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Using Dynamic Mode Decomposition to Extract Cyclic Behavior in the Stock Market

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Abstract

The presence of cyclic expansions and contractions in the economy has been known for over a century. The work reported here searches for similar cyclic behavior in stock valuations. The variations are subtle and can only be extracted through analysis of price variations of a large number of stocks. Koopman mode analysis is a natural approach to establish such collective oscillatory behavior. The difficulty is that even non-cyclic and stochastic constituents of a finite data set may be interpreted as a sum of periodic motions. However, deconvolution of these irregular dynamical facets may be expected to be *non-robust*, *i.e.,* to depend on specific data set. We propose an approach to differentiate robust and non-robust features in a time series; it is based on identifying robust features with *reproducible* Koopman modes, *i.e.,* those that persist between distinct sub-groupings of the data. Our analysis of stock data discovered four reproducible modes, one of which has period close to the number of trading days/year. To the best of our knowledge these cycles were not reported previously. It is particularly interesting that the cyclic behaviors persisted through the great recession even though phase relationships between stocks within the modes evolved in the intervening period.

Keywords: Econophysics, Business cycle, Stock markets, Dynamical system,

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Koopman operator, Dynamic mode decomposition, Complex system

1. Introduction

The existence of cyclic expansions and contractions in economic activity on a scale of 5-50 years has been long established [1]. Their study can be traced back to 1800's [2] and an early report of a cycle (of duration 7–11 years) was made by the French statistician Clement Juglar in 1860 [3, 4]. Subsequent studies established evidence for additional cycles including the Kitchin inventory cycle [5] of period 3–5 years, the Kuznets swing of duration of 15–25 years [6], and the Kondratiev wave (or long technological cycle) of period 45–60 years [7]. Much less is known about cyclic behavior in financial markets. The goal of the work reported here is to utilize daily variations in stock prices to search for such cyclic activity.

Speculations abound on precursors to and derivatives of economic changes; however, due to their qualitative nature, establishing such causalities is non-trivial. One quantifiable measure of activities related to the economy or to financial market activity is the valuation of stocks. The valuation of a stock emerges from a series of complex interactions between participants such as investors, traders, and banks under constrains imposed by regulations and exchange rules, and reflect their collective estimation of the performance of the entity and the overall economy.

Variation of the price of a single stock depends on a multitude of factors, a majority of which is specific to the commodity. Hence, the search for changes in the overall financial market using a single index or a small set of indices is unlikely to bear fruit. Indeed, it has been found that a very large number of wavelet modes are required to decompose financial market dynamics [8, 9]. A collective analysis of a large set of stocks is more likely to yield global market dynamics. Since little is known *a-priori* on relationships between individual stocks, empirical basis expansions, such as proper orthogonal decomposition (POD) [10, 11, 12, 13, 10, 14], is a natural approach to follow. In fact, POD and its variants, such as *singular spectrum analysis* (SSA) and its extension *Multivariate SSA* (M-SSA), have proved effective in the study of financial markets [15, 16, 17, 18, 19, 20]. SSA uses a short length sliding window to filter the original time series and constructs the trajectory matrix whose columns are lagged vectors with same length as the sliding window; M-SSA constructs a stacked matrix of multivariate time series from a set of individual times series. SSA and M-SSA perform singular value decomposition on the trajectory matrix, implicitly assuming that the original time series is stationary for the entire duration.

Koopman decomposition [21, 22, 23, 24] is a natural approach to search for collective oscillatory behavior. It is based on *Koopman operator theory* [25, 22, 24], which generalizes eigendecomposition to nonlinear systems. Koopman modes is a generalization of normal modes [24] and each mode represents a global collective motion in (the assumed) market dynamics. Importantly, spectral properties of market dynamics will be contained in the spectrum of the Koopman operator [22, 24]. A fast algorithm proposed by Schmid [25] and referred to as *dynamic mode decomposition* (DMD), can be used for computing approximately (a subset of) the Koopman spectrum from the time-series of valuations of a collection of stocks.

Each Koopman mode is associated with a complex eigenvalue whose real and imaginary parts represent the growth rate and frequency of the mode. Interestingly, Koopman eigenvalues may be used to differentiate robust characteristics of the underlying dynamics from those that depend on the specific data set [26]. The starting point is a search for *reproducible* Koopman modes, *i.e.,* those that persist in different sub-groupings of the time-series. The differentiation is based the following conjecture: *robust characteristics of the market dynamics can be associated with reproducible Koopman modes while non-robust features can be associated with non-reproducible modes.* Only robust features are expected to be useful in an analysis of financial market dynamics.

