Ch 1 to Ch 4

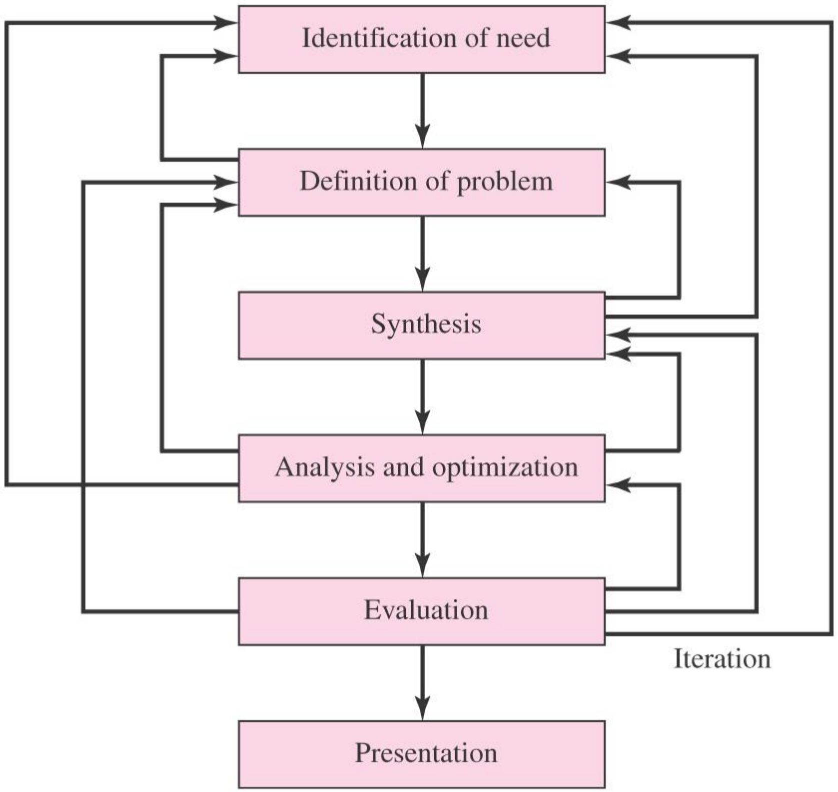
Introduction, Materials, Stress Analysis, Deflection and Stiffness

Design

- To formulate a plan for the satisfaction of a specified need
- Process requires innovation, iteration, and decision-making
- Communication-intensive
- Products should be
  - Functional
  - Safe
  - Reliable
  - Competitive
  - Usable
  - Manufacturable
  - Marketable

The Design Process

- Iterative in nature
- Requires initial estimation, followed by continued refinement
- Presentation is selling job.



Computational Tools

- Computer-Aided Engineering (CAE)
  - Any use of the computer and software to aid in the engineering process
  - Includes
    - Computer-Aided Design (CAD)
      - Drafting, 3-D solid modeling, etc.
    - Computer-Aided Manufacturing (CAM)
      - CNC toolpath, rapid prototyping, etc.
    - Engineering analysis and simulation
      - Finite element, fluid flow, dynamic analysis, motion, etc.
    - Math solvers
      - Spreadsheet, procedural programming language, equation solver, etc.

The Design Engineer's Professional Responsibilities

- Satisfy the needs of the customer in a competent, responsible, ethical, and professional manner.
- Some key advise for a professional engineer
  - Be competent
  - Keep current in field of practice
  - Keep good documentation
  - Ensure good and timely communication
  - Act professionally and ethically

Ethical Guidelines for Professional Practice

- National Society of Professional Engineers (NSPE) publishes a Code of Ethics for Engineers and an Engineers' Creed.
- www.nspe.org/ethics
- Six Fundamental Canons
- Engineers, in the fulfillment of their professional duties, shall:
  - Hold paramount the safety, health, and welfare of the public.
  - Perform services only in areas of their competence.
  - Issue public statements only in an objective and truthful manner.
  - Act for each employer or client as faithful agents or trustees.
  - Avoid deceptive acts.
  - Conduct themselves honorably, responsibly, ethically, and lawfully so as to enhance the honor, reputation, and usefulness of the profession.

Standards and Codes

- Standard
  - A set of specifications for parts, materials, or processes
  - Intended to achieve uniformity, efficiency, and a specified quality
  - Limits the multitude of variations
- Code
  - A set of specifications for the analysis, design, manufacture, and construction of something
  - To achieve a specified degree of safety, efficiency, and performance or quality
  - Does not imply absolute safety
- Various organizations establish and publish standards and codes for common and/or critical industries

Standards and Codes

Some organizations that establish standards and codes of particular interest to mechanical engineers:

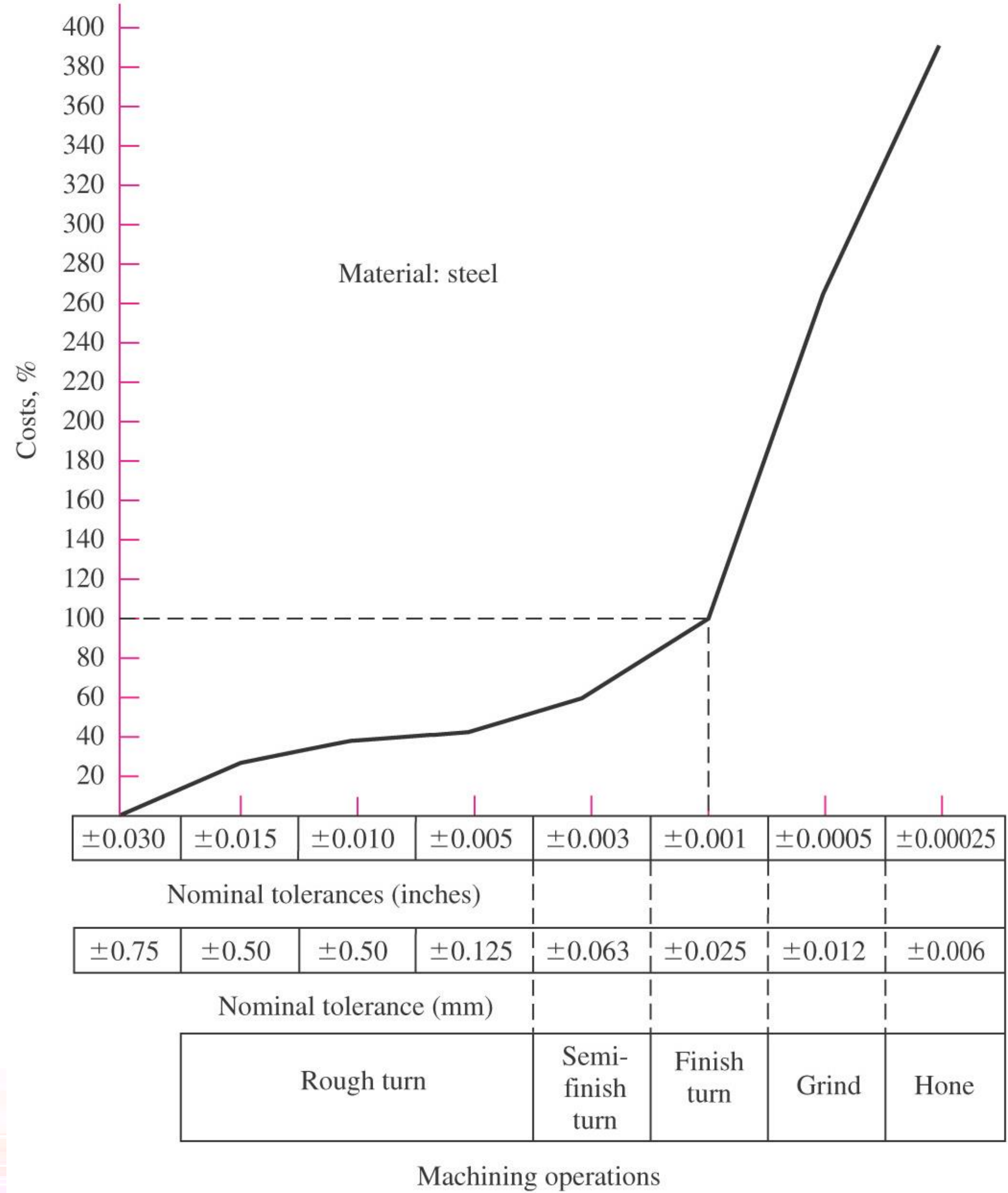
Aluminum Association (AA)
American Bearing Manufacturers Association (ABMA)
American Gear Manufacturers Association (AGMA)
American Institute of Steel Construction (AISC)
American Iron and Steel Institute (AISI)
American National Standards Institute (ANSI)
American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE)
American Society of Mechanical Engineers (ASME)
American Society of Testing and Materials (ASTM)
American Welding Society (AWS)
ASM International
British Standards Institution (BSI)
Industrial Fasteners Institute (IFI)
Institute of Transportation Engineers (ITE)
Institution of Mechanical Engineers (IMechE)
International Bureau of Weights and Measures (BIPM)
International Federation of Robotics (IFR)
International Standards Organization (ISO)
National Association of Power Engineers (NAPE)
National Institute for Standards and Technology (NIST)
Society of Automotive Engineers (SAE)

Economics

- Cost is almost always an important factor in engineering design.
- Use of standard sizes is a first principle of cost reduction.
- Table A-17 lists some typical preferred sizes.
- Certain common components may be less expensive in stocked sizes.

Tolerances

- Close tolerances generally increase cost
  - Require additional processing steps
  - Require additional inspection
  - Require machines with lower production rates

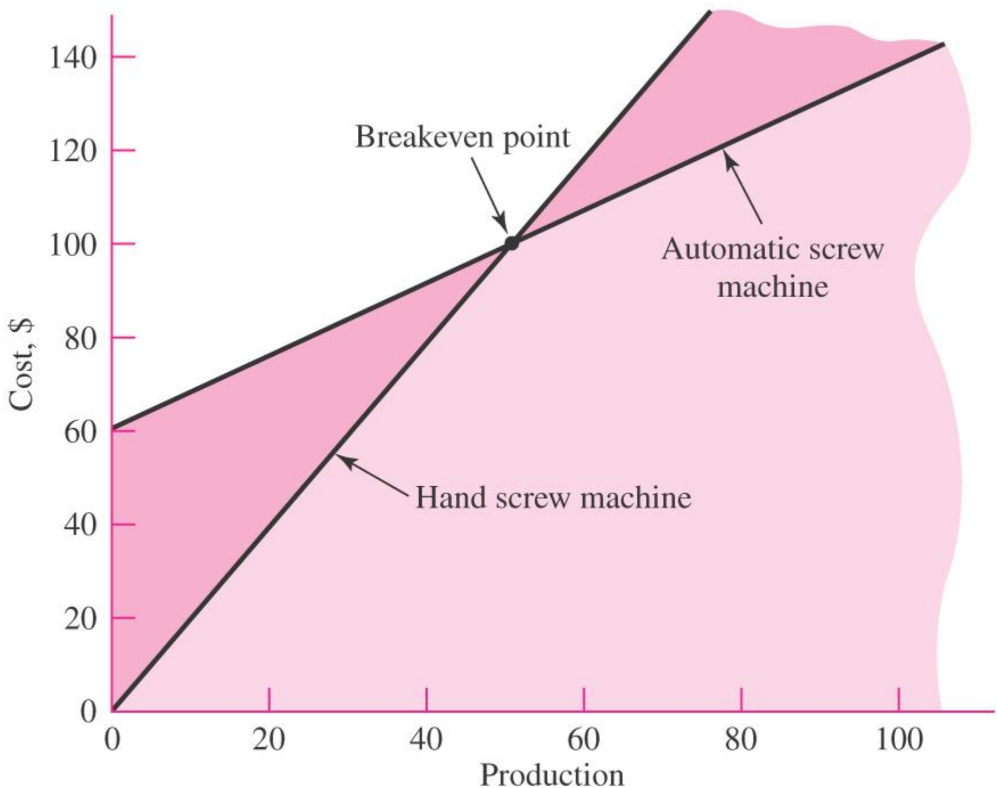


Breakeven Points

- A cost comparison between two possible production methods
- Often there is a breakeven point on quantity of production

EXAMPLE

- Automatic screw machine
  - 25 parts/hr
  - 3 hr setup
  - $20/hr labor cost
- Hand screw machine
  - 10 parts/hr
  - Minimal setup
  - $20/hr labor cost
- Breakeven at 50 units



Stress and Strength

- Stress
  - A state property at a specific point within a body
  - Primarily a function of load and geometry
  - Sometimes also a function of temperature and processing
  - σ and τ to denote normal stress and shear stress
- Strength
  - An inherent property of a material or of a mechanical element
  - Depends on treatment and processing
  - May or may not be uniform throughout the part
  - Examples: Ultimate strength, yield strength
  - *S* to denote strength, ex. S_y, S_u, S_e, S_{sy}

Uncertainty

- Common sources of uncertainty in stress or strength
  - Composition of material and the effect of variation on properties.
  - Variations in properties from place to place within a bar of stock.
  - Effect of processing locally, or nearby, on properties.
  - Effect of nearby assemblies such as weldments and shrink fits on stress conditions.
  - Effect of thermomechanical treatment on properties.
  - Intensity and distribution of loading.
  - Validity of mathematical models used to represent reality.
  - Intensity of stress concentrations.
  - Influence of time on strength and geometry.
  - Effect of corrosion.
  - Effect of wear.
  - Uncertainty as to the length of any list of uncertainties.

