omega_y\\ \omega\end{bmatrix} = \sum\mathbf{M}$$
$$I_z\frac{d\omega}{dt}+\Omega_x\Omega_y\left(I_y-I_x\right)=M_z$$

However, since $I_y \approx I_x$[6], the second term in the equation reduces to zero. So, to generate (or counteract) a torque, ω needs to be increased with time. Reaction wheels serve to purposes:

- Counter-act a disturbance torque (around the z-axis);
- Spacecraft rotation (or slew).

Simple, isn't it? Just apply an angular acceleration to arrive at the torque you need.

Now, onto the momentum wheel. This is almost exactly the same as the reaction wheel, but the nominal rotational speed is *not* zero (usually high and constant). Remembering the scalar components of the Euler equations, we have that

$$\boldsymbol{\omega}\times I\boldsymbol{\omega}=\begin{bmatrix}\Omega_x\\ \Omega_y\\ \omega\end{bmatrix}\times\begin{bmatrix}I_x & 0 & 0\\ 0 & I_y & 0\\ 0 & 0 & I_z\end{bmatrix}\cdot\begin{bmatrix}\Omega_x\\ \Omega_y\\ \omega\end{bmatrix} = \begin{bmatrix}\omega\Omega_y\left(I_z-I_y\right)\\ \omega\Omega_x\left(I_x-I_z\right)\\ \Omega_x\Omega_y\left(I_y-I_x\right)\end{bmatrix}$$

Where once again, the lower term is equal to zero. For the other two equations however, we see that if the spacecraft starts to rotate with angular velocity Ω_y for example, automatically a torque is generated around the x-axis. This wasn't the case for the reaction wheel, as that one didn't normally have any ω.

Now, so far, the calculations were with regards to a body-fixed system with the momentum wheel being the body. However, now consider the *spacecraft* being the body, rotating with angular velocities Ω_x, Ω_y and Ω_z.

[4] In this case, the body is *not* the spacecraft, but the wheel itself.

[5] In case you're wondering: why does this even matter, aren't we just looking at the reaction wheel? And if it does matter, why isn't there any Ω_z? The reason for this is that seen in an inertial system, the reaction wheel *will* be rotating around the x and y-axis, since it is fixed to the spacecraft itself. We can only control the velocity around the z-axis, hence we use a small ω for that, to indicate that that is the angular velocity of the reaction wheel, and not the angular velocity due to the spacecraft.

[6] Once again, these are the moments of inertia of the *wheel*, not the spacecraft. From symmetry, this equality holds.

However, the momentum wheel itself also has a constant angular momentum h around the z-axis:

$$\mathbf{H} = \begin{bmatrix} I_x\Omega_x \\ I_y\Omega_y \\ I_z\Omega_z + h \end{bmatrix}$$

$$\frac{d}{dt}\mathbf{H} + \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} \times \mathbf{H} = \sum \mathbf{M} \begin{cases} I_x\dot{\Omega}_x + \Omega_z\Omega_y\left(I_z - I_y\right) + h\Omega_y = M_x \\ I_y\dot{\Omega}_y + \Omega_z\Omega_x\left(I_x - I_z\right) - h\Omega_x = M_y \\ I_z\dot{\Omega}_z + \Omega_x\Omega_y\left(I_y - I_x\right) = M_z \end{cases}$$

Solving these differential equations gives you that a torque around the x-axis yields you an angular velocity around the y-axis equal to $\Omega_y = \frac{M_x}{h}$[7]. This angular velocity[8] is called **precession**. We see that we can reduce this precession by increasing the angular momentum of the momentum wheel. Still, it's pretty much the same result as what followed earlier, about how higher ω increased the stiffness.

Finally, onto the control moment gyro (see also figure 8.3.

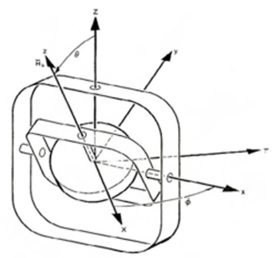


Figure 8.3: Control moment gyro.

We see that the control moment gyro rotates around two axes: the z and the x-axis. This rotation axis change allows for a control torque to be generated around a given direction.

Example Assume that the spacecraft rotates around x with velocity Ω_x and the CMG angular speed has a component ω_y along y and ω_z along z (so it's different from figure 8.3).

We then have

$$\boldsymbol{\omega} \times I\boldsymbol{\omega} = \begin{bmatrix} \Omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \times \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \cdot \begin{bmatrix} \Omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \omega_y\omega_z\left(I_z - I_y\right) \\ \Omega_x\omega_z\left(I_x - I_z\right) \\ \Omega_x\omega_y\left(I_y - I_x\right) \end{bmatrix}$$

So, if both ω_y and ω_z are nonzero, then you're generating a control torque around the x-axis (by constrast, the reaction wheel was only able to apply a control torque around the axis you're rotating the RW about).

Theorem: rules of thumb for preliminary wheel sizing

The wheel torque T needed to counter-act a given disturbance torque T_D shall include an appriate margin factor MF:

$$T = T_D\left(1 + MF\right) \tag{8.3}$$

The wheel torque T needed to slew the spacecraft with MMOI I a given angle θ in a given time t is

$$T = \frac{4I \cdot \theta}{t^2} \tag{8.4}$$

For wheels used to counter-act a periodical (sinusoidal) disturbance torque of maximum amplitude T_D, the maxim um possible stored momentum h per orbit is calculated by considering the root mean

[7] And similarly, $\Omega_x = \frac{M_y}{h}$.