The methodology is summarized in Figure 1. Our studies are conducted using valuations of the 567 stocks, each of whose market capitalization exceeded

7.5 billion dollars as of October 2014 and price history can be traced back to November 2002. The total market capitalization of these stocks is over 23 trillion dollars as of October 2014. The daily adjusted close prices were retrieved from “Yahoo! finance” (`http://finance.yahoo.com/`). Additional details of the filtering and choice of stocks, as well as data retrieval, is given in the Supplementary Materials. The 567 stocks are grouped according to sector and industry, and placed in a two-dimensional array to aid visualization. Dynamic mode decomposition of the data yield a large number of modes, the spectrum of the most significant of them is shown in Figure 1(a). The data is then partitioned into different subgroups, *e.g.,* the time-series for even and odd numbered trading days. We find only four common dynamic modes, which (in order of decreasing contributions to the data) are associated with periods 250 ± 4, 108 ± 2, 91 ± 1, and 76 ± 2 trading days respectively. To the best of our knowledge, these cycles shown in Figure 1(b), have not been reported before. The corresponding eigenfunctions capture the phase relationships between different stocks in the sample; for example, one stock (or a sector) may react to economic changes immediately, while others may lag the changes by a few weeks. Figure 1(c) shows the real part of one of the reproducible eigenfunctions. Finally, Figure 1(d) illustrates the daily variations in the valuations of Intel Corporation in one reproducible mode.

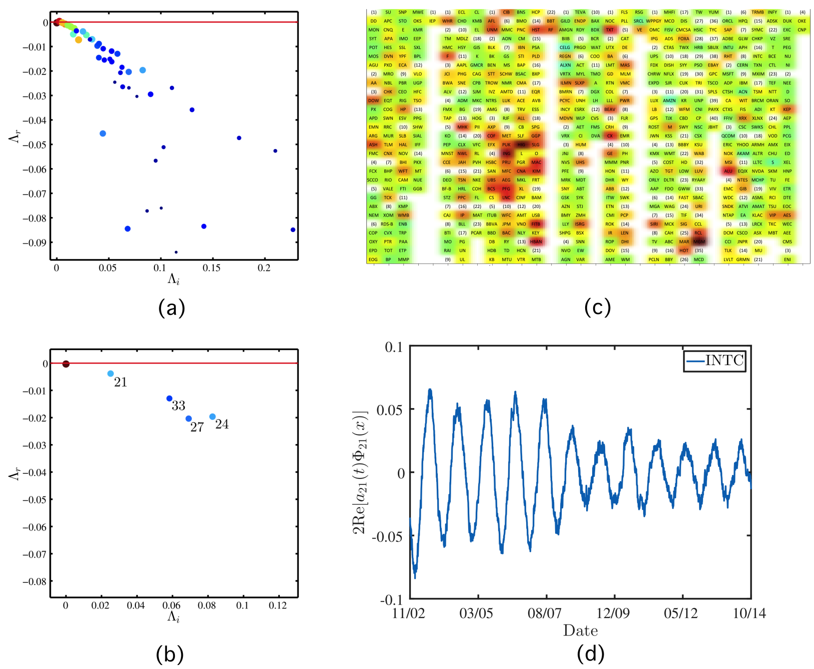

Figure 1: (a) The DMD spectrum, and (b) reproducible modes computed from the time series of the 567 stocks, each with a market capitalization of over 7.5 billion dollars as of October 2014 and a price history that can be traced back to November 2002. (c) The real part of one of the reproducible dynamic modes, and (d) the valuation of Intel Corporation under another reproducible mode.

2. Methods & Results

2.1. Koopman Decomposition

Denote the state of the market by a vector $\mathbf{z}$, whose constituents may include factors like the interest rate, expectations of economic growth, rate of unemployment, etc.; at the outset we recognize the precise form of $\mathbf{z}$ is unknown. Next, assume that market dynamics can be expressed as $\dot{\mathbf{z}} = \mathcal{F}(\mathbf{z})$; once again, its form is unknown. What we have available is "secondary" data, namely stock prices, which we expect to reflect changes in $\mathbf{z}$. Let us index the stocks in a pre-specified array $\mathbf{x}$, and express their values at time t through a field $u[\mathbf{z}](\mathbf{x}, t)$.

When the context is clear, we will simplify the notation by eliminating the state $\mathbf{z}$ from this expression.

