

## HW5\_EE341\_Fall2019

- 6.1
1. Active
  2. Saturation
  3. Active
  4. Saturation
  5. Active
  6. Cutoff

$$6.7 \quad 5 \times 10^{-3} = I_S e^{0.76/0.025} \quad (1)$$

$$I_C = I_S e^{0.70/0.025} \quad (2)$$

Dividing Eq. (2) by Eq. (1) yields

$$I_C = 5 \times 10^{-3} e^{-0.06/0.025}$$

$$= 0.45 \text{ mA}$$

For  $I_C = 5 \mu\text{A}$ ,

$$5 \times 10^{-6} = I_S e^{V_{BE}/0.025} \quad (3)$$

Dividing Eq. (3) by Eq. (1) yields

$$10^{-3} = e^{(V_{BE}-0.76)/0.025}$$

$$V_{BE} = 0.76 + 0.025 \ln(10^{-3})$$

$$= 0.587 \text{ V}$$

6.16 First we determine  $I_S$ ,  $\beta$ , and  $\alpha$ :

$$1 \times 10^{-3} = I_S e^{700/25}$$

$$\Rightarrow I_S = 6.91 \times 10^{-16} \text{ A}$$

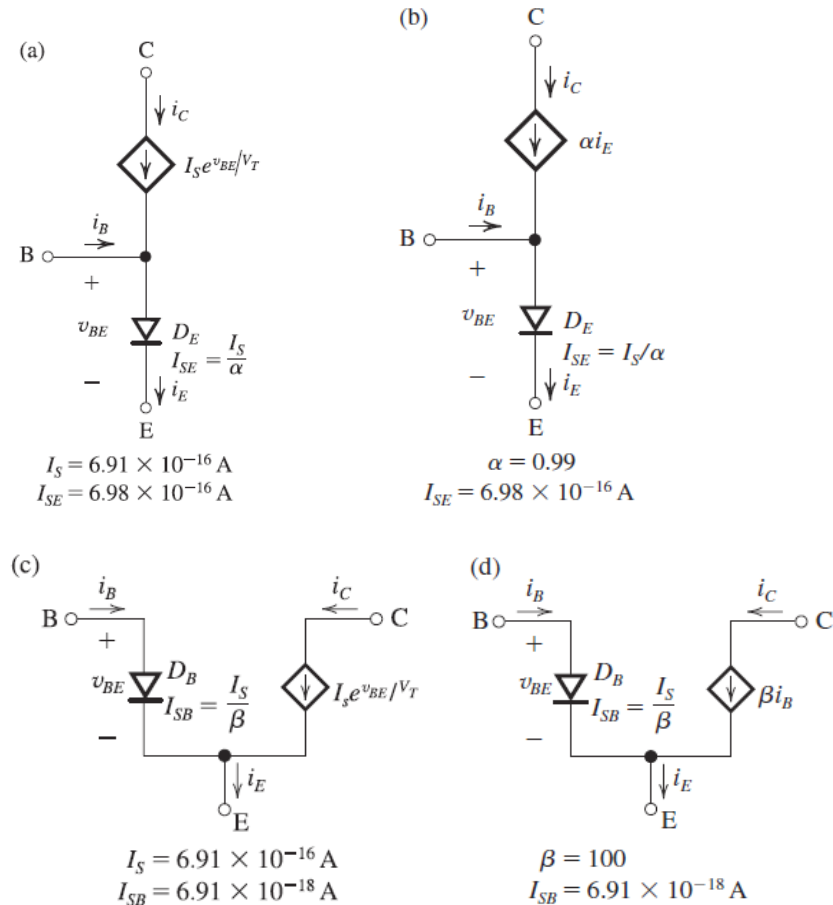
$$\beta = \frac{I_C}{I_B} = \frac{1 \text{ mA}}{10 \mu\text{A}} = 100$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