[8] Comparable to when you hold a wheel between your hands, you give it a spin and you find that you are rotating now as well, due to the torque.

square average of the disturbance torque over 1/4 of the orbital period P:

$$h = \frac{\sqrt{2}}{2} \cdot T_D \cdot \frac{P}{4} \tag{8.5}$$

For momentum wheels, if wheel axis is different to the disturbance torque axis, the gyroscopic stiffness of the wheel can be used to limit spacecraft rotations around an axis perpendicular to the wheel axis below a maximum allowed angle θ_a:

$$h = \frac{T_D}{\theta_a} \cdot \frac{P}{4} \tag{8.6}$$

Some of the formulas may seem a bit random at first, but make sense if you listen to Barry's advice and think about it: previous year, if you had to calculate the required torque necessary to slew a spacecraft θ radians in a time t, you'd take half the angle and half the time for

$$\frac{\theta}{2} = \frac{1}{2} \cdot \alpha \cdot \left(\frac{t}{2}\right)^2$$

$$\alpha = \frac{4 \cdot \theta}{t^2}$$

Then, the torque was simply calculated using

$$T = \alpha \cdot I = \frac{4I \cdot \theta}{t^2}$$

For the third formula, if we have a sinusoidal torque, our momentum storage will also have a sinusoidal shape (though it's shifted $\frac{1}{2}\pi$). So, the maximum storage will be accumulated during a period of $\frac{P}{4}$[9], and during this period the average torque equals $\frac{\sqrt{2}}{2} \cdot T_D$, and hence the maximum momentum storage equals $h = \frac{\sqrt{2}}{2} \cdot T_D \cdot \frac{P}{4}$. For the fourth formula, we have $\omega = \frac{d\theta}{dt} = \frac{T_D}{h}$,a nd hence:

$$d\theta = \frac{T_D}{h} \cdot dt$$

$$\int_0^{\theta_a} d\theta = \int_0^{P/4} \frac{T_D}{h} dt$$

$$\theta_a = \frac{T_D}{h} \frac{P}{4}$$

Thrusters for attitude control

The working of thrusters should be pretty obvious. Common types of thrusters are monopropellant (1 N), cold gas (≈ 0.1 N) and electric (≈ 1 mN). Note that obviously, propellant is needed as well.

THEOREM: RULES OF THUMB FOR THRUSTER SIZING

The thrust needed to counter act a torque T_D when the thrusters are located a distance L from each other, taking into account a margin factor MF:

$$F_T = \frac{T_D (1 + MF)}{L} \tag{8.7}$$

The thrust needed to provide the spacecraft with MMOI I a given slew rate ω in t seconds:

$$F_T = \frac{I \cdot \omega}{L \cdot t} \tag{8.8}$$

[9] Because when you start from zero momentum storage, you only increase your momentum for $\frac{P}{4}$ of the time.

The thrust needed to provide the spacecraft a slew angle θ in a direction perpendicular to the wheel axis where the wheel has an angular momentum h:

$$F_T = \frac{h \cdot \theta}{t \cdot L} \tag{8.9}$$

The thrust needed to dump a given wheel's angular momentum h in a time t:

$$F_T = \frac{h}{t \cdot L} \tag{8.10}$$

The thruster pulse life, based on the maximum number of pulses during life time N_{max} and the pulse duration t:

$$T_{tot} = N_{max} \cdot t \tag{8.11}$$

Magnetic torquer

If the local Earth's magnetic field is $\mathbf{B}$ and the coil dipole moment is $\mathbf{D}$, the generated torque $\mathbf{M}$ is

$$\mathbf{M} = \mathbf{D} \times \mathbf{B} \tag{8.12}$$

So, if $\mathbf{D}$ and $\mathbf{B}$ are perpendicular, the directions are related by the left-hand rule: your stretched-out fingers point in the direction of $\mathbf{D}$, you catch $\mathbf{B}$ with the palm of your hand and your thumb points in the direction of $\mathbf{M}$. Naturally, you can also use this to find $\mathbf{D}$ and $\mathbf{B}$. The magnitude of $\mathbf{D}$ is given by

$$D = N \cdot I \cdot A \tag{8.13}$$

Some characteristics of magnetic torquers are given in table 8.3.

Table 8.3: Characteristics of magnetic torquers

	Coil type	Torque rod type	Comments
Dipole moment	$\approx 5\,\mathrm{Am}^2$	$\approx 200\,\mathrm{Am}^2$	Coil with iron core
Usable torque	75-250µNm	2000-10000µNm	Nominal, in LEO
Mass	0.8 kg	5 kg	
Power consumption	1.5 W	3 W	At 28 V, 100% duty cycle
Typical application	Satellite mass < 100 kg	Satellite mass > 100 kg	

First, the scalar equation is

$$D = \frac{M}{B} = \frac{1000 \cdot 10^{-6}}{20 \cdot 10^{-6}} = 50\,\mathrm{Am}^2$$

Now, the direction: B points from North to South, so in opposite direction of the North arrow. Catching this with our left hand palm, and pointing our thumb downward, we see that D points towards the east.

A summary of the different actuators is given in table 8.4.

Now, only two things left to discuss. First, the ADCS processor: this computes the current attitude, predicts the future attitude, controls the attitude, allows the switch modes, reacts to ADCS failures, executes ground commands and distributes information to other subsystems or payload.

8.2.3 ADCS redundancy concepts

For ADCS, redundancy is key to risk managment technique. We need to avoid single points of failure. One example is the reaction wheel system depicted in figure 8.4. If one reaction wheel fails, 3-axis control is still possible by tuning the torques provided around different axes.

Table 8.4: Actuators.

Actuator type	Torque magnitude	Remarks
Thrusters	Small to very high	• Propellant needed on board
Magnetic torquer	A few mNm, only along two axes	• External torque • Cost-effective
Reaction wheel	$\approx 200\,\mathrm{mNm}$, variable	• Internal torque
Momentum wheel	$\approx 50\,\mathrm{mNm}$	• Internal torque • Provide also gyroscopic stiffness
Control moment gyro	Very high torques, but only temporarily	• Internal torque • Expensive

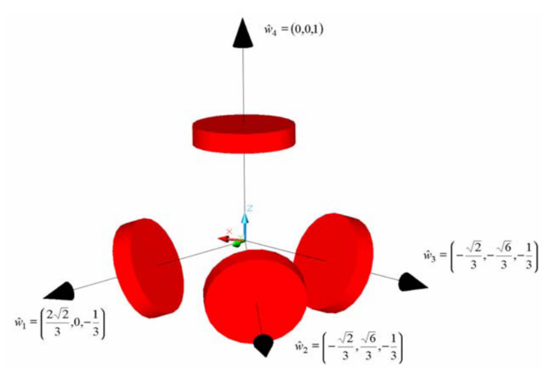


Figure 8.4: Redundant reaction wheels.

