Flatness and motion Planning: the Car with n-trailers

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Flatness and motion planning : the car with n trailers. *

P. ROUCHON[†] M. FLIESS[‡] J. LÉVINE[§] P. MARTIN[¶]

Abstract A solution of the motion planning without obstacles for the nonholonomic system describing a car with n trailers is proposed. This solution relies basically on the fact that the system is flat with the cartesian coordinates of the last trailer as linearizing output. The Frnet formulas are used to simplify the calculation. The 2-trailers case is treated in details and illustrated through parking simulations.

Key words Nonholonomic motion planning, dynamic feedback linearization, linearizing output, flatness, mobile robots.

1 Introduction

In [6, 5, 7, 8] a new point of view on the full linearization problem via dynamic feedback [1] is proposed by introducing the notion of flatness and linearizing output. The aim of this paper is to show that such a standpoint can be very useful for motion planning.

Roughly speaking, a control system is said to be (*differentially*) *flat* if the following conditions are satisfied:

1. there exists a finite set $y = (y_1, \dots, y_m)$ of variables which are differentially independent, i.e., which are not related by any differential equations.

2. the y_i's are differential functions of the system variables, i.e., are functions of the system variables (state, input, ...) and of a finite number of their derivatives.

3. Any system variable is a differential function of the y_i's, i.e., is a function of the y_i's and of a finite number of their derivatives.

We call $y = (y_1, \dots, y_m)$ a *flat* or *linearizing* output. Its number of components equals the number of independent input channels.

Notice however that the concept of flatness, which can be made quite precise via the language of *differential algebra* [6, 5, 7, 8], is best defined by not distinguishing between input, state, output and other variables.

For a "classic" dynamics,

$$\dot{x} = f(x, u), \quad x = (x_1, \dots, x_n), \quad u = (u_1, \dots, u_m), \tag{1}$$

flatness implies the existence of a vector-valued function h such that

$$y = h(x, u_1, \dots, u_1^{(\beta_1)}, \dots, u_m, \dots, u_m^{(\beta_m)}),$$

where $y = (y_1, \dots, y_m)$. The components of x and u are, moreover, given without any integration procedure by the vector-valued functions A and B:

$$\begin{array}{rcl} x & = & A(y_1, \dots, y_1^{(\alpha_1)}, \dots, y_m, \dots, y_m^{(\alpha_m)}) \\ u & = & B(y_1, \dots, y_1^{(\alpha_1+1)}, \dots, y_m, \dots, y_m^{(\alpha_m+1)}). \end{array} \tag{2}$$

The motion planning problem for (1) consists in finding the control $[0, T] \ni t \to u(t)$ steering the system from state $x = p$ at $t = 0$ to the state $x = q$ at $t = T$. When the system is flat, this problem is equivalent to find $[0, T] \ni t \to y(t)$ such that

$$p = A(y_1(0), \dots, y_1^{(\alpha_1)}(0), \dots, y_m(0), \dots, y_m^{(\alpha_m)}(0))$$

and

$$q = A(y_1(T), \dots, y_1^{(\alpha_1)}(T), \dots, y_m(T), \dots, y_m^{(\alpha_m)}(T)).$$

Since the mapping

$$\begin{array}{l} (y_1, \dots, y_1^{(\alpha_1)}, \dots, y_m, \dots, y_m^{(\alpha_m)}) \\ \quad \to A(y_1, \dots, y_1^{(\alpha_1)}, \dots, y_m, \dots, y_m^{(\alpha_m)}) \end{array}$$

is onto, in general, the problem consists in finding a smooth function $t \to y(t)$ with prescribed values for some of its derivatives at time 0 and time T and such that

$$[0, T] \ni t \to A(y_1(t), \dots, y_1^{(\alpha_1)}(t), \dots, y_m(t), \dots, y_m^{(\alpha_m)}(t))$$

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and

$$[0,T] \ni t \rightarrow \\ B(y_1(t), \ldots, y_1^{(\alpha_1+1)}(t), \ldots, y_m(t), \ldots, y_m^{(\alpha_m+1)}(t))$$

are well defined smooth functions.

