Mensuration

* Euler's Rule :- No. of Faces + No. of Vertices = No. of Edges + 2

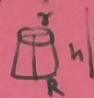
* Prism: LSA = Perimeter of Base × Height   Area = Area of base × Height   TSA = LSA + 2 × area of base

* Cuboid or Rectangular solid : LSA = $2(l+b).h$ TSA = $2(lb+lh+bh)$ Length of longest diagⁿ = $\sqrt{l^2+b^2+h^2}$
  Volume = lbh.

* Pyramid: Volume = $\frac{1}{3}$ × Area of base × Ht   LSA = $\frac{1}{2}$ × Perimeter of base × slant ht

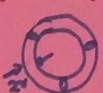
* Cone: $l^2 = r^2 + h^2$ Volume = $\frac{1}{3}\pi r^2 h$ Curved SA = $\pi r l$ TSA = $\pi r(l+r)$

* Frustum of cone : LSA = $\pi l(R+r)$ Volume = $\frac{1}{3}\pi h(R^2+r^2+Rr)$ $l^2 = (R-r)^2 + h^2$ TSA = LSA + $\pi(R^2+r^2)$
  $\frac{r}{R} = \frac{H-h}{H}$



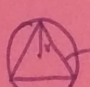
* Frustum of Pyramid : Volume = $\frac{1}{3} \times h(A_1 + A_2 + \sqrt{A_1 A_2})$ LSA = $\frac{1}{2}$ × Sum of perimeter of base & top × Slant height

* Torus: SA = $4\pi^2 r a$ Volume $2\pi^2 r^2 A$ $2\pi^2 r^2 A$



* Circumference of a circle is always greater than the perimeter of any inscribed polygon. Perimeter of any circumscribing polygon is always greater than the circumference of circle.

* 1 Feet = 12 inch   1 Inch = 2.54 cm

*  Eq Δ → $\sqrt{3}r$ = side of Δ

* Inradius of equilateral Δ is half of circumradius.

Coordinate Geometry

* Area of Δ :- (x_1, y_1) ... $= \frac{1}{2}\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$ ∴ If 3 points are collinear, area is 0.

* Area of quad :- (x_1, y_1) .... $= \frac{1}{2}\begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}$

* Internal division - $\frac{mx_2 + nx_1}{m+n} = x$ A —m— P (x,y) —n— B   External division $= \frac{mx_2 - nx_1}{m-n}$

* Centroid:- $\frac{x_1 + x_2 + x_3}{3}$

* St. Line Eqⁿˢ :-
  $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = m$
  $y = mx + c$
  $\frac{x}{a} + \frac{y}{b} = 1$ ← intercepts
  $ax + by + c = 0$, $a \neq 0$, $b \neq 0$

* l_1: $a_1x + b_1y + c_1 = 0$
  l_2: $a_2x + b_2y + c_2 = 0$

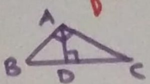
- Point of intersectⁿ = $\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$, $\frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$
- $\cos\theta = \left|\frac{a_1a_2 + b_1b_2}{\sqrt{(a_1^2 + b_1^2)}\sqrt{(a_2^2 + b_2^2)}}\right|$
- $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

* ∥ lines = $m_1 = m_2$

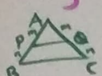
* ⊥ lines: $m_1m_2 = -1$

* 2 Δs are similar if,
  (i) 3 $\angle$s are respectively equal.
  (ii) 2 sides are proportional & included angle equal.

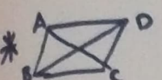
* Similar Δs
  Ratio of sides = Ratio of altitudes = Ratio of medians = Ratio of $\angle$ bisectors = Ratio of In-radii = Ratio of circumradii = $(\text{Ratio of area})^{1/2}$

*  $AD^2 = BD \cdot DC$ $\quad \Delta ABD \sim \Delta CAD$ $\frac{AD}{CD} = \frac{BD}{AD} \Rightarrow AD^2 = BD \cdot CD$.

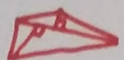
* Congruent conditions: SSS ; SAS ; ASA

*  $PQ \parallel BC$ $\quad \frac{AP}{PB} = \frac{AQ}{QC}$ $\quad \therefore$ converse is true as well.

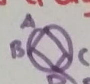
* Line joining mid-points of 2 sides of Δ is $\parallel$ to 3rd side & is half of 3rd side.

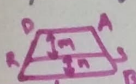
*  $AD \parallel BC$ $\quad$ Area ΔABC = Area ΔBDC.

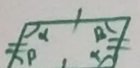
Quadrilaterals



* $\perp$s drawn to a digonal from opp. vertices are "offsets".

* Cyclic quad.  $A + D = 180°$ $\quad \angle CDE = \angle BAC$.

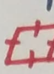
*  Trapezium $\quad RS = \frac{n}{m+n} \times \text{long side} + \frac{m}{m+n} \times \text{short side}$
  longer side

*  llgm $\therefore \alpha + \beta = 180°$ $\quad \therefore$ Each diagonal divides llgm into 2 congruent Δs.
  $\therefore$ Diagonals bisect each other.

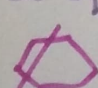
*  llgm
  Area:- PAB + PCD = PBC + PAD = Half of llgm ABCD

* If there is a llgm & a Δ with same base & between same $\parallel$ lines, area of Δ will be half of llgm.

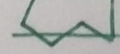
* Fig. formed by joining mid-points of any ~~llgm~~ quadrilateral in order is a llgm.

* Rhombus  $\therefore$ diganals bisect each other $\perp$ly. (converse is true as well)
  $\therefore$ 4 Δs formed by 2 bisecting diagonals are congruent.

* Largest rectange that can be inscribed in a circle will be square.

* Convex Polygon $\rightarrow$ Each interior $\angle$ less than 180° 
  Any st. line drawn pass only 2 side.

  Regular Polygon - Convex polygon, all sides & $\angle$s are equal.
  - can be inscribed in a circle.
  - Sum of $\angle$s = $(n-2) \times 180°$.

* Concave polygon -  more than 2 cutting

* Regular Polygon - If centre is joined with each vertica, we get n identical Δs. All these are isoscela Δs.
  - In hexagon, all are equilaterd Δs.
  - No. of diagonals = ${}^nC_2 - n$.

