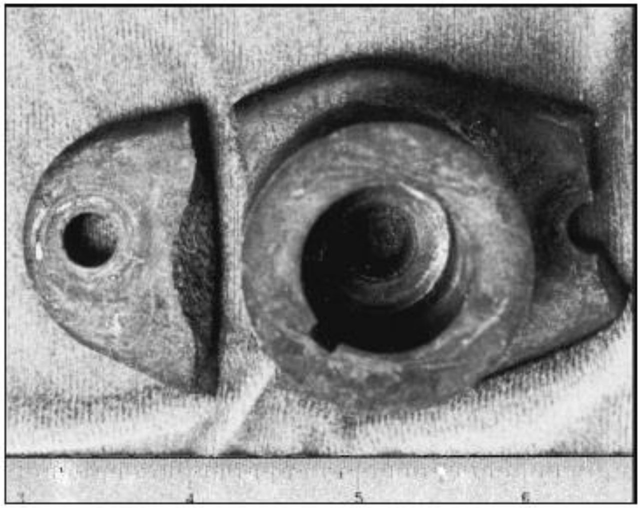
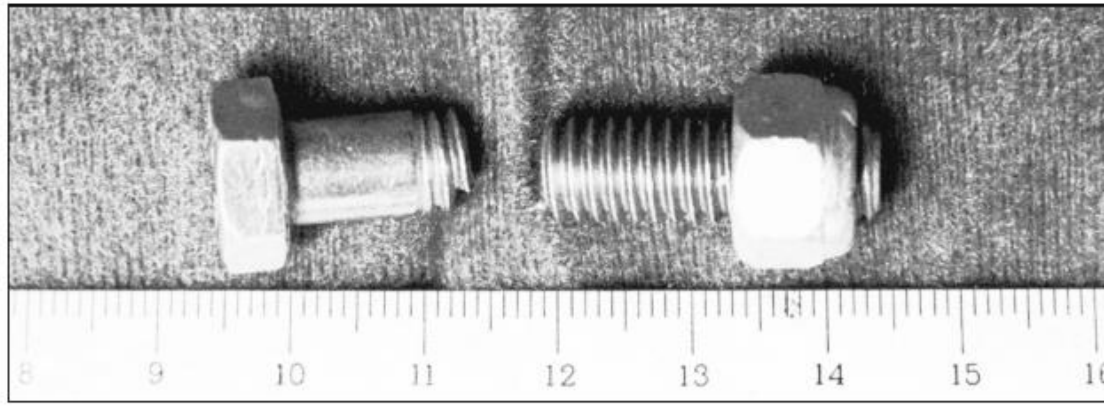
Ch 5

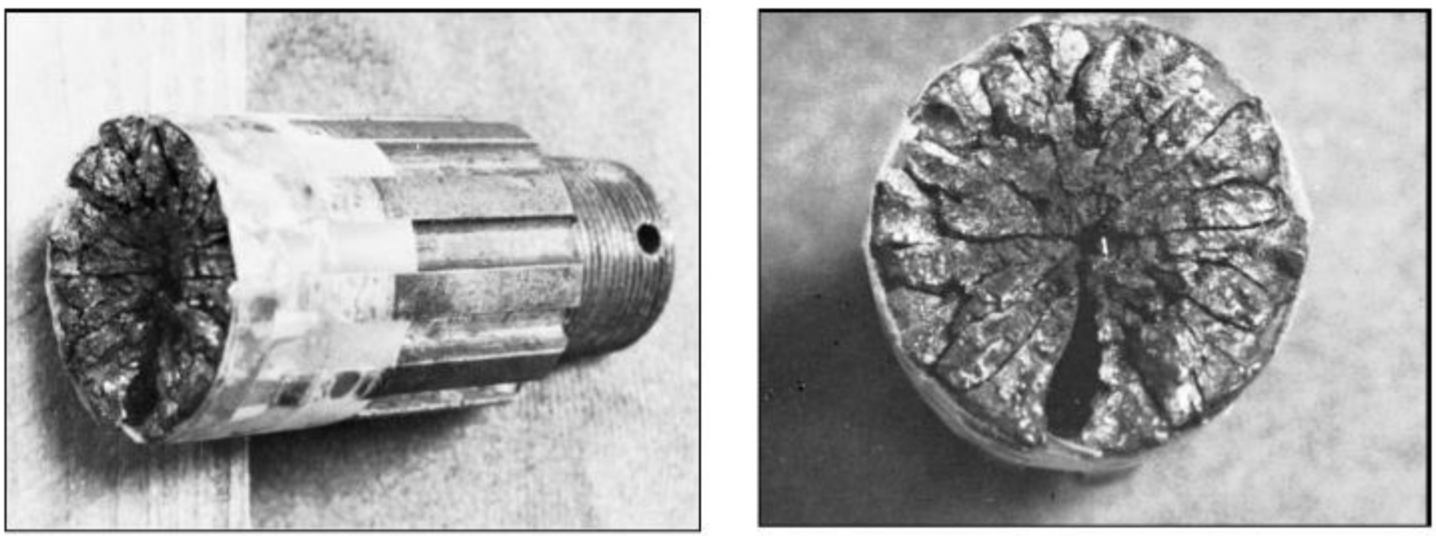
Failures Resulting from Static Loading



Impact failure of a lawn-mower blade driver hub.



Failure of an overhead-pulley retaining bolt on a weightlifting machine.



(a) (b)

Failure of truck driveshaft spline due to corrosion fatigue

Definition

- Strength: Property or characteristic of mechanical element
  - Results from the material identity, treatment, processing, loading
- Static load: stationary force or couple applied to a member
  - Unchanging in magnitude and location
  - Can produce tension, compression, shear, bending, torsion, or any combination
- Failure: can mean many possibility
  - In this chapter, we focus on the permanent distortion or separation
- In this chapter, we will predict failure based on strength and static loading
  - Develop failure theory for ductile and brittle material

Static Strength (靜態強度)

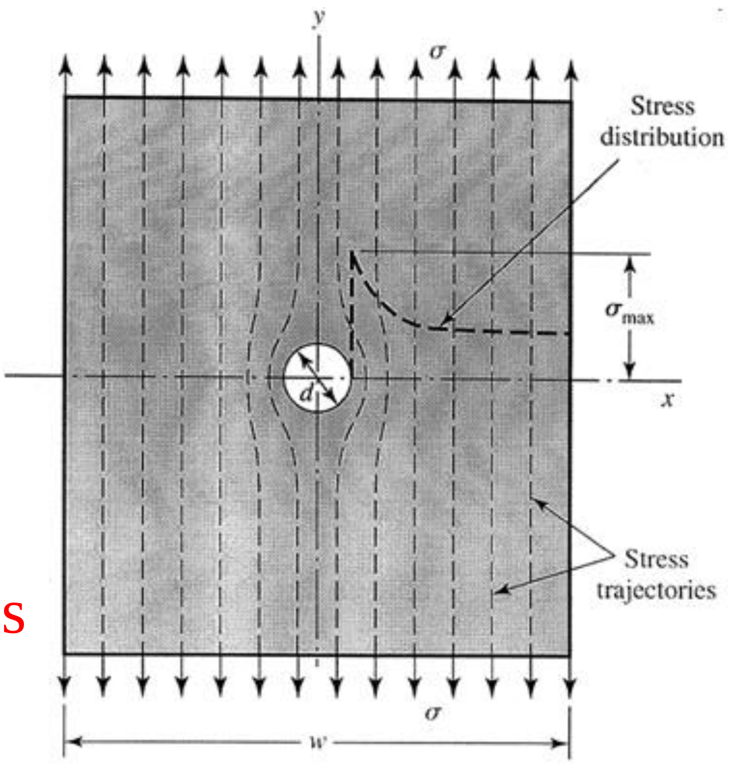
- 物體受到一個與時間無關的靜負載，且在該負載之下破壞，則在此負載下對應的應力值即為靜態強度
- 理想的狀況是：有大量的測試結果
  - 試片和最終所要使用的元件具有相同的熱處理、表面處理...
  - 測試時的負載和最終真實使用時的負載要相同
  - Experimental test data is better, but generally only warranted for large quantities or when failure is very costly (in time, expense, or life)
- Usually necessary to design using published strength values
  - Methods are needed to safely and efficiently use published strength values for a variety of situations

Stress Concentration

- Localized increase of stress near discontinuities
- K_t is Theoretical (Geometric) Stress Concentration Factor

$$K_t = \frac{\sigma_{\max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{\max}}{\tau_0}$$

- Stress concentration effect is commonly ignored for static loads on ductile materials
- Stress concentration must be included for brittle materials, since localized yielding may reach brittle failure rather than cold-working and sharing the load.



Static Failure Theories

- No universal theory of failure for the general case of material properties
- Structure metal behavior
  - Ductile
    - $\varepsilon_f \geq 0.05$
    - Identifiable yield strength
    - Often, S_y in tension = S_y in compression
  - Brittle
    - $\varepsilon_f \leq 0.05$
    - No identifiable S_y
    - Typically classified by ultimate tensile strengths, S_{ut} and ultimate compressive strengths, S_{uc}

Failure Theories

Ductile materials (yield criteria)

- Maximum shear stress (MSS) 最大剪應力法
- Distortion energy (DE) 畸變能量法
- Ductile Coulomb-Mohr (DCM) 韌性材料之庫倫摩爾法

Brittle materials (fracture criteria)

- Maximum normal stress 正向應立法
- Brittle Coulomb-Mohr (BCM) 脆性材料之庫倫摩爾法
- Modified Mohr (MM) 修正摩爾法

Maximum Shear Stress Theory (MSS)

- Theory: Yielding begins when the *maximum shear stress* in a stress element exceeds the maximum shear stress in a tension test specimen of the same material when that specimen begins to yield.
  - Tresca or Guest theory
- For ductile materials under a tension test
  - Slip line (Luder line) form at approximately 45° with the axis of the strip
  - Fracture lines are also seen at 45°
  - Why?
- https://www.youtube.com/watch?v=ybuLwFjXjAA

Maximum Shear Stress Theory (MSS)

- For a tension test specimen, the maximum shear stress is $\sigma_1/2$.
- At yielding, when $\sigma_1 = S_y$, the maximum shear stress is $S_y/2$.
- Could restate the theory as follows:
  - Theory: Yielding begins when the *maximum shear stress* in a stress element exceeds $S_y/2$.

Maximum Shear Stress Theory (MSS)

- For any stress element, use Mohr's circle to find the maximum shear stress. Compare the maximum shear stress to $S_y/2$.
- Ordering the principal stresses such that $\sigma_1 \geq \sigma_2 \geq \sigma_3$,

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} \geq \frac{S_y}{2} \qquad \text{or} \qquad \sigma_1 - \sigma_3 \geq S_y \tag{5–1}$$

- Incorporating a design factor n

$$\tau_{\max} = \frac{S_y}{2n} \qquad \text{or} \qquad \sigma_1 - \sigma_3 = \frac{S_y}{n} \tag{5–3}$$

- Or solving for factor of safety

$$n = \frac{S_y / 2}{\tau_{\max}}$$

Maximum Shear Stress Theory (MSS)

- To compare to experimental data, express $\tau_{\max}$ in terms of principal stresses and plot.
- To simplify, consider a plane stress state
- Let σ_A and σ_B represent the two non-zero principal stresses, then order them with the zero principal stress such that $\sigma_1 \geq \sigma_2 \geq \sigma_3$
- Assuming $\sigma_A \geq \sigma_B$ there are three cases to consider
  - Case 1: $\sigma_A \geq \sigma_B \geq 0$
  - Case 2: $\sigma_A \geq 0 \geq \sigma_B$
  - Case 3: $0 \geq \sigma_A \geq \sigma_B$

Maximum Shear Stress Theory (MSS)

- Case 1: $\sigma_A \geq \sigma_B \geq 0$
  - For this case, $\sigma_1 = \sigma_A$ and $\sigma_3 = 0$
  - Eq. (5–1) reduces to $\sigma_A \geq S_y$
- Case 2: $\sigma_A \geq 0 \geq \sigma_B$
  - For this case, $\sigma_1 = \sigma_A$ and $\sigma_3 = \sigma_B$
  - Eq. (5–1) reduces to $\sigma_A - \sigma_B \geq S_y$
- Case 3: $0 \geq \sigma_A \geq \sigma_B$
  - For this case, $\sigma_1 = 0$ and $\sigma_3 = \sigma_B$
  - Eq. (5–1) reduces to $\sigma_B \leq -S_y$

Maximum Shear Stress Theory (MSS)

- Plot three cases on principal stress axes
- Case 1: $\sigma_A \geq \sigma_B \geq 0$
  - $\sigma_A \geq S_y$
- Case 2: $\sigma_A \geq 0 \geq \sigma_B$
  - $\sigma_A - \sigma_B \geq S_y$
- Case 3: $0 \geq \sigma_A \geq \sigma_B$
  - $\sigma_B \leq -S_y$
- Other lines are symmetric cases
- Inside envelope is predicted safe zone