9 Spacecraft telecommunications

9.1 Introduction

The **telecommunications** (TC) subsystem is also called **Telemetry, Tracking & Command** (TT&C). Typical requirements are:

- Type of signal transmitted (voice, television, data, etc.);
- Capacity (bandwidth, frequencies);
- Coverage area served by satellite;
- Signal strength and quality;
- Connectivity (direct link to receiver or cross links);
- Availability (how frequent and long are the connections with the ground system);
- Life time (mission duration).

On the other hand, typical constrains are:

- Transmitter power;
- Receiver sensitivity;
- Interference;
- Environment;
- Regulations.

9.2 Fundamental principles

In tele-communications, information is typically exchanged in form of bits (*bi*nary digi*ts*). A fundamental equation is the Shannon-Hartley theorem:

Theorem: Shannon-Hartley theorem

> The channel capacity C in bit/s equals:
>
> $$C = B \cdot \log_2 \left(1 + \frac{S}{N}\right) \tag{9.1}$$

Where:

- C is the channel capacity in bit/s, i.e. the maximum theoretical data rate that the channel can transmit;
- B is the channel bandwidth in Hz, i.e. the maximum operating frequency minus the minimum operating frequency[1];
- S is the total received signal power over the bandwith in W;
- N is the total noise power over the bandwidth in W;
- S/N indicates the **Signal-to-Noise Ratio**, or SNR.

An ideal spacecraft TC system should have:

- A high data rate;
- A low error rate;
- A low (narrow) bandwidth;[2];
- A low power consumption;
- Simple hardware and software.

Of course, you can't have it all, because otherwise there'd be no point in spending two lectures on this subject.

[1] A signal is never transmitted using just one frequency, but always a range of frequencies is used, e.g. 10-12 GHz.

[2] A narrow bandwidth is preferable because the amount of available frequencies is not unlimited, thus it is expected that a spacecraft uses as few frequencies as possible (so as not to interfere with other peoples communications).

Now, guys working in telecoms have the strange habit of using decibels a lot; not just for sound intensity, but for literally everything. If you don't remember, the decibel is defined as

$$X\,[\mathrm{dB}] = 10 \cdot \log_{10}\left(\frac{X}{X_{ref}}\right) \tag{9.2}$$

Now, decibels have the advantage of allowing summation and subtraction rather multiplication and division:

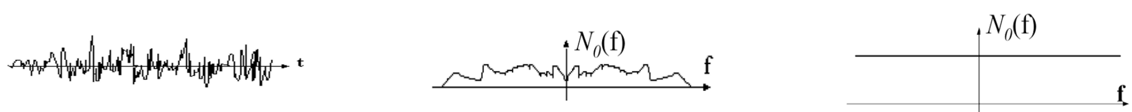
$$X = X_1 \cdot X_2 \rightarrow X\,[\mathrm{dB}] = 10 \cdot \log_{10}\left(\frac{X_1}{X_{ref}} \cdot \frac{X_2}{X_{ref}}\right) = 10 \cdot \log_{10}\left(\frac{X_1}{X_{ref}}\right) + 10 \cdot \log_{10}\left(\frac{X_2}{X_{ref}}\right) = X_1\,[\mathrm{dB}] + X_2\,[\mathrm{dB}]$$

I can't possibly see how this is worth introducing logarithms into your equations, but you know, at least less people know what you're doing.

X_{ref} can be just be 1 or a unit. For example, if $X_{ref} = 1\,\mathrm{W}$, we write dBW, and if we have $X_{ref} = 1\,\mathrm{mW}$, then we write dBm.

9.2.1 Noise

Electrical noise is generated by random thermal motions of atoms and electrons. Since we have electrical components on board, every signal has noise and each component in the transmission and reception chain contributes to noise. Noise can be represented in the time or frequency domain. In figure 9.1a, we see the noise signal as a function of time, whereas in figure 9.1b depicts the noise power spectral density (in W/Hz). The latter is more useful for our computations, as the area under the graph gives you the noise in watts. Now, we realize that 9.1b would be an absolutely awful integral to calculate, and hence we simplify it simply as depicted in figure 9.1c. From this graph, we see that the total noise is simply the noise density N_0 multiplied by the bandwidth B:



(a) Time domain. (b) Frequency domain. (c) Simplification of frequency domain.

Figure 9.1: Graphs of noise.

FORMULAS

$$N = N_0 \cdot B \tag{9.3}$$

Furthermore, the noise comes from thermal motions, and can, via the Boltzmann's constant (1.38×10^{-23} J/K) be related to the **system noise temperature**

FORMULAS

$$N_0 = k \cdot T_s \tag{9.4}$$

$$N = N_0 \cdot B = k \cdot T_s \cdot B \tag{9.5}$$

Which, if you think about it, should be familiar from Barry's lectures.

9.2.2 Amplifiers

An amplifier is a component that "amplifies" the power of all information passing through it: both signal and noise are amplified. Signal and noise power is multiplied by the amplifier power gain G. In an ideal amplifier, no additional noise is introduced and the Signal-to-Noise Ratio SNR remains constant. However, life isn't ideal and there is an additional noise produces by the amplifier:

$$N = G \cdot (k \cdot T + k \cdot T_n) = kG_1 \cdot (T + T_1)$$

Which will always be bigger than the noise input.

Of course, we can use multiple amplifiers in series. Since we have that the noise output of the first amplifier will equal

$$N_{o_1} = k \cdot G_1 \cdot (T + T_1)$$

We can do the same sort of trick for the second amplifier:

$$N_{o_2} = G_2 \cdot [k \cdot G_1 \cdot (T + T_1) + k \cdot T_2] = k \cdot G_1 \cdot G_2 \left[(T + T_1) + \frac{T_2}{G_1}\right]$$

Similar derivations can be done for the third amplifier, etc.
The initial noise and system noise are defined as follows:

Formulas

$$N_{o_2} = k \cdot G_1 \cdot G_2 \cdot \left[(\underbrace{T}_{\text{Initial noise}} + \underbrace{T_1) + \frac{T_2}{G_1}}_{\text{System noise}} \right] \tag{9.6}$$

Please note that if G_1 is very large, the influence of the second amplifier becomes very small. Hence, if G_1 is large enough, the first amplifier in a cascade is called a **low noise amplifier**, since it almost eliminates the effects of the noise produced by all other downstream elements.