Then we can determine  $I_{SE}$  and  $I_{SB}$ :

$$I_{SE} = \frac{I_S}{\alpha} = 6.98 \times 10^{-16} \text{ A}$$

$$I_{SB} = \frac{I_S}{\beta} = 6.91 \times 10^{-18} \text{ A}$$



6.28 (a) Refer to Fig. P6.28(a).

$$I_1 = \frac{10.7 - 0.7}{5 \text{ k}\Omega} = 2 \text{ mA}$$

Assuming operation in the active mode,

$$I_C = \alpha I_1 \simeq I_1 = 2 \text{ mA}$$

$$V_2 = -10.7 + I_C \times 5$$

$$= -10.7 + 2 \times 5 = -0.7 \text{ V}$$

Since  $V_2$  is lower than  $V_B$ , which is 0 V, the transistor is operating in the active mode, as assumed.

(d) Refer to Fig. P6.28(d). Since the collector is connected to the base with a 10-k $\Omega$  resistor and  $\beta$  is assumed to be very high, the voltage drop across the 10-k $\Omega$  resistor will be close to zero and the base voltage will be equal to that of the collector:

$$V_B = V_C$$

This also implies active-mode operation. Now,

$$V_E = V_B - 0.7$$

Thus,

$$V_E = V_C - 0.7$$

$$I_6 = \frac{V_E - (-10)}{3}$$

$$= \frac{V_C - 0.7 + 10}{3} = \frac{V_C + 9.3}{3} \quad (1)$$

Since  $I_B = 0$ , the collector current will be equal to the current through the 9.1-k $\Omega$  resistor,

$$I_C = \frac{+10 - V_C}{9.1} \quad (2)$$

Since  $\alpha \simeq 1$ ,  $I_C = I_E = I_6$  resulting in

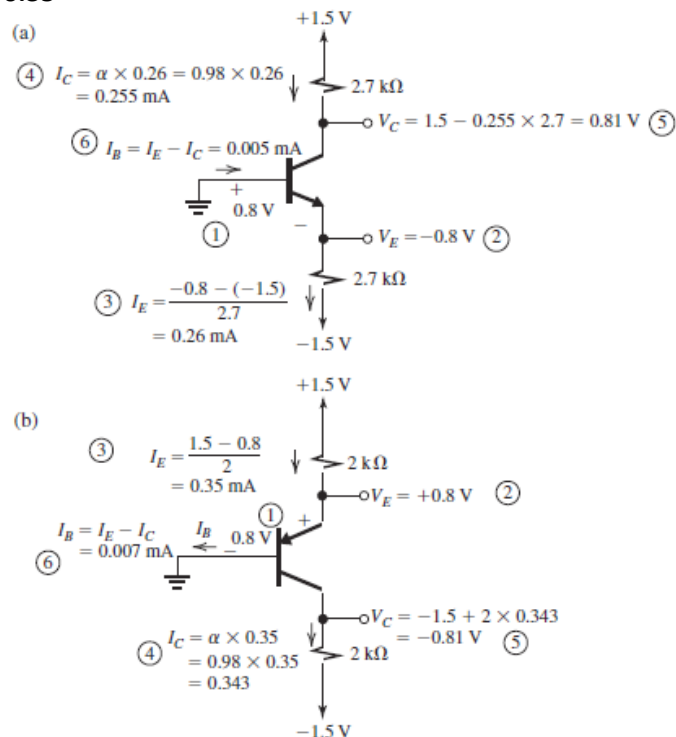
$$\frac{10 - V_C}{9.1} = \frac{V_C + 9.3}{3}$$