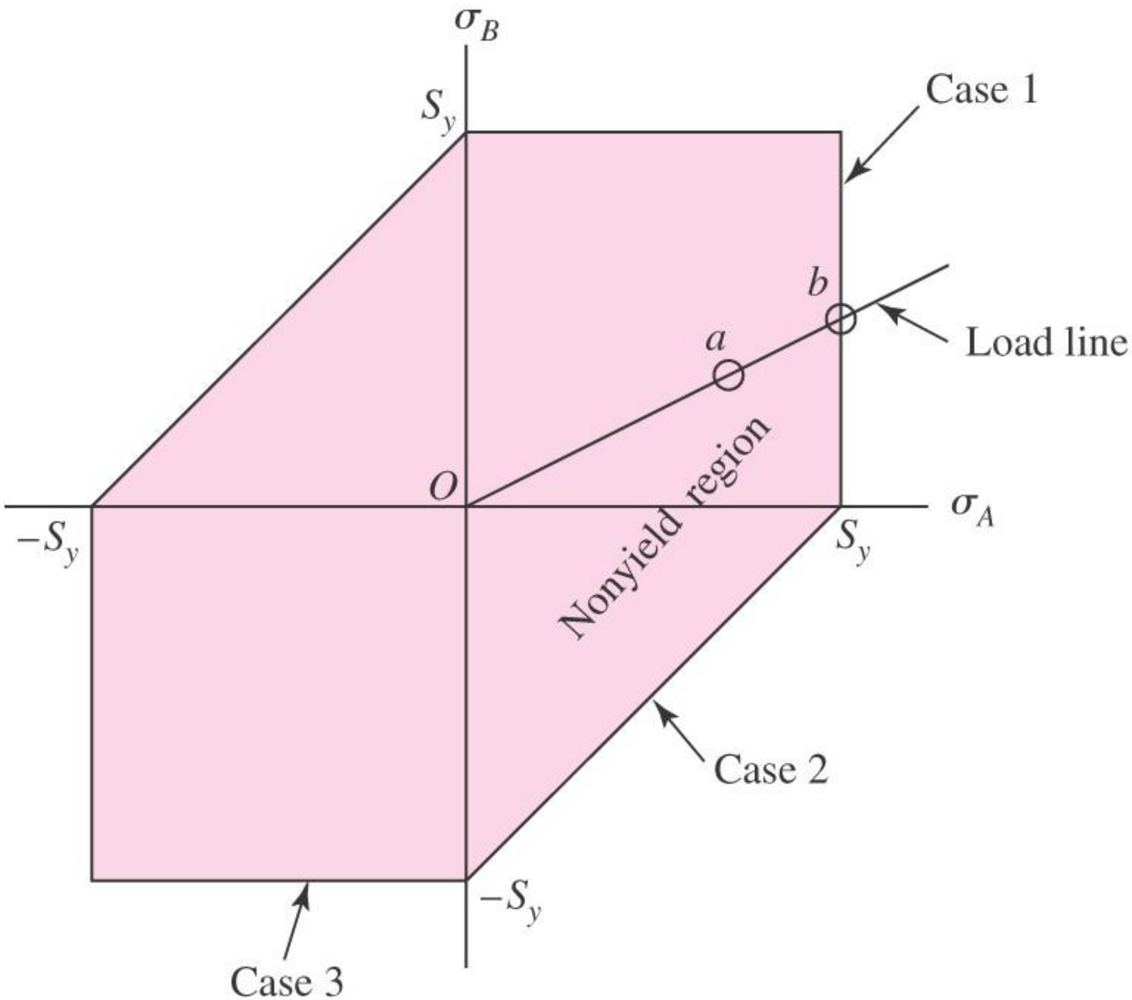
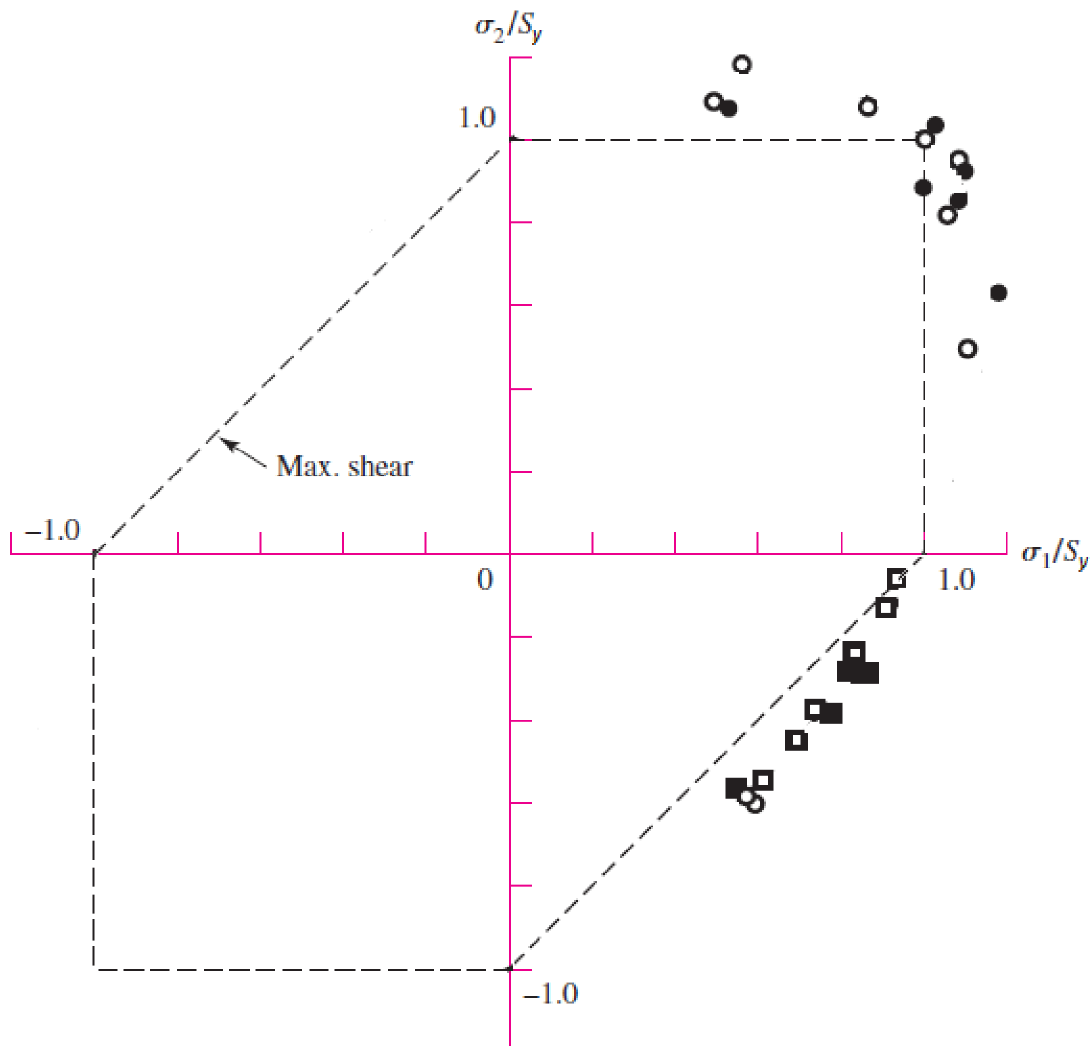


Fig. 5–7

Maximum Shear Stress Theory (MSS)

- Comparison to experimental data
- Conservative in all quadrants
- Commonly used for design situations



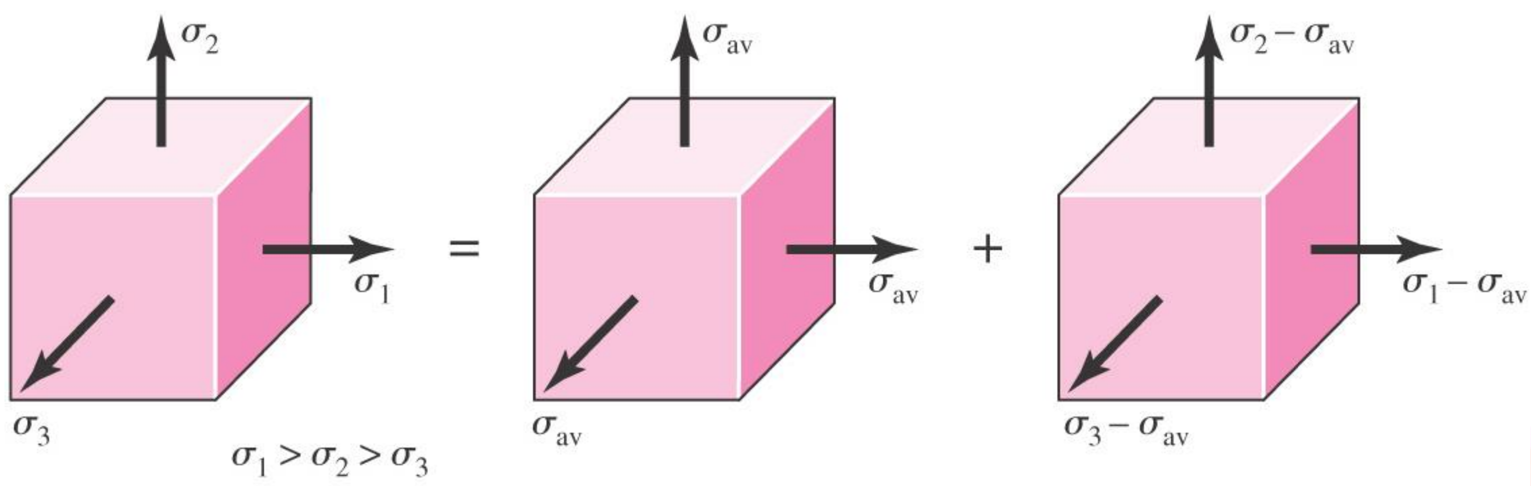
Shigley's Mechanical Engineering Design

Distortion Energy (DE) Failure Theory

- DE predicts that yielding occurs when the distortion strain energy per unit volume reaches or esceeds the distortion strain energy per unit volume for yield in simple tension or compression of the same material.
- Also known as:
  - Octahedral Shear Stress
  - Shear Energy
  - Von Mises
  - Von Mises – Hencky
- Originated from observation that ductile materials stressed hydrostatically (equal principal stresses) exhibited yield strengths greatly in excess of expected values.

Distortion Energy (DE) Failure Theory

- Theorizes that if strain energy is divided into hydrostatic volume changing energy and angular distortion energy, the yielding is primarily affected by the distortion energy.
- Theory: Yielding occurs when the *distortion strain energy* per unit volume reaches the distortion strain energy per unit volume for yield in simple tension or compression of the same material.



(*a*) Triaxial stresses (*b*) Hydrostatic component (*c*) Distortional component

Deriving the Distortion Energy

- Hydrostatic stress is average of principal stresses

$$\sigma_{av} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \qquad (a)$$

- Strain energy per unit volume, $u = \frac{1}{2}[\epsilon_1\sigma_1 + \epsilon_2\sigma_2 + \epsilon_3\sigma_3]$
- Substituting Eq. (3–19) for principal strains into strain energy equation,

$$\begin{aligned}
\epsilon_x &= \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \\
\epsilon_y &= \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \\
\epsilon_z &= \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]
\end{aligned} \qquad (3\text{–}19)$$

$$u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \qquad (b)$$

Deriving the Distortion Energy

$$u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \qquad (b)$$

- Strain energy for producing only volume change is obtained by substituting σ_{av} for σ_1, σ_2, and σ_3

$$u_v = \frac{3\sigma_{av}^2}{2E}(1 - 2\nu) \qquad (c)$$

- Substituting σ_{av} from Eq. (a),

$$u_v = \frac{1-2\nu}{6E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_3\sigma_1\right) \qquad (5\text{–}7)$$

- Obtain distortion energy by subtracting volume changing energy, Eq. (5–7), from total strain energy, Eq. (b)

$$u_d = u - u_v = \frac{1+\nu}{3E}\left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right] \qquad (5\text{–}8)$$

Deriving the Distortion Energy

$$u_d = u - u_v = \frac{1+\nu}{3E}\left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right] \quad (5\text{–}8)$$

- Tension test specimen at yield has $\sigma_1 = S_y$ and $\sigma_2 = \sigma_3 = 0$
- Applying to Eq. (5–8), distortion energy for tension test specimen is

$$u_d = \frac{1+\nu}{3E}S_y^2 \quad (5\text{–}9)$$

- DE theory predicts failure when distortion energy, Eq. (5–8), exceeds distortion energy of tension test specimen, Eq. (5–9)

$$\left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2} \geq S_y \quad (5\text{–}10)$$

Von Mises Stress

$$\left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2} \geq S_y \qquad (5\text{–}10)$$

- Left hand side is defined as *von Mises stress*

$$\sigma' = \left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2} \qquad (5\text{–}12)$$

