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Problem 2.38

$$P(\mu|D) \propto P(D|\mu) \times P(\mu)$$

$$P(\mu|D) \propto \prod_{i=1}^{n} \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}\right) \times \left(\frac{1}{\sqrt{2\pi}\sigma_0} e^{-\frac{(\mu-\mu_0)^2}{2\sigma_0{}^2}}\right)$$

$$P(\mu|D) \propto \exp\left[\left(\sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{2\sigma^2}\right) + \left(-\frac{(\mu-\mu_0)^2}{2\sigma_0{}^2}\right)\right]$$

$$= \exp\left[-\frac{1}{2}\left(\sum_{i=1}^{n} \frac{x_i{}^2+\mu^2-2x_i\mu}{2\sigma^2} + \frac{\mu^2+\mu_0{}^2-2\mu_0\mu}{2\sigma_0{}^2}\right)\right]$$

$$= \exp\left[-\frac{1}{2}\left(\frac{(\sum_{i=1}^{n} x_i{}^2-2x_i\mu)+N\mu^2}{2\sigma^2} + \frac{\mu^2+\mu_0{}^2-2\mu_0\mu}{2\sigma_0{}^2}\right)\right]$$

$$= \exp\left[-\left(\frac{\sum_{i=1}^{n} x_i{}^2-2x_i\mu}{2\sigma^2}\right) - \mu^2\left(\frac{1}{\sigma_0{}^2}+\frac{N}{\sigma^2}\right) - \left(\frac{\mu_0{}^2-2\mu_0\mu}{2\sigma_0{}^2}\right)\right]$$

$$= \exp\left[-\frac{\mu^2}{2}\left(\frac{1}{\sigma_0{}^2}+\frac{N}{\sigma^2}\right) + 2\mu\left(\frac{\sum_{i=1}^{n} x_i}{2\sigma^2}+\frac{\mu_0}{2\sigma_0{}^2}\right) - \left(\frac{\mu_0{}^2}{2\sigma_0{}^2}+\frac{\sum_{i=1}^{n} x_i{}^2}{2\sigma^2}\right)\right]$$

$$(\text{def}) = \exp\left[-\frac{1}{2\sigma_n{}^2}\left(\mu^2-2\mu\mu_n+\mu_n{}^2\right)\right] = \exp\left[-\frac{1}{2\sigma_n{}^2}(\mu-\mu_n)^2\right]$$

Matching the coefficients of μ_n and ${\sigma_n}^2$

${\boldsymbol{\sigma_n}}^{\mathbf{2}}$

$$-\frac{\mu^2}{2{\sigma_n}^2} = -\frac{\mu^2}{2}\left(\frac{1}{{\sigma_0}^2} + \frac{N}{\sigma^2}\right)$$

$$\frac{1}{{\sigma_n}^2} = \left(\frac{1}{{\sigma_0}^2} + \frac{N}{\sigma^2}\right)$$

$\boldsymbol{\mu_n}$

$$\frac{-2\mu\mu_n}{2{\sigma_n}^2} = 2\mu\left(\frac{\sum_{i=1}^{n} x_i}{2\sigma^2} + \frac{\mu_0}{2{\sigma_0}^2}\right)$$

$$\mu_n = \frac{1}{\left(\frac{1}{{\sigma_0}^2} + \frac{N}{\sigma^2}\right)}\left(\frac{\sum_{i=1}^{n} x_i}{\sigma^2} + \frac{\mu_0}{{\sigma_0}^2}\right)$$

$$\mu_n = \frac{{\sigma_0}^2\sigma^2}{(\sigma^2 + {\sigma_0}^2 N)}\left({\sigma_0}^2\frac{\sum_{i=1}^{n} x_i}{{\sigma_0}^2\sigma^2} + \sigma^2\frac{\mu_0}{{\sigma_0}^2\sigma^2}\right) = \frac{{\sigma_0}^2 N\mu_1}{(\sigma^2 + {\sigma_0}^2 N)} + \frac{\sigma^2\mu_0}{(\sigma^2 + {\sigma_0}^2 N)}$$