In this paper, we apply the method sketched here above to a class of systems studied in [13, 19, 15, 16] and describing the nonholonomic motion of a car with n trailers.

In section 2, we recall the basic state equations of this system and show that it is flat with the cartesian coordinates of the last trailer as linearizing output.In section 3, a geometric interpretation of the rolling without slipping relations is given. This allows us to simplify calculations and to bypass singularities via a natural parametrization and the Frnet formulas. A simple solution is then sketched for the general case when the number of trailers is arbitrary (see [8] for the complete proof). In section 4, detailed calculations and parking simulations are given for the 2-trailers case. In section 5, we give some comments on singularity and also some hints to take obstacles into account.

2 The system equations and flatness

We follow the modeling assumptions of [16]. The notations are summarized on figure 1. A basic model is the following :

$$\left\{ \begin{array}{rcl} \dot{x}_0 &=& \cos(\theta_0)\ u_1 \\ \dot{y}_0 &=& \sin(\theta_0)\ u_1 \\ \dot{\phi} &=& u_2 \\ \dot{\theta}_0 &=& \dfrac{1}{d_0}\tan(\phi)\ u_1 \\ && \text{for } i = 1, \ldots, n \\ \dot{\theta}_i &=& \dfrac{1}{d_i}\left(\displaystyle\prod_{j=1}^{i-1}\cos(\theta_{j-1}-\theta_j)\right)\ \sin(\theta_{i-1}-\theta_i)\ u_1 \end{array} \right. \tag{3}$$

where $(x_0, y_0, \phi, \theta_0, \ldots, \theta_n) \in \mathbb{R}^2 \times (S^1)^{n+2}$ is the state, (u_1, u_2) is the control and d_0, d_1, ..., d_n are positive parameters (lengths). Notice that, by using $\tan(\theta_i/2)$ instead of θ_i and $\tan(\phi/2)$ instead of ϕ, the equations become algebraic. The differential algebraic setting [4, 5] can thus be directly applied to this system.

In [6], we sketch the following result.

Proposition 1 *The car with n trailers described by (3) is a flat system. The linearizing output corresponds to the cartesian coordinates of the point P_n, the middle of the wheels axle of the last trailer :*

$$y = \begin{pmatrix} x_0 - \sum_{i=1}^n \cos(\theta_i) d_i \\ y_0 - \sum_{i=1}^n \sin(\theta_i) d_i \end{pmatrix}.$$

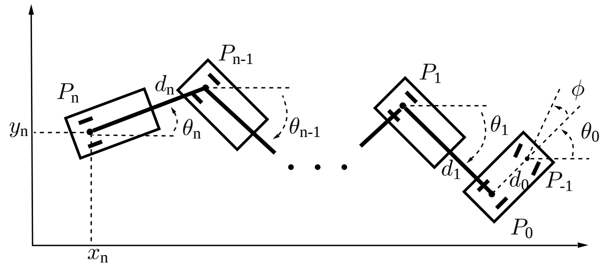

Figure 1: the kinematic car with n trailers

Flatness implies that, for generic states, the rank of the Lie algebra generated by the two vector fields associated to the control variables u_1 and u_2 is full (see [7]): this system is thus controllable around almost any state. In [11], it is proved that, for all states, the rank of this Lie algebra is full.