Uncertainty

- Stochastic method
  - Based on statistical nature of the design parameters
  - Focus on the probability of survival of the design's function (reliability)
  - Often limited by availability of statistical data
- Deterministic method
  - Establishes a *design factor*, n_d
  - Based on absolute uncertainties of a *loss-of-function parameter* and a *maximum allowable parameter*

$$n_d = \frac{\text{loss-of-function parameter}}{\text{maximum allowable parameter}} \qquad (1\text{–}1)$$

  - If, for example, the parameter is load, then

$$\text{Maximum allowable load} = \frac{\text{loss-of-function load}}{n_d} \qquad (1\text{–}2)$$

Reliability

- *Reliability*, *R* – The statistical measure of the probability that a mechanical element will not fail in use
- *Probability of Failure*, p_f – the number of instances of failures per total number of possible instances

$$R = 1 - p_f \qquad (1\text{–}4)$$

- Example: If 1000 parts are manufactured, with 6 of the parts failing, the reliability is

$$R = 1 - \frac{6}{1000} = 0.994 \qquad \text{or } 99.4\ \%$$

Reliability

- *Series System* – a system that is deemed to have failed if any component within the system fails
- The overall reliability of a series system is the product of the reliabilities of the individual components.

$$R = \prod_{i=1}^{n} R_i \qquad (1\text{-}5)$$

- Example: A shaft with two bearings having reliabilities of 95% and 98% has an overall reliability of

$$R = R_1\, R_2 \;= 0.95\,(0.98) = 0.93 \qquad \text{or } 93\%$$

Units

英制

- foot-pound-second (fps)

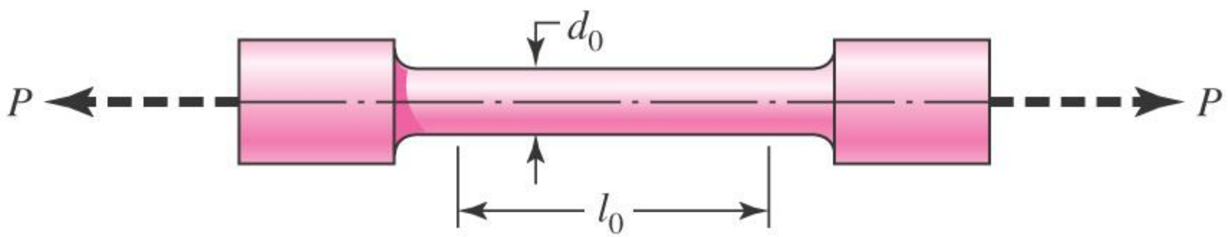
$$g = 32.1740 \frac{ft}{s^2}$$

- in-pound-second (ips)

$$g = 386 \frac{in}{s^2}$$

- Unit of force: lb
  - more precisely, lbf (pound-force)
  - 1000 lbf = kip

Standard Tensile Test



- Used to obtain material characteristics and strengths
- Loaded in tension with slowly increasing *P*
- Load and deflection are recorded

Stress and Strain

The *stress* is calculated from

$$\sigma = \frac{P}{A_0} \tag{2–1}$$

where $A_0 = \frac{1}{4}\pi d_0^2$ is the original cross-sectional area.

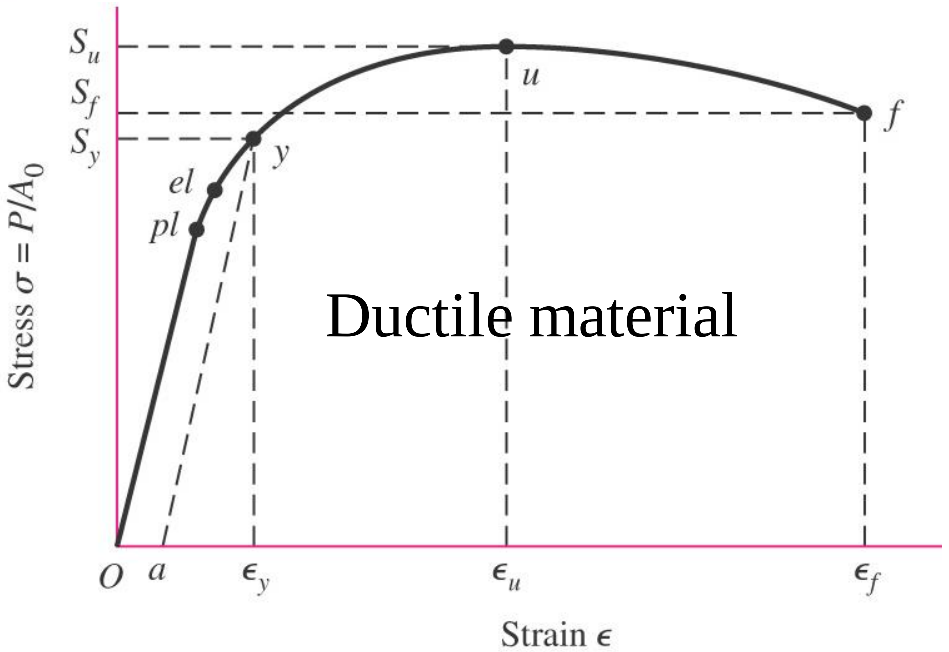
The *normal strain* is calculated from

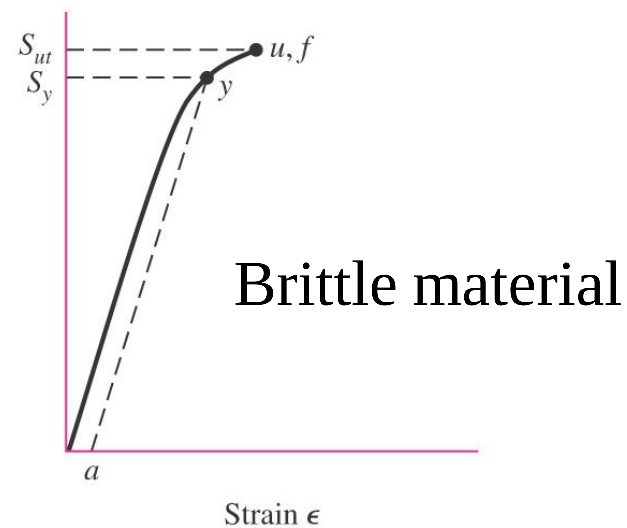
$$\epsilon = \frac{l - l_0}{l_0} \tag{2–2}$$

where l_0 is the original gauge length and l is the current length corresponding to the current P.

Stress-Strain Diagram

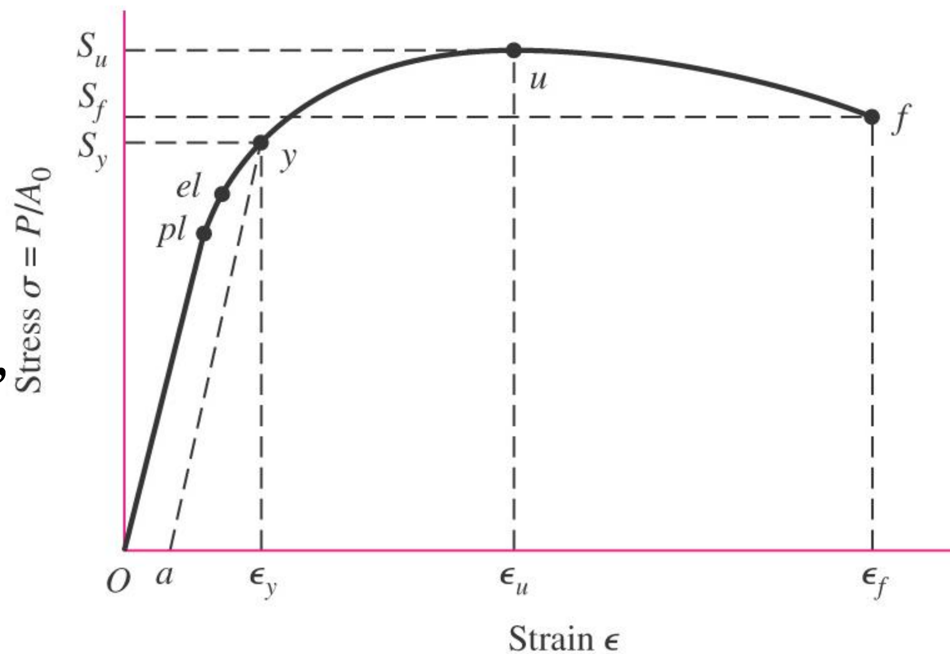
- Plot stress vs. normal strain
- Typically linear relation until the *proportional limit, pl*
- No permanent deformation until the *elastic limit, el*
- *Yield strength*, S_y , defined at point where significant plastic deformation begins, or where permanent set reaches a fixed amount, usually 0.2% of the original gauge length
- *Ultimate strength or tensile strength*, S_u , defined as the maximum stress on the diagram



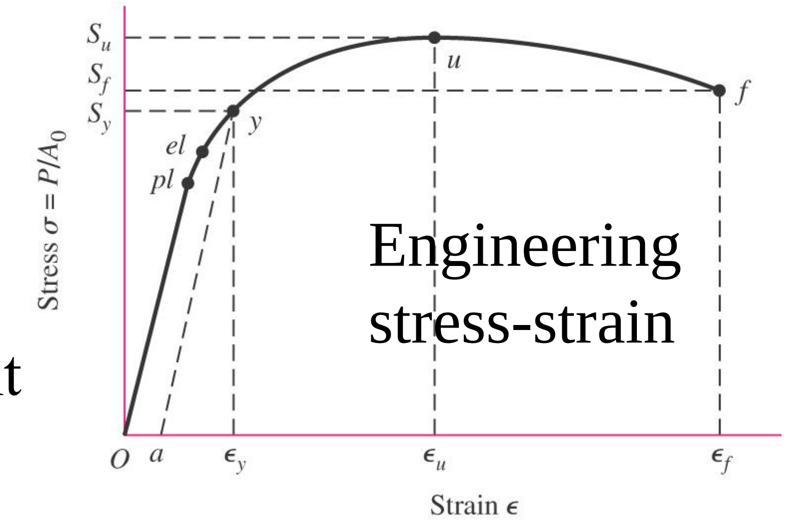


Elastic Relationship of Stress and Strain

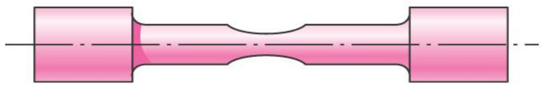
- Slope of linear section is *Young's Modulus*, or *modulus of elasticity*, *E*
- *Hooke's law* $\sigma = E\epsilon$
- *E* is relatively constant for a given type of material (e.g. steel, copper, aluminum)
- See Table A-5 for typical values
- Usually independent of heat treatment, carbon content, or alloying



True Stress-Strain Diagram



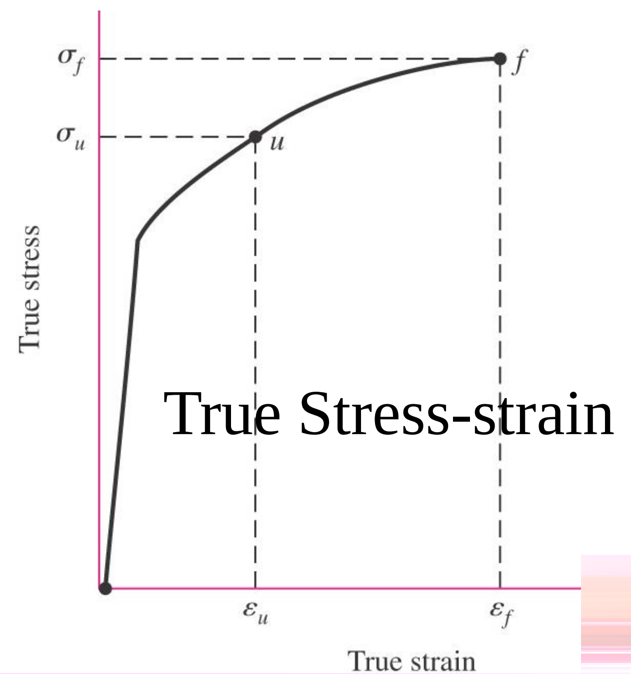
- *Engineering* stress-strain diagrams (commonly used) are based on original area.
- Area typically reduces under load, particularly during "necking" after point *u*.



- *True stress* is based on actual area corresponding to current *P*.
- *True strain* is the sum of the incremental elongations divided by the *current* gauge length at load *P*.

$$\varepsilon = \int_{l_0}^{l} \frac{dl}{l} = \ln \frac{l}{l_0}$$

- Note that true stress continually increases all the way to fracture.



Compression Strength

- Compression tests are used to obtain compressive strengths.
- Buckling and bulging can be problematic.
- For ductile materials, compressive strengths are usually about the same as tensile strengths, $S_{uc} = S_{ut}$.
- For brittle materials, compressive strengths, S_{uc} , are often greater than tensile strengths, S_{ut} .

Torsional Strengths

- Torsional strengths are found by twisting solid circular bars.
- Results are plotted as a *torque-twist diagram*.
- Shear stresses in the specimen are linear with respect to the radial location – zero at the center and maximum at the outer radius.
- Maximum shear stress is related to the angle of twist by

$$\tau_{\max} = \frac{Gr}{l_0}\theta \qquad (2\text{–}5)$$

  - θ is the angle of twist (in radians)
  - r is the radius of the bar
  - l_0 is the gauge length
  - G is the material stiffness property called the *shear modulus* or *modulus of rigidity*.

Torsional Strengths

- Maximum shear stress is related to the applied torque by

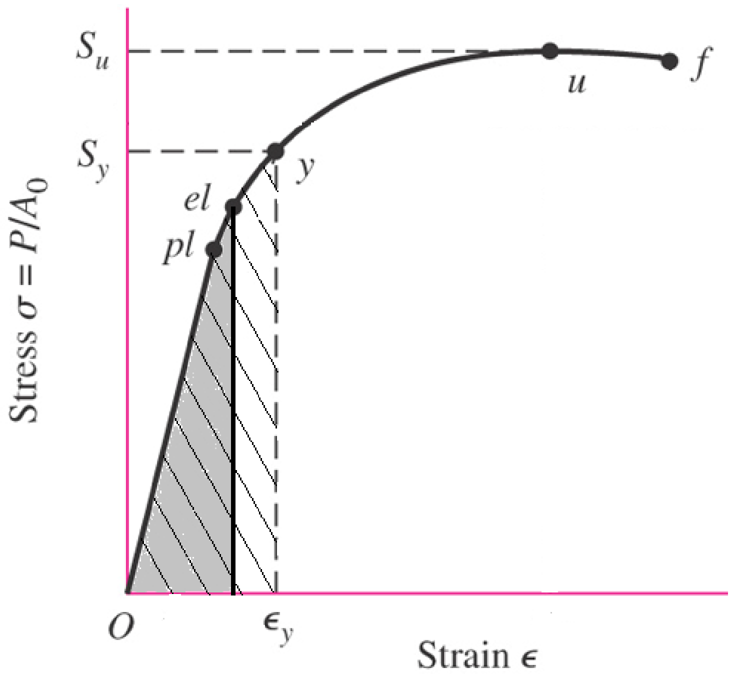
$$\tau_{\max} = \frac{Tr}{J} \qquad (2\text{–}6)$$

  - J is the polar second moment of area of the cross section
  - For round cross section, $J = \frac{1}{2}\pi r^4$
- *Torsional yield strength*, S_{sy} corresponds to the maximum shear stress at the point where the torque-twist diagram becomes significantly non-linear
- *Modulus of rupture*, S_{su} corresponds to the torque T_u at the maximum point on the torque-twist diagram

$$S_{su} = \frac{T_u r}{J} \qquad (2\text{–}7)$$

Resilience

- *Resilience* – Capacity of a material to absorb energy within its elastic range
- *Modulus of resilience*, u_R
  - Energy absorbed per unit volume without permanent deformation
  - Equals the area under the stress-strain curve up to the elastic limit
  - Elastic limit often approximated by yield point