- For plane stress, simplifies to

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} \qquad (5\text{–}13)$$

- In terms of *xyz* components, in three dimensions

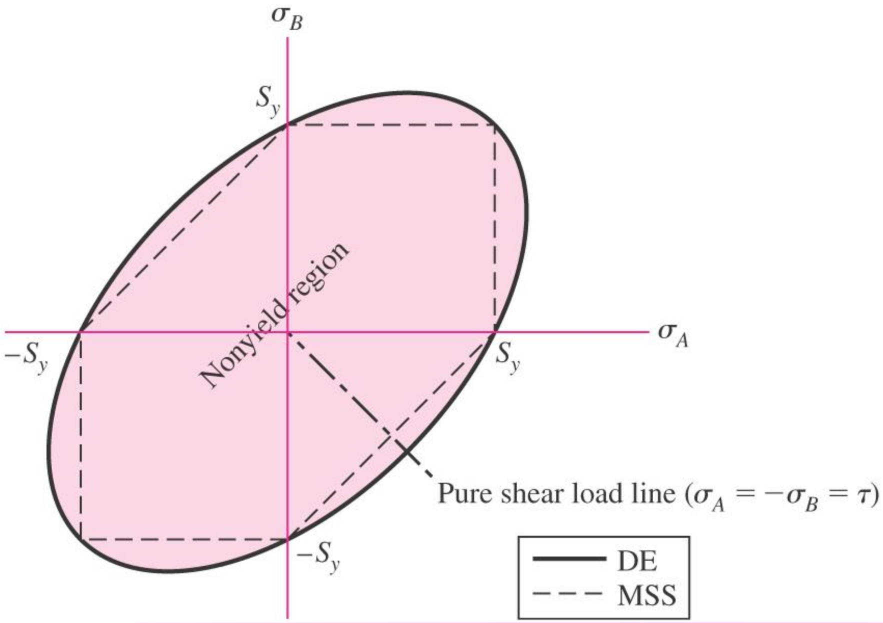
$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)\right]^{1/2} \qquad (5\text{–}14)$$

- In terms of *xyz* components, for plane stress

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2} \qquad (5\text{–}15)$$

Shear Strength Predictions

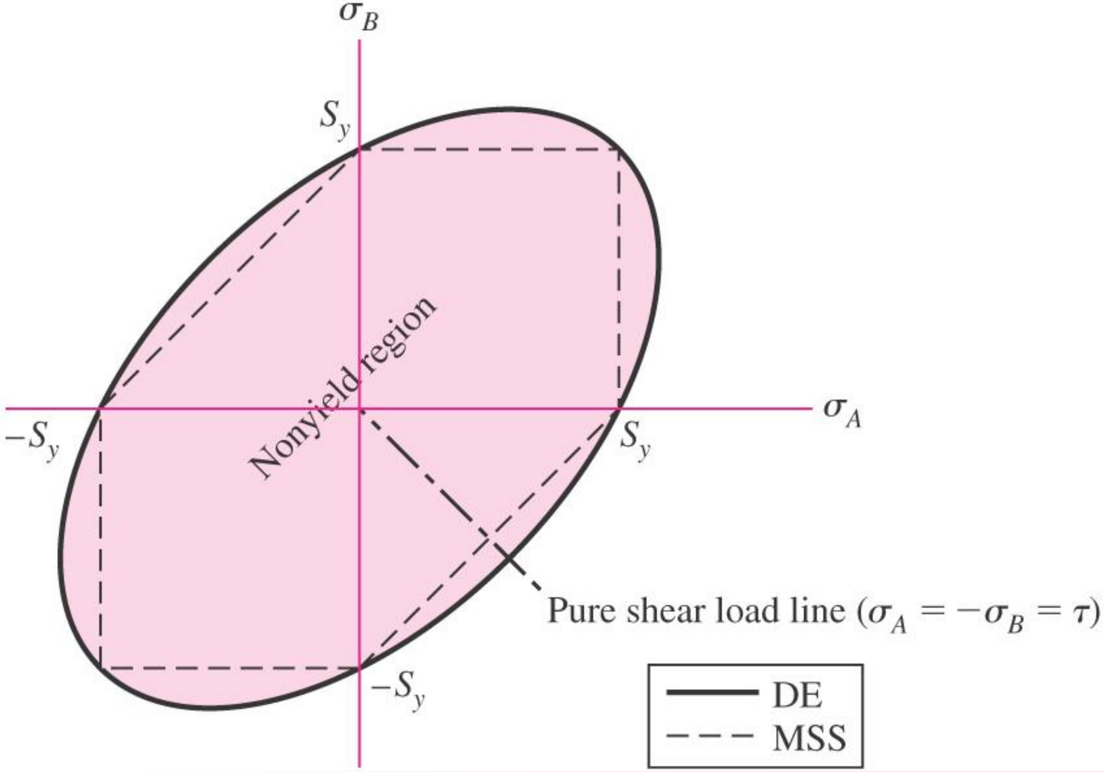
- For pure shear loading, Mohr's circle shows that $\sigma_A = -\sigma_B = \tau$
- Plotting this equation on principal stress axes gives load line for pure shear case
- Intersection of pure shear load line with failure curve indicates shear strength has been reached
- Each failure theory predicts shear strength to be some fraction of normal strength



Shear Strength Predictions

- For MSS theory, intersecting pure shear load line with failure line [Eq. (5–5)] results in

$$S_{sy} = 0.5 S_y \qquad (5\text{–}2)$$



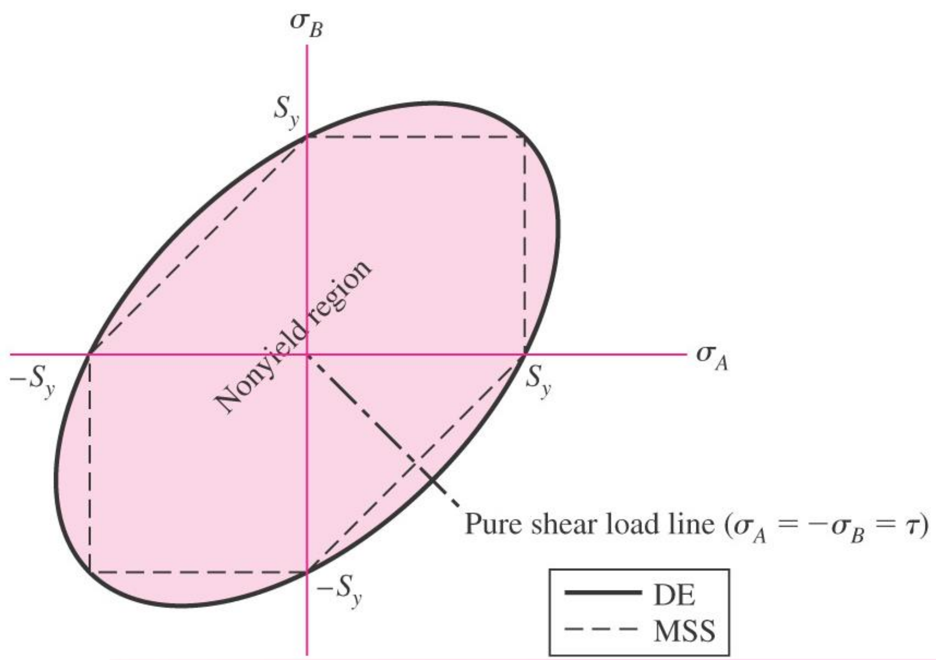
Shear Strength Predictions

- For DE theory, intersection pure shear load line with failure curve [Eq. (5–11)] gives

$$\left(3\tau_{xy}^2\right)^{1/2} = S_y \qquad \text{or} \qquad \tau_{xy} = \frac{S_y}{\sqrt{3}} = 0.577 S_y \qquad (5\text{–}20)$$

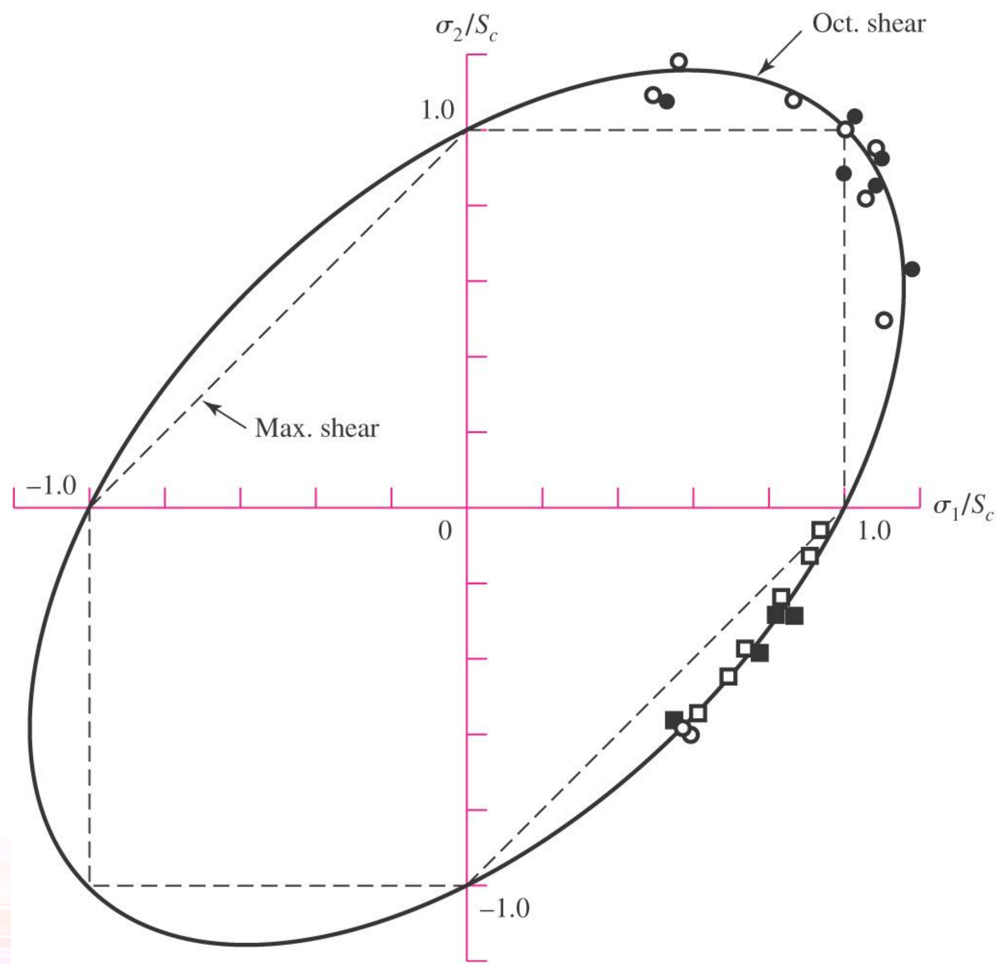
- Therefore, DE theory predicts shear strength as

$$S_{sy} = 0.577 S_y$$



DE Theory Compared to Experimental Data

- Plot von Mises stress on principal stress axes to compare to experimental data (and to other failure theories)
- DE curve is *typical* of data
- Note that *typical* equates to a 50% reliability from a design perspective
- Commonly used for analysis situations
- MSS theory useful for design situations where higher reliability is desired



Hydrostatic Stress & Volume Strain

- Small volume WxLxH, subjected to a normal strain in 3 direction

$$\varepsilon_x = \frac{dL}{L}\,;\ \varepsilon_y = \frac{dW}{W}\,;\ \varepsilon_z = \frac{dH}{H}$$

$$volume, V = LWH,$$

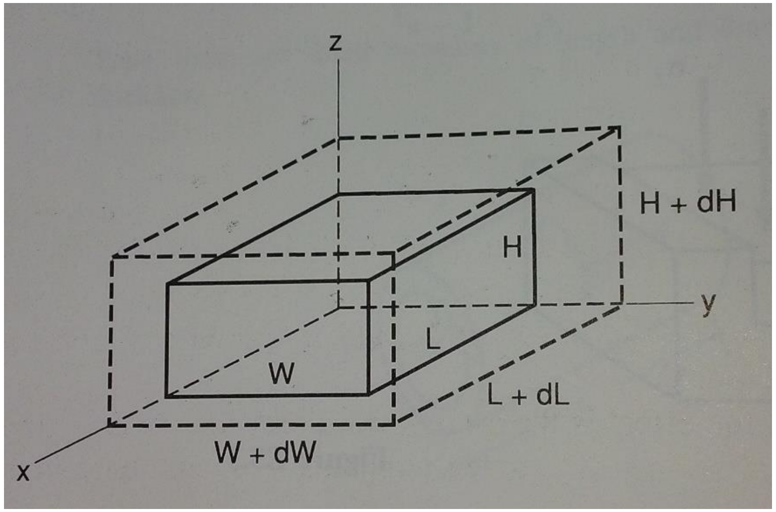
$$take\ total\ derivative\ of\ V$$

$$dV = \frac{\partial V}{\partial L}dL + \frac{\partial V}{\partial W}dW + \frac{\partial V}{\partial H}dH$$

$$divide\ both\ side\ by\ V$$

$$\frac{dV}{V} = \frac{dL}{L} + \frac{dW}{W} + \frac{dH}{H}$$

volume strain or dilitation: $\varepsilon_v \equiv \dfrac{dV}{V}$



Hydrostatic Stress & Volume Strain

- For an isotropic material

$$\begin{cases} \varepsilon_x = \dfrac{1}{E}\left[\sigma_x - v\left(\sigma_y + \sigma_z\right)\right] \\ \varepsilon_y = \dfrac{1}{E}\left[\sigma_y - v\left(\sigma_x + \sigma_z\right)\right] \\ \varepsilon_z = \dfrac{1}{E}\left[\sigma_z - v\left(\sigma_x + \sigma_y\right)\right] \end{cases}$$

$$\varepsilon_x = \frac{dL}{L}\,;\ \varepsilon_y = \frac{dW}{W}\,;\ \varepsilon_z = \frac{dH}{H}$$

$$\varepsilon_v = \frac{1-2v}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

