

# Evaluation of probabilistic photometric redshift estimation approaches for LSST

S.J. Schmidt<sup>1</sup>, A.I. Malz<sup>2,3</sup>, J.Y.H. Soo<sup>4</sup>, I.A. Almosallam<sup>5,6</sup>, M. Brescia<sup>7</sup>, S. Cavaudi<sup>7,8</sup>, J. Cohen-Tanugi<sup>9</sup>, A.J. Connolly<sup>10</sup>, P.E. Freeman<sup>11</sup>, M.L. Graham<sup>10</sup>, K. Iyer<sup>12</sup>, M.J. Jarvis<sup>13,14</sup>, J.B. Kalmbach<sup>15</sup>, E. Kovacs<sup>16</sup>, A.B. Lee<sup>11</sup>, G. Longo<sup>8</sup>, C. Morrison<sup>10</sup>, J. Newman<sup>17</sup>, E. Nourbakhsh<sup>1</sup>, E. Nuss<sup>9</sup>, T. Pospisil<sup>11</sup>, H. Tranin<sup>9</sup>, R. Zhou<sup>17</sup>, R. Izbicki<sup>18,19</sup>

(LSST Dark Energy Science Collaboration)

<sup>1</sup> Department of Physics, University of California, One Shields Ave., Davis, CA, 95616, USA

<sup>2</sup> Center for Cosmology and Particle Physics, New York University, 726 Broadway, New York, 10003, USA

<sup>3</sup> Department of Physics, New York University, 726 Broadway, New York, 10003, USA

<sup>4</sup> Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, UK

<sup>5</sup> King Abdulaziz City for Science and Technology, Riyadh 11442, Saudi Arabia

<sup>6</sup> Information Engineering, Parks Road, Oxford, OX1 3PJ, UK

<sup>7</sup> INAF-Capodimonte Observatory, Salita Moiariello 16, I-80131, Napoli, Italy

<sup>8</sup> Department of Physics E. Pancini, University Federico II, via Cinthia 6, I-80126, Napoli, Italy

<sup>9</sup> Laboratoire Univers et Particules de Montpellier, Université de Montpellier, CNRS, Montpellier, France

<sup>10</sup> Department of Astronomy, University of Washington, Box 351580, U.W., Seattle WA 98195, USA

<sup>11</sup> Department of Statistics & Data Science, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, PA 15213, USA

<sup>12</sup> Department of Physics and Astronomy, Rutgers, The State University of New Jersey, 136 Frelinghuysen Road, Piscataway, NJ 08854-8019 USA

<sup>13</sup> Astrophysics, Department of Physics, University of Oxford, Denys Wilkinson Building, Keble Road, Oxford, OX1 3RH, UK

<sup>14</sup> Department of Physics and Astronomy, University of the Western Cape, Bellville 7535, South Africa

<sup>15</sup> Department of Physics, University of Washington, Box 351560, Seattle, WA 98195, USA

<sup>16</sup> Argonne National Laboratory, Lemont, IL 60439, USA

<sup>17</sup> Department of Physics and Astronomy and the Pittsburgh Particle Physics, Astrophysics and Cosmology Center (PITT PACC), University of Pittsburgh, Pittsburgh, PA 15260, USA

<sup>18</sup> Department of Statistics, Federal University of Sao Carlos, Sao Carlos, Brazil

<sup>19</sup> External collaborator

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## ABSTRACT

Many scientific investigations of photometric galaxy surveys require redshift estimates, whose uncertainty properties are best encapsulated by photometric redshift (photo- $z$ ) posterior probability distribution functions (PDFs). A plethora of photo- $z$  PDF estimation methodologies abound, producing discrepant results with no consensus on a preferred approach. We present the results of a comprehensive experiment comparing twelve photo- $z$  algorithms on mock data produced for the Large Synoptic Survey Telescope (LSST) Dark Energy Science Collaboration (DESC). By supplying perfect prior information, in the form of the complete template library and a representative training set as inputs to each code, we demonstrate the impact of the assumptions underlying each technique on the output photo- $z$  PDFs. In the absence of a notion of true, unbiased photo- $z$  PDFs, we evaluate and interpret multiple metrics of the ensemble properties of the derived photo- $z$  PDFs as well as traditional reductions to photo- $z$  point estimates. We report systematic biases and overall over/underbreadth of the photo- $z$  PDFs of many popular codes, which may indicate avenues for improvement in the algorithms or implementations. Furthermore, we raise attention to the limitations of established metrics for assessing photo- $z$  PDF accuracy; though we identify the conditional density estimate (CDE) loss as a promising metric of photo- $z$  PDF performance in the case where true redshifts are available but true photo- $z$  PDFs are not, we emphasize the need for science-specific performance metrics.

**Key words:** galaxies: distances and redshifts – galaxies: statistics – methods: statistical

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### 1 INTRODUCTION

The current and next generations of large-scale galaxy surveys, including the Dark Energy Survey (DES, Abbott et al. 2005), the Kilo-Degree Survey (KiDS, de Jong et al. 2013), Hyper Suprime-Cam Survey (HSC, Aihara et al. 2018a,b), Large Synoptic Survey Telescope (LSST, Abell et al. 2009), Euclid (Laureijs et al. 2011), and Wide-Field Infrared Survey Telescope (WFIRST, Green et al. 2012), present a paradigm shift from spectroscopic to photometric galaxy catalogues of substantially larger size at a cost of lacking complete spectroscopically confirmed redshifts ( $z$ ).

Effective astrophysical inference using the catalogues resulting from these ongoing and upcoming missions, however, necessitates accurate and precise photometric redshift (photo- $z$ ) estimation methodologies. As an example, in order for photo- $z$  systematics to not dominate the statistical noise floor of LSST’s main cosmological sample of  $\sim 10^7$  galaxies, the LSST Science Requirements Document (SRD)<sup>1</sup> specifies that individual galaxy photo- $zs$  must have root-mean-square error  $\sigma_z < 0.02(1+z)$ ,  $3\sigma$  catastrophic outlier rate below 10%, and bias below 0.003. Specific science cases may have their own requirements on photo- $z$  performance that exceed those of the survey as a whole. In that vein, the LSST Dark Energy Science Collaboration (LSST-DESC) developed a separate SRD (The LSST Dark Energy Science Collaboration et al. 2018) that conservatively forecasts the constraining power of five cosmological probes, leading to even more stringent requirements on photo- $z$  performance, including those defined in terms of tomographically binned subsamples populations rather than individual galaxies.

Though the standard has long been for each galaxy in a photometric catalogue to have a photo- $z$  point estimate and Gaussian error bar; however, as use in precision cosmology measurements became more prevalent there was a realization that point-estimate photo- $z$ ’s were inadequate (Mandelbaum et al. 2008). The nontrivial mapping between broad band fluxes and redshift renders a simplistic point estimate inadequate to quantify the uncertainty landscape by neglecting degenerate redshift solutions. Far from a hypothetical situation, this degeneracy is a real consequence of the same deep imaging that enables larger galaxy catalogue sizes. The lower luminosity and higher redshift populations captured by deeper imaging introduce major physical systematics to photo- $zs$ , among them the Lyman break/Balmer break degeneracy, that did not affect shallower large area surveys like the Sloan Digital Sky Survey (SDSS, York et al. 2000) and Two Micron All Sky Survey (2MASS, Skrutskie et al. 2006).

To fully characterize such physical degeneracies, photometric galaxy catalogue data releases, (e. g. Erben et al. (2013), de Jong et al. (2017)), provided a more informative photo- $z$  data product, the photo- $z$  probability density function (PDF), that describes the redshift probability, commonly denoted as  $p(z)$ , as a function of a galaxy’s redshift, conditioned on the observed photometry. Early template-based methods such as Fernández-Soto et al. (1999) approximated the likelihood of photometry conditioned on redshift with the relative  $\chi^2$  values of template spectra. Not long

after, Bayesian adaptations of template-based approaches such as Benítez (2000) combined the estimated likelihoods with a prior to yield a posterior PDF of redshift conditioned on photometry. While the first data-driven photo- $z$  algorithms yielded a point estimate, Firth et al. (2003) estimated a photo- $z$  PDF using a neural net with realizations scattered within the photometric errors.

There are numerous techniques for deriving photo- $z$  PDFs, yet no one method has been established as clearly superior. In fact, in the absence of simulated data drawn from known redshift distributions, the very concept of a “true PDF” for an individual galaxy is somewhat ill-defined, and we must instead rely on measures of ensemble behaviour to characterize PDF quality (see § 4 for further discussion). This caveat aside, quantitative comparisons of photo- $z$  methods have been made before; the Photo- $z$  Accuracy And Testing (PHAT, Hildebrandt et al. 2010) effort focused on photo- $z$  point estimates derived from many photometric bands. Rau et al. (2015) introduced a new method for improving photo- $z$  PDFs using an ordinal classification algorithm. DES compared several codes for photo- $z$  point estimates and a subset with photo- $z$  PDF information (Sánchez et al. 2014) and examined summary statistics of photo- $z$  PDFs for tomographically binned galaxy subsamples (Bonnell et al. 2016).

This paper is distinguished by its focus on the evaluation criteria for photo- $z$  PDFs and interpretation thereof. This is a key project of the Photometric Redshifts working group of the LSST-DESC that aims to perform a comprehensive sensitivity analysis of photo- $z$  PDF techniques in order to ultimately select those that will become part of the LSST pipelines. These plans are laid out in the Science Roadmap (SRM)<sup>2</sup>. In this initial study, we focus on evaluating the performance of photo- $z$  PDF codes using PDF-specific performance metrics in a controlled experiment with complete and representative prior information (template libraries and training sets) to set a baseline for subsequent investigations. This approach probes how each code considered exploits the information content of the data versus prior information from template libraries and training sets.

The outline of the paper is as follows: in § 2 we present the simulated data set; in § 3 we describe the current generation codes employed in the paper; in § 4 we discuss the interpretation of photo- $z$  PDFs in terms of metrics of accuracy; in § 5 we show our results and compare the performance of the codes; in § 6 we offer our conclusions and discuss future extensions of this work.

### 2 DATA

In order to test the current generation of photo- $z$  PDF codes, we employ an existing simulated galaxy catalogue, described in detail in Section 2.1. The experimental conditions shared among all codes are motivated by the LSST SRD requirements and implemented for machine learning and template-based photo- $z$  PDF codes according to the procedures of Sections 2.3.1 and 2.3.2 respectively.

<sup>1</sup> available at <https://docushare.lsstcorp.org/docushare/dsweb/Get/LPM-17>

<sup>2</sup> Available at: [http://lsst-desc.org/sites/default/files/DESC\\_SRMs\\_V1\\_1.pdf](http://lsst-desc.org/sites/default/files/DESC_SRMs_V1_1.pdf)

114 **2.1 The Buzzard-v1.0 simulation**

115 Our mock catalogue is derived from the BUZZARD-highres-  
 116 v1.0 catalogue (DeRose et al. 2019, Wechsler et al., in prep).  
 117 BUZZARD is built on a dark matter-only N-body simulation  
 118 of  $2048^3$  particles in a  $400 \text{ Mpc h}^{-1}$  box. The lightcone was  
 119 constructed from smoothing and interpolation between a set  
 120 of time snapshots. Dark matter halos were identified using  
 121 the Rockstar software package (Behroozi et al. 2013) and  
 122 then populated with galaxies with a stellar mass and ab-  
 123 solute  $r$ -band magnitude in the SDSS system determined  
 124 using a sub-halo abundance matching model constrained to  
 125 match both projected two-point galaxy clustering statistics  
 126 and an observed conditional stellar mass function (Reddick  
 127 et al. 2013).

128 To assign a spectrum to each galaxy, the Adding Den-  
 129 sity Dependent Spectral Energy Distributions (SEDs) pro-  
 130 cedure (ADDSEDS, deRose in prep.)<sup>3</sup> was used. ADDSEDS uses  
 131 a sample of  $\sim 5 \times 10^5$  galaxies from the magnitude-limited  
 132 SDSS Data Release 6 Value Added Galaxy Catalogue (Blan-  
 133 ton et al. 2005) to train an empirical relation between abso-  
 134 lute  $r$ -band magnitude, local galaxy density, and SED. Each  
 135 SDSS spectrum is parameterized by five weights correspond-  
 136 ing to a weighted sum of five basis SED components using  
 137 the k-correct software package<sup>4</sup> (Blanton & Roweis 2007).

138 Correlations between SED and galaxy environment  
 139 were included so as to preserve the colour-density relation of  
 140 galaxy environment. The distance to the spatially projected  
 141 fifth-nearest neighbour was used as a proxy for local density  
 142 in the SDSS training sample. For each simulated galaxy,  
 143 a galaxy with similar absolute  $r$ -band magnitude and local  
 144 galaxy density was chosen from the training set, and that  
 145 training galaxy's SED was assigned to the simulated galaxy.  
 146 In Section 2.1.1, we critique the realism of this mock data.

147 **2.1.1 Caveats**

148 By necessity, BUZZARD does not contain all of the compli-  
 149 cating factors present in real data, and here we discuss the  
 150 most pertinent ways that this limitation affects our exper-  
 151 iment. BUZZARD includes only galaxies, not stars of AGN.  
 152 The catalogue-based construction excludes image-level ef-  
 153 fects, such as deblending errors, photometric measurement  
 154 issues, contamination from sky background (Zodiacal light,  
 155 scattered light, etc.), lensing magnification, and Galactic  
 156 reddening.