Koopman analysis [21, 22, 23, 24] of the system is conducted on the "observables" $u[\mathbf{z}](\mathbf{x}, t)$. Suppose the initial state $\mathbf{z}_0$ of the market evolves under $\mathcal{F}$ to a state $\mathbf{z}_t$ at time t. During this time interval, an initial function $u[\mathbf{z}_0](\mathbf{x}, t = 0)$ evolves to a function $u[\mathbf{z}_t](\mathbf{x}, t)$. The transformation between the functions is given by the Koopman (or composition) operator, defined as

$$U^t : u[\mathbf{z}_0](\mathbf{x}, t = 0) \to u[\mathbf{z}_t](\mathbf{x}, t). \tag{1}$$

Note that, unlike $\mathcal{F}$, which is finite-dimensional and may be nonlinear, the Koopman operator U^t is infinite-dimensional and linear [22, 24]. Interestingly, spectral properties of (the unknown) $\mathcal{F}$ are contained in the eigenspectrum of U^t. Eigenvalues of U^t form the *Koopman spectrum* and the corresponding eigenfunctions are referred to as *Koopman eigenfunctions*. Coefficients in the projection of the state $u[\mathbf{z}_t](\mathbf{x}, t)$ to the Koopman eigenfunctions form the *Koopman modes* of the state [22]. As the system evolves under $\mathcal{F}$, the k^{th} Koopman modes evolves as $\mathrm{e}^{\Lambda_k t}$, where Λ_k is the associated eigenvalue. Thus, the Koopman formalism is a natural approach to search for collective cyclic behavior in high dimensional time-series.

The challenge of establishing spectral properties of $\mathcal{F}$ reduces to computing the spectrum of U^t. Schmid [25] proposed a fast algorithm, referred to as *dynamic mode decomposition* (DMD), for computing approximations to (a subset of) the Koopman spectrum. The analysis, outlined in Supplementary Materials, is based on the computation of a matrix $\mathbf{A}$ from a series of snapshots of the secondary field $u[\mathbf{z}](\mathbf{x}, n\delta t)$ at equally spaced set of time points. The eigenvalues of $\mathbf{A}$, $\{\Lambda_k\}$, are approximations to (a subset of) Koopman eigenvalues and their eigenfunctions $\{\Phi_k(\mathbf{x})\}$, referred to as *dynamic modes*, are approximations to the corresponding Koopman eigenfunctions. Typically, DMD modes are ordered according to the contributions they make toward the spatio-temporal dynamics $u(\mathbf{x}, t)$; these contributions are referred to as "energies" of the modes and are defined in the Supplementary Materials. Below, DMD modes are assumed to be indexed in non-increasing order of their energies.

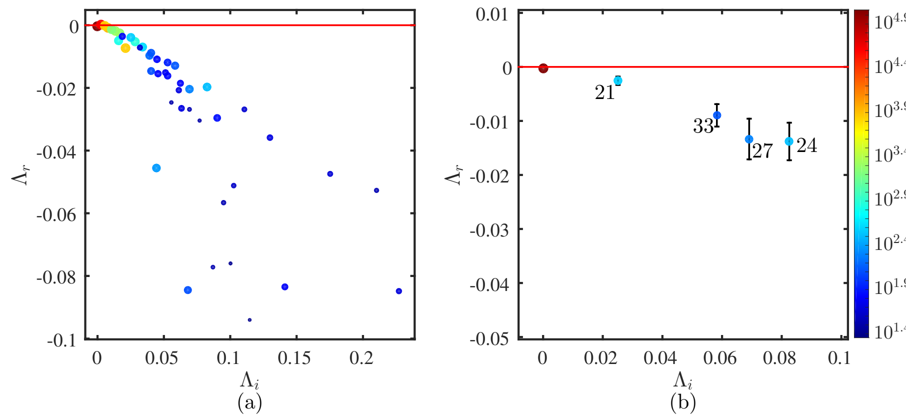

Figure 2: (a) DMD spectrum for the evolution of the 567 stocks during the 3000 trading days from November 2002 to October 2014. Only the 100 modes with the largest energies are selected and only those with non-negative imaginary parts are shown. (b) Reproducible modes, *i.e.,* those that remain (nearly) unchanged between several sub-groups of the time series. Mode numbers are given beside each point. The error bars quantify standard deviations of the real part of the eigenvalues for different sub-groupings of the time series. Deviations in the imaginary parts are significantly smaller. The colors assigned to the modes represent their energies according to the color bar or the right.

