TIADPE

Absolute positioning by triangulation and trilateration

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Outline

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Positioning

Fundamental in

- Location based services and mobile robotics

Different methods

1. Absolute positioning:
   With reference to beacons
2. Relative positioning:
   Continuous offset relative to initial absolute position
3. Hybrid positioning:
   Combination of absolute and relative positioning

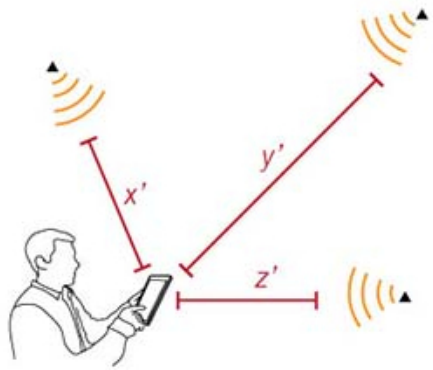
Absolute positioning

Triangulation

- Based on angle measurements relative to beacons
- Determine position and orientation (pose)

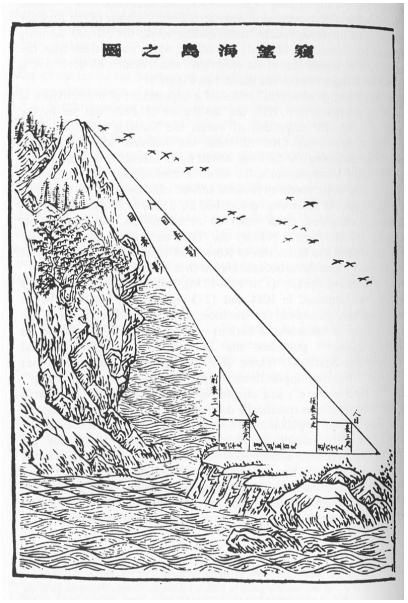
Trilateration

- Based on distance measurements relative to beacons
- Determine position



Outline

Triangulation in the history of surveying



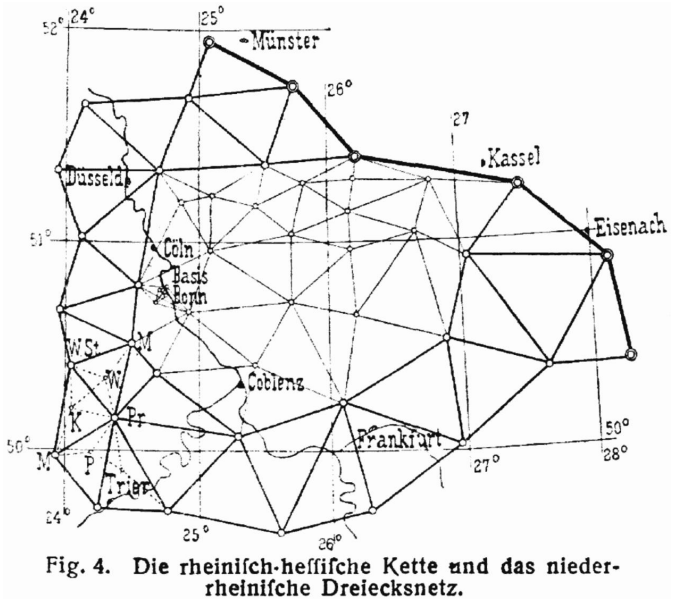


Fig. 4. Die rheinisch-hessische Kette und das niederrheinische Dreiecksnetz.

Figure : Left: 3rd century. Mid: 19th century. Right: 1929. (Wikipedia)

Triangulation methods

Triangulation methods

- Many methods exist each with own pros and cons
- Different approaches to (nearly) same problem

We will review a recent method

- **T**hree **o**bject **T**riangulation **al**gorithm (ToTal)

  (Pierlot and Droogenbroeck, 2014 [1])

- Companion website at: www2.ulg.ac.be/telecom/triangulation/
- (See also: http://www2.ulg.ac.be/telecom/publi/publications/pierlot/Pierlot2011ANewThreeObject/index.html)

Triangulation with three fixed and indexed beacons

ToTal (Pierlot and Droogenbroeck, 2014 [1])