157 The SEDs are five-component linear combinations of  
 158  $\sim 5 \times 10^5$  SDSS galaxies, so the sample contains only galax-  
 159 ies that resemble linear combinations of those for which  
 160 SDSS obtained spectra. The linear combination SEDs also  
 161 restrict the properties of the galaxy population to linear  
 162 combinations of the properties corresponding to five basis  
 163 templates, precluding the modeling of non-linear features  
 164 such as the full range of emission line fluxes relative to the  
 165 continuum. The only form of intrinsic dust reddening comes  
 166 from what is already present in the five basis SEDs via the  
 167 training set used to create the basis templates, and linear  
 168 combinations thereof do not span the full range of realistic  
 169 dust extinction observed in galaxy populations.

<sup>3</sup> <https://github.com/vipasu/addsed>

<sup>4</sup> <http://kcorrect.org>

170 While these idealized conditions limit the realism of our  
 171 mock data, they are irrelevant to the controlled experimental  
 172 conditions of this study, if anything assuring that differentia-  
 173 tion in the performance of the photo- $z$  PDF codes is due to  
 174 the inferential techniques rather than nuances in the data.

175 **2.2 LSST-like mock observations**

176 Given the SED, absolute  $r$ -band magnitude, and redshift,  
 177 we computed apparent magnitudes in the six LSST filter  
 178 passbands,  $ugrizy$ . We assigned magnitude errors in the six  
 179 bands using the simple model of Ivezić et al. (2008), assum-  
 180 ing achievement of the full 10-year depth, with a modifica-  
 181 tion of fiducial LSST total numbers of 30-second visits for  
 182 photometric error generation: we assume 60 visits in  $u$ -band,  
 183 80 visits in  $g$ -band, 180 visits in  $r$ -band, 180 visits in  $i$ -band,  
 184 160 visits in  $z$ -band, and 160 visits in  $y$ -band.

185 As a consequence of adding Gaussian-distributed pho-  
 186 tomatic errors, 2.0% of our galaxies exhibit a negative flux  
 187 in one or more bands, the vast majority of which are in the  
 188  $u$ -band. We deem such negative fluxes *non-detections* and  
 189 assign a placeholder magnitude of 99.0 in the catalogue to  
 190 indicate to the photo- $z$  PDF codes that such galaxies would  
 191 be “looked at but not seen” in multi-band forced photome-  
 192 try.

193 The full dataset thus covers 400 square degrees and con-  
 194 tains 238 million galaxies of redshift  $0 < z \leq 8.7$  down  
 195 to  $r = 29$ . Systematic inconsistencies with galaxy colors  
 196 at  $z > 2$  were observed, so the catalogue was limited to  
 197  $0 < z \leq 2.0$ . To obtain a catalogue matching the LSST  
 198 Gold Sample, we imposed an cut of  $i < 25.3$ , which gives a  
 199 signal-to-noise ratio  $\sim 30$  for most galaxies. In order for sta-  
 200 tistical errors to be subdominant to the systematic errors we  
 201 aim to probe, we further reduced the sample size to  $< 10^7$   
 202 galaxies by isolating  $\sim 16.8$  square degrees selected from five  
 203 separate spatial regions of the simulation. We refer to this  
 204 final set of galaxies as DC1, for the first LSST-DESC Data  
 205 Challenge.

206 **2.3 Shared prior information**

207 For the purpose of performing a controlled experiment that  
 208 compares photo- $z$  PDF codes on equal footing as a baseline  
 209 for a future sensitivity analysis, we take care to provide each  
 210 with maximally optimistic prior information. Redshift esti-  
 211 mation approaches built upon physical modeling and ma-  
 212 chine learning alike have a notion of prior information con-  
 213 sidered beyond the photometry of the data for which redshift  
 214 is to be constrained: that information is derived from a tem-  
 215 plate library for a model-based code and a training set for  
 216 a data-driven code. In this initial study, we seek to set a  
 217 baseline for a later comparison of the performance of photo-  
 218  $z$  PDF codes under incomplete and non-representative prior  
 219 information that will propagate differently in the space of  
 220 data-driven and model-based algorithms. However, for the  
 221 baseline case of perfect prior information, physical mod-  
 222 eling and machine learning codes can indeed be put on truly  
 223 equal footing. We outline the equivalent ways of providing  
 224 all codes perfect prior information below.

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### 225 2.3.1 Training and test set division

226 Following the findings of Bernstein & Huterer (2010), Masters et al. (2017) that only  $10^4$  spectra are necessary to  
 227 calibrate photo-zs to Stage IV requirements, we aimed to  
 228 set aside a randomly selected training set of  $3 - 5 \times 10^4$   
 229 galaxies,  $\sim 10\%$  of the full sample. After all cuts described  
 230 above, we designated the *DC1 training set* of 44 404 galaxies  
 231 for which observed photometry, true SEDs, and true red-  
 232 shifts would be provided to all codes and the blinded  
 233 *DC1 test set* of 399 356 galaxies for which photometry alone  
 234 would be provided to all codes and photo-z PDFs would be  
 235 requested. The LSST photometric filter transmission curves  
 236 were also considered public information that could be used  
 237 by any code.

### 288 2.3.2 Template library construction

289 We aimed to provide template-fitting codes with complete  
 290 yet manageable library of templates spanning the space of  
 291 SEDs of the DC1 galaxies. We constructed  $K = 100$  repre-  
 292 sentative templates from the  $\sim 5 \times 10^5$  SEDs of the SDSS  
 293 DR6 NYU-VAGC by using the five-dimensional vectors of  
 294 SED weight coefficients described above. After regularizing  
 295 the SED weight coefficients  $\in [0, 1]$ , we ran a simple K-  
 296 means clustering algorithm on the five-dimensional space  
 297 of regularized SED weight coefficients of the SDSS galaxy  
 298 sample. The resulting clusters were used to define Voronoi  
 299 cells in the space of weight coefficients, with centre pos-  
 300 i-  
 301 tions corresponding to weights for the k-correct SED com-  
 302 ponents, yielding the 100 *DC1 template set* to be provided  
 303 to all template-based codes. We did not, however, exclude  
 304 from consideration template-based codes that made modi-  
 305 fications in their use of these templates due to architecture  
 306 limitations (as opposed to knowledge of the experimental  
 307 conditions that could artificially boost the code's apparent  
 308 performance), with deviations noted in Section 3.

## 259 3 METHODS

260 Here we summarize the twelve photo-z PDF codes compared  
 261 in this study, summarized in Table 1, which include both es-  
 262 tablished and emerging approaches in template fitting and  
 263 machine learning. Though not exhaustive, this sample rep-  
 264 presents codes for which there was sufficient expertise within  
 265 the LSST-DESC Photometric Redshifts Working Group.  
 266 Some aspects of data treatment were left to the individual  
 267 code runners, for example, whether/how to split the avail-  
 268 able data with known redshifts into separate training and  
 269 validation sets. Another key difference is the treatment of  
 270 non-detections in one or more bands: some codes choose to  
 271 ignore a band, others replace the value with either an es-  
 272 timate for the detection limit, the mean of other values in  
 273 the training set, or another default value. There are varying  
 274 conventions among machine learning based codes for treat-  
 275 ment of non-detections, and no one prescription dominates  
 276 in the photo-z literature. The specific choices for each code  
 277 affect the results, and contribute to the implicit prior in-  
 278 fluencing their output. However, we remind the reader that  
 279 only 2.0 per cent of our sample has non-detections, almost  
 280 exclusively in the u-band, and thus should not dominate the

281 code performance differences. The authors welcome interest  
 282 from those outside LSST-DESC to have their codes assessed  
 283 in future investigations that build upon this one.

284 We describe the algorithms and implementations of the  
 285 model-based and data-driven codes in Sections 3.1 and 3.2  
 286 respectively, with a straw-person approach included in Sec-  
 287 tion 3.3.

### 288 3.1 Template-based Approaches

289 We test three publicly available and commonly used  
 290 template-based codes that share the standard physically mo-  
 291 tivated approach of calculating model fluxes for a set of tem-  
 292 plate SEDs on a grid of redshift values and evaluating a  $\chi^2$   
 293 merit function using the observed and model fluxes:

$$294 \chi^2(z, T, A) = \sum_i^{N_{\text{filt}}} \left( \frac{F_{\text{obs}}^i - A F_{\text{pred}}^i(T, z)}{\sigma_{\text{obs}}^i} \right)^2 \quad (1)$$

295 where  $A$  is a normalization factor,  $F_{\text{pred}}^i(T, z)$  is the flux  
 296 predicted for a template  $T$  at redshift  $z$ .  $F_{\text{obs}}^i$  is the observed  
 297 flux in a given band  $i$  and  $\sigma_{\text{obs}}^i$  is the observed flux error.  
 298  $N_{\text{filt}}$  is the total number of filters, in our case the six *ugrizy*  
 299 LSST filters. Specific implementation details of each code,  
 300 e. g. prior form and implementation, are described below.

#### 301 3.1.1 LePhare

302 Photometric Analysis for Redshift Estimate (LePhare<sup>5</sup>,  
 303 Arnouts et al. 1999; Ilbert et al. 2006) matches observed  
 304 colors with those predicted from a template set, which can  
 305 be semi-empirical or entirely synthetic, directly according  
 306 to the  $\chi^2$  form given in Equation 1. In words, the likeli-  
 307 hood is a sum of observed flux error  $\sigma_b^{\text{obs}}$ -weighted squared  
 308 differences between the observed flux  $F_b^{\text{obs}}$  and the normal-  
 309 ized predicted flux  $F_b^{\text{mod}}(T, z)$  in  $N_{\text{filt}}$  photometric filters  $b$ ,  
 310 which is the LSST *ugrizy* filters in this case. The reported  
 311 photo-z PDF is an arbitrary normalization of the likelihood  
 312 evaluated on the output redshift grid.

313 Here we use LePhare-v 2.2 with the DC1 template set  
 314 of Section 2.3.2.

#### 315 3.1.2 BPZ

316 Bayesian Photometric Redshift (BPZ<sup>6</sup>, Benítez 2000) de-  
 317 termines the likelihood  $p(C|z, T)$  of a galaxy's observed  
 318 colours  $C$  for a set of SED templates  $T$  at redshifts  
 319  $z$ . The BPZ likelihood is related to the  $\chi^2$  likelihood by  
 320  $p(C|z, T) \propto \exp[-\chi^2/2]$ . Given a Bayesian prior  $p(z, T|m_0)$   
 321 over apparent magnitude  $m_0$  and type  $T$ , and assuming  
 322 that the SED templates are spanning and exclusive, BPZ  
 323 constructs the redshift posterior  $p(z|C, m_0)$  by marginal-  
 324 izing over all SED templates with the form  $p(z|C, m_0) \propto$   
 325  $\sum_T p(C|z, T) p(z, T|m_0)$  (Eq. 3 from Benítez 2000), corre-  
 326 sponding to setting the parameter PROBS\_LITE=TRUE in the  
 327 BPZ parameter file. The BPZ prior is the product of an SED  
 328 template proportion that varies with apparent magnitude

5 <http://www.cfht.hawaii.edu/~arnouts/lephare.html>

6 <http://www.stsci.edu/~dcoe/BPZ/>

**Table 1.** List of photo-z PDF codes featured in this study

Published code	Type	Public source code
LePhare (Arnouts et al. 1999)	template fitting	<a href="http://www.cfht.hawaii.edu/~arnouts/lephare.html">http://www.cfht.hawaii.edu/~arnouts/lephare.html</a>
BPZ (Benítez 2000)	template fitting	<a href="http://www.stsci.edu/~dcoe/BPZ/">http://www.stsci.edu/~dcoe/BPZ/</a>
EAZY (Brammer et al. 2008)	template fitting	<a href="https://github.com/gbrammer/eazy-photoz">https://github.com/gbrammer/eazy-photoz</a>
ANNz2 (Sadeh et al. 2016)	machine learning	<a href="https://github.com/IftachSadeh/ANNZ">https://github.com/IftachSadeh/ANNZ</a>
FlexZBoost (Izbicki & Lee 2017)	machine learning	<a href="https://github.com/tospis/i/flexcode">https://github.com/tospis/i/flexcode</a> ; <a href="https://github.com/rizbicki/FlexCoDE">https://github.com/rizbicki/FlexCoDE</a>
GPz (Almosallam et al. 2016b)	machine learning	<a href="https://github.com/OxfordML/GPz">https://github.com/OxfordML/GPz</a>
METAPhoR (Cavuoti et al. 2017)	machine learning	<a href="http://dame.dsf.unina.it">http://dame.dsf.unina.it</a>
CMNN (Graham et al. 2018)	machine learning	N/A
SkyNet (Graff et al. 2014)	machine learning	<a href="http://ccforge.cse.rl.ac.uk/gf/project/skynet/">http://ccforge.cse.rl.ac.uk/gf/project/skynet/</a>
TPZ (Carrasco Kind & Brunner 2013)	machine learning	<a href="https://github.com/mgckind/MLZ">https://github.com/mgckind/MLZ</a>
Delight (Leistedt & Hogg 2017)	hybrid	<a href="https://github.com/ixkael/Delight">https://github.com/ixkael/Delight</a>
trainZ	machine learning	See Section 3.3

<sup>329</sup>  $p(T|m_0)$  and a prior  $p(z|T, m_0)$  over the expected redshift  
<sup>330</sup> as a function of apparent magnitude and SED template.

