Validation file for Equivalence Functions

```r
library(knitr)
opts_chunk$set(tidy.opts=list(width.cutoff=60),tidy=TRUE)
```

Tests against Literature, existing packages, and MiniTab

Below I reproduce some examples from the literature, compare TOSTER against other R packages that have equivalence test functions, and against Minitab. MiniTab software has implemented equivalence tests and power analysis for one-sample, independent, and paired t-tests. It is excellent - by far the most user-friendly software for anyone who prefers a commercial statistics program over an open source solution (it also has equivalence tests for 2x2 crossover designs, but not for correlations or meta-analysis). It also allows users to enter summary statistics, which is used here to compare the output of the equivalence functions in TOSTER against minitab. Note that when SD = 1, the mean difference equals the standardized difference. The MiniTab output is followed by the R function.

TOST for two independent samples

Example 1.1

From page 196 of Stegner, B. L., Bostrom, A. G., & Greenfield, T. K. (1996). Equivalence testing for use in psychosocial and services research: An introduction with examples. Evaluation and Program Planning, 19(3), 193-198. https://doi.org/10.1016/0149-7189(96)00011-0

Randomized controlled trial of two conditions of case management, with the main dependent variable being ratings on the Brief Psychiatric Rating Scale (BPRS). The means, sd's, and sample sizes are:

Control: M = 1.5679, SD = 0.4285, n = 64 Experimental: M = 1.6764, SD = 0.4748, n = 70

Calculated t-values for TOST are t1 = 5.383 and t2 = -2.616

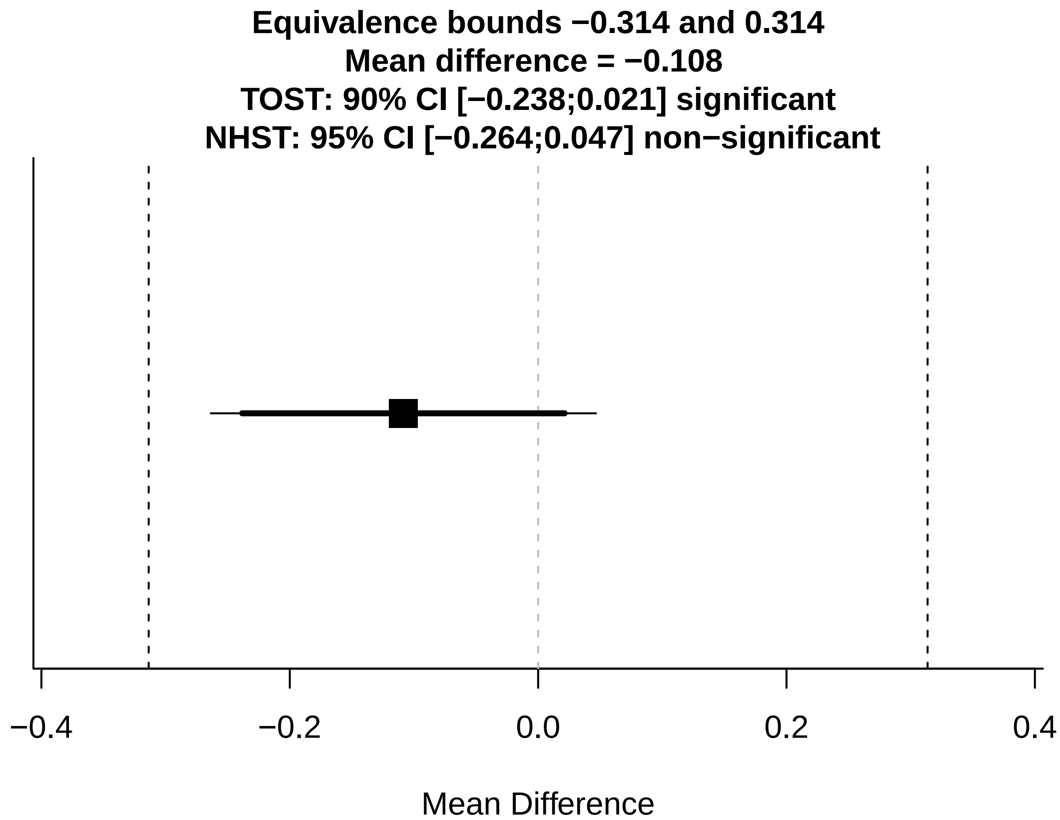
These results can be reproduced with TOSTER package and the TOSTtwo.raw R code:

```r
require(TOSTER)
```

```
## Loading required package: TOSTER
```

```r
TOSTtwo.raw(m1 = 1.5679, m2 = 1.6764, sd1 = 0.4285, sd2 = 0.4748,
    n1 = 64, n2 = 70, low_eqbound = -0.3136, high_eqbound = 0.3136,
    alpha = 0.05, var.equal = TRUE)
```

```
## Using alpha = 0.05 Student's t-test was non-significant, t(132) = -1.384005, p = 0.1686925

##

##
## Using alpha = 0.05 the equivalence test based on Student's t-test was significant, t(132) = 2.616216

##
```




```
## TOST results:
##   t-value 1   p-value 1 t-value 2    p-value 2  df
## 1  2.616216 0.004963596 -5.384227 1.612713e-07 132
##
## Equivalence bounds (raw scores):
##   low bound raw high bound raw
## 1       -0.3136         0.3136
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1             -0.2383608             0.02136078
```

Example 1.2

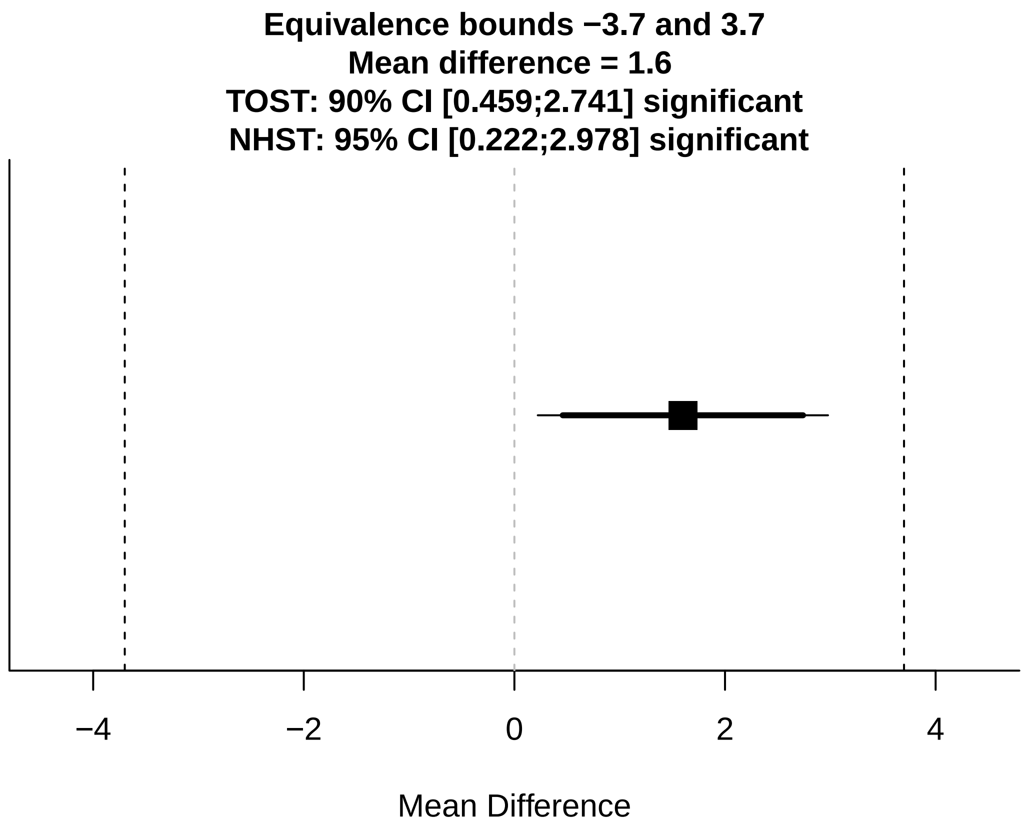
From page 225 of Limentani, G. B., Ringo, M. C., Ye, F., Bergquist, M. L., & MCSorley, E. O. (2005). Beyond the t-test: statistical equivalence testing. Analytical Chemistry, 77(11), 221-A.

Reproducing the 90% CI and significant equivalence test.

```
TOSTtwo.raw(m1 = 89.3, m2 = 87.7, sd1 = 1.9, sd2 = 1.3, n1 = 12,
    n2 = 12, low_eqbound = -3.7, high_eqbound = 3.7, alpha = 0.05,
    var.equal = TRUE)
```

```
## Using alpha = 0.05 Student's t-test was significant, t(22) = 2.407535, p = 0.02488925

##

##
```

```
## Using alpha = 0.05 the equivalence test based on Student's t-test was significant, t(22) = -3.15989,
##
```




```
## TOST results:
##   t-value 1    p-value 1 t-value 2   p-value 2 df
## 1  7.974961 3.109178e-08  -3.15989 0.002270472 22
##
## Equivalence bounds (raw scores):
##   low bound raw high bound raw
## 1          -3.7            3.7
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1              0.4588201                2.74118
```

Example 1.3

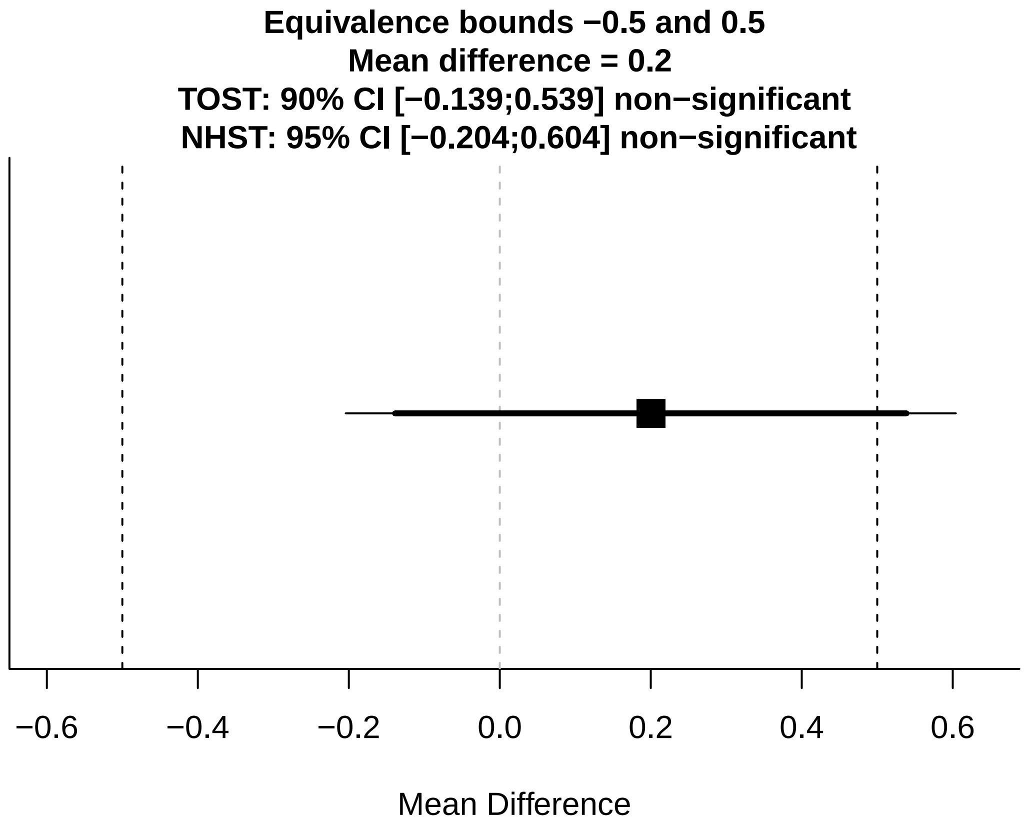
From Meyners, M. (2012). Equivalence tests - A review. Food Quality and Preference, 26(2), 231-245. https://doi.org/10.1016/j.foodqual.2012.05.003, page 237:

