MA0001 – Øving nr 4 Gruppe 4

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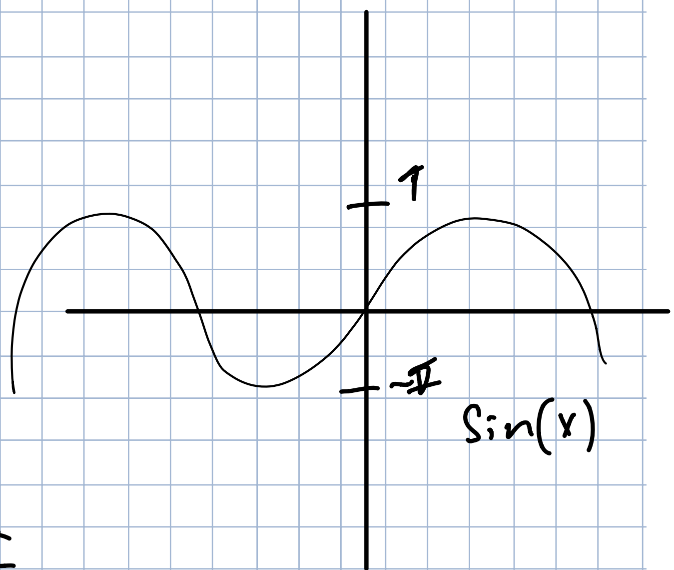
a) $f: \langle 0, 1] \to \mathbb{R}$ $f(x) = -\ln(x)$

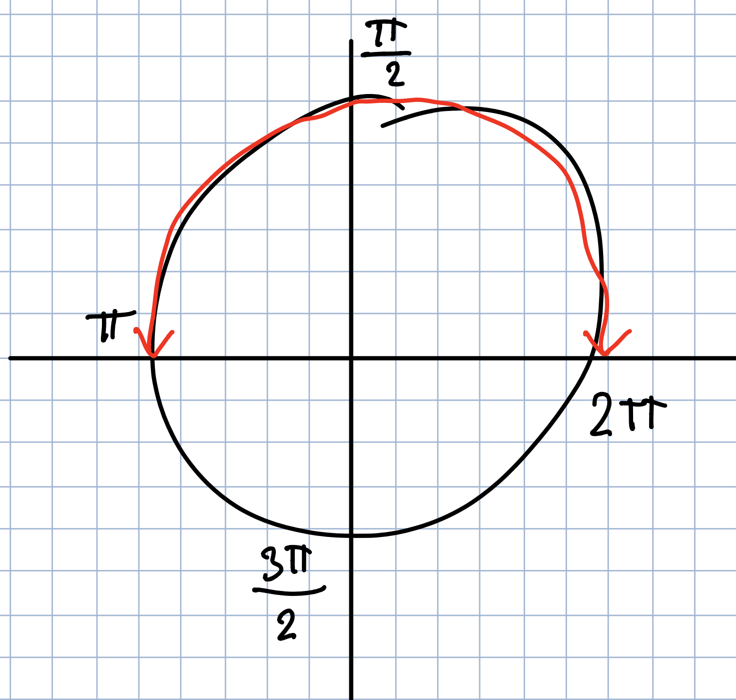
$V_f = [0, \infty\rangle$

b) $g(x) = -\ln(\sin(x))$



$D_g = \langle 2\pi n, 2\pi n + \pi\rangle$

$V_g = \langle 0, \infty\rangle$



$\boxed{2}$

$$a_n = \sqrt{n^2+9} - \sqrt{n^2-n+9}$$

$$= \frac{\left(\sqrt{n^2+9} - \sqrt{n^2-n+9}\right)\left(\sqrt{n^2+9} + \sqrt{n^2-n+9}\right)}{\sqrt{n^2+9} + \sqrt{n^2-n+9}}$$

$$= \frac{\left(\sqrt{n^2+9} - \sqrt{n^2-n+9}\right)^2}{\sqrt{n^2+9} + \sqrt{n^2-n+9}}$$

$$= \frac{n^2+9-(n^2-n+9)}{\sqrt{n^2+9} + \sqrt{n^2-n+9}}$$

$$= \frac{n}{\sqrt{n^2+9} + \sqrt{n^2-n+9}} \quad \Big| : n$$

$$= \frac{\frac{n}{n}}{\frac{\sqrt{n^2+9}}{n} + \frac{\sqrt{n^2-n+9}}{n}} = \frac{1}{\frac{\sqrt{n^2+9}}{n} + \frac{\sqrt{n^2-n+9}}{n}}$$

$$= \frac{1}{\sqrt{\frac{n^2+9}{n^2}} + \sqrt{\frac{n^2-n+9}{n^2}}}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{9}{n^2}} + \sqrt{1-\frac{1}{n}+\frac{9}{n^2}}}$$

$$= \frac{1}{\sqrt{1}+\sqrt{1}} = \frac{1}{2}$$

$\boxed{3}$

$$a_{n+1} = 2 - \frac{1}{a_n} \qquad a_0 = 2$$

Fikspunkt:

$$a = 2 - \frac{1}{a}$$

$$a^2 = 2a - 1$$

$$a^2 - 2a + 1 = 0$$

$$\frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 1}}{2}$$

$$\frac{2 \pm 0}{2}$$

$$a = \underline{\underline{1}}$$

Grenseverdi:

$$a_{n+1} = 2 - \frac{1}{a_n} \qquad a_0 = 2$$

$$L = 2 - \frac{1}{L}$$

$$L = \underline{\underline{1}}$$

$$a_1 = 2 - \frac{1}{2} = \frac{3}{2}$$

$$a_2 = 2 - a_1 = \frac{2}{1} - 1 \cdot \frac{2}{3}$$

$$= \frac{6}{3} - \frac{2}{3}$$

$$= \frac{4}{3}$$

$\vdots$

$\downarrow$

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Siden $a_0 = 2$, så vil

$\frac{1}{a_n}$ alltid være ≤ 1.

Da følger det at

$$\underline{\underline{a_{n+1} = 2 - \frac{1}{a_n} \geq 1}}$$

$\boxed{4}$

$$a_n \begin{cases} a_0 = 1 \\ a_1 = 2 \\ a_{n+1} = a_n + 2 \cdot a_{n-1}, \quad n \geq 1 \end{cases}$$

$a_2 = a_1 + 2 \cdot a_0 \quad = 2 + 2 \cdot 1 = \underline{4}$

$a_3 = a_2 + 2 \cdot a_1 \quad = 4 + 2 \cdot 2 = \underline{8}$

$a_4 = a_3 + 2 \cdot a_2 \quad = 8 + 2 \cdot 4 = \underline{16}$

$a_5 = a_4 + 2 \cdot a_3 \quad = 16 + 16 = 32$

$a_6 \quad = 64$

$a_7 \quad = 128$

$a_n = f(n) = 2^n$

$f(n) \rightarrow a_{n+1} = a_n + 2 \cdot a_{n-1}$

$a_{n+1} = 2^n + 2 \cdot 2^{n-1}$

$a_{n+1} = 2^n + 2^n$

$$\underline{\underline{a_{n+1} = 2 \cdot 2^n = 2^{n+1}}}$$