<sup>331</sup> Here we test BPZ-v 1.99.3 with the DC1 template set of  
<sup>332</sup> Section 2.3.2. To keep the number of free parameters man-  
<sup>333</sup> ageable, the DC1 template set is pre-sorted by the rest-frame  
<sup>334</sup>  $u - g$  colour and split into three broad classes of SED tem-  
<sup>335</sup> plate, equivalent to the E, Sp and Im/SB types in . The  
<sup>336</sup> Bayesian prior term  $p(T|m_0)$  was derived directly from the  
<sup>337</sup> DC1 training set, and the other term  $p(z|T, m_0)$  was cho-  
<sup>338</sup> sen to be the best fit for the eleven free parameters of the  
<sup>339</sup> functional form of Benítez (2000). We use template inter-  
<sup>340</sup> polation, creating two linearly interpolated templates between  
<sup>341</sup> each basis SED (sorted by rest-frame  $u - g$  colour) by set-  
<sup>342</sup> ting the parameter INTERP=2. Prior to running the code,  
<sup>343</sup> the non-detection placeholder magnitude was replaced with  
<sup>344</sup> an estimate of the one- $\sigma$  detection limit for the undetected  
<sup>345</sup> band as a proxy for a value close to the estimated sky noise  
<sup>346</sup> threshold.

### <sup>347</sup> 3.1.3 EAZY

<sup>348</sup> Easy and Accurate Photometric Redshifts from Yale (EAZY<sup>7</sup>,  
<sup>349</sup> Brammer et al. 2008) extends the basic  $\chi^2$  fit procedure that  
<sup>350</sup> defines template-fitting approaches. The algorithm models  
<sup>351</sup> the observed photometry with a linear combination of tem-  
<sup>352</sup> plate SEDs at each redshift. The best-fit SED is found by  
<sup>353</sup> simultaneously fitting one, two, or all of the templates via  $\chi^2$   
<sup>354</sup> minimization, which is distinct from marginalizing across all  
<sup>355</sup> templates. The minimized  $\chi^2$  likelihood at each redshift is  
<sup>356</sup> then combined with an apparent magnitude prior to obtain  
<sup>357</sup> the redshift posterior PDF. We note that the utilization of  
<sup>358</sup> the best-fit SED rather than a proper marginalization does  
<sup>359</sup> not lead to the correct posterior distribution, an implemen-  
<sup>360</sup> tation issue that has now been identified and will be ad-  
<sup>361</sup> dressed by the developers in the future. EAZY can account  
<sup>362</sup> for uncertainty in the template set by adding in quadrature  
<sup>363</sup> to the flux errors an empirically derived template error as a  
<sup>364</sup> function of redshift.

<sup>365</sup> The SED-independent apparent magnitude prior was  
<sup>366</sup> derived empirically from the DC1 training set. The EAZY ar-  
<sup>367</sup> chitecture cannot accept a template set other than the same  
<sup>368</sup> five basis templates employed by k-correct when construct-  
<sup>369</sup> ing the DC1 catalogue. However, EAZY does feature a flexible  
<sup>370</sup> all-templates mode, which fits the photometric data with

<sup>371</sup> a linear combination of the five basis templates. We set the  
<sup>372</sup> template error to zero since the same templates were in fact  
<sup>373</sup> used to produce the DC1 photometry.

## <sup>374</sup> 3.2 Machine Learning-based Approaches

We compared nine data-driven photo-z estimation ap-  
<sup>375</sup> proaches, eight of which are described in this section and one  
<sup>376</sup> of which is discussed in Section 3.3. Because the algorithms  
<sup>377</sup> differ more from one another and the techniques are rela-  
<sup>378</sup> tive newcomers to the astronomical literature, we provide  
<sup>379</sup> somewhat more detail about the implementations below.

### <sup>381</sup> 3.2.1 ANNz2

<sup>382</sup> ANNz2<sup>8</sup> (Sadeh et al. 2016) employs several machine learn-  
<sup>383</sup> ing algorithms, including artificial neural networks (ANN),  
<sup>384</sup> boosted decision tree, and k-nearest neighbour (KNN) re-  
<sup>385</sup> gression. In addition to accounting for errors on the input  
<sup>386</sup> photometry, ANNz2 uses the KNN-uncertainty estimate of  
<sup>387</sup> Oyaizu et al. (2008) to quantify uncertainty in the choice of  
<sup>388</sup> method over multiple runs. Using the Toolkit for Multivariate  
<sup>389</sup> Data Analysis with ROOT<sup>9</sup>, it can return the results  
<sup>390</sup> of running a single machine learning algorithm, a “best”  
<sup>391</sup> choice of the results from simultaneously running multiple  
<sup>392</sup> algorithms, or a combination of the results of multiple al-  
<sup>393</sup> gorithms weighted by their method uncertainties averaged  
<sup>394</sup> over multiple runs.

<sup>395</sup> In this study, we used ANNz2-v.2.0.4 to output only the  
<sup>396</sup> result of the ANN algorithm. Photo-z PDFs were produced  
<sup>397</sup> by running an ensemble of 5 ANNs with a 6 : 12 : 12 : 1  
<sup>398</sup> architecture corresponding to the 6  $ugrizy$  inputs, 2 hidden  
<sup>399</sup> layers with 12 nodes each, and 1 output of redshift. Each  
<sup>400</sup> of the five ANNs was trained with different random seeds  
<sup>401</sup> for the initialization of input parameters. Additionally, all  
<sup>402</sup> ANNs were trained on only a  $i \leq 25.3$  subsample of the DC1  
<sup>403</sup> training set, and half of the training set was reserved for  
<sup>404</sup> validation to prevent overfitting. Undetected galaxies were  
<sup>405</sup> excluded from the training set, and per-band non-detections  
<sup>406</sup> in the test set were replaced with the mean magnitude in  
<sup>407</sup> that band within the entire test set.

<sup>7</sup> <https://github.com/gbrammer/eazy-photoz>

<sup>8</sup> <https://github.com/IftachSadeh/ANNZ>

<sup>9</sup> <http://tmva.sourceforge.net/>

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### 408 3.2.2 Colour-Matched Nearest-Neighbours

409 The nearest-neighbours colour-matching photometric red-  
 410 shift estimator (CMNN, Graham et al. 2018) uses a training  
 411 set of galaxies with known redshifts that has equivalent or  
 412 better photometry than the test set in terms of quality and  
 413 filter coverage. For each galaxy in the test set, CMNN identifies  
 414 a colour-matched subset of training galaxies using a thresh-  
 415 old in the Mahalanobis distance  $D_M = \sum_j^{N_{\text{colours}}} (c_j^{\text{train}} -$   
 416  $c_j^{\text{test}})^2 / \delta c_{\text{test}}^2$  in the space of available colours  $c$ , with colour  
 417 measurement errors  $\delta c_{\text{test}}$  and  $N_{\text{colours}} = 5$  colors  $j$  defined  
 418 by the  $ugrizy$  filters, which defines the set of colour-matched  
 419 neighbours based on a value of the percent point function  
 420 (PPF). As an example, for  $N_{\text{fit}} = 5$  with PPF= 0.95, 95%  
 421 of all training galaxies consistent with the test galaxy will  
 422 have  $D_M < 11.07$ . Undetected bands are dropped, thereby  
 423 reducing the effective  $N_{\text{fit}}$  for that galaxy. The photo- $z$  PDF  
 424 of a given test set galaxy is the normalized distribution of  
 425 redshifts of its colour-matched subset of training set galax-  
 426 ies.

427 Here, we make two modifications to the implementation  
 428 of Graham et al. (2018) to comply with the controlled exper-  
 429 imental conditions. First, we do not impose non-detections  
 430 on galaxies fainter than the expected LSST 10-year limit-  
 431 ing magnitude or bright enough to saturate with LSST’s  
 432 CCDs, instead using all of the photometry for the DC1 test  
 433 and training sets. Second, we apply the initial colour cut  
 434 to the training set before calculating the Mahalanobis dis-  
 435 tance in order to accelerate processing and use a magnitude  
 436 pseudo-prior as in Graham et al. (2018), but for both we use  
 437 cut-off values corresponding to the DC1 training set galax-  
 438 ies’ colours and magnitudes.

439 We make an additional adaptation to enable the CMNN  
 440 algorithm to yield accurate photo- $z$  PDFs for all galaxies,  
 441 as the original Graham et al. (2018) algorithm is optimized  
 442 for photo- $z$  point estimates and is susceptible to less ac-  
 443 curate photo- $z$  PDFs for bright galaxies or those with few  
 444 matches in colour-space. We use PPF= 0.95 rather than  
 445 PPF= 0.68 to generate the subset of colour-matched train-  
 446 ing galaxies, whose redshifts are weighted by their inverse  
 447 Mahalanobis distances of the when composing the photo-  
 448  $z$  PDF rather than weighting all colour-matched training  
 449 galaxies equally. Additionally, when the number of colour-  
 450 matched training set galaxies is less than 20, the nearest 20  
 451 neighbours in color-space are used instead, and the output  
 452 photo- $z$  PDF is convolved with a Gaussian kernel of variance  
 453  $\sigma_{\text{train}}^2 (\text{PPF}_{20}/0.95)^2 - 1$  to account for the correspond-  
 454 ing growth of the effective PPF to include 20 neighbors.

### 455 3.2.3 FlexZBoost

456 **FlexZBoost**<sup>10</sup> (Izbicki & Lee 2017) is built on **FlexCode**, a  
 457 general-purpose methodology for converting any conditional  
 458 mean point estimator of  $z$  to a conditional density estima-  
 459 tor  $p(z|\mathbf{x}) \equiv f(z|\mathbf{x})$ , where  $\mathbf{x}$  here represents our photomet-  
 460 ric covariates and errors. **FlexZBoost** expands the unknown  
 461 function  $f(z|\mathbf{x}) = \sum_i \beta_i(\mathbf{x})\phi_i(z)$  using an orthonormal ba-  
 462 sis  $\{\phi_i(z)\}_i$ . By the orthogonality property, the expansion

463 coefficients  $\beta_i(\mathbf{x}) = \mathbb{E}[\phi_i(z)|\mathbf{x}] \equiv \int f(z|\mathbf{x})\phi_i(z)dz$  are thus  
 464 conditional means. The expectation value  $\mathbb{E}[\phi_i(z)|\mathbf{x}]$  of the  
 465 expansion coefficients conditioned on the data is equivalent  
 466 to the regression of the space of possible redshifts on the  
 467 space of possible photometry. Thus the expansion coeffi-  
 468 cients  $\beta_i(\mathbf{x})$  can be estimated from the data via regression  
 469 to yield the conditional density estimate  $\hat{f}(z|\mathbf{x})$ .

470 In this paper, we used **xgboost** (Chen & Guestrin  
 471 2016) for the regression; it should however be noted that  
 472 **FlexCode-RF**<sup>10</sup>, based on Random Forests, generally per-  
 473 forms better for smaller datasets. As our basis  $\phi_i(z)$ , we  
 474 choose a standard Fourier basis. The two tuning parame-  
 475 ters in our photo- $z$  PDF estimate are the number  $I$  of terms  
 476 in the series expansion and an exponent  $\alpha$  that we use to  
 477 sharpen the computed density estimates  $\tilde{f}(z|\mathbf{x}) \propto \hat{f}(z|\mathbf{x})^\alpha$ .  
 478 Both  $I$  and  $\alpha$  were chosen in an automated way by mini-  
 479 mizing the weighted  $L_2$ -loss function (Eq. 5 in Izbicki & Lee  
 480 2017) on a validation set comprised of a randomly selected  
 481 15% of the DC1 training set. While **FlexCode**’s lossless na-  
 482 tive encoding stores each photo- $z$  PDF using the basis co-  
 483 efficients  $\beta_i(\mathbf{x})$ , we discretized the final estimates into 200  
 484 linearly-spaced redshift bins  $0 < z < 2$  to match the consis-  
 485 tent output format of the experimental conditions.

### 486 3.2.4 GPz

487 **GPz**<sup>11</sup> (Almosallam et al. 2016a,b) is a sparse Gaussian pro-  
 488 cess based code, a scalable approximation of full Gaus-  
 489 sian Processes (Rasmussen & Williams 2006), that pro-  
 490 duces input-dependent variance estimates corresponding to  
 491 heteroscedastic noise. The model assumes a Gaussian pos-  
 492 terior probability  $p(z|\mathbf{x}) = \mathcal{N}(z|\mu(\mathbf{x}), \sigma(\mathbf{x})^2)$  of the out-  
 493 put redshift  $z$  given the input photometry  $\mathbf{x}$ . The mean  
 494  $\mu(\mathbf{x})$  and the variance  $\sigma(\mathbf{x})^2$  are modeled as functions  
 495  $f(\mathbf{x}) = \sum_{i=1}^m w_i \phi_i(\mathbf{x})$  linear combinations of  $m$  basis func-  
 496 tions  $\{\phi_i(\mathbf{x})\}_{i=1}^m$  with associated weights  $\{w_i\}_{i=1}^m$ . The de-  
 497 tails on how to learn the parameters of the model and the  
 498 hyper-parameters of the basis functions are described in Al-  
 499 mosallam et al. (2016b). **GPz**’s variance estimate is composed  
 500 of a model uncertainty term corresponding to sparsity of the  
 501 training set photometry and a noise uncertainty term en-  
 502 compassing noisy photometric observations, enabling quan-  
 503 tification of any need for more representative or more precise  
 504 training samples. **GPz** may also weight training set samples  
 505 by importance according to  $|z_{\text{spec}} - z_{\text{phot}}|/(1+z_{\text{spec}})$  to min-  
 506 imize the normalized photo- $z$  point estimate error, however,  
 507 this function may be adapted to photo- $z$  PDFs, pressuring  
 508 the model to dedicate more resources to test set galaxies  
 509 that are not well-represented in the training set.

