Opg. 2-1.2:

a) Diskret : $2 \sim 12$

b) Kontinuert: $11{,}5 \sim 12{,}5$

c) Diskret : 00000000 - 99999999

d) Kontinuert: $100 \sim 500$ pund i USA!
$50 \sim 150$ kg i DK

Opg. 2-2.2:

a) For en kontinuert stokastisk variabel er sandsynligheden for et enkelt punkt altid nul.

* Se nedenfor

$$\Rightarrow \Pr(X = \tfrac{1}{4}) = 0$$

b) $\Pr(X > \frac{3}{4}) \underset{\text{side 55}}{=} 1 - \Pr(X \leq \frac{3}{4}) = 1 - F_X(\frac{3}{4}) = \underline{\underline{0{,}125}}$

c) $\Pr(-0{,}5 < X \leq 0{,}5) \underset{\text{side 54}}{=} F_X(0{,}5) - F_X(-0{,}5)$

$$= 0{,}75 - 0{,}25 = \underline{\underline{0{,}5}}$$

* $\Pr(X = \frac{1}{4}) = \lim_{\delta \to 0} \Pr(\frac{1}{4} < X \leq \frac{1}{4} + \delta) = \lim_{\delta \to 0} F_X(\frac{1}{4} + \delta) - F_X(\frac{1}{4}) = 0$

Opg. 2-2.3: a) Der skal gælde; at $F_X(\infty) = 1$. (Side 54)

$$F_X(X=\infty) = A\{1 - e^{-(\infty - 1)}\}$$

$$= A\{1 - 0\} = 1$$

Altså må $\underline{\underline{A = 1}}$.

b) $F_X(2) = 1 \cdot (1 - e^{-(2-1)}) = \underline{\underline{0{,}6321}}$

c) $Pr(2 < X < \infty) = F_X(\infty) - F_X(2)$

$$= 1 - 0{,}6321 = \underline{\underline{0{,}3679}}$$

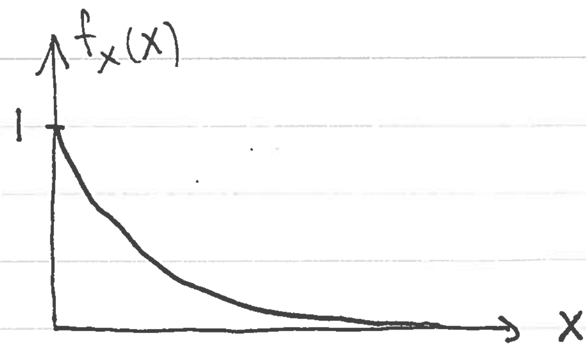
d) $Pr(1 < X \leq 3) = F_X(3) - F_X(1) = \underline{\underline{0{,}8647}}$.

Opg. 2-3.2: a) $f_X(x) = \dfrac{dF_X(x)}{dx} = \dfrac{d}{dx}\left[1 - e^{-(x-1)}\right] = e^{-(x-1)}$

for $1 < x < \infty$

$f_X(x) = 0$

for $-\infty < x \leq 1$



Opg. 2-3.2: Fortsat

b) $$Pr(2 < X \leq 3) = \int_2^3 f_X(x)\,dx$$

$$= \int_2^3 e^{-(x-1)}\,dx$$

$$= \left(-e^{-(3-1)}\right) - \left(-e^{-(2-1)}\right)$$

$$= -e^{-2} + e^{-1} = \underline{\underline{0{,}2325}}$$

c) $$Pr(x < 2) = \int_{-\infty}^{2} e^{-(x-1)}\,dx = \int_1^2 e^{-(x-1)}\,dx$$

$$= \left(-e^{-(2-1)}\right) - \left(-e^{-(1-1)}\right)$$

$$= -e^{-1} + e^{0} = 1 - e^{-1} = \underline{\underline{0{,}6321}}$$

Opg. 2-3.3: a) $f_X(x) = e^{-2|x|} \qquad -\infty < x < \infty$

$Y = X^2$ og $f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|$ (side 60)

$\Rightarrow f_Y(y) = \frac{1}{2\sqrt{y}}\left[f_X(\sqrt{y}) + f_X(-\sqrt{y})\right], y \geq 0$

(se ligning (2-6) på side 62)

$\Rightarrow f_Y(y) = \frac{1}{2\sqrt{y}}\left[e^{-2|\sqrt{y}|} + e^{-2|-\sqrt{y}|}\right]$