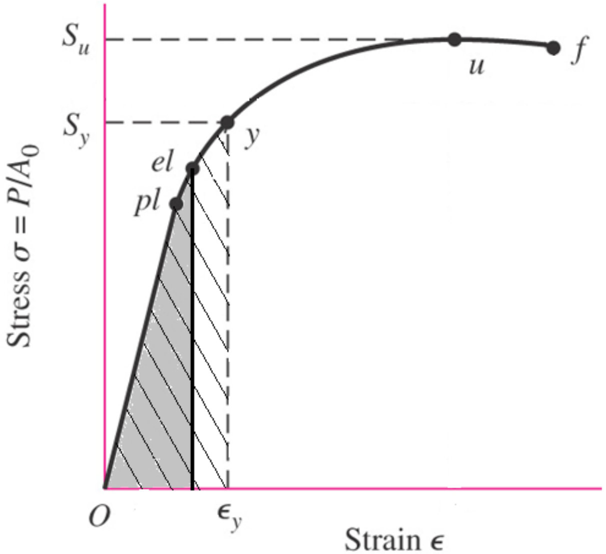
Resilience

- Area under curve to yield point gives approximation

$$u_R \cong \int_0^{\epsilon_y} \sigma \, d\epsilon$$

- If elastic region is linear,

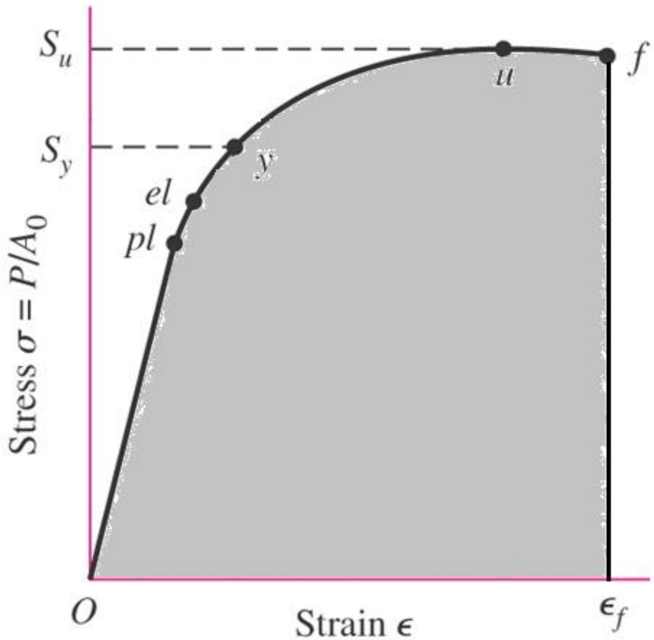
$$u_R \cong \frac{1}{2} S_y \epsilon_y = \frac{1}{2}(S_y)(S_y/E) = \frac{S_y^2}{2E}$$



- For two materials with the same yield strength, the less stiff material (lower E) has greater resilience

Toughness

- *Toughness* – capacity of a material to absorb energy without fracture
- *Modulus of toughness*, u_T
  - Energy absorbed per unit volume without fracture
  - Equals area under the stress-strain curve up to the fracture point



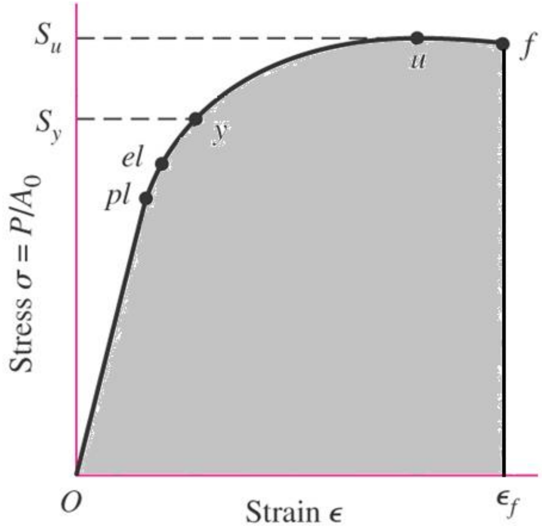
Toughness

- Area under curve up to fracture point

$$u_T = \int_0^{\epsilon_f} \sigma \, d\epsilon \qquad (2\text{–}10)$$

- Often estimated graphically from stress-strain data
- Approximated by using the average of yield and ultimate strengths and the strain at fracture

$$u_T \cong \left(\frac{S_y + S_{ut}}{2}\right)\epsilon_f \qquad (2\text{–}11)$$



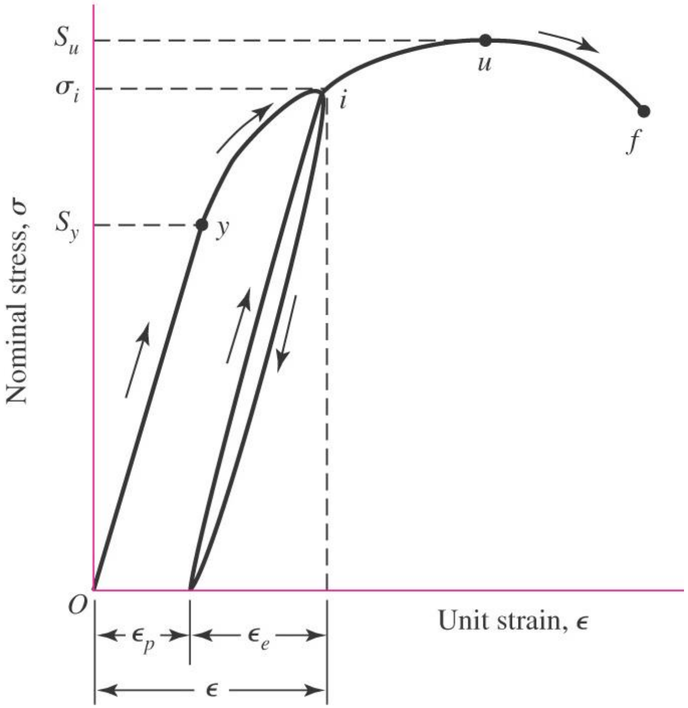
Shigley's Mechanical Engineering Design

Resilience and Toughness

- Measures of energy absorbing characteristics of a material
- Units are energy per unit volume
  - $\text{lbf}\cdot\text{in/in}^3$ or J/m^3
- Assumes low strain rates
- For higher strain rates, use impact methods (See Sec. 2-5)

Cold Work or Strain Hardened

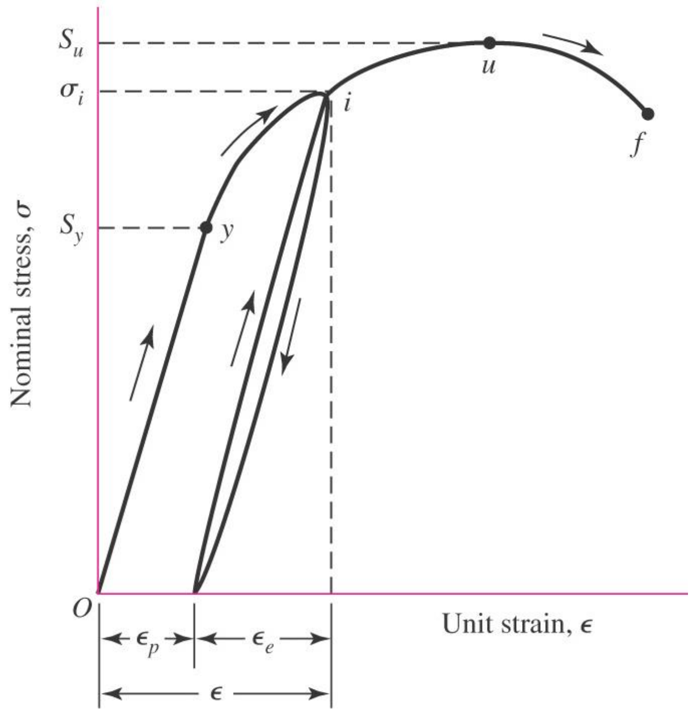
- *Cold work* – Process of plastic straining below recrystallization temperature in the plastic region of the stress-strain diagram
- Loading to point *i* beyond the yield point, then unloading, causes permanent plastic deformation, ϵ_p
- Reloading to point *i* behaves elastically all the way to *i*, with additional elastic strain ϵ_e



$$\epsilon = \epsilon_p + \epsilon_e \qquad \epsilon_e = \frac{\sigma_i}{E}$$

Cold Work

- The yield point is effectively increased to point *i*
- Material is said to have been *cold worked*, or *strain hardened*
- Material is less ductile (more brittle) since the plastic zone between yield strength and ultimate strength is reduced
- Repeated strain hardening can lead to brittle failure



Temperature Effects on Strengths

- Plot of strength vs. temperature for carbon and alloy steels
- As temperature increases above room temperature
  - S_{ut} increase slightly, then decreases significantly
  - S_y decreases continuously
  - Results in increased ductility

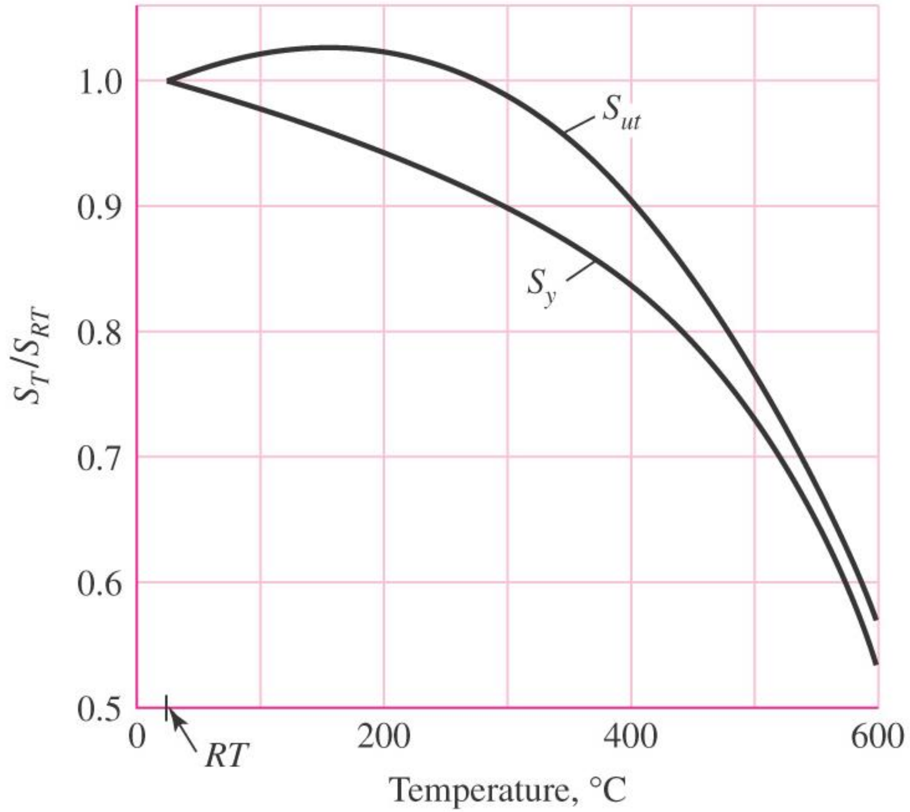


Fig. 2–9

Creep

- *Creep* – a continuous deformation under load for long periods of time at elevated temperatures
- Often exhibits three stages
  - 1st stage: elastic and plastic deformation; decreasing creep rate due to strain hardening
  - 2nd stage: constant minimum creep rate caused by the annealing effect
  - 3rd stage: considerable reduction in area; increased true stress; higher creep rate leading to fracture

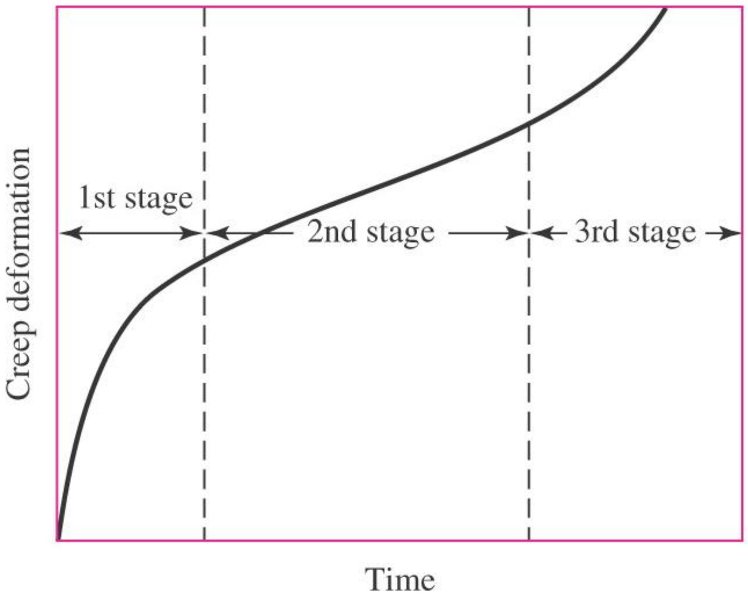


Fig. 2–10

Material Numbering Systems

- Common numbering systems
  - Society of Automotive Engineers (SAE)
  - American Iron and Steel Institute (AISI)
  - Unified Numbering System (UNS)
  - American Society for Testing and Materials (ASTM) for cast irons

UNS Numbering System

- UNS system established by SAE in 1975
- Letter prefix followed by 5 digit number
- Letter prefix designates material class
  - G – carbon and alloy steel
  - A – Aluminum alloy
  - C – Copper-based alloy
  - S – Stainless or corrosion-resistant steel

UNS for Steels

- For steel, letter prefix is G
- First two numbers indicate composition, excluding carbon content

G10	Plain carbon	G46	Nickel-molybdenum
G11	Free-cutting carbon steel with more sulfur or phosphorus	G48	Nickel-molybdenum
G13	Manganese	G50	Chromium
G23	Nickel	G51	Chromium
G25	Nickel	G52	Chromium
G31	Nickel-chromium	G61	Chromium-vanadium
G33	Nickel-chromium	G86	Chromium-nickel-molybdenum
G40	Molybdenum	G87	Chromium-nickel-molybdenum
G41	Chromium-molybdenum	G92	Manganese-silicon
G43	Nickel-chromium-molybdenum	G94	Nickel-chromium-molybdenum

- Second pair of numbers indicates carbon content in hundredths of a percent by weight
- Fifth number is used for special situations
- Example: G52986 is chromium alloy with 0.98% carbon

UNS for Aluminum Group

- For aluminum group, letter prefix is A
- The first number indicate the processing
  - A0: casting alloy
- The second number indicate the main alloy group
- The third number is used to modify the original alloy or to designate the impurity limit
- The last two numbers refer to other alloys used with the basic group

Table 2-1

Aluminum Alloy Designations

Aluminum 99.00% pure and greater	Ax1xxx
Copper alloys	Ax2xxx
Manganese alloys	Ax3xxx
Silicon alloys	Ax4xxx
Magnesium alloys	Ax5xxx
Magnesium-silicon alloys	Ax6xxx
Zinc alloys	Ax7xxx