- For hydrostatic stress,

$$\sigma_x = \sigma_y = \sigma_z = \sigma_{av}$$

$$\varepsilon_v = \frac{1-2v}{E}\left(3\sigma_{av}\right)$$

Bulk modulus

$$B = \frac{\sigma_{av}}{\varepsilon_v} = \frac{E}{3\left(1-2v\right)}$$

Example 5-1

A hot-rolled steel has a yield strength of $S_{yt} = S_{yc} = 100$ kpsi and a true strain at fracture of $\varepsilon_f = 0.55$. Estimate the factor of safety for the following principal stress states:

(a) $\sigma_x = 70$ kpsi, $\sigma_y = 70$ kpsi, $\tau_{xy} = 0$ kpsi

(b) $\sigma_x = 60$ kpsi, $\sigma_y = 40$ kpsi, $\tau_{xy} = -15$ kpsi

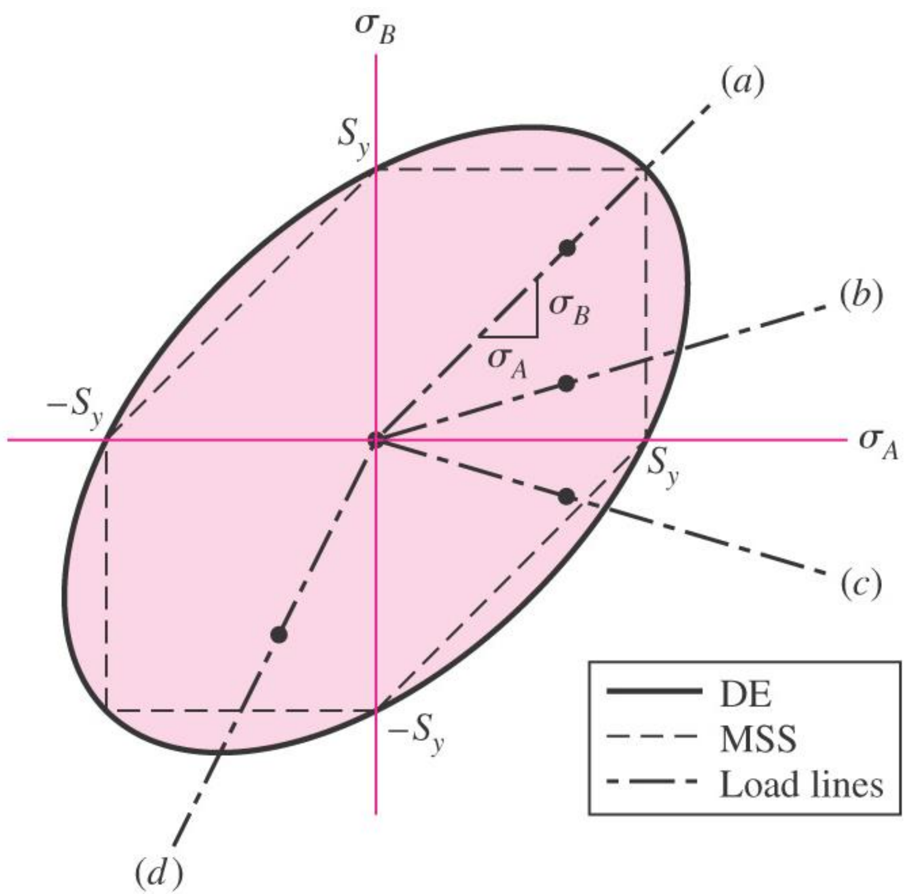
(c) $\sigma_x = 0$ kpsi, $\sigma_y = 40$ kpsi, $\tau_{xy} = 45$ kpsi

(d) $\sigma_x = -40$ kpsi, $\sigma_y = -60$ kpsi, $\tau_{xy} = 15$ kpsi

(e) $\sigma_1 = 30$ kpsi, $\sigma_2 = 30$ kpsi, $\sigma_3 = 30$ kpsi

Example 5-1

For each case, except case (*e*), the coordinates and load lines in the σ_A, σ_B plane are shown in Fig. 5–11. Case (*e*) is not plane stress. Note that the load line for case (*a*) is the only plane stress case given in which the two theories agree, thus giving the same factor of safety.

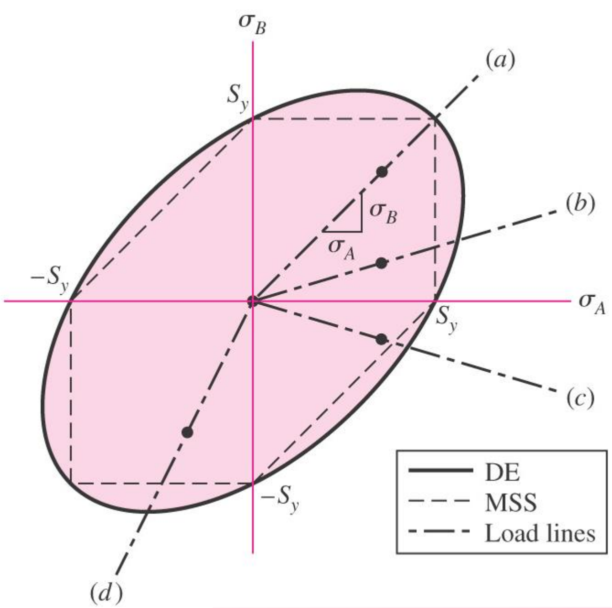


Example 5-1

A tabular summary of the factors of safety is included for comparisons.

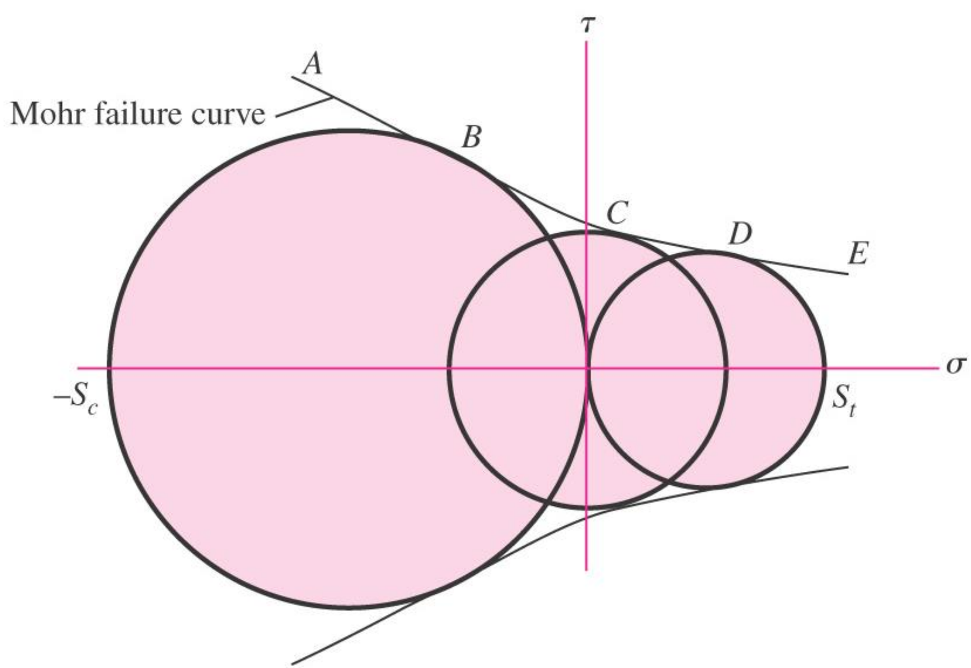
	(a)	(b)	(c)	(d)	(e)
DE	1.43	1.70	1.14	1.70	∞
MSS	1.43	1.47	1.02	1.47	∞

Since the MSS theory is on or within the boundary of the DE theory, it will always predict a factor of safety equal to or less than the DE theory, as can be seen in the table.



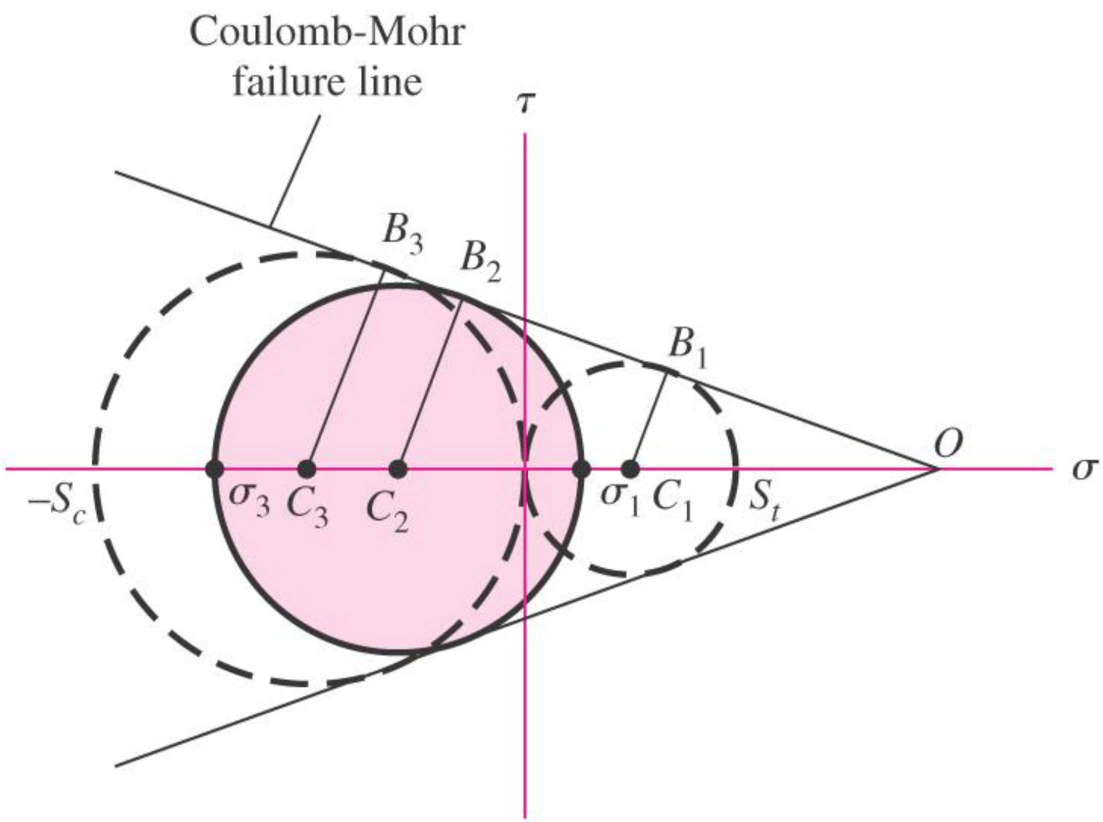
Mohr Theory

- Some materials have compressive strengths different from tensile strengths
- *Mohr theory* is based on three simple tests: tension, compression, and shear
- Plotting Mohr's circle for each, bounding curve defines failure envelope



Coulomb-Mohr Theory

- Curved failure curve is difficult to determine analytically
- *Coulomb-Mohr theory* simplifies to linear failure envelope using only tension and compression tests (dashed circles)



Coulomb-Mohr Theory

- From the geometry, derive the failure criteria

$$\frac{B_2C_2 - B_1C_1}{OC_2 - OC_1} = \frac{B_3C_3 - B_1C_1}{OC_3 - OC_1}$$

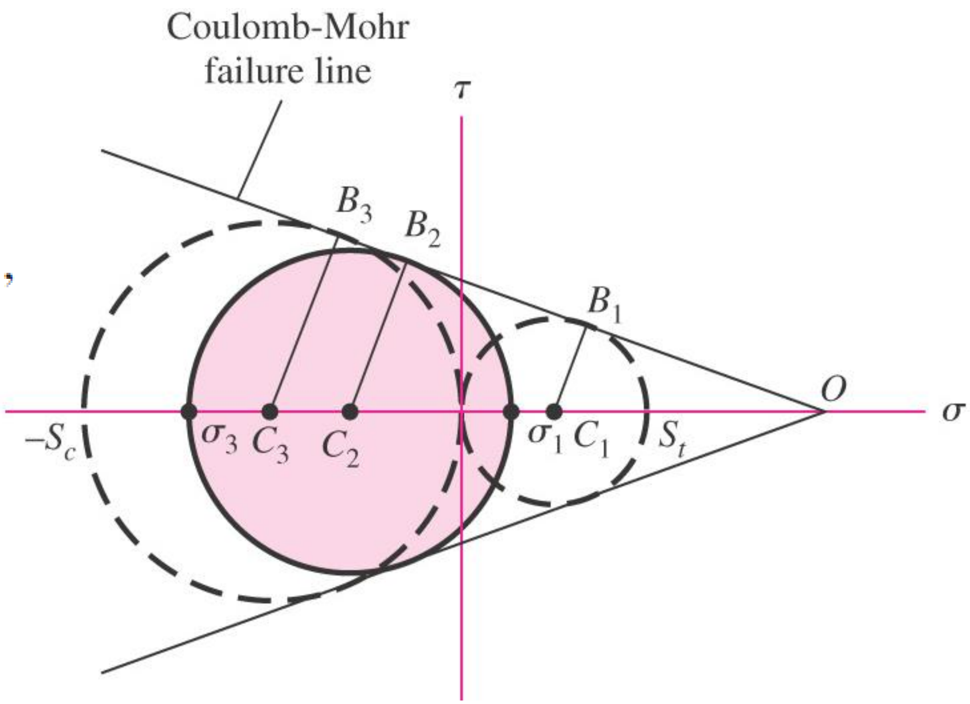
$$\frac{B_2C_2 - B_1C_1}{C_1C_2} = \frac{B_3C_3 - B_1C_1}{C_1C_3}$$