510 To smooth the long tail in the distribution of magni-  
 511 tude errors, we use the log of the magnitude errors, im-  
 512 proving numerical stability and eliminating the need for  
 513 constraints on the optimization process. Unobserved mag-  
 514 nitudes  $x_u = \mu_u + \Sigma_{uo}\Sigma_{oo}^{-1}(x_o - \mu_o)$  were imputed from  
 515 observed magnitudes  $x_o$  and the training set mean  $\mu$  and  
 516 covariance  $\Sigma$  using a linear model. This is the optimal ex-  
 517 pected value of the unobserved variables given the observed  
 518 ones under the assumption that the distribution is jointly  
 519 Gaussian; note that this reduces to a simple average if the

10 <https://github.com/tpospisi/flexcode>;  
<https://github.com/rizbicki/FlexCoDE>

11 <https://github.com/OxfordML/GPz>

520 covariates are independent with  $\Sigma_{uo} = 0$ . We reserved for  
 521 validation 20% of the training set and used the Variable  
 522 Covariance option in GPz with 200 basis functions (see Al-  
 523 mosallam et al. (2016b) for details), neglecting to apply cost-  
 524 sensitive learning options.

### 525 3.2.5 METAPhOr

526 Machine-learning Estimation Tool for Accurate Photometric  
 527 Redshifts (METAPhOr<sup>12</sup>, Cavuoti et al. 2017) is based on  
 528 the Multi Layer Perceptron with Quasi Newton Algorithm  
 529 (MLPQNA) with the least square error model and Tikhonov  
 530  $L_2$ -norm regularization (Hofmann & Mathé 2018). Photo-z  
 531 PDFs are generated by running  $N$  trainings on the same  
 532 training set, or  $M$  trainings on  $M$  different random sam-  
 533 plings of the training set. Upon regression of the test set,  
 534 the photometry  $m_{ij}$  of each test set galaxy  $j$  in filter  $i$  is  
 535 perturbed according to  $m'_{ij} = m_{ij} + \alpha_i F_{ij} \epsilon$  in terms of  
 536 the standard normal random variable  $\epsilon \sim \mathcal{N}(0, 1)$ , a mul-  
 537 tiplicative constant  $\alpha_i$  permitting accommodation of multi-  
 538 survey photometry, and a bimodal function  $F_{ij}$  composed of  
 539 a polynomial fit of the mean magnitude errors on the binned  
 540 bands plus a constant term representing the threshold be-  
 541 low which the polynomial's noise contribution is negligible  
 542 (Brescia et al. 2018).

543 In this work, we used a hierarchical KNN to replace  
 544 non-detections with values based on their neighbors. The  
 545 usual cross-validation of redshift estimates and PDFs was  
 546 also omitted for this study.

### 547 3.2.6 SkyNet

548 SkyNet<sup>13</sup> (Graff et al. 2014) employs a neural network based  
 549 on a second order conjugate gradient optimization scheme  
 550 (see Graff et al. 2014, for further details). The neural net-  
 551 work is configured as a standard multilayer perceptron with  
 552 three hidden layers and one input layer with 12 nodes cor-  
 553 responding to the 6 photometric magnitudes and their mea-  
 554 surement errors. We use SkyNet as a regressor for photo-z  
 555 point estimation and as a classifier for photo-z PDF estima-  
 556 tion.

557 The regressor used a standard  $\chi^2$  error function with a  
 558 single linear node as the output layer and 10 nodes with a  
 559 tanh activation function for each hidden layer. The classifier  
 560 used a cross-entropy error function with a 20:40:40 node (all  
 561 rectified linear units) architecture for each hidden layer and  
 562 an output layer of 200 nodes corresponding to 200 bins for  
 563 the PDF, with a softmax activation function to enforce the  
 564 normalization condition that the probabilities sum to unity.  
 565 While previous implementations of the code (see Appendix  
 566 C.3 of Sánchez et al. 2014; Bonnett 2015) implement a  
 567 sliding bin smoothing, no such procedure was used in this  
 568 study.

569 We pre-whitened the data by pegging the magnitudes  
 570 to (45,45,40,35,42,42) and errors to (20,20,10,5,15,15) for  
 571 *ugrizy* filters, respectively. To avoid over-fitting, 30% of the  
 572 training set was reserved for validation, and training was  
 573 halted as soon as the error rate began to increase on the

574 validation set. The weights were randomly initialized based  
 575 on normal sampling.

### 576 3.2.7 TPZ

577 Trees for Photo-z(TPZ<sup>14</sup>, Carrasco Kind & Brunner 2013;  
 578 Carrasco Kind & Brunner 2014) uses prediction trees and  
 579 random forest techniques to estimate photo-z PDFs. TPZ re-  
 580 cursively splits the training set into branch pairs based on  
 581 maximizing information gain among a random subsample of  
 582 features, to minimize correlation between the trees, termi-  
 583 nating only when a newly created leaf meets a criterion, such  
 584 as a leaf size minimum or a variance threshold. The regions  
 585 in each terminal leaf node correspond to a subsample of the  
 586 training set with similar properties. Bootstrap samples from  
 587 the training set photometry and errors are used to build a  
 588 set of prediction trees.

589 To run TPZ, we replaced non-detections with an approxi-  
 590 mation of the  $1\sigma$  detection threshold based on the error fore-  
 591 cast of the 10-year LSST data, i. e.  $dm = 2.5 \log(1 + N/S)$   
 592 where  $dm \sim 0.7526$  magnitudes for  $N/S = 1$ . We cali-  
 593 brated TPZ with the Out-of-Bag cross-validation technique  
 594 (Breiman et al. 1984; Carrasco Kind & Brunner 2013) to  
 595 evaluate its predictive validity and determine the relative  
 596 importance of the different input attributes. We grew 100  
 597 trees to a minimum leaf size of 5 using the *ugri* magnitudes,  
 598 all  $u - g, g - r, r - i, i - z, z - y$  colours, and the associated  
 599 errors, as the  $z$  and  $y$  magnitudes did not show significant  
 600 correlation with the redshift in our cross-validation. We par-  
 601 titioned our redshift space into 200 bins and smoothed each  
 602 individual PDF with a smoothing scale of twice the bin size.

### 603 3.2.8 Delight

604 Delight<sup>15</sup> (Leistedt & Hogg 2017) is a hybrid technique that  
 605 infers photo-zs with a data-driven model of latent SEDs and  
 606 a physical model of photometric fluxes as a function of red-  
 607 shift. Generally, machine learning methods rely on represen-  
 608 tative training data with shared photometric filters, while  
 609 template based methods rely on a complete library of tem-  
 610 plates based on physical models constructed. Delight aims  
 611 to take the best aspects of both approaches by construct-  
 612 ing a large collection of latent SED templates (or physical  
 613 flux-redshift models) from training data, with a template  
 614 SED library as a guide to the learning of the model, thereby  
 615 circumventing the machine learning prerequisite of represen-  
 616 tative training data in the same photometric bands and the  
 617 template fitting requirement of detailed galaxy SED models.  
 618 It models noisy observed flux  $\hat{\mathbf{F}} = \mathbf{F} + \mathbf{F}_b$  as a sum of a noise-  
 619 less flux plus a Gaussian processes  $\mathbf{F}_b \sim \mathcal{GP}(\mu^F, k^F)$  with  
 620 zero mean function  $\mu^F$  and a physically motivated kernel  $k^F$   
 621 that induces realistic correlations in flux-redshift space.

622 From a template-fitting perspective, each test set galaxy  
 623 has a posterior  $p(z|\hat{\mathbf{F}}) \approx \sum_i p(\hat{\mathbf{F}}|z, T_i)p(z|T_i)p(T_i)$  of red-  
 624 shift  $z$  conditioned on noisy flux  $\hat{\mathbf{F}}$ , where  $p(z|T_i)p(T_i)$  cap-  
 625 tures prior information about the redshift distributions and  
 626 abundances of the galaxy templates  $T_i$ . As in traditional  
 627 template fitting, each likelihood  $p(\hat{\mathbf{F}}|\mathbf{F})$  relates the noisy flux

12 <http://dame.dsfs.unina.it>

13 <http://ccpforge.cse.rl.ac.uk/gf/project/skynet/>

14 <https://github.com/mgckind/MLZ>

15 <https://github.com/ixkael/Delight>

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$\hat{\mathbf{F}}$  with the noiseless flux  $\mathbf{F}$  predicted by the model of a linear combination of templates, carefully constructed to account for model uncertainties and different normalization of the same SED, plus the Gaussian process term.

The machine learning approach appears in the inclusion of a pairwise comparison term  $p(\mathbf{F}|z, z_j, \hat{\mathbf{F}}_j)$  for the prediction of model flux  $\mathbf{F}$  at a model redshift  $z$  with respect to training set galaxy  $j$  with redshift  $z_j$  and observed flux  $\hat{\mathbf{F}}_j$ . Thus the photo- $z$  posterior  $p(\hat{\mathbf{F}}|z, T_i) = \int p(\hat{\mathbf{F}}|\mathbf{F})p(\mathbf{F}|z, z_j, \hat{\mathbf{F}}_j)d\mathbf{F}$  may be interpreted as the probability that the training and the target galaxies have the same SED at different redshifts. The flux prediction  $p(\mathbf{F}|z, z_j, \hat{\mathbf{F}}_j)$  of the training galaxy at redshift  $z$  is modeled via the Gaussian process described above; more detail is provided in Leistedt & Hogg (2017).

In this study, the default settings of `Delight` were used, with the exception that the PDF bins were set to be linearly-spaced rather than logarithmic. The Gaussian process was trained using the full DC1 training set. We used the full DC1 template set with a flat prior in magnitude and SED type. Photometric uncertainties from the inputs are propagated into the code, while non-detections for each band are set to the mean of the respective bands.

### 3.3 trainZ: a pathological photo- $z$ estimator

We also consider a pathological photo- $z$  PDF estimation method, dubbed `trainZ`, which assigns each test set galaxy a photo- $z$  PDF equal to the normalized redshift distribution  $N(z)$  of the training set, according to

$$p(z|\{z_j\}) \equiv \frac{1}{N_{\text{train}}} \sum_{i=1}^{N_{\text{train}}} \begin{cases} 1 & \text{if } z_k \leq z_i < z_{k+1} \\ 0 & \text{otherwise} \end{cases}. \quad (2)$$

Unlike the other methods, the `trainZ` estimator is *independent of the photometric data*, effectively performing a KNN procedure with  $k = N_{\text{train}}$ .

Though `trainZ` is strongly vulnerable to a nonrepresentative training set, it should optimize performance metrics probing the ensemble properties of the galaxy sample, modulo Poisson error due to small sample size, as the training set and test set are drawn from the same underlying population. We will demonstrate its performance under the metrics of Section 4 and discuss it as an illustrative experimental control case in Section 6.1 to highlight the limitations of our evaluation criteria for photo- $z$  PDFs.

## 4 ANALYSIS

The goal of this study is to evaluate the degree to which photo- $z$  PDFs of each method can be trusted for a generic analysis. The overloaded “ $p(z)$ ” is a widespread abuse of notation that obfuscates this goal, so we will dedicate some attention to breaking it apart. Galaxies have redshifts  $z$  and photometric data  $d$  drawn from a joint probability space  $p(z, d)$  in nature. As a result, each observed galaxy  $i$  has a true posterior photo- $z$  PDF  $p(z|d_i)$  as well as a true likelihood  $p(d|z_i)$ . There are a number of metrics that can be used to test the accuracy of a photo- $z$  posterior as an estimator of a true photo- $z$  posterior if the true photo- $z$  PDF is known. However, the true photo- $z$  PDF is in general not accessible

unless the photometry is in fact drawn from the true photo- $z$  likelihoods, which is not how most existing datasets have been constructed. Instead, data is often taken from empirical observations, or generated from mock catalogs derived from N-body simulations of the Universe (as is the case in this study). In these cases there is no “true PDF” for an individual object given its observed properties, as for real data the true redshift distribution is unknown, and for simulations the photometric data is usually generated by empirical or semi-analytic relations tied to the underlying dark matter distribution and local environment. As a consequence, most measures of PDF fidelity will need to rely on ensemble properties of the similar objects, or the entire sample. We do discuss a metric that can measure performance of individual PDF quality, the conditional density estimate loss, in § 4.2. We also mention a potential future project addressing this issue in § 6.2.

Before describing the metrics appropriate to the DC1 data set, we outline the philosophy behind our choices. A photo- $z$  PDF estimator derived by method  $H$  must be understood as a posterior probability distribution

$$\hat{p}_i^H(z) \equiv p(z|d_i, I_D, I_H), \quad (3)$$

conditioned not only on the photometric data  $d_i$  for that galaxy but also on parameters encompassing a number of things that will differ depending on the method  $H$  used to produce it, namely the often implicit assumptions  $I_H$  necessary for the method to be valid and any inputs  $I_D$  it takes as prior information, such as a template library or training set. Because of this, direct comparison of photo- $z$  PDFs produced by different methods is in some sense impossible; even if they share the same external prior information  $I_D$ , by definition they cannot be conditioned on the same assumptions  $I_H$ , otherwise they would not be distinct methods at all. We call  $I_H$  the *implicit prior* specific to the method, though some aspects of its nature may be discerned.

In this study, we isolate the effect of differences in prior information  $I_H$  specific to each method by using a single training set  $I_D^{\text{ML}}$  for all machine learning-based codes and a single template library  $I_D^T$  for all template-based codes. These sets of prior information are carefully constructed to be representative and complete, so we have  $I_D \equiv I_D^{\text{ML}} \equiv I_D^T$  for every method  $H$ . Under this assumption, a ratio of posteriors of codes is in effect a ratio of the implicit posteriors  $p(z|d_i, I_H')$  since the external prior information  $I_D$  is present in the numerator and denominator. Thus comparisons of  $\hat{p}_i^H(z)$  isolate the effect of the method used to obtain the estimator, which should enable interpretation of the differences between estimated PDFs in terms of the specifics of the method implementations.