9.2.3 Ground station elements

A typical ground station is made of receiving antenna, an amplifier and a telemetry receiver, connected through cables, as depicted in figure 9.2.

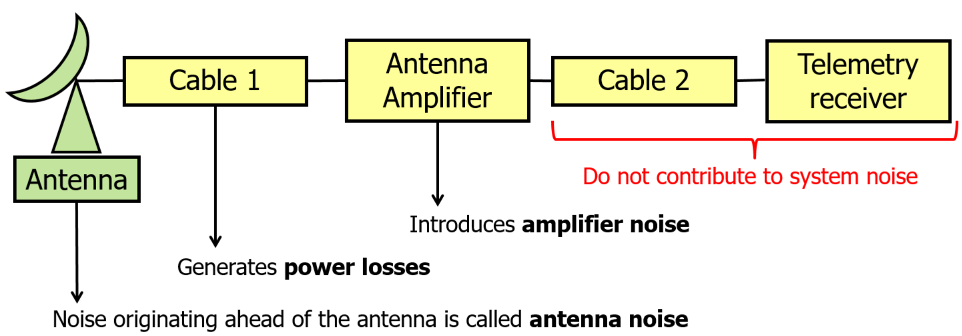


Figure 9.2: Ground station elements.

Just as before, if the antenna gain is high enough, it works as a low-noise amplifier[3]. Let's take a closer look at all three contributions to the system noise:

- The **antenna noise** generates ahead of the antenna aperture, and is therefore caused by sources external to the ground station, e.g. galactic noise, Sun or Earth noise (if Sun or Earth is in the antenna main beam or side lobe[4] The intensities of the noises are depicted in figure 9.3, where line number D

[3]Please note that an amplifier increases both the signal and the noise, and thus has no effect directly on the SNR. Its function in the system is thus only to work as low-noise amplifier to reduce the effects of noise produced by downstream components, not to improve signal quality.

[4]You may think but who'd ever point his signal at the Sun, we don't have any receiving antennas there lol. What is meant is when you are the receiving antenna on the Earth, and the signal comes from the same direction as the solar rays, as if you're looking into the Sun (this can also happen if you're a satellite that's currently at the dark side of the Earth, then you also looking into the Sun if you're far enough from the Earth). This distorts the signal. Similarly, if you're a spacecraft antenna and the receiving signal comes from Earth, the Earth radiation also distorts the signal a bit.

especially applies when the antenna beamwidth is narrow (it's based on a beamwidth of 0.5°). Sum the ones applicable up and you have your antenna noise temperature.

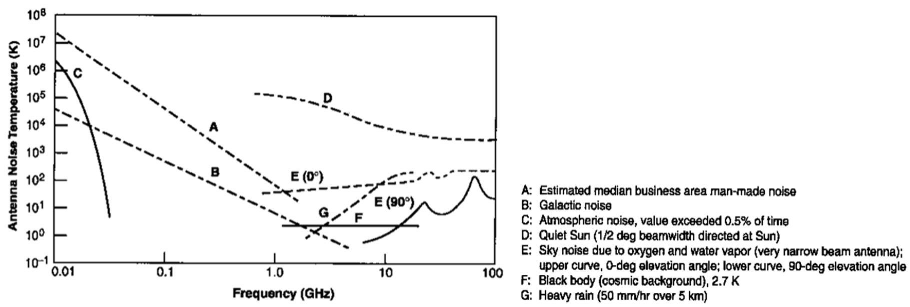


Figure 9.3: Antenna noise temperature.

- Some signal power is normally lost in connecting cables (especially long ones). The **loss factor** L of a cable is the ratio of output to input signal power (naturally $L \leq 1$). When a noise at a given temperature T_0 enters the cable, the output noise is T_0/L. The temperature of the noise introduced by the cable is therefore

FORMULAS

$$T_{cable} = \frac{T_0}{L} - T_0 = T_0 \cdot \left(\frac{1-L}{L}\right) \tag{9.7}$$

- The amplifier noise temperature T_n is usually expressed in terms of **noise figure** F, which is defined as the ratio of the output noise temperatue of the amplifier to the input noise temperature, when a noise at a given (reference) temperature T_0 enters the amplifier, i.e. $F = \frac{T_0+T_n}{T_0}$. From this, we decude:

FORMULAS

$$T_n = T_0 \cdot (F-1) \tag{9.8}$$

Note that the amplifier and cable usually use the same T_0, with a typical reference temperature being 290 K.

Summing this all together, we can write that the system noise temperature of a ground station equals:

FORMULAS

$$T_{sys} = T_{ant} + T_0 \cdot \left(\frac{1-L}{L}\right) + T_0 \cdot (F-1) \tag{9.9}$$

Please note that the contributions of the ground station elements to the system noise differ for different frequencies, as depicted in the table in figure 9.4.

9.3 *Link budget*

A signal faces great challenges on its way from the transmitter to the receiver, from line losses to transmission path losses. In this section, it will be the goal to establish a (preliminary) formula in which we can establish for each part of the road what the expected noise level will be. This is called the **link budget**.

Looking at figure 9.5, we can distinguish three phases of the transmission: the spacecraft part, the ground station part, and the part in between. Let's look at them one-by-one.

9.3.1 *Spacecraft transmission losses*

First, from the transmitter to the antenna, we naturally have a loss factor L_l. Furthermore, if the antenna transmits the signal isotropically in all directions, then the power flux density at a distance S from the

Noise Temperature	Frequency [GHz]					
	Downlink			Crosslink	Uplink	
	0.2	*2-12*	*20*	*60*	*0.2-20*	*40*
Antenna Noise [K]	150	25	100	20	290	290
(Cable Loss Factor)$^{-1}$ [dB]	0.5	0.5	0.5	0.5	0.5	0.5
Cable Loss Noise [K]	35	35	35	35	35	35
Receiver Noise Figure [dB]	0.5	1.0	3.0	5.0	3.0	4.0
Receiver Noise [K]	36	75	289	627	289	438
System Noise [K]	**221**	**135**	**424**	**682**	**614**	**763**

Figure 9.4: Noise values for typical ground stations.

