Innleveringsoppgaver

1 Regn ut grenseverdien

$$\lim_{x\to 0}\frac{\sqrt{x^2+3}-\sqrt{3}}{x}.$$

2 Bruk skviseteoremet til å regne ut grenseverdiene.

a) $\lim_{x\to -\infty} e^{x}\sin(e^{-x})$.

b) $\lim_{x\to \pi} \sin(x)\cos\left(\frac{1}{x-\pi}\right)$.

3 En funksjon $f\colon A\to\mathbb{R}$ kalles *kontinuerlig* i et punkt $a\in A$ dersom

$$\lim_{x\to a} f(x) = f(a).$$

Bestem konstanten k slik at funksjonen

$$f(x)=\begin{cases} x^2+4 & \text{for } x\geq 0,\\ \frac{\sin(kx)}{4x} & \text{for } x<0,\end{cases}$$

er kontinuerlig i $x=0$.

Anbefalte øvingsoppgaver

Fra Avsnitt 3.1 (side 101) i *Calculus for Biology and Medicine*, 3. utgave av Claudia Neuhauser.

- 37, 39, 41, 47, 49, 51.

Fra Avsnitt 3.3 (side 113).

- 1, 3, 7, 13, 15.

Fra Avsnitt 3.4 (side 118–119).

- 5, 7, 9, 11, 13, 15, 19.

OBS: Disse oppgaven skal *ikke* leveres inn!

MA0001 – Øving nr 5 Gruppe 4

Magnus L. Holtet

$\boxed{1}$

$$\lim_{x \to 0} \frac{\sqrt{x^2+3} - \sqrt{3}}{x}$$

$$\lim_{x \to 0} \frac{\left(\sqrt{x^2+3} - \sqrt{3}\right) \cdot \left(\sqrt{x^2+3} + \sqrt{3}\right)}{x\left(\sqrt{x^2+3} + \sqrt{3}\right)}$$ | Bruker Konjugatsetningen

$$\lim_{x \to 0} \frac{x^2+3-3}{x\left(\sqrt{x^2+3} + \sqrt{3}\right)}$$ | $: x^2$

$$\lim_{x \to 0} \frac{\frac{x^2}{x^2}}{\frac{x\left(\sqrt{x^2+3} + \sqrt{3}\right)}{x^2}}$$

$$\lim_{x \to 0} \frac{1}{\frac{\sqrt{x^2+3}}{x} + \frac{\sqrt{3}}{x}} = \frac{1}{\sqrt{1+\frac{3}{x^2}} + \frac{\sqrt{3}}{x}} = \underline{\underline{0}}$$

$\boxed{2}$

a)

$$\lim_{x \to -\infty} e^{x} \sin(e^{-x})$$

$$-1 \leq \sin(e^{-x}) \leq 1 \quad \Big| \cdot e^{x}$$

$$-e^{x} \leq e^{x} \cdot \sin(e^{-x}) \leq e^{x}$$

$$\lim_{x \to -\infty} -e^{x} \leq \lim_{x \to -\infty} e^{x} \cdot \sin(e^{-x}) \leq \lim_{x \to -\infty} e^{x}$$

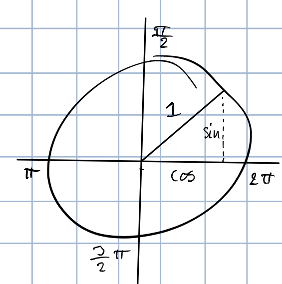
$$0 \leq \lim_{x \to -\infty} e^{x} \cdot \sin(e^{-x}) \leq 0$$

$$\lim_{x \to -\infty} e^{x} \cdot \sin(e^{-x}) = \underline{\underline{0}}$$

b)

$$\lim_{x \to \pi} \sin(x) \cos\left(\frac{1}{x-\pi}\right)$$

$$-1 \leq \cos\left(\frac{1}{x-\pi}\right) \leq 1 \quad \Big| \cdot \sin(x)$$



$$\lim_{x \to \pi} -\sin(x) \leq \lim_{x \to \pi} \sin x \cdot \cos \frac{1}{x-\pi} \leq \sin x$$

$$0 \leq \lim_{x \to \pi} \sin x \cdot \cos \frac{1}{x-\pi} \leq 0$$

$$\lim_{x \to \pi} \sin x \cdot \cos \frac{1}{x-\pi} = \underline{\underline{0}}$$

$\boxed{3}$

$$f(x) = \begin{cases} x^2 + 4 & x \geq 0 \\ \dfrac{\sin(kx)}{4x} & x < 0 \end{cases}$$

$$f(0) = 0^2 + 4 = 4$$

$$\lim_{x \to 0} \frac{\sin(k \cdot x)}{4x} = 4 \qquad z = k \cdot x$$

$$\lim_{x \to 0} \frac{\sin(z)}{\frac{4 \cdot z}{k}} = 4$$

$$\frac{k}{4} \cdot \lim_{x \to 0} \frac{\sin z}{z} = 4$$

$$k = \underline{\underline{16}}$$