The exact implementation of the metrics theoretically depends on the parametrization of the photo- $z$  PDFs, which may differ across codes and can affect the precision of the estimator (Malz et al. 2018). Even considering a single method under the same parametrization, such as the 200-bin  $0 < z < 2$  piecewise constant function used here, the exact bin definitions will affect the result. The piecewise constant format is chosen because of its established presence in the literature, and the choice of 200 bins was motivated by the approximate number of columns expected to be available for

storage of photo- $z$  PDFs for the final LSST Project tables.<sup>16</sup> We will discuss the choice of photo- $z$  PDF parameterization further in Section 6.

This analysis is conducted using the `qp`<sup>17</sup> software package (Malz et al. 2018) for manipulating and calculating metrics of univariate PDFs. We present the metrics of photo- $z$  PDFs that address our goals in the sections below. Section 4.1 outlines aggregate metrics of a catalogue of photo- $z$  PDFs, and Section 4.2 presents a metric of individual photo- $z$  PDFs in the absence of true photo- $z$  PDFs. Though the outmoded practices should not be encouraged, those seeking a connection to previous comparison studies will find metrics of redshift point estimate reductions of photo- $z$  PDFs in Appendix B and metrics of a science-specific summary statistics heuristically derived from photo- $z$  PDFs in Appendix A.

#### 4.1 Metrics of photo- $z$ PDF ensembles

Because LSST’s photo- $z$  PDFs will be used for many scientific applications, some of which require accuracy of each individual catalog entry, we consider several metrics that probe the population-level performance of the photo- $z$  PDFs. Because we have the true redshifts but not true photo- $z$  PDFs for comparison, we remind the reader of the Cumulative Distribution Function (CDF)

$$\text{CDF}[f, q] \equiv \int_{-\infty}^q f(z) dz, \quad (4)$$

of a generic univariate PDF  $f(z)$ , which is used as the basis for several of our metrics.

A quantile of a distribution is the value  $q$  at which the CDF of the distribution is equal to  $Q$ ; percentiles and quartiles are familiar examples of linearly spaced sets of 100 and 4 quantiles, respectively. The quantile-quantile (QQ) plot serves as a graphical visualization for comparing two distributions, where the quantiles of one distribution are plotted against the quantiles of the other distribution, providing an easy way to qualitatively assess the consistency between an estimated distribution and a true distribution. The closer the QQ plot is to diagonal, the closer the match between the distributions.

The probability integral transform (PIT)

$$\text{PIT} \equiv \text{CDF}[\hat{p}, z_{\text{true}}] \quad (5)$$

is the CDF of a photo- $z$  PDF evaluated at its true redshift, and the distribution of PIT values probes the average accuracy of the photo- $z$  PDFs of an ensemble of galaxies. The distribution of PIT values is effectively the derivative of the QQ plot. A catalogue of accurate photo- $z$  PDFs should have a PIT distribution that is uniform  $U(0, 1)$ , and deviations from flatness are interpretable: overly broad photo- $z$  PDFs induce underrepresentation of the lowest and highest PIT values, whereas overly narrow photo- $z$  PDFs induce overrepresentation of the lowest and highest PIT values. Catastrophic outliers with a true redshift outside the support of its photo- $z$  PDF have  $\text{PIT} \approx 0$  or  $\text{PIT} \approx 1$ .

The PIT distribution has been used to quantify the performance of photo- $z$  PDF methods in the past (e. g. Bordoloi et al. 2010; Polsterer et al. 2016; Tanaka et al. 2018). Tanaka et al. (2018) use the histogram of PIT values as a diagnostic indicator of overall code performance, while Freeman et al. (2017) independently define the PIT and demonstrate how its individual values may be used both to perform hypothesis testing (via, e. g. the KS, CvM, and AD tests; see below) and to construct quantile-quantile plots. Following Kodra & Newman (in prep.) we define the PIT-based catastrophic outlier rate as the fraction of galaxies with  $\text{PIT} < 0.0001$  or  $\text{PIT} > 0.9999$ , which should total 0.0002 for an ideal uniform distribution.

We evaluate a number of quantitative metrics derived from the visually interpretable QQ plot and PIT histogram, built on the Kolmogorov-Smirnov (KS) statistic

$$\text{KS} \equiv \max_z \left( | \text{CDF}[\hat{f}, z] - \text{CDF}[\tilde{f}, z] | \right), \quad (6)$$

interpretable as the maximum difference between the CDFs of an approximating univariate distribution  $\hat{f}(z)$  and a reference distribution  $\tilde{f}(z)$ . We also consider two variants of the KS statistic. A cousin of the KS statistic, the Cramer-von Mises (CvM) statistic

$$\text{CvM}^2 \equiv \int_{-\infty}^{+\infty} (\text{CDF}[\hat{f}, z] - \text{CDF}[\tilde{f}, z])^2 d\text{CDF}[\tilde{f}, z] \quad (7)$$

is the mean-squared difference between the CDFs of an approximate and true PDF. The Anderson-Darling (AD) statistic

$$\text{AD}^2 \equiv N_{\text{tot}} \int_{-\infty}^{+\infty} \frac{(\text{CDF}[\hat{f}, z] - \text{CDF}[\tilde{f}, z])^2}{\text{CDF}[\tilde{f}, z](1 - \text{CDF}[\tilde{f}, z])} d\text{CDF}[\tilde{f}, z] \quad (8)$$

is a weighted mean-squared difference featuring enhanced sensitivity to discrepancies in the tails of the distribution. In anticipation of a substantial fraction of galaxies having PIT of 0 or 1, a consequence of catastrophic outliers, we evaluate the AD statistic with modified bounds of integration (0.01, 0.99) to exclude those extremes in the name of numerical stability.

#### 4.2 Conditional Density Estimate (CDE) Loss: a metric of individual photo- $z$ PDFs

The BUZZARD simulation process precludes testing the degree to which samples from our photo- $z$  posteriors reconstruct the space of  $p(z, \text{data})$ . To the knowledge of the authors, there is only one metric that can be used to evaluate the performance of individual photo- $z$  PDF estimators in the absence of true photo- $z$  posteriors. Using the notation introduced in Section 3.2.3, the conditional density estimation (CDE) loss is defined as

$$L(f, \hat{f}) \equiv \int \int (f(z|\mathbf{x}) - \hat{f}(z|\mathbf{x}))^2 dz dP(\mathbf{x}) \quad (9)$$

in terms of the photometry  $\mathbf{x}$ , an analogue to the familiar root-mean-square-error used in conventional regression. We estimate the CDE loss via

$$\hat{L}(f, \hat{f}) = \mathbb{E}_{\mathbf{x}} \left[ \int \hat{f}(z|\mathbf{X})^2 dz \right] - 2\mathbb{E}_{\mathbf{x}, Z} \left[ \hat{f}(Z|\mathbf{X}) \right] + K_f, \quad (10)$$

<sup>16</sup> See, e. g. the LSST Data Products Definition Document, available at: <https://ls.st/dpdd>

<sup>17</sup> <http://github.com/aimalz/qp/>

where the first term is the expectation value of the photo- $z$  posterior with respect to the marginal distribution of the photometric covariates  $\mathbf{X}$ , the second term is the expectation value with respect to the joint distribution of  $\mathbf{X}$  and the space  $Z$  of all possible redshifts, and the third term  $K_f$  is a constant depending only upon the true conditional densities  $f(z|\mathbf{x})$ . We may estimate these expectations empirically on the test or validation data (Eq. 7 in Izbicki et al. 2017) without knowledge of the true densities.

## 5 RESULTS

We begin with a demonstrative visual inspection of the photo- $z$  PDFs produced by each code for individual galaxies. Figure 1 shows the photo- $z$  PDFs for four galaxies chosen as examples of photo- $z$  PDF archetypes: a narrow unimodal PDF, a broad unimodal PDF, a bimodal PDF, and a multimodal PDF. We reiterate that under our idealized experimental conditions, differences between codes are the isolated signature of the implicit prior due to the method by which the photo- $z$  PDFs were derived.

The most striking differences between codes are due to small-scale features induced by the interaction between the shared piecewise constant parameterization of 200 bins  $0 < z < 2$  of Section 4 and the smoothing conditions or lack thereof in each algorithm. The  $\text{dz} = 0.01$  redshift resolution is sufficient to capture the broad peaks of faint galaxies’ photo- $z$  PDFs with large photometric errors but is too broad to resolve the narrow peaks for bright galaxies’ photo- $z$  PDFs with small photometric errors. This observation is consistent with the findings of Malz et al. (2018) that the piecewise constant parameterization underperforms in the presence of small-scale structures.

However, the shared small-scale features of ANNz2, METAPhoR, CMNN, and SkyNet are a result of various weighted sums of the limited number of training set galaxies with colors similar to those of the test set galaxy in question, with behavior closer to classification than regression in the case of ANNz2. The settings used on GPz in this work forced broadening of the single Gaussian to cover the multimodal redshift solutions of the other codes.

### 5.1 Performance on photo- $z$ PDF ensembles

The histogram of PIT values, QQ plot, and QQ difference plot relative to the ideal diagonal are provided in Figure 2, showcasing the biases and trends in the average accuracy of the photo- $z$  PDFs for each code. The high QQ values (more high than low PIT values) of BPZ, CMNN, Delight, EAZY, and GPz indicate photo- $z$  PDFs biased low, and the low QQ values (more low than high PIT values) of SkyNet and TPZ indicate photo- $z$  PDFs biased high. The gray shaded band marks the  $2\sigma$  variance in PIT values found using the trainZ algorithm with a bootstrap resampling of the training set and a sample size of 30,000 galaxies. This represents a very conservative estimate of the representative training sample size, and thus an approximate minimal error significance compared to ideal performance. Deviations in the PIT histograms outside of this range show that significant biases are present for some codes.

The PIT histograms of Delight, CMNN, SkyNet, and TPZ

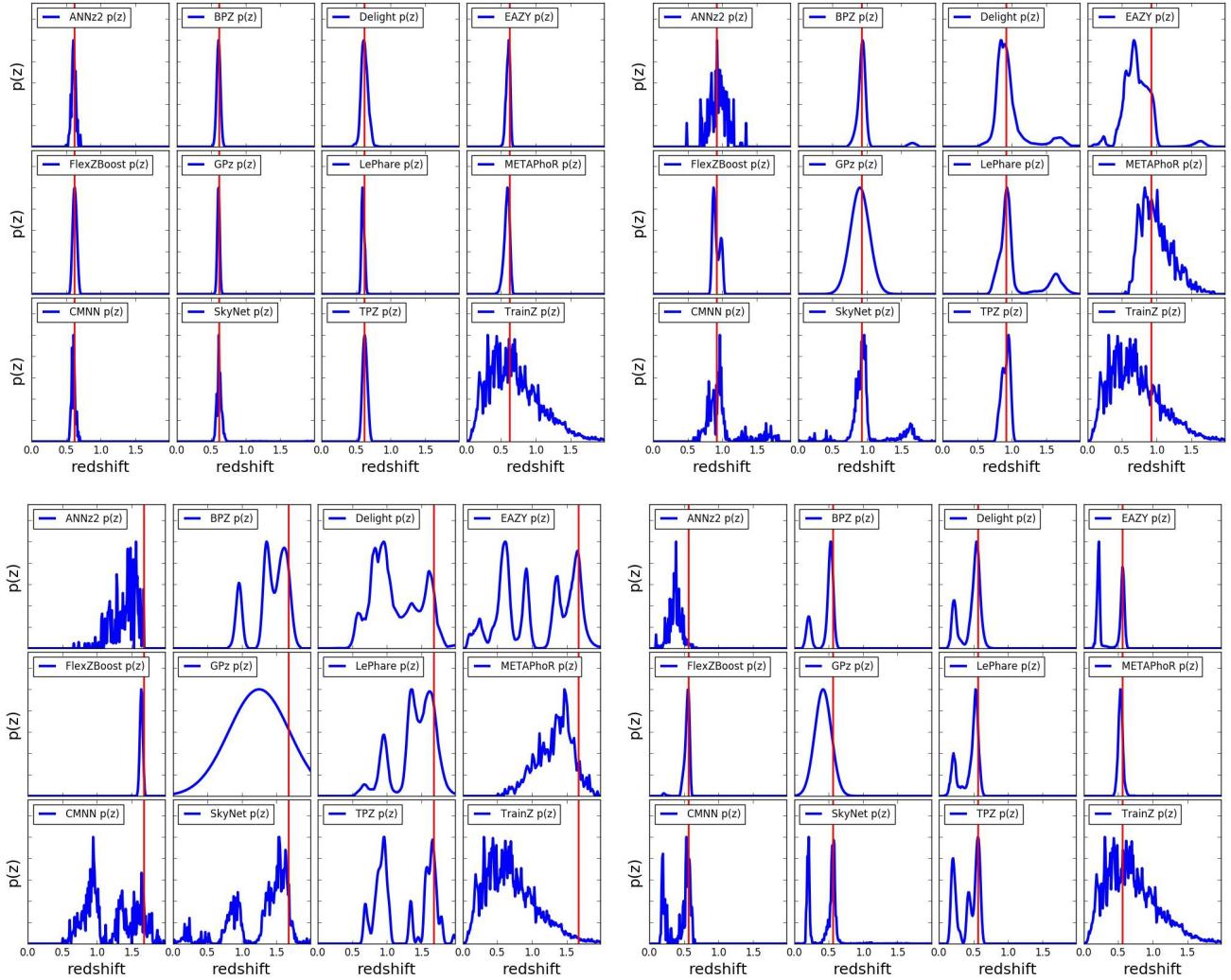
**Table 2.** The catastrophic outlier rate as defined by extreme PIT values. We expect a value of 0.0002 for a proper Uniform distribution. An excess over this small value indicates true redshifts that fall outside the non-zero support of the  $p(z)$ .