In $\mathbf{x}$, stocks are arranged according to the sectors “Basic Materials,” “Conglomerates,” “Consumer Goods,” “Financial,” “Healthcare,” “Industrial Goods,” “Services,” “Technology,” and “Utilities.” Within sectors, entities are sub-grouped by industry, and finally, within each industry, ordered according to descending market capitalization. The input data consist of end-of-the-day valuations of the 567 stocks over the 3000 trading days from November 2002 to October 2014. $u(k,t)$ is the return (*i.e.,* , logarithm of the price) of the k^{th} stock “de-trended” by removing the linear growth (calculated using a linear regression on its time series).

The DMD spectrum consists either of real eigenvalues or pairs of complex conjugate eigenvalues (Supplementary Materials). Figure 2(a) shows the eigenvalues of the 100 dynamic modes with the largest energies. Only those with non-negative imaginary parts are shown. (Energies of modes corresponding to conjugate pairs of eigenvalues are identical.) Color coding given on the right as well as the size of a dot rely on the energy of the mode. The largest energy is contained in the time-averaged mode $\Phi_0(\mathbf{x})$, the remainder distributed among a large number of dynamic modes.

Figure 3 shows energies of the dynamic modes with the 36 largest values. Note that mode energies [except for the real mode $\Phi_0(\mathbf{x})$] appear in pairs, as required for complex conjugate pairs (see Supplementary Materials).

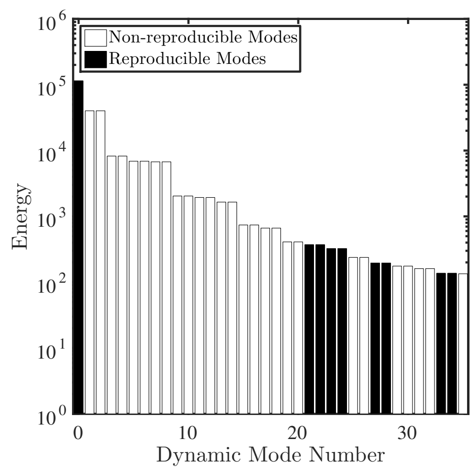

Figure 3: Energies of 36 dynamic modes with the largest energies. Reproducible modes are shown in black.

2.2. Reproducible Modes

Our goal is to search for cyclic changes in the markets. In particular, such oscillations should present in *any* subset of the time series. The data is also expected to contain constituents that depend on the subset, *i.e.,* those that are not robust. It is necessary to identify and eliminate these non-robust components from consideration. Toward this end, we associate robust features of financial markets with dynamic modes that persist in different sub-groupings of the time series.

In order to implement the robustness criterion, we partition the stock data from the 3000 trading days into the following subgroups: (1) odd trading days,

(2) even trading days, (3) trading days divisible by 3, (4) trading days with a remainder 1 when divided by 3, and (5) trading days with a remainder 2 when divided by 3. The determination of whether a dynamic mode is to be considered as reproducible is made using the proximity of the eigenvalues and the corresponding eigenfunctions between the subgroups. The precise conditions used to establish reproducible modes are given in Supplementary Materials.

We find only four reproducible dynamic modes which are shown in Figure 2(b), and whose energies are shown in black in Figure 3. Note that many high-energy dynamic modes are not reproducible. This is to be expected in financial markets due to the high-dimensionality of the dynamics and the presence of stochasticity.

Interestingly, the eigenvalues of the four reproducible modes are maintained when the data is partitioned into trading days 1-1000, 1001-2000, and 2001-3000; however, the corresponding dynamic modes are not close to those of the full data set. This observation suggests that the robust cyclic behavior was maintained through the great recession of 2008, even though the phase relationships between different stocks changed in the interim.

Figures 4 and 5 show the real and imaginary parts of the reproducible dynamic mode $\Phi_{21}(\mathbf{x})$, whose period is 250 ± 4 trading days. Here the standard deviation is derived from the periods computed from the different sub groupings of the data. The period of the mode is approximately the number of trading days in a year and it likely reflects seasonal changes in the market.

As discussed before, the trading symbols are grouped first by sectors, then by industries, and ordered according to descending market capitalization. The notation used is as follows: the number "(1)" indicates the start of a new sector, and a different number (*e.g.,* (6)), indicates the beginning of the corresponding industry within the sector. The detailed numbering of industries within the 9 sectors can be found in Supplementary Materials. The remaining reproducible modes are also given in the Supplementary Materials.