NUMBERS

* Multiplicand $\times$ Multiplier = Product ($3 \times 4 = 12$)
* $\frac{32}{5}$: 32 → dividend, 5 → divisor; 6 → quotient, 2 → remainder
* Rational Number ⇒ terminating or recurring decimal, p/q, $q \neq 0$
* Irrational Number → Non-terminating Non-recurring
* Every prime number greater than 3 can be written in the form of $6k+1$ or $6k-1$.
* 1 is neither prime nor composite. There are 15 prime no.s between 1 and 50 and 10 prime nos between 50 and 100.
* Product of any no. of odd numbers is always odd. Product of any no. of numbers when there is atleast one even number is even.
* Perfect No. → sum of ALL its factors excluding itself (but including 1) is equal to the number itself
* Hierarchy → Brackets → of → Division, Multiplication, Addition, Subtraction
* A number is divisible by 2^n or 5^n ($2, 4, 8 \ldots$ / $5, 25, 125 \ldots$), if a number formed by the last n digits taken in that order (from left to right) is divisible by 2^n or 5^n respectively.
* <u>Divisibility</u> :-
  - 6 → both divisibly by 2 & 3
  - 7 → difference b/w no. of tens in the number and twice the units is divisible by 7.

    4558 → 16 → 455 → 439   Not divisible by 7
    439 → 18 → 43 → 25
  - 9 → sum of digits is a multiple of 9
  - 11 → sum of alternate numbers is same or they differ by multiple of 11.

    785345 → 16 − 16 = 0 divisible
  - 19 → sum of no. of tens in the number and twice the unit digit is divisible by 19.

    4579 → 18 + 457 = 475
    475 → 10 + 47 = 57 → divisible

* $0.\overline{37} = \frac{37}{99}$, $0.\overline{225} = \frac{225}{999}$ (3 digits ⇝ 3 9's)
* $0.1\overline{36} = x \Rightarrow 10x = 1 + 0.\overline{36} = 1 + 36/99 \Rightarrow x = \ldots$

  $0.1\overline{36} = \frac{136-1}{990} = 135/990$

* $N = a^p b^q c^r \ldots$, a, b, c are prime numbers, p, q & r are +ve integers
  - No. of factors $= (p+1)(q+1)(r+1)\ldots$ (including 1 & the no. itself)
  - No. of ways of expressing the given number as a product of two $= \frac{1}{2}\{(p+1)(q+1)(r+1)\ldots\}$
  - If all $p, q, r..$ are even the numerator is odd and the no. is a perfect square

    as a product of 2 different factors $= \frac{1}{2}\{((p+1)(q+1)(r+1)\ldots) - 1\}$ → excluding $\sqrt{N} \times \sqrt{N}$

    " " " " 2 factors $= \frac{1}{2}\{(p+1)(q+1)(r+1)\ldots) + 1\}$ → including $\sqrt{N} \times \sqrt{N}$
  - Sum of all factors of a number $= \frac{a^{p+1}-1}{a-1} \times \frac{b^{q+1}-1}{b-1} \times \frac{c^{r+1}-1}{c-1} \ldots$
  - Product of all factors of a no. $= N^{d/2}$ → whether or not N is a perfect square; $d \to (p+1)(q+1)(r+1)$
  - No. of ways of writing product of 2 coprimes $= 2^{n-1}$, n → no. of diff prime factors
  - No. of coprimes to N, that are less than $N = N(1 - 1/a)(1 - 1/b)(1 - 1/c)\ldots \Rightarrow \phi(N)$
  - → Sum of coprimes to N that are less that $N = \frac{N}{2} \times \phi(N)$

* No. of arrangements of n items of which are p are of one type, q are of second type, r are of 3rd type & rest all distinct is :- $\dfrac{n!}{p!\,q!\,r!}$. Note - not valid for any no. of selecting - only when all selected.

* No. of arrangements of n distinct items where each item can be used any no. of times.(repitition allowed) $= n^r$.

* Out of n given things, no. of ways of selecting 1 or more things at a time $= {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n - 1$.

* No. of ways of dividing $(p+q)$ items into 2 groups of p & q items respectively is $\dfrac{(p+q)!}{p!\,q!}$. can be extended to any no. of items

- No. of ways of dividing $2p$ items into 2 equal groups of p each, where 2 groups have distinct identity $= \dfrac{(2p)!}{(p!)^2}$

- If 2 groups do not have distinct identity $= \dfrac{(2p)!}{2!\,(p!)^2}$.

* Circular Permutation:-

If clockwise & anti-clockwise arrangement different $= (n-1)!$

If " " " same $= \frac{1}{2}(n-1)!$

* If all the possible n-digit numbers using n distinct digits are formed, sum of all the numbers is equal to

$$(n-1)! \times (\text{sum of } n \text{ digits}) \times (111\dots1)\ n \text{ times}$$

* No. of diagonals of n sided regular polygon $= {}^nC_2 - n$.

* No. of integral solutions of eq^n :-

$$x_1 + x_2 + x_3 \dots + x_n = s \qquad s \geq 0 \qquad s \geq n$$

No. of +ve integral sol^n $= {}^{s-1}C_{n-1}$

No. of non-negative integral sol^n $= {}^{n+s-1}C_{n-1}$.

* Derangement:-

No. of ways of placing letters in envelopes such that no letter is placed in corresponding envelope is

$$D_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!}\right]$$

* Total no. of ways in which a selection can be made by taking some or all out of $p+q+r\dots$ things where p are alike of one kind, q alike of a 2nd kind & so on is

$$\{(p+1)(q+1)(r+1)\dots\} - 1.$$

* ${}^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$

* ${}^nP_r = r\,{}^{n-1}P_{r-1} + {}^{n-1}P_r$

Geometry

* $\alpha < 90°$ Acute angle | $90° < \alpha < 180°$ Obtuse | $180° < \alpha < 360°$ Reflex Angle
* $\alpha + \beta = 90°$ $\alpha, \beta \rightarrow$ Complementary Angles | $\alpha + \beta = 180°$ $\alpha, \beta \rightarrow$ Supplementary $\angle$s
* Any point on $\perp$ bisector is equidistant from both ends. $AQ = BQ$.
* $l \parallel m$ n is called transversal. $\alpha + \beta = 180°$.
* $l \parallel m \parallel n$ $\frac{AB}{BC} = \frac{DE}{EF}$.   * $x + y = z$.
* Altitude — Orthocentre; $\perp$ bisector — Circumcentre; Median (divides in 2 equal areas) — Centroid; Angular bisector — Incentre
* $\alpha \rightarrow 60°$ Equilateral Δ Vertical $\angle$, base $\leftarrow$ Isosceles Δ
* In Isosceles Δ, $\perp$ drawn to base bisects base as well as vertical $\angle$. (Altitude $\parallel$ $\perp$ bisector $\parallel$ $\angle$ bisector)
* Eq. Δ Altitude = $\perp$ bisector = $\angle$ bisector = Median = $\sqrt{3}a/2$
* Sum of any 2 sides of a Δ is greater than 3rd side.
  Difference of any 2 sides of a Δ is less than 3rd side.
* If $a < b < c \Rightarrow \alpha < \beta < \gamma$ & Vice versa.
* Acute $\angle$ Δ $AC^2 = AB^2 + BC^2 - 2 \cdot BC \cdot BD$