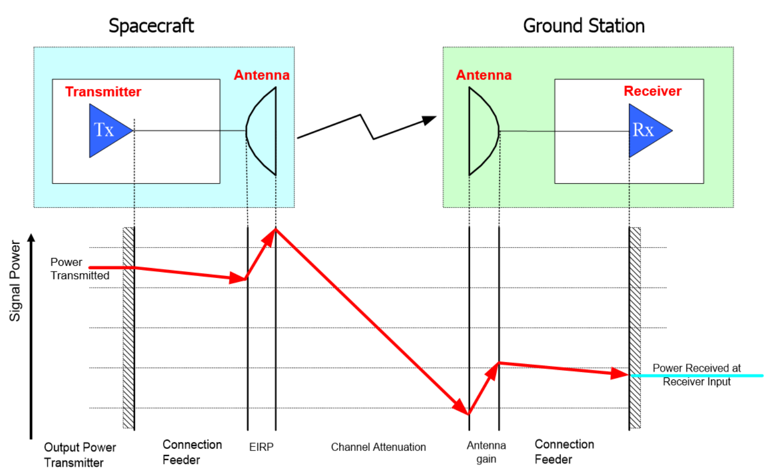


Figure 9.5: Link budget.

antenna is logically equal to[5]

$$W_f = \frac{P \cdot L_l}{4\pi S^2}$$

However, there's also an antenna gain you may have heard of before. The transmitting antenna gain G_t is defined as

FORMULAS

$$G_t = \frac{\text{Power flux density to center of antenna coverage (point A)}}{\text{Power flux density of an ideal isotropic antenna}} \tag{9.10}$$

See also figue 9.6.

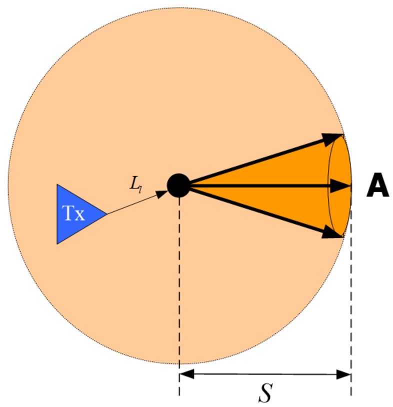


Figure 9.6: EIRP.

The transmitting antenna gain means the power is more concentrated, and hence increases the received flux:

$$W_f = \frac{P \cdot L_l \cdot G_t}{4\pi S^2} = \frac{EIRP}{4\pi S^2}$$

Where $EIRP = P \cdot L_l \cdot G_t$ is the **effective isotropic radiated power**.

9.3.2 *Transmission path losses*

While travelling from s/c to ground, the signal is attenuated by various external factors. The most important of these, the space loss, will be discussed later though. However, L_a is the loss factor that accounts for other transmission path losses, and hence:

$$W_f = \frac{P \cdot L_l \cdot G_t \cdot L_a}{4\pi S^2} = \frac{EIRP \cdot L_a}{4\pi S^2}$$

Values of L_a can be looked up in figure 9.7. Please note that these are in dB; that the atmospheric attenuation shows the absorption peaks due to water vapour (21 GHz) and oxygen (60 GHz, 130 GHz); that the rain attenuation depends on the elevation angle at which the spacecraft is seen by the ground station.

9.3.3 *Ground station losses*

Let C be the power received by the ground station antenna in Watts, with A_r being the physical apperture area of the antenna ($\pi D_r^2/4$) times the antenna efficiency η:

$$C = W_f \cdot A_r = \left(\frac{P \cdot L_l \cdot G_t \cdot L_a}{4\pi S^2}\right) \cdot \left(\frac{\pi \cdot D_r^2}{4}\eta\right) = \frac{P \cdot L_l \cdot G_t \cdot L_a \cdot D_r^2 \cdot \eta}{16 S^2}$$

[5]Just the net power divided by the area of a sphere with radius S.

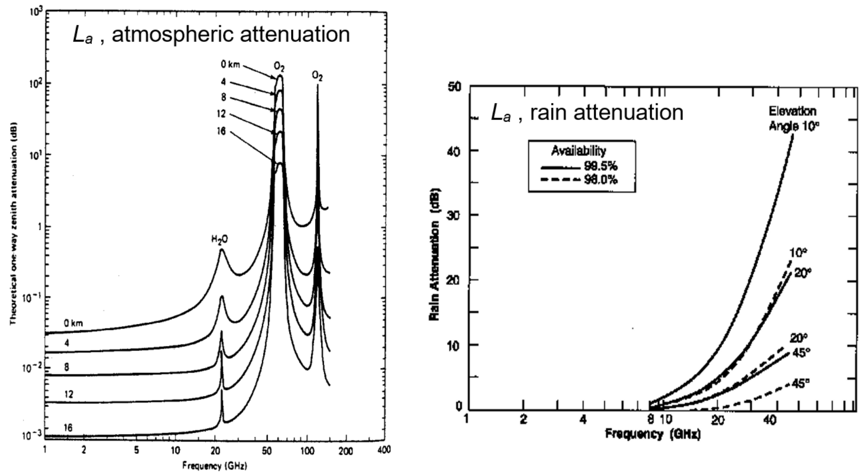


Figure 9.7: Atmospheric and rain attuenation.

