

# IMITATOR 2.5: A Tool for Analyzing Robustness in Scheduling Problems

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**Abstract.** The tool IMITATOR implements the *Inverse Method (IM)* for Timed Automata (TAs). Given a TA  $\mathcal{A}$  and a tuple  $\pi_0$  of reference valuations for timings, *IM* synthesizes a constraint around  $\pi_0$  where  $\mathcal{A}$  behaves in the same discrete manner. This provides us with a quantitative measure of robustness of the behavior of  $\mathcal{A}$  around  $\pi_0$ . The new version IMITATOR 2.5 integrates the new features of stopwatches (in addition to standard clocks) and updates (in addition to standard clock resets), as well as powerful algorithmic improvements for state space reduction. These new features make the tool well-suited to analyze the robustness of solutions in several classes of preemptive scheduling problems.

**Keywords:** Real-Time Systems, Parametric Timed Automata, Stopwatches

## 1 Motivation

IMITATOR 2.5 (for *Inverse Method for Inferring Time Abstract behavior*) is a tool for parameter synthesis in the framework of real-time systems based on the inverse method *IM* for Parametric Timed Automata (PTAs). Different from CEGAR-based methods, this algorithm for parameter synthesis makes use of a “good” parameter valuation  $\pi_0$  instead of a set of “bad” states [3]. IMITATOR takes as input a network of PTAs with stopwatches and a reference valuation  $\pi_0$ ; it synthesizes a constraint  $K$  on the parameters such that (1)  $\pi_0 \models K$  and (2) for all parameter valuation  $\pi$  satisfying  $K$ , the trace set (i.e., the discrete behavior) of  $\mathcal{A}$  under  $\pi$  is the same as for  $\mathcal{A}$  under  $\pi_0$ . This provides the system with a criterion of *robustness* (see, e.g., [14]) around  $\pi_0$ .



**Fig. 1.** Functional view of IMITATOR

**History and New Features** A basic implementation named IMITATOR has first been proposed, under the form of a Python script calling HYTECH [11]. The tool has then been entirely rewritten in IMITATOR.II [2], under the form of a standalone OCaml program. A number of case studies containing up to 60 timing parameters could be efficiently verified in the purely timed framework.

Since [2], we extended the input formalism to PTAs equipped with *stop-watches*: clocks can now be stopped for some time while others keep growing. Also, we added clock updates: clocks can now be set to arbitrary linear combinations of other clocks, parameters and discrete variables. These extensions allow us to consider larger classes of case studies, such as scheduling problems.

## 2 Architecture and Features

IMITATOR is available under the GNU GPL license. The core of the program is written in OCaml, and interacts with the Parma Polyhedra Library (PPL) [6]. Exact arithmetics with unbounded precision is used.

IMITATOR takes as input a network of PTAs with stopwatches. The input syntax, inspired by HYTECH, allows the use of clocks (or stopwatches), rational-valued discrete variables, and parameters (i.e., unknown constants) to be used altogether in linear terms, within guards, invariants and updates.

A constraint is output in text format; furthermore, the set of traces computed by the analysis can be output under a graphical form (using Graphviz) for reasonably sized case studies (up to a few thousands reachable states).

IMITATOR implements in particular the following algorithms:

**Full reachability analysis** Given a PTA, it computes the reachability graph.

**Inverse method** Given a PTA and a reference parameter valuation  $\pi_0$ , it computes a constraint  $K$  on the parameter guaranteeing the same time-abstract behavior as under  $\pi_0$  (see Fig. 1).

IMITATOR 2.5 makes use of several algorithmic optimizations. In particular, we implemented a technique that merges any two states sharing the same discrete part and such that the union of their constraint on the clocks and parameters is convex [5]. This optimization preserves the correctness of all our algorithms; better, the output constraint is then always weaker or equal, i.e., covers a set of parameter valuations larger or equal. It behaves particularly well in the framework of scheduling problems, where the state space is drastically reduced. Actually, most of the scheduling examples we consider run out of memory without this merging technique.

