EE341 Fall 2019 HW 6

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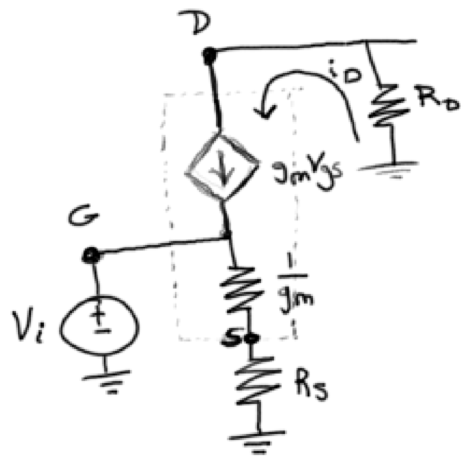
7.30



$$i_D = g_m V_{gs}$$

$$v_i = i_D\left(\frac{1}{g_m} + R_s\right)$$

$$v_s = i_D R_s$$

$$V_D = -i_D R_D \quad \because \quad \frac{0 - V_D}{R_D} = i_D$$

$$\therefore \frac{V_s}{V_i} = \frac{i_D R_s}{i_D\left(\frac{1}{g_m} + R_s\right)} = \frac{R_s}{\frac{1}{g_m} + R_s} = \boxed{\frac{1}{\frac{1}{g_m R_s} + 1}}$$

$$\therefore \frac{V_D}{V_i} = \frac{-i_D R_D}{i_D\left(\frac{1}{g_m} + R_s\right)} = \boxed{-\frac{R_D}{R_s}\,\frac{1}{\frac{1}{g_m R_s} + 1}}$$

7.33

(a)

$V_{ov} = V_{GS} - V_t = 1.5 - 1.0 = 0.5\,V$

Since $V_{DS} \geq V_{ov}$, the NMOS is saturated.

$I_D = \frac{1}{2} K_n V_{ov}^2 \rightarrow 0.5 = \frac{1}{2} \cdot 4 \cdot 0.5^2 = 0.5.$

VERIFIED

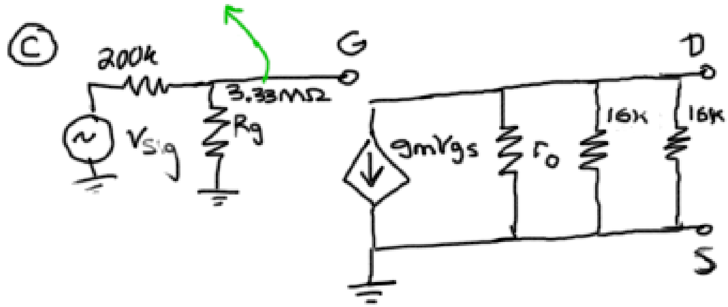
(b)

$$r_o = \frac{|V_A|}{I_D} = \frac{100V}{0.5mA} = \boxed{200\,k\Omega}$$

$$g_m = \sqrt{2k_n I_D} = \sqrt{2 \cdot 4 \cdot 0.5} = \boxed{2\,m\mho}$$

$10M \| 5M = 3.33M$

(c)



(d)

$$R_{in} = R_G = \boxed{3.33M\Omega}$$

$$V_{gs} = V_{sig} \frac{R_g}{R_g + R_{sig}} \rightarrow \frac{V_{gs}}{V_{sig}} = \frac{R_g}{R_g + R_{sig}} = \boxed{0.94}$$

$$\frac{V_o}{V_{gs}} = -g_m (r_o \| 16k \| 16k) = \boxed{-15.4}$$

$$\frac{V_o}{V_{sig}} = \frac{V_o}{V_{gs}} \cdot \frac{V_{gs}}{V_{sig}} = -15.4 \cdot 0.94 = \boxed{-14.48}$$

7.121

(a) $I_D = \frac{1}{2} k_n V_{ov}^2$

$$V_{ov} = \sqrt{\frac{2I_D}{k_n}} = 0.4\,V = V_{GS} - V_t \rightarrow V_{GS} = 1.2\,V$$