Photo- $z$ Code	fraction PIT < $10^{-4}$ or > 0.999
ANNz2	0.0265
BPZ	0.0192
Delight	0.0006
EAZY	0.0154
FlexZBoost	0.0202
GPz	0.0058
LePhare	0.0486
METAPhoR	0.0229
CMNN	0.0034
SkyNet	0.0001
TPZ	0.0130
trainZ	0.0002

feature an underrepresentation of extreme values, indicative of overly broad photo- $z$  PDFs, while the overrepresentation of extreme values for METAPhoR indicate overly narrow photo- $z$  PDFs. These five codes in particular have a free parameter for bandwidth, which may be responsible for this vulnerability, in spite of the opportunity for fine-tuning with perfect prior information. FlexZBoost’s “sharpening” parameter (described in Section 3.2.3) played a key role in diagonalizing the QQ plot, indicating a common avenue for improvement in the approaches that share this type of parameter. On the other hand, the three purely template-based codes, BPZ, EAZY, and LePhare, do not exhibit much systematic broadening or narrowing, which may indicate that complete template coverage effectively defends from these effects.

Close inspection of the extremes at PIT values of 0 and 1 reveal spikes in the first and last bin of the PIT histogram for some codes in Figure 2, corresponding to “catastrophic outliers” where the true redshift lies outside of the support of the  $p(z)$ . The catastrophic outlier rates are provided in Table 2. As expected, trainZ achieves precisely the 0.0002 value expected of an ideal PIT distribution. ANNz2, FlexZBoost, LePhare, and METAPhoR have notably high catastrophic outlier rates  $> 0.02$ , exceeding 100 times the ideal PIT rate, meriting further investigation.

Figure 3 displays the values of the KS, CvM, and AD test statistics calculated by comparing the PIT distribution and a uniform distribution  $U(0, 1)$ . This plot highlights the relative rather than absolute numbers. METAPhoR and LePhare perform well under the AD but poorly under the KS and CvM due to their high catastrophic outlier rates. ANNz2 and FlexZBoost are the top scorers under these metrics of the PIT distribution. ANNz2’s strong performance can be attributed to an aspect of the training process in which training set galaxies with a PIT that more closely matches the percentiles of the DC1 training set’s redshift distribution are upweighted; in effect, these quantile-based metrics were part of the algorithm itself that may or may not serve it well under more realistic experimental conditions. Similar to what was done for the PIT histograms in Figure 2 we create bootstrap training samples of 30,000 galaxies for use with trainZ in order to estimate a conservative statistical floor that we would expect in real data. No code reaches this



**Figure 1.** The individual photo- $z$  PDFs (blue) distributions produced by the twelve codes (small panels) on four exemplary galaxies' photometry (large panels) with different true redshifts (red). The photo- $z$  PDFs of all codes share some features for the example galaxies due to physical color degeneracies and photometric errors: tight unimodal  $p(z)$  (upper left), broad unimodal  $p(z)$  (upper right), bimodal  $p(z)$  (lower right), and complex/multimodal  $p(z)$  (lower left). The diverse algorithms and implementations induce differences in small-scale structure and sensitivity to physical systematics.

idealized floor, indicating that all codes suffer some degradation from the ideal when employing their implicit priors, though ANNz2, FlexZBoost, and GPz are within a factor of two.

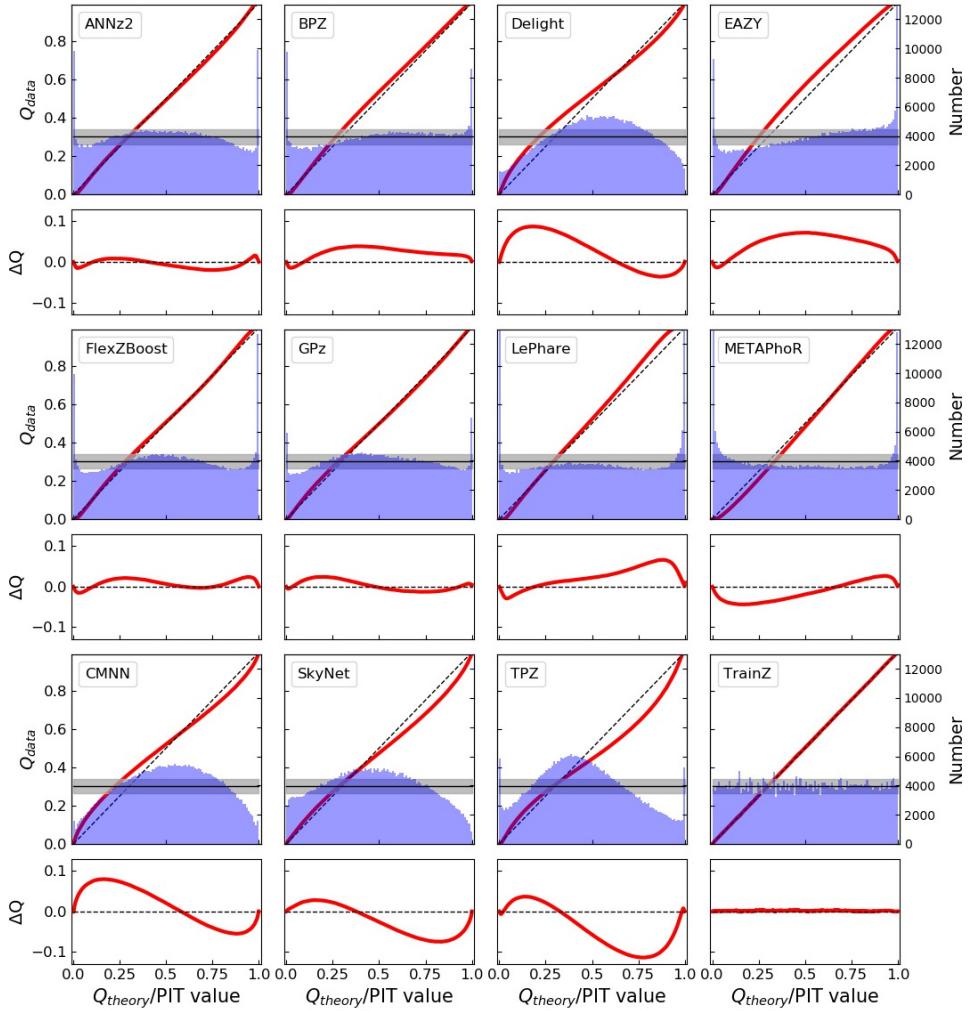
## 5.2 Performance on individual photo- $z$ PDFs

The values of the CDE loss statistic of individual photo- $z$  PDF accuracy are provided in Table 3. It is worth noting that strong performance on the CDE loss should imply strong performance on the other metrics, though the inverse is not necessarily true. Thus the CDE loss is the most effective metric for generic science cases.

This metric is the only one that can appropriately penalize trainZ and indicates strong performance for CMNN and FlexZBoost, the latter of which is optimized for this metric.

**Table 3.** CDE loss statistic of the individual photo- $z$  PDFs for each code. A lower value of the CDE loss indicates more accurate individual photo- $z$  PDFs, with CMNN and FlexZBoost performing best under this metric.

Photo- $z$ Code	CDE Loss
ANNz2	-6.88
BPZ	-7.82
Delight	-8.33
EAZY	-7.07
FlexZBoost	-10.60
GPz	-9.93
LePhare	-1.66
METAPhOr	-6.28
CMNN	-10.43
SkyNet	-7.89
TPZ	-9.55
trainZ	-0.83



**Figure 2.** The QQ plot (red) and PIT histogram (blue) of the photo- $z$  PDF codes (panels) along with the ideal QQ (black dashed diagonal) and ideal PIT (gray horizontal) curves, as well as a difference plot for the QQ difference from the ideal diagonal (lower inset). The gray shaded region indicates the  $2\sigma$  range from a bootstrap resampling of the training set with a size of 30,000 galaxies using `trainZ`. The twelve codes exhibit varying degrees of four deviations from perfection: an overabundance of PIT values at the centre of the distribution indicate a catalogue of overly broad photo- $z$  PDFs, an excess of PIT values at the extrema indicates a catalogue of overly narrow photo- $z$  PDFs, catastrophic outliers manifest as overabundances at PIT values of 0 and 1, and asymmetry indicates systematic bias, a form of model misspecification. Values in excess of the  $2\sigma$  shaded region show that for some codes these errors will be significant given expected training sample sizes.

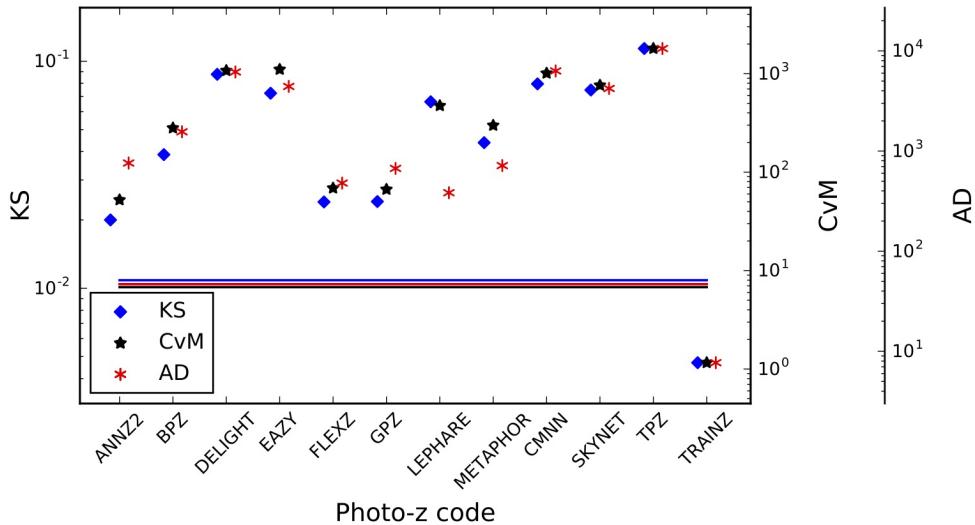
## 956 6 DISCUSSION AND FUTURE WORK

957 In contrast with other photo- $z$  PDF comparison papers that  
 958 have aimed to identify the “best” code for a given survey, we  
 959 have focused on the somewhat more philosophical questions  
 960 of how to assess photo- $z$  PDF methods and how to interpret  
 961 differences between codes in terms of photo- $z$  PDF per-  
 962 formance. In Section 6.1, we reframe the strong performance of  
 963 our pathological photo- $z$  PDF technique, `trainZ`, as a cau-  
 964 tionary tale about the importance of choosing appropriate  
 965 comparison metrics. In Section 6.2, we outline the experi-  
 966 ments we intend to build upon this study. In Section 6.3, we  
 967 discuss the enhancements of the mock data set that will be  
 968 necessary to enable the future experiments.

### 969 6.1 Interpretation of metrics

970 We remind the reader that contributed codes were given a  
 971 goal of obtaining accurate photo- $z$  PDFs, not an accurate  
 972 stacked estimator of the redshift distribution, so we do not  
 973 expect the same codes to necessarily perform well for both  
 974 classes of metrics. Indeed, the codes were optimized for their  
 975 interpretation of our request for “accurate photo- $z$  PDFs,”  
 976 and we expect that the implementations would have been  
 977 adjusted had we requested optimization of the traditional  
 978 metrics of Appendices A and B.

979 Furthermore, our metrics are not necessarily able to  
 980 assess the fidelity of individual photo- $z$  PDFs relative to  
 981 true posteriors: in the absence of a “true PDF” from which  
 982 redshifts are drawn, it is difficult to construct metrics to  
 983 measure performance for individual galaxies rather than



**Figure 3.** A visualization of the Kolmogorov-Smirnov (KS, blue diamond), Cramer-von Mises (CvM, black star), and Anderson-Darling (AD, red asterisk) statistics for the PIT distributions. There is generally good agreement between these statistics, with differences corresponding to the codes with outstanding catastrophic outlier rates, a reflection in the differences in how each statistic weights the tails of the distribution. Horizontal lines indicate the level of uncertainty found by bootstrapping a training set sample of 30,000 galaxies using `trainZ`, none of the codes reach this conservative ideal floor in expected uncertainty.

ensembles, with the exception of the CDE Loss metric of section 4.2, where knowledge of the true densities is not required in computing the Loss function for an individual galaxy. A lack of appropriate metrics more sophisticated than the CDE Loss remains an open issue for science cases requiring accurate individual galaxy PDFs. The metric-specific performance demonstrated in this paper implies that we may need multiple photo-z PDF approaches tuned to each metric in order to maximize returns over all science cases in large upcoming surveys.

The `trainZ` estimator of Section 3.3, which assigns every galaxy a photo-z PDF equal to  $N(z)$  of the training set, is introduced as an experimental control or null test to demonstrate this point via *reductio ad absurdum*. Because our training set is perfectly representative of the test set,  $N(z)$  should be identical for both sets down to statistical noise. We make the alarming observation that `trainZ` outperforms all codes on the PIT-based metrics, and all but one code on the  $N(z)$  based statistics.