## 3 Application to Robustness Analysis in Scheduling

Due to the aforementioned state space reduction and the use of stopwatches, IMITATOR 2.5 becomes an interesting tool for synthesizing robust conditions for scheduling problems. Let us illustrate this on a preemptive jobshop example

given in [1]. The jobshop scheduling problem is a generic resource allocation problem in which common resources (“machines”) are required at various time points (and for given duration) by different tasks. For instance, one needs to use a machine  $m_1$  for  $d_1$  time units, machine  $m_2$  for  $d_2$  time units, and so on. The goal is to find a way (“schedule”) to allocate the resources such that all tasks terminate as early as possible (“minimal makespan”). Let us consider the jobshop problem  $\{J_1, J_2\}$  for 2 jobs and 3 machines with:  $J_1 = (m_1, d_1), (m_2, d_2), (m_3, d_3)$  and  $J_2 = (m_2, d'_2)$  with  $d_1 = 3, d_2 = 2, d_3 = 4, d'_2 = 5$ . There are many possible schedules (two of them are depicted in Fig. 2 in Appendix). In [1], this problem is modeled as a product  $\mathcal{A}$  of TAs with stopwatches, each TA modeling a job. Each schedule corresponds to a branch in the reachability tree of  $\mathcal{A}$ . The makespan value corresponds to the duration of the shortest branch, here 9.

Let us explain how to analyze the robustness of the valuation  $\pi_0 : \{d_2 = 2, d'_2 = 5\}$  with respect to the makespan value 9. We first consider a parametric version of  $\mathcal{A}$  where  $d_2$  and  $d'_2$  become parameters. In the same spirit as in [9], we add an observer  $\mathcal{O}$ , which is a TA synchronized with  $\mathcal{A}$ , that fires a transition labeled *DEADLINE* as soon as a schedule spends more than 9 time units. We then use IMITATOR (instead of a CEGAR-like method as in [9]) with  $\mathcal{A} \parallel \mathcal{O}$  as a model input and  $\pi_0$  as a valuation input. This yields the constraint  $K$ :  $7 > d'_2 \wedge 3 > d_2 \wedge d'_2 + d_2 \geq 7$ . (For a geometrical representation of  $K$ , see Fig. 3 of Appendix.) By the *IM* principle, the set of traces (i.e., discrete runs) of  $\mathcal{A} \parallel \mathcal{O}$  is always the same, for any point  $(d_2, d'_2)$  of  $K$ . (This set of traces is depicted under the form of a tree in Fig. 4 in Appendix.) Since the makespan for  $\pi_0$  is 9, we know that some branches of the tree do not contain any *DEADLINE* label (these branches end at node *s73* in Fig. 4). This holds for each point  $(d_2, d'_2)$  of  $K$ . In other words, the makespan of the system is (at most) 9, everywhere in  $K$ .

Another example of application of IMITATOR 2.5 to a scheduling problem, originating from [8], is given in Appendix 4 (using the cartography method [4]). All case studies and experiments (performed on Ubuntu 11.10 equipped with an Intel Core2 2.93 GHz processor with 2 GiB RAM) are described in a research report [15] (Source code, binaries, all case studies and logs are available in <http://www.lsv.ens-cachan.fr/Software/imitator/>).

## 4 Related Work

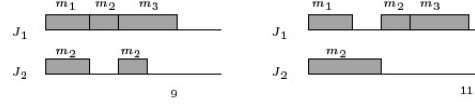
The use of models such as PTAs and parametric Time Petri Nets (TPNs) for solving scheduling problems has received attention in the past few years. For example, Roméo [13] performs model checking for parametric TPNs with stopwatches, and synthesizes parameter valuations satisfying TCTL formulæ. An extension of UPPAAL allows parametric model checking [7], although the model itself remains non-parametric. The approach most related to IMITATOR 2.5 is [9, 12], where the authors infer parametric constraints guaranteeing the feasibility of a schedule, using PTAs with stopwatches. The main difference between [9, 12] and IMITATOR relies in our choice of the inverse method, rather than a CEGAR-based method. First results obtained on the same case studies are uncomparable

(although similar in form), which seems to indicate that the two methods are complementary. The problem of finding the schedulability region was attacked in analytic terms in [8]. The size of our examples is modest, compared to those treated by such analytic methods. However, in many scheduling problems, no analytic solution exists (see, e.g., [16]), and exhaustive simulation is exponential in the number of jobs. In such cases, symbolic methods as ours and those of [9, 12] are useful to treat critical real-life examples of small size. We are thus involved in a project [10] with an industrial partner with first interesting results.

