arules - A Computational Environment for Mining Association Rules and Frequent Item Sets

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Abstract

Mining frequent itemsets and association rules is a popular and well researched approach for discovering interesting relationships between variables in large databases. The R package **arules** presented in this paper provides a basic infrastructure for creating and manipulating input data sets and for analyzing the resulting itemsets and rules. The package also includes interfaces to two fast mining algorithms, the popular C implementations of Apriori and Eclat by Christian Borgelt. These algorithms can be used to mine frequent itemsets, maximal frequent itemsets, closed frequent itemsets and association rules.

1 Introduction

Mining frequent itemsets and association rules is a popular and well researched method for discovering interesting relations between variables in large databases. Piatetsky-Shapiro (1991) describes analyzing and presenting strong rules discovered in databases using different measures of interest. Based on the concept of strong rules, Agrawal, Imielinski, and Swami (1993) introduced the problem of mining association rules from transaction data as follows.

Let $I = \{i_1, i_2, \ldots, i_n\}$ be a set of n binary attributes called *items*. Let $\mathcal{D} = \{t_1, t_2, \ldots, t_m\}$ be a set of transactions called the *database*. Each transaction in \mathcal{D} contains a subset of the items in I. A *rule* is defined as an implication of the form $X \Rightarrow Y$ where $X, Y \subseteq I$ and $X \cap Y = \emptyset$. The sets of items (for short *itemsets*) X and Y are called *antecedent* (left-hand-side or LHS) and *consequent* (right-hand-side or RHS) of the rule.

For convenience we introduce $\mathcal{X} = \{X_1, X_2, \dots, X_l\}$ for sets of itemsets with length l. Analogous, we define \mathcal{R} for sets of rules.

To select interesting rules from the set of all possible rules, constraints on various measures of significance and interest can be used. The best-known constraints are minimum thresholds on support and confidence. Support is defined on an itemset as the proportion of transactions in the data set which contain the itemset. All itemsets which have a support above a set minimum support threshold are called frequent itemsets. Finding frequent itemsets can be seen as a simplification of the unsupervised learning problem called "mode finding" or "bump hunting" (Hastie, Tibshirani, and Friedman, 2001). For these problems each item is seen as a variable. The goal is to find prototype values so that the probability density evaluated at these values is sufficiently large. However, for practical applications with a large number of variables, probability estimation will be unreliable and computationally too expensive. This is why in practice frequent itemsets are used instead of probability estimation.

Confidence is defined on rules as $\operatorname{conf}(X\Rightarrow Y)=\operatorname{supp}(X\cup Y)/\operatorname{supp}(X)$. This can be interpreted as an estimate of the probability P(Y|X), the probability of finding the RHS of the rule in transactions under the condition that these transactions also contain the LHS (see e.g., Hipp, Güntzer, and Nakhaeizadeh, 2000). Association rules are required to satisfy both constraints, minimum support and minimum confidence, at the same time.

At medium to low support values, often a great number of frequent itemsets are found in a database. However, since the definition of support enforces that all subsets of a frequent itemset have to be also frequent, it is sufficient to only mine all maximal frequent itemsets, defined as frequent itemsets which are not proper subsets of any other frequent itemset (Zaki, Parthasarathy, Ogihara, and Li, 1997b). Another approach to reduce the number of mined itemsets is to only mine frequent closed itemsets. An itemset is closed if no proper superset of the itemset is contained in each transaction in which the itemset is contained (Pasquier, Bastide, Taouil, and Lakhal, 1999; Zaki, 2004). Frequent closed itemsets are a superset of the maximal frequent itemsets. Their advantage over maximal frequent itemsets is that in addition to be able to infer all frequent itemsets, they also preserve the support information for all frequent itemsets which can be important for computing additional interest measures after the mining process is finished (e.g., confidence for rules generated from the found itemsets, or all-confidence (Omiecinski, 2003)).

A practical solution to the problem of finding too many association rules with support and confidence is to further filter or rank found rules using additional interest measures. A popular measure for this purpose is lift (Brin, Motwani, Ullman, and Tsur, 1997). Lift is defined on rules as $lift(X \Rightarrow Y) = \frac{\sup(X \cup Y)}{\sup(X)\sup(Y)}$. It can be interpreted as the deviation of the support of the whole rule from the support expected under independence given the supports of the LHS and the RHS. Greater lift values indicate stronger associations.

In the last decade research on algorithms to solve the frequent itemset problem has been abundant. Goethals and Zaki (2004) compare the currently fastest algorithms. Among these algorithms are the implementations of the Apriori and Eclat algorithms by Borgelt (2003) interfaced in the package arules. The two algorithms use very different mining strategies. Apriori, developed by Agrawal and Srikant (1994), is a level-wise, breadth-first algorithm which counts transactions. In contrast, Eclat (Zaki et al., 1997b) employs equivalence classes, depth-first search and set intersection instead of counting. The algorithms can be used to mine frequent itemsets, maximal frequent itemsets and closed frequent itemsets. The implementation of Apriori can additionally be used to generate association rules.

The R (R Development Core Team, 2005) package **arules** presented in this paper provides the infrastructure needed to create and manipulate input data sets for the mining algorithms and for analyzing the resulting itemsets and rules. Since it is common to work with large sets of rules and itemsets, the package uses sparse matrix representation to minimize memory usage. The infrastructure provided by the package was also created to explicitly facilitate easy extensions, both for interfacing new algorithms and for adding new types of interest measures and associations.

The rest of the paper is organized as follows: In the next section, we give an overview of the data structure implemented in the package **arules**. In Section 2 we introduce the functionality of the classes to handle transaction data and associations. In Section 3 we describe the way mining algorithms are interfaced in **arules** using the already implemented interfaces for Apriori and Eclat as examples. In Section 4 we present some auxiliary methods for support counting, rule induction and sampling available in **arules**. We provide several examples in Section 5. The first two examples show typical R sessions for preparing, analyzing and manipulating a transaction data set, and for mining association rules. The third example demonstrates how **arules** can be extended to integrate a new interest measure. We conclude with a summary of the features and advantage of the package **arules** as a computational environment for mining association rules and frequent itemsets.

2 Data structure overview

To enable the user to represent and work with input and output data of association rule mining algorithms in R, a well thought out structure is necessary which can deal in an efficient way with large amounts of sparse binary data. The S4 class structure implemented in the package **arules** is presented in Figure 1.

