SIA for Model 3 using the Taylor series approach

ODE system

$$X'[t] = \frac{r_g * X[t]}{(1+\mathrm{Exp}[a*(\mathrm{Inhib}[t]-b)])}$$

$$\mathrm{Inhib}'[t] = \frac{r_g * X[t]}{(1+\mathrm{Exp}[a*(\mathrm{Inhib}[t]-b)])} - r_d * \mathrm{Inhib}[t]$$

Initial conditions: X[0]=4.45, Inhib[0]=0

Only X[t] is observed and all parameters are assumed to be unknown

First four derivatives for X[t] for when t = 0

$$X'[0] = \frac{4.45\, r_g}{1+e^{-a\,b}}$$

$$X''[0] = -\frac{19.8025\, a\, e^{-a\,b}\, r_g^2}{\left(1+e^{-a\,b}\right)^3} + \frac{4.45\, r_g^2}{\left(1+e^{-a\,b}\right)^2}$$

$$X'''[0] = \frac{176.242\, a^2\, e^{-2\,a\,b}\, r_g^3}{\left(1+e^{-a\,b}\right)^5} - \frac{59.4075\, a\, e^{-a\,b}\, r_g^3}{\left(1+e^{-a\,b}\right)^4} - \frac{88.1211\, a^2\, e^{-a\,b}\, r_g^3}{\left(1+e^{-a\,b}\right)^4} + \frac{4.45\, r_g^3}{\left(1+e^{-a\,b}\right)^3} - \frac{4.45\, a\, e^{-a\,b}\, r_g \left(-\frac{4.45\, r_d\, r_g}{1+e^{-a\,b}} - \frac{19.8025\, a\, e^{-a\,b}\, r_g^2}{\left(1+e^{-a\,b}\right)^3} + \frac{4.45\, r_g^2}{\left(1+e^{-a\,b}\right)^2}\right)}{\left(1+e^{-a\,b}\right)^2}$$

$$\begin{aligned}
X''''[0] = &-\frac{2352.83\, a^3\, e^{-3\,a\,b}\, r_g^4}{\left(1+e^{-a\,b}\right)^7} + \frac{969.332\, a^2\, e^{-2\,a\,b}\, r_g^4}{\left(1+e^{-a\,b}\right)^6} + \frac{2352.83\, a^3\, e^{-2\,a\,b}\, r_g^4}{\left(1+e^{-a\,b}\right)^6} - \\
&\frac{118.815\, a\, e^{-a\,b}\, r_g^4}{\left(1+e^{-a\,b}\right)^5} - \frac{352.485\, a^2\, e^{-a\,b}\, r_g^4}{\left(1+e^{-a\,b}\right)^5} - \frac{392.139\, a^3\, e^{-a\,b}\, r_g^4}{\left(1+e^{-a\,b}\right)^5} + \\
&\frac{4.45\, r_g^4}{\left(1+e^{-a\,b}\right)^4} + \frac{118.815\, a^2\, e^{-2\,a\,b}\, r_g^2 \left(-\frac{4.45\, r_d\, r_g}{1+e^{-a\,b}} - \frac{19.8025\, a\, e^{-a\,b}\, r_g^2}{\left(1+e^{-a\,b}\right)^3} + \frac{4.45\, r_g^2}{\left(1+e^{-a\,b}\right)^2}\right)}{\left(1+e^{-a\,b}\right)^4} - \\
&\frac{17.8\, a\, e^{-a\,b}\, r_g^2 \left(-\frac{4.45\, r_d\, r_g}{1+e^{-a\,b}} - \frac{19.8025\, a\, e^{-a\,b}\, r_g^2}{\left(1+e^{-a\,b}\right)^3} + \frac{4.45\, r_g^2}{\left(1+e^{-a\,b}\right)^2}\right)}{\left(1+e^{-a\,b}\right)^3} - \frac{59.4075\, a^2\, e^{-a\,b}\, r_g^2 \left(-\frac{4.45\, r_d\, r_g}{1+e^{-a\,b}} - \frac{19.8025\, a\, e^{-a\,b}\, r_g^2}{\left(1+e^{-a\,b}\right)^3} + \frac{4.45\, r_g^2}{\left(1+e^{-a\,b}\right)^2}\right)}{\left(1+e^{-a\,b}\right)^3} - \\
&\frac{1}{\left(1+e^{-a\,b}\right)^2} 4.45\, a\, e^{-a\,b}\, r_g \left(\frac{176.242\, a^2\, e^{-2\,a\,b}\, r_g^3}{\left(1+e^{-a\,b}\right)^5} - \frac{59.4075\, a\, e^{-a\,b}\, r_g^3}{\left(1+e^{-a\,b}\right)^4} - \right. \\
&\frac{88.1211\, a^2\, e^{-a\,b}\, r_g^3}{\left(1+e^{-a\,b}\right)^4} + \frac{4.45\, r_g^3}{\left(1+e^{-a\,b}\right)^3} - r_d \left(-\frac{4.45\, r_d\, r_g}{1+e^{-a\,b}} - \frac{19.8025\, a\, e^{-a\,b}\, r_g^2}{\left(1+e^{-a\,b}\right)^3} + \frac{4.45\, r_g^2}{\left(1+e^{-a\,b}\right)^2}\right) - \\
&\left. \frac{4.45\, a\, e^{-a\,b}\, r_g \left(-\frac{4.45\, r_d\, r_g}{1+e^{-a\,b}} - \frac{19.8025\, a\, e^{-a\,b}\, r_g^2}{\left(1+e^{-a\,b}\right)^3} + \frac{4.45\, r_g^2}{\left(1+e^{-a\,b}\right)^2}\right)}{\left(1+e^{-a\,b}\right)^2} \right)
\end{aligned}$$

Analysis of Taylor series coefficients

1. Use X’[0] to solve for the parameter combination a*b:

a*b-> $\text{Log}\left[-\frac{20.\ X'[0]}{20.\ X'[0]-89.\ r_g}\right]$

2. Substitute new solution for a*b into X’’[0] and solve for r_g:

$$r_g \to -\frac{224719.\ a\ X'[0]^3}{224719.\ X'[0]^2-1.\times10^6\ a\ X'[0]^2-1.\times10^6\ X''[0]}$$

3. Substitute new solutions for a*b and r_g into X’’’[0] and solve for r_d:

$$r_d \to \left(4.49438\times10^6\ a\ X'[0]^4 - 9.\ a^2\ X'[0]^4 + 8.98876\times10^6\ X'[0]^2\ X''[0] - 2.\times10^7\ a\ X'[0]^2\ X''[0] - 6.\times10^7\ X''[0]^2 + 2.\times10^7\ X'[0]\times X'''[0]\right) \Big/ \left(4.49438\times10^6\ X'[0]^3 - 9.\ a\ X'[0]^3 - 2.\times10^7\ X'[0]\times X''[0]\right)$$

4. Substitute all new solutions into X’’’’[0] which leaves a as the only unknown parameter and thus a is identifiable

3. Since a, is identifiable, r_d can now be identified as well as r_g

4. Since both a and a*b are now identifiable, b can now be classed as identifiable and thus all parameters have been identified and Model 3 is structurally identifiable