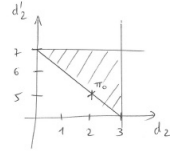
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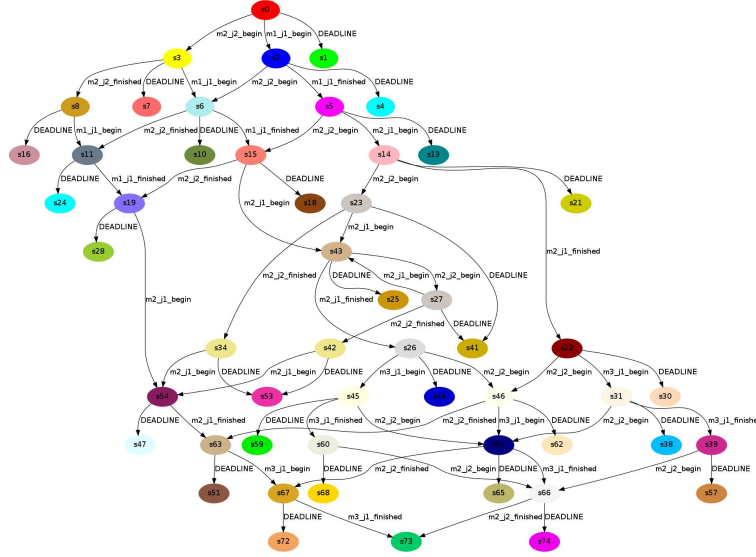
## Appendix: Case Study from [1]



**Fig. 2.** Schedules for jobshop example (with makespan 9 (left) and 11 (right)) [1]



**Fig. 3.** Geometrical representation of  $K$  around  $\pi_0 : (2, 5)$



**Fig. 4.** Graphics of the trace set output by IMITATOR on the jobshop example

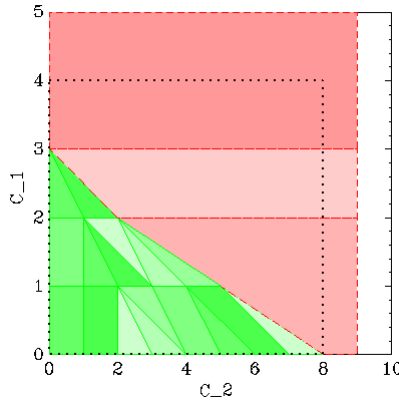
## Appendix: Case Study from [8]

We present here what we can obtain from IMITATOR 2.5 with the Behavioral Cartography (BC) output. By iterating *IM* over all the integers points inside a finite rectangle  $V_0$ , one is able to decompose (most of) the parametric space included into  $V_0$  into behavioral tiles<sup>1</sup>.

For a given tile, we ensure that for every valuation of the parameters in this tile, the behavior of  $\mathcal{A}$  is the same, therefore, we only have to test the schedulability of a single point for each tile to test whether or not  $\mathcal{A}$  is schedulable for any point in this tile. We apply this method on the example presented in [8].

We have three periodic tasks  $\tau_1, \tau_2$  and  $\tau_3$  with periods of 3, 8 and 20 respectively and deadlines of 3, 8 and 20 respectively. We are interested in finding the set of computation times of each task such that the system is schedulable. The interested reader can find the full details in [8].

We set  $V_0$  as  $C_1 \in [0, 3]$ ,  $C_2 \in [0, 8]$  and  $C_3 \in [0, 20]$  because the system is unschedulable if there exists a task  $\tau_i$  such that  $C_i > T_i$ . BC outputs the set of tiles and, we check for one point of each tile whether the system is schedulable or not. The result for this example is given in Figure 5, for  $C_1, C_2$  left as parameters and  $C_3$  set to 0, for the sake of lisibility but the method works for an arbitrary number of parameters.



**Fig. 5.** Schedulability, in green the system is schedulable, in red, unschedulable

It is interesting to notice that, we covered the full parameter space and, therefore solved the parametric schedulability problem by simply checking the schedulability of 20 valuations of the parameters, as there is only 20 tiles.

<sup>1</sup> Actually, one can take a finer discretization step to ensure a better coverability of  $V_0$