For input data the classes transactions and tidLists (transaction ID lists, an alternative way to represent transaction data) are provided. The output of the mining algorithms comprises the

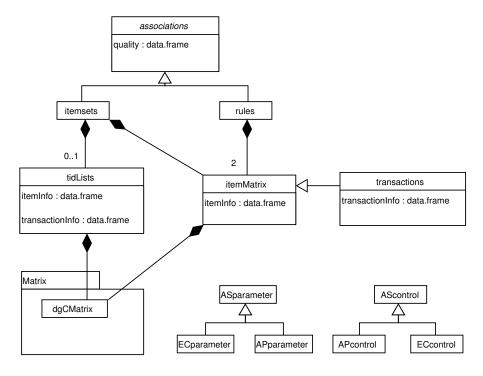


Figure 1: UML class diagram of the **arules** package.

classes itemsets and rules representing a set of itemsets or a set of rules, respectively. Both classes directly extend a common virtual class called associations which provides a common interface. In this structure it is easy to add a new type of associations by adding a new class that extends associations.

Items in associations and transactions are implemented by the itemMatrix class which provides a facade for the sparse matrix implementation dgCMatrix from package Matrix (Bates and Maechler, 2005).

To control the behavior of the mining algorithms, the two classes ASparameter and AScontrol are used. Since each algorithm can use additional algorithm-specific parameters, we implemented for each interfaced algorithm its own set of control classes. We used the prefix AP for Apriori and EC for Eclat. In this way, it is easy to extend the control classes when interfacing a new algorithm.

2.1 Set representation

From the definition of the association rule mining problem we saw that transactions are a set \mathcal{D} containing transactions, each transaction being an itemset. Very similar are sets of associations. A set of itemsets \mathcal{X} contains itemsets and a set of rules \mathcal{R} contains tuples of itemsets, one for the LHS and on for the RHS of each rule.

Sets of itemsets (including transactions) can be represented as binary incidence matrices with columns equal to the number of different items and rows equal to the number of different itemsets. The matrix entries represent the presence (1) or absence (0) of an item in a particular itemset. An example of a binary incidence matrix is shown in Figure 2.

Since a typical frequent itemsets and transaction data (e.g., supermarket transactions) only contains a small number of items compared to the total number of available items, the binary incidence matrix will in general be very sparse with many items and a very large number of rows. A natural representation for such data is a sparse matrix format. For our implementation we chose the dgCMatrix which is defined in the R package Matrix implemented by Bates and Maechler (2005).

		items									
		i ₁	i ₂	i ₃		i _n	Σ				
	X ₁	0	1	0		1	7				
	X_2	0	1	0		1	2				
w	X ₁ X ₂ X ₃ X ₄	0	1	0		0	3				
itemsets	X_4	0	0	0		0	1				
terr		-									
.=					•						
					•		-				
	X_{m-1}	1	0	0		1	11				
	X_{m-1} X_m	0	0	1		1	5				
	Σ	22	127	8		98					

Figure 2: Example of a set of itemsets represented as a binary incidence matrix.

The dgCMatrix is a compressed, sparse, column-oriented matrix which contains the indices of the rows unequal to zero, the pointers to the initial indices of elements in each column and the nonzero elements of the matrix. Since the package Matrix does not provide efficient subset selection functionality which directly works on the dgCMatrix data structure, we implemented a suitable function in C and interfaced it as the subset selection method ([). Despite the column orientation of the dgCMatrix, it is more convenient to work with incidence matrices which are row-oriented. This makes the most important manipulation, e.g., selecting a subset of transactions from a data set for mining, more comfortable and efficient. Therefore, we implemented the class itemMatrix providing a row-oriented facade to the dgCMatrix which stores a transposed incidence matrix. At this level also the constraint that the incidence matrix is binary (and not real valued as the dgCMatrix) is enforced. The data structure of the dgCMatrix class is not intended to be directly accessed by the end user of arules. The interfaces of itemMatrix can be used without knowledge of how the internal representation of the data works. However, if necessary, the dgCMatrix can be directly accessed by developers to add functionality to arules (e.g., to develop new types of associations or interest measures or to efficiently compute a distance matrix between itemsets for clustering). In this case, the dgCMatrix should be accessed using the coercion mechanism from itemMatrix to dgCMatrix with as.

In addition to the sparse matrix, itemMatrix stores item labels (e.g., name of the items) and handles the necessary mapping between the item label and the corresponding column number in the incidence matrix. Optionally, itemMatrix can also store additional information on items. For example, the category hierarchy in a supermarket setting can be stored which enables the analyst to select only transactions (or as we later see also rules and itemsets) which contain items from a certain category (e.g., all dairy products).

For itemMatrix, basic matrix operations including dim and subset selection ([) are implemented. The first element of dim and [represents itemsets or transactions and the second elements represents items. For example, on a transaction data set in variable data the subset selection 'data[1:10, 16:20]' selects a matrix containing the first 10 transactions and items 16 to 20.

Since itemMatrix represents sets of itemsets, a length method is provided to get the number of itemsets in the set. Technically, length returns the number of rows in the matrix which is equal to the first element returned by dim. arules also provides set operations including union, intersect and setequal. These and many other methods are only possible if the used itemMatrix objects are compatible, i.e., if matrices have the same number of columns and the items are in the same order.

With combine, several (compatible) itemMatrix objects can be combined. duplicated, unique and match are also available. To get the actual number of items in the itemsets stored in the itemMatrix, the size method is used. It returns a vector with the number of items (ones) for each element in the set (row sum in the matrix). Obtaining the sizes from the sparse representation is

a very efficient operation, since it can be calculated directly from the vector of column pointers in the dgCMatrix. For a purchase incidence matrix, size will produce a vector of length of the number of transactions in the matrix and each element of the vector contains the number of items in the corresponding transaction. This information can be used, e.g., to select or filter unusually long or short transactions.

The itemFrequency method calculates the frequency for each item in a itemMatrix. Conceptually, the item frequencies are the column sums of the binary matrix. Technically, column sums can be implemented for sparse representation efficiently by just tabulating the vector of row numbers of the non-zero elements in the dgCMatrix. Item frequencies can be used for many purposes. For example, they are needed to compute interest measures. itemFrequency is also used by the method itemFrequencyPlot to produce a bar plot of item count frequencies or support. Such a plot gives a quick overview of a set of itemsets and shows which are the most important items in terms of occurrence frequency.

Coercion from and to matrix and list primitives is provided where names and dimnames are used as item labels. For the coercion form itemMatrix to list there exist two possibilities. The usual coercion to list with the keyword as results in a list of vectors of character strings, each containing the item labels of the items in the corresponding row of the itemMatrix. The actual conversion is done by the method LIST with its default behavior (argument decode set to TRUE). If LIST is called with the argument decode set to FALSE, the result is a list of integer vectors with column numbers for items instead of the item labels. For many computations it is often useful to work with such a list and later use the item column numbers to go back to the original itemMatrix for, e.g., subsetting columns. For decoding items later, a decode method is also available.

