Error-Control Coding Tutorial

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Introduction

- Shannon developed information theory in 1948.

Shannon proved that

For any given communication channel has a capacity C, as long as the transmission rate R is less than C, then it is possible to design a virtually error-free (or the error probability is arbitrary small) communication system using error control codes.

- Before Shannon, people thought noise prevented error free communication,
- Shannon's theorem showed that the channel noise limited the transmission rate, not the error probability.
- Shannon's contribution proved the existence of such codes, but he did not tell us how to find them.
- Error control coding is a discipline under the branch of applied mathematics called information theory.

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Introduction

- In 1960s, the researchers who were looking for the proper codes split into two groups:
  - The algebraists concentrated on a class of codes called *block codes*
  - The probabilists discovered the other class of codes called *convolutional codes*.
- In 1970s, these two research paths merged, efficient decoding algorithm were developed.

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An illustration of the digital communication system

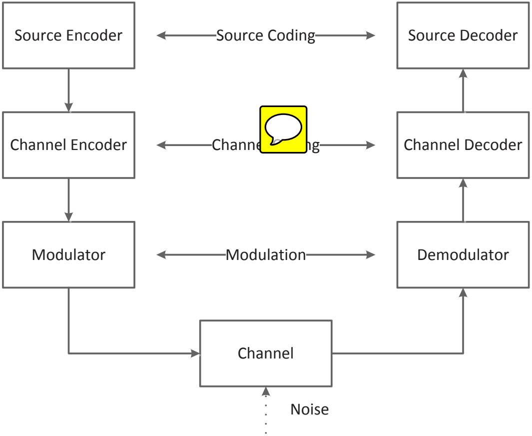

Figure: The digital communication system

- Source Encoder-Decoder pair: designate to reduce the source output to a Minimal Representation, i.e., data compression.
- Channel Encoder-Decoder:
  - Channel encoder adds redundant bits to the source bit stream to create channel codeword.
  - Channel decoder uses the redundant bits to detect and/or correct as many errors as the particular error control code is capable.
- Communication channel introduces errors. The channel can be
  - e.g., radio, twisted wire pair, coaxial cable, fiber optic cable, magnetic tape, optical discs, or any other noisy medium.
- Error control code: A set of codewords used with an encoder and decoder to detect errors, correct errors, or both detect and correct errors.

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Random Errors

- Memoryless channel: The noise affects each transmitted symbol independently, as such as BSC (Binary symmetric channel).
  - Transmission errors occur randomly in the received sequence.
  - The memoryless channels are called *random-error channels*.
  - e.g., many satellite channels are random error channels, and most line-of-sight transmission facilities are primarily affected by random errors.
  - Codes designed to correct random errors are called *random-error correcting codes*.

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Channel with Memory

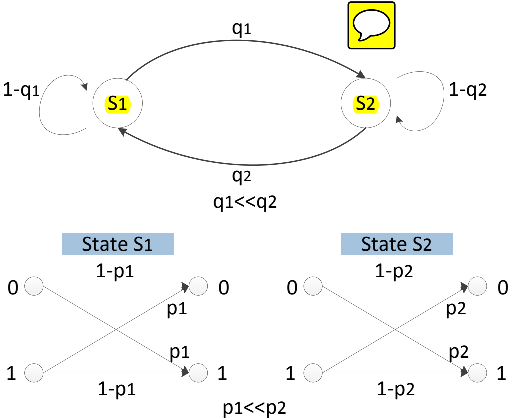

Figure: Illustration of channel with memory

Burst Errors

- Channel with memory: The noise is not independent from transmission to transmission.
  - Channel has 'good state' and 'bad state'. The channel is in 'good state' most of time but occasionally switched to 'bad state'.
    - In 'good state', transmission error occurs seldom.
    - In 'bad state', transmission error are highly probable.
    - e.g., deep-fading caused by multipath transmission,
  - For example, mobile telephony channel is a burst-error channel, error bursts caused by signal fading due to multipath transmission.
  - Codes designed to correct burst errors are called *burst-error correcting codes*.
- Some channel contain a combination of both random and burst errors, are called *compound channels.*

