Actions

$$E = \frac{u^n_{\overline{t}t}}{u''(t_n)} = \frac{u(t_n+\tau) - 2u(t_n) + u(t_n-\tau)}{\tau^2 u''(t_n)} = \left[u(t) = e^{i\omega t}\right] =$$

$$= \frac{e^{i\omega(t_n+\tau)} - 2e^{i\omega t_n} + e^{i\omega(t_n-\tau)}}{-\omega^2\tau^2 e^{i\omega t_n}} = \frac{e^{i\omega\tau} - 2 + e^{-i\omega\tau}}{-\omega^2\tau^2} = \frac{\cos\omega\tau + i\sin\omega\tau - 2 + \cos\omega\tau - i\sin\omega\tau}{-\omega^2\tau^2} =$$

$$= -\frac{2(\cos\omega\tau - 1)}{\omega^2\tau^2} = \frac{4\sin^2\frac{\omega\tau}{2}}{\omega^2\tau^2} = \left(\frac{2}{\omega\tau}\right)^2 \sin^2\frac{\omega\tau}{2}$$