Proof Denote by (x_i, y_i) the cartesian coordinates of P_i, the middle of the wheels axle of trailer i:

$$\begin{array}{rcl} x_i &=& x_0 - \sum_{j=1}^i \cos(\theta_j) d_j \\ y_i &=& y_0 - \sum_{j=1}^i \sin(\theta_j) d_j. \end{array}$$

A straightforward calculation shows that $\tan(\theta_i) = \dfrac{\dot{y}_i}{\dot{x}_i}$. for $i = 0, \ldots, n$. Since $x_i = x_{i+1} + d_{i+1}\cos(\theta_{i+1})$ and $y_i = y_{i+1} + d_{i+1}\sin(\theta_{i+1})$ for $i = 0, \ldots, n-1$, θ_n, x_{n-1}, y_{n-1}, θ_{n-1}, ..., θ_1, x_0, y_0) and θ_0 are functions of (x_n, y_n) and its derivatives up to the order $n+1$. Since $u_1 = \dot{x}_0/\cos(\theta_0)$, $d_0\dot{\theta}_0/u_1 = \tan(\phi)$ and $u_2 = \dot{\phi}$, the entire state and the control are functions of the output (x_n, y_n) and its derivatives up to order $n+3$. ∎

Clearly the calculations sketched above lead to functions having singularities when, e.g., the derivatives $\dot{x}_i$ become zero. In the next section, we shall see that this kind of singularities can be ignored. We will see, in section 5, that the control trajectories constructed below respect necessarily these angular constraints.

3 The basic geometric construction and the Frnet formula

Throughout this section, we assume that the angles $\theta_i - \theta_{i-1}$ $(i = 1, \ldots, n)$ and ϕ belong to $]-\pi/2, \pi/2[$. This corresponds to a natural physical limitation (the impossibility of the trailer i to be in front of trailer $i-1$).

Proposition 1 means that the trajectory of the entire system can be derived (without integration) from the

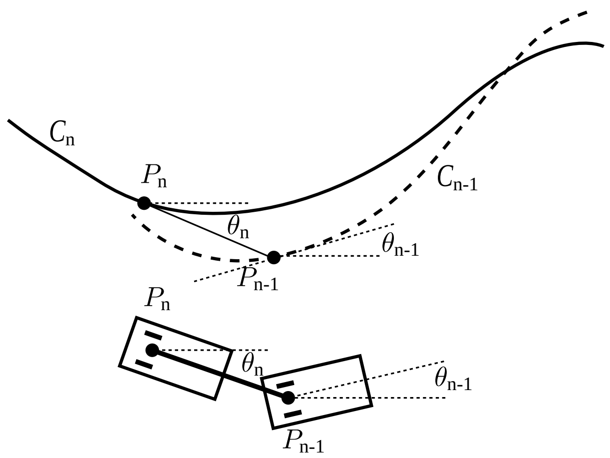

Figure 2: the geometric interpretation of the rolling without slipping condition for the last trailer n.

trajectory of the point P_n. Assume that P_n follows the smooth curve $\mathcal{C}_n$. The rolling without slipping conditions imply that the velocity of P_n, which is tangent to $\mathcal{C}_n$, is colinear to the direction of the straight line passing through P_n and P_{n-1}. Denote by τ_n the tangent vector of length 1 to the curve $\mathcal{C}_n$ at P_n which is equal to $\overrightarrow{P_nP_{n-1}}/d_n$. One can choose a natural parametrization of $\mathcal{C}_n$, $s_n \to P_n(s_n)$ (s_n is the arc length), such that $\tau_n = \dfrac{dP_n}{ds_n}$. Denote by κ_n the signed curvature of $\mathcal{C}_n$ (function of s_n). It is defined by the Frnet formulas (see [3, page 51])

$$\frac{d\tau_n}{ds_n} = \kappa_n(s_n)\nu_n, \quad \frac{d\nu_n}{ds_n} = -\kappa_n(s_n)\tau_n$$

where ν_n is the normal vector to $\mathcal{C}_n$ such that (τ_n, ν_n) admits a positive orientation. Then, as displayed on figure 2, we have

$$P_{n-1} = P_n + d_n\tau_n.$$

Thus the curve $\mathcal{C}_{n-1}$ generated by P_{n-1}, is given through the parametrization s_n (not natural in general)

