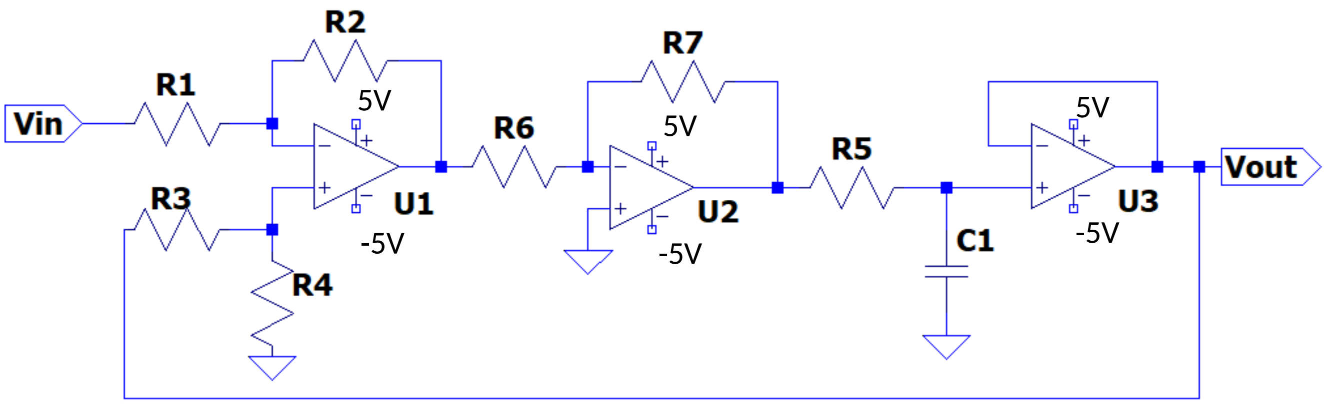
REPORT

Experiment 1: RC circuit with unit feedback



	rise time (s)	delay time (s)	steady-state error (V)
Theoretical Result			**0.5**
Experiment Result	**0.285**	**0.080**	**0.530**
Simulation Result	**0.2197**	**0.0737**	**0.5000**

注意事項:請參考投影片第 5 頁的定義

1. Calculate the **closed loop transfer function**.

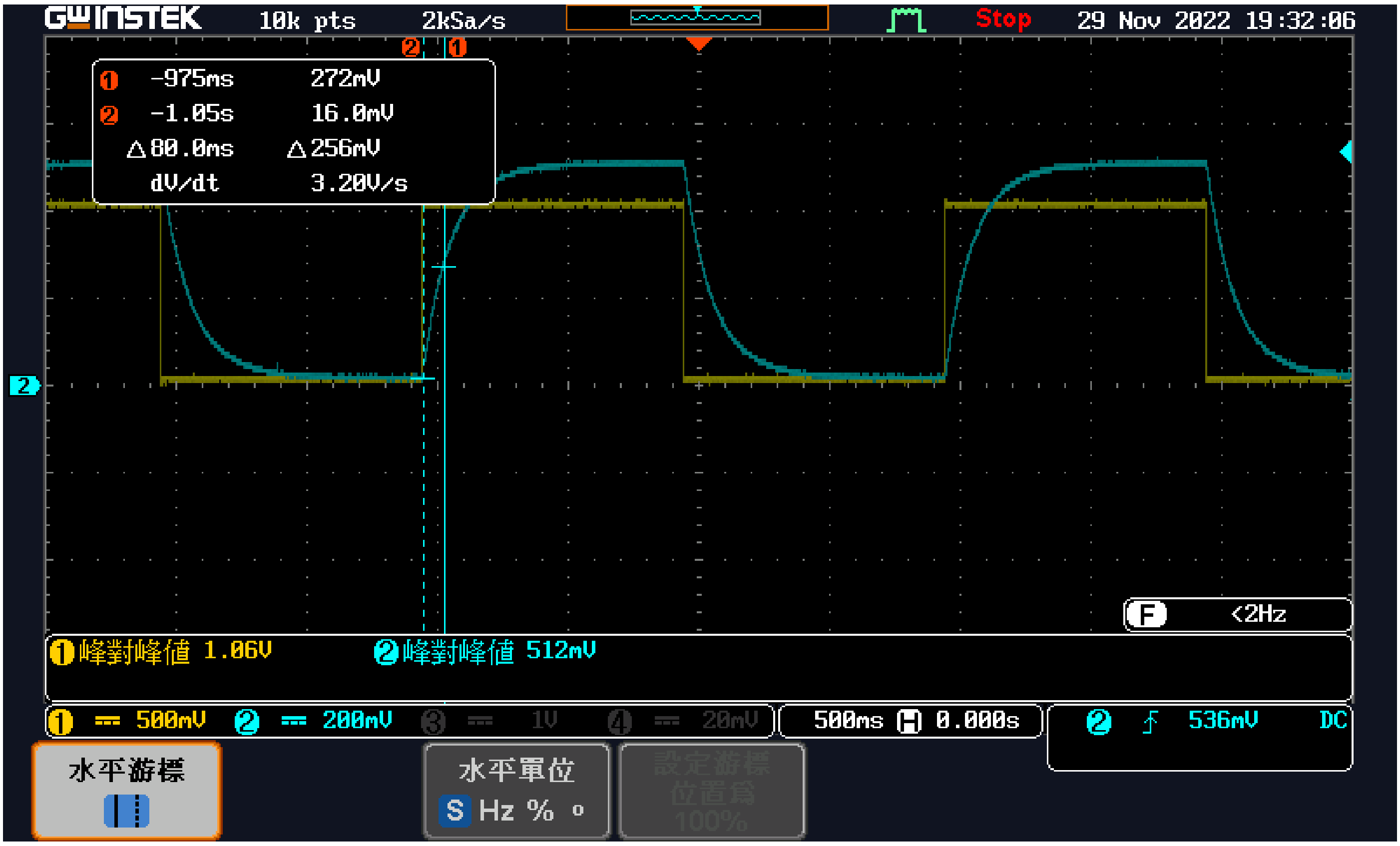
$$V_{out} = -1 \times (V_{out} - V_{¿}) \times \frac{1}{0.2s+1}$$