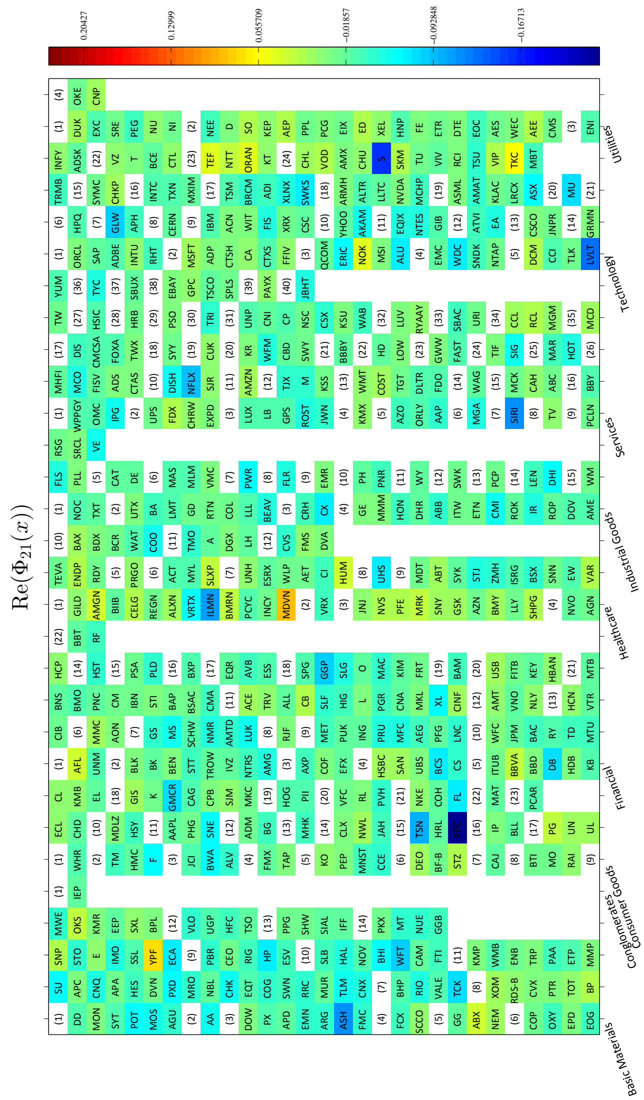

Figure 4: The real part of the reproducible dynamic mode $\Phi_{21}(\mathbf{x})$ with period of approximate 250 trading days, *i.e.,* a year.

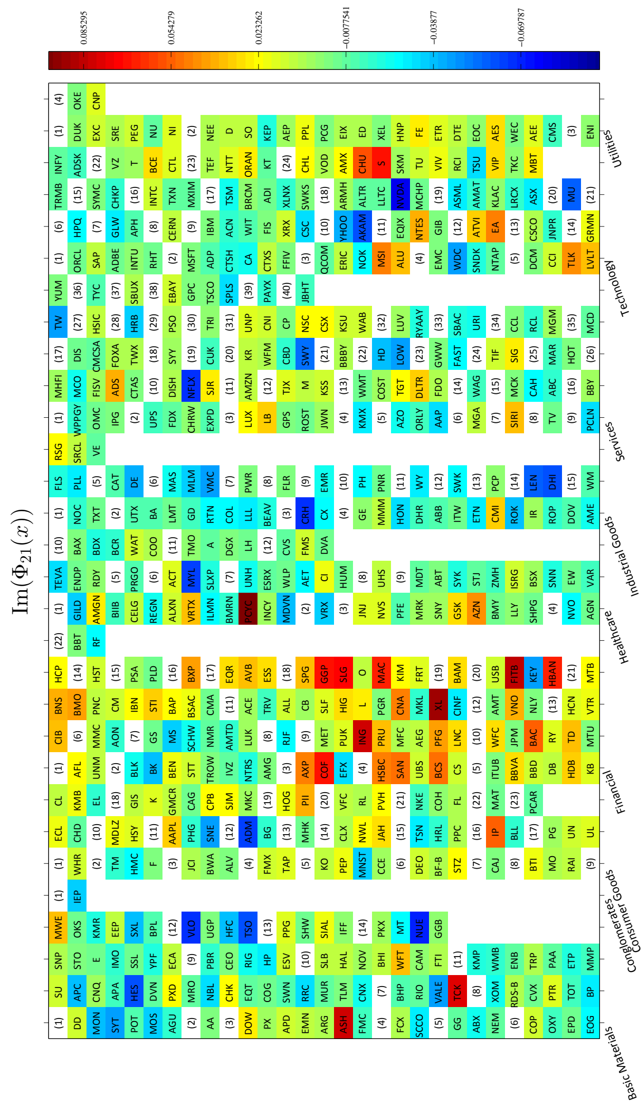

Figure 5: The imaginary part of the reproducible dynamic mode $\Phi_{21}(\mathbf{x})$.