$B_1C_1 = S_t/2$, $B_2C_2 = (\sigma_1 - \sigma_3)/2$,

and $B_3C_3 = S_c/2$

$$\frac{\dfrac{\sigma_1 - \sigma_3}{2} - \dfrac{S_t}{2}}{\dfrac{S_t}{2} - \dfrac{\sigma_1 + \sigma_3}{2}} = \frac{\dfrac{S_c}{2} - \dfrac{S_t}{2}}{\dfrac{S_t}{2} + \dfrac{S_c}{2}}$$

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1 \qquad (5\text{–}22)$$



Coulomb-Mohr Theory

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1$$

- To plot on principal stress axes, consider three cases
- Case 1: $\sigma_A \geq \sigma_B \geq 0$ For this case, $\sigma_1 = \sigma_A$ and $\sigma_3 = 0$
  - Eq. (5−22) reduces to

$$\sigma_A \geq S_t \qquad (5\text{–}23)$$

- Case 2: $\sigma_A \geq 0 \geq \sigma_B$ For this case, $\sigma_1 = \sigma_A$ and $\sigma_3 = \sigma_B$
  - Eq. (5-22) reduces to

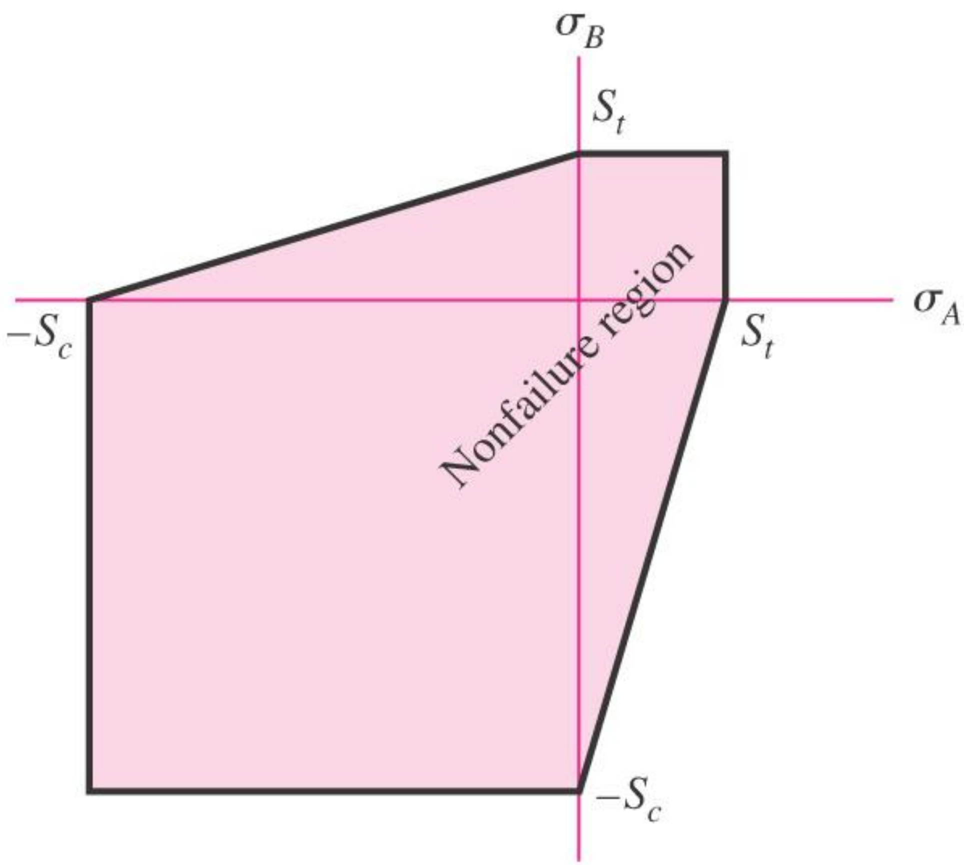
$$\frac{\sigma_A}{S_t} - \frac{\sigma_B}{S_c} \geq 1 \qquad (5\text{–}24)$$

- Case 3: $0 \geq \sigma_A \geq \sigma_B$ For this case, $\sigma_1 = 0$ and $\sigma_3 = \sigma_B$
  - Eq. (5−22) reduces to

$$\sigma_B \leq -S_c \qquad (5\text{–}25)$$

Coulomb-Mohr Theory

- Plot three cases on principal stress axes
- Similar to MSS theory, except with different strengths for compression and tension



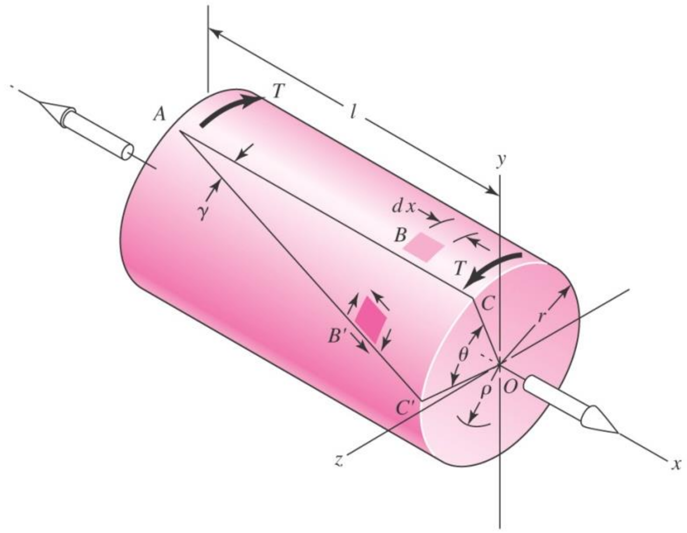
Coulomb-Mohr Theory

- Intersect the pure shear load line with the failure line to determine the shear strength
- Since failure line is a function of tensile and compressive strengths, shear strength is also a function of these terms.

$$S_{sy} = \frac{S_{yt} S_{yc}}{S_{yt} + S_{yc}} \tag{5–27}$$

Example 5-2

A 25-mm-diameter shaft is statically torqued to 230 N · m. It is made of cast 195-T6 aluminum, with a yield strength in tension of 160 MPa and a yield strength in compression of 170 MPa. It is machined to final diameter. Estimate the factor of safety of the shaft.



Example 5-4

The cantilevered tube shown in Fig. 5–17 is to be made of 2014 aluminum alloy treated to obtain a specified minimum yield strength of 276 MPa. We wish to select a stock-size tube from Table A–8 using a design factor $n_d = 4$. The bending load is $F = 1.75$ kN, the axial tension is $P = 9.0$ kN, and the torsion is $T = 72$ N · m. What is the realized factor of safety?

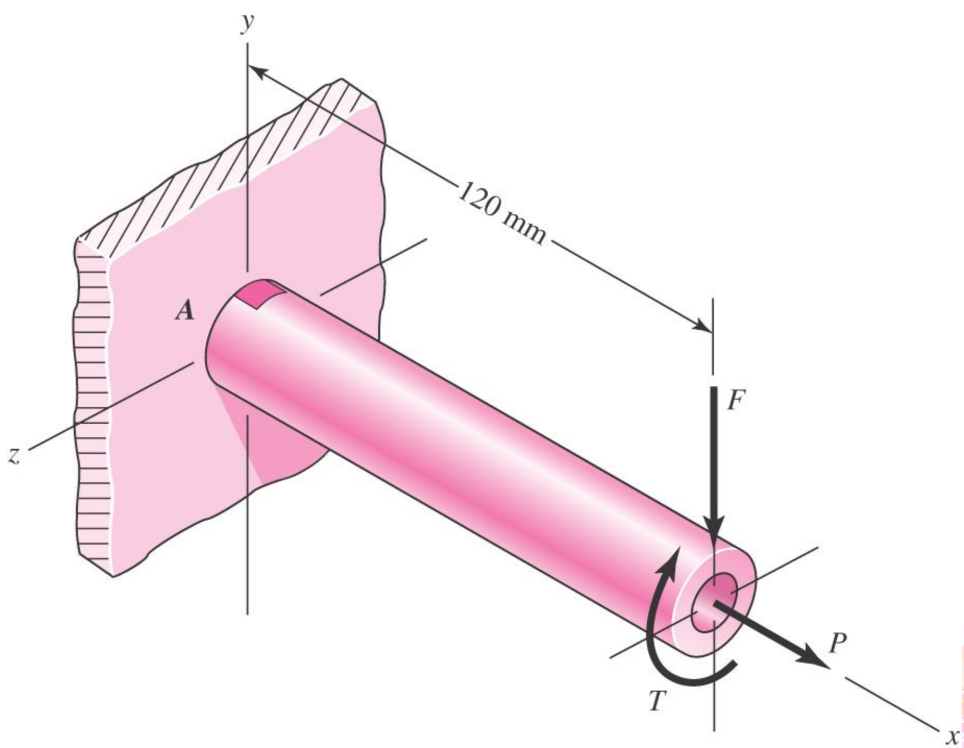


Table A-8: Properties of Round Tubing

w_a = unit weight of aluminum tubing, lbf/ft
w_s = unit weight of steel tubing, lbf/ft
m = unit mass, kg/m
A = area, in^2 (cm^2)
I = second moment of area, in^4 (cm^4)
J = second polar moment of area, in^4 (cm^4)
k = radius of gyration, in (cm)
Z = section modulus, in^3 (cm^3)
d, t = size (OD) and thickness, in (mm)

Size, in	w_a	w_s	A	I	k	Z	J
$1 \times \frac{1}{8}$	0.416	1.128	0.344	0.034	0.313	0.067	0.067
$1 \times \frac{1}{4}$	0.713	2.003	0.589	0.046	0.280	0.092	0.092
$1\frac{1}{2} \times \frac{1}{8}$	0.653	1.769	0.540	0.129	0.488	0.172	0.257
$1\frac{1}{2} \times \frac{1}{4}$	1.188	3.338	0.982	0.199	0.451	0.266	0.399
$2 \times \frac{1}{8}$	0.891	2.670	0.736	0.325	0.664	0.325	0.650
$2 \times \frac{1}{4}$	1.663	4.673	1.374	0.537	0.625	0.537	1.074
$2\frac{1}{2} \times \frac{1}{8}$	1.129	3.050	0.933	0.660	0.841	0.528	1.319
$2\frac{1}{2} \times \frac{1}{4}$	2.138	6.008	1.767	1.132	0.800	0.906	2.276
$3 \times \frac{1}{4}$	2.614	7.343	2.160	2.059	0.976	1.373	4.117
$3 \times \frac{3}{8}$	3.742	10.51	3.093	2.718	0.938	1.812	5.436
$4 \times \frac{3}{16}$	2.717	7.654	2.246	4.090	1.350	2.045	8.180
$4 \times \frac{3}{8}$	5.167	14.52	4.271	7.090	1.289	3.544	14.180

Size, mm	m	A	I	k	Z	J
12×2	0.490	0.628	0.082	0.361	0.136	0.163
16×2	0.687	0.879	0.220	0.500	0.275	0.440
16×3	0.956	1.225	0.273	0.472	0.341	0.545
20×4	1.569	2.010	0.684	0.583	0.684	1.367
25×4	2.060	2.638	1.508	0.756	1.206	3.015
25×5	2.452	3.140	1.669	0.729	1.336	3.338
30×4	2.550	3.266	2.827	0.930	1.885	5.652
30×5	3.065	3.925	3.192	0.901	2.128	6.381
42×4	3.727	4.773	8.717	1.351	4.151	17.430
42×5	4.536	5.809	10.130	1.320	4.825	20.255
50×4	4.512	5.778	15.409	1.632	6.164	30.810
50×5	5.517	7.065	18.118	1.601	7.247	36.226

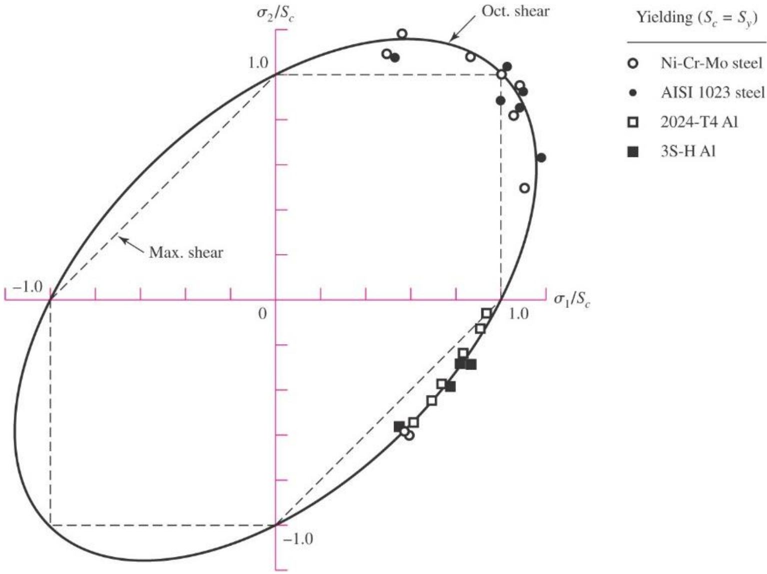
Failure of Ductile Materials Summary

$S_{yt}=S_{yc}$

- For design purpose, MSS is easy, quick to use and conservative.
- Distortion energy is a better predictor of failure
  - The best to learn why a part fail
  - A larger factor of safety may be warranted