$$s_n \to P_{n-1}(s_n) = P_n(s_n) + d_n\tau_n(s_n).$$

This parametrization is regular since the vector $\dfrac{dP_{n-1}}{ds_n}$ is always different from zero: its length is equal to $\sqrt{1+d_n^2\kappa_n^2}$. Thus

$$\tan(\theta_{n-1} - \theta_n) = d_n\kappa_n \tag{4}$$

and the arc length of $\mathcal{C}_{n-1}$, s_{n-1}, is given by $ds_{n-1} = \sqrt{1+d_n^2\kappa_n^2}\ ds_n$. Additional calculations give the curvature κ_{n-1} of $\mathcal{C}_{n-1}$:

$$\kappa_{n-1} = \frac{1}{\sqrt{1+d_n^2\kappa_n^2}}\left(\kappa_n + \frac{d_n}{1+d_n^2\kappa_n^2}\ \frac{d\kappa_n}{ds_n}\right). \tag{5}$$

With one derivation, we have obtained, without any singularity, the curve $\mathcal{C}_{n-1}$ with the natural parametrization $s_{n-1} \to P_{n-1}(s_{n-1})$.

The same method can be utilized for deriving $\mathcal{C}_{n-2}$ from $\mathcal{C}_{n-1}$. By iterating this process, we see that the curve $\mathcal{C}_i$ generated by P_i $(i = -1, \ldots, n-1)$ is derived from the curve $\mathcal{C}_n$ via $(n-i)$ differentiations with respect to s_n. We have sketched the proof of the following result (more details are given in [8]).

Proposition 2 *Consider the car with n trailers of figure 1 the motion of which is described by (3). Assume that the curve $\mathcal{C}_n$ followed by P_n is smooth and admits a natural parametrization $s_n \to P_n(s_n)$. Then, the state $(x_0, y_0, \phi, \theta_0, \ldots, \theta_n)$ is a smooth function of P_n, $\dfrac{dP_n}{ds_n}$, κ_n (signed curvature of $\mathcal{C}_n$), $\dfrac{d\kappa_n}{ds_n}$, ..., $\dfrac{d^n\kappa_n}{ds_n^n}$, i.e., the state $(x_0, y_0, \phi, \theta_0, \ldots, \theta_n)$ is then a smooth function of s_n.*

According to proposition 2, it suffices to express s_n as a time function $s_n = \sigma(t)$, where $t \to \sigma(t)$ is C^1, to obtain the state dependence with respect to time. The first control is given by

$$u_1(t) = \left(\cos(\theta_0(\sigma(t)))\tfrac{dx_0}{ds_n}(\sigma(t)) + \sin(\theta_0(\sigma(t)))\tfrac{dy_0}{ds_n}(\sigma(t))\right)\ \dot\sigma(t)$$

and the second one by

$$u_2(t) = \frac{d\phi}{ds_n}(\sigma(t))\ \dot\sigma(t).$$

Via this method, we always calculate control trajectories that are, at least, C^0. Notice that the parametrization of $\mathcal{C}_n$ with respect to t is not necessarily everywhere regular but this has no consequence in our problem.

The geometric construction of figure 2 can also be found in [10] where it serves for a car with a single trailer: the time t is replaced by an arc length s; a linear approximation of the differential equation in s is considered around the configurations $\theta_n - \theta_{n-1} \approx \ldots \approx \theta_1 - \theta_0 \approx \phi \approx 0$; the parking of a slightly different system with one trailer is analyzed.

4 The car with 2 trailers

We now restrict to the particular case $n = 2$. We show how the previous analysis can be employed to solve

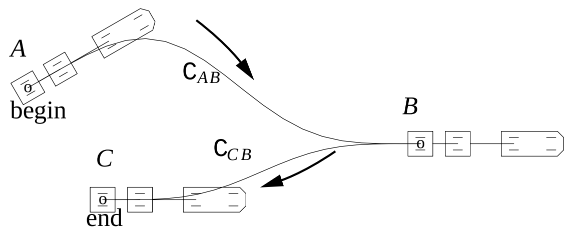

Figure 3: parking the car with 2-trailers from A to B via C.

