Construction of New Codes

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Concatenated codes

- Concatenated coding, devised by Forney in 1966, is a powerful technique for constructing long powerful codes from short component codes.
- Single level concatenation often uses a non-binary code as an outer code and a binary code as an inner code.

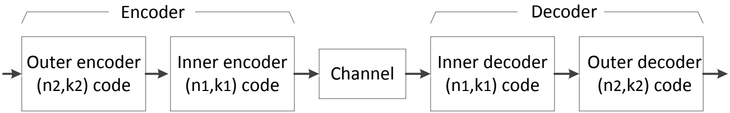

Figure: Communication system using a concatenated code.

Concatenated codes

- The primary reason for using a concatenated code is to achieve higher reliability with less implementation complexity than a single coding operation.
- Concatenated codes are effective against a mixture of random errors and bursts.
- Concatenated codes are widely used in both communication and digital data storage system to achieve higher reliability with reduced decoding complexity.
- Concatenation can also be multilevel, with multiple outer codes concatenated with multiple inner codes.

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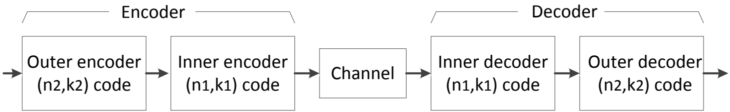
Concatenated codes Encoding



- A simple concatenated code is formed from two codes: an (n_1, k_1) binary code C_1 and an (n_2, k_2) non-binary code C_2 with symbols from $\mathrm{GF}(2^{k_1})$.
- The each symbol of the code vector in C_2 can be represented by k_1 bits.
- $k_1 k_2$ information bits are divided into k_2 components with k_1 bits each.
  1. These k_2 components are encoded based on coding rule of C_2 to an n_2-component codeword.
  2. Each k_1 bits are encoded into an n_1-component codeword in C_1.

Concatenated codes Encoding

- Each block of a simple concatenated code has $n_1 n_2$ bits in total.
- The encoded information is transmitted, one C_1 codeword at a time.
- C_1 and C_2 are called inner code and outer coder.
- If the minimum distance of C_1 and C_2 is d_1 and d_2, respectively, then the minimum distance of the concatenated code is at least $d_1 d_2$.

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Concatenated Codes Decoding

- The concatenated code of C_1 and C_2 is decoded in two steps:
  1. Each C_1 is decoded when it arrives. The check parity bits are removed, leaving n_2 components with k_1-bits each.
  2. The n_2-components codeword is decoded based on decoding method of C_2.

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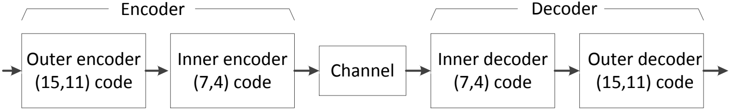
An Example of Concatenated Codes



- The Concatenation of the $(15, 11)$ RS code with symbols from $\mathrm{GF}(2^4)$ and the $(7, 4)$ binary Hamming code.
- Each code symbol of the RS code is represented by a byte of four binary bits. i.e., each 4-bit byte is encoded into $(7, 4)$ Hamming code.
- The resulted concatenated code is a $(105, 44)$ binary code.
- As the minimum distance of Hamming code $(7, 4)$ is 3 and the minimum distance of RS code $(15, 11)$ is 5, the minimum distance of the concatenated code is 15.

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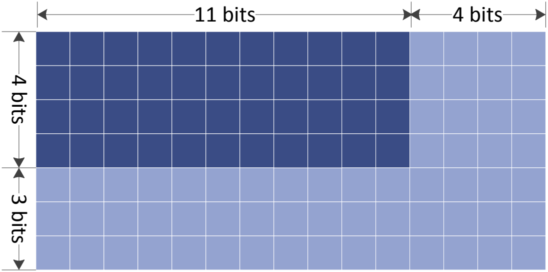

Figure: A concatenated code of $C_{RS}(15, 11)$ and Hamming code $C_b(7, 4)$.

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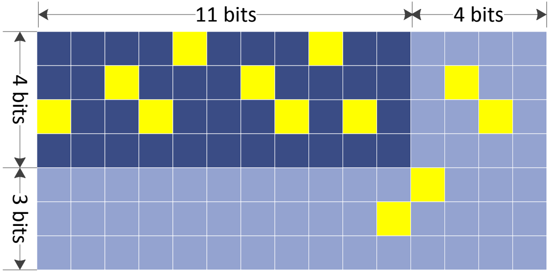

Figure: Errors happen in the concatenated code of $C_{RS}(15, 11)$ and Hamming code $C_b(7, 4)$.

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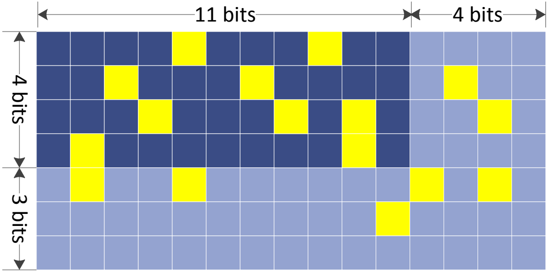

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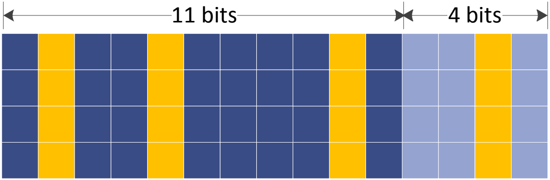

Figure: Using Error detection by Hamming code $C_b(7,4)$ and error correction by $C_{RS}(15,11)$.