2.3. Dynamics

Under general conditions, dynamic modes $\{\Phi_k(\mathbf{x})\}$ form a complete basis of the dynamics of the secondary field [22, 25]. Thus, $u(\mathbf{x}, t)$ can be expanded as

$$u(\mathbf{x}, t) = \sum_{k=0}^{N-1} a_k(t)\Phi_k(\mathbf{x}), \tag{2}$$

where N is the number of DMD modes. Since dynamic modes are only approximations to Koopman modes, the time evolution $a_k(t)$'s are not precisely of the form $\mathrm{e}^{\Lambda_k t}$; they need to be evaluated through a projection. The DMD matrix $\mathbf{A}$, defined in Supplementary Materials, is real but is generally not symmetric [25]. Consequently, its eigenvalues are not necessarily real and dynamic modes are not necessarily orthonormal. The computation of $a_k(t)$ requires the eigendecomposition of the operator $\mathbf{A}^\dagger$ conjugate to $\mathbf{A}$. Its eigenvalues are Λ_k^* and the corresponding eigenfunctions are denoted $\{\tilde{\Phi}_k(\mathbf{x})\}$. They are orthonormal to the dynamic modes; *i.e.*, $\sum_{\mathbf{x}} \tilde{\Phi}_m(\mathbf{x})\Phi_n(\mathbf{x}) = \delta_{mn}$. Consequently,

$$a_k(t) = \sum_{\mathbf{x}} \tilde{\Phi}_k(\mathbf{x}) u(\mathbf{x}, t). \tag{3}$$

Figure 6(a) shows the time evolution $a_{21}(t)$ of the reproducible dynamic mode $\Phi_{21}(\mathbf{x})$. The power spectrum, Figure 6(b), shows that the dynamics contains a small broadband component, due to the fact that dynamic modes are only approximations to the Koopman modes. The scatter plot of the real vs. imaginary parts of $a_{21}(t)$ in Figure 6(c) exhibits irregular dynamics. Interestingly, the "angular" progression of the phase-space orbit is highly regular. To illustrate this point, we define

$$\theta(t) = \tan^{-1} \frac{\mathrm{Im}(a(t))}{\mathrm{Re}(a(t))}, \tag{4}$$

where $\theta(t)/2\pi$ returns to its original value after a *period* $T = 2\pi/\,\mathrm{Im}(\Lambda)$. Figure 6(d) shows that the dynamics of $\theta_{21}(t)$ is highly regular while that of the magnitude $|a_{21}(t)|$ is irregular. Spatial structures $\Phi_k(\mathbf{x})$'s and time evolutions $a_k(t)$'s for the remaining reproducible modes can be found in the Supplementary Materials.

The time series of the valuation of any stock contains cyclic components associated with the four reproducible modes. For example, Figure 7 shows these components for the Exxon-Mobil Corporation (XOM), the Wells-Fargo & Comapany (WFC), and the Intel Corporation (INTC) for the mode $\Phi_{21}(\mathbf{x})$. It is interesting to speculate if a trading strategy can be developed to exploit these highly regular and robust oscillations.