To illustrate the approach, we consider the data presented by Bi (2005) of 100 consumers each from two cities A and B, giving overall liking scores on a 9-point scale. The equivalence margin is set to d = 0.5, i.e. a difference of 0.5 or less in mean values is considered negligible. With mean values of lA = 7.1 and lB = 6.9 and variances s2A = 2.0 and s2B = 2.2, the corresponding t-statistic is 0.976, and the p-values for the two one-sided test problems become p1 = 0.0717 and p2 = 0.0003.

We can use the TOST procedure for an independent t-test to reproduce the p-values and 90% CI:

```
TOSTtwo.raw(m1 = 7.1, m2 = 6.9, sd1 = sqrt(2), sd2 = sqrt(2.2),
    n1 = 100, n2 = 100, low_eqbound = -0.5, high_eqbound = 0.5,
    alpha = 0.05, var.equal = TRUE)
```

```
## Using alpha = 0.05 Student's t-test was non-significant, t(198) = 0.9759001, p = 0.3303048

##

##
## Using alpha = 0.05 the equivalence test based on Student's t-test was non-significant, t(198) = -1.46

##
```




```
## TOST results:
##   t-value 1    p-value 1 t-value 2  p-value 2  df
## 1   3.41565 0.0003859877  -1.46385 0.07241035 198
##
## Equivalence bounds (raw scores):
##   low bound raw high bound raw
## 1          -0.5            0.5
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1             -0.1386793              0.5386793
```

This outcome matches the result of (page 237): The 90% CI is [0.139; 0.539] with a p-value of approximately 0.07.

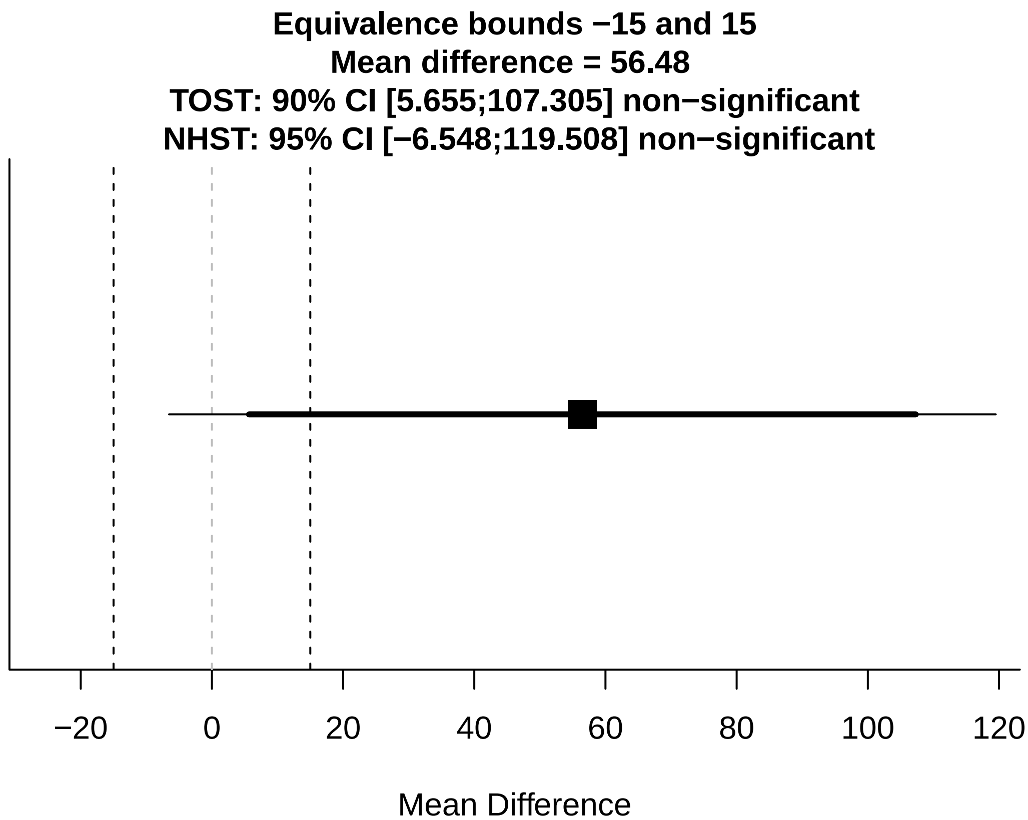
Example 1.4

From Table 1 in Quertemont, E. (2011). How to Statistically Show the Absence of an Effect. Psychologica Belgica, 51(2), 109. https://doi.org/10.5334/pb-51-2-109

Example 1: t(8) = 1.52; p = 0.92 Example 2: t(298) = -2.23; p = 0.013

```
# Example 1
TOSTtwo.raw(m1 = 459.09, m2 = 402.61, sd1 = 47.53, sd2 = 38.42,
    n1 = 5, n2 = 5, low_eqbound = -15, high_eqbound = 15, alpha = 0.05,
    var.equal = TRUE)
```

```
## Using alpha = 0.05 Student's t-test was non-significant, t(8) = 2.066441, p = 0.07262966

##

##
## Using alpha = 0.05 the equivalence test based on Student's t-test was non-significant, t(8) = 1.5176

##
```

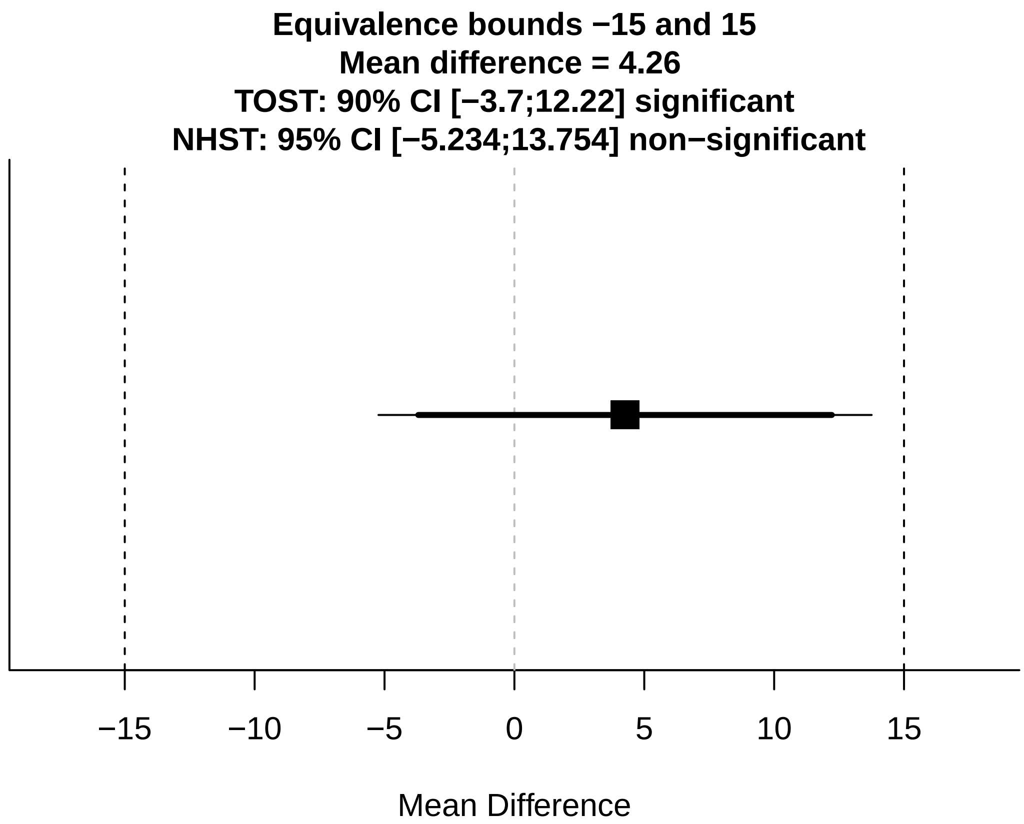



```
## TOST results:
##   t-value 1  p-value 1 t-value 2 p-value 2 df
## 1  2.615247 0.01543878  1.517634  0.916208  8
##
## Equivalence bounds (raw scores):
##   low bound raw high bound raw
## 1           -15             15
##
```

```
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1               5.654791               107.3052
```

```
# Example 2
TOSTtwo.raw(m1 = 407.24, m2 = 402.98, sd1 = 41.77, sd2 = 41.79,
    n1 = 150, n2 = 150, low_eqbound = -15, high_eqbound = 15,
    alpha = 0.05, var.equal = TRUE)
```

```
## Using alpha = 0.05 Student's t-test was non-significant, t(298) = 0.8830225, p = 0.3779359

##

##
## Using alpha = 0.05 the equivalence test based on Student's t-test was significant, t(298) = -2.22621

##
```




```
## TOST results:
##   t-value 1    p-value 1 t-value 2 p-value 2  df
## 1  3.992257 4.123961e-05 -2.226212 0.0133738 298
##
## Equivalence bounds (raw scores):
##   low bound raw high bound raw
## 1           -15             15
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1              -3.700077               12.22008
```

These results perfectly match those in Table 1 of Quertemont (2011).

Example 1.5

MiniTab by default uses Welch's t-test when testing for equivalence, which does not require the assumption of equal variances, which matters with unequal sample sizes. TOSTER also presents Welch's t-test by default. Note that minitab rounds degrees of freedom down to a whole number. This is a conservative procedure. TOSTER does not. The difference is rarely noticable.

MiniTab Result Two-Sample Equivalence Test

*Method

Test mean = mean of Test Reference mean = mean of Reference Equal variances were not assumed for the analysis.