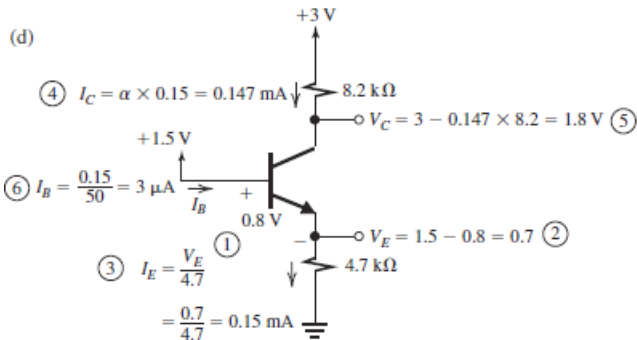
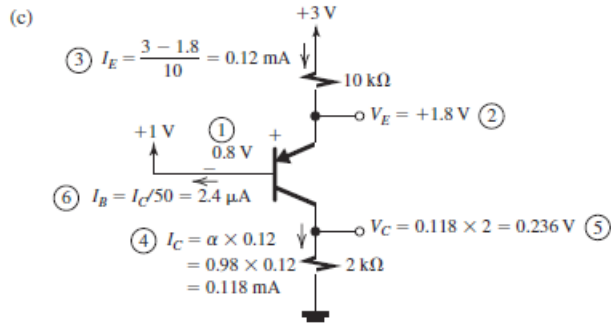
$$\Rightarrow V_C = -4.5 \text{ V}$$

and

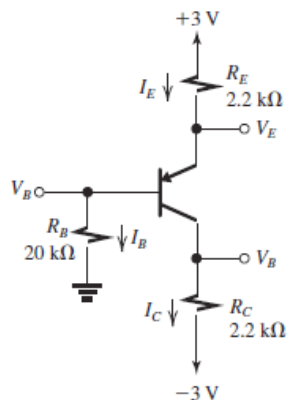
$$I_6 = \frac{V_C + 9.3}{3} = \frac{-4.5 + 9.3}{3} = 1.6 \text{ mA}$$

### 6.35





6.59



Assume active-mode operation:

$$I_E = \frac{3 - V_{EB}}{R_E + \frac{R_B}{\beta + 1}}$$

$$I_E = \frac{3 - 0.7}{2.2 + \frac{20}{51}} = 0.887 \text{ mA}$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.887}{51} = 0.017 \text{ mA}$$

$$I_C = I_E - I_B = 0.887 - 0.017 = 0.870 \text{ mA}$$

$$V_B = I_B R_B = 0.017 \times 20 = 0.34 \text{ V}$$

$$V_E = V_B + V_{EB} = 0.34 + 0.7 = 1.04 \text{ V}$$

$$V_C = -3 + I_C R_C = -3 + 0.87 \times 2.2 = -1.09 \text{ V}$$

Thus,  $V_C < V_B + 0.4$ , which means active-mode operation, as assumed. The maximum value of  $R_C$  that still guarantees active-mode operation is that which causes  $V_C$  to be 0.4 V above  $V_B$ : that is,  $V_C = 0.34 + 0.4 = 0.74 \text{ V}$ .

Correspondingly,

$$R_{C\max} = \frac{0.74 - (-3)}{0.87} = 4.3 \text{ k}\Omega$$

6.66 (a)  $\beta = \infty$

The circled numbers below indicate the order of the analysis steps.

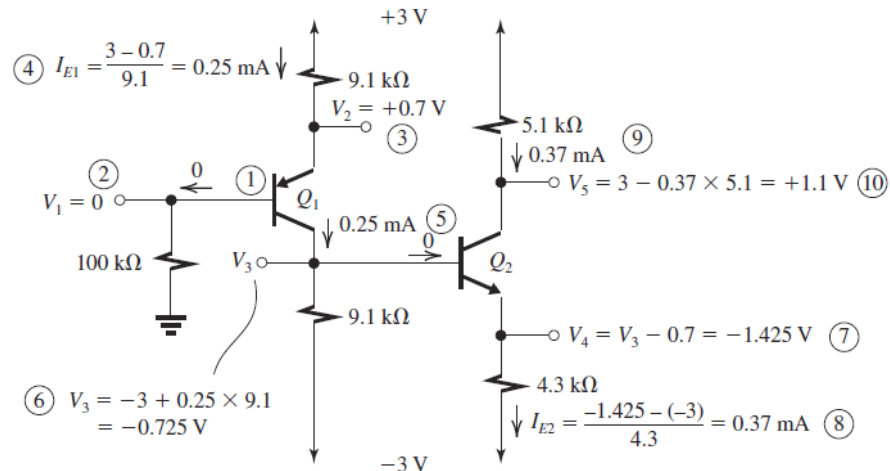


Figure 1 ( $\beta = \infty$ )