Now, we have the receiving antenna gain G_r to consider. This is defined as the ratio of its effective aperture area A_r to the effective area of an isotropic antenna, equal be definition to $\frac{\lambda^2}{4\pi}$, where λ is the wavelength of the transmitted signal, with $\lambda = \frac{c}{f}$. We thus have:

$$\begin{aligned} G_r &= \frac{A_r}{\left(\frac{\lambda^2}{4\pi}\right)} = \frac{\left(\frac{\pi \cdot D_r^2}{4}\right)\eta}{\left(\frac{\lambda^2}{4\pi}\right)} = \frac{\pi^2 \cdot D_r^2}{\lambda^2}\eta \\ D_r^2\eta &= G_r\frac{\lambda^2}{\pi^2} \end{aligned}$$

Hence, we can write:

$$C = \frac{P \cdot L_l \cdot G_t \cdot L_a \cdot D_r^2 \cdot \eta}{16S^2} = \frac{P \cdot L_l \cdot G_t \cdot L_a \cdot G_r}{16S^2} \cdot \frac{\lambda^2}{\pi^2}$$

Now, we define the space loss to equal $\left(\frac{\lambda}{4\pi S}\right)^2$. Making appropriate substitution, we end up at

$$C = \frac{P \cdot L_l \cdot G_t \cdot L_a \cdot G_r}{16S^2} \cdot \frac{\lambda^2}{\pi^2} = P \cdot L_l \cdot G_t \cdot L_a \cdot G_r \cdot L_s$$

Two final losses: the antenna pointing loss L_{pr} accounts for pointing errors of the transmitting and receiving antenna, and can be calculated (in decibel) for each antenna as:

$$L_{pr}\,[\text{dB}] = -12 \cdot \left(\frac{e_t}{\alpha_{1/2}}\right)^2$$

Where e_t is the pointing offset angle in degree, and $\alpha_{1/2}$ the antenna half-power beamwidth in degree (which I mentioned before as well[6]). Finally, the reception feeder loss L_r accounts for any other losses in the reception chain. The received power thus becomes

$$C = P \cdot L_l \cdot G_t \cdot L_a \cdot G_r \cdot L_s \cdot L_{pr} \cdot L_r$$

Now, the received energy per bit is simply the total received power times the duration of a single bit of data:

FORMULAS

$$E_b = C \cdot \frac{1}{R} = \frac{P \cdot L_l \cdot G_t \cdot L_a \cdot G_r \cdot L_s \cdot L_{pr} \cdot L_r}{R} \tag{9.11}$$

[6] Just beware that whereas the accuracy for example needs to be 15°, the beamwidth itself is twice this value, so 30°.

We can now finally obtain the signal-to-noise ratio:

FORMULA: link equation

$$SNR = \frac{E_b}{N_0} = \frac{E_b}{k \cdot T_s} = \frac{P \cdot L_l \cdot G_t \cdot L_a \cdot G_r \cdot L_s \cdot L_{pr} \cdot L_r}{R \cdot k \cdot T_s} \tag{9.12}$$

Now, as some of the losses were presented in decibels, the equations becomes in decibels:

FORMULA: link equation

$$\begin{aligned} \frac{E_b}{N_0}\,[\mathrm{dB}] = &\quad 10 \cdot \log_{10} P + 10 \cdot \log_{10} L_l + 10 \cdot \log_{10} G_t + 10 \cdot \log_{10} L_a + 10 \cdot \log_{10} G_r \\ + &\quad 10 \cdot \log_{10} L_s + 10 \cdot \log_{10} L_{pr} + 228.6 - 10 \cdot \log_{10} R - 10 \cdot \log_{10} T_s \end{aligned} \tag{9.13}$$

The $+228.6$ is from $10 \cdot \log_{10} k = 10 \cdot \log_{10} 1.38 \cdot 10^{-23} = -228.6\,\mathrm{dBW}$.

10 Spacecraft telecommunications

10.1 Telecommunications technologies

The main processes a signal goes through are depicted in figure 10.1. Some abbreviations are Mod. (modulator), Ampl. (amplifier), LNA (low-noise amplifier), Demod. (demodulator) and FEC (forward error correction).

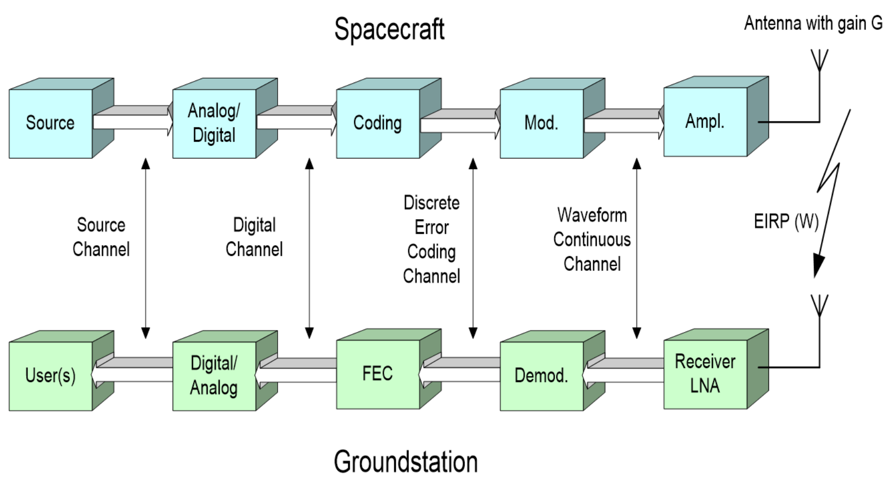


Figure 10.1: Path of a signal.

The main performance parameters of spacecraft antennas are:

- Peak gain (G_{peak}): maximum antenna gain (usually at beam center, remember the antenna pattern);
- Half-power beamwidth ($\alpha_{1/2}$: angle (in degree) where the gain is half the peak value (again, remember the antenna pattern);
- Antenna efficiency: ratio of effective aperture area to physical aperture area;
- Frequency limits: highest and lowest transmittable frequencies (in GHz);
- Polarization: variation of electric field vector direction over one wave period:
  - Linear polarization: electric field direction is constant;
  - Circular polarization: electric field direction rotates of a complete circle.
- Bandwidth B: expressed as percent bandwidth ($\%B$) or fractional bandwidth (B):

$$\%B = 2 \cdot \frac{f_H - f_L}{f_H + f_L}, \quad B = \frac{f_H}{f_L} \tag{10.1}$$

Where f_H is the upper band frequency and f_L the lower band frequency, in the same units of course.

