EE341 Fall 2019 HW 6
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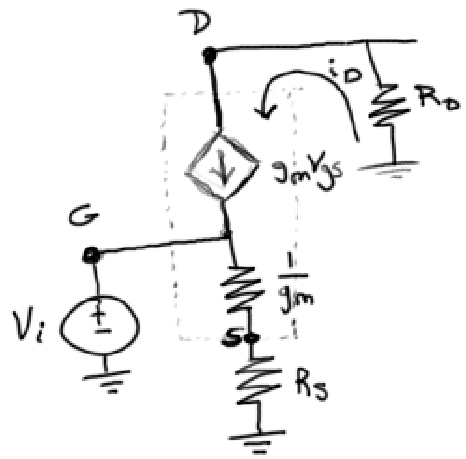
Updated: November 13, 2019

7.30



$$i_D = g_m V_{gs}$$

$$v_i = i_D\left(\frac{1}{g_m} + R_s\right)$$

$$v_S = i_D R_S$$

$$V_D = -i_D R_D \quad \because \quad \frac{0 - V_D}{R_D} = i_D$$

$$\therefore \frac{V_S}{V_i} = \frac{i_D R_S}{i_D\left(\frac{1}{g_m} + R_s\right)} = \frac{R_S}{\frac{1}{g_m} + R_s} = \boxed{\frac{1}{\frac{1}{g_m R_S} + 1}}$$

$$\therefore \frac{V_D}{V_i} = \frac{-i_D R_D}{i_D\left(\frac{1}{g_m} + R_S\right)} = \boxed{-\frac{R_D}{R_S}\frac{1}{\frac{1}{g_m R_S} + 1}}$$

7.33

(a)

$V_{ov} = V_{GS} - V_t = 1.5 - 1.0 = 0.5\,V$

Since $V_{DS} \geq V_{OV}$, the NMOS is saturated.

$I_D = \frac{1}{2} K_n V_{ov}^2 \rightarrow 0.5 = \frac{1}{2} \cdot 4 \cdot 0.5^2 = 0.5.$

VERIFIED

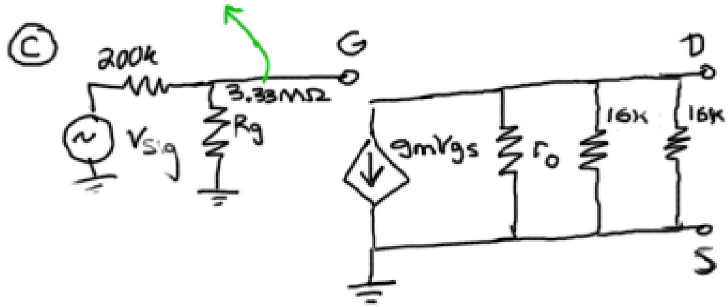
(b)

$$r_o = \frac{|V_A|}{I_D} = \frac{100V}{0.5mA} = \boxed{200\,k\Omega}$$

$$g_m = \sqrt{2k_n I_D} = \sqrt{2 \cdot 4 \cdot 0.5} = \boxed{2\,m\mho}$$

$10M \| 5M = 3.33M$

(c)



(d)

$$R_{in} = R_G = \boxed{3.33M\Omega}$$

$$V_{gs} = V_{sig}\frac{R_g}{R_g + R_{sig}} \rightarrow \frac{V_{gs}}{V_{sig}} = \frac{R_g}{R_g + R_{sig}} = \boxed{0.94}$$

$$\frac{V_o}{V_{gs}} = -g_m(r_o \| 16k \| 16k) = \boxed{-15.4}$$

$$\frac{V_o}{V_{sig}} = \frac{V_o}{V_{gs}} \cdot \frac{V_{gs}}{V_{sig}} = -15.4 \cdot 0.94 = \boxed{-14.48}$$