(b)  $\beta = 100$

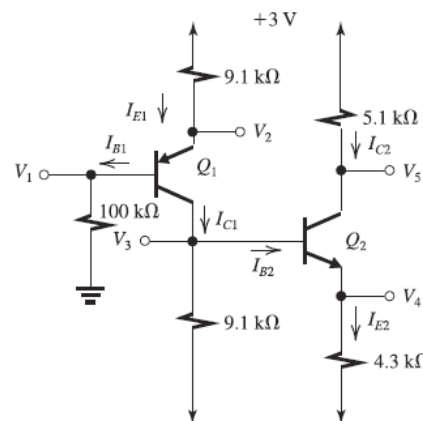


Figure 2 ( $\beta = 100$ )

By reference to Fig. 2 on page 26, we can write an equation for the loop containing the EBJ of  $Q_1$  as follows:

$$3 = I_{E1} \times 9.1 + 0.7 + I_{B1} \times 100$$

Substituting  $I_{B1} = I_E/(\beta + 1) = I_E/101$  and rearranging, we obtain

$$I_{E1} = \frac{3 - 0.7}{9.1 + \frac{100}{101}} = 0.228 \text{ mA}$$

Thus,

$$I_{B1} = \frac{I_{E1}}{101} = 0.0023 \text{ mA}$$

$$V_2 = V_1 + 0.7 = 0.93 \text{ V}$$

$$I_{C1} = \alpha I_{E1} = 0.99 \times 0.228 = 0.226 \text{ mA}$$

Then we write a node equation at  $C_1$ :

$$I_{C1} = I_{B2} + \frac{V_3 - (-3)}{9.1}$$

Substituting for  $I_{C1} = 0.226$  mA,  $I_{B2} = I_{E2}/101$ , and  $V_3 = V_4 + 0.7 = -3 + I_{E2} \times 4.3 + 0.7$  gives

$$0.226 = \frac{I_{E2}}{101} + \frac{-3 + 4.3I_{E2} + 0.7 + 3}{9.1}$$

$$= \frac{I_{E2}}{101} + \frac{4.3I_{E2}}{9.1} + \frac{0.7}{9.1}$$

$$\Rightarrow I_{E2} = 0.31 \text{ mA}$$

$$I_{B2} = 0.0031$$

$$I_{C1} - I_{B2} = 0.226 - 0.0031 = 0.223 \text{ mA}$$

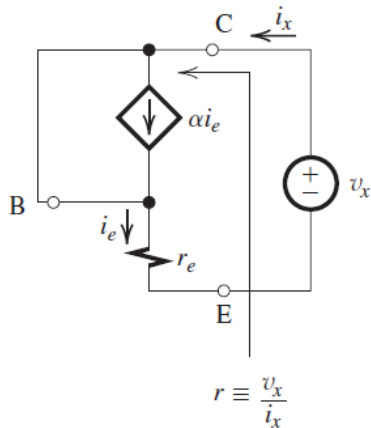
$$V_3 = -3 + 0.223 \times 9.1 = -0.97 \text{ V} \simeq -1 \text{ V}$$

$$V_4 = V_3 - 0.7 = -1.7 \text{ V}$$

$$I_{C2} = \alpha I_{E2} = 0.99 \times 0.31 = 0.3 \text{ mA}$$

$$V_5 = +3 - 0.3 \times 5.1 = +1.47 \text{ V}$$

**7.51** Replacing the BJT with the  $T$  model of Fig. 7.26(b), we obtain the circuit shown below.



Since  $v_x$  appears across  $r_e$  and  $i_x = i_e = \frac{v_x}{r_e}$ , the small-signal resistance  $r$  is given by