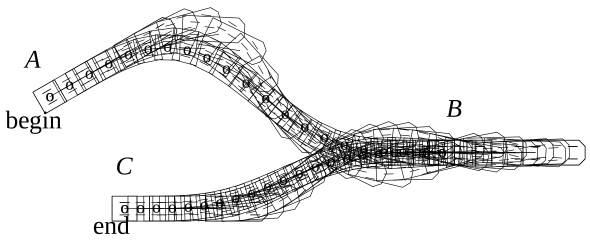

Figure 4: the successive motions of the car with 2-trailers.

the parking problem. The simulations of figures 3 and 4 have been written in MATLAB.

The car and its trailers start from A with orientation as displayed on figure 3. We want to find a path that steers the system to B. We consider the point C and the two curves $\mathcal{C}^{AC}$ and $\mathcal{C}^{BC}$ of figure 3 defined by their natural parametrizations $[0, L^{AC}] \ni s \rightarrow P^{AC}(s)$ and $[0, L^{BC}] \ni s \rightarrow P^{BC}(s)$, respectively: $P^{AC}(0) = A$, $P^{AC}(L^{AC}) = C$, $P^{BC}(0) = B$ and $P^{BC}(L^{BC}) = C$. The curvatures are denoted by κ^{AC} and κ^{BC}.

These two curves are followed by the point P_2 from A to B via C. Thus according to (4) and (5), the initial and final configurations in A and B impose $\frac{d^r \kappa^{AC}}{ds^r}(0) = 0$ and $\frac{d^r \kappa^{BC}}{ds^r}(0) = 0$ for $r = 0,\ 1,\ 2$. Moreover the contact of t $\mathcal{C}^{AC}$ and $\mathcal{C}^{BC}$ at C must be of order ≥ 4: $\frac{d^r \kappa^{AC}}{ds^r}(L^{AC}) = 0$ and $\frac{d^r \kappa^{BC}}{ds^r}(L^{BC}) = 0$ for $r = 0,\ 1,\ 2$. It is straightforward to construct curves satisfying such conditions (take, e.g., polynomial curves of degree $\leq$ 9).

From proposition 2, we know that if the point P_2 follows $\mathcal{C}^{AC}$ and $\mathcal{C}^{BC}$ as displayed on figure 3, then the initial and final states will be as desired. Take a C^1 function $[0, T] \ni t \rightarrow s(t) \in [0, L^{AC}]$ such that $s(0) = 0$, $s(T) = L^{AC}$ and $\dot{s}(0) = \dot{s}(T) = 0$. This leads to smooth control trajectories $[0, T] \ni t \rightarrow u_1(t) \geq 0$ and $[0, T] \ni t \rightarrow u_2(t)$ steering the system from A at time $t = 0$ to C at time $t = T$. Similarly, $[T, 2T] \ni t \rightarrow s(t) \in [0, L^{BC}]$ such that $s(T) = L^{BC}$, $s(2T) = 0$ and $\dot{s}(T) = \dot{s}(2T) = 0$ leads to control trajectories $[T, 2T] \ni t \rightarrow u_1(t) \leq 0$ and $[T, 2T] \ni t \rightarrow u_2(t)$ steering the system from C to B. We obtain the motions displayed on figure 4.