Finally, the image method can be used to produce a level plot of an itemMatrix which is useful for quick visual inspection. For transaction data sets (e.g., point-of-sale data) such a plot can be very helpful for checking if the data set contains structural changes (e.g., items were not offered or out-of-stock during part of the observation period) or to find abnormal transactions (e.g., transactions which contain almost all items may point to recording problems). Spotting such problems in the data can be very helpful for data preparation.

2.2 Transaction data

The main application of association rules is for market basket analysis where large transaction data sets are mined. In this setting each transaction contains the items which were purchased at one visit to a retail store (see e.g., Berry and Linoff, 1997). Transaction data are normally recorded by point-of-sale scanners and consists of tuples of the form:

All tuples with the same transaction ID form a single transaction which contains all the items given by the item IDs in the tuples. Additional information denoted by the dots might be available. For example, the customer ID might be available via a loyalty program in a supermarket. Further information on transactions (e.g., time, location), on the items (e.g., category, price) or on the customer (socio-demographic variables as age, gender, etc.) might be available.

For mining, the transaction data is first transformed into a binary purchase incidence matrix with columns equal to the number of different items and rows equal to the number of different transactions. The matrix entries represent the presence (1) or absence (0) of an item in a particular transaction. This format is often called the *horizontal* database layout (Zaki, 2000). Alternatively, transaction data can be represented in a *vertical* database layout in the form of *transaction ID lists* (Zaki, 2000). In this format for each item a list of IDs of the transactions the item is contained in is stored. An example of a transaction data set in horizontal and vertical layout is depicted in Figure 3. Depending on the algorithm, one of the layouts is used for mining. In **arules** both layouts are implemented as the classes transactions and tidLists. Similar to transactions also tidLists

	ı	ı	ite	ems			1				
		i ₁	i ₂	i ₃		i _n	transaction ID lists				
	t ₁	0	1	0		1	i ₁ t ₅ , t ₁₄ , t ₄₂ ,				
	t_2	0	1	0		1	i_2 $t_1, t_2, t_3,$				
ည	t_3	0	1	0		0	i ₃ t ₂₁₂ , t ₈₀₅ , t ₂₂₁₀ ,				
transactions	t_4	0	0	0		0	ν				
ısa							ite .				
Izar	•		•	•	٠.	•					
_	•	•		•	•	•					
	t _{m-1}	1	0	0	•••	1	i_{n-1} $t_1, t_{14}, t_{42}, t_{401}, \dots$				
	t _m	0	0	1		1	i_n $t_1, t_2, t_{54}, t_{100}, \dots$				
			(a	a)			(b)				

Figure 3: Example of a set of transaction represented in (a) horizontal layout and (b) vertical layout.

uses a sparse representation to store its lists. Objects of classes transactions and tidLists can be directly converted into each other by coercion.

The class transactions directly extends itemMatrix and inherits its basic matrix and set functionality (e.g., subset selection, combine, union). In addition, transactions has a slot to store further information for each transaction in form of a data.frame. The slot can hold arbitrary named vectors with length equal to the number of stored transactions. In **arules** the slot is currently used to store transaction IDs, however, it can also be used to store user IDs, revenue or profit, or other information on each transaction. With this information subsets of transactions (e.g., only transactions of a certain user or exceeding a set profit) can be selected.

Objects of class transactions can be easily created by coercion from matrix or list. If names or dimnames are available in these data structures, they are used as item labels or transaction IDs accordingly. To import data from a file, the read.transactions function is provided. This function reads files structured as shown above and also the very common format with one line per transaction and the items separated by a predefined character. Finally, the method inspect can be used to inspect transactions (e.g., an interesting transaction selected with subset selection).

Another important application of mining association rules has been proposed by Piatetsky-Shapiro (1991) and Srikant and Agrawal (1996) for discovering interesting relationships between the values of categorical and quantitative (metric) attributes. For mining associations rules, non-binary attributes have to be mapped to binary attributes. The straight forward mapping method is to transform the metric attributes into k ordinal attributes by building categories (e.g., an attribute income might be transformed into a ordinal attribute with the three categories: "low", "medium" and "high"). Then, in a second step, each categorical attribute with k categories is represented by k binary dummy attributes which correspond to the items used for mining. An example application using questionnaire data can be found in Hastie et al. (2001).

The typical representation for data with categorical and quantitative attributes in R is a data.frame. First, a domain expert has to create useful categories for all metric attributes. This task is supported in R by functions as $\operatorname{cut}(x, \ldots)$, etc. The second step, the generation of binary dummy items, is automated in package **arules** by the coerce method from data.frame to transactions. In this process, the original attribute names and categories are preserved as additional item information and can be used to select itemsets or rules which contain items referring to a certain original attributes. The resulting transactions object can be mined and analyzed the same way as market basket data, see the example in Section 5.1.

2.3 Associations: itemsets and sets of rules

The result of mining transaction data in **arules** are **associations**. Conceptually, associations are sets of objects denoted by $\mathcal{X} = \{X_1, X_2, \dots, X_l\}$. Each object in the set describes the relationship between some items (e.g., as an itemset or a rule) and has values for different measures of quality assigned. Such quality measures can be measures of significance (e.g., support) or measures of interest (e.g., confidence, lift) or other measures (e.g., revenue covered by the association).

All types of association have a common interface in arules comprising the following methods:

- A summary method to produces a short overview of the set and inspect method to display individual associations,
- a length method for getting the number of elements in the set,
- sorting the set using the values of different quality measures (SORT),
- subset extraction ([and the subset method),
- set operations (union, intersect and setequal)
- handling of duplicated elements and matching (duplicated, unique and match).

Currently implemented association in **arules** are sets of itemsets (e.g., used for frequent itemsets of their closed or maximal subset) and sets of rules (e.g., association rules). Both classes, itemsets and rules, directly extend the virtual class associations and provide the interface described above. Class itemsets contains one itemMatrix object to store the items as a binary matrix where each

Class itemsets contains one itemMatrix object to store the items as a binary matrix where each row in the matrix represents an itemset. In addition, it may contain transaction ID lists as an object of class tidLists. Note that when representing transactions, tidLists stores for each item a transaction list, but here it stores for each itemset a list of transaction IDs in which the itemset appears. Such lists are currently only returned by eclat.

Class rules consists of two itemMatrix objects representing the left-hand-side (LHS) and the right-hand-side (RHS) of the rules, respectively.

The items in the associations and the quality measures can be accessed and manipulated in a safe way using accessor and replace methods for quality, items, lhs and rhs. In addition the association classes have built-in validity checking which ensures that all elements have a matching dimension.