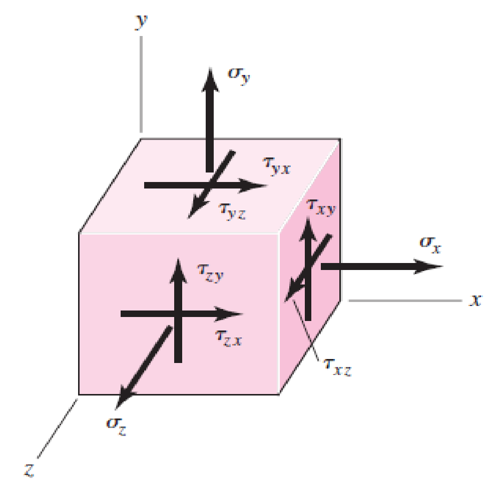
Load and Stress Analysis

Stress

- *Normal stress* is normal to a surface, designated by σ
- *Tangential shear stress* is tangent to a surface, designated by τ
- Normal stress acting outward on surface is *tensile stress*
- Normal stress acting inward on surface is *compressive stress*
- U.S. Customary units of stress are pounds per square inch (psi)
- SI units of stress are newtons per square meter (N/m^2)
- 1 N/m^2 = 1 pascal (Pa)

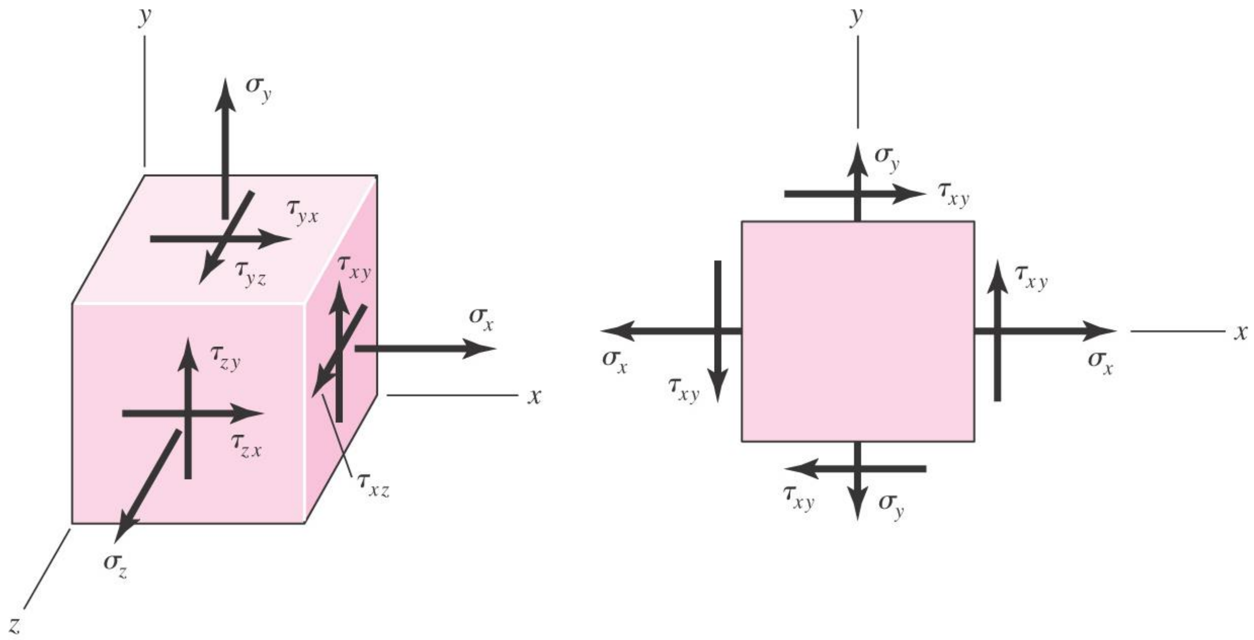
Stress Components

- Rectangular coordinate
- 6 faces→12 shear components and 6 normal components
- Based on force balance, forces acting on opposite facets must have the same magnitude and point to opposite direct
- 6 faces→6 shear components and 3 normal components
- Based on torque balance, two shear components acting on two facets but pointing towards a common edge have the same magnitude $\tau_{yx} = \tau_{xy}, \tau_{xz} = \tau_{zx}, \tau_{zy} = \tau_{yz}$
    - Only 3 independent shear components
- Total, only 6 independent stress components: $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{zx}, \tau_{yz}$

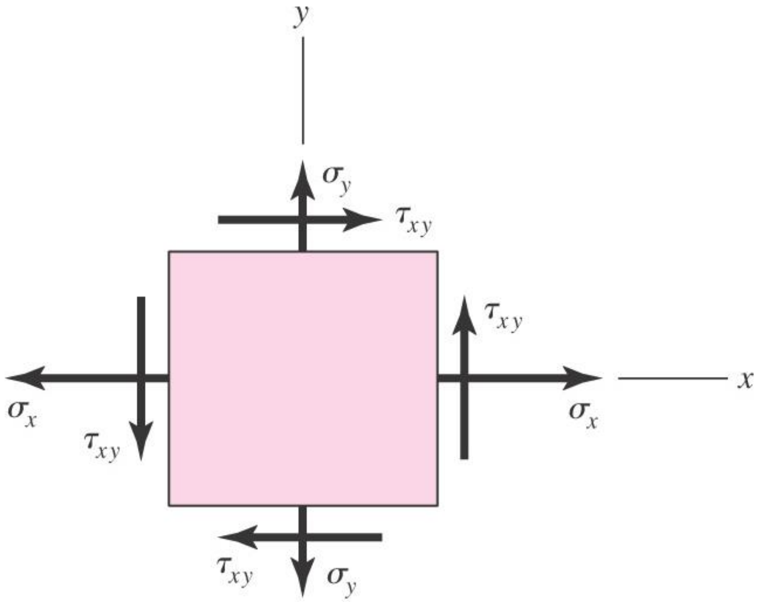


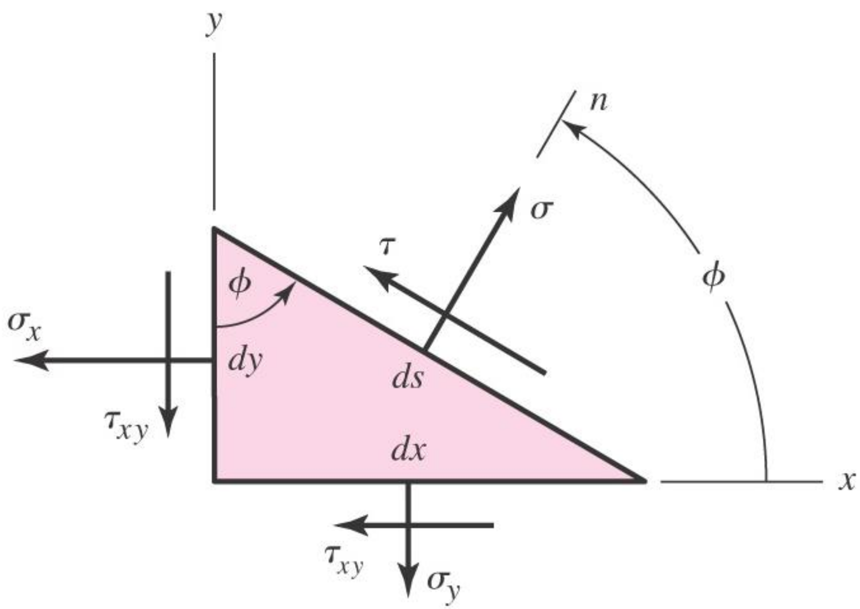
Plane Stress

- *Plane stress* occurs when stresses on one surface are zero
- Assume no stress in z direction $\sigma_z = \tau_{zx} = \tau_{zy} = 0$



Plane-Stress Transformation Equations





- Cutting plane stress element at an arbitrary angle and balancing stresses gives *plane-stress transformation equations*

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\phi + \tau_{xy} \sin 2\phi \qquad (3\text{–}8)$$

$$\tau = -\frac{\sigma_x - \sigma_y}{2} \sin 2\phi + \tau_{xy} \cos 2\phi \qquad (3\text{–}9)$$

Principal Stresses for Plane Stress

- Differentiating Eq. (3-8) with respect to ϕ and setting equal to zero maximizes σ and gives

$$\tan 2\phi_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad (3\text{–}10)$$

- The two values of $2\phi_p$ are the *principal directions.*
- The stresses in the principal directions are the *principal stresses.*
- The principal direction surfaces have zero shear stresses.
- Substituting Eq. (3-10) into Eq. (3-8) gives expression for the non-zero principal stresses.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (3\text{–}13)$$

- Note that there is a third principal stress, equal to zero for plane stress.

Extreme-value Shear Stresses for Plane Stress

- Performing similar procedure with shear stress in Eq. (3-9),
  - Differentiate Eq. 3-9 with respect to ϕ and set to 0

$$\tan 2\phi_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

- The two extreme-value shear stresses are

$$\tau_1, \tau_2 = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- the maximum shear stresses are found to be on surfaces that are $\pm 45^{\circ}$ from the principal directions.

$$\phi_s = \phi_p \pm 45^{\circ}$$

- Surface containing the maximum shear stresses also contain equal normal stresses

$$\sigma = \frac{\sigma_x + \sigma_y}{2}$$

Mohr's Circle for Plane Stress

- A graphical method for visualizing the stress state at a point
- Represents relation between x-y stresses and principal stresses
- Parametric relationship between σ and τ (with 2ϕ as parameter)

Construction of Mohr's circle

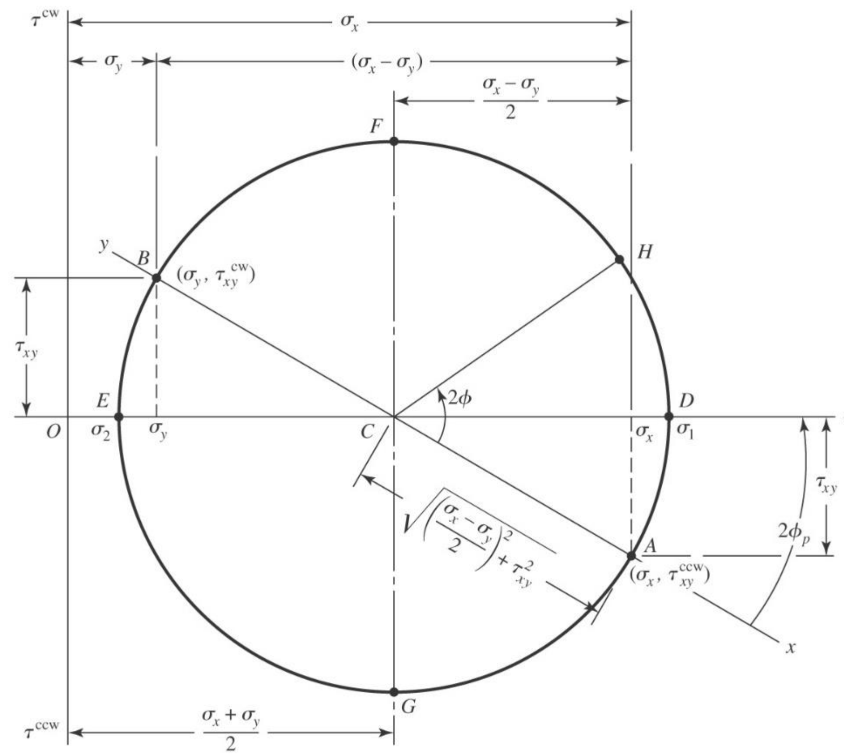
- x, y axis are σ, τ
- The center of the Mohr's circle,

$$C = \left(\frac{\sigma_x + \sigma_y}{2}, 0 \right)$$

- 找出原來stress state 的點 $A = \left(\sigma_x, \tau_{xy} \right)$
- 以CA為半徑做圓

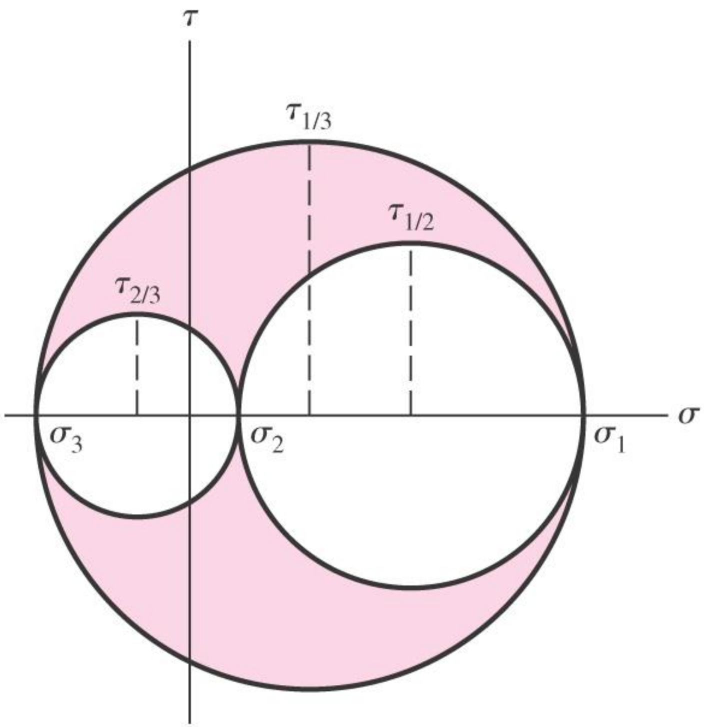
$$R = \sqrt{\left(\sigma_x - \frac{\sigma_x + \sigma_y}{2} \right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

- 旋轉θ的stress state
  - 從CA以相同的方向旋轉2θ，得到在圓周上的H點
- 可從圖上得出 $\sigma_x^{'}, \tau_{xy}^{'}$
- 可從圖上得出principal stresses 和 max shear stress



General Three-Dimensional Stress

- All stress elements are actually 3-D.
- Plane stress elements simply have one surface with zero stresses.
- For cases where there is no stress-free surface, the principal stresses are found from the roots of the cubic equation