Descriptive Statistics

Variable N Mean StDev SE Mean Test 86 2,5 1,3 0,14018 Reference 102 2,3 1,1 0,10892

Difference: Mean(Test) - Mean(Reference)

Difference SE 95% CI Equivalence Interval 0,20000 0,17752 (-0,093626; 0,49363) (-0,4; 0,4)

CI is not within the equivalence interval. Cannot claim equivalence.

Test

Null hypothesis: Difference < or = -0,4 or Difference > or = 0,4 Alternative hypothesis: -0,4 < Difference < 0,4 alpha level: 0,05

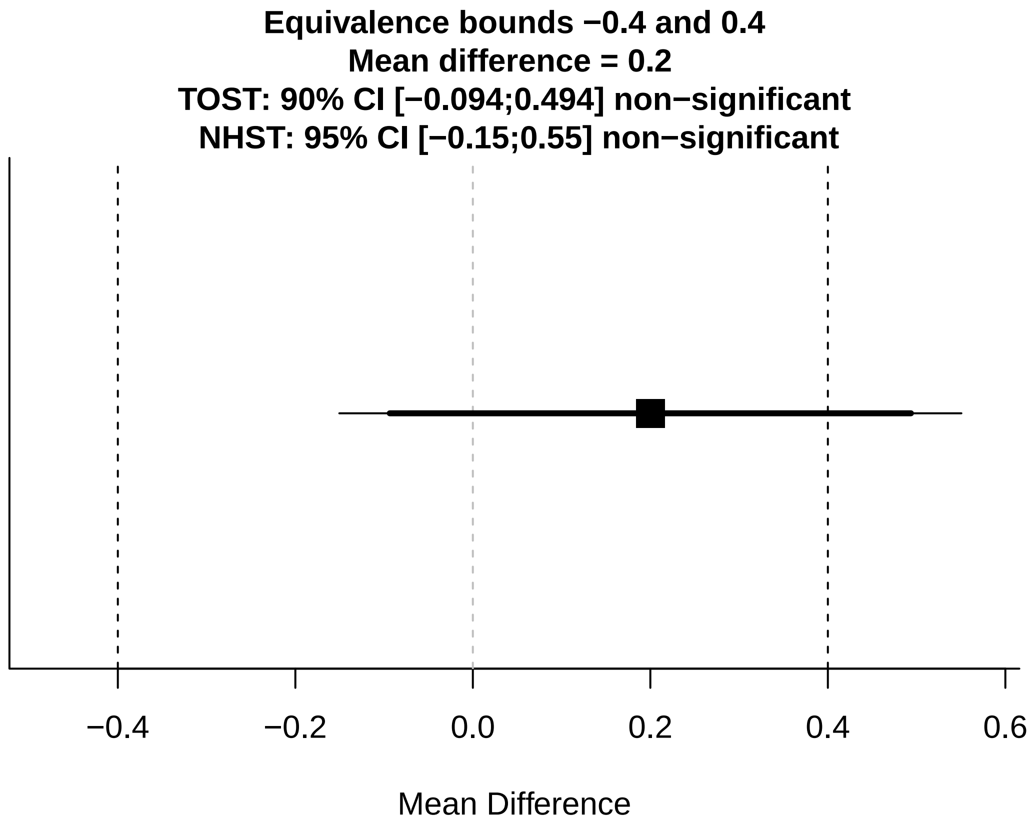
Null Hypothesis DF T-Value P-Value Difference < or = -0,4 167 3,3799 0,000 Difference > or = 0,4 167 -1,1266 0,131

The greater of the two P-Values is 0,131. Cannot claim equivalence.

Reproduce results in TOSTER

```r
TOSTtwo.raw(m1 = 2.5, m2 = 2.3, sd1 = 1.3, sd2 = 1.1, n1 = 86,
    n2 = 102, low_eqbound = -0.4, high_eqbound = 0.4, alpha = 0.05)
```

```
## Using alpha = 0.05 Welch's t-test was non-significant, t(167.2924) = 1.126624, p = 0.2615151

##

##
## Using alpha = 0.05 the equivalence test based on Welch's t-test  was non-significant, t(167.2924) = 

##
```




```
## TOST results:
##   t-value 1    p-value 1 t-value 2 p-value 2       df
## 1  3.379871 0.0004512547 -1.126624 0.1307575 167.2924
##
## Equivalence bounds (raw scores):
##   low bound raw high bound raw
## 1          -0.4            0.4
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1            -0.09362298               0.493623
```

Example 1.6

Now for the same data as example 1.5, but assuming equal variances:

Results MiniTab Two-Sample Equivalence Test

Method

Test mean = mean of Test Reference mean = mean of Reference Equal variances were assumed for the analysis.

Descriptive Statistics

Variable N Mean StDev SE Mean Test 86 2,5 1,3 0,14018 Reference 102 2,3 1,1 0,10892

Pooled StDev = 1,19556

Difference: Mean(Test) - Mean(Reference)

Difference SE 95% CI Equivalence Interval 0,20000 0,17502 (-0,089331; 0,48933) (-0,4; 0,4)

CI is not within the equivalence interval. Cannot claim equivalence.

Test

Null hypothesis: Difference < or = -0,4 or Difference > or = 0,4 Alternative hypothesis: -0,4 < Difference < 0,4 alpha level: 0,05

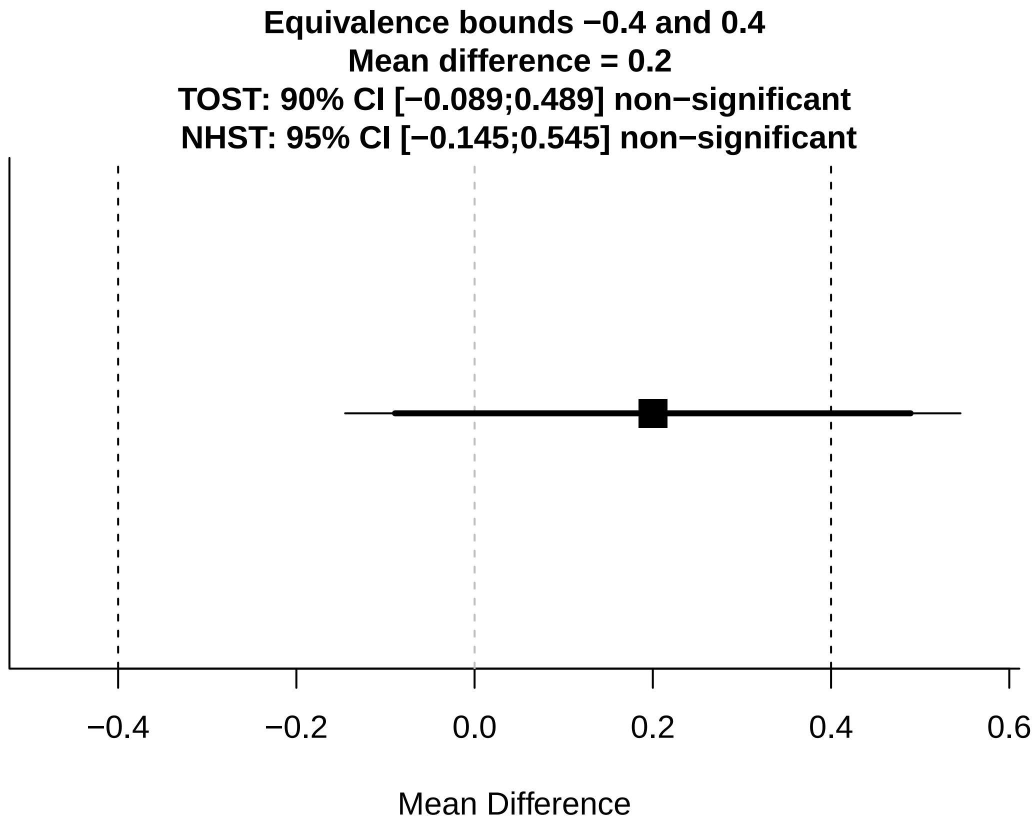
Null Hypothesis DF T-Value P-Value Difference < or = -0,4 186 3,4281 0,000 Difference > or = 0,4 186 -1,1427 0,127

The greater of the two P-Values is 0,127. Cannot claim equivalence.

Reproducing results with TOSTER

```
TOSTtwo.raw(m1 = 2.5, m2 = 2.3, sd1 = 1.3, sd2 = 1.1, n1 = 86,
    n2 = 102, low_eqbound = -0.4, high_eqbound = 0.4, alpha = 0.05,
    var.equal = TRUE)
```

```
## Using alpha = 0.05 Student's t-test was non-significant, t(186) = 1.142695, p = 0.254634

##

##
## Using alpha = 0.05 the equivalence test based on Student's t-test was non-significant, t(186) = -1.14

##
```




```
## TOST results:
##   t-value 1    p-value 1 t-value 2 p-value 2  df
## 1  3.428085 0.0003742019 -1.142695  0.127317 186
##
## Equivalence bounds (raw scores):
##   low bound raw high bound raw
## 1          -0.4            0.4
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1            -0.08933128              0.4893313
```