It is simple to add new quality measures to existing associations. Since the quality slot holds a data.frame, additional columns with new quality measures can be added. These new measures can then be used to sort or select associations using the SORT or the subset methods. Adding a new type of associations to arules is easy as well. One has only to implement a new class extending the virtual associations class and provide the interface described above.

3 Mining algorithm interfaces

In package **arules** we interface free reference implementations of Apriori and Eclat by Christian Borgelt (Borgelt and Kruse, 2002; Borgelt, 2003). The code is called directly from R by the functions apriori and eclat and the data objects are directly passed from R to the C code and back without writing to external files. The implementations can mine association rules, frequent itemsets, and closed and maximal frequent itemsets.

The input format of the data for the apriori and eclat functions is transactions or a data format which can be coerced to transactions (e.g., matrix or list). The algorithm parameters are divided into two groups represented by the arguments parameter and control. The mining parameters (parameter) change the characteristics of the mined itemsets or rules (e.g., the minimum support) and the control parameters (control) influence the performance of the algorithm (e.g., an initial

sorting of the items with respect to their frequency). These arguments have to be instances of the classes APparameter and APcontrol for the function apriori or ECparameter and ECcontrol for the function eclat, respectively. Alternatively, data which can be coerced to these classes (e.g., NULL which will give the default values or a named list with names equal to slot names to change the default values) can be passed. In these classes, each slot specifies a different parameter and the values. The default values are equal to the defaults of the stand-alone C programs (Borgelt, 2004) except that by default the more common original support definition (instead of the support of only the antecedent) is used for the specified minimum support required.

For apriori the appearance feature implemented by Christian Borgelt can also be used. With argument appearance of function apriori one can specify which items have to or cannot appear in itemsets or rules. For more information on this feature we refer to the Apriori manual (Borgelt, 2004).

The output of the functions apriori and eclat is an object of a class extending associations which contains the sets of mined associations and can be further analyzed using the methods provided for these classes.

It is straightforward to interface additional algorithms which use an incidence matrix or transaction ID list representation as input. The necessary steps are:

- 1. Adding interface code to the algorithm, preferably by directly calling into the native implementation language (rather than using files for communication), and an R function calling this interface.
- 2. Implementing extensions for parameter and control.

There exist many different algorithms which solve the frequent and closed frequent itemset problems. Each algorithm has specific strengths which can be important for very large databases. Such algorithms, e.g. kDCI, LCM, FP-Growth or Patricia, are discussed in Goethals and Zaki (2003). The source code is available on the internet and can be interfaced in the future for **arules**.

4 Auxiliary methods

In **arules** several helpful methods are implemented for support counting, rule induction, sampling, etc. In the following we will discuss some of these methods.

4.1 Counting support for itemsets

Normally, itemset support is counted during mining the database with a set minimum support. During this process all frequent itemsets plus some infrequent candidate itemsets are counted (or support is determined by other means. This procedure might take some time, especially for low minimum support values.

If only the support information for a single or a few itemsets is needed, we might not want to mine the database for all frequent itemsets. We also do not know in advance how high (or low) to set the minimum support to still get the support information for the itemset in question.

For this problem, **arules** contains the method **support** which determines the support for a set of given sets of items (as an itemMatrix) by means of transaction ID set intersection (using tidLists). Transaction ID set intersection is used by several fast mining algorithms (e.g., by Eclat (Zaki et al., 1997b)). The support of an itemset is determined by intersecting the transaction ID sets of its subsets. In the simplest case the transaction IDs for all items which occur in an itemset are intersected. The support count is then the size of the intersection set.

In addition to determining the support of a few itemsets without mining all frequent itemsets, support is also useful for finding the support of infrequent itemsets with a support so low that mining is infeasible due to combinatorial explosion.

4.2 Rule induction

A part of the association rule mining problem is the generation (or induction) of a set of rules \mathcal{R} from a set of frequent itemsets \mathcal{X} . The implementation of the Apriori algorithm used in **arules** already contains a rule induction engine and returns per default the set of association rules of the form $X \Rightarrow Y$ which satisfy given minimum support and minimum confidence. Following the definition of Agrawal et al. (1993) Y is restricted to single items.

In some cases it is necessary to separate mining itemsets and generating rules from itemsets. For example, if only rules stemming from a subset of all frequent itemsets are interesting for the user. The Apriori implementation efficiently generates rules by reusing the data structures built during mining the frequent itemsets. However, if Apriori is used to only return itemsets or Eclat or some other algorithm is used to mine itemsets, the data structure needed for rule induction is lost. The complete set of frequent itemsets contains all information needed to induce association rules, since the support information for all frequent itemsets and its subsets (which are also frequent) is available. However, for efficient induction, a suitable data structure which allows fast look-ups of support values, has to be rebuilt.

If rules need to be induced from an arbitrary set of itemsets, not all needed support information to calculate confidence is available. For example, if all available information is a itemset containing five items and we want to induce rules, we need the support of the itemset (which we might know), but also the support of all subsets of length four. The missing support information has to be counted from the database. If we want to induce rules efficiently for a given set of itemsets, we also have to store support values in a suitable data structure to avoid counting itemsets more than once.

The method ruleInduction provided in arules reuses the counting mechanism, the data structure of the rule induction engine of the C implementation of Apriori to induce rules for a given confidence from an arbitrary set of itemsets \mathcal{X} in the following way:

- 1. Reduce the database to only the items which occur in \mathcal{X} ,
- 2. determine the lowest support of an itemset in \mathcal{X} ,
- 3. reuse the implementation of Apriori to mine the set of all rules \mathcal{R} from the reduced database using the given confidence and the lowest itemset support found above,
- 4. remove the rules from \mathcal{R} which can not be generated from the itemsets in \mathcal{X} .

Most time is spent on the rule filtering step since here the set of items in each mined rule in \mathcal{R} has to be matched against the items in each itemset in \mathcal{X} . However, if only a small number of distinct items occurs in \mathcal{X} , ruleInduction is reasonably fast.

Alternatively, the appearance of items in rules can be directly controlled with the argument appearance for Apriori (see Borgelt, 2004) which might be faster in some cases.

4.3 Sampling from transactions

Taking samples from large databases for mining is a powerful technique. This technique is especially useful if the original database does not fit into main memory, but the sample does. However, even if the database fists into main memory, sampling can provide an enormous speed-up for mining at the cost of only little degradation of accuracy.

Mannila, Toivonen, and Verkamo (1994) proposed sampling with replacement for association rule mining and quantify the estimation error due to sampling. Using Chernov bounds on the binomial distribution (the number of transactions which contains a given itemset in a sample), the authors argue that in theory even relatively small samples should provide good estimates for support.