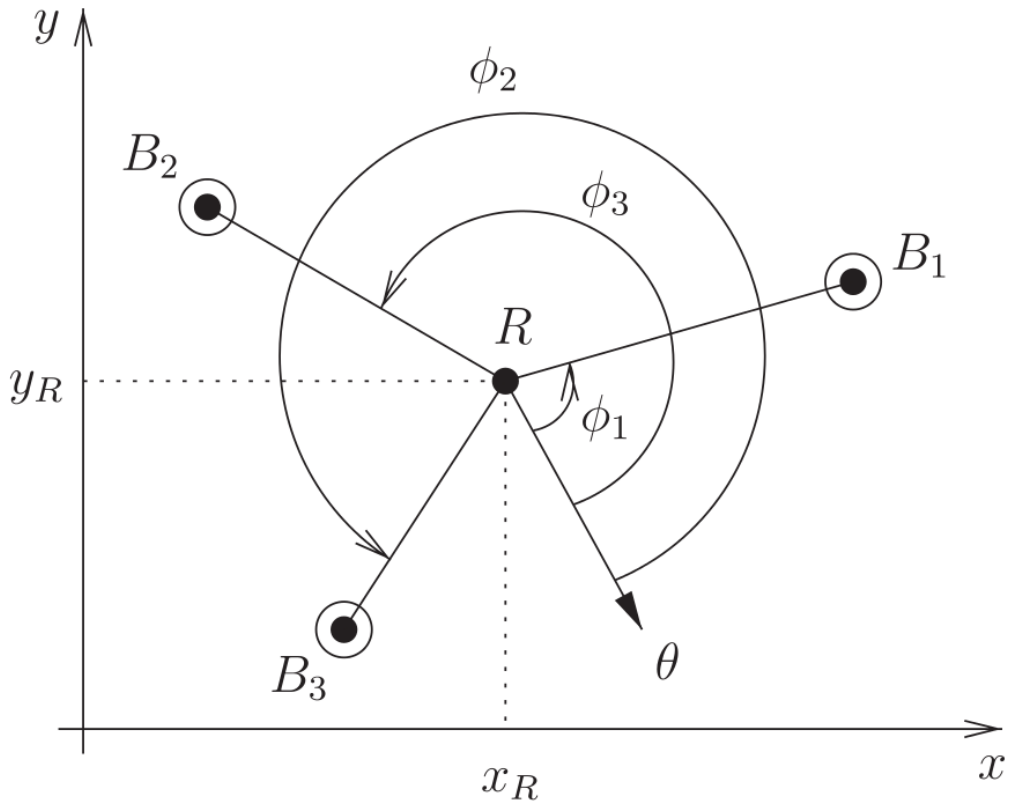


Figure : Find position $(x_R, y_R) \in \mathbb{R}^2$ and orientation $\theta \in \mathbb{R}$ of object R. NOTE: ϕ_2 and ϕ_3 are mixed up.

Two beacons constrains locus to a circle

ToTal (Pierlot and Droogenbroeck, 2014 [1])