10.1.1 Types of antennas

There are several types of antennas used in spacecraft. The first one is the famous parabolic antenna. These one allow for high gain, and can generate shaped beams by shaping the contour. They can be used as phased array antennas (coordinated phase shifts for constructive interference).

The gain can be calculated using

$$G_{peak} = \frac{\pi^2 \cdot D^2}{\lambda^2}\eta \tag{10.2}$$

$$G_{peak}\,[\text{dB}] = 20\log_{10} D + 20\log_{10} f + 17.8 \tag{10.3}$$

Please note, the second equation *only* applies for an efficiency of 55%. If it's something else, you need to alter the 17.8 Furthermore, the half-power beamwidth is equal to

$$\alpha_{1/2}\,[\mathrm{deg}] = \frac{21}{f \cdot D} \tag{10.4}$$

Additional characteristics are given in figure 10.2 (you don't have to know them by heart, of course).

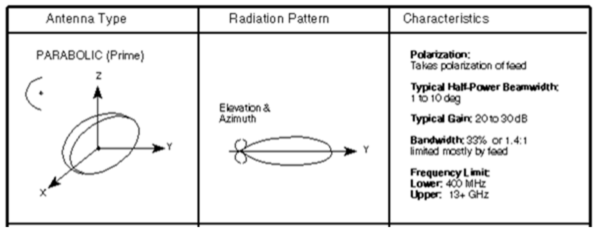


Figure 10.2: Characteristics of parabolic antennas.

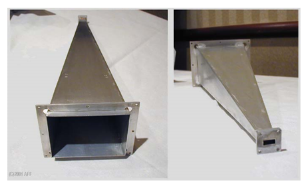


Figure 10.3: Horn antenna.

Now, onto the horn antennas, as depicted in figure 10.3. These are simple and directional antennas, and can provide global Earth coverage at high frequencies. The gain equals:

$$G_{peak} = \frac{\pi^2 \cdot D^2}{\lambda^2}\eta \tag{10.5}$$

$$G_{peak}\,[\mathrm{dB}] = 20\log_{10}\left(\frac{\pi D}{\lambda}\right) - 2.8 \tag{10.6}$$

Where the second equation once again only applies for $\eta = 0.52$. In addition D now is the diameter of the circle with the same area as the rectangular shape depicted in figure 10.3. Furthermore:

$$\alpha_{1/2}\,[\mathrm{deg}] = \frac{225}{\pi D/\lambda} \tag{10.7}$$

Where λ is the wavelength. Additional characteristics are given in figure 10.4 (you don't have to know them by heart, of course).

Finally, the helical antennas have low gain, and circular polarization only. It is used for global coverage at low frequency ($< 4\,\mathrm{GHz}$):

$$G_{peak}\,[\mathrm{dB}] = 10\log_{10}\left(\frac{\pi^2 D^2 L}{\lambda^3}\right) + 10.3 \tag{10.8}$$

With λ the wavelength, D the helix diameter and L the helix length. Again, this is only applicable for $\eta = 0.70$. Furthermore,

$$\alpha_{1/2}\,[^\circ] = \frac{52}{\sqrt{\pi^2 D^2 L/\lambda^3}} \tag{10.9}$$

Additional characteristics are given in figure 10.5.

Of course, other antenna shapes are also possible.

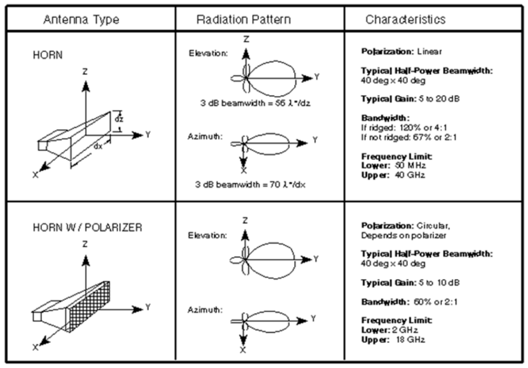


Figure 10.4: Characteristics of horn antennas.

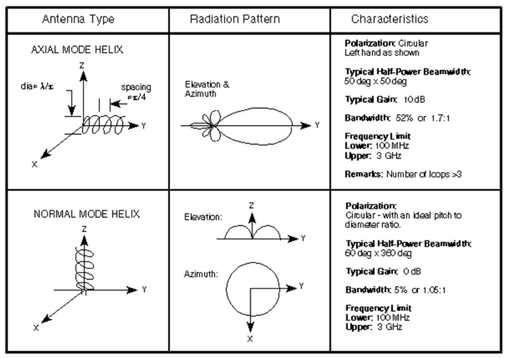


Figure 10.5: Characteristics of helical antennas.

10.1.2 *Frequency bands*

You may remember the frequency bands depicted in figure 10.6 <INSERT TABLE ON SLIDE 12>. S and X bands are the most commonly used. Also note how the frequencies for downlink and uplink are usually slightly different; this is to guarantee that the antenna can distinguish the two. Finally, from all these formulas on beamwidth, note how a larger antenna or higher frequency decrease the angle of the beamwidth.

10.1.3 *Modulation and demodulation*

Now, onto the modulators and demodulators. Typical spacecraft signals are characterized by low frequencies (in the order of Hz to kHz). Frequencies for communication however, are much higher. The signal needs a **Mo**dulation-**Dem**odulation process (Modem):

- Modulation: before transmission, the information contained in a low-frequency signal is encoded into a high-frequency carrier signal;
- Demodulation: in the receiver, the low-frequency signal is recovered (decoded) from the carrier signal[1].

Both the low-frequency and high-frequency signal can be analog or digital. This leads to the possibilities shown in figure 10.7. In this chapter, we'll focus only on modulators that have an analog signal as end-result (so only analog-analog and digital-analog). First, let's focus on some analog-analog converters.

10.1.4 *Coding*

Coding can be used for several purposes: encryption, compression or to add redundancy to the signal (so that errors can be detected and corrected). Now, two implementation options are possible for coding:

[1]Just to clarify: the signal in between the modulator and demodulator is called the carrier (as it carries the signal, as if it were be).