  Obtuse $\angle$ Δ $AC^2 = AB^2 + BC^2 + 2 \cdot BC \cdot BD$
* The internal bisector of an $\angle$ bisects the opposite side in ratio of other sides. $AB/AC = BD/DC$
* median $2(AD^2 + BD^2) = AB^2 + AC^2$ (Useful for calculating medians)
* Circumcentre is equidistant from vertices. Circumradius, Circumcircle.
* Incentre is equidistant from 3 sides of Δ. $\angle BIC = 90° + A/2$. Incircle, Inradius
* If internal bisector of one $\angle$ & external bisectors of other two $\angle$s are drawn, they meet at a point called Excentre. There will be total 3 Excentres – one corresponding to the internal bisector of each $\angle$.
*

	Circumcentre	Orthocentre
Acute	Inside	Inside
Right	Hypotenuse	Vertex
Obtuse	Outside	Obtuse

* In a right $\angle$ Δ, length of meridian drawn to hypotenuse is equal to half the hypotenuse. This median = Circumradius.
* medians $\frac{AO}{OD} = \frac{CO}{OE} = \frac{BO}{OF} = \frac{2}{1}$.
* In equilateral Δ, centroid, orthocentre, circumcentre & Incentre all coincide.
* In Isosceles Δ, centroid, orthocentre, circumcentre & Incentre all lie on median.
* Equilateral $\Delta \rightarrow \frac{\text{Circumradius}}{\text{Inradius}} = 2/1$ Area = $4/1$.
* Medians All 6 Δs have equal area.

NUMBERS

* Successive Division:-

$$\begin{array}{ccc} 3 & 4 & 7 \leftarrow \text{divisor} \\ \downarrow & \downarrow & \\ 2 & 3 & 1 \leftarrow \text{remainder.} \end{array}$$

∴ Form $\underbrace{3\times4\times7}_{\text{product}} + 23$

$((1\times4)+3)\times3+2$

* Largest power of number (a) in N! (prime)

$$\begin{array}{r|l} 5 & 216 \\ 5 & 43 \\ 5 & 8 \\ & 1 \end{array}$$

$\Rightarrow 43+8+1 = 52.$

* $a^n-b^n \rightarrow$ always divisible by $a-b$
  - when n is even, divisible by $a+b$.
  - when n is odd, not divisible by $a+b$.

* $a^n+b^n \rightarrow$ never divisible by $a-b$
  - If n is odd, divisible by $a+b$. Not divisible if n is even.

* Product of any 2 consecutive no.s divisible by 2.
* Product of any 3 consecutive nos divisible by 3.
* Out of any n consecutive integers, exactly one number will be divided by n and product of n consecutive integer will be divisible by $n!$.

* Tens digit of a perfect square ending with 6 is always odd.
* If a perfect square ends with 5, tens digit is 2.
* Product of an m digit and an n digit numbers will have either $(m+n-1)$ or $(m+n)$ digits.
* Twin prime numbers are prime nos which differ by 2.
* n^{th} root of a M digit number will M/n digits, if $[M/n]$ is an integer or $\left[\frac{M}{n}\right]+1$ if $\left[\frac{M}{n}\right]$ is not integer. $\left[\frac{M}{n}\right] \rightarrow$ greatest integer less than M.

$t_n = 2\left[\frac{1}{n} - \frac{1}{n+1}\right]$

* $\frac{1}{1+2^{1/3}+2^{2/3}} = 2^{1/3} - 1^{1/3}$

* 3 people:- same dir^n

Time taken to meet 1st time $= LCM\left(\frac{L}{a-b}, \frac{L}{b-c}\right)$

* Time taken to meet first time at starting point $= LCM\left(\frac{L}{a}, \frac{L}{b}, \frac{L}{c}\right)$

* **Clocks**

Minute hand:- 360° in 1 hr = 6° per min

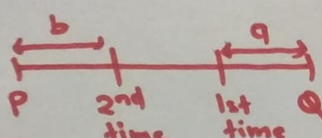
Hour hand:- 30° per hour = $\frac{1}{2}$° per min.

* In 12 hours, 2 hands of a clock

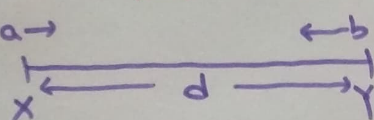
0° → 11 times = Every $\frac{12}{11}$ hours = $1\frac{1}{11}$ hours = $65\frac{5}{11}$ mins

180° → 11 times

90° or any other ∠ → 22 times

* 

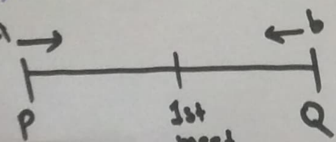
PQ = (3a − b)

* 

1st meet distance covered = d

2nd onwards = 2d.

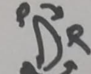
a → X → Y
Y → X
X → Y
⋮

* 

After 1st meet a took t_1 to reach Q

b took t_2 to reach P

∴ 1st meet, $t = \sqrt{t_1 t_2}$

Same for semi-circular as well. 

Time & Work

* $\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$.

Number Systems

* Base/Scale of Notation/ Radix

Binary - 0, 1　　Octal - 8　　Duo-decimal - 12

Septenuary - 7　　Decimal - 10　　Hexa-decimal - 16

* Conversion decimal to any other base :-

$8\,\underline{|\,23}$
$\;\;2 - 5$

$(23)_{10} = (25)_8$

$2\,|\,23$
$2\,|\,11 - 1$
$2\,|\,5 - 1$
$2\,|\,2 - 1$

$(23)_{10} = (2111)_2$

$0.375 \times 2 = 0.75$　　0
$0.75 \times 2 = 1.5$　　1
$0.5 \times 2 = 1$　　1

$(0.375)_{10} = (0.011)_2$

* Binary to Octal

$00(\underline{1}\,\underline{001}\,\underline{100})_2$　Groups of 3
$\;\;\;1 \quad 1 \quad 4$

Binary to Hexa-decimal - Groups of 4.

