An in-depth study of the syntactic analysis of graphs using search matching algorithms, to the degree of sub graphs

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1 Graph Basics

1.1 Graph Overview

A Graph in Mathematics and Computer Science is defined as a pair $G = (V, E)$, where V is the set of vertices and E is the set of edges, formed by pairs of vertices with each other. Figure 1 demonstrates the structural attributes of a simple graph.

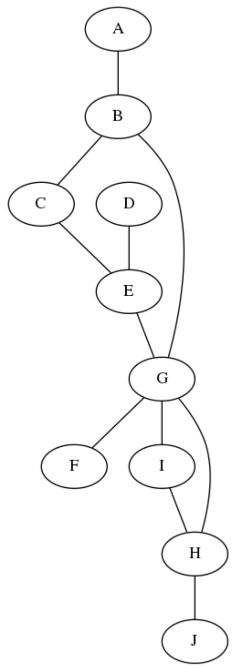

Figure 1: Representation of a graph

Graphs can either directed or they can undirected. This means that the edges in the graph could have an abscence of direction as in the 1 above, or they could have a direction showing from which vertice the edge is coming from, and to which vertice the edge is going to. Figure 2 demonstrates a directed graph, commonly known as a digraph. This characteristic is demonstrated by the edge 3, that goes from node B to node G, and also by edge 2 that goes from node B to node C.

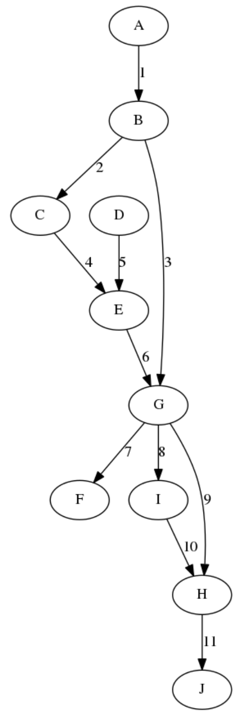

Figure 2: Representation of a digraph

Note that the undirected graph mentioned above is commonly referred to as a bigraph because the direction of its edges could be percieved as to going in both direction as it is not specifed.

In this paper, the focus is primarily on directed graphs, and they shall be referred to as digraphs from here onward.

1.2 Digraph

A Digraph in mathematics is defined as is a pair of disjoint vertices and edges (V, E), and their repective mappings that comprises of two components, namely the initial vertix and terminal vertice of each edge i.e. each edge has a initial vertix:

$$Ei \to Vi \tag{1}$$

and a terminal vertix:

$$Vj \to Ej \tag{2}$$

for some vertices Vi,Vj in V and edges Ei,Ej in E [7] refer to the figure 2.

1.3 Graph representation

Graphs are represented in a variaty of ways, from adjacency lists, incident matrices and adjacency matrices. The algorithms that are studied in this paper make us of adjacency matrices and adjacency list representations of graphs.

1.3.1 Adjacency matrices

An adjacency matrices is a nxn matrix A, with $A(i, j) = 1iff(i, j)E$ [9]. This means that wherever there is an edge in the graph, it is denoted by a 1 in the matrix, places in the matrix where there is an absence of an edge, are denoted by 0.

Figure 3 depicts the association between a graph and its adjacency matrix.

	A	B	C	D	E	F
A	-	1	1	1	1	
B	1	-		1	1	
C	1		-			1
D	1	1		-		1
E	1	1			-	1
F			1	1	1	-

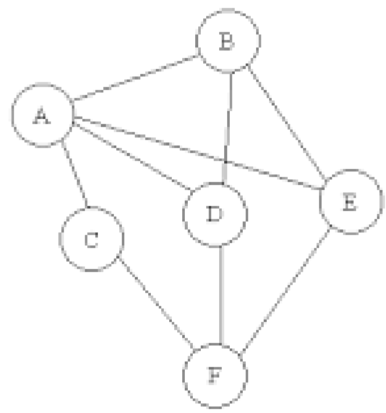

Figure 3: Representation of a graph and its associated adjacency matrix

1.3.2 Adjacency list

An Adjacency list is vertices of a graph, of which each vertice is connected to the list. The vertices in an adjacency list point to their own list of edges that they are connected to(i.e. the list contains the edges that connectect them to other vertices). Figure 4 depicts an example of an adjacency list.