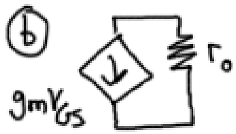
$$R_G = \boxed{10\,M\Omega}$$

$$R_S = \frac{-V_{GS} + 5V}{0.4\,mA} = \boxed{9.5\,k\Omega}$$

$$V_D = +0.8\,V \underbrace{+V_S + V_{ov}}_{\text{Signal}} = 0\,V$$

$$R_D = \frac{5V - V_D}{0.4\,mA} = \boxed{12.5\,k\Omega}$$

(b)



$$g_m = k_n I_D = 5 \cdot 0.4 = \boxed{2\,m\mho}$$

$$r_o = \frac{|V_A|}{I_D} = \frac{40V}{0.4\,mA} = \boxed{100\,k\Omega}$$

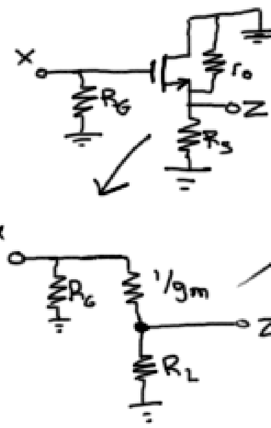
(c)

$$G_v = \frac{v_o}{v_i} \cdot \frac{v_i}{v_{sig}} = -\left(g_m \left(R_L \,//\, R_D \,//\, r_o\right)\right) \cdot \frac{R_G}{R_{sig} + R_G}$$

$$= 2\,m\mho \cdot -\left(10\,k\Omega \,//\, 12.5\,k\Omega \,//\, 100\,k\Omega\right) \cdot \frac{10\,M\Omega}{1\,M\Omega + 10\,M\Omega}$$

$$\boxed{= -9.60}$$

(d)

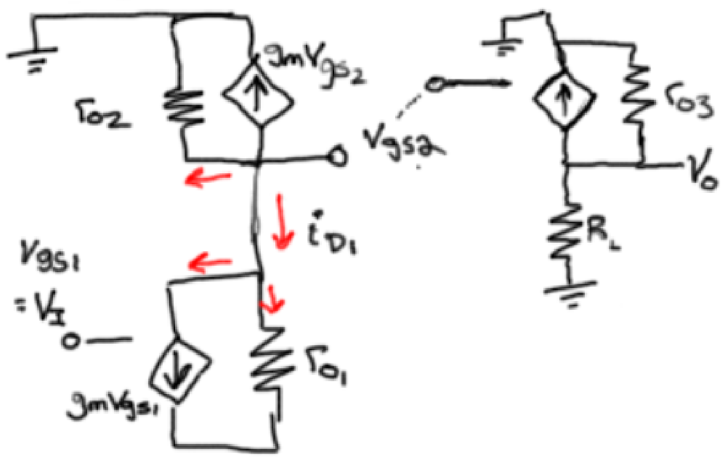


$R_L = R_S // r_o = 8.68\,k\Omega$

$$\frac{V_Z}{V_X} = \frac{R_L}{1/g_m + R_L} = \boxed{0.95}$$

$$R_o = \frac{1}{g_m} = \boxed{0.5\,k\Omega}$$

8.19



LEFT-SIDE

KCL:

Q1 ANALYSIS

$$i_{D1} = i_{ro1} + g_{m1} V_{gs1} = \frac{V_{gs2}}{r_{o1}} + g_{m1} V_{gs1}$$

Q2 ANALYSIS

$$\frac{V_{gs2}}{r_{o2}} = -i_{D1} - g_{m2} V_{gs2}$$

$$V_{gs2} = -(i_{D1} + g_{m2} V_{gs2}) r_{o2} = -\left(\frac{V_{gs2}}{r_{o1}} + g_{m1} V_{gs1} + g_{m2} V_{gs2}\right) r_{o2}$$

$$V_{gs2} + \frac{r_{o2}}{r_{o1}} V_{gs2} + g_{m2} V_{gs2} r_{o2} = -g_{m1} V_{gs1} r_{o2}$$

$$\star\; V_{gs2}\left(1 + \frac{r_{o2}}{r_{o1}} + g_{m2} r_{o2}\right) = -g_{m1} V_{gs1} r_{o2}$$