7.121

(a) $I_D = \frac{1}{2} k_n V_{ov}^2$

$$V_{ov} = \sqrt{\frac{2I_D}{k_n}} = 0.4\,V = V_{GS} - V_t \rightarrow V_{GS} = 1.2\,V$$

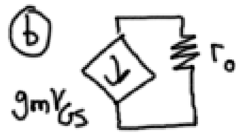
$$R_G = \boxed{10\,M\Omega}$$

$$R_S = \frac{-V_{GS} + 5V}{0.4\,mA} = \boxed{9.5\,k\Omega}$$

$$V_D = +0.8\,V \underbrace{+V_S + V_{ov}}_{\text{Signal}} = 0\,V$$

$$R_D = \frac{5V - V_D}{0.4\,mA} = \boxed{12.5\,k\Omega}$$

(b)



$$g_m = k_n I_D = 5 \cdot 0.4 = \boxed{2\,m\mho}$$

$$r_o = \frac{|V_A|}{I_D} = \frac{40V}{0.4\,mA} = \boxed{100\,k\Omega}$$

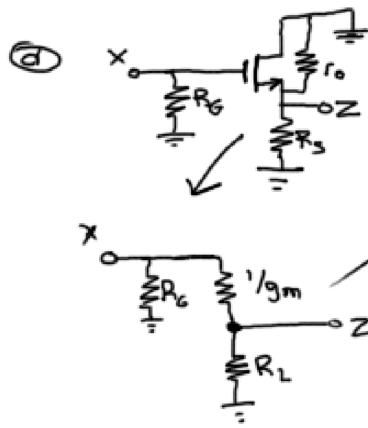
(c)

$$G_v = \frac{V_o}{V_i} \cdot \frac{V_i}{V_{sig}} = -\left(g_m \left(R_L \,//\, R_D \,//\, r_o\right)\right) \cdot \frac{R_G}{R_{sig} + R_G}$$

$$= 2\,m\mho \cdot -\left(10\,k\Omega \,//\, 12.5\,k\Omega \,//\, 100\,k\Omega\right) \cdot \frac{10\,M\Omega}{1\,M\Omega + 10\,M\Omega}$$

$$\boxed{= -9.60}$$

(d)

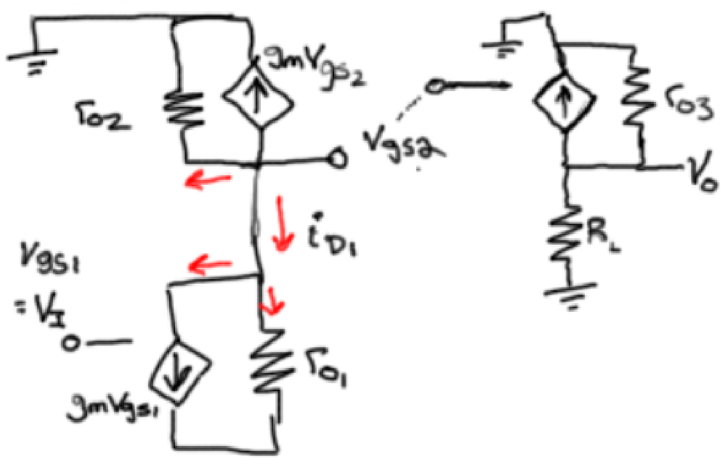


$R_L = R_S // r_o = 8.68k\Omega$

$$\frac{V_z}{V_x} = \frac{R_L}{1/g_m + R_L} = \boxed{0.95}$$

$$R_o = \frac{1}{g_m} = \boxed{0.5k\Omega}$$

8.19



LEFT-SIDE

KCL:

$$i_{D1} = i_{ro1} + g_{m1}V_{gs1} = \frac{V_{gs2}}{r_{o1}} + g_{m1}V_{gs1}$$

Q_1 ANALYSIS

Q_2 ANALYSIS

$$\frac{V_{gs2}}{r_{o2}} = -i_{D1} - g_{m2}V_{gs2}$$

$$V_{gs2} = -(i_{D1} + g_{m2}V_{gs2})r_{o2} = -\left(\frac{V_{gs2}}{r_{o1}} + g_{m1}V_{gs1} + g_{m2}V_{gs2}\right)r_{o2}$$

$$V_{gs2} + \frac{r_{o2}}{r_{o1}}V_{gs2} + g_{m2}V_{gs2}r_{o2} = -g_{m1}V_{gs1}r_{o2}$$