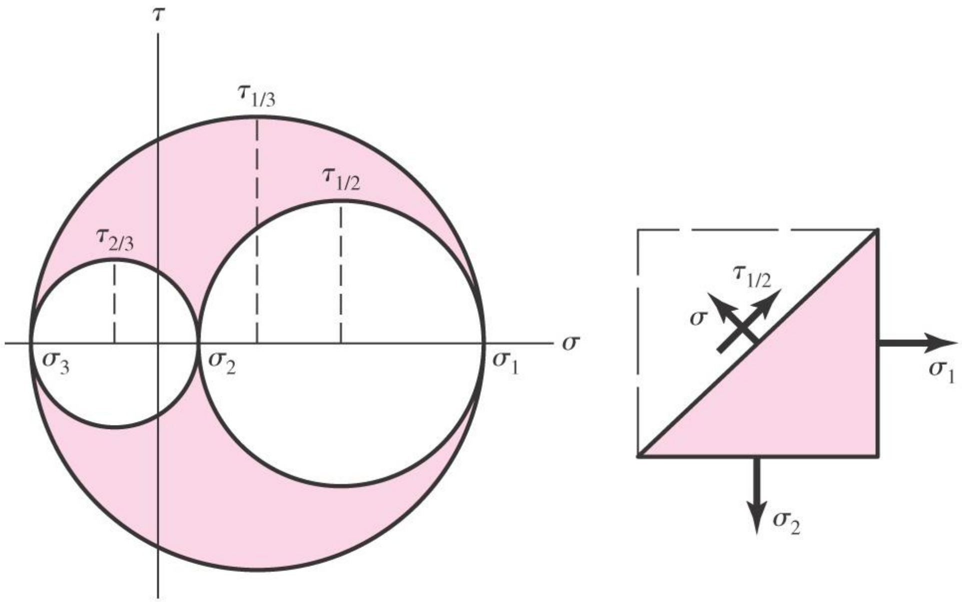
$$\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + (\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2)\sigma$$
$$- (\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2) = 0 \qquad (3\text{–}15)$$

General Three-Dimensional Stress

- Always three extreme shear values

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2} \qquad \tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2} \qquad \tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2} \qquad (3\text{–}16)$$

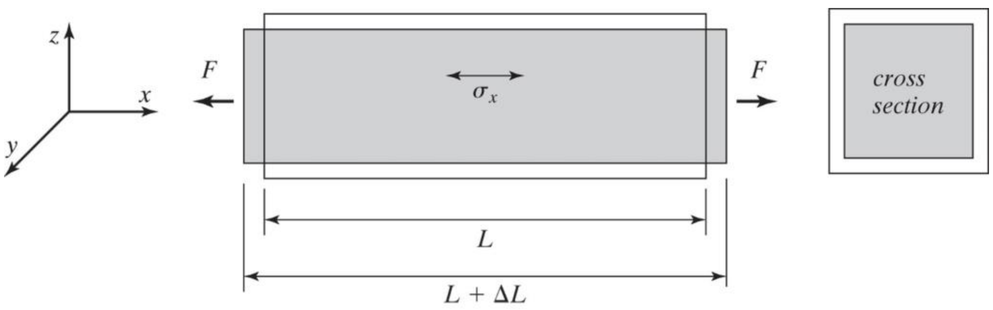
- *Maximum Shear Stress* is the largest
- Principal stresses are usually ordered such that $\sigma_1 > \sigma_2 > \sigma_3$, in which case $\tau_{max} = \tau_{1/3}$



Elastic Strain

- *Hooke's law* $\sigma = E\epsilon$
- E is Young's modulus, or modulus of elasticity
- Tension in on direction produces negative strain (contraction) in a perpendicular direction.
  - Poisson's ratio

$$\nu \equiv \left| \frac{\text{transverse elongation}}{\text{longitudinal elongation}} \right|$$



- For axial stress in x direction,

$$\epsilon_x = \frac{\sigma_x}{E} \qquad \epsilon_y = \epsilon_z = -\nu\frac{\sigma_x}{E}$$

- The constant of proportionality ν is *Poisson's ratio*
- See Table A-5 for values for common materials.

Elastic Strain

- For a stress element undergoing σ_x, σ_y, and σ_z, simultaneously,

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$
$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \qquad (3\text{–}19)$$
$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

Elastic Strain

- Hooke's law for shear:

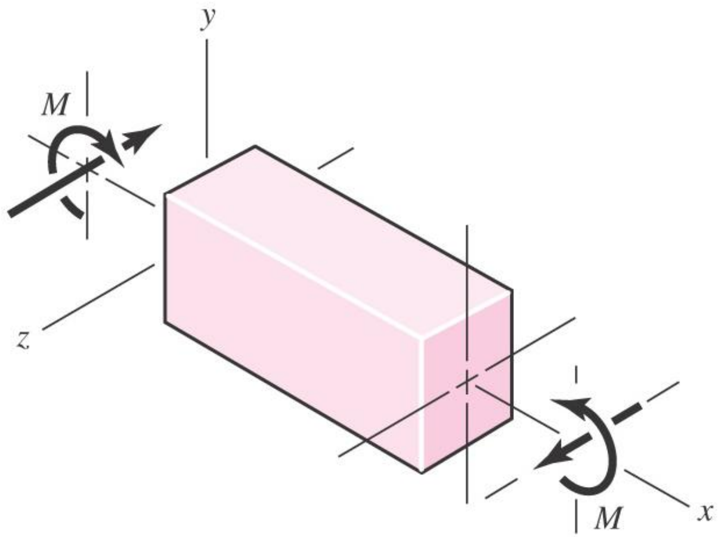
$$\tau = G\gamma \qquad (3\text{–}20)$$

- *Shear strain* γ is the change in a right angle of a stress element when subjected to pure shear stress.
- G is the *shear modulus of elasticity* or *modulus of rigidity*.
- For a linear, isotropic, homogeneous material,

$$E = 2G(1 + \nu) \qquad (3\text{–}21)$$

Normal Stresses for Beams in Bending

- Straight beam in positive bending
- *x* axis is *neutral axis*
- *xz* plane is *neutral plane*
- *Neutral axis* is coincident with the *centroidal axis* of the cross section



Normal Stresses for Beams in Bending

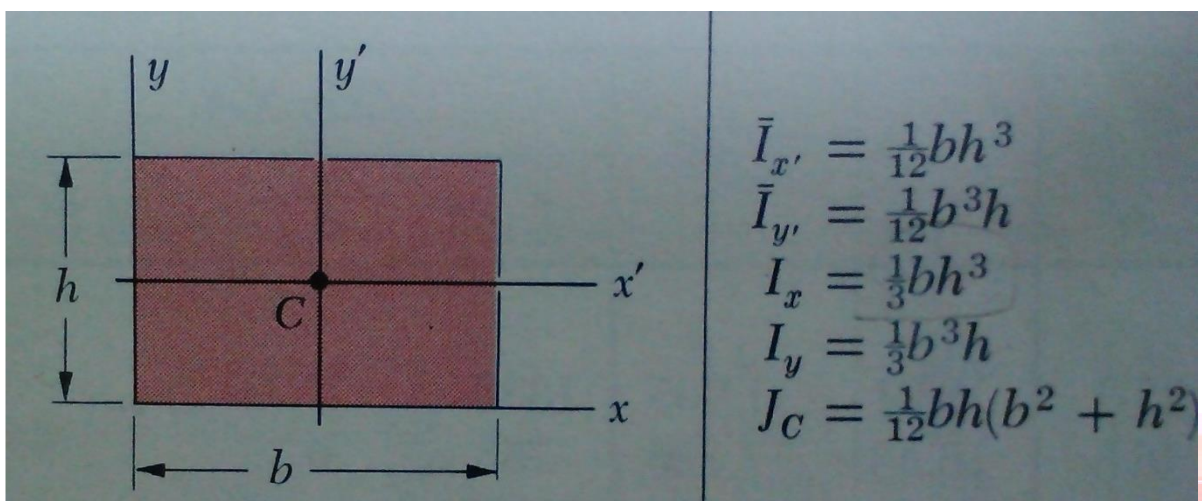
- Bending stress varies linearly with distance from neutral axis, *y*

$$\sigma_x = -\frac{My}{I} \qquad (3\text{–}24)$$

- *I* is the *second-area moment* about the *z* axis

$$I = \int y^2 dA \qquad (3\text{–}25)$$

$$I = \int_w \int_{h=\frac{-t}{2}}^{\frac{t}{2}} h^2 dA$$



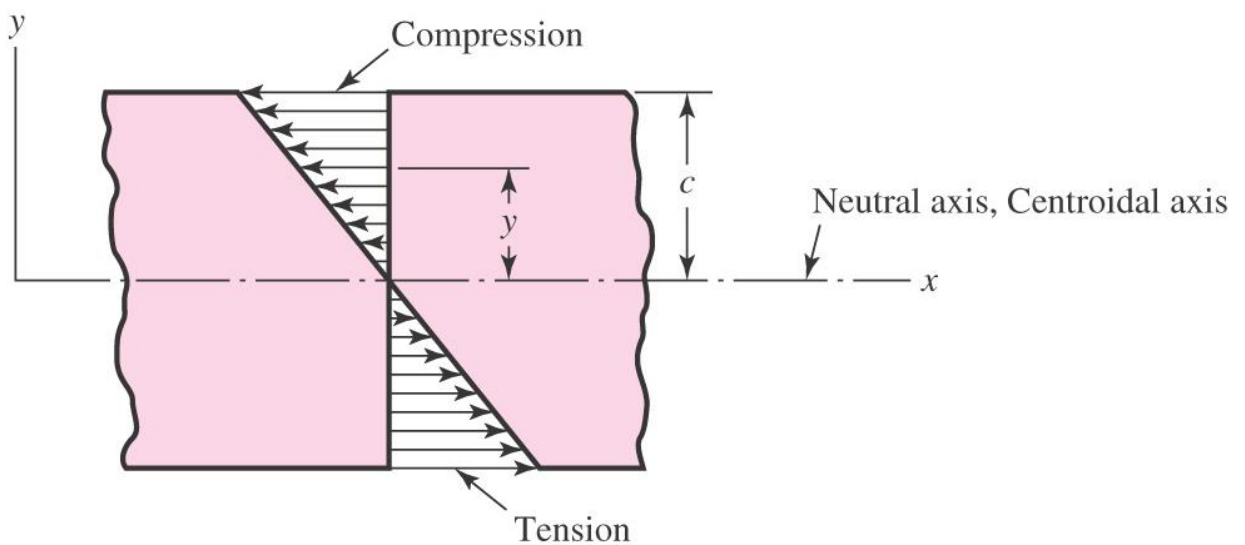
Normal Stresses for Beams in Bending

- Maximum bending stress is where y is greatest.

$$\sigma_{\max} = \frac{Mc}{I} \qquad (3\text{–}26a)$$

$$\sigma_{\max} = \frac{M}{Z} \qquad (3\text{–}26b)$$

- c is the magnitude of the greatest y
- $Z = I/c$ is the *section modulus*



Assumptions for Normal Bending Stress

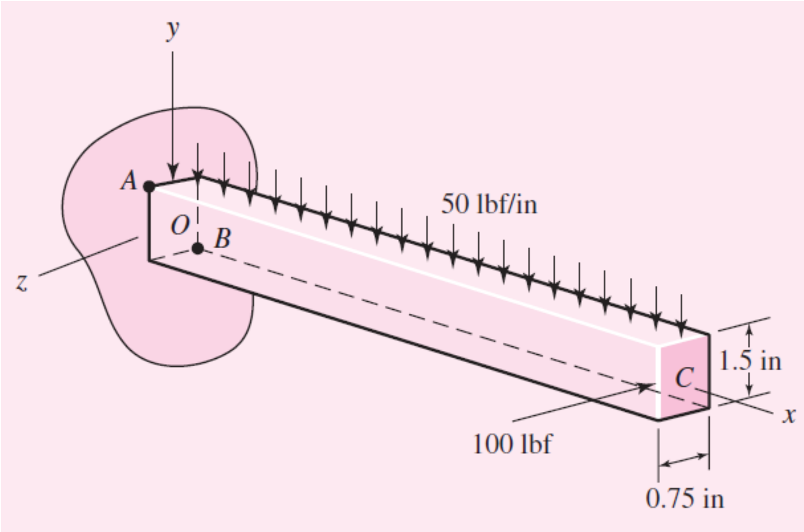
- Pure bending (though effects of axial, torsional, and shear loads are often assumed to have minimal effect on bending stress)
- Material is isotropic and homogeneous
- Material obeys Hooke's law
- Beam is initially straight with constant cross section
- Beam has axis of symmetry in the plane of bending
- Proportions are such that failure is by bending rather than crushing, wrinkling, or sidewise buckling
- Plane cross sections remain plane during bending

Two-Plane Bending

- Consider bending in both *xy* and *xz* planes
- Cross sections with one or two planes of symmetry only

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

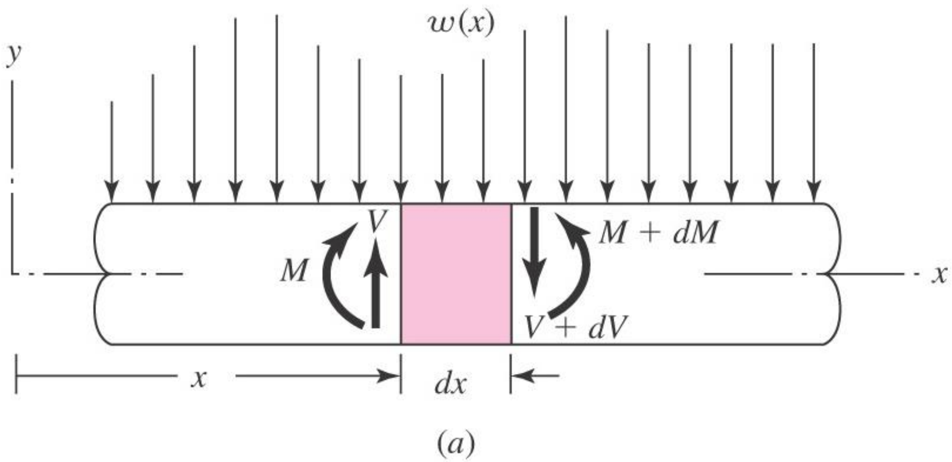
- For solid circular cross section, the maximum bending stress is



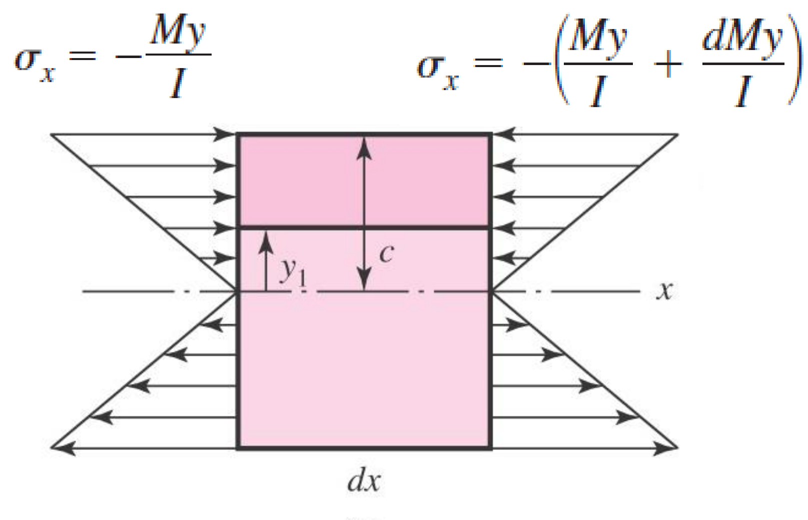
(a)

$$\sigma_m = \frac{Mc}{I} = \frac{(M_y^2 + M_z^2)^{1/2}(d/2)}{\pi d^4/64} = \frac{32}{\pi d^3}(M_y^2 + M_z^2)^{1/2} \qquad (3\text{–}28)$$