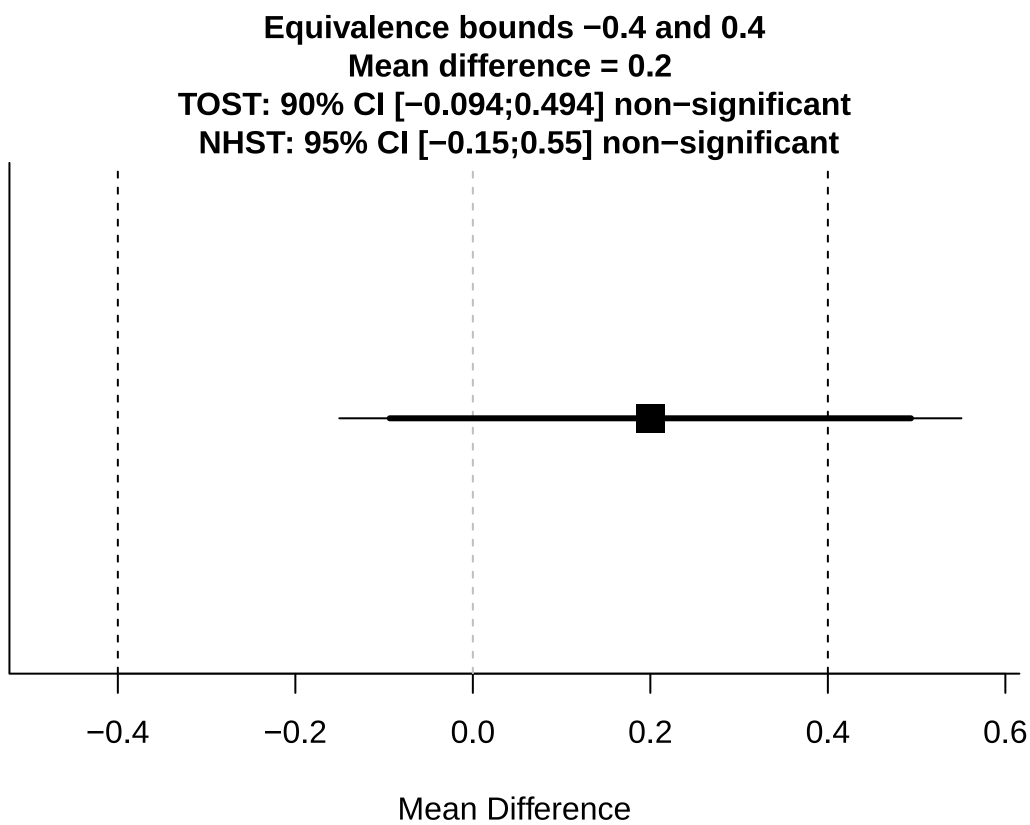
```
TOSTtwo.raw(m1 = 2.5, m2 = 2.3, sd1 = 1.3, sd2 = 1.1, n1 = 86,
    n2 = 102, low_eqbound = -0.4, high_eqbound = 0.4, alpha = 0.05)
```

```
## Using alpha = 0.05 Welch's t-test was non-significant, t(167.2924) = 1.126624, p = 0.2615151
```

```
##
```

```
##
## Using alpha = 0.05 the equivalence test based on Welch's t-test  was non-significant, t(167.2924) = 
```

```
##
```




```
## TOST results:
##   t-value 1    p-value 1 t-value 2 p-value 2       df
## 1  3.379871 0.0004512547 -1.126624 0.1307575 167.2924
##
## Equivalence bounds (raw scores):
##   low bound raw high bound raw
## 1          -0.4            0.4
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1            -0.09362298               0.493623
```

Paired t-test

Example 2.1

We can compare the TOSTpaired.raw function against the equivalence package.

Paired t-test in equivalence package (from example in tost function) and TOSTpaired.raw.

```
require(equivalence)
```

```
## Loading required package: equivalence
```

```
## Loading required package: lattice
```

```
## Loading required package: boot
```

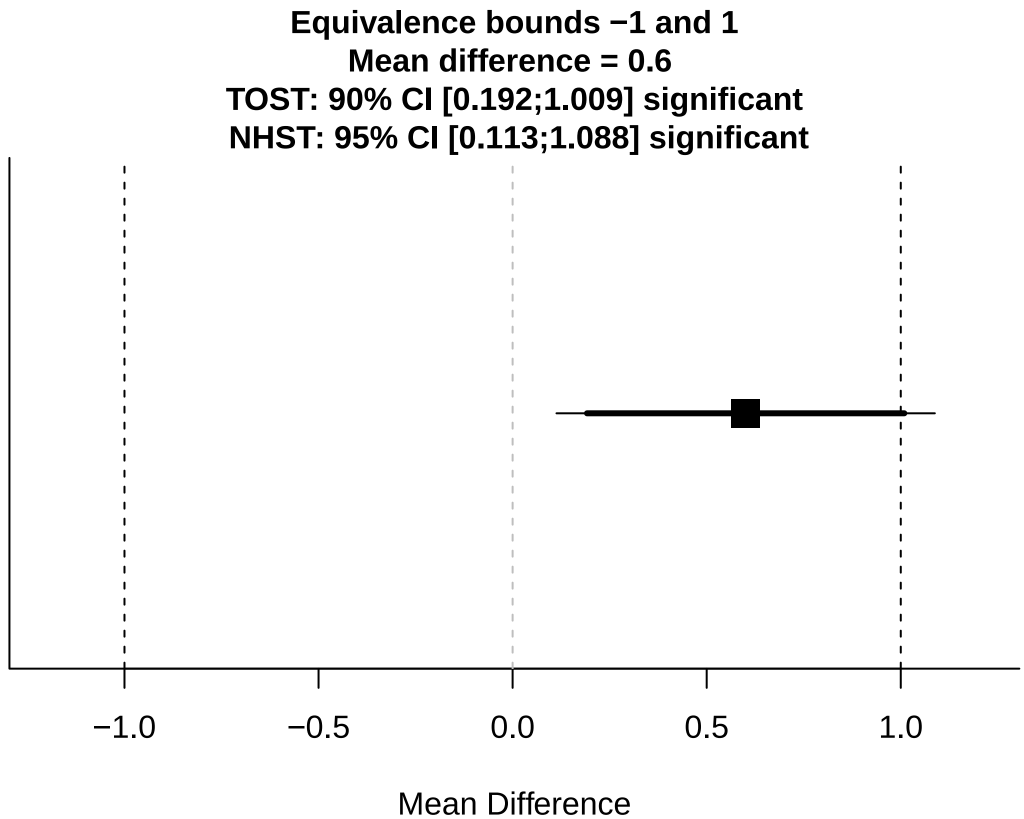
```
##
## Attaching package: 'boot'
## The following object is masked from 'package:lattice':
##
##     melanoma
## Loading required package: PairedData
## Loading required package: MASS
## Loading required package: gld
## Loading required package: mvtnorm
## Loading required package: ggplot2
##
## Attaching package: 'PairedData'
## The following object is masked from 'package:base':
##
##     summary
```

```r
data(ufc)
# Remove missing data
ufc <- ufc[complete.cases(ufc[, 9:10]), ]

tost(ufc$Height.m.p, ufc$Height.m, epsilon = 1, paired = TRUE)
```

```
##
##  Paired TOST
##
## data:  ufc$Height.m.p and ufc$Height.m
## df = 376
## sample estimates:
## mean of the differences
##               0.6003913
##
## Epsilon: 1
## 95 percent two one-sided confidence interval (TOST interval):
##  0.1916419 1.0091407
## Null hypothesis of statistical difference is: not rejected
## TOST p-value: 0.05389588
```

```r
TOSTpaired.raw(n = length(ufc$Height.m.p), m1 = mean(ufc$Height.m.p,
    na.rm = TRUE), m2 = mean(ufc$Height.m, na.rm = TRUE), sd1 = sd(ufc$Height.m.p,
    na.rm = TRUE), sd2 = sd(ufc$Height.m, na.rm = TRUE), r12 = cor(ufc$Height.m,
    ufc$Height.m.p, use = "pairwise.complete.obs"), low_eqbound = -1,
    high_eqbound = 1, alpha = 0.05)
```

```
## Using alpha = 0.05 the NHST t-test was significant, t(376) = 2.42201, p = 0.01590709
##
##
## Using alpha = 0.05 the equivalence test was significant, t(376) = -16.99448, p = 9.130218e-49
##
```




```
## TOST results:
##   t-value 1    p-value 1 t-value 2    p-value 2  df
## 1   21.8385 3.655235e-69 -16.99448 9.130218e-49 376
##
## Equivalence bounds (raw scores):
##   low bound raw high bound raw
## 1            -1              1
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1              0.1916419               1.009141
```

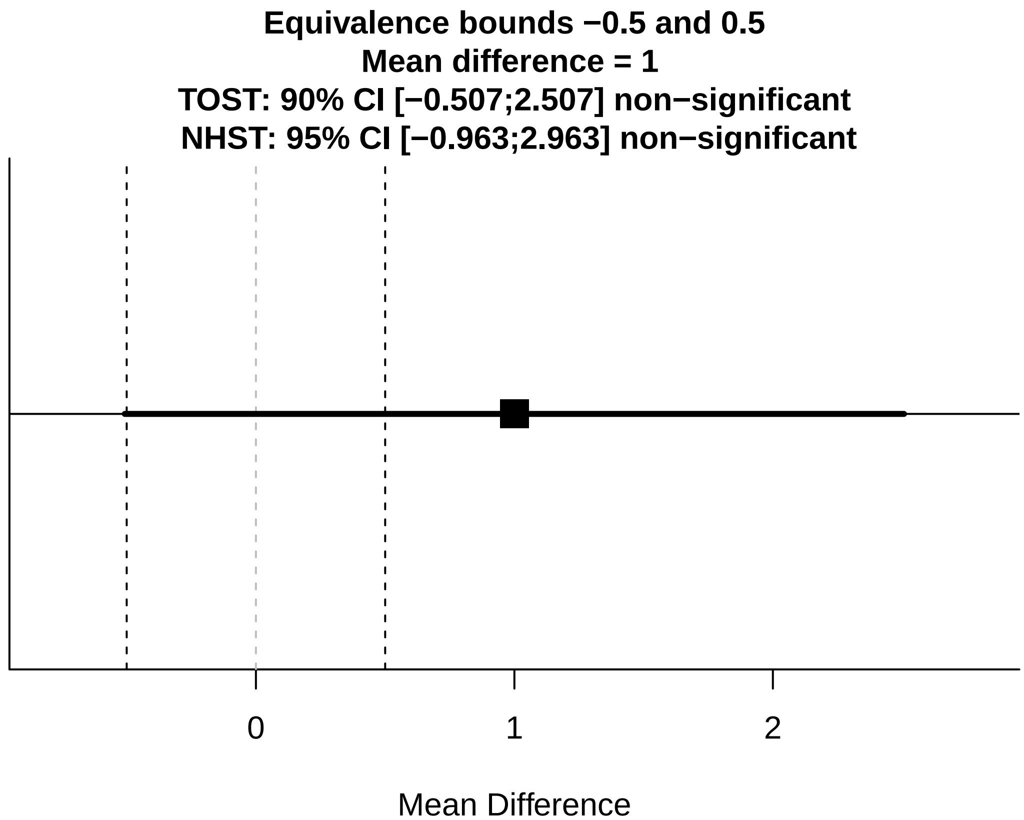
Example 2.2

Quertemont also provides an example for paired t-tests in the appendix (page 124): Mdif = 1, SDif = 1.58, d = .5, t = 0.71, df = 4, p = .74 with a 95% CI from -.96 to + 2.96.

```
morning <- c(3, 4, 4, 5, 4)
evening <- c(1, 4, 1, 4, 5)

TOSTpaired.raw(n = 5, m1 = mean(morning), m2 = mean(evening),
    sd1 = sd(morning), sd2 = sd(evening), r12 = cor(morning,
        evening), low_eqbound = -0.5, high_eqbound = 0.5, alpha = 0.05)
```

```
## Using alpha = 0.05 the NHST t-test was non-significant, t(4) = 1.414214, p = 0.2301996
##
```

```
##
## Using alpha = 0.05 the equivalence test was non-significant, t(4) = 0.2961796, p = 0.6090832
##
```




```
## TOST results:
##   t-value 1  p-value 1 t-value 2 p-value 2 df
## 1  2.532248 0.03225513 0.2961796 0.6090832  4
##
## Equivalence bounds (raw scores):
##   low bound raw high bound raw
## 1          -0.5            0.5
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1             -0.5074433               2.507443
```

This output matches the calculations in Quertemont (2011).