★ $$V_{gs2}\left(1 + \frac{r_{o2}}{r_{o1}} + g_{m2}r_{o2}\right) = -g_{m1}V_{gs1}r_{o2}$$

RIGHT-SIDE

$$V_o = -I_3(r_{o3} \| R_L)$$
$$= -g_{m3} V_{gs3} (r_{o3} \| R_L)$$
$$\star \quad = -g_{m3} V_{gs2} (r_{o3} \| R_L)$$

Q3 ANALYSIS

$$\frac{V_{gs2}}{V_{gs1}} = \frac{-g_{m1} r_{o2}}{1 + \frac{r_{o2}}{r_{o1}} + g_{m2} r_{o2}}$$ LEFT-SIDE GAIN

$$\frac{V_o}{V_{gs2}} = -g_{m3}(r_{o3} \| R_L)$$ RIGHT-SIDE GAIN

$$\frac{V_o}{V_{gs1}} = \frac{V_o}{V_{gs2}} \cdot \frac{V_{gs2}}{V_{gs1}} = \frac{g_{m1} g_{m3} (r_{o3} \| R_L) r_{o2}}{1 + \frac{r_{o2}}{r_{o1}} + g_{m2} r_{o2}}$$ TOTAL GAIN

DIVIDE OUT r_{o2}:

$$\frac{v_o}{v_{gs1}} = \frac{g_{m1} g_{m3} (r_{o3} \| R_L)}{\frac{1}{r_{o2}} + \frac{1}{r_{o1}} + g_{m2}}$$

ASSUME LARGE r_o's:

$$\frac{V_o}{V_{gs1}} = \frac{g_{m1} g_{m3} R_L}{g_{m2}} = \frac{V_o}{V_I}$$

SINCE Q_2 & Q_3 HAVE SAME k_n':

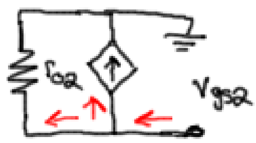
$$\frac{V_o}{V_I} = \frac{g_{m1} k_n' \frac{W_3}{L} V_{ov3} R_L}{k_n' \frac{W_2}{L} V_{ov2}}$$ where $V_{ov2} = V_{ov3}$

$$= \frac{g_{m1} R_L W_3}{W_2}$$

SO,

$$\boxed{\frac{V_o}{V_I} = g_{m1} R_L \frac{W_3}{W_2}}$$

FIND Q2 RESISTANCE



$$i_{D2} = \frac{v_{gs2}}{r_{o2}} + g_{m2} v_{gs2}$$

$$i_{D2} = v_{gs2}\left(\frac{1}{r_{o2}} + g_{m2}\right)$$

$$\frac{v_{gs2}}{i_{D2}} = \frac{1}{\frac{1}{r_{o2}} + g_{m2}} = \boxed{r_{o2} // g_{m2}}$$

VOLTAGE GAIN OF Q1

$$\frac{v_{gs2}}{v_{gs1}} = \frac{-g_{m1} r_{o2}}{1 + \frac{r_{o2}}{r_{o1}} + g_{m2} r_{o2}}$$

$$= \frac{-g_{m1}}{\frac{1}{r_{o2}} + \frac{1}{r_{o1}} + g_{m2}}$$

$$\boxed{= -g_{m1}\left(r_{o1} \| r_{o2} \| g_{m2}\right)}$$