The CDE loss and point estimate metrics appropriately penalize `trainZ`'s naivete. As shown in Appendix B, `trainZ` has identical `ZPEAK` and `ZWEIGHT` values for every galaxy, and thus the photo-z point estimates are constant as a function of true redshift, i. e. a horizontal line at the mode and mean of the training set distribution respectively. The explicit dependence on the individual posteriors in the calculation of the CDE loss, described in Section 5.2, distinguishes this metric from those of the photo-z PDF ensemble and stacked estimator of the redshift distribution, despite their prevalence in the photo-z literature.

In summary, context is crucial to defending against deceptively strong performers such as `trainZ`; the best photo-z PDF method is the one that most effectively achieves our science goals, not the one that performs best on a metric that does not reflect those goals. In the absence of a single scientific motivation or the information

necessary for a principled metric definition, we must consider many metrics and be critical of the information transmitted by each.

## 6.2 Extensions to the experimental design

The work presented in this paper is only a first step in assessing photo-z PDF approaches and moving toward an improved photometric redshift estimator. Here we discuss the next steps for subsequent investigations.

This initial paper explored code performance in idealized conditions with perfect catalog-based photometry and representative training data. A top priority for a follow-up study is to test realistic forms of incomplete, erroneous, and non-representative template libraries and training sets as well as the impact of other forms of external priors that must be ingested by the codes, major concerns in Newman et al. (2015); Masters et al. (2017). We plan to perform a full sensitivity analysis on a realistically incomplete training set of spectroscopic galaxies, modeling the performance of spectrographs, emission-line properties, and expected signal-to-noise to determine which potential training set galaxies are most likely to be excluded. In addition to outright redshift failures we will model the inclusion of a small number of high-confidence yet false redshifts due to emission line misidentification or noise spikes. Section 4 discussed the difficulty in evaluating PDF accuracy for individual objects. We are planning a project to generate “true PDF” distributions using generative adversarial networks((GANs) Goodfellow et al. 2014) trained on the simulation data. This dataset will enable a test of PDF accuracy for individual galaxies rather than resorting to ensembles, as required for many tests in the current work.

Appendix A only addresses the stacked estimator of the redshift distribution of the entire galaxy catalogue rather than subsets in bins, tomographic or otherwise. The effects

of tomographic binning scheme will be explored in a dedicated future paper, including propagation of redshift uncertainties in a set of fiducial tomographic redshift bins in order to estimate impact on cosmological parameter estimation.

Sequels to this study will also address some shortcomings of our experimental procedure. The fixed redshift grid shared between the codes may have unfairly penalized codes with a different native parameterization, as precision is lost when converting between formats. Performance on sharply peaked photo-z PDFs may have been suppressed across all codes due to the insufficient resolution of the redshift grid. In light of the results of Malz et al. (2018), in future analyses we plan to switch from a fixed grid to the quantile parameterization or to permit each code to use its native storage format under a shared number of parameters.

### 6.3 Realistic mock data

To make optimal use of the LSST data for cosmological and other astrophysical analyses of the Science Roadmap, future investigations that build upon this one will require a more sophisticated set of galaxy photometry and redshifts. This initial paper explored a data set that was constructed at the catalog level, with no inclusion of the complications that come from measuring photometry from images. Future data challenges will move to catalogs constructed from mock images, including the complications of deblending, sensor inefficiencies, and heterogeneous observing conditions, all anticipated to affect the measured colours of LSST’s galaxy sample (Dawson et al. 2016).

The DC1 galaxy SEDs were linear combinations of just five basis SED templates, but a next generation of data for photo-z PDF investigations must include a broader range of physical properties. Though we only considered  $z < 2$  here, LSST 10-year data will contain  $z > 2$  galaxies, plagued by fainter apparent magnitudes and anomalous colours due to stellar evolution. A subsequent study must also have a data set that includes low-level active galactic nuclei (AGN) features in the SEDs, which perturb colours and other host galaxy properties. An observational degeneracy between the Lyman break of a  $z \sim 2 - 3$  galaxy from the Balmer break of a  $z \sim 0.2 - 0.3$  galaxy is a known source of catastrophic outliers (Massarotti et al. 2001) that was not effectively included in this study. To gauge the sensitivity of photo-z PDF estimators to catastrophic outliers, our data set must include realistic high-redshift galaxy populations.

The overarching plan describing everything laid out in this section is described in more detail in the LSST-DESC Science Roadmap (see Footnote in Section 1).

## 7 CONCLUSION

This paper compares twelve photo-z PDF codes under controlled experimental conditions of representative and complete prior information to set a baseline for an upcoming sensitivity analysis. This work isolates the impact on metrics of photo-z PDF accuracy due to the estimation technique as opposed to the complications of realistic physical systematics of the photometry. Though the mock data set of this investigation did not include true photo-z posteriors for

comparison, we interpret deviations from perfect results given perfect prior information as the imprint of the implicit assumptions underlying the estimation approach.

We evaluate the twelve codes under science-agnostic metrics both established and emerging to stress-test the ensemble properties of photo-z PDF catalogues derived by each method. In appendices, we also present metrics of point estimates and a prevalent summary statistic of photo-z PDF catalogues used in cosmological analyses to enable the reader to relate this work to studies of similar scope. We observe that no one code dominates in all metrics, and that the standard metrics of photo-z PDFs and the stacked estimator of the redshift distribution can be gamed by a very simplistic procedure that asserts the prior over the data. We emphasize to the photo-z community that metrics used to vet photo-z PDF methods must be scrutinized to ensure they correspond to the quantities that matter to our science.

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Author contributions are listed below.

S.J. Schmidt: Led the project. (conceptualization, data curation, formal analysis, investigation, methodology, project administration, resources, software, supervision, visualization, writing – original draft, writing – review & editing)

A.I. Malz: Contributed to choice of metrics, implementation in code, and writing. (conceptualization, methodology, project administration, resources, software, visualization, writing – original draft, writing – review & editing)

J.Y.H. Soo: Ran ANNz2 and Delight, updated abstract, edited sections 1 through 6, added tables in Methods and Results, updated references.bib and added references throughout the paper

I.A. Almosallam: vetted the early versions of the data set and ran many photo-z codes on it, applied GPz to the final version and wrote the GPz subsection

M. Brescia: main ideator of METAPHOR and of MLPQNA; modification of METAPHOR pipeline to fit the LSST data structure and requirements

S. Cavuoti: Contributed to choice and test of metrics, ran METAPHOR, minor text editing

J. Cohen-Tanugi: contributed to running code, analysis discussion, and editing, reviewing the paper

A.J. Connolly: Developed the colour-matched nearest-neighbours photo-z code; participated in discussions of the analysis.

P.E. Freeman: Contributed to choice of CDE metrics and to implementation of FlexZBoost

M.L. Graham: Ran the colour-matched nearest-neighbours photo-z code on the Buzzard catalog and wrote the relevant piece of Section 2; participated in discussions of the analysis.

K. Iyer:

assisted in writing metric functions used to evaluate codes

M.J. Jarvis: Contributed text on AGN to Discussion section and portions of GPz work

J.B. Kalmbach: Worked on preparing the figures for the paper.

E. Kovacs: Ran simulations, discussed data format and properties for SEDs, dust, and ELG corrections

A.B. Lee: Co-developed FlexZBoost and the CDE loss statistic, wrote text on the work, and supervised the development of FlexZBoost software packages  
 G. Longo: Scientific advise, test and validation of the modified METAPHOR pipeline, text of the METAPHOR section  
 C. Morrison: Managerial support; Discussions with authors regarding metrics and style; Some coding contribution to metric computation.  
 J. Newman: Contributions to overall strategy, design of metrics, and supervision of work done by Rongpu Zhou  
 E. Nourbakhsh: Ran and optimized TPZ code and wrote a subsection of Section 2 for TPZ  
 E. Nuss: contributed to running code, analysis discussion, and editing/reviewing the paper  
 T. Pospisil: Co-developed FlexZBoost software and CDE loss calculation code  
 H. Tranin: contributed to providing SkyNet results and writing the relevant section  
 R. Zhou: Optimized and ran EAZY and contributed to the draft  
 R. Izbicki: Co-developed FlexZBoost and the CDE loss statistic, and wrote software for FlexZBoost

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In addition to packages cited in the text, analyses performed in this paper used the following software packages: **Numpy** and **Scipy** (Oliphant 2007), **Matplotlib** (Hunter 2007), **Seaborn** (Waskom et al. 2017), **minFunc** (Schmidt 2005), **pySkyNet** (Bonnett 2016), and **photUtils** from the LSST simulations package (Connolly et al. 2014).

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## APPENDIX A: EVALUATION OF THE REDSHIFT DISTRIBUTION

Perhaps the most popular application of photo-z PDFs is the estimation of the overall redshift distribution  $N(z)$ , a quantity that enters some cosmological calculations and the true value of which is known for the DC1 data set and will be

denoted as  $\tilde{N}(z)$ . In terms of the prior information provided to each method, the true redshift distribution satisfies the tautology  $\tilde{N}(z) = p(z|I_D)$  due to our experimental set-up; because the DC1 training and template sets are representative and complete,  $I_D$  represents a prior that is also equal to the truth. In this ideal case of complete and representative prior information, the method that would give the best approximation to  $\tilde{N}(z)$  would be one that neglects all the information contained in the photometry  $\{d_i\}_{N_{tot}}$  and gives every galaxy the same photo-z PDF  $\hat{p}_i(z) = \tilde{N}(z)$  for all  $i$ ; the inclusion of any information from the photometry would only introduce noise to the optimal result of returning the prior. This is the exact estimator, **trainZ**, that we have described in Section 3.3, and which will serve as an experimental control.

### A1 Metrics of the stacked estimator of the redshift distribution

Though alternatives exist (Malz & Hogg prep), “stacking” according to

$$\hat{N}^H(z) \equiv \frac{1}{N_{tot}} \sum_i^{N_{tot}} \hat{p}_i^H(z) \quad (\text{A1})$$

is the most widely accepted method for obtaining  $\hat{N}^H(z)$  as an estimator of the redshift distribution from photo-z PDFs derived by a method  $H$ . Though the use of the stacked estimator of the redshift distribution is not formally correct, we use it under the untested assumption that the response of our metrics of  $\hat{N}^H(z)$  will be analogous to the same metrics applied to a principled estimator of the redshift distribution.

As  $N(z)$  is itself a univariate PDF, we apply the metrics of the previous sections to it as well. We additionally calculate the first three moments

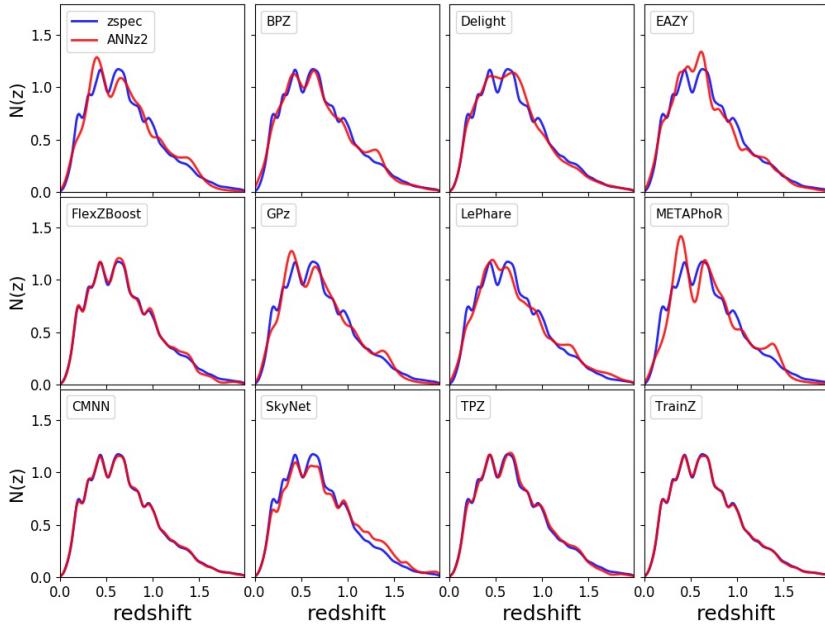
$$\langle z^m \rangle \equiv \int_{-\infty}^{\infty} z^m N(z) dz \quad (\text{A2})$$

of the estimated redshift distribution  $\hat{N}^H(z)$  for each code and compare them to the moments of the true redshift distribution  $\tilde{N}(z)$ . Under the assumption that the stacked estimator is unbiased, a superior method minimizes the difference between the true and estimated moments.

### A2 Performance on the stacked estimator of the redshift distribution

Figure A1 shows the stacked estimator  $\hat{N}(z)$  of the redshift distribution for each code compared to the true redshift distribution  $\tilde{N}(z)$ , where the stacked estimator has been smoothed for each code in the plot using a kernel density estimate (KDE) with a bandwidth chosen by Scott’s Rule (Scott 1992) in order to minimize visual differences in small-scale features; the quantitative statistics, however, are calculated using the empirical CDF which is not smoothed.

Many of the codes, including all the model-fitting approaches and **ANNz2**, **GPz**, **METAPhOr**, and **SkyNet** from the data-driven camp, overestimate the redshift density at  $z \sim 1.4$ . This behavior is a consequence of the 4000 Å break passing through the gap between the  $z$  and  $y$  filters, which



**Figure A1.** The smoothed stacked estimator  $\hat{N}(z)$  of the redshift distribution (red) produced by each code (panels) compared to the true redshift distribution  $\tilde{N}(z)$  (blue). Varying levels of agreement are seen among the codes, with the smallest deviations for CMNN, FlexZBoost, TPZ, and trainZ.

induces a genuine discontinuity in the  $z - y$  colour as a function of redshift that can sway the photo- $z$  PDF estimates in the absence of bluer spectral features.