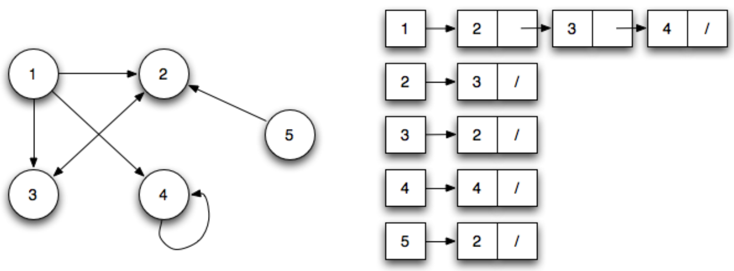

Figure 4: Representation of a graph and its associated adjacency list

1.4 Supergraphs and subgraphs

Let G_A be be graph define as follows G_A = (VA,EB) and let G_B be another graph that is defined as follows G_B = (VB,EB) where VA,VB are sets of vertices and EA,EB are sets of edges. In graph theory, a graph G_A is said to be a subgraph of graph G_B, and graph G_B is said to be a supergraph of graph G_A if all the vertices and edges that are in graph G_A are also in graph G_B, that is [3] 1) VA ¡= VB, and 2) Every edge of G_A is also an edge in G_B.

The figure 5 below depicts this relation.

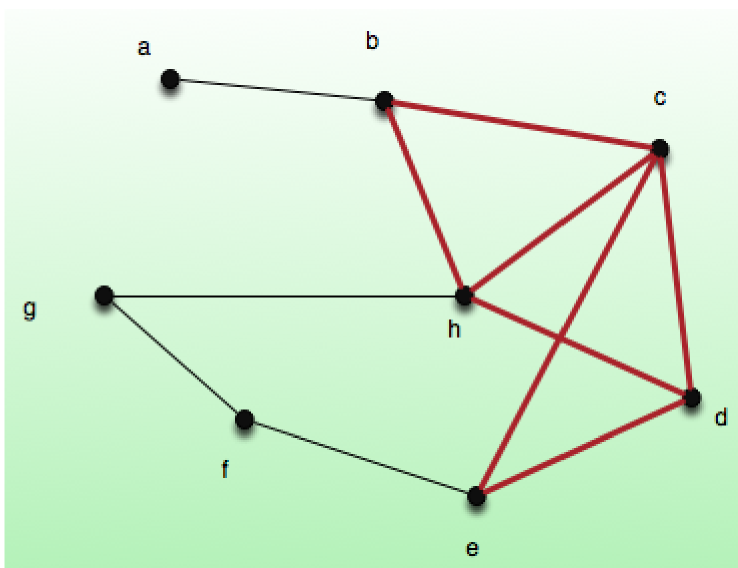

Figure 5: Representation of a super graph and its subgraph that is depicted in red

1.5 Graph Isomorphism

Two graphs are said to be isomorphic if they are syntactically similar to each other iff there is a bijection between their respective nodes which make each edge of G1 correspond to exactly one edge of G2, and vice versa[12], i.e. the graphs are structurally the same to each other. This property is demonstrated in figure 6. The two graphs look very different, but when they are further inspected, it is evident that the two are a representation of the same data scheme or even the same graph that has been rearranged.

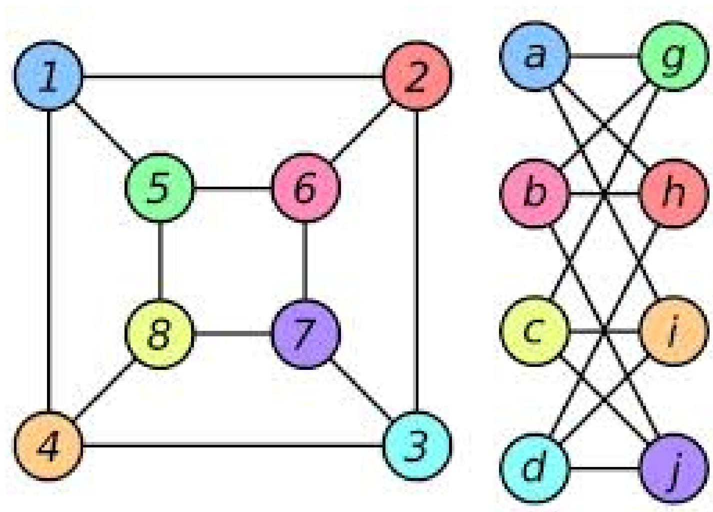

Figure 6: Representation of a super graph and its subgraph that is depicted in red