Statistics

* $A.M. = \dfrac{x_1 + x_2 + \dots + x_n}{n}$　　$G.M. = (x_1 x_2 \dots x_n)^{1/n}$　　$H.M. = \dfrac{n}{1/x_1 + 1/x_2 + \dots \frac{1}{x_n}}$
* Algebraic sum of deviations about mean is 0. $\Sigma(x_i - \bar{x}) = 0$.
* For any 2 +ve numbers :- $AM \geq GM \geq HM$　　$(GM)^2 = (HM)(AM)$
* Median - middle value when sorted in decreasing/increasing order.
* Mode - value occurring highest no. of times.
* In symmetric distribution, Mean = Median = Mode.
* In moderately symmetric distribution, Mode = 3 Median - 2 Mean.
* Q_1 = size of $\left(\frac{n+1}{4}\right)^{th}$ term　　$Q_3 = 3 \times Q_1$ size　　$QD = \dfrac{Q_3 - Q_1}{2}$.
  $Q_2 \rightarrow$ median.
* M.D. = Mean Deviation = $\dfrac{\sum_{i=1}^{n} |x_i - A|}{n}$.
* Standard Deviation = $\sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x_i^2}{n} - (\bar{x})^2} = \sqrt{\text{Variance}}$
* The sum of deviations is least when taken about Median.
* Quartile deviation of 2 +ve no.s a & b = $\left|\dfrac{a-b}{2}\right|$.
* S.D. $(x_1, x_2, x_3, \dots, x_n) = 6$
  S.D. $(2x_1 + 3, 2x_2 + 3, 2x_3 + 3, \dots 2x_n + 3) = 12$

Numbers

* $(n-1)!$ is not divisible by n, when n is prime.
* $(x^4 + x^2 + 1) = (x^2 + x + 1)(x^2 - x + 1)$
* Any no. of the form 2^{4k} leaves remainder 1, when divided by 3 or 5.
* Remainder of 10^a, when a is any odd +ve integer number divided by 11 is always 10.
* 76^n will always end in 76 for any +ve integral value of n.
* x^n will end in 76, whenever x is even and n is a multiple of 20.

Logarithms

$\boxed{e = 2.718}$

* $a^x = N \Rightarrow \log_a N = x$.
* $\log_a N = \underline{abc}.\underline{xyz}$ — abc → characteristic, xyz → mantissa
* Characteristic of the logarithm of a no. greater than unity is +ve and is less than by one than the no. of digits in its integral part.
  Ex → characteristic of $\log 4758$ is 3.
* Characteristic of the logarithm of a no. less than one is –ve. Its magnitude is 1 more than no. of zeroes immediately after decimal point.
  Characteristic of $\log 0.0034$ will be -3 or $\bar{3}$.
* Mantissa are same for the logarithms of all no.s which have same significant digits.

$\boxed{\log 2 = 0.301}$

* Ex → $\log 0.02 = -2 + 0.301 = -1.699$
* $\log_a a = 1$
  $\log_a 1 = 0$
  $\log_a (mn) = \log_a m + \log_a n$
  $\log_a (m/n) = \log_a m - \log_a n$
  $\log_a m^p = p \log_a m$

$\log_a b = 1/\log_b a$

$\log_a m = \log_b m / \log_b a$

$\log_{a^q} m^p = p/q \log_a m$

$a^{\log_a N} = N$.

$\log_{1/a} a = \log_{a^{-1}} a = -\log_a a = -1$.

$a^{\log_b c} = c^{\log_b a}$

Permutations & Combinations

* $^nP_r = \frac{n!}{(n-r)!}$ * $^nP_n = n!$ * $0! = 1$ * $^nC_r = \frac{n!}{r!(n-r)!} = {}^nC_{n-r}$
* $^nP_r = n.{}^{n-1}P_{r-1} = n(n-1)\,{}^{n-2}P_{r-2} = n(n-1)(n-2)\,{}^{n-3}P_{r-3} \ldots$
* $^{n-1}P_r = (n-r)\,{}^{n-1}P_{r-1}$ * $^nP_r = {}^{n-1}P_r + r.{}^{n-1}P_{r-1}$
* $1! + 2\times 2! + 3\times 3! + \ldots + n.n! = {}^{n+1}P_{n+1} - 1! = (n+1)! - 1$
* $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
* $^nC_x = {}^nC_y \Rightarrow x = y$ or $x + y = n$
* $n.{}^{n-1}C_{r-1} = (n-r+1)\,{}^nC_{r-1}$. * $^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1}$.
* If n is even, greatest value of ^nC_r is $^nC_{n/2}$.
  If n is odd, greatest value of ^nC_r is $^nC_{\frac{n+1}{2}}$ or $^nC_{\frac{n-1}{2}}$.
* $^nC_r/{}^nC_{r-1} = \frac{n-r+1}{r}$.
* $^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$
* $^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots = 2^{n-1}$.
* $^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_n = 2^{2n}$
* $^nC_n + {}^{n+1}C_n + {}^{n+2}C_n + \ldots + {}^{2n-1}C_n = {}^{2n}C_{n+1}$

* Translation of Axes: $x = \underset{\text{new}}{\underset{\downarrow}{X}} + h$ $y = \underset{\text{new}}{\underset{\downarrow}{Y}} + k$ origin shifted to (h,k)

* Rotation of Axes: $x = X\cos\theta - Y\sin\theta$ $y = X\sin\theta + Y\cos\theta$

  $X = x\cos\theta + y\sin\theta$ $Y = -x\sin\theta + y\cos\theta$

* Distance b/w $\parallel$ lines $= \left|\dfrac{c_1 - c_2}{\sqrt{a^2+b^2}}\right|$

* Circles $\rightarrow C_1, C_2$ Eq^n of common chord $\Rightarrow C_1 = C_2$ (solve this)

* Circle eq^n = $(x-a)^2 + (y-b)^2 = r^2$

* Pair of St. lines :- $(a_1x + b_1y + c_1)(a_2x + b_2y + c_2) = 0$

  $\Rightarrow ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0.$

Trigonometry

* $\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \mathrm{cosec}^2\theta$

* $\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$

* $\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

  $\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

  $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$ $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