$$r \equiv \frac{v_x}{i_x} = \frac{v_x}{i_e} = r_e$$

**7.54** Refer to Fig. P7.54.

$$\alpha = \frac{\beta}{\beta + 1} = \frac{200}{201} = 0.995$$

$$I_C = \alpha \times I_E = 0.995 \times 10 = 9.95 \text{ mA}$$

$$V_C = I_C R_C = 9.95 \times 0.1 \text{ k}\Omega = 0.995 \text{ V} \simeq 1 \text{ V}$$

Replacing the BJT with its hybrid- $\pi$  model results in the circuit shown below.

$$g_m = \frac{I_C}{V_T} \simeq \frac{10 \text{ mA}}{0.025 \text{ V}} = 400 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{400} = 0.5 \text{ k}\Omega$$

$$R_{ib} = r_\pi = 0.5 \text{ k}\Omega$$

$$R_{in} = 10 \text{ k}\Omega \parallel 0.5 \text{ k}\Omega = 0.476 \text{ k}\Omega$$

$$\frac{v_\pi}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{0.476}{0.476 + 1} = 0.322 \text{ V/V}$$

$$\frac{v_o}{v_\pi} = -g_m R_C = -400 \times 0.1 = -40 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = -40 \times 0.322 = -12.9 \text{ V/V}$$

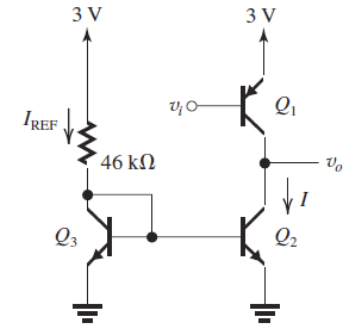
$$\mathbf{8.49} \text{ (a) } I_{REF} = I_{C3} = \frac{3 - V_{BE3}}{46 \text{ k}\Omega}$$

$$I_{REF} = \frac{3 - 0.7}{46}$$

$$= 0.05 \text{ mA}$$

$$\Rightarrow I_{C2} = 5I_{C3}$$

$$I_{C2} = I = 0.25 \text{ mA} \Rightarrow I = 0.25 \text{ mA}$$



$$(b) |V_A| = 50 \text{ V} \Rightarrow r_{o1} = \frac{|V_A|}{I} = \frac{30}{0.25} = 120 \text{ k}\Omega$$

$$r_{o2} = \frac{30}{0.25} = 120 \text{ k}\Omega$$

Total resistance at the collector of  $Q_1$  is equal to  $r_{o1} \parallel r_{o2}$ , thus

$$r_{\text{tot}} = 120 \text{ k}\Omega \parallel 120 \text{ k}\Omega = 60 \text{ k}\Omega$$

$$(c) g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.25}{0.025} = 10 \text{ mA/V}$$

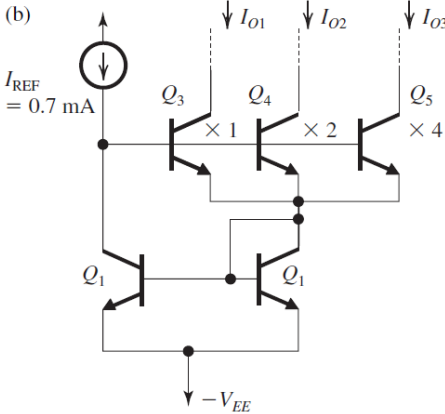
$$r_{\pi1} = \frac{\beta}{g_m} = \frac{50}{10} = 5 \text{ k}\Omega$$

$$(d) R_{\text{in}} = r_{\pi1} = 5 \text{ k}\Omega$$

$$R_o = r_{o1} \parallel r_{o2} = 120 \text{ k}\Omega \parallel 120 \text{ k}\Omega = 60 \text{ k}\Omega$$

$$A_v = -g_{m1}R_o = -10 \times 60 = -600 \text{ V/V}$$

$$8.85 (a) I_{O1} = I_{O2} = \frac{1}{2} \frac{I_{\text{REF}}}{1 + \frac{2}{\beta^2}}$$