Zaki, Parthasarathy, Li, and Ogihara (1997a) built upon the theoretic work by Mannila et al. (1994) and show that for an itemset X with support $\tau = \text{supp}(X)$ and for an acceptable relative

error of support ϵ (an accuracy of $1 - \epsilon$) at a given confidence level 1 - c, the needed sample size n can be computed by

$$n = \frac{-2\ln(c)}{\tau\epsilon^2}. (1)$$

Depending on its support, for each itemset a different sample size is appropriate. As a heuristic, the authors suggest to use the user specified minimum support threshold for τ . This means that for itemsets close to minimum support, the given error and confidence level hold while for more frequent itemsets the error rate will be less. However, with this heuristic the error rate for itemsets below minimum support can exceed ϵ at the given confidence level and thus some infrequent itemset might appear as frequent in the sample.

Zaki et al. (1997a) also evaluated sampling in practice on several data sets and conclude that sampling not only speeds mining up considerably, but also the errors are considerably smaller than theoretically obtained by Chernov bounds and thus samples smaller than obtained by formula 1 are often sufficient.

Another way to obtain the required sample size for association rule mining is progressive sampling (Parthasarathy, 2002). This approach starts with a small sample and uses progressively larger samples until model accuracy does not improve significantly anymore. Parthasarathy (2002) defines a proxy for model accuracy improvement by using a similarity measure between two sets of associations. The idea is that since larger samples will produce more accurate results, the similarity between two sets of associations of two consecutive samples is low if accuracy improvements are high and increases with decreasing accuracy improvements. Thus increasing sample size can be stopped if the similarity between consecutive samples reaches a "plateau."

Toivonen (1996) presents an application of sampling to reduce the needed I/O overhead for very large databases which do not fit into main memory. The idea is to use a random sample from the data base to mine frequent itemsets at a support threshold below the set minimum support. The support of these itemsets is then counted in the whole database and the infrequent itemsets are discarded. If the support threshold to mine the sample is picked low enough, almost all frequent itemsets and their support will be found in one pass over the large database.

In **arules** sampling is implemented by the method **sample** which provides all capabilities of the standard sampling method in R (e.g., sampling with or without replacement and probability weights).

5 Examples

In this section we show the basic functionality of arules with some examples.

5.1 Example 1: Analyzing and preparing a transaction data set

In this example, we show how a data set can be analyzed and manipulated before associations are mined. This is important for finding problems in the data set which could make the mined associations useless or at least inferior to associations mined on a properly prepared data set. For the example, we look at the Epub transaction data contained in package arules. This data set contains downloads of documents from the Electronic Publication platform of the Vienna University of Economics and Business Administration available via http://epub.wu-wien.ac.at from January 2003 to August 2005.

First, we load arules and the data set.

> library("arules")

Loading required package: stats4 Loading required package: Matrix

```
> data("Epub")
> Epub

transactions in sparse format with
3307 transactions (rows) and
436 items (columns)
```

> summary(Epub)

We see that the data set consists of 3307 transactions and is represented as a sparse matrix with 3307 rows and 436 columns which represent the items. Next, we use the summary method to get more information about the data set.

```
transactions as itemMatrix in sparse format with 3307 rows (elements/itemsets/transactions) and 436 columns (items)

most frequent items:
doc_11d doc_4c6 doc_2cd doc_71 doc_24e (Other)
194 115 106 102 93 5323
```

element (itemset/transaction) length distribution:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2359	491	201	95	46	24	17	12	11	7	9	4	4	3	1	3
17	18	19	20	22	24	25	28	34	38	74	79				
2	3	5	2	1	1	1	1	1	1	1	1				

```
Min. 1st Qu. Median Mean 3rd Qu. Max. 1.000 1.000 1.000 1.794 2.000 79.000
```

includes extended transaction information - examples:

```
transactionIDs TimeStamp
1 session_4795 2003-01-01 19:59:00
2 session_4797 2003-01-02 06:46:01
3 session_479a 2003-01-02 09:50:38
```

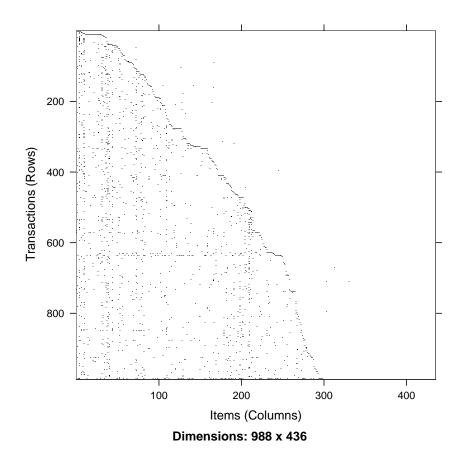
The summary method displays the most frequent items in the data set, information about the transaction length distribution and that the data set contains some extended transaction information. We see that the data set contains transaction IDs and in addition time stamps (using class POSIXct) for the transactions. The additional information can be used for analyzing the data set.

We selected only the year part of the time stamps. For 2003, the first year in the data set we have 988 transactions. We can select the corresponding transactions and inspect the structure using a level-plot.

```
> Epub_2003 <- Epub[year == "2003"]
> length(Epub_2003)
```

[1] 988

> image(Epub_2003)



The plot is a direct visualization of the binary incidence matrix where the the dark dots represent the ones in the matrix. From the plot we see that the items in the data set are not evenly distributed. In fact, the white area to the top right side suggests, that in the beginning of 2003 only very few items were available (less than 50) and then during the year more items were added until it reached a number of around 300 items. Also, we can see that there are two transactions in the data set which contain a very high number of items (denser horizontal lines). These transactions need further investigation since they could originate from data collection problems (e.g., a web robot downloading many documents from the publication site). To find the very long transactions we can use the size method and select very long transactions (containing more than 20 items).

> transactionInfo(Epub_2003[size(Epub_2003) > 20])

TimeStamp	${\tt transactionIDs}$	
003-04-29 12:30:38	session_56e2	301
003-08-17 17:16:12	session_6308	580
003-12-29 19:35:35	session 72dc	896

We found three long transactions and printed the corresponding transaction information. Of course, size can also be used in a similar fashion to remove long or short transactions.

Transactions can be inspected using the inspect method. Since the long transactions identified above would result in a very long printout, we will inspect the first 5 transactions in the subset for 2003.