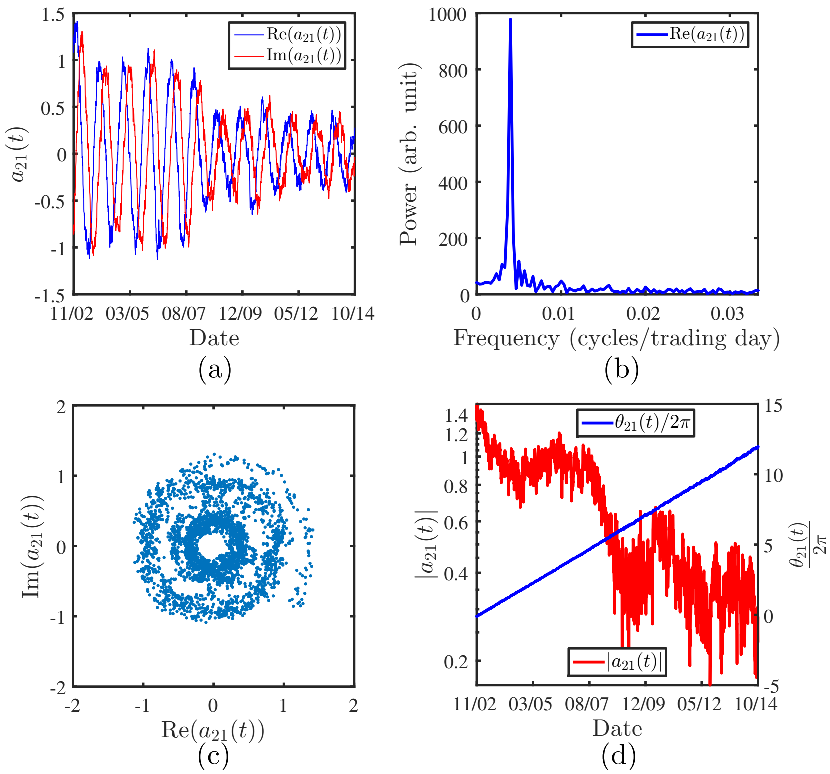

Figure 6: (a) The time evolution of the coefficient $a_{21}(t)$ of the reproducible dynamic mode $\Phi_{21}(\mathbf{x})$ whose period is approximately 250 trading days. (b) Power spectrum of $a_{21}(t)$ shows that, unlike for Koopman modes, the dynamics of $a_{21}(t)$ is not precisely periodic. (c) The points of $a_{21}(t)$ in the complex plane exhibit significant irregularities. (d) However, the behavior of the phase angle, defined as $\theta_{21}(t) = \tan^{-1}\left[\mathrm{Im}(a_{21}(t))/\,\mathrm{Re}(a_{21}(t))\right]$, evolves smoothly; the modulus $|a_{21}(t)|$ fluctuates in time.

We finally inquire if the cyclic behaviors can be observed on single indices. One such quantity may be the average $\bar{u}(t)$ of the 567 stock valuations, weighted by their market capitalizations. Figure 8(a) shows its evolution and Figure 8(b) shows the power spectrum. The arrows in Figure 8(b) are the frequencies of the four reproducible modes. It is clear that the robust cyclic behavior of the collection of stocks cannot be extracted from $\bar{u}(t)$.

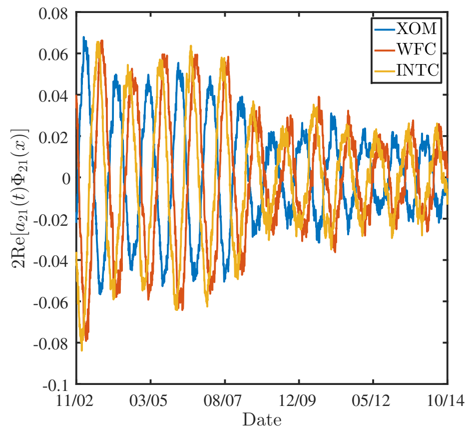

Figure 7: Evolution of the valuation of the Exxon-Mobil Corporation (XOM), the Wells-Fargo & Comapany (WFC), and the Intel Corporation (INTC) associated with the reproducible mode $\Phi_{21}(\mathbf{x})$.

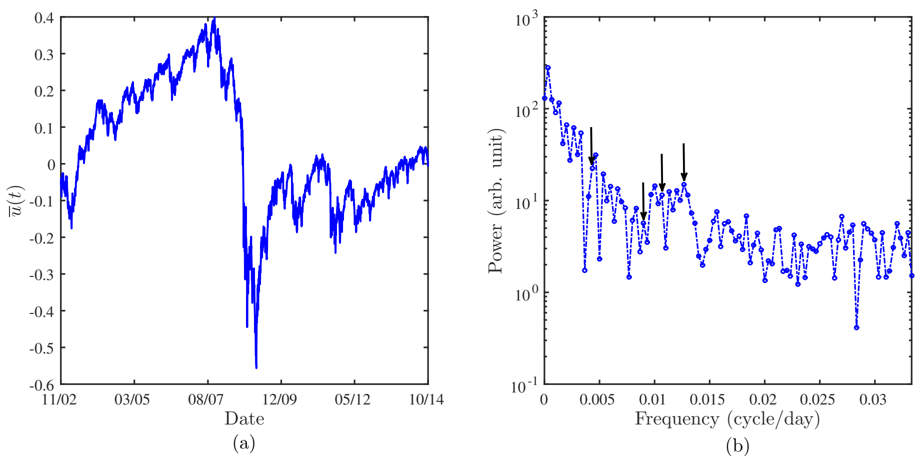

Figure 8: (a) The dynamics $\bar{u}(t)$ of the mean return of the 567 stocks weighted by their market capitalizations. (b) The power spectrum of $\bar{u}(t)$. Frequencies of the four reproducible modes are labeled by arrows.