Example 2.3

MiniTab does not allow users to perform equivalence tests for paired samples t-test based on summary data, but this equivalence test can be calculated based on the raw data.

Data: DV1<-c(4,2,4,3,5,4,3,4,5,4,2,3,4,5,5) DV2<-c(3,4,2,3,5,4,3,4,5,3,4,3,2,3,4)

Results MiniTab Equivalence Test with Paired Data: DV1, DV2

Method

Test mean = mean of DV1 Reference mean = mean of DV2

Descriptive Statistics

Variable N Mean StDev SE Mean DV1 15 3,8000 1,0142 0,26186 DV2 15 3,4667 0,91548 0,23637

Difference: Mean(DV1) - Mean(DV2)

Difference StDev SE 95% CI Equivalence Interval 0,33333 1,23443 0,31873 (-0,22804; 0,89471) (-0,8; 0,8)

CI is not within the equivalence interval. Cannot claim equivalence.

Test

Null hypothesis: Difference < or = -0,8 or Difference > or = 0,8 Alternative hypothesis: -0,8 < Difference < 0,8 alpha level: 0,05

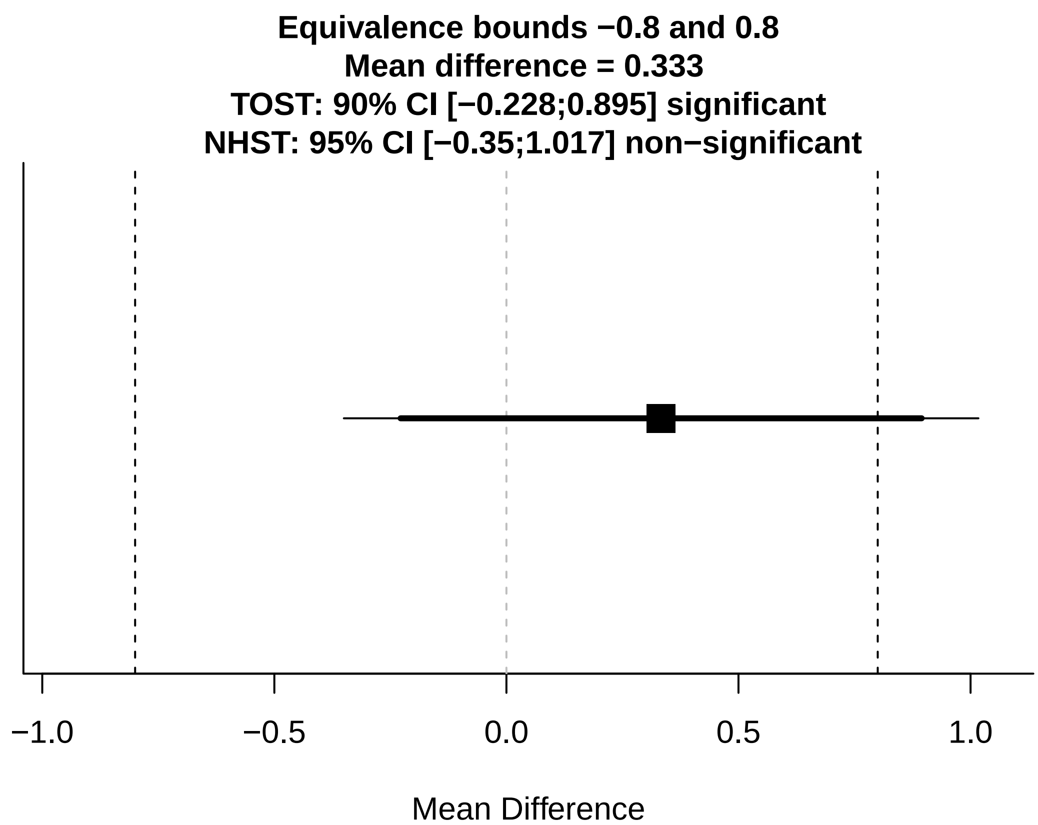
Null Hypothesis DF T-Value P-Value Difference < or = -0,8 14 3,5558 0,002 Difference > or = 0,8 14 -1,4642 0,083

The greater of the two P-Values is 0,083. Cannot claim equivalence.

Reproduce Results with TOSTER

```
DV1 <- c(4, 2, 4, 3, 5, 4, 3, 4, 5, 4, 2, 3, 4, 5, 5)
DV2 <- c(3, 4, 2, 3, 5, 4, 3, 4, 5, 3, 4, 3, 2, 3, 4)

TOSTpaired.raw(n = length(DV1), m1 = mean(DV1), m2 = mean(DV2),
    sd1 = sd(DV1), sd2 = sd(DV2), r12 = cor(DV1, DV2), low_eqbound = -0.8,
    high_eqbound = 0.8, alpha = 0.05)
```

```
## Using alpha = 0.05 the NHST t-test was non-significant, t(14) = 1.045825, p = 0.3133636

##

##
## Using alpha = 0.05 the equivalence test was significant, t(14) = -2.052562, p = 0.02964983

##
```




```
## TOST results:
##   t-value 1    p-value 1 t-value 2  p-value 2 df
## 1  4.144212 0.0004963511 -2.052562 0.02964983 14
##
## Equivalence bounds (raw scores):
##   low bound raw high bound raw
## 1          -0.8            0.8
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1             -0.2280449              0.8947115
```

One-sample t-test

Example 3.1

Result MiniTab One-Sample Equivalence Test

Method

Target = 0

Descriptive Statistics

N Mean StDev SE Mean 100 -0,3 1 0,10000

Difference: Mean - Target

Difference SE 95% CI Equivalence Interval -0,30000 0,10000 (-0,46604; 0) (-0,5; 0,5)

CI is within the equivalence interval. Can claim equivalence.

Test

Null hypothesis: Difference < or = -0,5 or Difference > or = 0,5 Alternative hypothesis: -0,5 < Difference < 0,5 alpha level: 0,05

Null Hypothesis DF T-Value P-Value Difference < or = -0,5 99 2,0000 0,024 Difference > or = 0,5 99 -8,0000 0,000

The greater of the two P-Values is 0,024. Can claim equivalence.

Reproduce with TOSTER:

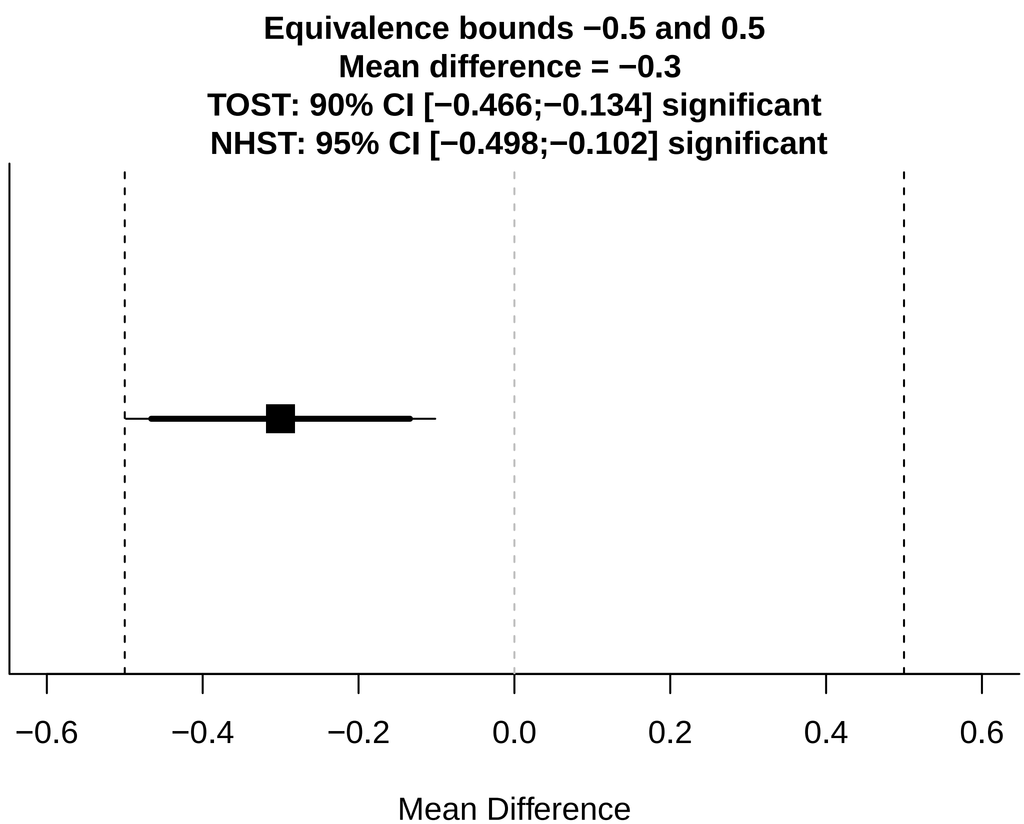
```
TOSTone.raw(m = -0.3, mu = 0, sd = 1, n = 100, low_eqbound = -0.5,
    high_eqbound = 0.5, alpha = 0.05)
```

```
## Using alpha = 0.05 the NHST one-sample t-test was significant, t(99) = -3, p = 0.003415508
```

```
##
```

```
##
## Using alpha = 0.05 the equivalence test was significant, t(99) = 2, p = 0.02411985
```

```
##
```