RIGHT-SIDE

$$V_o = -I_3(r_{o3} \| R_L)$$
$$= -g_{m3} V_{gs3} (r_{o3} \| R_L)$$
$$\star\ = -g_{m3} V_{gs2} (r_{o3} \| R_L)$$

Q3 ANALYSIS

$$\frac{V_{gs2}}{V_{gs1}} = \frac{-g_{m1} r_{o2}}{1 + \frac{r_{o2}}{r_{o1}} + g_{m2} r_{o2}}$$ LEFT-SIDE GAIN

$$\frac{V_o}{V_{gs2}} = -g_{m3}(r_{o3} \| R_L)$$ RIGHT-SIDE GAIN

$$\frac{V_o}{V_{gs1}} = \frac{V_o}{V_{gs2}} \cdot \frac{V_{gs2}}{V_{gs1}} = \frac{g_{m1} g_{m3} (r_{o3} \| R_L) r_{o2}}{1 + \frac{r_{o2}}{r_{o1}} + g_{m2} r_{o2}}$$ TOTAL GAIN

DIVIDE OUT r_{o2}:

$$\frac{v_o}{v_{gs1}} = \frac{g_{m1} g_{m3} (r_{o3} \| R_L)}{\frac{1}{r_{o2}} + \frac{1}{r_{o1}} + g_{m2}}$$

ASSUME LARGE r_o's:

$$\frac{V_o}{v_{gs1}} = \frac{g_{m1} g_{m3} R_L}{g_{m2}} = \frac{V_o}{V_I}$$

SINCE Q_2 & Q_3 HAVE SAME k_n':

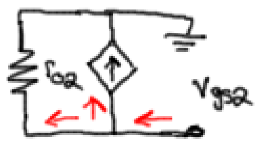
$$\frac{V_o}{V_I} = \frac{g_{m1} k_n' \frac{W_3}{L} V_{ov3} R_L}{k_n' \frac{W_2}{L} V_{ov2}}$$ where $V_{ov2} = V_{ov3}$

$$= \frac{g_{m1} R_L W_3}{W_2}$$

SO,

$$\boxed{\frac{V_o}{V_I} = g_{m1} R_L \frac{W_3}{W_2}}$$

FIND Q2 RESISTANCE



$$i_{D2} = \frac{v_{gs2}}{r_{o2}} + g_{m2} v_{gs2}$$

$$i_{D2} = v_{gs2}\left(\frac{1}{r_{o2}} + g_{m2}\right)$$

$$\frac{v_{gs2}}{i_{D2}} = \frac{1}{\frac{1}{r_{o2}} + g_{m2}} = \boxed{r_{o2} // g_{m2}}$$

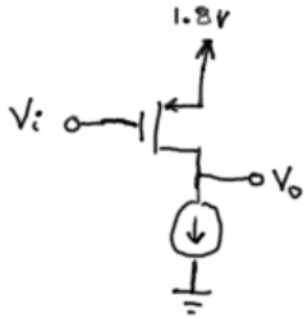
VOLTAGE GAIN OF Q1

$$\frac{v_{gs2}}{v_{gs1}} = \frac{-g_{m1} r_{o2}}{1 + \frac{r_{o2}}{r_{o1}} + g_{m2} r_{o2}}$$