> inspect(Epub_2003[1:5])

Most transactions contain one item. Only transaction 4 contains three items. For further inspection transactions can be converted into a list with:

```
> as(Epub_2003[1:5], "list")
$session_4795
[1] "doc_154"

$session_4797
[1] "doc_3d6"

$session_479a
[1] "doc_16f"

$session_47b7
[1] "doc_f4" "doc_11d" "doc_1a7"

$session_47bb
[1] "doc_83"
```

Finally, transaction data in horizontal layout can be converted to transaction ID list in vertical layout using coercion.

```
> Epub_tidLists <- as(Epub, "tidLists")
> Epub_tidLists

tidLists in sparse format for
    436 items/itemsets (rows) and
    3307 transactions (columns)
```

For performance reasons the transaction ID list is also stored in a sparse matrix. To get a list, coercion to list can be used.

```
> as(Epub_tidLists[1:3], "list")
```

```
$doc_154
[1] "session_4795" "session_6082" "session_60dd" "session_67db" "session_769c"
[6] "session_7ee3" "session_bd9d" "session_c591" "session_ce9f"
$doc_3d6
 [1] "session_4797"
                        "session_4893"
                                          "session_48f4"
                                                             "session_4ca3"
 [5] "session_wu4450a"
                        "session_52c6"
                                          "session_5712"
                                                             "session_58e3"
 [9] "session_5984"
                        "session_5b20"
                                          "session_5c20"
                                                             "session_5dc0"
[13] "session_5eac"
                        "session_wu4a129"
                                          "session_6599"
                                                             "session_673d"
                                                             "session_708a"
[17] "session_683e"
                                          "session_6f2f"
                        "session_wu4d25a"
[21] "session_7a0c"
                        "session_7de5"
                                           "session_89db"
                                                             "session_9227"
[25] "session_9941"
                        "session_a4d7"
                                           "session_a8c0"
                                                             "session_c3c4"
[29] "session_c546"
                        "session_ca44"
$doc_16f
[1] "session_479a" "session_56e2" "session_630c" "session_72dc" "session_8b3e"
[6] "session_91ab" "session_a202" "session_a7b9"
```

In this representation each item has an entry which is a vector of all transactions it occurs in. Transaction ID list can be directly used as input for mining algorithms which use such a vertical database layout to mine associations.

In the next example, we will see how a data set is created and rules are mined.

5.2 Example 2: Preparing and mining a questionnaire data set

As a second example, we prepare and mine questionnaire data. We use the Adult data set from the UCI machine learning repository (Blake and Merz, 1998) provided by package **arules**. This data set is similar to the data used by Hastie et al. (2001). The data originates from the U.S. census bureau database and contains 48842 instances with 14 attributes like age, work class, education, etc. In the original applications of the data, the attributes were used to predict the income level of individuals. We added the attribute income with levels small and large, representing an income of $\leq \$50,000$ and > \$50,000, respectively. This data is included in arules as the data set arules and arules are the data set arules are the data

```
> data("Adult")
> dim(Adult)
[1] 48842
             15
> Adult[1:2, ]
             workclass fnlwgt education education-num
  age
                                                           marital-status
1
  39
             State-gov 77516 Bachelors
                                                    13
                                                            Never-married
  50 Self-emp-not-inc 83311 Bachelors
                                                    13 Married-civ-spouse
       occupation relationship race sex capital-gain capital-loss
     Adm-clerical Not-in-family White Male
                                                    2174
                                                                    0
2 Exec-managerial
                        Husband White Male
                                                       0
                                                                    0
 hours-per-week native-country income
              40 United-States
1
                                small
2
              13 United-States
                                small
```

Adult contains a mixture of categorical and metric attributes and needs some preparations before it can be transformed into transaction data suitable for association mining. First, we remove

the two attributes fnlwgt and education-num. The first attribute is a weight calculated by the creators of the data set from control data provided by the Population Division of the U.S. census bureau. The second removed attribute is just a numeric representation of the attribute education which is also part of the data set.

```
> Adult[["fnlwgt"]] <- NULL
> Adult[["education-num"]] <- NULL</pre>
```

Next, we need to map the four remaining metric attributes (age, hours-per-week, capital-gain and capital-loss) to ordinal attributes by building suitable categories. We divide the attributes age and hours-per-week into suitable categories using knowledge about typical age groups and working hours. For the two capital related attributes, we create a category called None for cases which have no gains/losses. Then we further divide the group with gains/losses at their median into the two categories Low and High.

```
> Adult[["age"]] <- ordered(cut(Adult[["age"]], c(15, 25, 45, 65,
+ 100)), labels = c("Young", "Middle-aged", "Senior", "Old"))
> Adult[["hours-per-week"]] <- ordered(cut(Adult[["hours-per-week"]],
+ c(0, 25, 40, 60, 168)), labels = c("Part-time", "Full-time",
+ "Over-time", "Workaholic"))
> Adult[["capital-gain"]] <- ordered(cut(Adult[["capital-gain"]],
+ c(-Inf, 0, median(Adult[["capital-gain"]][Adult[["capital-gain"]]] >
+ 0]), 1e+06)), labels = c("None", "Low", "High"))
> Adult[["capital-loss"]] <- ordered(cut(Adult[["capital-loss"]],
+ c(-Inf, 0, median(Adult[["capital-loss"]][Adult[["capital-loss"]]] >
+ 0]), 1e+06)), labels = c("none", "low", "high"))
```

Now, the data can be automatically recoded as a binary incidence matrix by coercing the data set to transactions.

```
> Adult_transactions <- as(Adult, "transactions")
> Adult_transactions

transactions in sparse format with
48842 transactions (rows) and
115 items (columns)
```

The remaining 13 categorical attributes were automatically recoded into 115 binary items. During encoding the item labels were generated in the form of <variable name> = <category label>.

```
> summary(Adult_transactions)
```

```
transactions as itemMatrix in sparse format with 48842 rows (elements/itemsets/transactions) and 115 columns (items)
```

```
most frequent items:
```

```
      capital-loss = none
      capital-gain = None

      46560
      44807

      native-country = United-States
      race = White

      43832
      41762

      workclass = Private
      (Other)

      33906
      401333
```

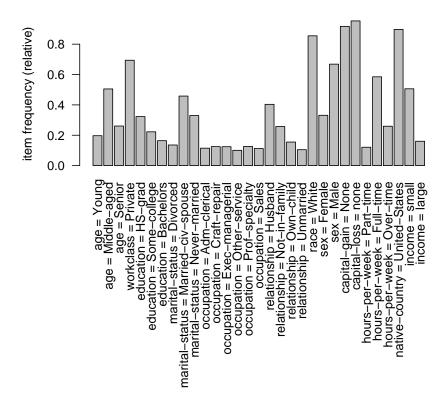
```
element (itemset/transaction) length distribution:
         10
                      12
                11
                            13
        971
   19
             2067 15623 30162
   Min. 1st Qu.
                  Median
                            Mean 3rd Qu.
                                             Max.
          12.00
                   13.00
                           12.53
                                    13.00
                                            13.00
includes extended item information - examples:
             labels variables
                                     levels
                                      Young
        age = Young
```