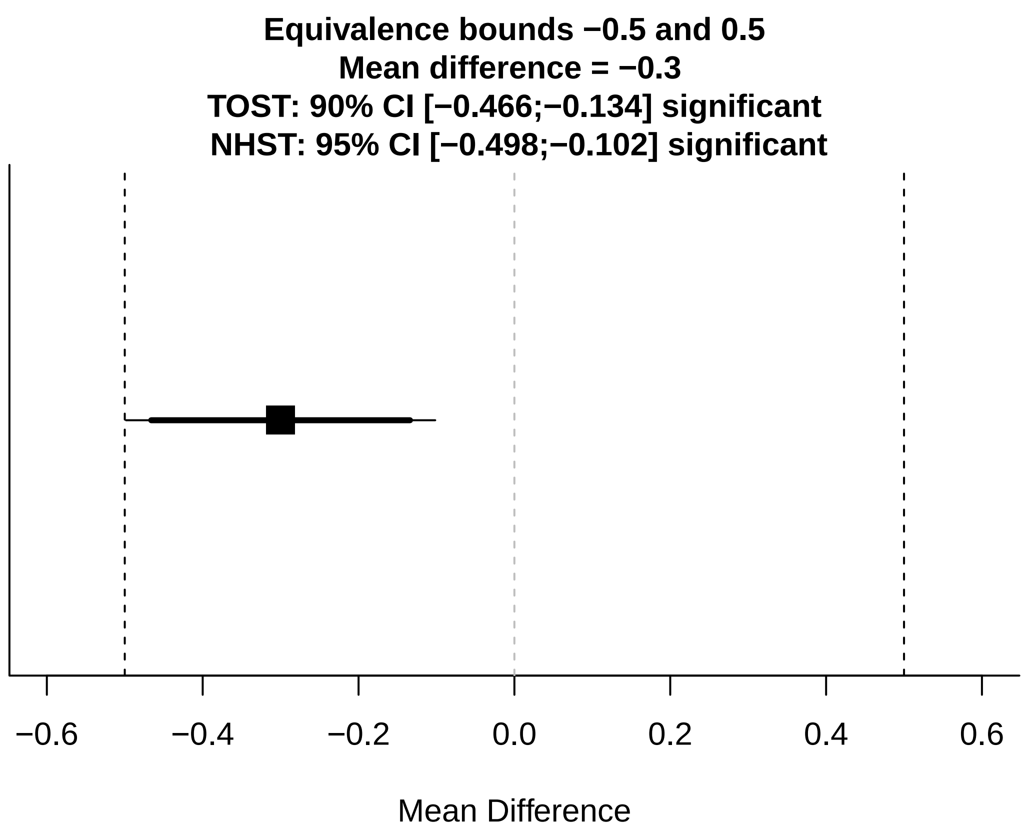


```
## TOST results:
##   t-value 1  p-value 1 t-value 2    p-value 2 df
## 1         2 0.02411985        -8 1.200152e-12 99
##
## Equivalence bounds (raw scores):
##   low bound raw high bound raw
## 1          -0.5            0.5
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1             -0.4660391             -0.1339609
```

Which should be identical to the TOSTone test (because when SD = 1, unstandardized scores equal standardized scores)

```r
TOSTone(m = -0.3, mu = 0, sd = 1, n = 100, low_eqbound_d = -0.5,
    high_eqbound_d = 0.5, alpha = 0.05)
```

```
## Using alpha = 0.05 the NHST one-sample t-test was significant, t(99) = -3, p = 0.003415508

##

##
## Using alpha = 0.05 the equivalence test was significant, t(99) = 2, p = 0.02411985

##
```




```
## TOST results:
##   t-value 1  p-value 1 t-value 2    p-value 2 df
```

```
## 1         2 0.02411985        -8 1.200152e-12 99
##
## Equivalence bounds (Cohen's d):
##   low bound d high bound d
## 1        -0.5          0.5
##
## Equivalence bounds (raw scores):
##   low bound raw high bound raw
## 1          -0.5            0.5
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1             -0.4660391             -0.1339609
```

Example 3.2

MiniTab Result One-Sample Equivalence Test

Method

Target = 0

Descriptive Statistics

N Mean StDev SE Mean 100 1 1,2 0,12000

Difference: Mean - Target

Difference SE 95% CI Equivalence Interval 1,0000 0,12000 (0; 1,1992) (-0,8; 0,8)

CI is not within the equivalence interval. Cannot claim equivalence.

Test

Null hypothesis: Difference < or = -0,8 or Difference > or = 0,8 Alternative hypothesis: -0,8 < Difference < 0,8 alpha level: 0,05

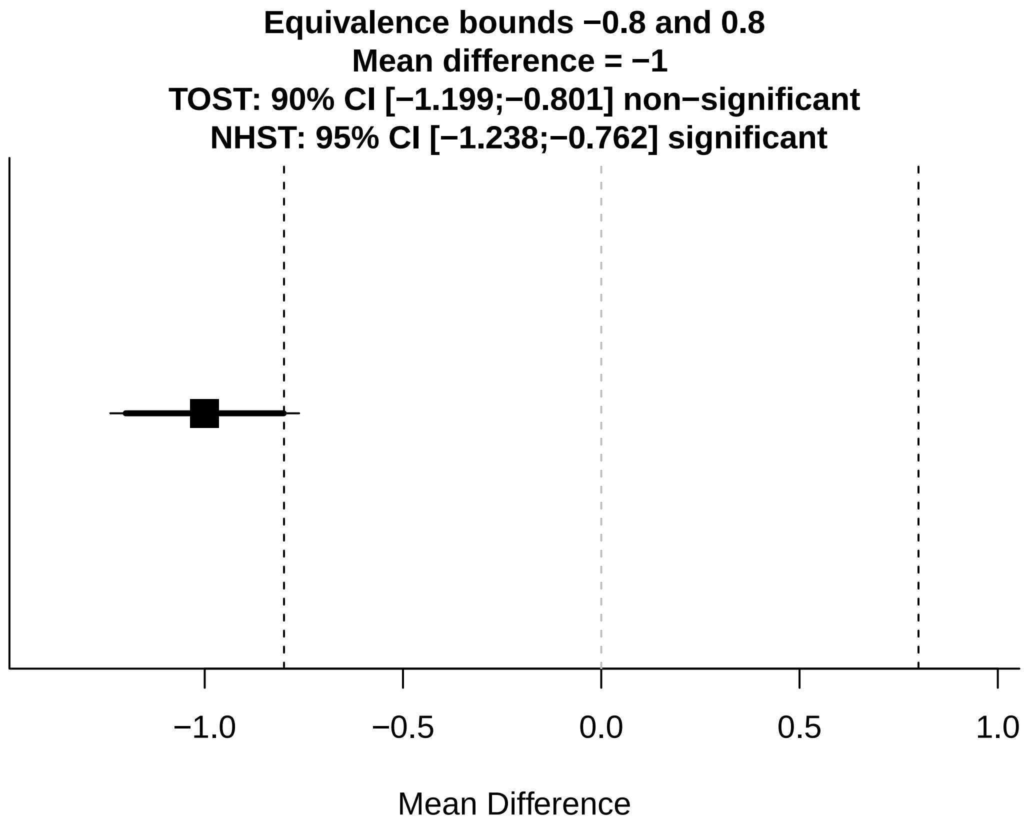
Null Hypothesis DF T-Value P-Value Difference < or = -0,8 99 15,000 0,000 Difference > or = 0,8 99 1,6667 0,951

The greater of the two P-Values is 0,951. Cannot claim equivalence.

Reproducing the results with TOSTER

```r
TOSTone.raw(m = -1, mu = 0, sd = 1.2, n = 100, low_eqbound = -0.8,
    high_eqbound = 0.8, alpha = 0.05)
```

```
## Using alpha = 0.05 the NHST one-sample t-test was significant, t(99) = -8.333333, p = 4.611473e-13

##

##
## Using alpha = 0.05 the equivalence test was non-significant, t(99) = -1.666667, p = 0.9506292

##
```




```
## TOST results:
##   t-value 1 p-value 1 t-value 2    p-value 2 df
## 1 -1.666667 0.9506292       -15 1.553349e-27 99
##
## Equivalence bounds (raw scores):
##   low bound raw high bound raw
## 1          -0.8            0.8
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1              -1.199247             -0.8007531
```

Equivalence test for correlations

Example 4.1

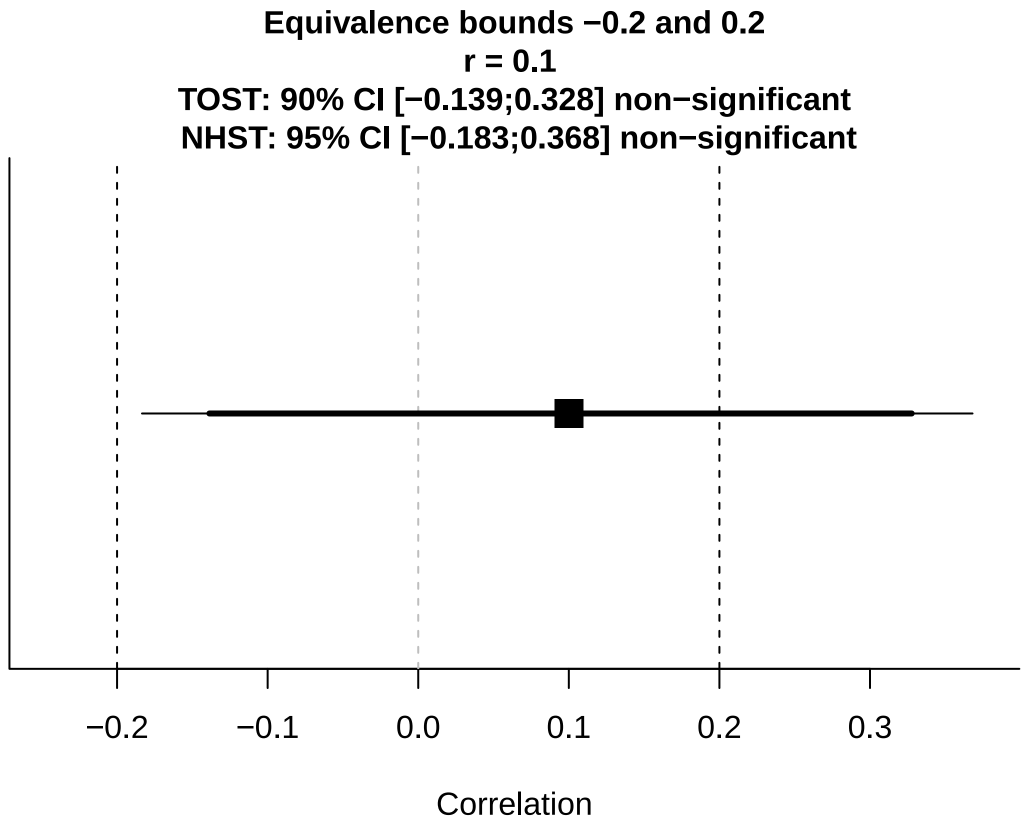
Quertemont (2011) provides an example in the appendix (page 126): Suppose a sample with N = 50, r = .1, and d = .2, and calculated that p(Z < -0.70) = .24. The confidence interval for r is -0.18 to 0.37

We can reproduce this result using the TOSTr function:

```
TOSTr(n = 50, r = 0.1, low_eqbound_r = -0.2, high_eqbound_r = 0.2,
    alpha = 0.05)
```

```
## Using alpha = 0.05 the NHST t-test was non-significant, p = 0.4895926
```

```
##
##
## Using alpha = 0.05 the equivalence test was non-significant, p = 0.2413396
##
```




```
## TOST results:
##    p-value 1 p-value 2
## 1 0.01886717 0.2413396
##
## Equivalence bounds (r):
##   low bound r high bound r
## 1        -0.2          0.2
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1             -0.1386915              0.3277112
```

The resulting p-value and 95% CI match the calculations by Quertemont (2011).