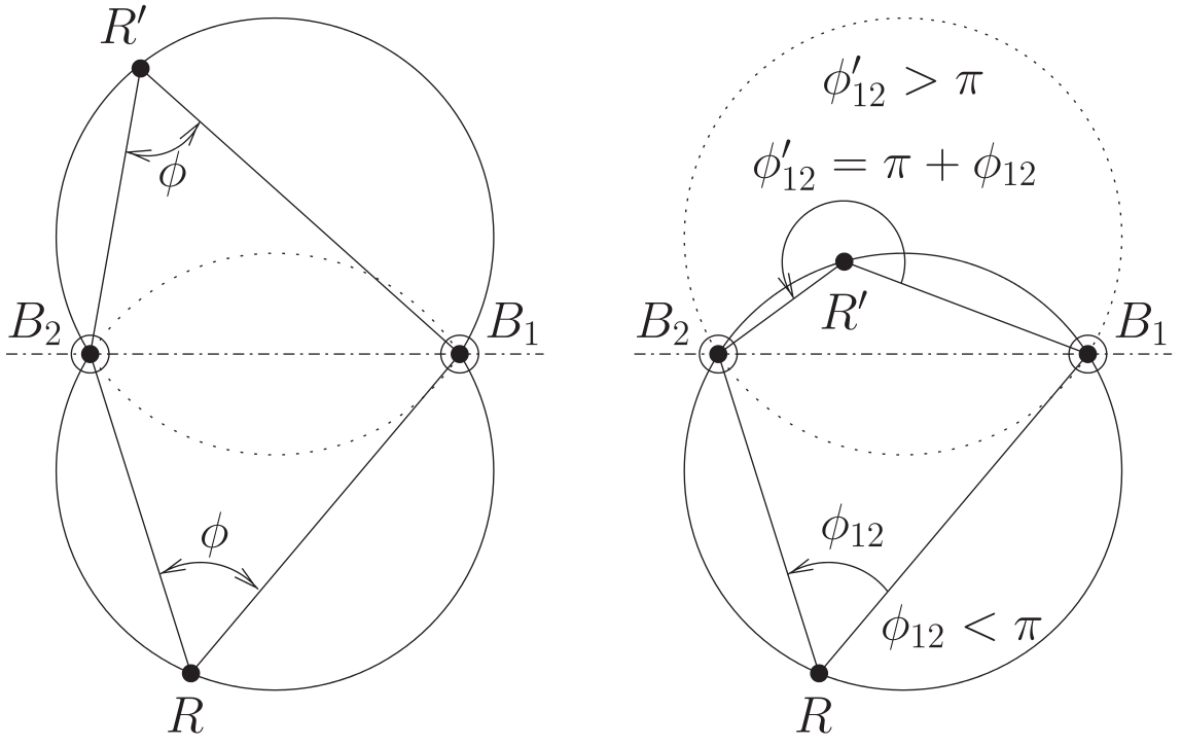


Figure : Left: For const. ϕ the potential positions of R are on arcs of **two** symmetric circles (solid). Right: Remove symmetry ambiguity by defining angles between beacons to be CCW, e.g. $\phi_{12} = \phi_2 - \phi_1$. On the remaining circle: $\phi_{12} < \pi$ on lower part and $\phi_{12} > \pi$ on upper part.

Three beacons constrains locus to a point

ToTal (Pierlot and Droogenbroeck, 2014 [1])