Shear Stresses for Beams in Bending



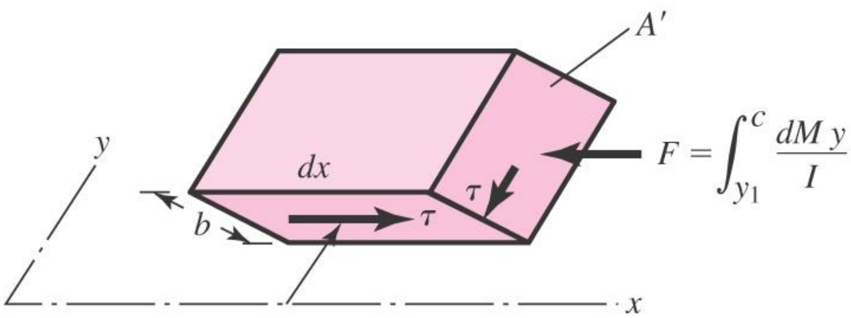
(a)



(b)

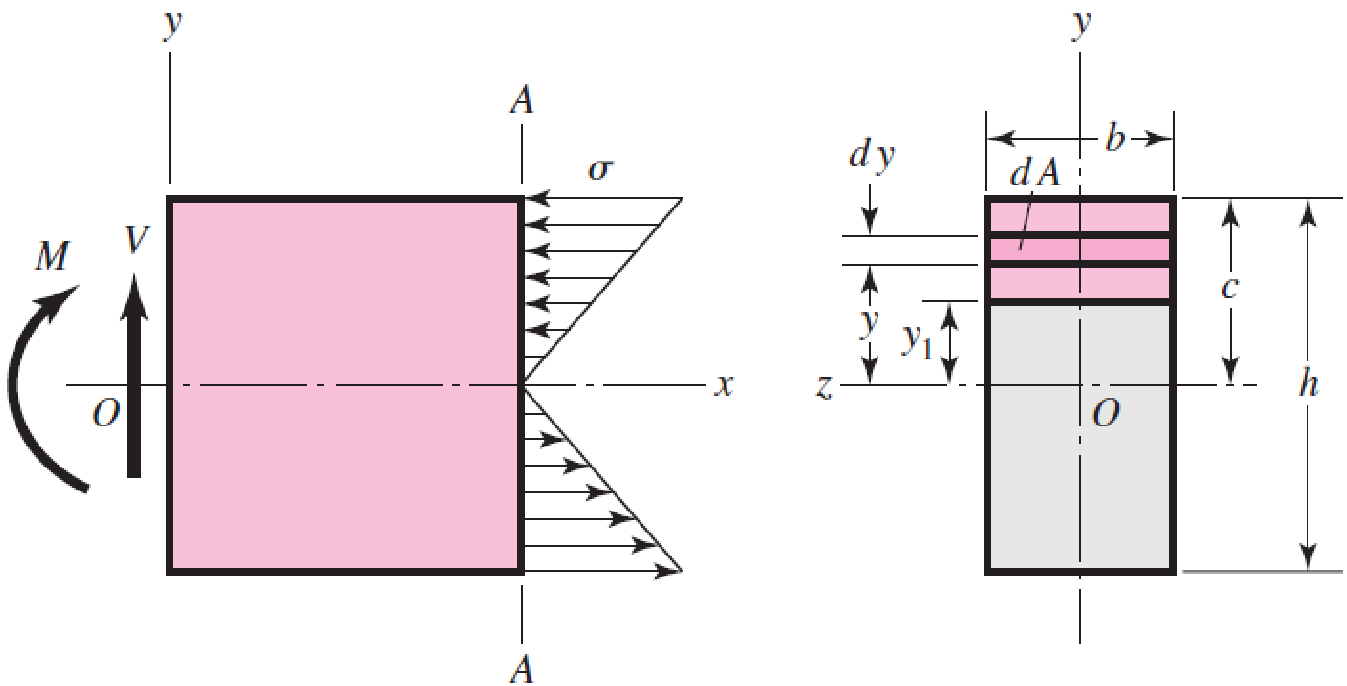
$$\tau b\,dx = \int_{y_1}^{c} \frac{(dM)y}{I} dA$$

$$\tau = \frac{V}{Ib} \int_{y_1}^{c} y dA$$



(c)

Transverse Shear Stress

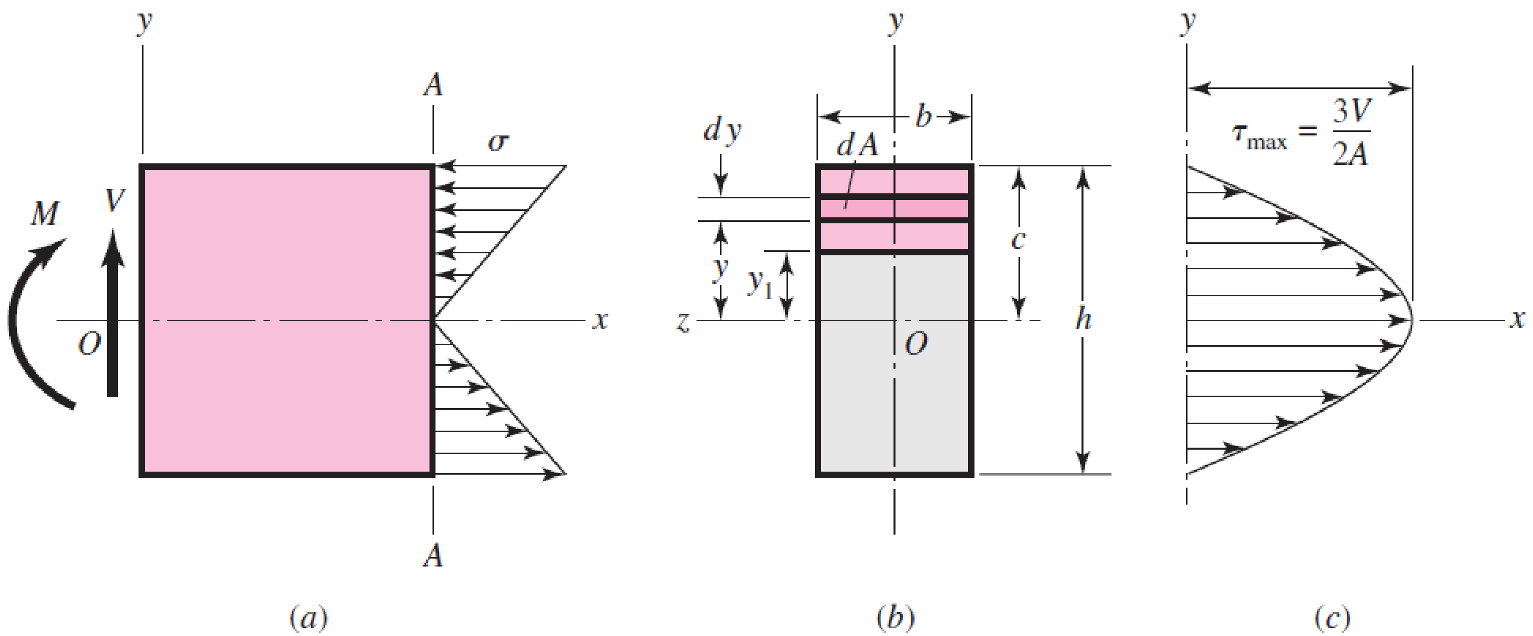


$$Q = \int_{y_1}^{c} y\,dA = \bar{y}'A' \qquad (3\text{–}30)$$

$$\tau = \frac{VQ}{Ib} \qquad (3\text{–}31)$$

- Transverse shear stress is always accompanied with bending stress.

Transverse Shear Stress in a Rectangular Beam

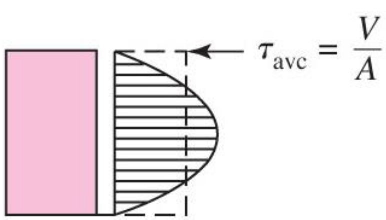
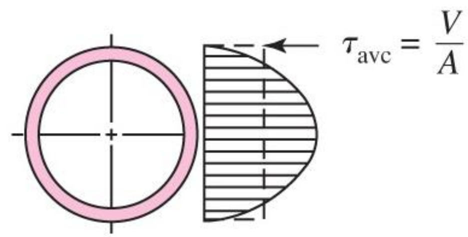
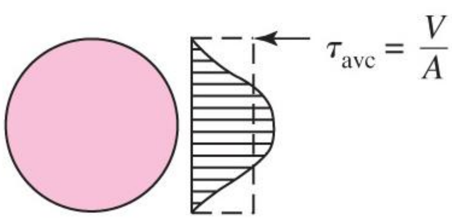
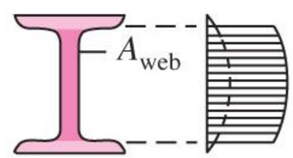


$$Q = \int_{y_1}^{c} y\,dA = b\int_{y_1}^{c} y\,dy = \left.\frac{by^2}{2}\right|_{y_1}^{c} = \frac{b}{2}\left(c^2 - y_1^2\right)$$

$$\tau = \frac{VQ}{Ib} = \frac{V}{2I}\left(c^2 - y_1^2\right) \qquad I = \frac{Ac^2}{3}$$

$$\tau = \frac{3V}{2A}\left(1 - \frac{y_1^2}{c^2}\right) \qquad (3\text{–}33)$$

Maximum Values of Transverse Shear Stress

Beam Shape	Formula	Beam Shape	Formula
 Rectangular	$\tau_{max} = \frac{3V}{2A}$	 Hollow, thin-walled round	$\tau_{max} = \frac{2V}{A}$
 Circular	$\tau_{max} = \frac{4V}{3A}$	 Structural I beam (thin-walled)	$\tau_{max} \doteq \frac{V}{A_{web}}$

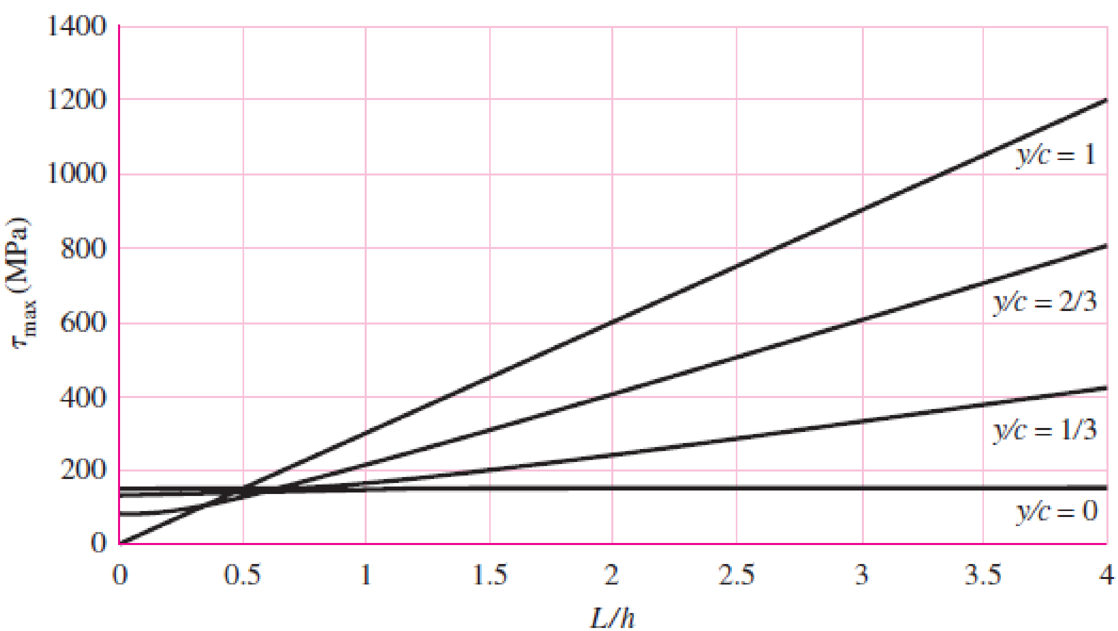
Significance of Transverse Shear Compared to Bending

- Example: Cantilever beam, rectangular cross section
- Maximum shear stress, including bending stress (*My/I*) and transverse shear stress (*VQ/Ib*),

$$\tau_{\max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{3F}{2bh}\sqrt{4(L/h)^2(y/c)^2 + \left[1-(y/c)^2\right]^2}$$

Figure 3–19

Plot of maximum shear stress for a cantilever beam, combining the effects of bending and transverse shear stresses.