The figure shows the required circuit. Observe that the output transistor is split into three transistors having base-emitter junctions with area ratio 1:2:4. Thus

$$I_{O1} = \frac{0.1}{1 + \frac{2}{\beta^2}} = \frac{0.1}{1 + \frac{2}{50^2}} = 0.0999 \text{ mA}$$

$$I_{O2} = \frac{0.2}{1 + \frac{2}{50^2}} = 0.1998 \text{ mA}$$

$$I_{O4} = \frac{0.4}{1 + \frac{2}{50^2}} = 0.3997 \text{ mA}$$

### 8.96

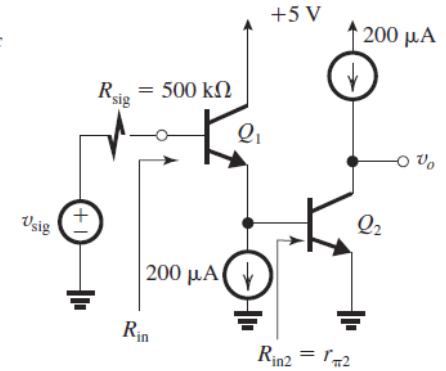
Each of  $Q_1$  and  $Q_2$  is operating at an  $I_C$  approximately equal to  $200 \mu\text{A}$ . Thus for both devices,

$$g_m = \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$r_e \simeq \frac{1}{g_m} = 0.125 \text{ k}\Omega$$

$$r_{\pi} = \frac{\beta}{g_m} = \frac{100}{8} = 12.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{50}{0.2} = 250 \text{ k}\Omega$$



$$(a) R_{\text{in}2} = r_{\pi2} = 12.5 \text{ k}\Omega$$

$$R_{\text{in}} = (\beta_1 + 1)[r_{e1} + (r_{\pi2} \parallel r_{o1})]$$

$$= 101[0.125 + (12.5 \parallel 250)]$$

$$= 1.215 \text{ M}\Omega$$

$$\frac{v_{b1}}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \frac{1.215}{1.215 + 0.5} = 0.71 \text{ V/V}$$

$$\frac{v_{e1}}{v_{b1}} = \frac{r_{\pi2} \parallel r_{o1}}{(r_{\pi2} \parallel r_{o1}) + r_{e1}} = 0.99 \text{ V/V}$$

$$\frac{v_o}{v_{b1}} = -g_{m2}r_{o2} = -8 \times 250 = -2000 \text{ V/V}$$

$$G_v = \frac{v_o}{v_{\text{sig}}} = 0.71 \times 0.99 \times -2000 = -1405 \text{ V/V}$$

(b) Increasing the bias current by a factor of 10 (i.e., to 2 mA) results in

$$g_m = 80 \text{ mA/V}$$

$$r_e = 0.0125 \text{ k}\Omega$$

$$r_\pi = 1.25 \text{ k}\Omega$$

$$r_o = 25 \text{ k}\Omega$$

$$R_{in2} = r_{\pi2} = 1.25 \text{ k}\Omega$$

$$R_{in} = 101[0.0125 + (1.25 \parallel 25)] = 121.5 \text{ k}\Omega$$

Thus,  $R_{in}$  has been reduced by a factor of 10.

$$\frac{v_{b1}}{v_{sig}} = \frac{121.5}{121.5 + 500}$$

$$= 0.2 \text{ V/V (considerably reduced)}$$

$$\frac{v_{e1}}{v_{b1}} = \frac{(1.25 \parallel 25)}{(1.25 \parallel 25) + 0.0125}$$

$$= 0.99 \text{ V/V (unchanged)}$$

$$\frac{v_o}{v_{b1}} = -g_{m2}r_o = -80 \times 25$$

$$= -2000 \text{ V/V (unchanged)}$$

$$G_v = \frac{v_o}{v_{sig}} = 0.2 \times 0.99 \times -2000 = -396 \text{ V/V}$$

which has been reduced by a factor of 3.5! All this reduction in gain is a result of the reduction in  $R_{in}$ .

