Homework 5

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1. a $\nabla f(x) = 6x$ 1. b $\nabla\nabla f(x) = 6$

2. a $\nabla f(x) = < 6\, x_1 x_2{}^3, 9x_1{}^2 x_2{}^2 + 2x_2 x_3{}^3, 3x_2{}^2 x_3{}^2 >$

$$2.\text{b } H = \nabla\nabla f(x) = \begin{pmatrix} 6x_2{}^3 & 18x_1x_2{}^2 & 0 \\ 18x_1x_2{}^2 & 18x_1{}^2x_2 + 2x_3{}^3 & 6x_2x_3{}^2 \\ 0 & 6x_2x_3{}^2 & 6x_2{}^2x_3 \end{pmatrix}$$

$$3.\text{a } \nabla\text{f(x)} = \frac{1}{2}(x^T Q^T + x^T Q) - b^T$$

$$3.\text{b } \nabla\nabla\text{f(x)} = \frac{1}{2}(Q + Q^T)$$

4.

$$F(x_1, x_2, \lambda) = x_1{}^2 + x_1{}^2 x_3{}^2 + 2x_1x_2 + x_2{}^4 + 8x_2 + \lambda\,(2x_1 + 5x_2 + x_3 - 3\,)$$

$$\nabla F(x_1, x_2, \lambda) =< 2x_1 + 2x_1x_3{}^2 + 2x_2 + \lambda\,(2), 2x_1 + 4x_2{}^3 + 8 + \lambda\,(5), 2x_3x_1{}^2 + \lambda >$$

$$< 0{,}0{,}2 >^T \rightarrow \nabla F(x_1, x_2, \lambda) =< 2\lambda, 8 + 5\lambda, \lambda > = < 0{,}0{,}0 > \rightarrow$$
$$(There\ is\ no\ unique\ solution\ for\ \lambda)$$

$$< 0{,}0{,}3 >^T \rightarrow \nabla F(x_1, x_2, \lambda) =< 2\lambda, 8 + 5\lambda, \lambda > = < 0{,}0{,}0 > \rightarrow$$

(There is no unique solution for λ)

$$< 1{,}0{,}1 >^T \rightarrow \nabla F(x_1, x_2, \lambda) =< 4 + 2\lambda, 10 + 5\lambda, 2 + \lambda > = < 0{,}0{,}0 > \rightarrow$$
$$(\lambda\ is\ 2\ which\ means\ it\ is\ statitionary\ point)$$

$$< 1{,}0{,}1 >^T \rightarrow \text{H} = \begin{pmatrix} 2 + 2x_3{}^2 & 2 & 4x_3x_1 \\ 2 & 12x_2{}^2 & 0 \\ 4x_3x_1 & 0 & 2x_1{}^2 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 4 \\ 2 & 0 & 0 \\ 4 & 0 & 2 \end{pmatrix}$$

Finding eigen values of H

Det (H-λI)=0

$$\text{Det }(H-\lambda I) = 12\lambda + 6\lambda^2 - 8 - \lambda^3 = -(\lambda+2)(\lambda^2 - 8\lambda + 4) =$$

$$-(\lambda+2)((\lambda-4)^2 - 12) = 0$$

$$\text{So } \lambda = -2, \ \pm\sqrt{12} + 4$$

Which mean there are both positive and negative eigenvalues which means

$$< \ 1,0,1 \ >^T \ \textit{is saddle point}$$

5.

$$f(x) = 2{x_1}^2 + {x_2}^2 \quad g(x) = x_1 + x_2 - 1$$

$$F(x_1, x_2, \lambda) = \ 2{x_1}^2 + {x_2}^2 \quad + \lambda\,(x_1 + x_2 - 1\,)$$

$$\nabla f(x) = < 4x_1, 2x_2 >$$

$$\nabla g(x) = < 1,1 >$$

$$\frac{\partial F(x)}{\partial x_1} = 4x_1 + \lambda \quad \frac{\partial F(x)}{\partial x} = 2x_2 + \ \lambda \quad \frac{\partial F(x)}{\partial \lambda} = x_1 + x_2 - 1$$

$$4x_1 + \lambda \ = 0\,, \ \ 2x_2 + \ \lambda = 0 \ \ , x_1 + x_2 - 1 = 0$$

$$x_1 = \frac{-\lambda}{4} \quad , \quad x_2 = \frac{-\lambda}{2} \quad \rightarrow \quad \frac{-\lambda}{4} + \frac{-\lambda}{2} = 1$$

$$\lambda = -\frac{4}{3} \quad \rightarrow \quad x_1 = \frac{1}{3}\,, \quad x_2 = \frac{2}{3}$$

$$H = \nabla\nabla F(x_1, x_2, \lambda) = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$$

Det(H)=4x2-0x0=8

Because Det (H)>0 and $\frac{\partial}{\partial x_1}\frac{\partial F}{\partial x_1} > 0$ *then the point* $(\frac{1}{3}, \frac{2}{3})$ is local minimum