$= \frac{1}{\sqrt{y}} e^{-2|\sqrt{y}|}$

b) $Pr(Y>2) = \int_2^\infty f_Y(y)\,dy$

$= \int_2^\infty \frac{1}{\sqrt{y}} e^{-2|\sqrt{y}|}\,dy = \int_2^\infty \frac{1}{\sqrt{y}} e^{-2\sqrt{y}}\,dy$

Substitution: $u = \sqrt{y} \rightarrow u^2 = y$

$\Rightarrow \frac{dy}{du} = 2u \Rightarrow dy = 2u\,du$

$Pr(Y>2) = \int_{\sqrt{2}}^\infty \frac{1}{u} e^{-2u} \cdot 2u \cdot du = 2\int_{\sqrt{2}}^\infty e^{-2u}$

$= 2e^{-2\sqrt{2}}$

Opg. 2-4.1: a) $E[X] = \bar{X} = \int_1^\infty x f_X(x)\,dx$

$$= \int_1^\infty x e^{-(x-1)} dx$$

Løses ved integration-by-parts og substitution

Lad $u = x$, $dv = e^{-(x-1)} dx$, $du = dx$

$$\Rightarrow \frac{dv}{dx} = e^{-(x-1)}$$

Ved substitution fås

$$v = \int \frac{dv}{dx} dx = \int e^{-(x-1)} dx = -e^{-(x-1)}$$

Integration by parts.

$$\int x e^{-(x-1)} dx = \int u\,dv = uv - \int v\,du$$

$$= -x \cdot e^{-(x-1)} - \int -e^{-(x-1)} dx$$

$$= -x \cdot e^{-(x-1)} + \int e^{-(x-1)} dx$$

$$= -x \cdot e^{-(x-1)} - e^{-(x-1)} = -(x+1) e^{-(x-1)}$$

$$\bar{X} = \int_1^\infty x e^{-(x-1)} dx = \left(-(\infty+1) e^{-(\infty-1)}\right) - \left(-(1+1) e^{-(2-1)}\right)$$

$$= 0 + 2 \cdot 1 = \underline{\underline{2}}$$

Opg 2-4.1: b) $\overline{X^2} = \int_1^\infty x^2 f_X(x)dx$

$$= \int_1^\infty x^2 e^{-(x-1)} dx$$

Ved integration by parts og substitution fås

$$\int x^2 e^{-(x-1)} dx = -x^2 e^{-(x-1)} - 2(x+1) e^{-(x-1)}$$

$$\overline{X^2} = \int_1^\infty x^2 e^{-(x-1)} dx$$

$$= \left(-\infty^2 e^{-(\infty-1)} - 2(\infty+1) e^{-(\infty-1)}\right) - \left(-1^2 e^{-(1-1)} - 2(1+1) e^{-(1-1)}\right)$$

$$= (0-0) - (-1\cdot 1 - 2\cdot 2\cdot 1)$$

$$= -(-5) = \underline{5}$$

c) $\sigma_x^2 = \overline{X^2} - \bar{X}^2 = 5 - 2^2 = \underline{\underline{1}}$

Opg. 2-5.1: $\bar{X} = 5$, $\sigma^2 = 16$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\bar{X})^2/2\sigma^2}$$

$$= \frac{1}{\sqrt{2\pi}\cdot 4} \cdot e^{-(x-5)^2/(2\cdot 16)}$$

Husk, at $\boxed{F_X(x) = \phi\left(\frac{x-\bar{X}}{\sigma}\right)}$ ← givet ved tabelopslag