ANNz2, GPz, and METAPhoR show possible signs of over-training: showing exaggerated peaks and troughs relative to the training set. Further investigation on overtraining is needed, if present this is an obstacle that may be overcome with adjustment of the implementation.

As expected, trainZ perfectly recovers the true redshift distribution: as the training sample is selected from the same underlying distribution as the test set, the redshift distributions are identical, up to Poisson fluctuations due to the finite number of sample galaxies. CMNN is also in excellent agreement for similar reasons: with a representative training sample of galaxies spanning the colour-space, the sum of the colour-matched neighbour redshifts should return the true redshift distribution. FlexZBoost and TPZ also perform superb recovery of the true redshift distribution, with only a slight deviation at  $z \sim 1.4$ . Our metrics, however, cannot discern whether these four approaches, as well as Delight, are spared the  $z \sim 1.4$  degeneracy in  $\hat{N}(z)$  because they have more effectively used information in the data or if the impact is simply washed out by the stacked estimator's effective average over the test set galaxy sample. See Appendix B for further discussion of the  $z \sim 1.4$  issue.

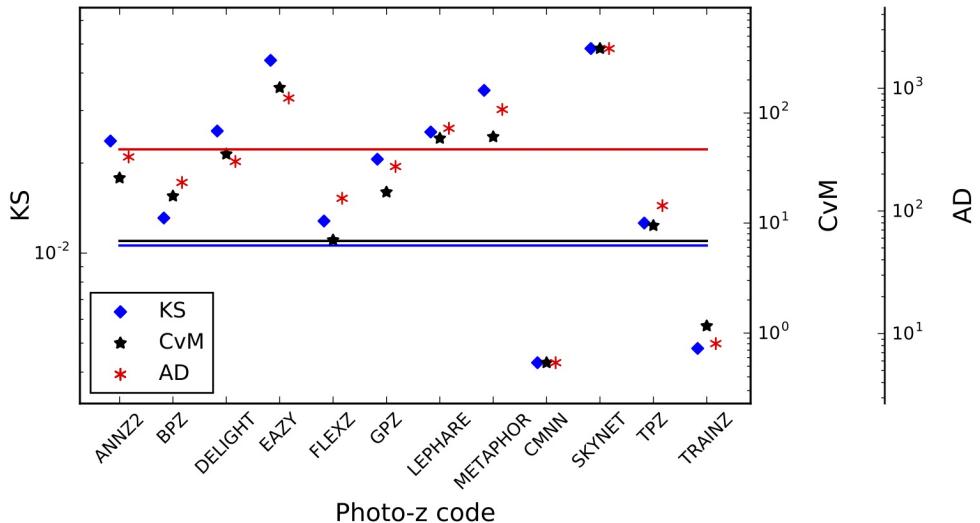
Figure A2 shows the quantitative Kolmogorov-Smirnov (KS), Cramer-Von Mises (CvM), and Anderson Darling (AD) test statistics for each of the codes for the  $\hat{N}(z)$  based measures. The horizontal lines show the result of a bootstrap resampling of the training set using 30,000 samples for trainZ. As mentioned in Section 5.1 this represents a

conservative idealized limit on expected performance for a modest-sized representative training set of galaxies. The AD bootstrap statistic is elevated due to this statistic's sensitivity to distribution tails. The stacked estimators of the redshift distribution for CMNN and trainZ best estimate  $\tilde{N}(z)$  under these metrics, and BPZ, GPz, and TPZ are within a factor of two of the conservative limit for all statistics. The only codes that do especially poorly are EAZY, LePhare, METAPhoR, and SkyNet. It is unsurprising that CMNN scores well, as with a nearly complete and representative training set choosing neighbouring points in color/magnitude space to construct an estimator should lead to excellent agreement in the final  $\hat{N}(z)$ .

It is, however, surprising that TPZ does well on  $\hat{N}(z)$  given its poor performance on the ensemble photo- $z$  PDFs, especially knowing that TPZ was optimized for photo- $z$  PDF ensemble metrics rather than the stacked estimator of the redshift distribution. A possible explanation is the choice of smoothing parameter chosen during validation, which affects photo- $z$  PDF widths as well as overall redshift bias and could be modified to improve performance under the photo- $z$  PDF metrics.

The first three moments of the stacked  $\hat{N}(z)$  distribution relative to the empirical estimate of the truth distribution are given in Table A1. Accuracy of the moments varies widely between codes, raising concerns about the propagation to cosmological analyses.

SkyNet exhibits redshift bias in Figure A1 and is a clear outlier in the first moment of  $\hat{N}(z)$  in Table A1. The SkyNet algorithm employs a random subsampling of the training set without testing that the subset is representative of the



**Figure A2.** A visualization of the Kolmogorov-Smirnov (KS, blue diamond), Cramer-von Mises (CvM, black star), and Anderson-Darling (AD, red asterisk) statistics for the  $\hat{N}(z)$  distributions. Horizontal lines indicate the statistic values (including uncertainty) achieved using `trainZ` via bootstrap resampling a training set containing 30,000 redshifts. We make the reassuring observation that these related statistics do not disagree significantly with one another. `CMNN` actually outperforms the control case, `trainZ`, and several codes are within a factor of two of this conservative idealized limit. `SkyNet` scores poorly due to an overall bias in its redshift predictions.

**Table A1.** Moments of the stacked estimator  $\hat{N}(z)$  of the redshift distribution. Most of the codes considered recover the moments of  $\hat{N}(z)$

Moments of $\hat{N}(z)$	mean	variance	skewness
Estimator			
Empirical “truth”	0.701	0.630	0.671
ANNz2	0.702	0.625	0.653
BPZ	0.699	0.629	0.671
Delight	0.692	0.609	0.638
EAZY	0.681	0.595	0.619
FlexZBoost	0.694	0.610	0.631
GPz	0.696	0.615	0.639
LePhare	0.718	0.668	0.741
METAPhORe	0.705	0.628	0.657
CMNN	0.701	0.628	0.667
SkyNet	0.743	0.708	0.797
TPZ	0.700	0.619	0.643
<code>trainZ</code>	0.699	0.627	0.666

full population, and the implementation used here does not upweight rarer low- and high-redshift galaxies, as in Bonnett (2015), suggesting a possible cause that may be addressed in future work.

## APPENDIX B: Photo-z POINT ESTIMATION AND METRICS

While this work assumes that science applications value the information of the full photo-z PDF, we present conventional metrics of photo-z point estimates as a quick and dirty diagnostic tool and to facilitate direct comparisons to historical studies.

### B1 Reduction of photo-z PDFs to point estimates

Though we acknowledge that many of the codes can also return a native photo-z point estimate, we put all codes on equal footing by considering two generic photo-z point estimators, the mode  $z_{PEAK}$  and main-peak-mean  $z_{WEIGHT}$  (Dahlen et al. 2013), a weighted mean within the bounds of the main peak, as identified by the roots of  $p(z) - 0.05 \times z_{PEAK}$ . Though  $z_{WEIGHT}$  neglects information in a secondary peak of a, say, bimodal distribution, it avoids the pitfall of reducing the photo-z PDF to a redshift between peaks where there is low probability.

### B2 Metrics of photo-z point estimates

We calculate the commonly used point estimate metrics of the overall intrinsic scatter, bias, and catastrophic outlier rate, defined in terms of the standard error  $e_z \equiv (z_{PEAK} - z_{true}) / (1 + z_{true})$ . Because the standard deviation of the photo-z residuals is sensitive to outliers, we define the scatter in terms of the Interquartile Range (IQR), the difference between the 75th and 25th percentiles of the distribution of  $e_z$ , imposing the scaling  $\sigma_{IQR} = IQR / 1.349$  to ensure that the area within  $\sigma_{IQR}$  is the same as that within one standard deviation from a standard Normal distribution. We also resist the effect of catastrophic outliers by defining the bias  $b_z$  as the median rather than mean value of  $e_z$ . The catastrophic outlier rate  $f_{out}$  is defined as the fraction of galaxies with  $e_z$  greater than  $\max(3\sigma_{IQR}, 0.06)$ .

For reference, Section 3.8 of the LSST Science Book (Abell et al. 2009) uses the standard definitions of these parameters in requiring

- RMS scatter  $\sigma < 0.02(1 + z_{true})$
- bias  $b_z < 0.003$
- catastrophic outlier rate  $f_{out} < 10\%$  .

**B3 Comparison of photo- $z$  point estimate metrics**

Figure B1 shows both point estimates for all codes both  $z_{PEAK}$  and  $z_{WEIGHT}$ . Point density is shown with mixed contours to emphasize that most of the galaxies do fall close to the  $z_{phot} = z_{spec}$  line, while points trace the details of the catastrophic outlier populations.

The finite grid spacing of the photo- $z$  PDFs induces some discretization in  $z_{PEAK}$ . The features perpendicular to the  $z_{phot} = z_{spec}$  line are due to the 4000Å break passing through the gaps between adjacent filters. Even the strongest codes feature populations far from the  $z_{phot} = z_{spec}$  line representing a degeneracy in the space of colours and redshifts.

The intrinsic scatter, bias, and catastrophic outlier rate are given in Table B1. Perhaps unsurprisingly, performance under these metrics largely tracks that of the metrics of Section 4 of the photo- $z$  PDFs from which the point estimates were derived. All twelve codes perform at or near the goals of the LSST Science Requirements Document<sup>18</sup> and Graham et al. (2018), which is encouraging if not unexpected for  $i < 25$ .

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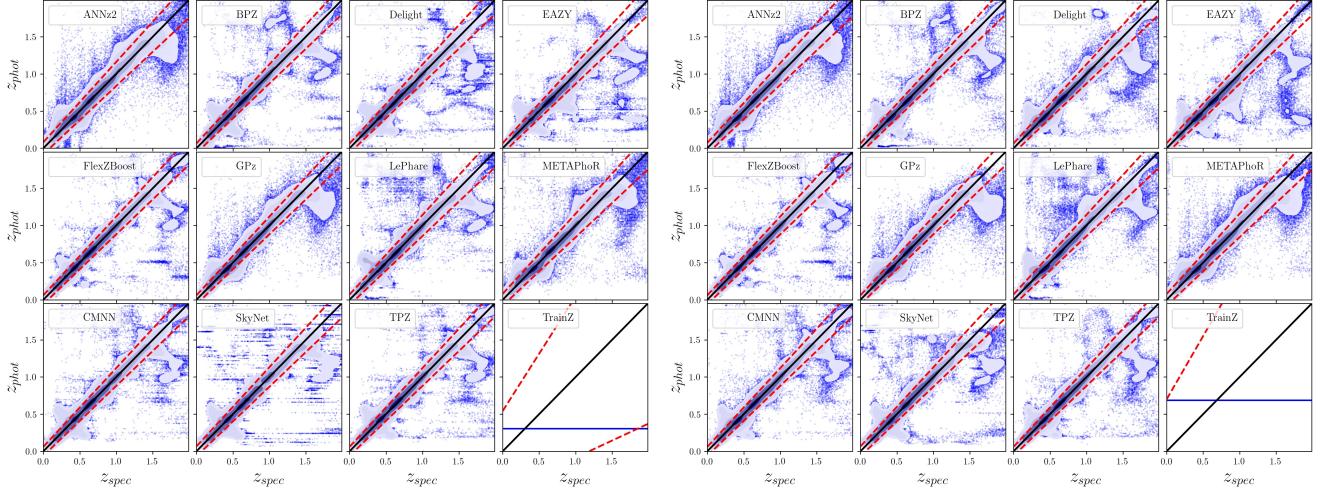
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**Figure B1.** The density of photo- $z$  point estimates (contours) reduced from the photo- $z$  PDFs with outliers (blue) beyond the outlier cutoff (red dashed lines), via the mode ( $z_{PEAK}$ , left panel) and main-peak-mean ( $z_{WEIGHT}$ , right panel). The **trainZ** estimator (lower right sub-panels) has a shared  $z_{PEAK}$  and  $z_{WEIGHT}$  for the entire test set galaxy sample.

**Table B1.** Photo- $z$  point estimate statistics

Photo- $z$ PDF Code	$Z_{PEAK}$		$Z_{WEIGHT}$			
	$\frac{\sigma_{IQR}}{(1+z)}$	median	outlier fraction	$\frac{\sigma_{IQR}}{(1+z)}$	median	outlier fraction
ANNz2	0.0270	0.00063	0.044	0.0244	0.000307	0.047
BPZ	0.0215	-0.00175	0.035	0.0215	-0.002005	0.032
Delight	0.0212	-0.00185	0.038	0.0216	-0.002158	0.038
EAZY	0.0225	-0.00218	0.034	0.0226	-0.003765	0.029
FlexZBoost	0.0154	-0.00027	0.020	0.0148	-0.000211	0.017
GPz	0.0197	-0.00000	0.052	0.0195	0.000113	0.051
LePhare	0.0236	-0.00161	0.058	0.0239	-0.002007	0.056
METAPhoR	0.0264	0.00000	0.037	0.0262	0.001333	0.048
CMNN	0.0184	-0.00132	0.035	0.0170	-0.001049	0.034
SkyNet	0.0219	-0.00167	0.036	0.0218	0.000174	0.037
TPZ	0.0161	0.00309	0.033	0.0166	0.003048	0.031
<b>trainZ</b>	0.1808	-0.2086	0.000	0.2335	0.022135	0.000