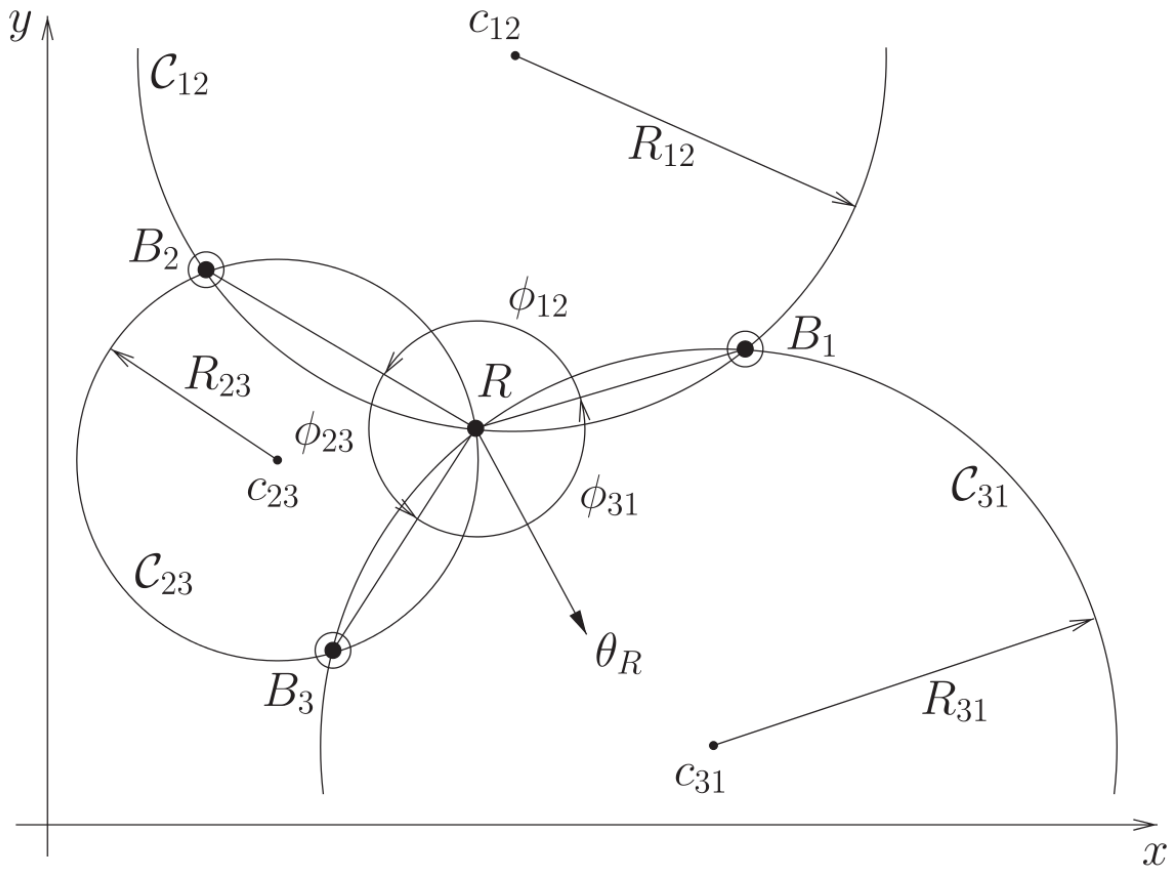


Figure : R lie on the unique intersection between $\mathcal{C}_{12}$, $\mathcal{C}_{23}$, and $\mathcal{C}_{31}$. All angles and the corresponding notation is CCW and relative to R.

Determining the locus point by power centers

ToTal (Pierlot and Droogenbroeck, 2014 [1])

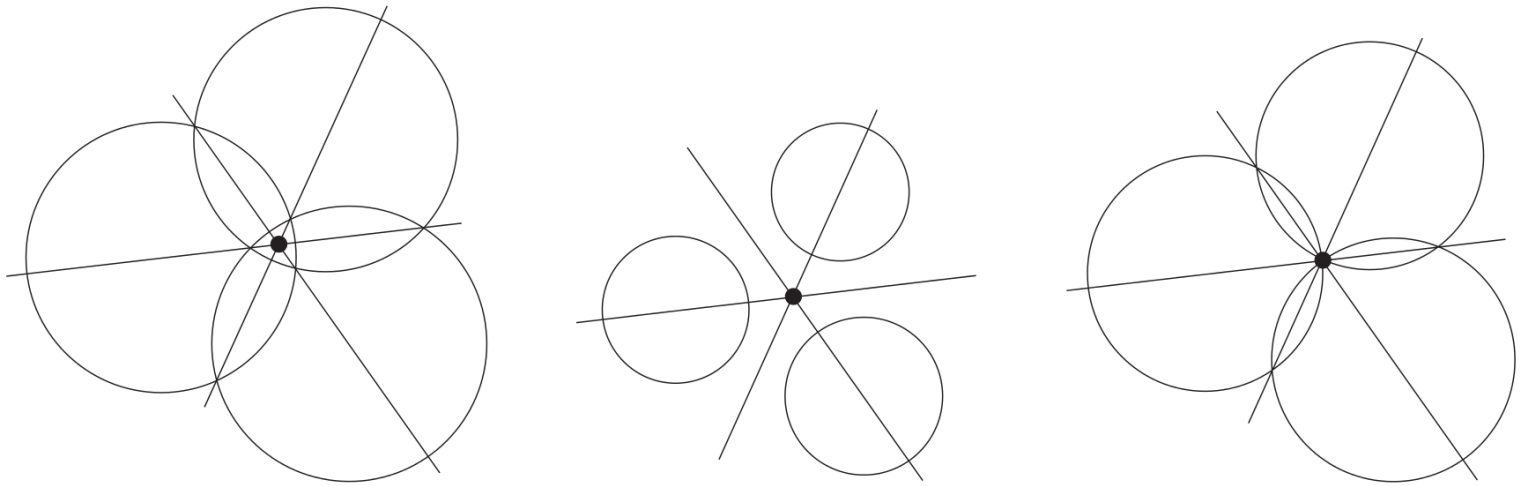
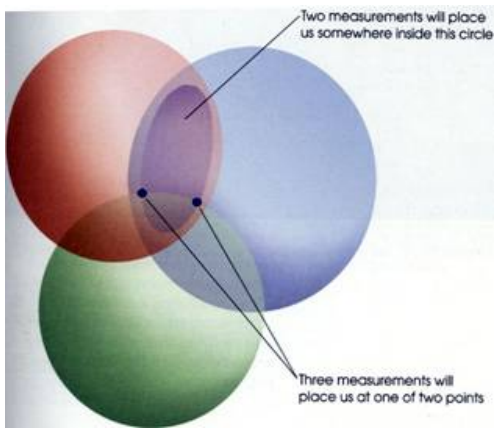