$S_{yt}\neq S_{yc}$

- Mohr theory is the best
  - Require 3 modes of test
  - Graphical construction of failure locus
- Coulomb-Mohr theory is easier
  - Require tension and compression test
  - Easier to construct failure locus
  - Easier to use, calculate



Failure Theories for Brittle Materials

- Experimental data indicates some differences in failure for brittle materials.
- Failure criteria is generally ultimate fracture rather than yielding
- Compressive strengths are usually larger than tensile strengths

Maximum Normal Stress Theory

- Theory: Failure occurs when the maximum principal stress in a stress element exceeds the strength.
- Predicts failure when

$$\sigma_1 \geq S_{ut} \qquad \text{or} \qquad \sigma_3 \leq -S_{uc} \tag{5–28}$$

- For plane stress,

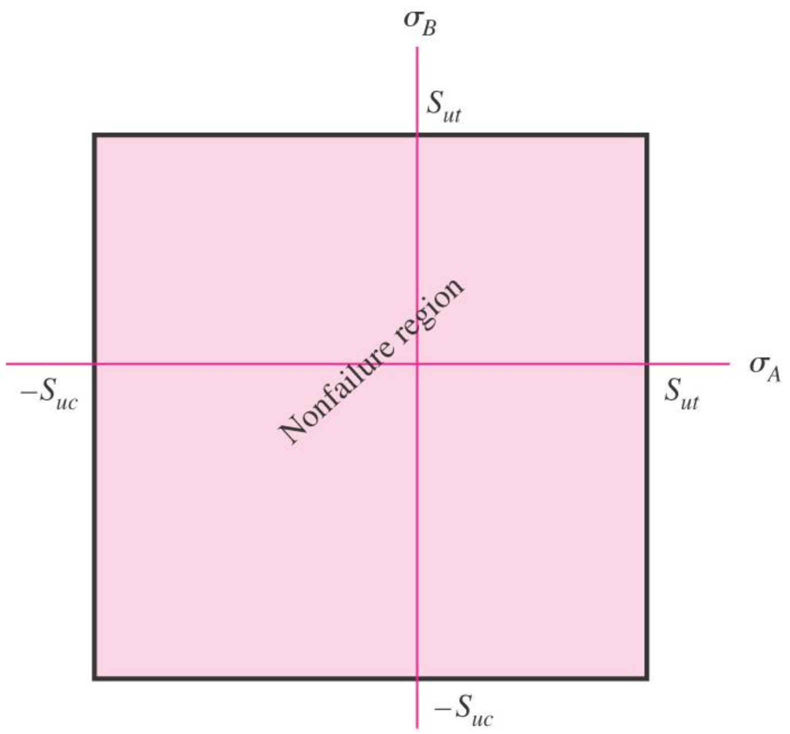
$$\sigma_A \geq S_{ut} \qquad \text{or} \qquad \sigma_B \leq -S_{uc} \tag{5–29}$$

- Incorporating design factor,

$$\sigma_A = \frac{S_{ut}}{n} \qquad \text{or} \qquad \sigma_B = -\frac{S_{uc}}{n} \tag{5–30}$$

Maximum Normal Stress Theory

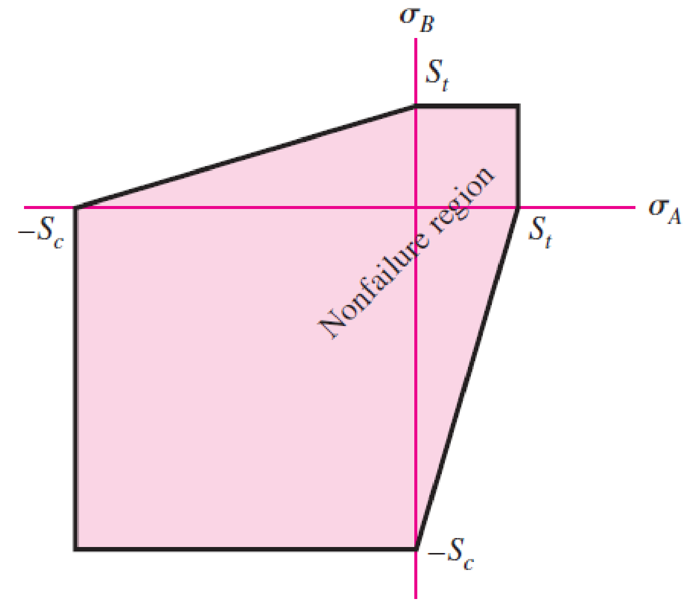
- Plot on principal stress axes
- Unsafe in part of fourth quadrant
- Not recommended for use
- Included for historical comparison



Brittle Coulomb-Mohr (BCM)

- Same as previously derived, using ultimate strengths for failure
- Failure equations dependent on quadrant

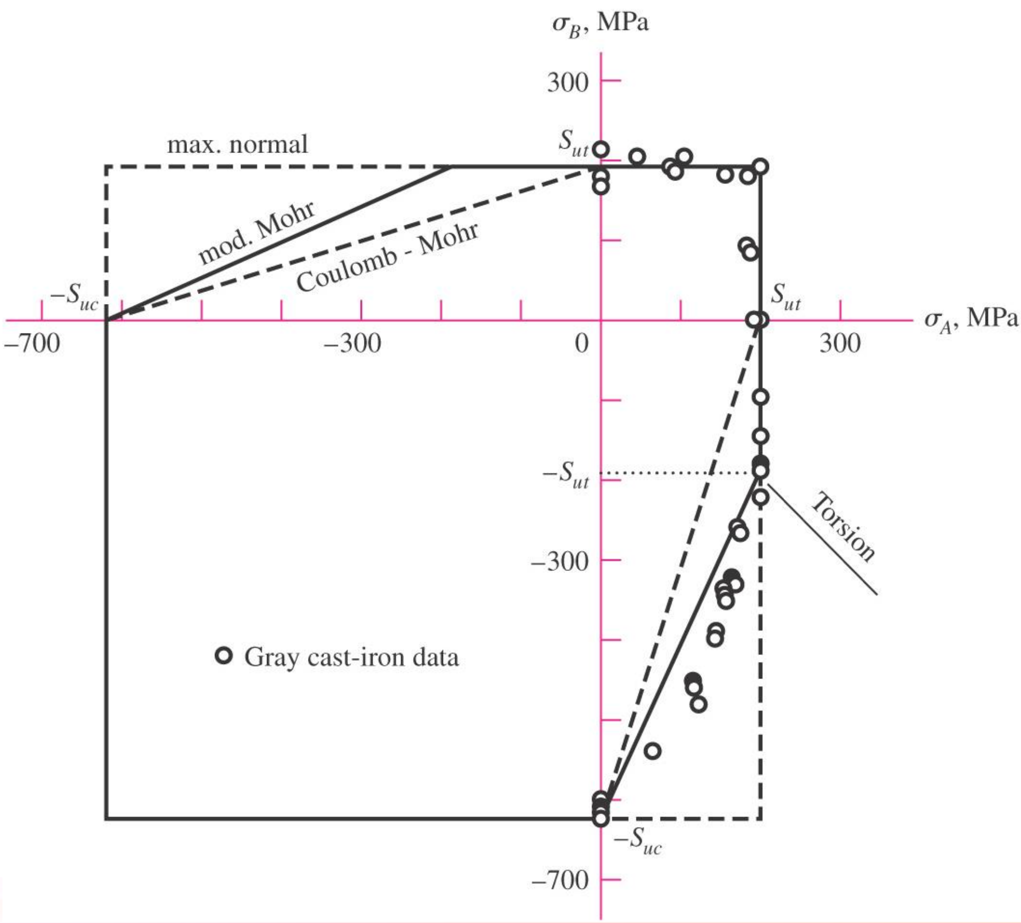
Quadrant condition	Failure criteria	
$\sigma_A \geq \sigma_B \geq 0$	$\sigma_A = \dfrac{S_{ut}}{n}$	(5–31*a*)
$\sigma_A \geq 0 \geq \sigma_B$	$\dfrac{\sigma_A}{S_{ut}} - \dfrac{\sigma_B}{S_{uc}} = \dfrac{1}{n}$	(5–31*b*)
$0 \geq \sigma_A \geq \sigma_B$	$\sigma_B = -\dfrac{S_{uc}}{n}$	(5–31*c*)



- Coulomb-Mohr
- For BCM, only need to change St, Sc to Sut, Suc

Brittle Failure Experimental Data

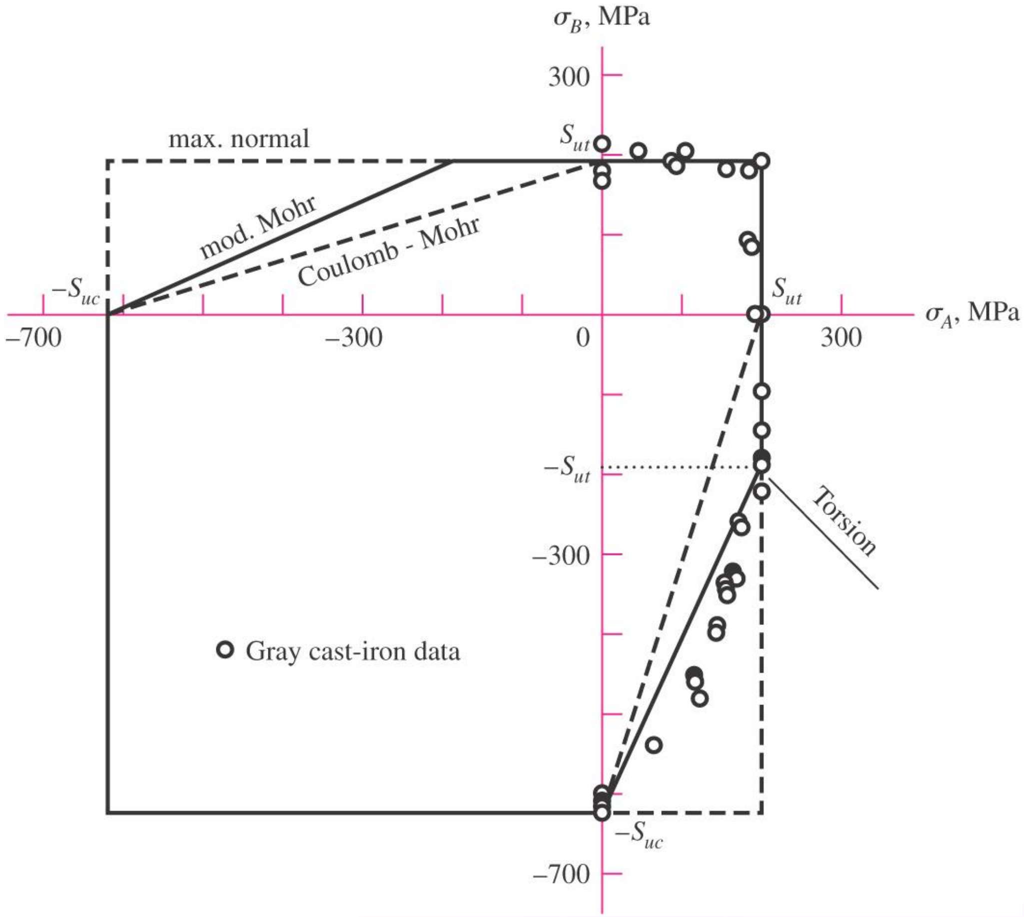
- Coulomb-Mohr is conservative in 4th quadrant
- *Modified Mohr* criteria adjusts to better fit the data in the 4th quadrant



Modified-Mohr

Quadrant condition	Failure criteria	
$\sigma_A \geq \sigma_B \geq 0$	$\sigma_A = \dfrac{S_{ut}}{n}$	(5–32*a*)
$\sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left\lvert \dfrac{\sigma_B}{\sigma_A} \right\rvert \leq 1$	$\sigma_A = \dfrac{S_{ut}}{n}$	(5–32*a*)
$\sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left\lvert \dfrac{\sigma_B}{\sigma_A} \right\rvert > 1$	$\dfrac{(S_{uc} - S_{ut})\,\sigma_A}{S_{uc}S_{ut}} - \dfrac{\sigma_B}{S_{uc}} = \dfrac{1}{n}$	(5–32*b*)
$0 \geq \sigma_A \geq \sigma_B$	$\sigma_B = -\dfrac{S_{uc}}{n}$	(5–32*c*)