2 age = Middle-aged

The summary of the transaction data set gives a rough overview showing the most frequent items, the length distribution of the transactions and the extended item information which shows which variable and which value were used to create each binary item. In the first example we see that the item with label age = middle-aged was generated by variable age and level middle-aged. To see which items are important in the data set we can use the itemFrequencyPlot. To reduce

the number of items, we only plot the item frequency for items with a support greater than 10%.

age Middle-aged



Next, we call the function apriori to find all rules (the default association type for apriori) with a minimum support of 0.5% and a confidence of 0.7.

```
> rules <- apriori(Adult_transactions, parameter = list(support = 0.005,
+ confidence = 0.7))</pre>
```

```
parameter specification:
 confidence minval smax arem aval originalSupport support minlen maxlen target
        0.7
                      1 none FALSE
                                              TRUE
                                                     0.005
                                                                 1
 FALSE
algorithmic control:
 filter tree heap memopt load sort verbose
    0.1 TRUE TRUE FALSE TRUE
                                      TRUE
apriori - find association rules with the apriori algorithm
version 4.21 (2004.05.09)
                                 (c) 1996-2004
                                                 Christian Borgelt
set item appearances ...[0 item(s)] done [0.00s].
set transactions ...[115 item(s), 48842 transaction(s)] done [0.11s].
sorting and recoding items ... [71 item(s)] done [0.01s].
creating transaction tree ... done [0.14s].
checking subsets of size 1 2 3 4 5 done [0.95s].
writing ... [114922 rule(s)] done [0.04s].
creating S4 object ... done [0.29s].
> rules
set of 114922 rules
```

First, the function prints the used parameters. Apart from the specified minimum support and minimum confidence, all parameters have the default values. It is important to note that with parameter maxlen, the maximum size of mined frequent itemsets, is by default restricted to 5. Longer association rules are only mined if maxlen is set to a higher value. After the parameter settings, the output of the C implementation of the algorithm with timing information is displayed. The result of the mining algorithm is a set of 114922 rules. For an overview of the mined rules the method summary can be used. It shows the number of rules, the most frequent items contained in the left-hand-side and the right-hand-side and their respective length distributions and summary statistics for the quality measures returned by the mining algorithm.

```
> summary(rules)
set of 114922 rules
rule length distribution (lhs + rhs):
          2
                       4
    1
                3
        341 5198 28692 80687
    4
  Min. 1st Qu. Median
                            Mean 3rd Qu.
                                            Max.
  1.000
          4.000
                  5.000
                           4.651
                                            5.000
                                   5.000
summary of quality measures:
    support
                                            lift
                       confidence
                            :0.7000
Min.
        :0.005016
                    \mathtt{Min}.
                                      Min.
                                             : 0.7635
 1st Qu.:0.007268
                    1st Qu.:0.8503
                                      1st Qu.: 1.0025
 Median :0.011547
                    Median :0.9153
                                      Median: 1.0371
        :0.024740
                            :0.8945
                                             : 1.2506
 Mean
                    Mean
                                      Mean
 3rd Qu.:0.023525
                    3rd Qu.:0.9542
                                      3rd Qu.: 1.1616
```

Max.

:1.0000

Max.

:0.953278

Max.

:30.4376

As typical for association rule mining, the number of found rules is huge. To analyze these rules, for example, the method subset can be used to produce a subset of rules which contain items which resulted form the variable income in the right-hand-side of the rule and the lift measure exceeds 1.4.

```
> rules.sub <- subset(rules, subset = rhs %in% "income" & lift >
+ 1.4)
```

We can then inspect the three rules with the highest lift value (using the SORT method).

```
> inspect(SORT(rules.sub, by = "lift")[1:3])
```

```
support confidence
                                                                                        lift
1 {occupation = Exec-managerial,
   race = White,
   sex = Male,
                                        => {income = large} 0.005568978 0.7101828 4.423766
    capital-gain = High}
2 {marital-status = Married-civ-spouse,
   occupation = Exec-managerial,
   race = White,
    capital-gain = High}
                                        => {income = large} 0.005057123 0.7077364 4.408527
3 {education = Some-college,
    occupation = Adm-clerical,
   relationship = Unmarried,
                                        => {income = small} 0.005487081 0.7382920 1.458724
    capital-loss = none}
```

Using such subset selection and sorting a set of associations can be analyzed even if it is huge.

5.3 Example 3: Extending arules with a new interest measure

In this example, we show how easy it is to add a new interest measure. As the interest measure we chose *all-confidence* introduced by Omiecinski (2003). All-confidence is defined on itemsets X as:

$$\operatorname{all-confidence}(X) = \frac{\operatorname{supp}(X)}{\max(\operatorname{supp}(I \subset X))}$$
 (2)

This measure has the property $\operatorname{conf}(I\Rightarrow Z\backslash I)\geq \operatorname{all-confidence}(X)$ for all $I\subset X$. This means that all possible rules generated from itemset X must at least have a confidence given by the itemset's all-confidence value. Omiecinski (2003) shows that the support in the denominator of equation 2 must stem from a single item and thus can be simplified to $\max(\sup(i\in X))$.

To obtain an itemset to calculate all-confidence for, we mine frequent itemsets from the previously used Adult data set using the Eclat algorithm.

```
> data("Adult_transactions")
> fsets <- eclat(Adult_transactions, parameter = list(support = 0.05),
+ control = list(verbose = FALSE))</pre>
```

For the denominator of all-confidence we need to find all mined single items and their corresponding support values. In the following we create a named vector where the names are the column numbers of the items and the values are their support.

```
> single_items <- fsets[size(items(fsets)) == 1]
> single_support <- quality(single_items)$support
> names(single_support) <- unlist(LIST(items(single_items), decode = FALSE))
> head(single_support, n = 5)
```

```
66 63 111 60 8
0.9532779 0.9173867 0.8974243 0.8550428 0.6941976
```

Next, we can calculate the all-confidence using formula 2 for all itemsets. The single item support needed for the denomination is looked up from the named vector single_support and the resulting measure is added to the set's quality data frame.

```
> itemset_list <- LIST(items(fsets), decode = FALSE)
> all_conf <- quality(fsets)$support/sapply(itemset_list, function(x) max(single_support[as.chara=))
> quality(fsets) <- cbind(quality(fsets), all_conf)</pre>
```

The new quality measure is now part of the set of itemsets.