Frequency Band	Frequency Range (GHz)		Service
	Uplink	Downlink	
UHF	0.2 – 0.45	0.2 – 0.45	Military
L	1.635 – 1.66	1.535 – 1.56	Maritime/Nav Telephone
S	2.65 – 2.69	2.5 – 2.54	Broadcast, Telephone
C	5.9 – 6.4	3.7 – 4.2	Domestic, Comsat
X	7.9 – 8.4	7.25 – 7.75	Military, Comsat
Ku	14.0 – 14.5	12.5 – 12.75	Domestic, Comsat
Ka	27.5 – 31.0	17.7 – 19.7	Domestic, Comsat
SHF/EHF	43.5 – 45.5	19.7 – 20.7	Military, Comsat
SHF/EHF	49	38	Internet Data, Telephone, Trunking
V	~60		Satellite Crosslinks

Figure 10.6: Frequency bands.

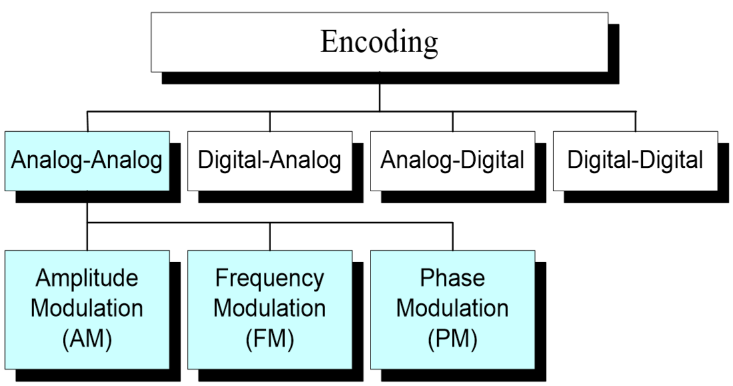


Figure 10.7: Types of modulation.

- Bi-directional: when an error is detected, the signal is transmitted again, until the message is without errors (obviously you only send the part of the message with the errors in it);
- Uni-directional: the detected errors are corrected by the receiver and the signal does not need to be re-transmitted. Whilst this may cause a wrong correction of the error, if your spacecraft for example goes to Neptune, this is much more convenient, as a signal takes like 1.5 hour to get from Neptune to Earth. Then sinding a signal back to ask for a retransmission, and then waiting for it to arrive back takes a really, really long time. An example of this uni-directional coding is Forward Error Coding (FEC).

The Bit Error Rate is the probability that a single bit of data is corrupted over a given period of time. For example, a BER of 10^{-3} means that 1 out of 10^3 bits is corrupted. Considering a frame made of N biys, the probability that it arrives with no errors equals

$$P_1 = (1 - BER)^N \tag{10.10}$$

The probability P_2 that the frame arrives with one or more errors is then simply

$$P_2 = 1 - P_1 = 1 - (1 - BER)^N \tag{10.11}$$

Allowed BER value is usually a key mission requirement. Typical values are in the order of 10^{-6}, but sometimes up to 10^{-9}.

10.1.5 A simple coding method: block coding

Now, assume a signal message made of N bits, so that the number of possible messages is $m = 2^N$. Each possible message is associated to a codeword of n bits. The code rate R is the ratio of useful bits of data generated by the code. A code is more efficient when R is closer to 1:

$$R = \frac{N}{n} = \frac{\log_2 M}{n} \tag{10.12}$$

For example, consider messages of 3 bits, and we use codewords of 6 bits for each of those messages. Then simply

$$R = \frac{3}{6} = \frac{1}{2} \tag{10.13}$$

So that for each 2 bits of data generated, only 1 is useful.

10.1.6 Decoding techniques

The received error may have one or more bits corrupted due to errors. For a given received signal, the decoder looks for the codeword that is most similar to it (the one that has most common bits to the received message). The actual message is then extracted from the codeword. So, for example, looking at the table in figure 10.8, where we have determined the codewords such that each codeword differs from others by at least 3 bits. So, for example, if we receive 000100, we can see this codeword as 000000, as that one is most similar to it, and hence our initial message was 000.

Other coding methods are for example the (X,Y) Reed-Solomon code, which has codewords of $6 \cdot X$ bits, out of which $6 \cdot Y$ bits are the actual message. For example, a (64,40) Reed-Solomon code has codewords of 384 bits, out of which 240 bits are the actual message. So, the code rate is simply $R = \frac{40}{64} = \frac{5}{8}$. Now, how low you want your BER wants to be affects your required SNR (this logically becomes higher when your requirement on the BER becomes more strict). Now, for a given BER, the required SNR for a given modulation technique can be found in figure 10.9. It is apparent that by using different techniques (e.g. BPSK), power can be saved compared to techniques such as FSK. This saved power is called coding gain and can be used to reduce transmitter power, antenna size, increase range, etc.

Message **u**	Codeword **x**
0 0 0	0 0 0 0 0 0
1 0 0	1 0 0 0 1 1
0 1 0	0 1 0 1 0 1
0 0 1	0 0 1 1 1 0
1 1 0	1 1 0 1 1 0
1 0 1	1 0 1 1 0 1
0 1 1	0 1 1 0 1 1
1 1 1	1 1 1 0 0 0

Figure 10.8: Coding of a message.

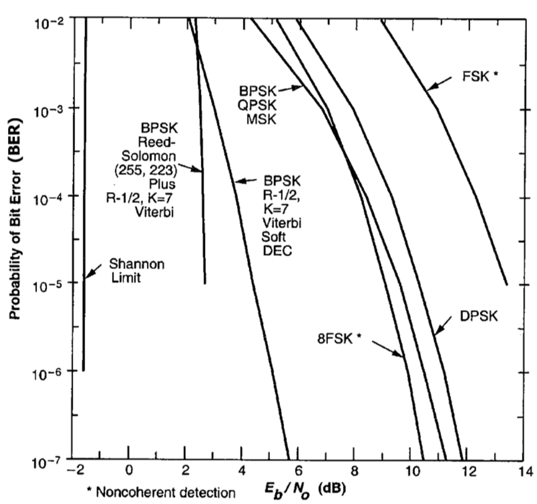


Figure 10.9: Coding gain.