$$= \frac{-g_{m1}}{\frac{1}{r_{o2}} + \frac{1}{r_{o1}} + g_{m2}}$$

$$\boxed{= -g_{m1}\left(r_{o1} \,||\, r_{o2} \,||\, g_{m2}\right)}$$

8.30



$1.8 - V_{SD} \geq V_{OV}$] SATURATION REGION

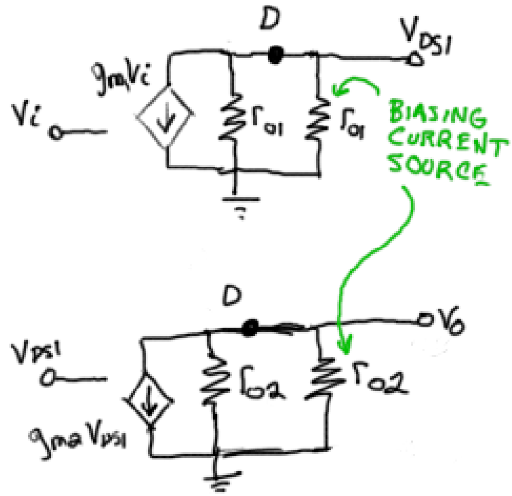
$1.8 - V_{SD} \geq 0.2$

$1.6 \geq V_{SD}$

HIGHEST INSTANTANEOUS VOLTAGE

$\boxed{V_{SD} = 1.6V}$

8.42



$r_{o1} = r_{o2}$ since $V_{A1} = |V_{A2}|$

$$\frac{V_{DS1}}{r_o \| r_o} = -g_{m1} V_i$$

$$\frac{V_{DS1}}{V_i} = -\frac{1}{2} g_{m1} r_o$$

Q1 ANALYSIS

$$\frac{V_o}{r_o \| r_o} = -g_{m2} V_{DS1}$$

$$\frac{V_o}{V_{DS1}} = -\frac{1}{2} g_{m2} r_o$$

Q2 ANALYSIS

$$\frac{V_o}{V_i} = \frac{V_o}{V_{DS1}} \cdot \frac{V_{DS1}}{V_i} = \boxed{\frac{1}{4} g_{m1} g_{m2} r_o^2}$$

FIND $|V_{ov}|$ IF $|V_A| = 5V$ & $A_v = 400$

$r_o = \frac{|V_A|}{I_D}$ $\quad g_{m1} = g_{m2} = \frac{2I_D}{V_{ov}}$

$I_D = \frac{1}{2} k_n V_{ov}^2$

$$A_v = 400 = \frac{1}{4}\left(\frac{2I_D}{V_{ov}}\right)^2\left(\frac{V_A}{I_D}\right)^2$$

$$= \frac{1}{4} \cdot \frac{4V_A^2}{V_{ov}^2}$$

$$= \frac{V_A^2}{V_{ov}^2}$$

$$V_{ov} = \sqrt{V_A^2/400} = \frac{V_A}{\sqrt{400}} = \frac{V_A}{20}$$

$$|V_{ov}| = \frac{5}{20} = \boxed{0.2\,V}$$

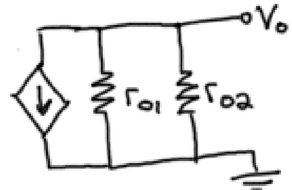
8.44

FIND I_{REF}:

$$I_{REF} = I_{D3}$$

$$I_{D3} = \frac{1}{2} k_p' \frac{W}{L} V_{OV}^2$$

$$= \frac{1}{2} 100 \frac{W}{L} \left(V_{SG} - |V_t|\right)^2$$



$$R_o = 100k\Omega = r_{o1} \| r_{o2} = \frac{|V_A|}{I_{D1}} \| \frac{|V_A|}{I_{D2}}$$

$$= \frac{1}{2} \frac{|V_A|}{I_{D2}}$$

Same

$$R_o = \frac{|V_A|}{2 I_{D2}} \rightarrow I_{D2} = \frac{5}{2 \cdot 100k\Omega} = 0.025mA$$

$$I_{REF} = I_{D3} = I_{D2} = \boxed{0.025mA}$$