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Performance Measurement

- To measure the system performance regarding errors, there defines:
  - *Bit-Error-Rate* (*BER*): The probability of bit error. (typically $< 10^{-4}$)
    - It is useful indicator of system performance on an independent error channel,
    - It has little meaning on bursty, or dependent error channel.
  - *Message-Error-Rate* (*MER*): The probability of message error.
    - Usually like operator wants the message error-free and does not care of BER.
  - *Undetected Message Error Rate* (*UMER*): The probability that error detection decoder fails. (often required $< 10^{-16}$)
    - Such event occurs when the error pattern introduced by the channel such that the transmitted codeword is converted to another valid codeword.
    - So the decoder cannot tell the difference and regards the codword as error free.

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Energy Per Bit

- *Energy per bit* (E_b): The amount of energy contained in one bit.
    - It is very important as almost all channel impairments can be overcome by increasing the energy per bit.
    - Unit of Energy per bit is *Joules* per bit. E_b can expressed by

$$E_b = P_{tx} T_b = \frac{P_{tx}}{R}$$

where P_{tx} is the transmission power (in Watts), T_b is the time to transmit one bit, R is the bit rate (in bit per second)

    - If transmission power is fixed, the E_b can be increased by lowering the bit rate.
    - Lowering the bit rate increases the robustness.

Coding Gain

- *Coding Gain*: The difference (in dB) in the required signal to noise ratio to maintain reliable communications after coding is employed.

$$G(dB) = \left(\frac{E_b}{N_0}\right)_u (dB) - \left(\frac{E_b}{N_0}\right)_c (dB)$$

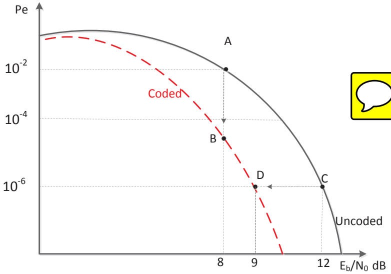

Figure: Coded vs. uncoded error performance Comparison

Code Rate

- *Code Rate*: Assuming an encoder takes k information bits and add r redundant bits (also called parity check bits), for total of $n = k + r$ bits per codeword, the code rate is k/n.
  - This code is called a (n, k) error control code.
  - The added parity check bits are overhead, so system designer prefer a code having higher coding gain with fewer parity bits.

k bits	$n - k$ bits

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What error control coding can do

- Reduce the cost of communication systems.
  - e.g. transmitter power is expensive specially in satellite transponders. coding can reduce the satellite power
- Extend the wireless network coverage and overcome deep fading.
  - e.g. in WiMAX network, the far away CPEs or CPEs suffers deep fading, BSs can choose low coding rate to keep the reliable transmission.
- In some circumstance, it can prolong the network life of wireless sensor networks (WSN).
  - e.g. in WSN, To save energy to prolong the network life is a critial topic in the battery driven wireless sensors. The channel characteristics determines whether to FEC or ARQ.
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What Error Control Coding Cannot Do

- In the Gaussian noise channel, Shannon's capacity formula sets a lower limit on the signal to noise ratio to maintain a reliable communication.
- Shannon's lower limit depends on whether the channel is *power limited* or *bandwidth limited*.
  - The deep space channel is an example of a power limited channel because bandwidth is an abundant resource compared to transmission power.
  - Telephone channels are considered bandwidth-limited because telephony operator adheres to a strict channel bandwidth.
- For a strictly power-limited channel, Shannon's limit is about -1.6 dB.
  - To ensure reliable communications E_b/N_0 must be at least -1.6 dB, no matter how powerful an error control code is.

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How Error Control Coding Works

k bits	$n - k$ bits

- Assuming in a binary block code, there are k bits information bits. There are 2^k uncoded source message. Each message maps to one of the codewords in the codebook.
- $n - k$ bits are redundant bits.
- With n-bits of a encoded codeword, there are 2^n codes in the codebook.
- When the channel introduces errors, a valid codeword can be changed to any one of the 2^n n-bit words.
- The job of decoder is to find a valid codeword that is closet to the received n-bit word.

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