a) $\Pr(X > 0) = 1 - \Pr(X \leq 0)$

$$= 1 - F_X(0) = \left(1 - \int_{-\infty}^{0} f_X(x)\,dx\right)$$

$$= 1 - \phi\left(\frac{0-5}{4}\right)$$ ← normalisering

$$= 1 - \phi(-1{,}25)$$ ← symmetri: $\phi(-x) = 1 - \phi(x)$

$$= 1 - (1 - \phi(1{,}25))$$

$$= +\phi(1{,}25) = \underline{\underline{0{,}8944}}$$ ← Tabel D

b) $\Pr(0 < X \leq 5) = F_X(5) - F_X(0)$

$$= \phi\left(\frac{5-5}{4}\right) - \phi\left(\frac{0-5}{4}\right)$$

$$= \phi(0) - \phi(-1{,}25)$$

$$= \phi(0) - [1 - \phi(1{,}25)]$$

$$= 0{,}5 - 1 + 0{,}8944 = \underline{\underline{0{,}3944}}$$

Opg. 2-5.1: Fortsat

c) $Pr(X \geq 10) = 1 - Pr(X \leq 10)$

$$= 1 - F_X(10)$$

$$= 1 - \phi\left(\frac{10-5}{4}\right)$$

$$= 1 - \phi(1{,}25)$$

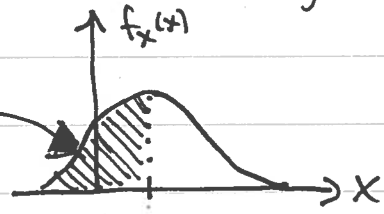
$$= 1 - 0{,}8944$$

$$= \underline{\underline{0{,}1056}}$$

Opg. 2-5.3:

a) $Pr(X \leq 1) = 0{,}5 \Rightarrow \overline{X} = 1$

Husk på, at Gauss fordelingen jo er symmetrisk:



$F_X(X=1)$

$$= \int_{-\infty}^{1} f_X(x)\,dx = 0{,}5 \quad 1$$

Det gælder altid for ~~en~~ Gauss, at

$Pr(X \leq \overline{X}) = 0{,}5 =$ "det halve af arealet under f_X"

Opg 2-5.3: Fortsat

b) $Pr(X > 5) = 0{,}0228$

$= 1 - Pr(X \leq 5)$

$= 1 - \Phi\left(\frac{5-1}{\sigma}\right)$

$\Rightarrow \Phi\left(\frac{5-1}{\sigma}\right) = 1 - 0{,}0228 = 0{,}9772$

Ved tabel opslag finder vi, at

$\Phi(2) = 0{,}9772 = \Phi\left(\frac{5-1}{\sigma}\right)$

Så vi har, at

$$2 = \frac{5-1}{\sigma} = \frac{4}{\sigma} \Leftrightarrow \sigma = \frac{4}{2} = 2$$

Dermed er variansen $\underline{\underline{\sigma^2 = 4}}$

Opg. 2-5.3 : Fortsat

c) $Pr(X \leq 3) = \Phi\left(\frac{3-1}{2}\right) = \Phi(1) = \underline{\underline{0{,}8413}}$

Opg. 2-7.3: Sæt $\bar{\tau} = 6$, så $f_\tau(\tau) = \frac{1}{6} e^{-\tau/6}, \tau > 0$

(Se eksponentialfordelingen side 92-93)

a) $Pr(\tau \leq 6) = \int_0^6 \frac{1}{6} e^{-\tau/6} d\tau = \underline{\underline{0{,}632}}$

b) $Pr(\tau > 10) = 1 - F_\tau(10)$

$= 1 - (1 - e^{-10/6}) = \underline{\underline{0{,}189}}$

c) $Pr(5 < \tau \leq 6) = F_\tau(6) - F_\tau(5) = \underline{\underline{0{,}0667}}$

Opg. 2-8.1:

a) $Pr\{T > 10 \mid T > 5\}$

$= \frac{Pr\{T > 10, T > 5\}}{Pr\{T > 5\}} = \frac{Pr\{T > 10\}}{Pr\{T > 5\}} = \frac{1 - F_T(10)}{1 - F_T(5)}$

$= 0{,}367$

Opg. 2-8.1: Fortsat

b) ① $F(\tau \mid \tau > 3) = P_r(T \leq \tau \mid T > 3)$

$$= \frac{P_r\{T \leq \tau,\ T > 3\}}{P_r\{T > 3\}}$$

$$= \frac{P_r\{3 < T < \tau\}}{P_r\{T > 3\}} = \frac{F(\tau) - F(3)}{1 - F(3)}.$$

② $f(\tau \mid \tau > 3) = \frac{d}{d\tau} F(\tau \mid \tau > 3)$

$$= \frac{f(\tau)}{1 - f(3)} = \frac{1/5 \cdot e^{-\tau/5}}{e^{-3/5}}$$

③ $E(\tau \mid \tau > 3) = \int_3^\infty \tau \cdot f(\tau \mid \tau > 3)\, d\tau$

$$= \frac{1}{5 e^{-3/5}} \int_3^\infty \tau\, e^{-\tau/5}\, d\tau$$

$$= \underline{\underline{8 \text{ år}}}$$