Example 4.2

The R code from Goertzen & Cribbie, page 536, was used:

```
# Running an original two t test procedure for equivalence
equivint <- 0.3
corxy <- 0.02
```

```r
n <- 100
alpha <- 0.05

zei <- log((1 + equivint)/(1 - equivint))/2
zcorxy <- log((1 + corxy)/(1 - corxy))/2
equivt1_fz <- (zcorxy - zei)/(1/sqrt(n - 3))
pvalue1_fz <- pnorm(equivt1_fz)
equivt2_fz <- (zcorxy + zei)/(1/sqrt(n - 3))
pvalue2_fz <- 1 - pnorm(equivt2_fz)
ifelse(pvalue1_fz <= alpha & pvalue2_fz <= alpha, decis_fz <- "The null hypothesis that the correlation
    decis_fz <- "The null hypothesis that the correlation between var1 and var2 falls outside of the eq
```

```
## [1] "The null hypothesis that the correlation between var1 and var2 falls outside of the equivalence
```

```r
pvalue1_fz
```

```
## [1] 0.002176282
```

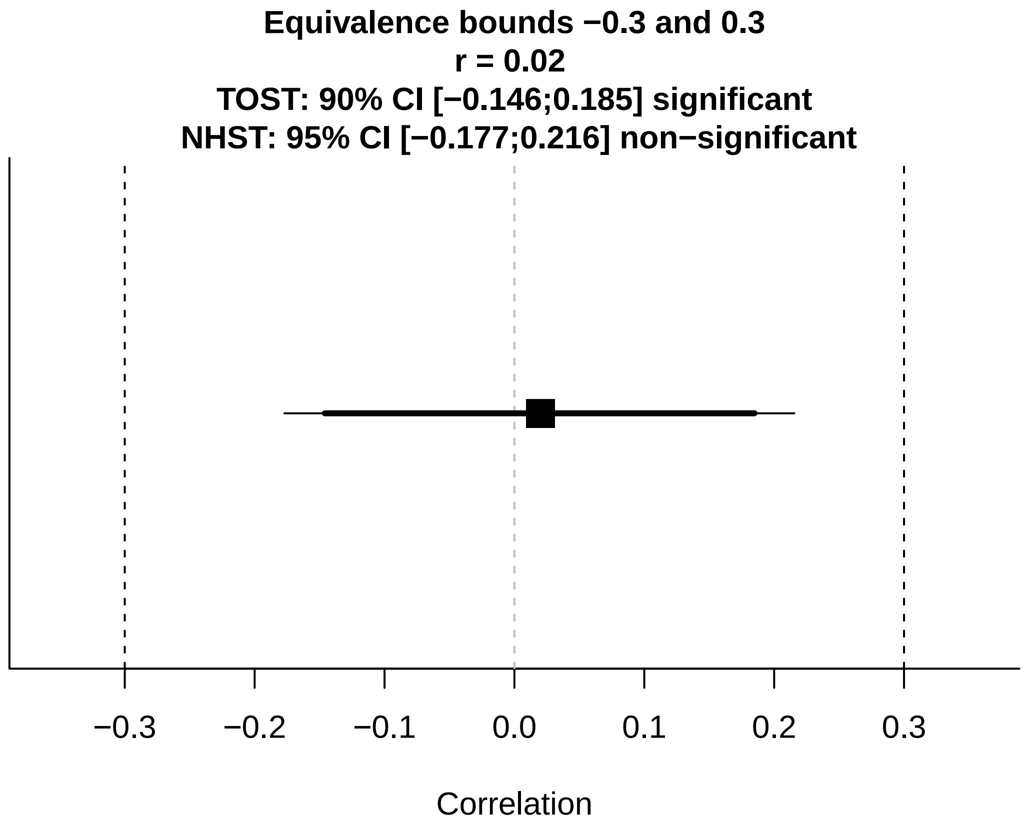
```r
pvalue2_fz
```

```
## [1] 0.0005863917
```

Which is identical to the p-values observed in the spreadsheet, and returned by the the TOSTr function:

```r
TOSTr(n = 100, r = 0.02, low_eqbound_r = -0.3, high_eqbound_r = 0.3,
    alpha = 0.05)
```

```
## Using alpha = 0.05 the NHST t-test was non-significant, p = 0.8434322

##

##
## Using alpha = 0.05 the equivalence test was significant, p = 0.002176282

##
```




```
## TOST results:
##      p-value 1   p-value 2
## 1 0.0005863917 0.002176282
##
## Equivalence bounds (r):
##   low bound r high bound r
## 1        -0.3          0.3
##
## TOST confidence interval:
##   Lower Limit 90% CI raw Upper Limit 90% CI raw
## 1              -0.145957              0.1848622
```

Meta-analaysis based on Cohen's d

Example 5.1

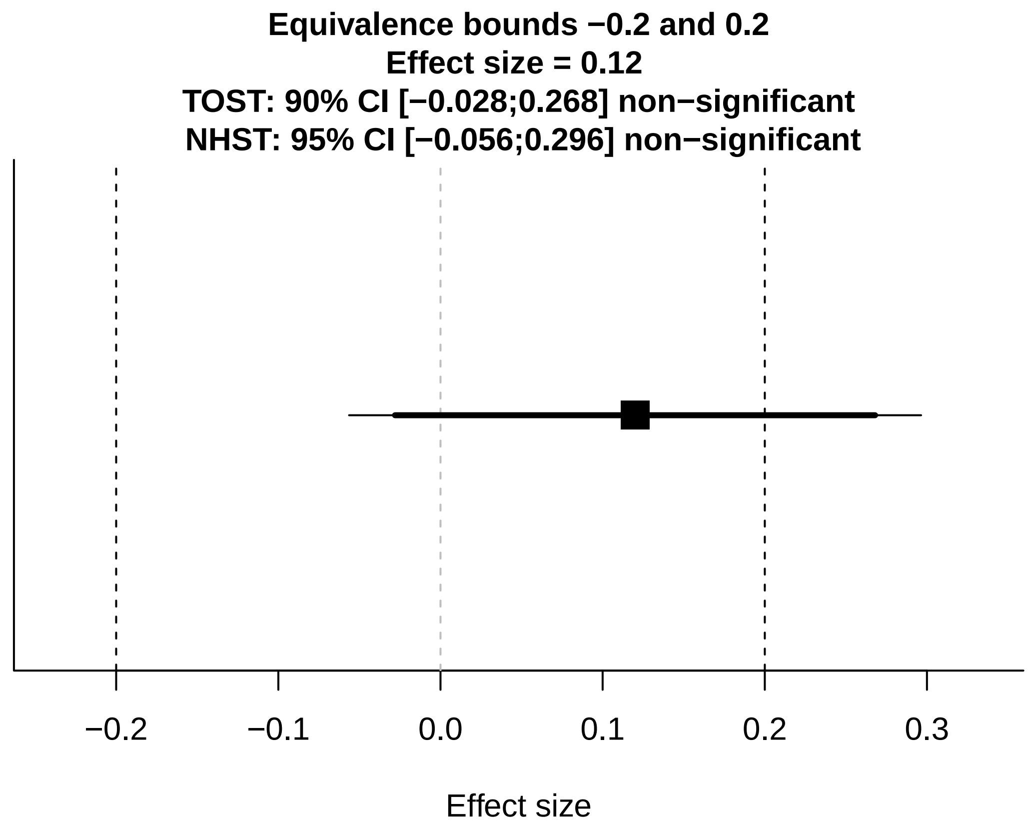
We can reproduce an example for a meta-analysis from From Rogers, J. L., Howard, K. I., & Vessey, J. T. (1993). Using significance tests to evaluate equivalence between two experimental groups. Psychological Bulletin, 113(3), page 557.

For example, we obtain the following calculations for the cognitive versus behavioral contrast. Traditional z: z = ES/SE = 0.12/0.09 = 1.333, p = .091. Traditional confidence interval: ES ? (za/2)(SE) = 0.12 ? (1.96)(0.09), or -0.056 to 0.296. Equivalence z: z1 = (ES + 0.20)/SE = (0.12 + 0.20)/0.09 = 3.556, p = .000. z2 = (ES - Q.2Q)/SE = (0.12 - 0.20)/0.09 = -0.889, p= .187. So we table the larger p value of 0.187. Equivalence confidence interval: ES ? (za)(SE) = 0.12 ? (1.645)(0.09), or -0.028 to 0.268.

We can recreate this with the TOSTmeta function:

```
TOSTmeta(ES = 0.12, var = 0.0081, se = 0.09, low_eqbound_d = -0.2,
    high_eqbound_d = 0.2, alpha = 0.05)
```

```
## Using alpha = 0.05 the meta-analysis was non-significant, Z = 1.333333, p = 0.1824224
##
##
## Using alpha = 0.05 the equivalence test was non-significant, Z = -0.8888889, p = 0.1870314
##
```




```
## TOST results:
##   Z-value 1    p-value 1  Z-value 2 p-value 2
## 1  3.555556 0.0001885906 -0.8888889 0.1870314
##
## Equivalence bounds (Cohen's d):
##   low bound d high bound d
## 1        -0.2          0.2
##
## TOST confidence interval:
##   Lower Limit 90% CI Upper Limit 90% CI
## 1        -0.02803683          0.2680368
```

TOST for two proportions

Example 6.1

Walker, E., & Nowacki, A. S. (2011), applies an equivalence test for the difference between two proportions to data from Staszewski, S., Keiser, P., Montaner, J., Raffi, F., Gathe, J., Brotas, V., … & Tortell, S. (2001), comparing the response rates of two treatments for HIV.