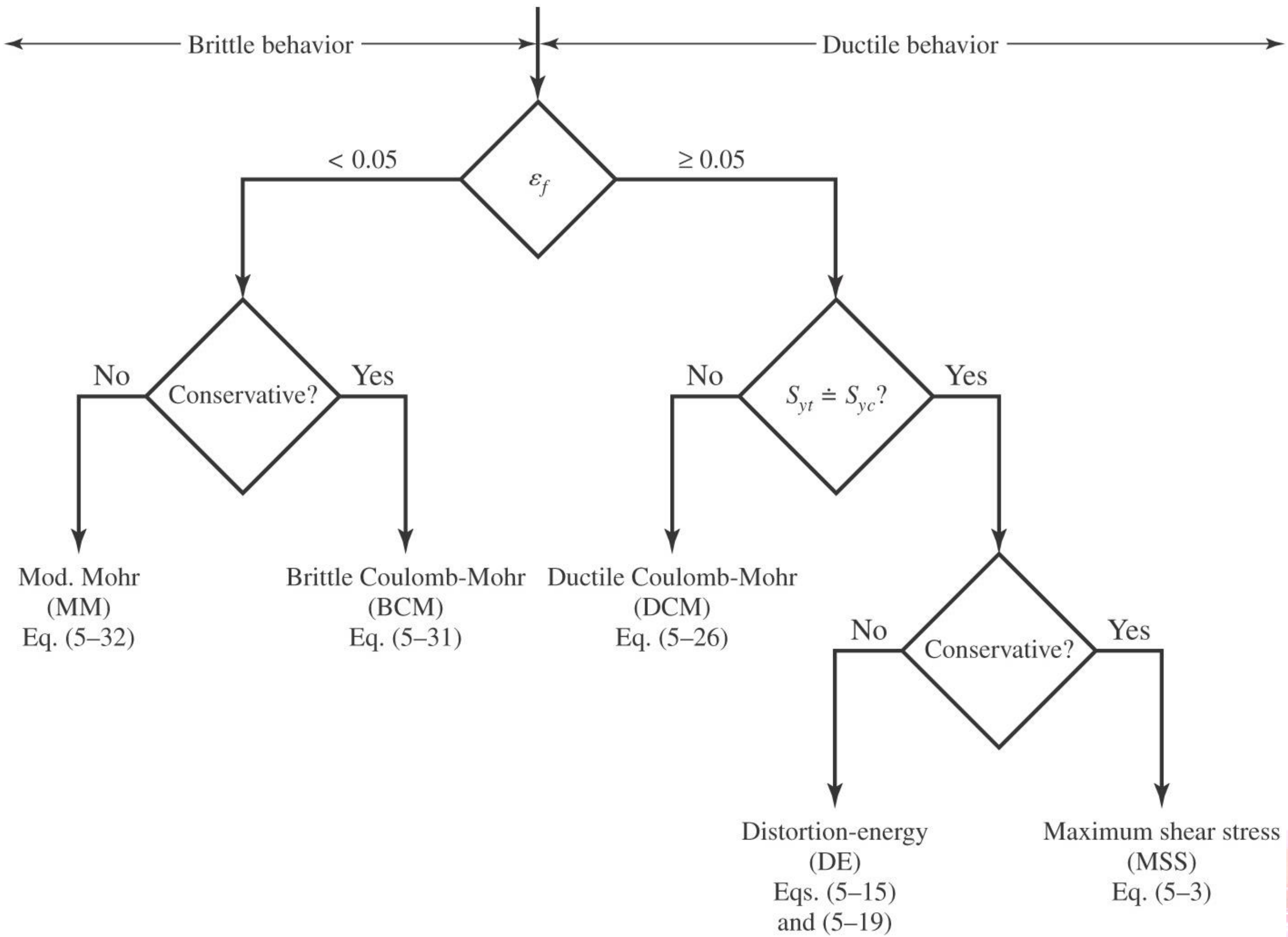
Brittle Failure Experimental Data



Selection of Failure Criteria

- First determine ductile vs. brittle
- For ductile
  - MSS is conservative, often used for design where higher reliability is desired
  - DE is typical, often used for analysis where agreement with experimental data is desired
  - If tensile and compressive strengths differ, use Ductile Coulomb-Mohr
- For brittle
  - Mohr theory is best, but difficult to use
  - Brittle Coulomb-Mohr is very conservative in 4^{th} quadrant
  - Modified Mohr is still slightly conservative in 4^{th} quadrant, but closer to typical

Selection of Failure Criteria in Flowchart Form



Introduction to Fracture Mechanics

- *Linear elastic fracture mechanics* (*LEFM*) analyzes crack growth during service
- Assumes cracks can exist before service begins, e.g. flaw, inclusion, or defect
- Attempts to model and predict the growth of a crack
- Stress concentration approach is inadequate when notch radius becomes extremely sharp, as in a crack, since stress concentration factor approaches infinity
- Ductile materials often can neglect effect of crack growth, since local plastic deformation blunts sharp cracks
- *Relatively brittle* materials, such as glass, hard steels, strong aluminum alloys, and steel below the ductile-to-brittle transition temperature, benefit from fracture mechanics analysis

Stress Concentration Factors vs Cracks

Figure A–15–3

Notched rectangular bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

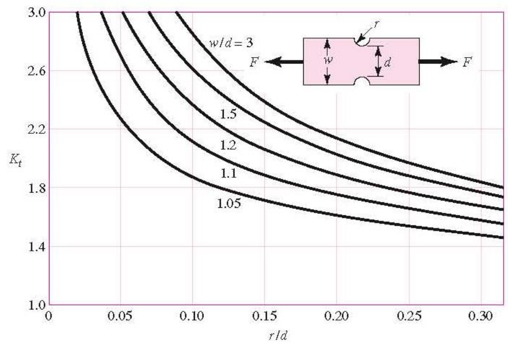


Figure A–15–5

Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

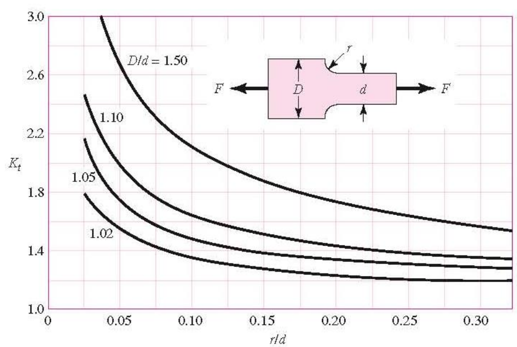
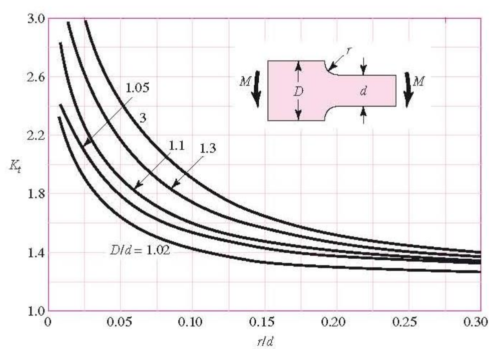
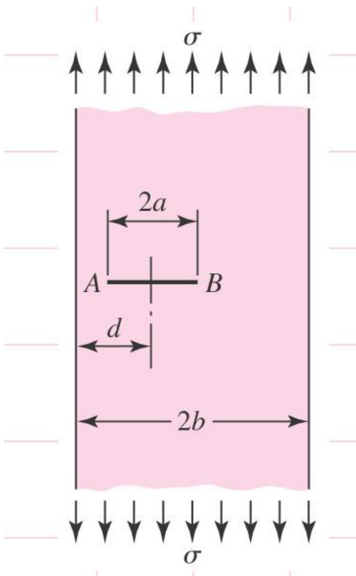


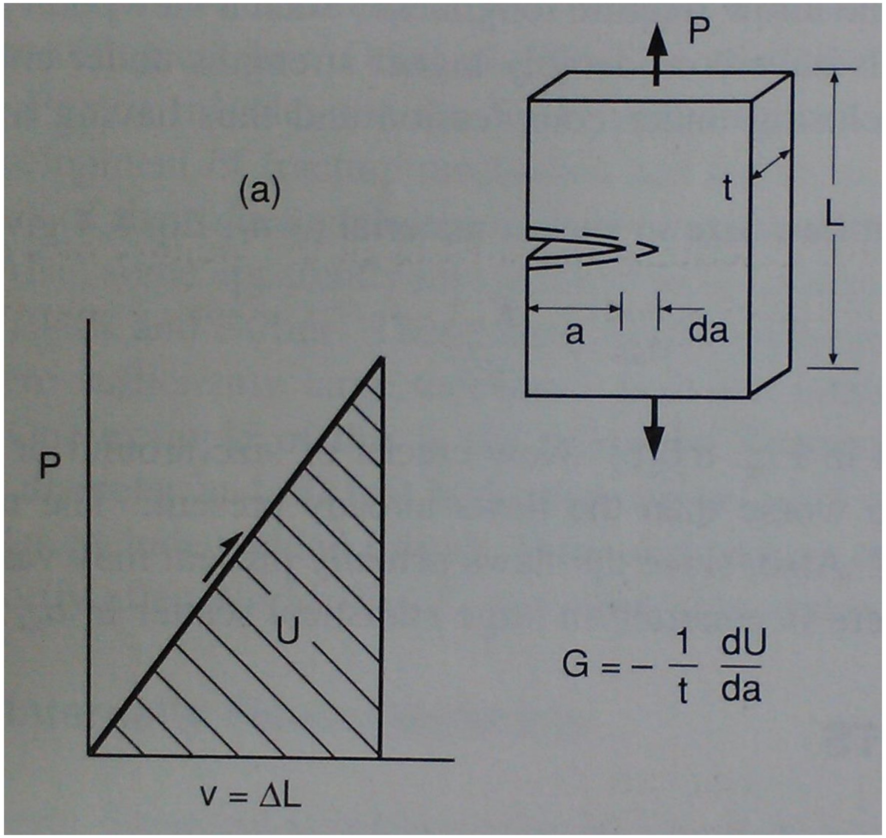
Figure A–15–6

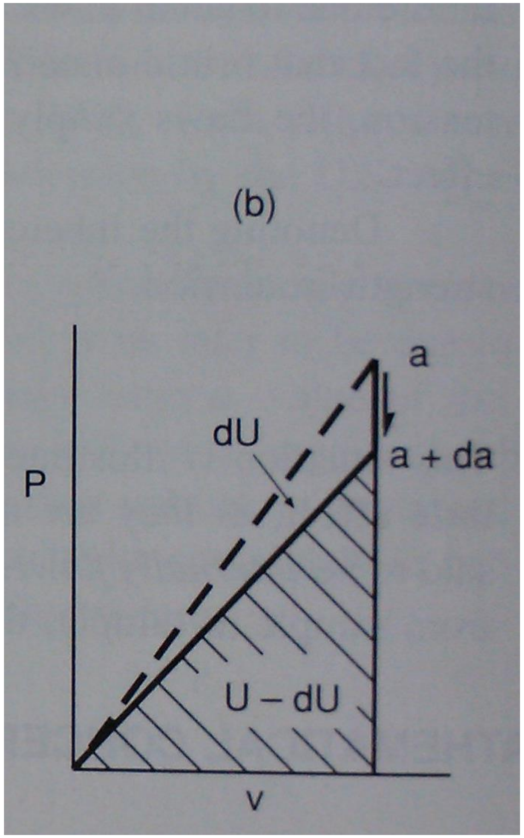
Rectangular filleted bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, t is the thickness.





Strain Energy Release Rate

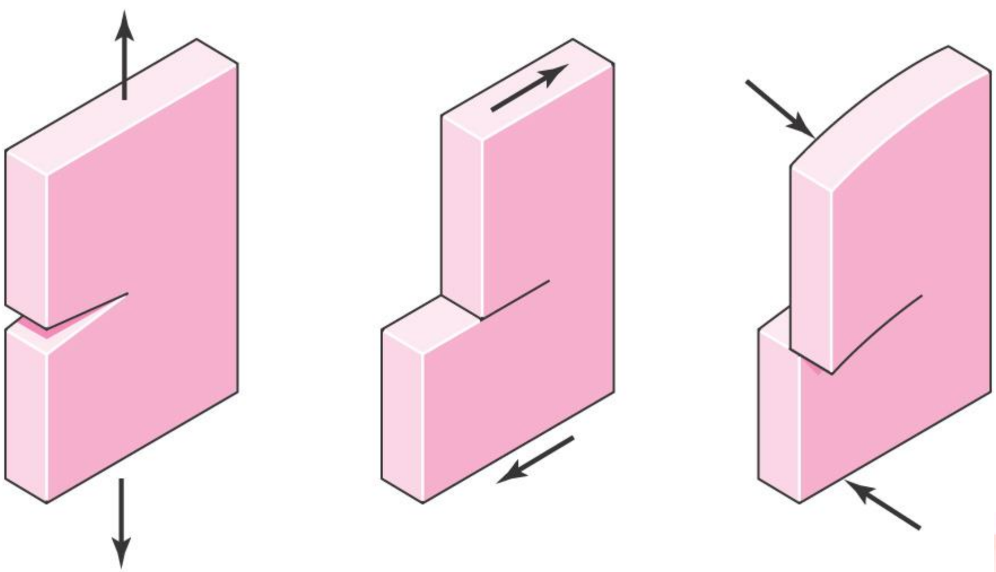




G: energy per unit crack area required to extend the crack

Crack Modes and the Stress Intensity Factor

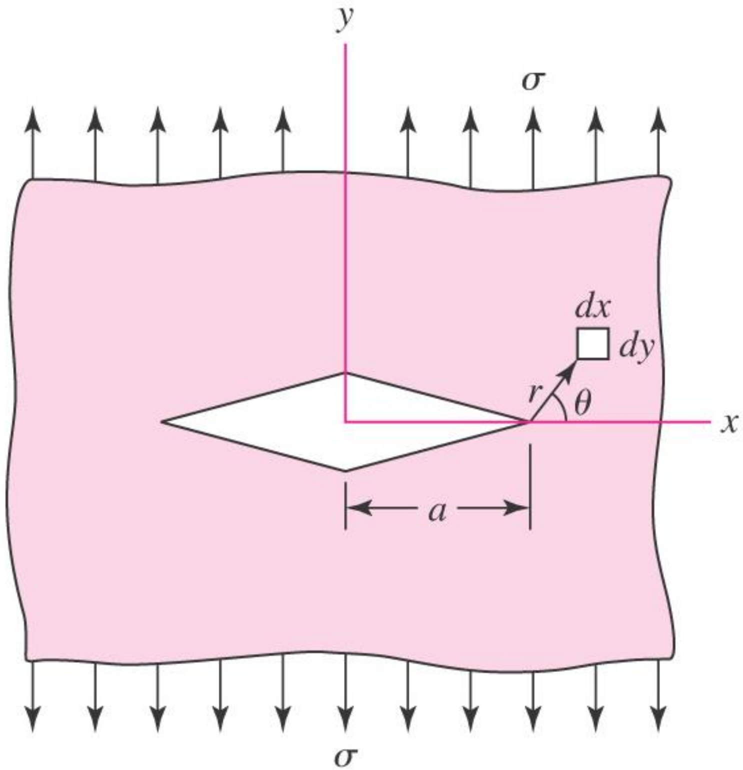
- Three distinct modes of crack propagation
  - *Mode I: Opening crack mode*, due to tensile stress field
  - *Mode II: Sliding mode*, due to in-plane shear
  - *Mode III: Tearing mode*, due to out-of-plane shear
- Combination of modes possible
- Opening crack mode is most common, and is focus of this text