Let us detail the calculation of the control trajectories for the motion from A to C. Similar calculations can be done for the motion from C to B. Assume that $\mathcal{C}_2 = \mathcal{C}^{AC}$ is given via the regular parametrization, $y = f(x)$ ((x, y) are the cartesian coordinates). Denote by s_i the arc length of curve $\mathcal{C}_i$, $i = 0, 1, 2$. Then $ds_2 = \sqrt{1 + (df/dx)^2}\, dx$ and the curvature of $\mathcal{C}_2$ is given by

$$\kappa_2 = \frac{d^2 f/dx^2}{(1 + (df/dx)^2)^{3/2}}.$$

According to (5),

$$\kappa_1 = \frac{1}{\sqrt{1 + d_2^2 \kappa_2^2}} \left(\kappa_2 + \frac{d_2}{1 + d_2^2 \kappa_2^2} \frac{d\kappa_2}{ds_2} \right)$$

and $ds_1 = \sqrt{1 + d_2^2 \kappa_2^2}\, ds_2$. Similarly,

$$\kappa_0 = \frac{1}{\sqrt{1 + d_1^2 \kappa_1^2}} \left(\kappa_1 + \frac{d_1}{1 + d_1^2 \kappa_1^2} \frac{d\kappa_1}{ds_1} \right)$$

and $ds_0 = \sqrt{1 + d_1^2 \kappa_1^2}\, ds_1$. Thus u_1 is given explicitly by

$$u_1 = \frac{ds_0}{dt} = \sqrt{1 + d_1^2 \kappa_1^2}\, \sqrt{1 + d_2^2 \kappa_2^2}\, \sqrt{1 + (df/dx)^2}\, \dot{x}(t)$$

where $[0, T] \ni t \rightarrow x(t)$ is any increasing smooth time function such that $(x(0), f(x(0)))$ (resp. $(x(T), f(x(T)))$) are the coordinates of A (resp. C) and $\dot{x}(0) = \dot{x}(T) = 0$. Since $\tan(\phi) = d_0 \kappa_0$, we get

$$u_2 = \frac{d\phi}{dt} = \frac{d_0}{1 + d_0^2 \kappa_0^2} \frac{d\kappa_0}{ds_0}\, u_1$$

5 Remarks

The relative position of the car with respect to the trailer is defined by $\theta_1 - \theta_0$ and is directly related to the curvature κ_1 of $\mathcal{C}_1$ by

$$\tan(\theta_0 - \theta_1) = d_1 \kappa_1.$$

The angle $\theta_0 - \theta_1$ always remains in $] - \pi/2, \pi/2[$. This property is general: all the trajectories obtained via a smooth curve $\mathcal{C}_n$ are such that $\theta_{i-1} - \theta_i$ ($i = 1, \ldots, n$) and ϕ always remain in $] - \pi/2, \pi/2[$.

When $\theta_0 - \theta_1 = -\pi/2$ or $\pi/2$ the curve $\mathcal{C}_1$ is no longer twice differentiable (infinite curvature). This corresponds to a true singularity. For arbitrary n, the singularities that cannot be canceled by this geometric construction appear when, for at least one $i \in \{1, \ldots, n\}$, $\theta_{i-1} - \theta_i = -\pi/2$ or $\pi/2$ or when $\phi = -\pi/2$ or $\pi/2$.

In this paper, we are not actually concerned with obstacles. The fact that the internal configuration depends only on the curvature results from the general following property: a plane curve is entirely defined (up to rotation and translation) by its curvature. For the n-trailer case, the angles $\theta_n - \theta_{n-1}$, ..., $\theta_1 - \theta_0$ and ϕ describing the relative configuration of the system are only functions of κ_n and its first n-derivatives with respect to s_n.

Consequently, limitations due to obstacles can be expressed up to a translation (defined by P_n) and a rotation (defined by the tangent direction $\frac{dP_n}{ds_n}$) via κ_n and its first n-derivatives. Such considerations can be of some help in finding a curve $\mathcal{C}_n$ avoiding collisions. More details on obstacle avoidance can be found in [12] where a car without trailer is considered.

6 Conclusion

The concept of flatness, which has been illustrated by this example of motion planning, may be utilized in many industrial applications, such as the crane [9], aircraft control [2, 14] and chemical reactors [18, 17].

Acknowledgment

The MATLAB programs used for the simulations of figures 3 and 4 can be obtained upon request and via electronic mail from the first author (rouchon@cas.ensmp.fr).

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