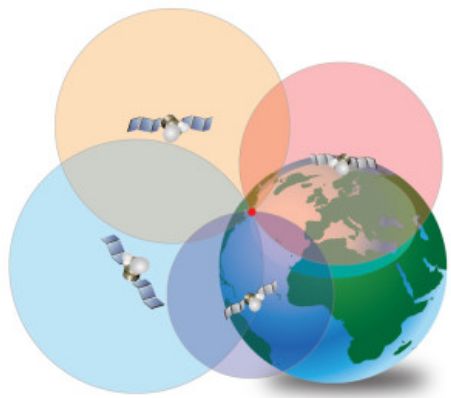
Figure : Power centers for three 3-locus-circle configurations, i.e. points with equal power relative to all three circles. Left: Most interesting situation as power center coincide with unique circle intersection point; hence, the circles' intersection point can be computed as the power center, i.e. intersection point of three power lines. Power of point $p = \{x, y\}$ wrt. to center $\mathcal{C} = \{x_c, y_c\}$ is $\mathcal{P}_{\mathcal{C},p} = (x - x_c)^2 + (y - y_c)^2 - R^2$

Outline

Trilateration overview

- Trilateration is a method to determine the **position** of an object based on simultaneous **distance** measurements from **three** fixed and indexed beacons
- Trilateration is applied in, e.g. surveying and navigation, including global positioning systems (GPS)
- Trilateration can be used to find the position of a device and thereby be an integral part in location based services