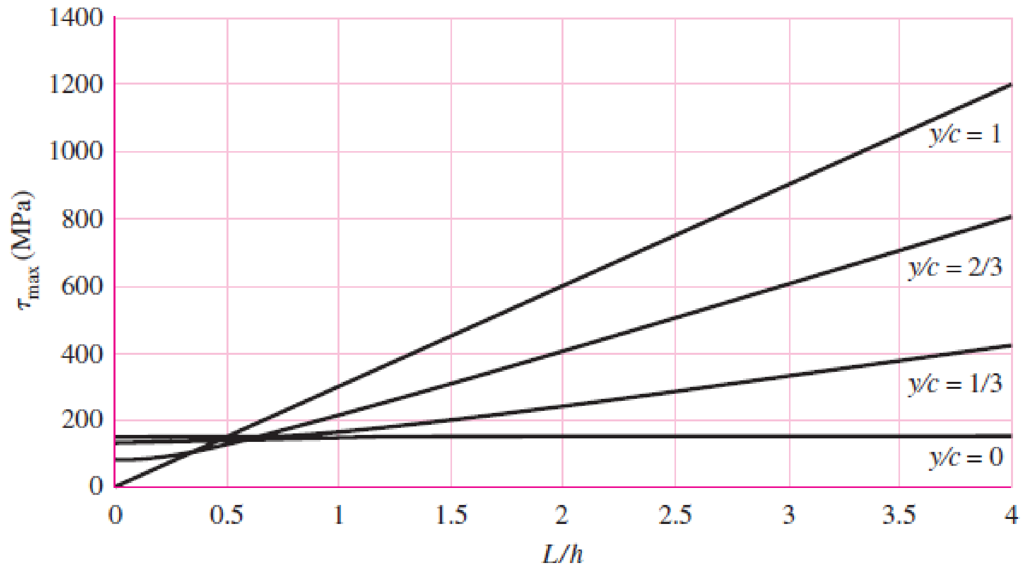


Significance of Transverse Shear Compared to Bending

- Critical stress element (largest τ_{max}) will always be either
  - Due to bending, on the outer surface (y/c=1), where the transverse shear is zero
  - Or due to transverse shear at the neutral axis (y/c=0), where the bending is zero
- Transition happens at some critical value of L/h
- Valid for any cross section that does not increase in width farther away from the neutral axis.
  - Includes round and rectangular solids, but not I beams and channels

Figure 3–19

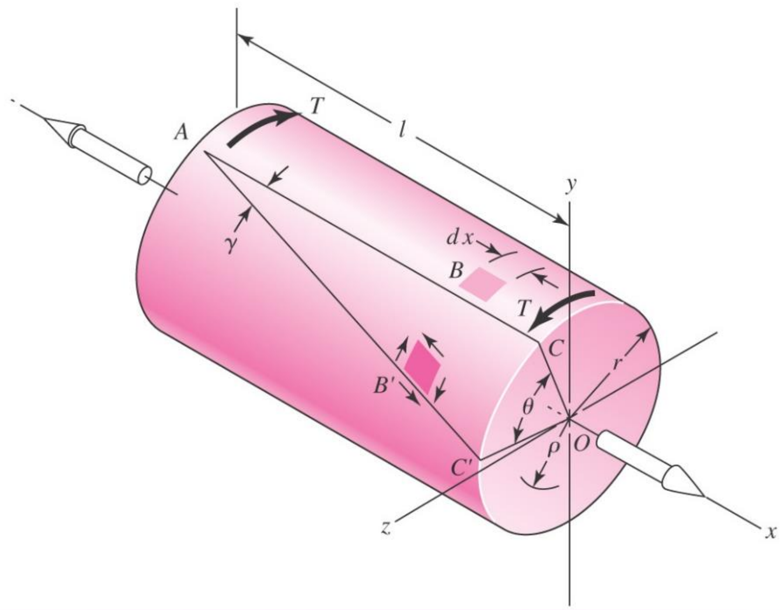
Plot of maximum shear stress for a cantilever beam, combining the effects of bending and transverse shear stresses.



Torsion

- *Torque vector* – a moment vector collinear with axis of a mechanical element
- A bar subjected to a torque vector is said to be in *torsion*
- *Angle of twist*, in radians, for a solid round bar

$$\theta = \frac{Tl}{GJ} \qquad (3\text{–}35)$$



Shigley's Mechanical Engineering Design

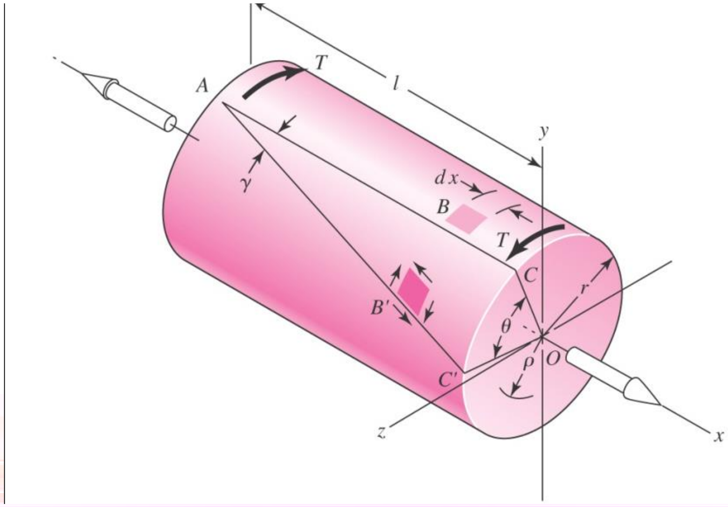
Torsional Shear Stress

- For round bar in torsion, torsional shear stress is proportional to the radius ρ

$$\tau = \frac{T\rho}{J} \tag{3-36}$$

- Maximum torsional shear stress is at the outer surface

$$\tau_{\max} = \frac{Tr}{J} \tag{3-37}$$



Assumptions for Torsion Equations

- Equations (3-35) to (3-37) are only applicable for the following conditions
  - Pure torque
  - Remote from any discontinuities or point of application of torque
  - Material obeys Hooke's law
  - Adjacent cross sections originally plane and parallel remain plane and parallel
  - Radial lines remain straight
    - Depends on axisymmetry, so does not hold true for noncircular cross sections
- Consequently, only applicable for round cross sections

Torsional Shear in Rectangular Section

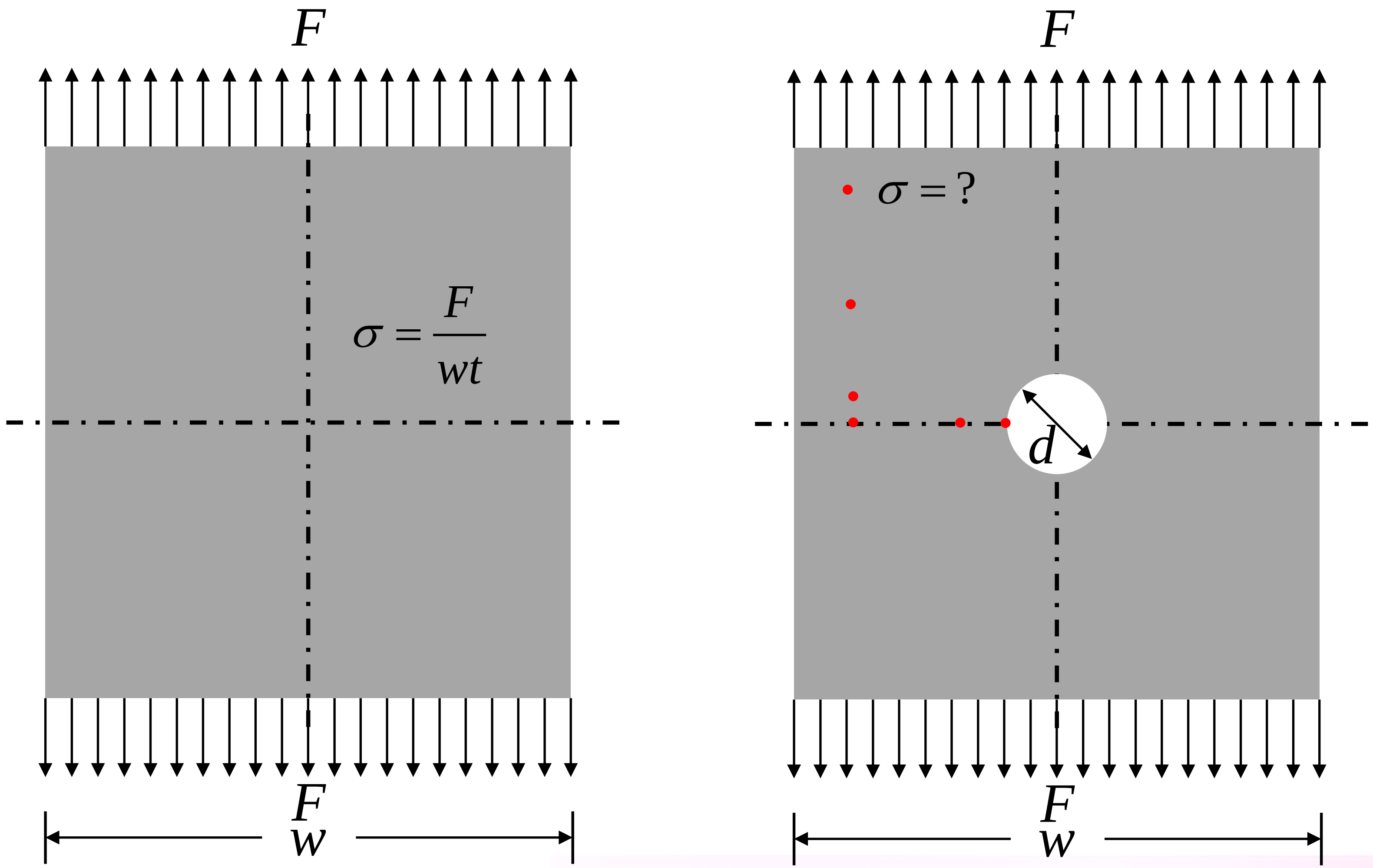
- Shear stress does not vary linearly with radial distance for rectangular cross section
- Shear stress is zero at the corners
- Maximum shear stress is at the middle of the longest side
- For rectangular *b* x *c* bar, where *b* is longest side

$$\tau_{\max} = \frac{T}{\alpha b c^2} \doteq \frac{T}{bc^2}\left(3 + \frac{1.8}{b/c}\right) \qquad (3\text{–}40)$$

$$\theta = \frac{Tl}{\beta b c^3 G} \qquad (3\text{–}41)$$

b/c	1.00	1.50	1.75	2.00	2.50	3.00	4.00	6.00	8.00	10	∞
α	0.208	0.231	0.239	0.246	0.258	0.267	0.282	0.299	0.307	0.313	0.333
β	0.141	0.196	0.214	0.228	0.249	0.263	0.281	0.299	0.307	0.313	0.333

Stress Concentration



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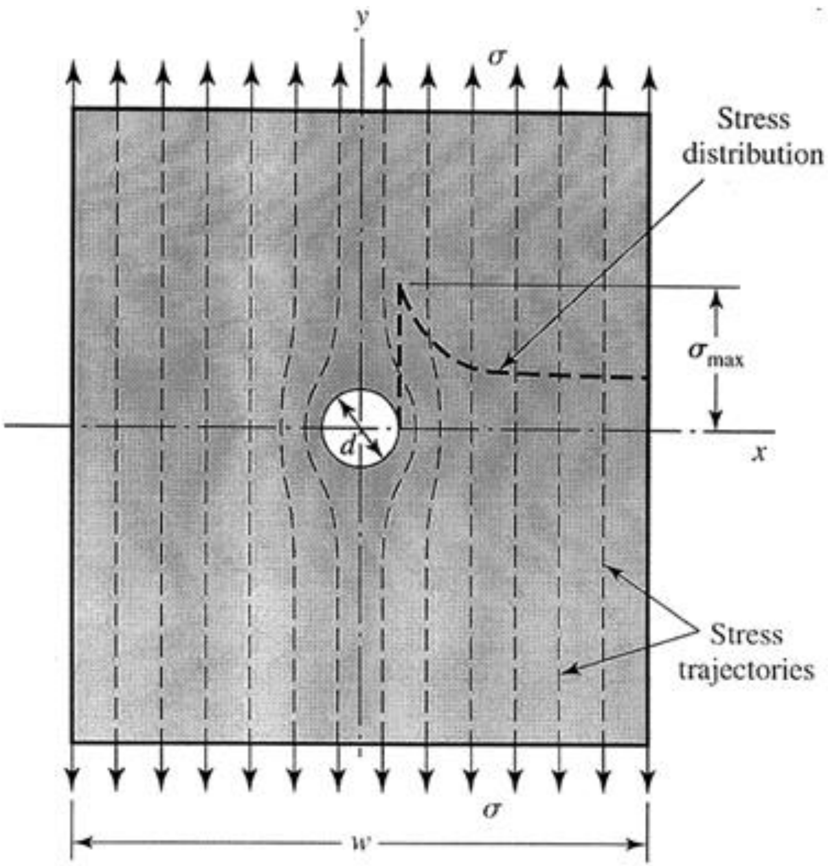
Stress Concentration

- Localized increase of stress near discontinuities
- K_t is Theoretical (Geometric) Stress Concentration Factor

$$K_t = \frac{\sigma_{\max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{\max}}{\tau_0}$$



Theoretical Stress Concentration Factor

- Graphs available for standard configurations
- See Appendix A-15 and A-16 for common examples
- Many more in *Peterson's Stress-Concentration Factors*
- Note the trend for higher *Kt* at sharper discontinuity radius, and at greater disruption
- Stress concentration effect is commonly ignored for static loads on ductile materials

Figure A–15–1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.

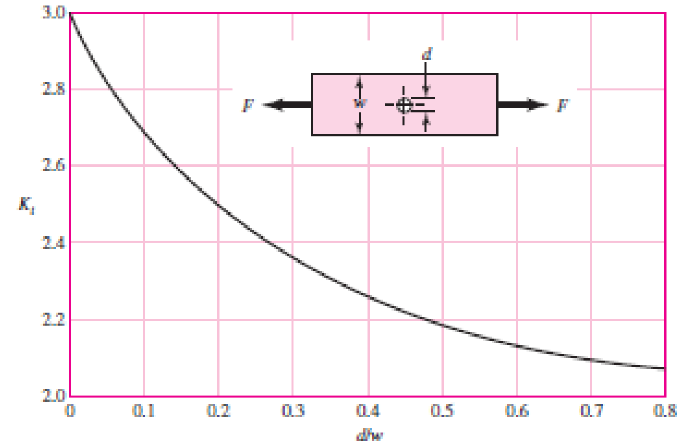
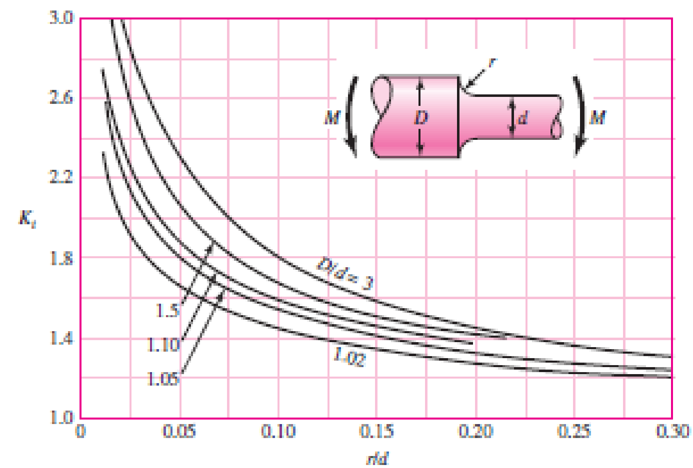