**8.98** From Fig. P8.98 we see that

$$I_{E2} = 10 \text{ mA}$$

$$I_{E1} = \frac{I_{E2}}{\beta_2 + 1} \simeq \frac{10}{100} = 0.1 \text{ mA}$$

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{25 \text{ mV}}{10 \text{ mA}} = 2.5 \Omega$$

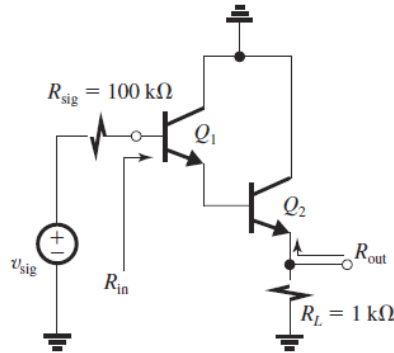
$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \Omega$$

The Darlington follower circuit prepared for small-signal analysis is shown in the figure.

$$R_{in} = (\beta + 1)[r_{e1} + (\beta_2 + 1)(r_{e2} + R_L)]$$

$$= 101[0.25 + (101)(0.0025 + 1)]$$

$$= 10.25 \text{ M}\Omega$$



$$R_{out} = r_{e2} + \frac{r_{e1} + R_{sig}/(\beta_1 + 1)}{\beta_2 + 1}$$

$$= 2.5 + \frac{250 + \frac{100 \times 10^3}{101}}{101} = 14.8 \Omega$$

With  $R_L$  removed,

$$G_{vo} = \frac{v_o}{v_{sig}} = 1$$

With  $R_L$  connected,

$$G_v = \frac{v_o}{v_{sig}} = G_{vo} \frac{R_L}{R_L + R_{out}}$$

$$= 1 \times \frac{1}{1 + 0.0148} = 0.985$$

**8.101** Refer to Fig. P8.101. All transistors are operating at  $I_E = 0.5 \text{ mA}$ . Thus,

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \Omega$$

(a) Refer to Fig. P8.101(a).

$$\frac{v_o}{v_{sig}} = -\frac{\alpha \times \text{Total resistance in collector}}{\text{Total resistance in emitter}}$$

$$= \frac{-\alpha \times 10 \text{ k}\Omega}{\frac{10 \text{ k}\Omega}{\beta + 1} + r_e}$$

For

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

$$G_v = \frac{-0.99 \times 10}{\frac{10}{101} + 0.05} = -66.4 \text{ V/V}$$

(d) Refer to Fig. P8.101(d).

$$R_{in} \text{ (at the base of } Q_1) = (\beta_1 + 1)[r_{e1} + r_{\pi2}]$$

where

$$r_{e1} = 50 \Omega$$

$$r_{\pi2} = (\beta + 1)r_{e2} = 101 \times 50 = 5.05 \text{ k}\Omega$$

Thus,

$$R_{in} = 101(0.05 + 5.05) = 515 \text{ k}\Omega$$

$$\frac{v_{b1}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{515}{515 + 10} = 0.98 \text{ V/V}$$

$$\frac{v_{b2}}{v_{b1}} = \frac{r_{\pi2}}{r_{\pi2} + r_{e1}} = \frac{5.05}{5.05 + 0.05} = 0.98 \text{ V/V}$$

$$\frac{v_o}{v_{b2}} = -g_{m2} \times 10 \text{ k}\Omega$$

$$= -20 \times 10 = -200 \text{ V/V}$$

$$G_v = \frac{v_o}{v_{sig}} = 0.98 \times 0.98 \times -200 = -194 \text{ V/V}$$