Position: Multiple possibilities on locus circle

Distance from one beacon

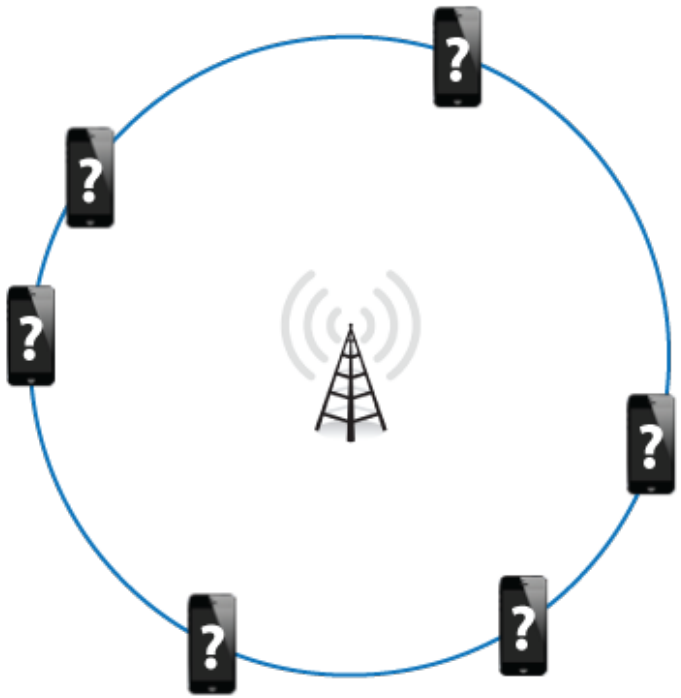


Figure : Device located on the circumference

Position: Two possibilities at intersection points

Distance from two beacons

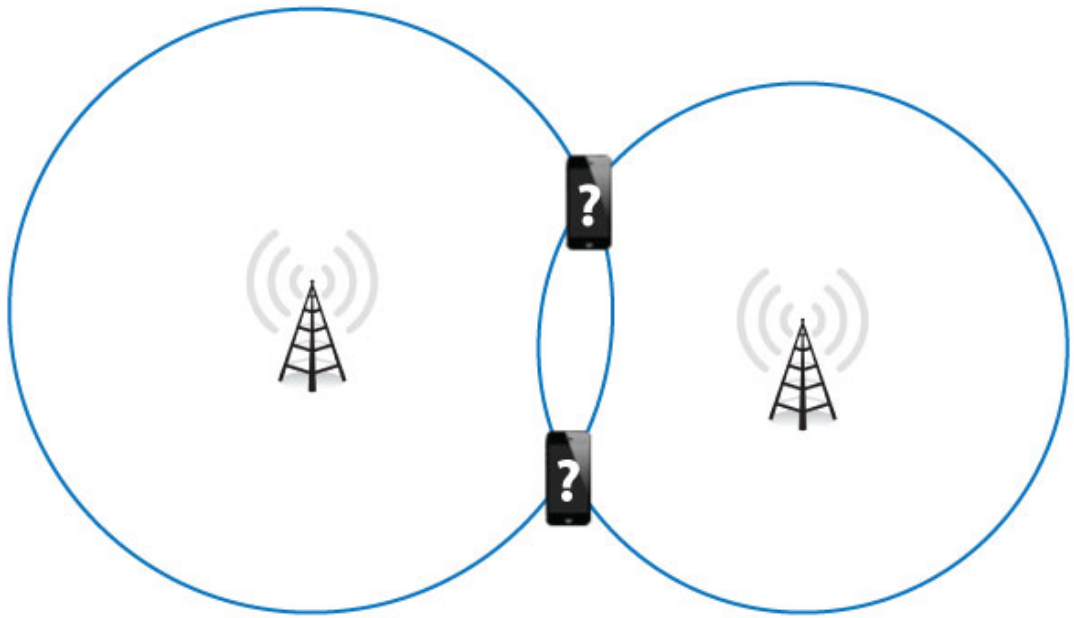


Figure : Device located at one of the two intersection points

Position: Single possibility at intersection point

Distance from three or more beacons

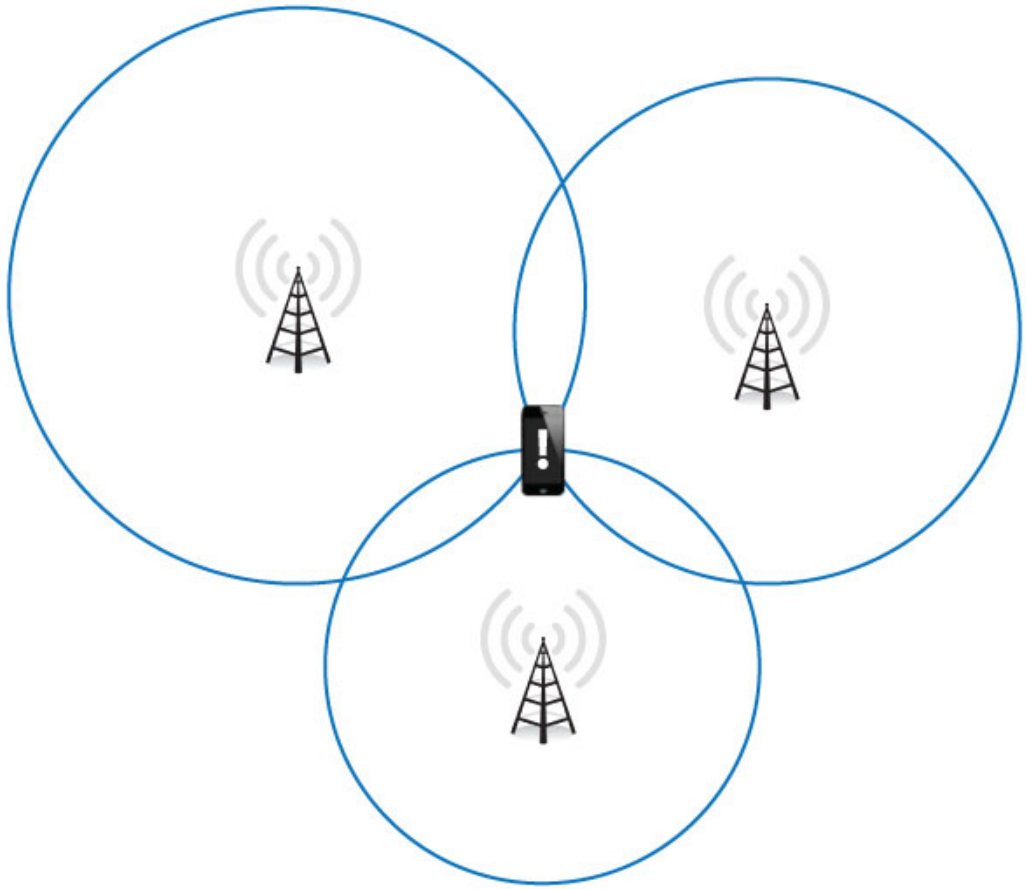


Figure : Device located at the common intersection point

References I

[1] Pierlot, V. and M. V. Droogenbroeck (2014). A new three object triangulation algorithm for mobile robot positioning. *IEEE Transactions on Robotics 30*(3), 566–577.