  $\cot(A+B) = \dfrac{\cot A\cot B - 1}{\cot A + \cot B}$ $\cot(A-B) = \dfrac{\cot A\cot B + 1}{\cot B - \cot A}$

* $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

  $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

  $\tan(A+B+C) = \dfrac{\Sigma\tan A - \pi\tan A}{1 - \Sigma\tan A\tan B}$

* $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

* $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

  $= 1 - 2\sin^2\theta$

  $= \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

* $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

* $\cot 2\theta = \dfrac{\cot^2\theta - 1}{2\cot\theta}$

* $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

  $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

  $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

  $\cot 3\theta = \dfrac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1}$

* $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

  $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

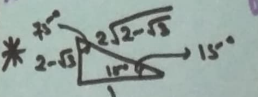
  $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

  $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

* $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$

* cos rule: $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

* Max. value of $a\cos x + b\sin x$ is $\sqrt{a^2+b^2}$.

* 

$\rightarrow 15°$ $\sin 15°$, $\cos 15°$, $\tan 15°$
$\sin 75°$ . . . .

* Pythagorean

  (3, 4, 5) (5, 12, 13) (7, 24, 25) (8, 15, 17)

  (6, 8, 10)

  — — — — — — — —

  (11, 60, 61) (12, 35, 37)

* $(1 + \tan 60°)(1 + \tan 165°) = 2.$ $\underbrace{60° \quad 165°}_{225°}$

Simple Interest - Compound Interest

$\log 2 = 0.301$

* Simple Interest :- $A = P\left(1 + \frac{nr}{100}\right)$

  Compound Interest :- $A = P\left(1 + \frac{r}{100}\right)^n$

* Compounding more than once a year:-

  $A = P\left(1 + \frac{r}{k \cdot 100}\right)^{kn}$ compounding k times a year

  $r \rightarrow$ nominal rate of interest   effective rate of interest =

* If compounding is done every moment,

  $A = P \cdot e^{\frac{nr}{100}}$ $n \rightarrow$ no. of years $e = 2.718$.

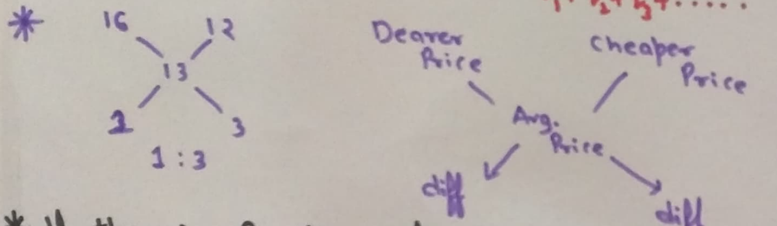
* Difference b/w C.I. & S.I. on a certain sum for 2 years $= \frac{Pr^2}{(100)^2}$

* C.I. for k^{th} year $= P\left(1 + \frac{r}{100}\right)^{k-1} \times \frac{r}{100}$.

Averages - Mixtures - Alligations

* Average $= P + \frac{1}{n} \sum_{i=1}^{n} (Q_i - P)$ $P \rightarrow$ any no.

* Weighted Average $= \frac{p_1 q_1 + p_2 q_2 + p_3 q_3 + \dots}{q_1 + q_2 + q_3 + \dots}$

* 

* If there is P volume of pure liquid initially and in each operation Q volume is taken out and replaced by Q volume of water, at the end of n such operations concⁿ of liquid in solⁿ is

  $$\left(\frac{P-Q}{P}\right)^n = k.$$

Time & Distance

* Kmph $\rightarrow$ m/sec : $5/18$ m/sec $\rightarrow$ Kmph : $18/5$.

* Average Speed $= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

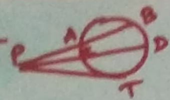
* Boats & Streams:

  $V_s \rightarrow$ Speed still $V_c \rightarrow$ current

  $V_u = V_s - V_c$ $V_d = V_s + V_c$ $V_s = \frac{V_u + V_d}{2}$ $V_c = \frac{V_d - V_u}{2}$.

* Races & Circular Track:-

	Same Dirⁿ	Opp. Dirⁿ	
Time taken to meet 1st time ever	$\frac{L}{a-b}$	$\frac{L}{a+b}$	$L \rightarrow$ length of circular track
Time taken to meet 1st time at starting point	$\mathrm{LCM}\left(\frac{L}{a}, \frac{L}{b}\right)$	$\mathrm{LCM}\left(\frac{L}{a}, \frac{L}{b}\right)$	

*  PAB, PCT → Secant
PT → Tangent $\quad PA \cdot PB = PC \cdot PD = PT^2$

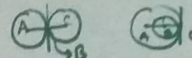
* 2 Tangents can be drawn to the circle from any point outside the circle. (Equal length). Tangent is ⊥ to radius.

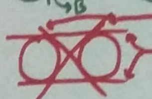
* ⊥ drawn from centre of a circle to a chord bisects it. 

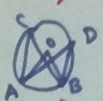
* 2 chords that are equal in length will be equidistant from the centre.

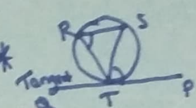
* One and only one circle passes through any 3 given non-collinear points.

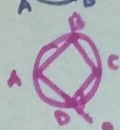
* When there are 2 intersecting circle, line joining the 2 circles will ⊥ly bisect the line joining the point of intersection.

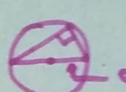
*  A, B, C are collinear. Point of contact, centra of 2 circles are collinear.

*  Direct common tangent $= \sqrt{\left(\text{dist. b/w centre}\right)^2 - (r_1 - r_2)^2}$
Transverse common tangent $= \sqrt{\left(\text{dist. b/w centre}\right)^2 - (r_1 + r_2)^2}$

*  $\angle ACB = \angle ADB = \frac{1}{2}\angle AOB$
O → centre of circle

*  $\angle STP = \angle SRT$
∠ b/w tangent & chord = ∠ by chord in alt. segment

*  $\angle ABC = \angle CDO$
cyclic quad. $\quad \angle ABC + \angle ADC = 180°$

*  diameter.

* Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \frac{1}{2}bh = \frac{1}{2}ab\sin C = \frac{abc}{4R} = rs = r(r+2R)$
$s = \frac{a+b+c}{2}$ $\quad = \frac{1}{2}bc\sin A$
Only true for right ∠ Δ

Eq. $\Delta = \frac{\sqrt{3}}{4}a^2$ $\quad$ Isosceles $\Delta = \frac{b}{4}\sqrt{4a^2 - b^2}$

* Area of Quad. = $\frac{1}{2}$ × one diagonal × sum of offsets drawn to that diagⁿ.

* Area of Cyclic quad = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ $\quad s = \frac{a+b+c+d}{2}$.

* Area of Trapezium = $\frac{1}{2}$ × sum of ∥ sides × distance b/w them.

* Area of ∥gm = base × height = Pdt of 2 sides × sin of included ∠

* Rhombus : Area = $\frac{1}{2}$ × pdt of diagonals $\quad$ Perimeter = 4 × side

* Area of Regular Polygon : $\frac{1}{2}$ × Perimeter × ⊥ dist. from centre of polygon to any side.

* Area of Ellipse = πab $\quad$ Perimeter = $\pi(a+b)$

* Inradius of a Δ is always less than half of smallest altitude.

* In a cyclic quadrilateral, if a, b, c, d are the lengths of 4 consecutive sides & diagonals are d_1, d_2 ∴ $d_1 d_2 = ac + bd$.

* Right ∠ Δ, with fixed length of hypotenuse, area is max. when Δ is isosceles.

* Area of Right ∠ Δ, with inradius r, circumradius R = $r(r+2R)$.
$r = \frac{a+b-c}{2}$ $\quad$ Ex. a = 3, b = 4, c = 5; r = 1, R = 2.5 (xn also True)

* For any Δ, sides are inversely ∝ to corresponding altitudes.

* $(x-p)(x-q)<0$ satisfied by all values between p and q.
  $(x-p)(x-q)>0$ not satisfied by any value b/w p and q.
* $(a-x)^m (b+x)^n$ $m, n > 0$ (integer)
  max. value occurs when $\frac{a-x}{m} = \frac{b+x}{n}$
* $a, b > 0$
  If $m<0$ or $m>1$ $\left(\frac{a+b}{2}\right)^m \leq \frac{a^m+b^m}{2}$ Equality holds if $a=b$.
  If $m=0$ or 1 $\left(\frac{a+b}{2}\right)^m = \frac{a^m+b^m}{2}$.
  If $0<m<1$ $\left(\frac{a+b}{2}\right)^m \geq \frac{a^m+b^m}{2}$ Equality holds if $a=b$.
  Note:- can be extended to 3 or more quantities.
* $\frac{(x+a)(x+b)}{(x+c)}$ Min. value $= a-c+b-c+2\sqrt{(a-c)(b-c)}$
  x is $\sqrt{(a-c)(b-c)} - c$.

Progression & Series

AP
* $T_n = n^{th}$ term $= a+(n-1)d$ $S_n = \frac{n}{2}[2a+(n-1)d]$
  $= \frac{n}{2}(a+l)$
* 3 terms in AP :- $(a-d)$, a, $(a+d)$
  4 terms :- $(a-3d)$, $(a-d)$, $(a+d)$, $(a+3d)$
  5 terms - $(a-2d)$, $(a-d)$, a, $(a+d)$, $(a+2d)$
* **GP**
  $T_n = ar^{n-1}$ $S_n = \frac{a(r^n-1)}{r-1} = \frac{r \times \text{Last Term} - \text{First Term}}{r-1}$
* G.M. $= \sqrt[n]{a_1 \cdot a_2 \cdot a_3 \ldots a_n}$
* For any 2 unequal +ve no., $\frac{a+b}{2} > \sqrt{ab}$ A.M. > G.M.
* 3 terms : $\frac{a}{r}$ a ar 4 terms: $\frac{a}{r^3}$ $\frac{a}{r}$ ar ar^3
* If $|r|<1$ $S_\infty = \frac{a}{1-r}$.

H.P.

$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d} \ldots$ reciprocals in A.P

* Sum of first n natural numbers $= \frac{n(n+1)}{2}$
  Sum of squares of first n natural nos $= \frac{n(n+1)(2n+1)}{6}$
  Sum of cubes of first n natural nos $= \left[\frac{n(n+1)}{2}\right]^2$
* $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$
* $\frac{1}{a(a+d)(a+2d)} = \frac{1}{2d^2}\left[\frac{1}{a} - \frac{2}{a+d} + \frac{1}{a+2d}\right]$
* $\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} = \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right)$
* $\sqrt{1 + \frac{1}{n^2} + \frac{1}{(n+1)^2}} = 1 + \frac{1}{n} - \frac{1}{n+1}$
* $t_n = \frac{1+2+3+\ldots+n}{1^3+2^3+3^3+\ldots+n^3}$
  $t_n = 2\left[\frac{1}{n} - \frac{1}{n+1}\right]$
* $\frac{1}{1+2^{1/3}+2^{2/3}} = 2^{1/3} - 1^{1/3}$

Quadratic Equation

* $ax^2+bx+c=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ Roots : sum $= -b/a$ Pdt. $= c/a$

$(b^2-4ac) \rightarrow$ discriminant

* $ax^2+bx+c=0$ Roots :- α, β

If roots $1/\alpha$, $1/\beta$: eq^n $\rightarrow$ $a(\frac{1}{x})^2+b(1/x)+c=0$ x replaced with $1/x$.

If roots $(\alpha+k), (\beta+k)$: eq^n $\rightarrow$ $a(y-k)^2+b(y-k)+c=0$ x replaced with $(y-k)$.

If roots $(\alpha-k), (\beta-k)$: eq^n $\rightarrow$ $a(y+k)^2+b(y+k)+c=0$ x replaced with $y+k$.

If roots $k\alpha$, $k\beta$: eq^n $\rightarrow$ $a(y/k)^2+b(y/k)+c=0$ x replaced with y/k.

If roots α/k, β/k : eq^n $\rightarrow$ $a(ky)^2+b(ky)+c=0$ x replaced with ky.

* $ax^2+bx+c=0$ If $a>0$, min. value $= (4ac-b^2)/4a$ $x = -b/2a$

If $a<0$, max. value $= (4ac-b^2)/4a$ $x=-b/2a$.

* If $p(x)$ (polynomial) is divided by $(ax+b)$, then the remainder is $p(-b/a)$. $ax+b=0 \Rightarrow x=-b/a$

Degree of remainder is always less than the degree of divisor.

* $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots + a_1 x + a_0$.