Interleaving

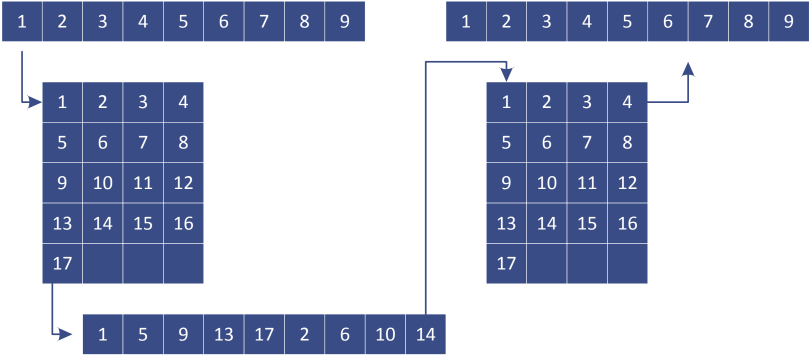

Figure: An illustration of interleaving.

Why interleaving is needed?

- Interleaving can reduce the effect of the burst errors.
- The essence of interleaving is that it randomizes and distributes a burst over many received vectors, thus it reduces the size of the error event per received vector.

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Why interleaving is needed?

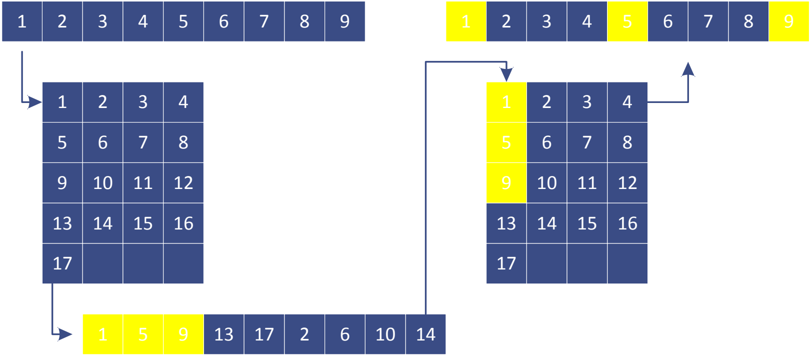

Figure: Interleaving randomizes a burst error.

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Concatenated codes with Interleaving

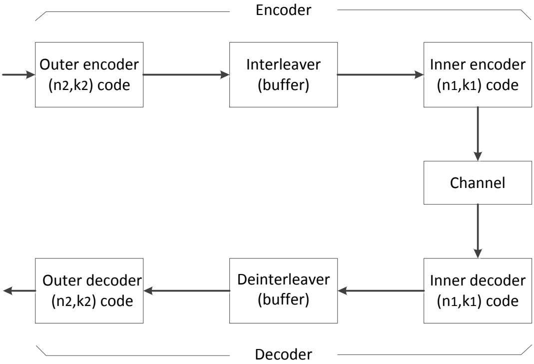

Figure: An interleaved concatenated coding system.

Concatenated codes with Interleaving

- Interleave is often used in concatenated code.
- Let the outer code C_2 be an (n_2, k_2) block code with symbols from GF(2^m). The inner code C_1 be an (n_1, k_1) binary linear code with $k_1 = \lambda m$.
- A message of k_2 m-bit components (i.e., $k_2 m$ bits) is first encoded into an n_2-component codeword in C_2.
- This codeword is temporarily stored in a buffer as a row in a matrix.
- After λ outer codewords have been formed, the buffer stores a matrix of size $\lambda \times n_2$.
- Each column of the matrix has λm bits. These λm bits can be encoded into an n_1-bit codeword in C_1.
- A completed interleaved code matrix has $n_1 n_2$ bits.
- Each encoded column is transmitted serially.
- The outer code is interleaved by a depth of λ.

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Concatenated codes with Interleaving

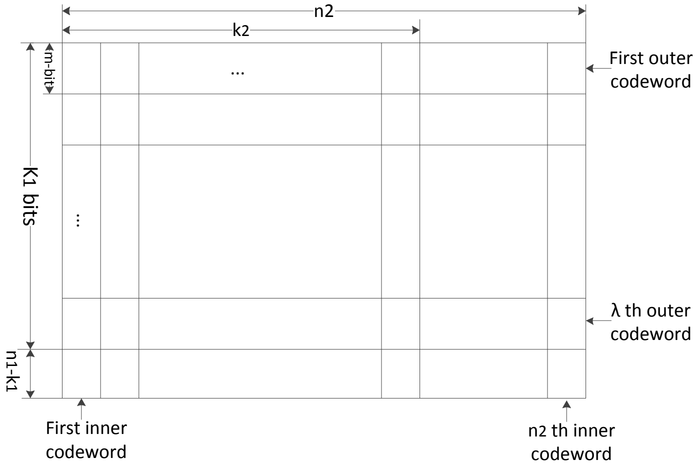

Figure: An interleaved concatenated code matrix.

Concatenated codes with Interleaving

Continuing...

- When each inner codeword is received and decoded, the errors are corrected or detected. The parity check bits are removed.
- The decoded λ bytes are stored in a receiver buffer as a column in a $\lambda \times n_2$ matrix.
- After decoding n_2 inner codes is finished, the receiver buffer contains a $\lambda \times n_2$ matrix.
- Then each row of the $\lambda \times n_2$ matrix can be decoded based on the outer code C_2.
- After λ outer codes have been decoded, the size of the matrix becomes $\lambda m \times k_2 = k_1 \times k_2$

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