Welch's power spectral density estimate

Description

Welch's method for estimating the power spectra is carried out by dividing the time signal into successive blocks, forming the periodogram for each block, and then averaging. MatDeck contains a function called powspectwelch() specifically for this purpose.
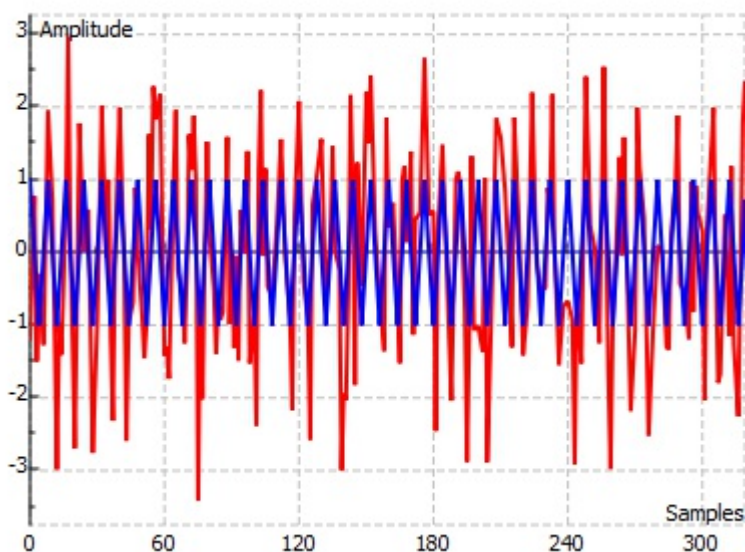
Calculation

MatDeck's function powspectwelch() has the following arguments: input the signal as a vector for which the power spectral density is estimated, window function is determined by string name of the window used, and the number of samples used to generate the result which is also number of samples to perform fft() within each of the periodogram functions. The last two arguments are: the block length in samples, and number of overlapping samples between consecutive blocks. These two arguments define the overall number of periodograms which are averaged.

Example I

In the following, we illustrate how to use powspectwelch() on several illustrative test signals. The first test signal is obtained by combining a cosine signal given by a normalized frequency, and additive white Gaussian noise -AWGN.

$n := \text{ynodes}(x1, 0, 319, 320)$ Sample index vector

$x := \cos((\pi/4) \cdot n)$ Cosine signal

$\text{noise} := \text{normrandvec}(0, 1, 320)$ Additive white Gaussian noise

$xs := x + \text{noise}$

$g1 := \text{join mat cols}(n, xs)$ Graph of signal with noise

$g2 := \text{join mat cols}(n, x)$ Graph of original cosine signal

We prepare the other arguments and we calculate the power spectral density of the noisy signal defined above. We plot the obtained power spectra versus a normalized frequency, which means that one in spectral graph represents half of the sampling rate.

$\text{nfft} := 512$ Number of samples for periodogram

$\text{block} := 100$ Block length

$\text{nover} := 50$ Overallping samples between segments

$\text{Pxx} := \text{powspectwelch}(\text{xs}, \text{"rectangular"}, \text{nfft}, \text{block}, \text{nover})$ Welch's method, rectangular window

$\text{f} := \text{ynodes}(\text{f1}, 0, 1 - 1/\text{nfft}, \text{nfft}/2 + 1)$ Frequency axis

$\text{Pxxgr} := \text{join mat cols}(\text{f} \cdot \pi, 10 \log 10(\text{Pxx}))$ Power spectrum graph
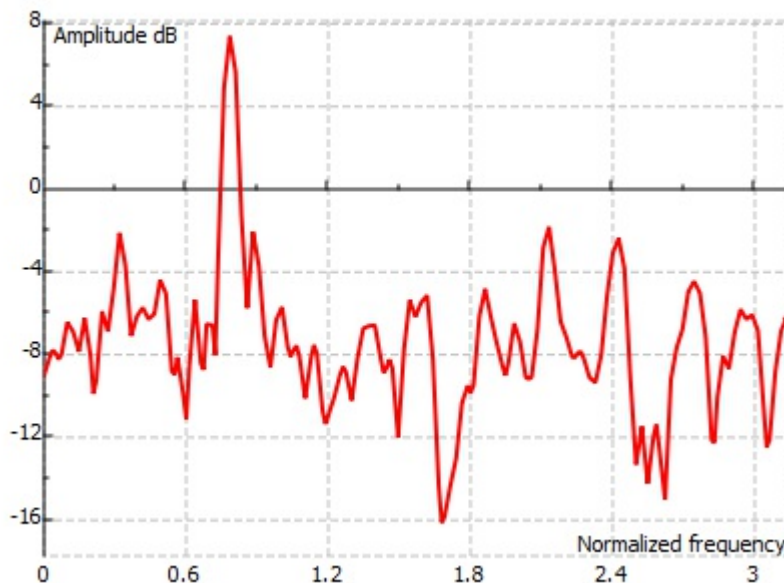
Example II

In the following example we calculate the power spectrum of noisy signal which consists of two sinusoidal functions. We generate a test signal, by combining the noisy signal above with a additional sine wave. The power spectrum is displayed in the following graph. The amplitude of the peaks in the power spectrum differ for the same amount as the signal amplitudes differ in the domain of samples. The Hamming window is used to calculate the power spectra estimate.

$\text{xg} := \text{xs} + 0.5 \sin((\pi/3) \cdot \text{n})$ Noisy signal with two cosine/sine waves

$\text{nfft1} := 512$ Length of the result in samples

$\text{block1} := 100$ Length of blocks in samples

$\text{nover1} := 25$ Number of overlapping samples between consecutive blocks

$\text{Pxxg} := \text{powspectwelch}(\text{xg}, \text{"rectangular"}, \text{nfft1}, \text{block1}, \text{nover})$ Power spectrum, Hamming window

$\text{Pxxggr} := \text{join mat cols}(\text{f} \cdot \pi, 10 \log 10(\text{Pxxg}))$ Power spectrum graph