Sum of roots $= S_1 = -\frac{a_{n-1}}{a_n}$

Sum of pdt of roots taken 2 at a time $= S_2 = \frac{a_{n-2}}{a_n}$

Sum of pdts of roots taken $m (m\leq n)$ at a time $= S_m = \sum (-1)^m \frac{a_{n-m}}{a_n}$

* If coefficients are real, and complex number Z_1 is a root of $f(x)=0$, then the conjugate of Z_1, $\bar{Z}_1$ is also a root of $f(x)=0$.

$\Downarrow$

Any eq^n of an odd degree must have at least one real root.

* No. of +ve roots is at most equal to no. of sign changes.

If there are k sign changes, no. of +ve roots could be $k, k-2, k-4, \ldots$.

-ve roots can be found out from $f(-x)=0$.

* $a_1x^2+b_1x+c_1=0$ $a_2x^2+b_2x+c_2=0$.

One common root condition: $(c_1a_2-c_2a_1)^2 = (b_1c_2-b_2c_1)(a_1b_2-a_2b_1)$

Inequalities & Modulus

* a, b real numbers

If $a>b$, $c>0$; then $ac>bc$

$a<b$, $c>0$; $\Rightarrow$ $ac<bc$

$a>b$, $c<0$; $\Rightarrow$ $ac<bc$

$a<b$, $c<0$; $\Rightarrow$ $ac>bc$.

$a>b$ $\Rightarrow -a<-b$.

* $a, b \rightarrow$ +ve $a>b$

$\frac{1}{a}<1/b$

$a/c > b/c$ $c>0$

$\frac{a}{c}<b/c$ $c<0$

* A, G, H are arithematic mean, geometric mean and harmonic mean of n +ve real numbers, then $A \geq G \geq H$, equality only when the numbers are equal.

* If $a+b=$ constant ($a, b \rightarrow$ +ve nos.) ; max (ab) when $a=b$.

If $ab=$ constant ; min $(a+b)$ when $a=b$.

* For any +ve number $x \geq 1$

$2 \leq (1+1/x)^x < 2.8$

* For any +ve number, $x+1/x \geq 2$.

* $|x+y| \leq |x|+|y|$ * $|x \cdot y| = |x| \cdot |y|$

* $||x|-|y|| \leq |x-y|$

* $-|x| \leq x \leq |x|$

* $a_1x + b_1y + c_1z = m$
  $a_2x + b_2y + c_2z = n$

  If $\frac{a_1}{a_2} = \frac{c_1}{c_2}$: then y can be uniquely determined.

* Linearly Independent Eq^n :- $I_3 \neq I_1 + kI_2$ $I_1, I_2, I_3 \to$ 3 eq^{ns}

* $\frac{a}{b}$ $a \to$ antecedent, $b \to$ consequent

* $\frac{a}{b}$ ∴ $a > b$ ratio of greater inequality
  $a < b$ ratio of less inequality
  $a = b$ ratio of equality

* $a : b :: c : d$ — b, c: means; a, d: Extremes

$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$ Invertendo

$\Rightarrow \frac{a}{c} = \frac{b}{d}$ Alternendo

$\Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$ Componendo

$\Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$ Dividendo

$\Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$ Compnendo - Dividendo

* $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = \frac{a+c+e+\dots}{b+d+f+\dots}$

* $a : b :: b : c \Rightarrow ac = b^2$ ∵ c is the 3rd proportion of a & b.
  - a, b, c are in continued proportion
  - b is the mean proportion of a & c.

* $\frac{a}{b} \xrightarrow{\text{Triplicate}} a^3/b^3$ $\frac{a}{b} \xrightarrow{\text{duplicate}} \frac{a^2}{b^2}$ $\frac{a}{b} \xrightarrow{\text{Sub-duplicate}} \frac{\sqrt{a}}{\sqrt{b}}$.

* When 2 articles are sold at <u>same price</u>, such that ther is <u>profit</u> of p% on one article and <u>loss</u> of p% on other, then net result of transaction is LOSS.

$$\text{LOSS \%} = \frac{p^2}{100}.$$

ALL	✓	✗
NO	✓	✓
Some	✗	✗
Some not	✗	✓

* The middle term must be distributed at least once.
* If one premise is -ve, the conclusion must be -ve.
* If one premise is particular, the conclusion must be particular
* If both premisa are -ve or particular, conclusion can not be drawn.

* If (whenever) p, then q $p \Rightarrow q$ ∵ $-q \Rightarrow -p$
* Only if p, then q $q \Rightarrow p$ ∵ $-p \Rightarrow -q$
* Unless p, then q Neg $p \Rightarrow q$ ∵ Neg $q \Rightarrow p$

<u>Negation</u>

If p, then q $\to$ p & $-q$

Unless p, then q $\to$ $-p$ & $-q$

Only if p, then q $\to$ $-p$ & q

Neg (p OR q) is same as Neg p & Neg q	Neg (p and q) is same as Neg p or Neg q.

Functions

* Cardinality (Order) of a set :- no. of distinct elements
* Subset :- $A \subseteq B$ Proper subset :- $A \subset B$
* Empty set is a subset of every subset.
  If cardinality is n, total no. of subsets $= 2^n$.
* Power set :- Set of all subsets.
  Cardinality :- 2^n.
* Disjoint sets :- $A \cap B = \phi$. $\phi \to$ null set.
* $A - B$ = Set of elements of A, which do not belong to B.
* Complement of a Set :- A' or $A^c = U - A$ $U \to$ universal set.
* Symmetric difference of 2 sets :-

  $A \Delta B = (A-B) \cup (B-A)$

* $A \cup A = A$, $A \cap A = A$
  $A \cup B = B \cup A$
  $A \cup (B \cup C) = (A \cup B) \cup C$
  $A \cap (B \cap C) = (A \cap B) \cap C$
  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
  $C - (C - A) = C \cap A$

  $C - (A \cup B) = (C - A) \cap (C - B)$
  $C - (A \cap B) = (C - A) \cup (C - B)$
  $(A^c)^c = A$
  $(A \cup B)^c = A^c \cap B^c$
  $(A \cap B)^c = A^c \cup B^c$
  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

* Cartesian pdt. of 2 sets :- Ordered pairs
  - If $O(A) = m$, $O(B) = n$, $O(A \times B) = mn$
  - $A \times B \neq B \times A$
  - $n(A \times B) = n(B \times A)$