2.4. Are Reproducible Modes Artifacts of Dynamic Mode Decomposition?

The presence of the four modes in several sub-groupings of the data, and the persistence of their periodicity through the great recession lends credence to the assertion that reproducible modes are reflections of financial market dynamics.

However, we wish to verify that they are not artifacts of the computation of dynamic modes. We test this using two model data sets.

- The first contains 3000 steps from each of 567 realizations of the same Brownian process. Thus, the dynamics of each synthetic stock is assumed to follow the same stochastic process. We conduct dynamic mode decomposition on this synthetic data in a search for reproducible dynamic modes.
- The second data set is identical to the first, except for the inclusion of a periodic drift in the stochastic process. The drift for different entities have the same amplitude and period but random initial phases. Once again, we search for reproducible modes in the simulated data.

The Brownian process in the first simulation has an annualized standard deviation of 0.1. Dynamic mode decomposition of the 567 distinct trajectories was implemented using the same subgroups and thresholds as in the analysis of financial market data. No reproducible modes were found. The second set of simulations contained, in addition, a sinusoidal drift term $A \sin(\frac{2\pi t}{T} + \phi)$, where the period T was chosen to be 250 trading days and the phase shift ϕ for each "stock" was random. A reproducible mode of period T could be identified for $A > A_{cr} = 0.056$. No other reproducible modes were found.

3. Discussion and Conclusions

The existence of economic cycles has been known for over a century. The goal of the work reported here was to search for similar cyclic behavior in the dynamics of the stock valuations. Since they have not been reported previously, one expects their consequences to be subtle; in particular, it is unrealistic to expect to extract them from the time series of a single index or a small set of indices. This motivated our choice to study the collective dynamics of stocks with the highest market capitalizations.

Koopman decomposition is a natural approach to extract spectral properties of stock valuations. However, the ability to differentiate robust and non-robust dynamical characteristics is critical to complete the analysis. We implemented the differentiation by associating robust features of the market dynamics with reproducible dynamic modes, *i.e.,* modes that persist in distinct sub-groupings of the time series. Our analysis identified four reproducible modes with periods 250 ± 4, 108 ± 2, 91 ± 1, and 76 ± 2 trading days. The period of the first of these is a year, and likely reflects seasonal changes in stock valuations. That we identified this period lends credence to our analysis. As illustrated in Figure 7 using the time series for Exxon-Mobil Corporation (XOM), Wells-Fargo & Comapany (WFC), and Intel Corporation (INTC), reproducible modes constitute part of the valuations of individual stocks. At this point we cannot determine if the robust cycles are caused by changes in economic activities or have their origins in trading rules and practices. It was particularly interesting that the cyclic behavior persisted through the great recession, even though the form of the dynamic mode, or equivalently the phase relationships between different stocks, changed over the intervening period. It will be of interest to speculate if a trading strategy can be developed to exploit these highly regular and robust oscillations.

As seen in Figure 3, the four reproducible modes contribute only a very small fraction of the energy of the market dynamics. For this reason, the modes can only be identified through a composite analysis of the dynamics of a large number of stock valuations. Studies of single indices, such as the example shown in Figure 8(b), are not sufficient to extract these modes. In our exposition, we interpreted the remaining variations as due to non-robust dynamics; *i.e.,* contributed by non-reproducible dynamic modes. However, we are unlikely to know the dynamics $\mathcal{F}$ underlying the market; we may thus interpret (at least part of) these fluctuations as resulting from stochastic processes.

Analysis of the synthetic data introduced in Section 2.4 supports our contention that the observed robust cyclic behavior is not an artifact of the dynamic mode decomposition. The simulations also show that in order for a cyclic behavior to be observable, its amplitude need to be larger than a critical value. Thus, there may be additional smaller amplitude cyclic features of the markets.

Intra-day variations in currency exchange rates [27] and stock prices [28] are governed by non-stationary stochastic processes. In addition, while autocorrelation functions for increments in the return decay rapidly, those for the absolute values of the increments decay slowly [29]. It will be of interest to determine if these intra-day variations contain cyclic behaviors as well. We are currently investigating this issue.

Acknowledgments

We wish to thank Robert Azencott for discussions and important suggestions to our work.

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Author Contributions Statements

J.-C.H, G.H.G., and S.R. developed the technique for separating robust and non-robust constituents of spatio-temporal dynamics. J.-C.H. proposed its application to financial markets and performed the computations. J.L.M. contributed important insights to the project. J.-C.H. and G.H.G. wrote the paper.

Additional Information

The authors declare no competing financial interests.