> summary(fsets)

set of 5908 itemsets

most frequent items:

element (itemset/transaction) length distribution:

```
1 2 3 4 5
36 303 1078 2103 2388
```

```
Min. 1st Qu. Median Mean 3rd Qu. Max. 1.000 4.000 4.000 4.101 5.000 5.000
```

summary of quality measures:

```
support
                    all_conf
Min.
      :0.05004
                 Min. :0.05249
1st Qu.:0.06230
                 1st Qu.:0.06986
Median :0.08124
                 Median: 0.09349
Mean :0.11114
                 Mean :0.13111
3rd Qu.:0.12554
                 3rd Qu.:0.14326
Max.
      :0.95328
                 Max.
                        :1.00000
```

includes transaction ID lists: FALSE

It can be used to manipulate the set. For example, we can look at the itemsets which contain an item related to education and sort them by all-confidence (we filter itemsets of length 1 first, since they have per definition a all-confidence of 1).

```
hours-per-week = Full-time} 0.2090209 0.3572453
2 {education = HS-grad,
  income = small} 0.1807051 0.3570388
3 {workclass = Private,
  education = HS-grad} 0.2391794 0.3445408
```

All-confidence is implemented in arules as the method all_confidence.

5.4 Example 4: Sampling

In this example, we show how sampling can be used in arules. We use again the Adult data set.

```
> data("Adult_transactions")
> Adult_transactions

transactions in sparse format with
  48842 transactions (rows) and
  115 items (columns)
```

To calculate a reasonable sample size n, we use the formula developed by Zaki et al. (1997a) and presented in Section 4.3. We choose a minimum support of 5%. As an acceptable error rate for support ϵ of we choose 10% and as the confidence level (1-c) we choose 90%.

```
> supp <- 0.05
> epsilon <- 0.1
> c <- 0.1
> n <- -2 * log(c)/(supp * epsilon^2)
> n

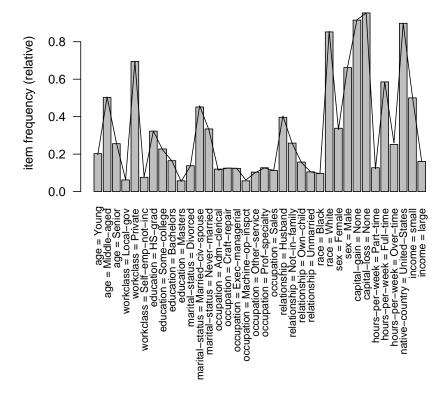
[1] 9210.34
```

The resulting sample size is considerably smaller than the original data base. With sample we produce a sample of size n with replacement from the database.

```
> Adult.sample <- sample(Adult_transactions, n, replace = TRUE)</pre>
```

The resulting sample can be compared with the database (the population) using an item frequency plot. The item frequencies in the sample are displayed as bars and the item frequencies in the original database are represented by the line. For better readability of the labels, we only display frequent items in the plot and reduce the label size with the parameter <code>cex.names</code>.

```
> items.frequent <- itemFrequency(Adult_transactions) > supp
> itemFrequencyPlot(Adult.sample[, items.frequent], population = Adult_transactions[,
+ items.frequent], cex.names = 0.8)
```



To compare the speed-up reached by sampling we use the Eclat algorithm to mine frequent itemsets on both, the database and the sample and compare the system time (in seconds) used for mining.

Mining the sample instead of the whole data base results in a speed-up factor of:

```
> t[1]/t.sample[1]
```

[1] 4.272727

To evaluate the accuracy for the itemsets mined from the sample, we analyze the difference between the two sets.

```
> itemsets
```

```
set of 5908 itemsets
```

> itemsets.sample

set of 5948 itemsets

The two sets have roughly the same size. To check if the sets contain similar itemsets, we match the sets and see what fraction of frequent itemsets found in the database were also found in the sample.

```
> match <- match(itemsets, itemsets.sample)
> length(match[!is.na(match)])/length(itemsets)
[1] 0.9807041
```

Almost all frequent itemsets were found using the sample. The summaries of the support of the frequent itemsets which were not found in the sample and the itemsets which were frequent in the sample although they were infrequent in the database give:

> summary(quality(itemsets[which(is.na(match))])\$support)

```
Min. 1st Qu. Median Mean 3rd Qu. Max. 0.05004 0.05049 0.05105 0.05133 0.05178 0.05460
```

> summary(quality(itemsets.sample[-match[!is.na(match)]])\$support)

```
Min. 1st Qu. Median Mean 3rd Qu. Max. 0.05005 0.05060 0.05114 0.05150 0.05212 0.05527
```

This shows that only itemsets with support very close to the minimum support were falsely missed or found.

For the frequent itemsets which were found in the database and in the sample, we can calculate accuracy from the the error rate.

```
> supp.itemsets <- quality(itemsets[!is.na(match)])$support
> supp.itemsets.sample <- quality(itemsets.sample[match[!is.na(match)]])$support
> accuracy <- 1 - abs(supp.itemsets.sample - supp.itemsets)/supp.itemsets
> summary(accuracy)

Min. 1st Qu. Median Mean 3rd Qu. Max.
0.8706 0.9653 0.9800 0.9755 0.9906 1.0000
```

The summary shows that sampling resulted in finding the support of itemsets with high accuracy. Thus for extremely large databases or for application where mining time is important, sampling provides a powerful technique.

6 Summary and outlook

With package **arules** we provide the basic infrastructure which enables us to mine associations and analyze and manipulate the results. Previously, in R there was no such infrastructure available. The main features of **arules** are:

- Efficient implementation using sparse matrices.
- Simple and intuitive interface to manipulate and analyze transaction data, sets of itemsets and rules with subset selection and sorting.
- Interface to two fast mining algorithms.
- Flexibility in terms of adding new quality measures, and additional item and transaction descriptions which can be used for selecting transactions and analyzing resulting associations.
- Extensible data structure to allow for easy implementation of new types of associations and interfacing new algorithms.

There are several interesting possibilities to extend **arules**. For example, it would be very useful to interface algorithms which use statistical measures to find "interesting" itemsets (which are not necessarily frequent itemsets as used in an association rule context). Such algorithms include implementations of the χ^2 -test based algorithm by Silverstein, Brin, and Motwani (1998) or the baseline frequency approach by DuMouchel and Pregibon (2001).

Another interesting extension would be to interface synthetic data generators for fast evaluation and comparison of different mining algorithms. The best known generator for transaction data for mining association rules was developed by Agrawal and Srikant (1994). Alternatively data can be generated by simple probabilistic models as done by Hahsler, Hornik, and Reutterer (2005).

Finally, similarity measures between itemsets and rules can be implemented for **arules**. With such measures distance based clustering and visualization of associations is possible (see e.g., Strehl and Ghosh, 2003)).

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