* Relation :-
  Domain of R = Range of R^{-1}
  Range of R = Domain of R^{-1}

* Function :-
  Each element of A associate with exactly one element of set B.
  $(a, b) \in f$ $a \to$ pre-image
  $b \to$ image / Range
  $A \to B$ $A \to$ domain
  $B \to$ co-domain

* Range $\subseteq$ co-domain
  $f \subseteq A \times B$
  $n(A) = m$, $n(B) = n$; no. of function $= n^m$.
  $f(xy) = f(x) \cdot f(y)$ $f(x) = x^n$

For any real fun^n $f(x)$

$\frac{f(x) + f(-x)}{2}$ Even

$\frac{f(x) - f(-x)}{2}$ Odd

Functions :- continued

- many-one
- onto :- Range = co-domain
  (Surjection) $n(A) \geq n(B)$
  $n(A) = m$, $n(B) = 2$, no. of onto function $= 2^m - 2$
  $n(A) = m$, $n(B) = n$, $= n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m \ldots$
  $(n \leq m)$
- into
- bjection :- one-one & onto
  $n(A) = n(B)$
  If $n(A) = n$, no. of bijections $= n!$

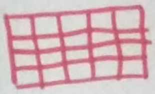
* constant function :- $f(x) = k$
* Identity function :- $f(x) = x$
* Inverse function :- has to be bijective
  $f(a) = b \Rightarrow a = f^{-1}(b)$
* $f(x+y) = f(x) + f(y)$ $f(x) = kx$
  $f(x+y) = f(x) \cdot f(y)$ $f(x) = a^x$
  $f(xy) = f(x) + f(y)$ $f(x) = k \ln(x)$

NUMBERS

* $a^3+b^3=(a+b)(a^2-ab+b^2)$ * $a^3-b^3=(a-b)\ a^2+ab+b^2$
* $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ $\therefore$ $a^3+b^3+c^3=3abc$, if $a+b+c=0$
* $\frac{15}{6}\Rightarrow$ remainder 3 $=\frac{7}{6}+\frac{8}{6}$ $=\frac{-14}{6}+\frac{29}{6}$
  $\quad 1 \quad 2 \qquad -2+5$

* $\frac{11}{8}\Rightarrow$ remainder 3 $\therefore\ \frac{22}{8}\Rightarrow$ remainder $3\times2=6$
* Product of 2 numbers = LCM × HCF *LCM is a multiple of HCF.
* HCF of fractions = $\frac{\text{HCF of numerators}}{\text{LCM of denominators}}$ LCM of fraction = $\frac{\text{LCM of numerators}}{\text{HCF of denominators}}$
* Any number which when divided by p, q or r leaving the same remainder in each case will be of form $k(\text{LCM of } p,q,r)+s$.
* Any number which when divided by p,q or r leaving respective remainders of s,t and u wher $p-s=q-r=t-u=v$, will be like form
  $k(\text{LCM of } p,q,r)-v$.
* divided by 7 remainder 3, divided by 5 remainder 2
  larger → $7k+3\Rightarrow 7k+1$ is divisible by 5
  $\Rightarrow$ smallest value of $k=2$. $\because 7k+3=17$
  Form $\Rightarrow$ LCM of 7 & 5, i.e. $35m+17$
* Largest number with which numbers p,q or r are divided giving remainder s,t and q respectively will be HCF of $(p-s),(q-t),(r-u)$.
* Largest no. with which if we divide p,q,r remainders are same is HCF of $(p-q)$ and $(p-r)$.
* Last digit of powers of any number follow a cyclic pattern.
* Remainders of power of a number also follow cyclic patten. $3^1/4\to3$ $3^2/4\to1$ $3^3/4\to3$
* Remainder Theorem-
  When $f(x)$, a polynomial function in x is divided by $x-a$, the remainder is $f(a)$. $2^{69}/9\Rightarrow(2^3)^{23}/2^3+1$ $\therefore(-1)^{23}\Rightarrow$ remaindr $-1+9=8$
  $2^{94}/15=2^2\times(2^4)^{23}/2^4-1=4$.
* Remainders of Power:- (Last 2 digits)
  - If a ends in 0, all higher powers have atleast 2 0s.
  - " " " " 5, powers end in 25 or 75 depending tens digit is even or odd.
  - If a ends in 1,3,7,9, powers follow a cycle of 20 (or some factors of 20)
  - If a ends in, 4,8, " " " " " "
  - If a ends in $4k+2$, → 02,04,08,16 …., 52,04,08 ⇒ all subsequent seria are same.
* Remainder when N is divided by $D_n=99\ldots9$ (n times) or 10^n-1, we start at right end of N, group the digits n at a time, add all to get S_n. Remainder → $\frac{S_n}{D_n}$.
* $(a+b)^n={}^nC_0a^n+{}^nC_1a^{n-1}b+{}^nC_2a^{n-2}b^2+\ldots+{}^nC_nb^n$
* If p is prime, and HCF$(a,p)=1$, then $a^{p-1}-1$ is a multiple of p.
* If p is prime, $(p-1)!+1$ is a multiple of p.

* In a polygon of n sides, no. of points of intersection of diagonals when taken in pairs & lie inside the polygon is $^nC_4$.

*  $n \times n$ No. of squares $= 1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$

  No. of rectangles $= {}^{n+1}C_2 \times {}^{n+1}C_2$.

* Out of n things, r goes to wrong place, $(n-r)$ goes to right place

  $^nC_{n-r}\,D_r$ $D_r = r!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots+(-1)^r\frac{1}{r!}\right]$

Probability

* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
* Multiplication theorem of Probability

  $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$
* If A & B are independent, $P(A \cap B) = P(A) \times P(B)$.
* A, B, C are 3 events such that each of 3 pairs A,B; B,C; C,A are independent. A, B, C are said to be pairwise independent.
* Mutual Independence : $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$
* Baye's Theorem $= \frac{p_i q_i}{p_1 q_1 + p_2 q_2 + \ldots + p_n q_n}$

* Coin:-

  Total no. of outcomes $= 2^n$

  Probability of getting exactly r-numbered of heads, $= \frac{^nC_r}{2^n}$.
* Cards:-

  Suites:- Clubs, hearts, diamonds, spade

  Honours - Ace, King, Queen, Jack.
* Leap Year :- 2004, 2008, 2400 (✓) 2100, 2200 (X)
* Odds in favour $= \frac{\text{No. of favourable outcomes}}{\text{No. of unfavourable outcomes}}$

  Odds in against $=$ 1/odds in favour.