(*a*) Mode I (*b*) Mode II (*c*) Mode III

Mode I Crack Model

- Stress field on *dx dy* element at crack tip



$$\sigma_x = \sigma\sqrt{\frac{a}{2r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \quad (5\text{–}34a)$$

$$\sigma_y = \sigma\sqrt{\frac{a}{2r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \quad (5\text{–}34b)$$

$$\tau_{xy} = \sigma\sqrt{\frac{a}{2r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \quad (5\text{–}34c)$$

$$\sigma_z = \begin{cases} 0 & \text{(for plane stress)} \\ \nu(\sigma_x + \sigma_y) & \text{(for plane strain)} \end{cases} \quad (5\text{–}34d)$$

Stress Intensity Factor

- Common practice to define *stress intensity factor*

$$K_I = \sigma\sqrt{\pi a} \quad (5\text{–}35)$$

- Incorporating K_I, stress field equations are

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \quad (5\text{–}36a)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \quad (5\text{–}36b)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \quad (5\text{–}36c)$$

$$\sigma_z = \begin{cases} 0 & \text{(for plane stress)} \\ \nu(\sigma_x + \sigma_y) & \text{(for plane strain)} \end{cases} \quad (5\text{–}36d)$$

Stress Intensity Modification Factor

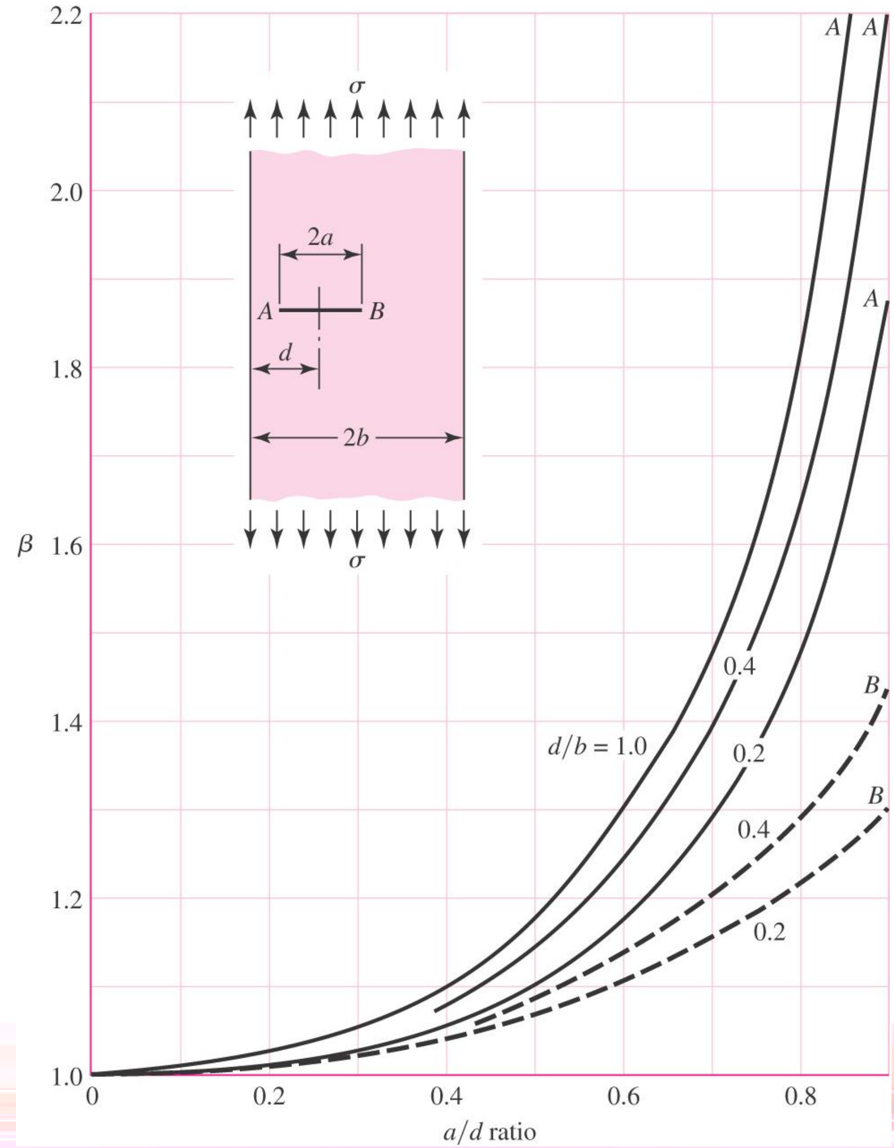
- Stress intensity factor K_I is a function of geometry, size, and shape of the crack, and type of loading
- For various load and geometric configurations, a *stress intensity modification factor* β can be incorporated

$$K_I = \beta\sigma\sqrt{\pi a} \qquad (5\text{–}37)$$

- Tables for β are available in the literature
- Figures 5–25 to 5–30 present some common configurations

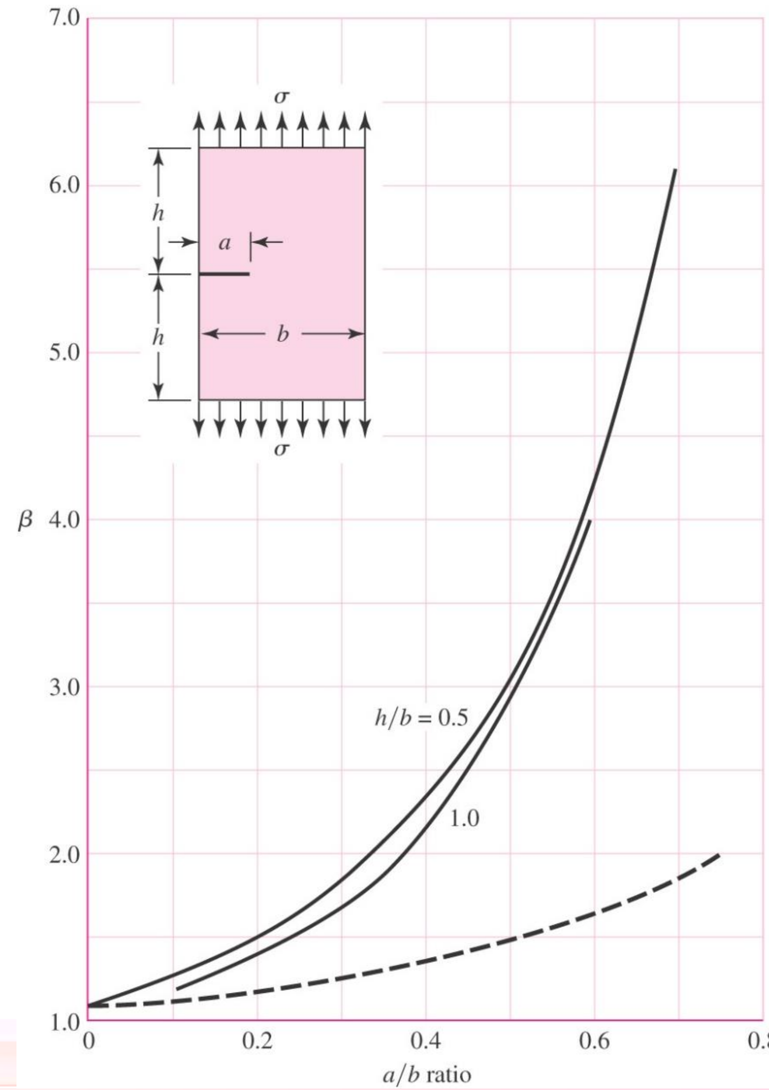
Stress Intensity Modification Factor

- Off-center crack in plate in longitudinal tension
- Solid curves are for crack tip at *A*
- Dashed curves are for tip at *B*



Stress Intensity Modification Factor

- Plate loaded in longitudinal tension with crack at edge
- For solid curve there are no constraints to bending
- Dashed curve obtained with bending constraints added



Shigley's Mechanical Engineering Design

Fracture Toughness

- Crack propagation initiates when the stress intensity factor reaches a critical value, the *critical stress intensity factor* K_{Ic}
- K_{Ic} is a material property dependent on material, crack mode, processing of material, temperature, loading rate, and state of stress at crack site
- Also know as *fracture toughness* of material
- Fracture toughness for plane strain is normally lower than for plain stress
- K_{Ic} is typically defined as *mode I, plane strain fracture toughness*

Typical Values for K_{Ic}

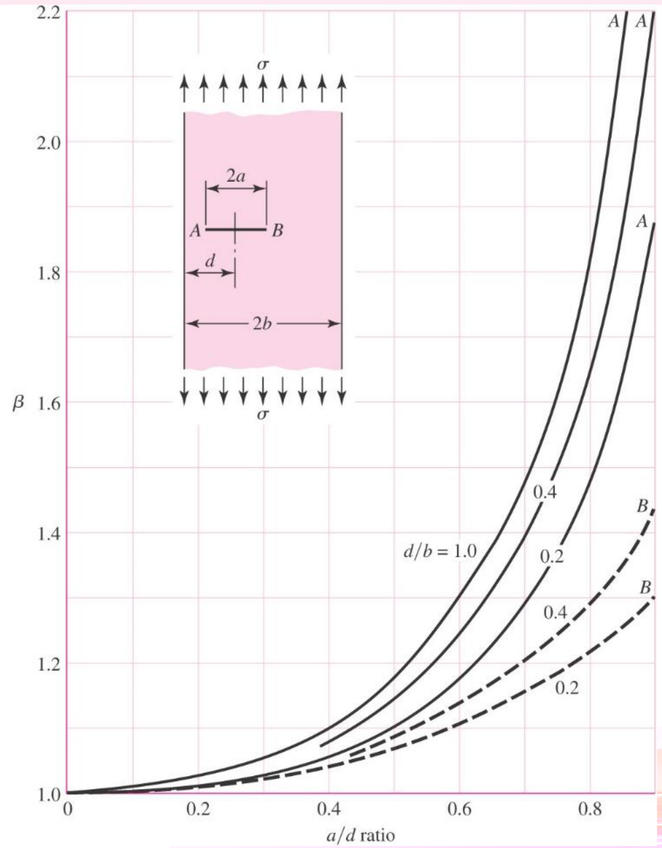
Table 5–1

Values of K_{Ic} for Some Engineering Materials at Room Temperature

Material	K_{Ic}, MPa$\sqrt{m}$	S_y, MPa
Aluminum		
2024	26	455
7075	24	495
7178	33	490
Titanium		
Ti-6AL-4V	115	910
Ti-6AL-4V	55	1035
Steel		
4340	99	860
4340	60	1515
52100	14	2070

Example 5-6

A steel ship deck plate is 30 mm thick and 12 m wide. It is loaded with a nominal uniaxial tensile stress of 50 MPa. It is operated below its ductile-to-brittle transition temperature with K_{Ic} equal to 28.3 MPa. If a 65-mm-long central transverse crack is present, estimate the tensile stress at which catastrophic failure will occur. Compare this stress with the yield strength of 240 MPa for this steel.



Example 5-7

A plate of width 1.4 m and length 2.8 m is required to support a tensile force in the 2.8-m direction of 4.0 MN. Inspection procedures will detect only through-thickness edge cracks larger than 2.7 mm. The two Ti-6AL-4V alloys in Table 5–1 are being considered for this application, for which the safety factor must be 1.3 and minimum weight is important. Which alloy should be used?