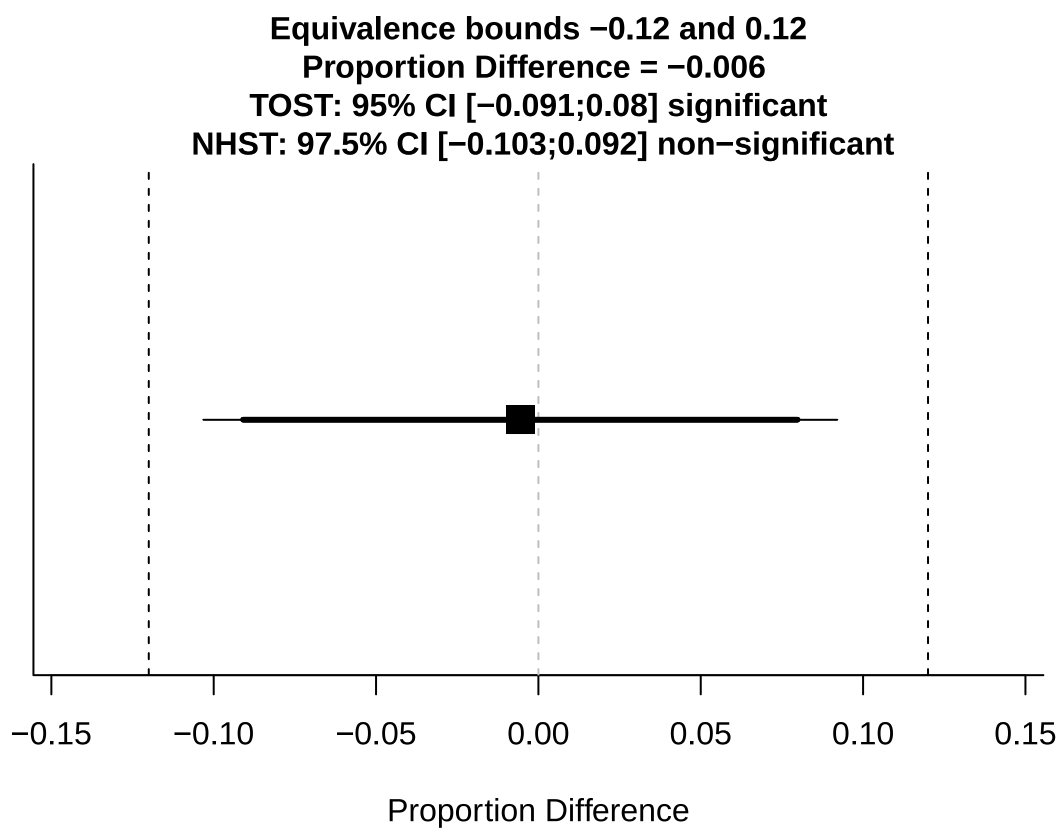
The following proportions were reported: prop1 = 133/262, prop2 = 136/265. The equivalence bounds were set to a proportion difference of +/- 12%. The confidence level for the TOST CI was set to 95%, so alpha was (1-TOST.CI)/2 = 0.025

Walker, E., & Nowacki, A. S. (2011) reports the following confidence intervals for the TOST: Lower = -0.09, upper = 0.08.

We can recreate these confidence intervals with the `TOSTtwo.prop` function.

```
TOSTtwo.prop(prop1 = 133/262, prop2 = 136/265, n1 = 262, n2 = 265,
    low_eqbound = -0.12, high_eqbound = 0.12, alpha = 0.025,
    plot = TRUE)
```

```
## Using alpha = 0.025 Fishers exact z-test was non-significant, z = -0.1279861, p = 0.89816

##

##
## Using alpha = 0.025 the equivalence test based on Fishers exact z-test was significant, z = 2.627386

##
```



```
##
## TOST results:
##   z-value 1   p-value 1 z-value 2   p-value 2
## 1  2.627386 0.004302179 -2.883359 0.001967296
##
## Equivalence bounds:
##   low bound high bound
## 1     -0.12       0.12
##
## TOST confidence interval:
##   Lower Limit 95% CI Upper Limit 95% CI
## 1         -0.0909329         0.07978498
```

Example 6.2

O'reilly, R., Bishop, J., Maddox, K., Hutchinson, L., Fisman, M., & Takhar, J. (2007), compared the clinical efficiency of face-to-face- vs telepsychiatry. They tested the proportions of both the return to normal levels on a symptom inventory, and hospitalization in the first year after treatment.

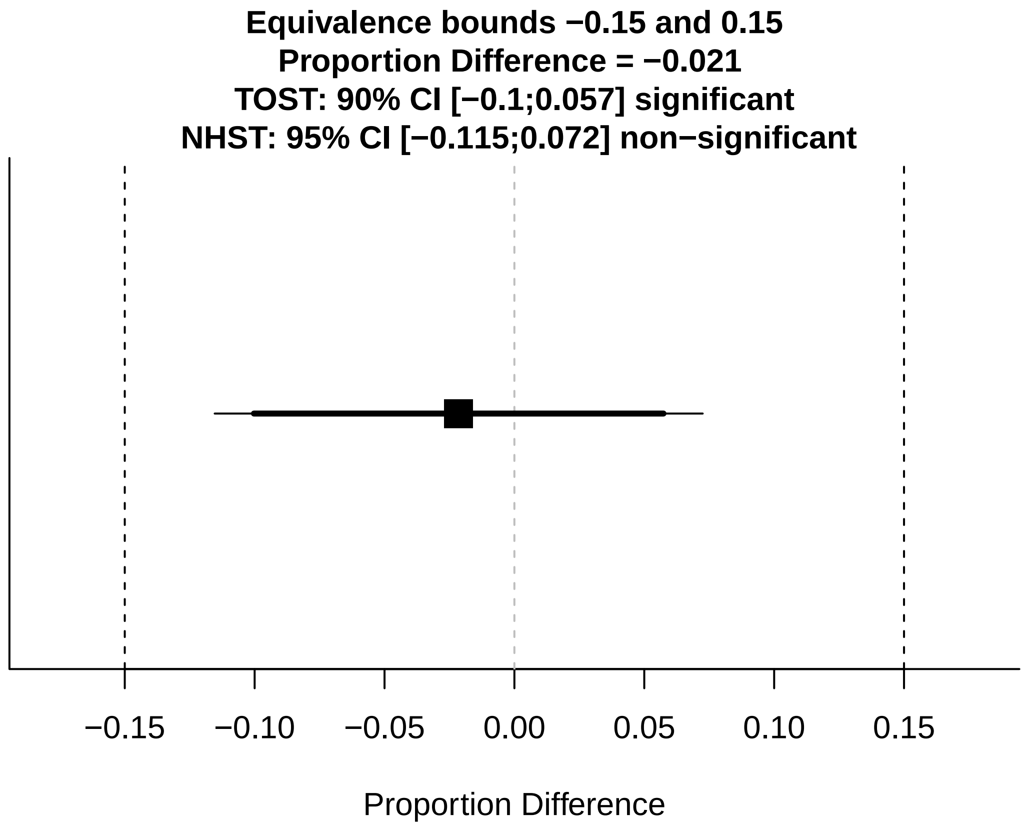
For return to normal levels on symptom inventory… The following proportions were reported: prop1 = 29/148, prop2 = 30/138. The equivalence bounds were set to a proportion difference of +/- 15%. The confidence level for the TOST CI was set to 90%, so alpha was (1-TOST.CI)/2 = 0.05

The authors only report the lower end of the interval: Lower = -0.10

We can recreate this value with the `TOSTtwo.prop` function.

```r
TOSTtwo.prop(prop1 = 29/148, prop2 = 30/138, n1 = 148, n2 = 138,
    low_eqbound = -0.15, high_eqbound = 0.15, alpha = 0.05, plot = TRUE)
```

```
## Using alpha = 0.05 Fishers exact z-test was non-significant, z = -0.4474218, p = 0.6545706

##

##
## Using alpha = 0.05 the equivalence test based on Fishers exact z-test was significant, z = 2.682079,

##
```




```
##
## TOST results:
##   z-value 1   p-value 1 z-value 2    p-value 2
## 1  2.682079 0.003658309 -3.576923 0.0001738315
##
## Equivalence bounds:
##   low bound high bound
## 1     -0.15       0.15
##
## TOST confidence interval:
##   Lower Limit 90% CI Upper Limit 90% CI
## 1         -0.1002848         0.05739407
```

For hospitalization post treatment… The following proportions were reported: prop1 = 18/246, prop2 = 15/224. The equivalence bounds were set to a proportion difference of +/- 10%. The confidence level for the TOST CI was set to 90%, so alpha was (1-TOST.CI)/2 = 0.05

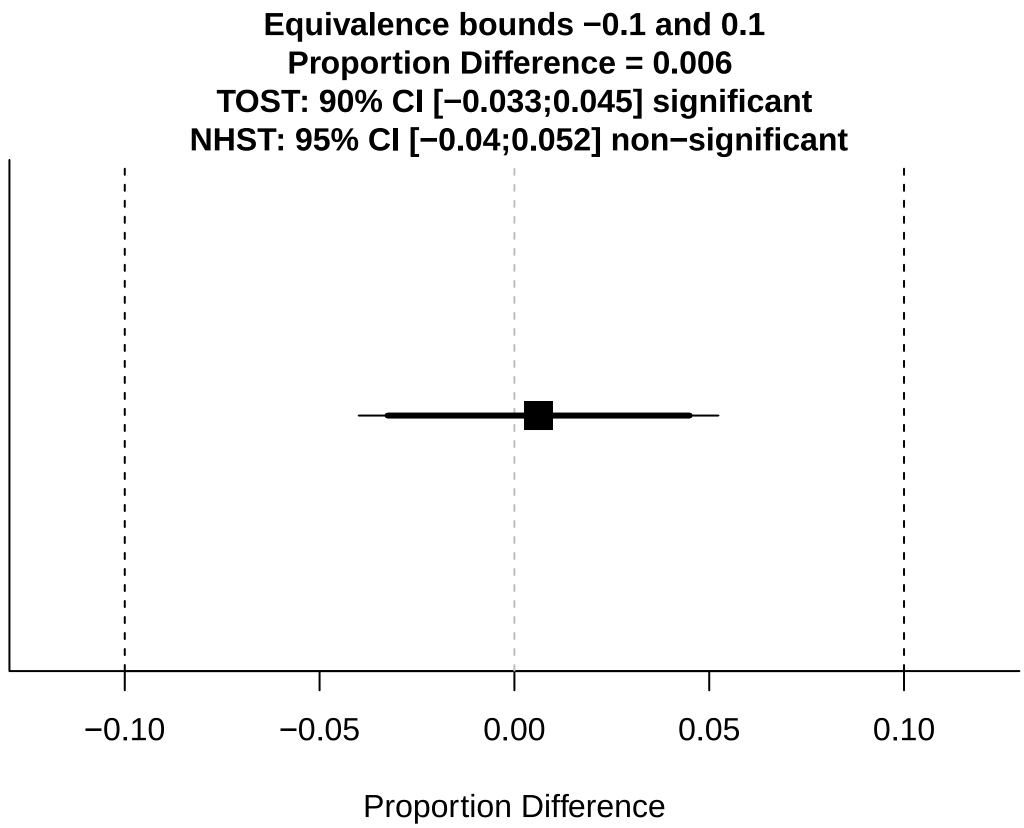
The authors only report the lower end of the interval: Lower = -0.03

We can recreate this value with the `TOSTtwo.prop` function.

```
TOSTtwo.prop(prop1 = 18/246, prop2 = 15/224, n1 = 246, n2 = 224,
    low_eqbound = -0.1, high_eqbound = 0.1, alpha = 0.05, plot = TRUE)
```

```
## Using alpha = 0.05 Fishers exact z-test was non-significant, z = 0.2635423, p = 0.7921326

##

##
```

```
## Using alpha = 0.05 the equivalence test based on Fishers exact z-test was significant, z = -3.982726
##
```




```
##
## TOST results:
##   z-value 1    p-value 1 z-value 2   p-value 2
## 1   4.50981 3.244282e-06 -3.982726 3.40647e-05
##
## Equivalence bounds:
##   low bound high bound
## 1      -0.1        0.1
##
## TOST confidence interval:
##   Lower Limit 90% CI Upper Limit 90% CI
## 1        -0.03253001          0.0449429
```