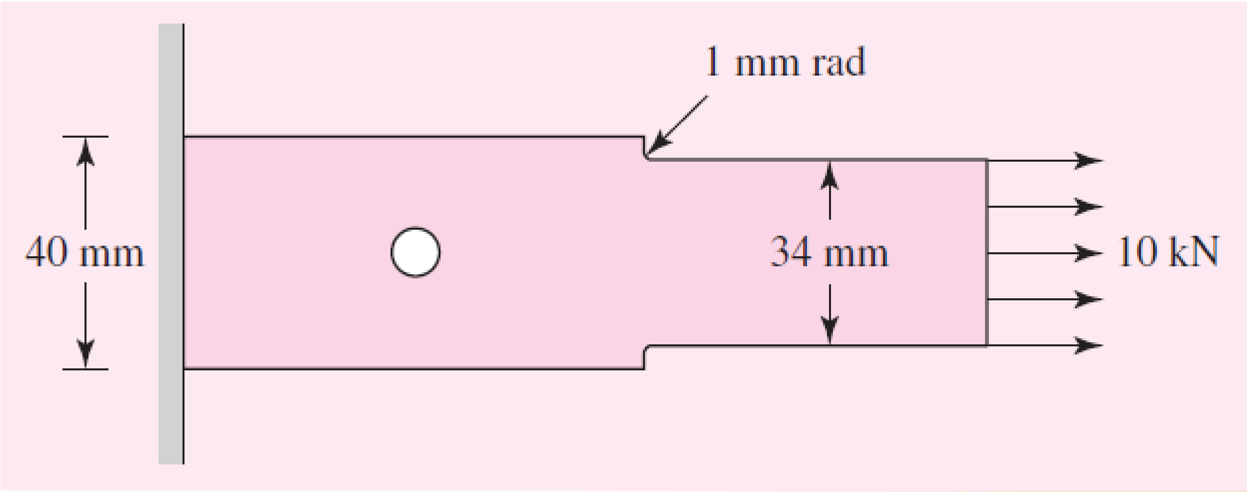
Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

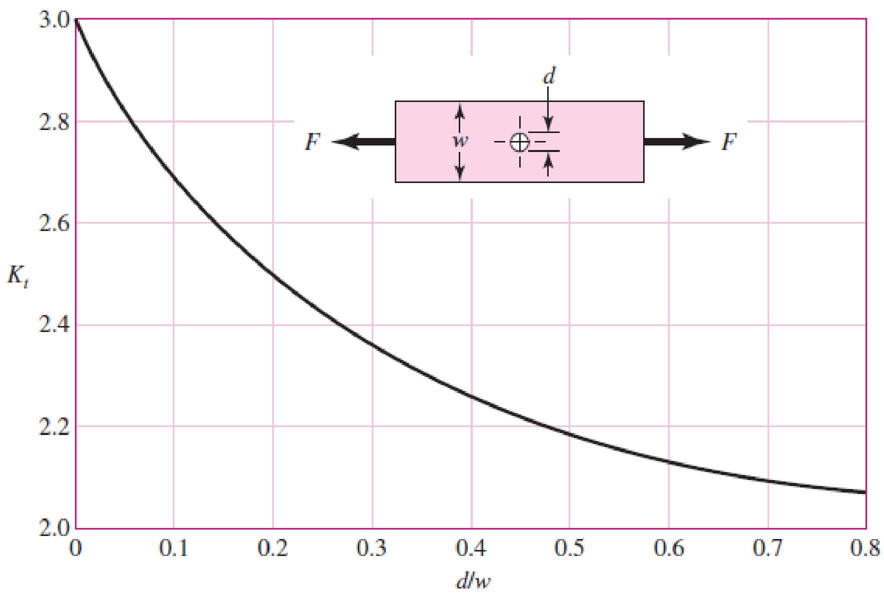


Example 3-13

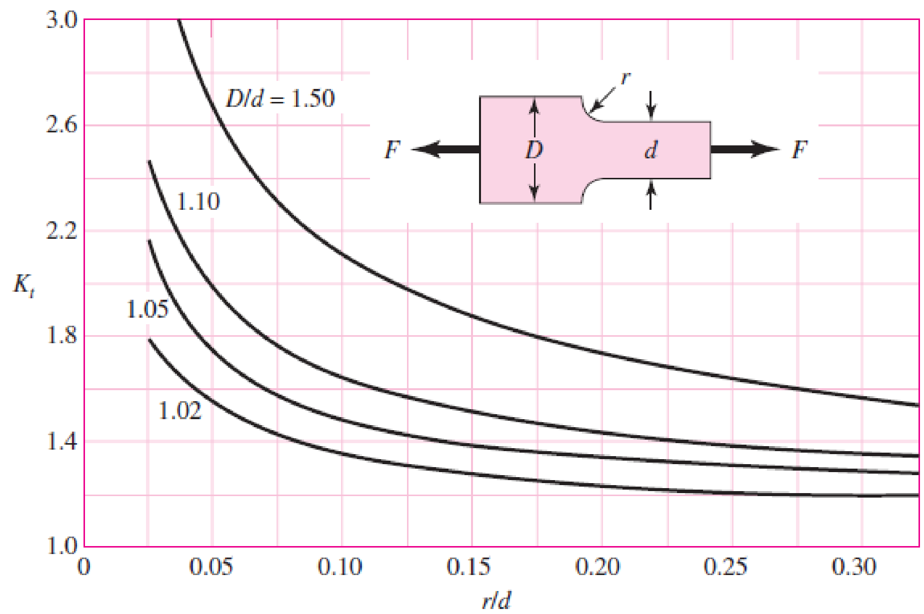
The 2-mm-thick bar shown in Fig. 3–30 is loaded axially with a constant force of 10 kN. The bar material has been heat treated and quenched to raise its strength, but as a consequence it has lost most of its ductility. It is desired to drill a hole through the center of the 40-mm face of the plate to allow a cable to pass through it. A 4-mm hole is sufficient for the cable to fit, but an 8-mm drill is readily available. Will a crack be more likely to initiate at the larger hole, the smaller hole, or at the fillet?



Stress Concentration Factor: Table A15



$$\sigma_o = \frac{F}{A}\ ;\ A = (w-d)t$$

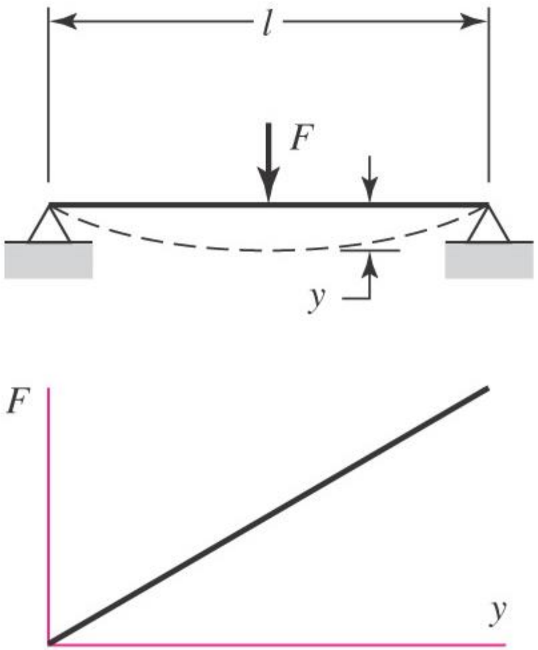


$$\sigma_o = \frac{F}{A}\ ;\ A = dt$$

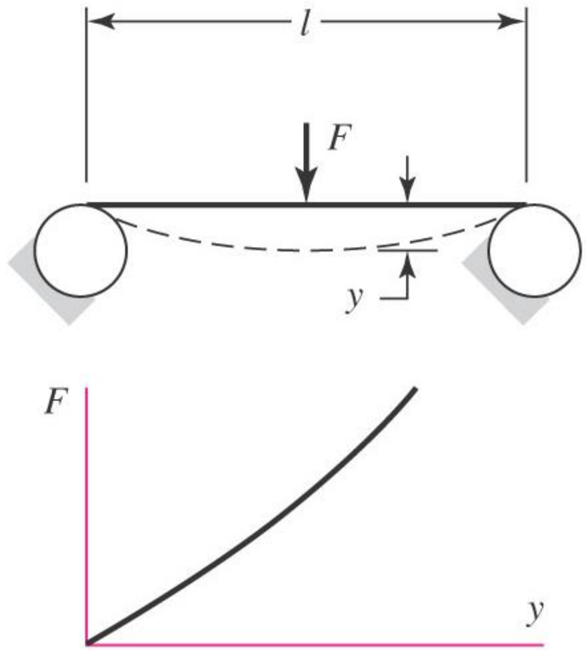
Force vs Deflection

- *Elasticity* – property of a material that enables it to regain its original configuration after deformation
- *Spring* – a mechanical element that exerts a force when deformed

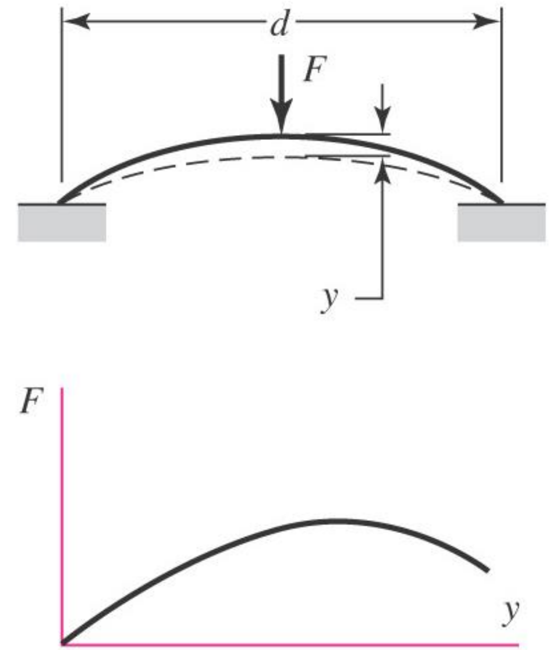
Linear spring



Nonlinear stiffening spring



Nonlinear softening spring



Spring Rate

- Relation between force and deflection, $F = F(y)$
- *Spring rate*

$$k(y) = \lim_{\Delta y \to 0} \frac{\Delta F}{\Delta y} = \frac{dF}{dy} \qquad (4\text{–}1)$$

- For linear springs, k is constant, called *spring constant*

$$k = \frac{F}{y} \qquad (4\text{–}2)$$

Axially-Loaded Stiffness

- Total extension or contraction of a uniform bar in tension or compression

$$\delta = \frac{Fl}{AE} \tag{4–3}$$

- Spring constant, with $k = F/\delta$

$$k = \frac{AE}{l} \tag{4–4}$$

Torsionally-Loaded Stiffness

- Angular deflection (in radians) of a uniform solid or hollow round bar subjected to a twisting moment *T*

$$\theta = \frac{Tl}{GJ} \qquad (4\text{–}5)$$

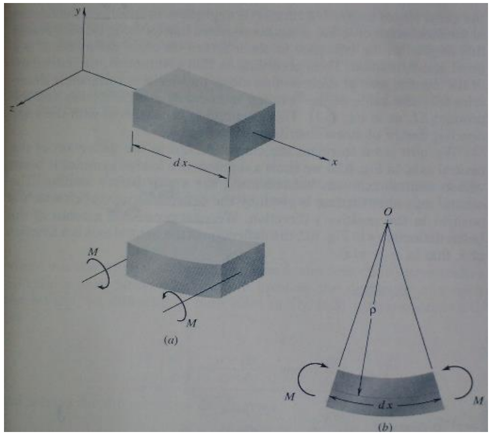
- Converting to degrees, and including $J = \pi d^4/32$ for round solid

$$\theta = \frac{583.6Tl}{Gd^4} \qquad (4\text{–}6)$$

- Torsional spring constant for round bar

$$k = \frac{T}{\theta} = \frac{GJ}{l} \qquad (4\text{–}7)$$

Deflection Due to Bending



- Curvature of beam subjected to bending moment M

$$\frac{1}{\rho} = \frac{M}{EI}$$

- From mathematics, curvature of plane curve

$$\frac{1}{\rho} = \frac{\dfrac{d^2v}{dx^2}}{\left[1+\left(\dfrac{dv}{dx}\right)^2\right]^{3/2}} = \frac{M}{EI_z}$$

- If the slope is very small, the denominator of Eq. approaches unity.

$$\frac{d^2v}{dx^2} = \frac{M(x)}{EI_z}$$

Deflection Due to Bending

$$\frac{q}{EI} = v''''$$

$$\frac{V}{EI} = -v'''$$

$$\frac{M}{EI} = v''$$

$$\tan\theta \approx \theta = v'$$

Strain Energy

- External work done on elastic member in deforming it is transformed into *strain energy*, or *potential energy*.
- Strain energy equals product of average force and deflection.

$$U = \frac{F}{2}y = \frac{F^2}{2k} \tag{4-15}$$

Some Common Strain Energy Formulas

- For axial loading, applying $k = AE/l$ from Eq. (4-4),

$$U = \frac{F^2 l}{2AE} \qquad (4\text{–}16)$$

or

$$U = \int \frac{F^2}{2AE}\,dx \qquad (4\text{–}17)$$

tension and compression

- For torsional loading, applying $k = GJ/l$ from Eq. (4-7),

$$U = \frac{T^2 l}{2GJ} \qquad (4\text{–}18)$$

or

$$U = \int \frac{T^2}{2GJ}\,dx \qquad (4\text{–}19)$$

torsion

Some Common Strain Energy Formulas

- For direct shear loading,

$$U = \frac{F^2 l}{2AG} \qquad (4\text{–}20)$$

or

$$U = \int \frac{F^2}{2AG}\,dx \qquad (4\text{–}21)$$

} direct shear

- For bending loading,

$$U = \frac{M^2 l}{2EI} \qquad (4\text{–}22)$$

or

$$U = \int \frac{M^2}{2EI}\,dx \qquad (4\text{–}23)$$

} bending

Some Common Strain Energy Formulas

- For transverse shear loading,

$$U = \frac{CV^2 l}{2AG} \quad \text{(4–24)}$$

or

$$U = \int \frac{CV^2}{2AG}\,dx \quad \text{(4–25)}$$

} transverse shear

where *C* is a modifier dependent on the cross sectional shape.

Table 4–1

Strain-Energy Correction Factors for Transverse Shear

Source: Richard G. Budynas, *Advanced Strength and Applied Stress Analysis,* 2nd ed., McGraw-Hill, New York, 1999.

Beam Cross-Sectional Shape	Factor C
Rectangular	1.2
Circular	1.11
Thin-walled tubular, round	2.00
Box sections†	1.00
Structural sections†	1.00

†Use area of web only.

Summary of Common Strain Energy Formulas

$$\left.\begin{aligned} U &= \frac{F^2 l}{2AE} \\ U &= \int \frac{F^2}{2AE}\,dx \end{aligned}\right\} \text{ tension and compression}$$

$$\left.\begin{aligned} U &= \frac{T^2 l}{2GJ} \\ U &= \int \frac{T^2}{2GJ}\,dx \end{aligned}\right\} \text{ torsion}$$

$$\left.\begin{aligned} U &= \frac{F^2 l}{2AG} \\ U &= \int \frac{F^2}{2AG}\,dx \end{aligned}\right\} \text{ direct shear}$$

$$\left.\begin{aligned} U &= \frac{M^2 l}{2EI} \\ U &= \int \frac{M^2}{2EI}\,dx \end{aligned}\right\} \text{ bending}$$

$$\left.\begin{aligned} U &= \frac{CV^2 l}{2AG} \\ U &= \int \frac{CV^2}{2AG}\,dx \end{aligned}\right\} \text{ transverse shear}$$