$$\Longrightarrow (0.2s+1)V_{out} = -V_{out} + V_{¿}$$

$$\Longrightarrow (0.2s+2)V_{out} = V_{¿}$$

$$\Longrightarrow \frac{V_{out}}{V_{¿}} = \frac{1}{0.2s+2}\ \#$$

2. V_{out} and Vin waveform (1 pic):

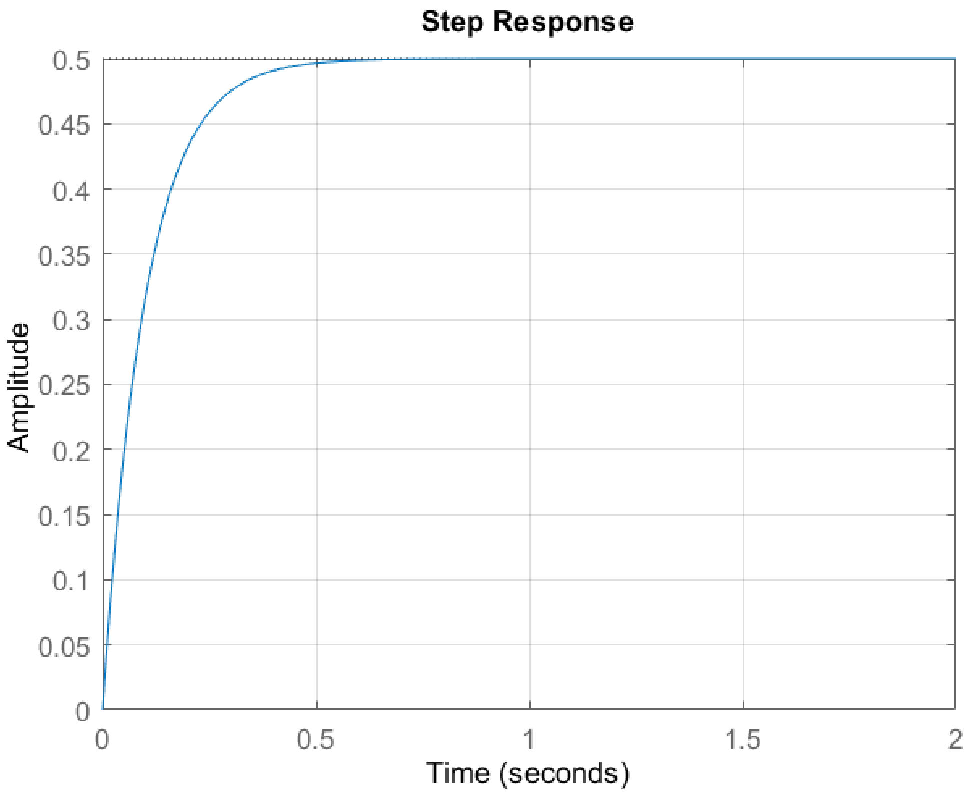

GW INSTEK
10k pts
2kSa/s
Stop
29 Nov 2022 19:32:06
1 -975ms 272mV
2 -1.05s 16.0mV
△80.0ms △256mV
dV/dt 3.20V/s
<2Hz
1 峰對峰值 1.06V
2 峰對峰值 512mV
1 500mV
2 200mV
3 1V
4 20mV
500ms 0.000s
2 536mV DC
水平游標
水平單位
S Hz % °
設定游標
位置爲
100%

3. Simulation

Closed loop transfer function from command window:

```
sys =

      1
  ---------
  0.2 s + 2
```

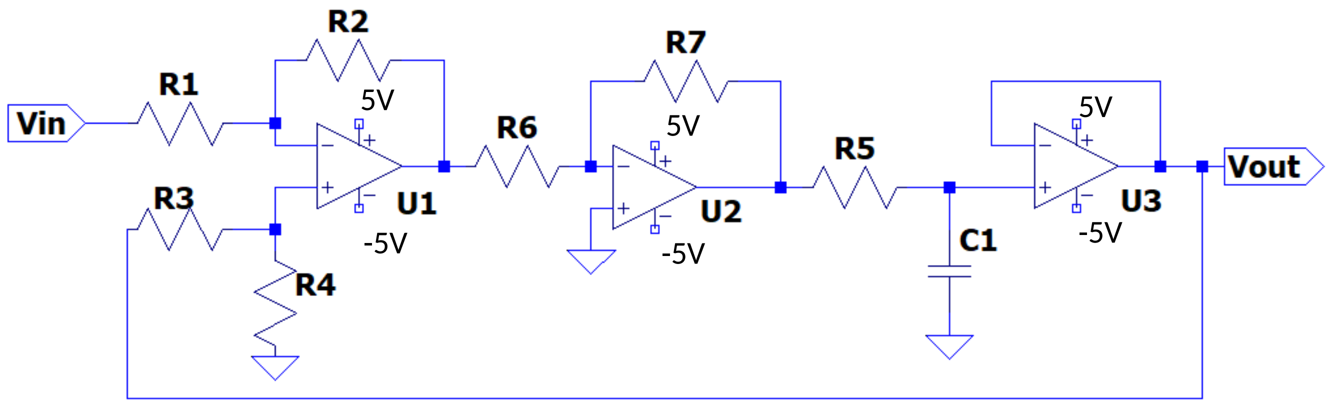
The unit step response(1 pic):




```
ans =

  struct with fields:

        RiseTime: 0.2197
    SettlingTime: 0.3912
     SettlingMin: 0.4523
     SettlingMax: 0.5000
       Overshoot: 0
      Undershoot: 0
            Peak: 0.5000
        PeakTime: 1.0546
```

```
delayTime =

    0.0737


sserror =

    0.5000
```

Experiment 2: RC circuit with P-controller and unit feedback



	rise time (s)	delay time (s)	steady-state error (V)
Theoretical Result			**0.0909**
Experiment Result	**0.050**	**0.014**	**0.1**
Simulation Result	**0.0399**	**0.0134**	**0.0909**

注意事項:請參考投影片第5頁的定義

1. Calculate K_p and the **closed loop transfer function**.

Assume V_2 is the output of U1. The relation of input and output of difference amplifier:

$$V_2 = V_{out} - V_{¿}$$

Assume V_3 is the output of U2. The relation of input and output of inverting amplifier:

$$\frac{0-V_2}{R_6} + \frac{0-V_3}{R_7} = 0$$

$$\Longrightarrow V_3 = \frac{-R_7}{R_6} V_2$$

Substitute $V_2 = V_{out} - V_{¿}$:

$$¿\frac{100}{10}\left(V_{¿} - V_{out}\right)$$

$$¿10\left(V_{¿} - V_{out}\right)$$

The relation of input and output of plant:

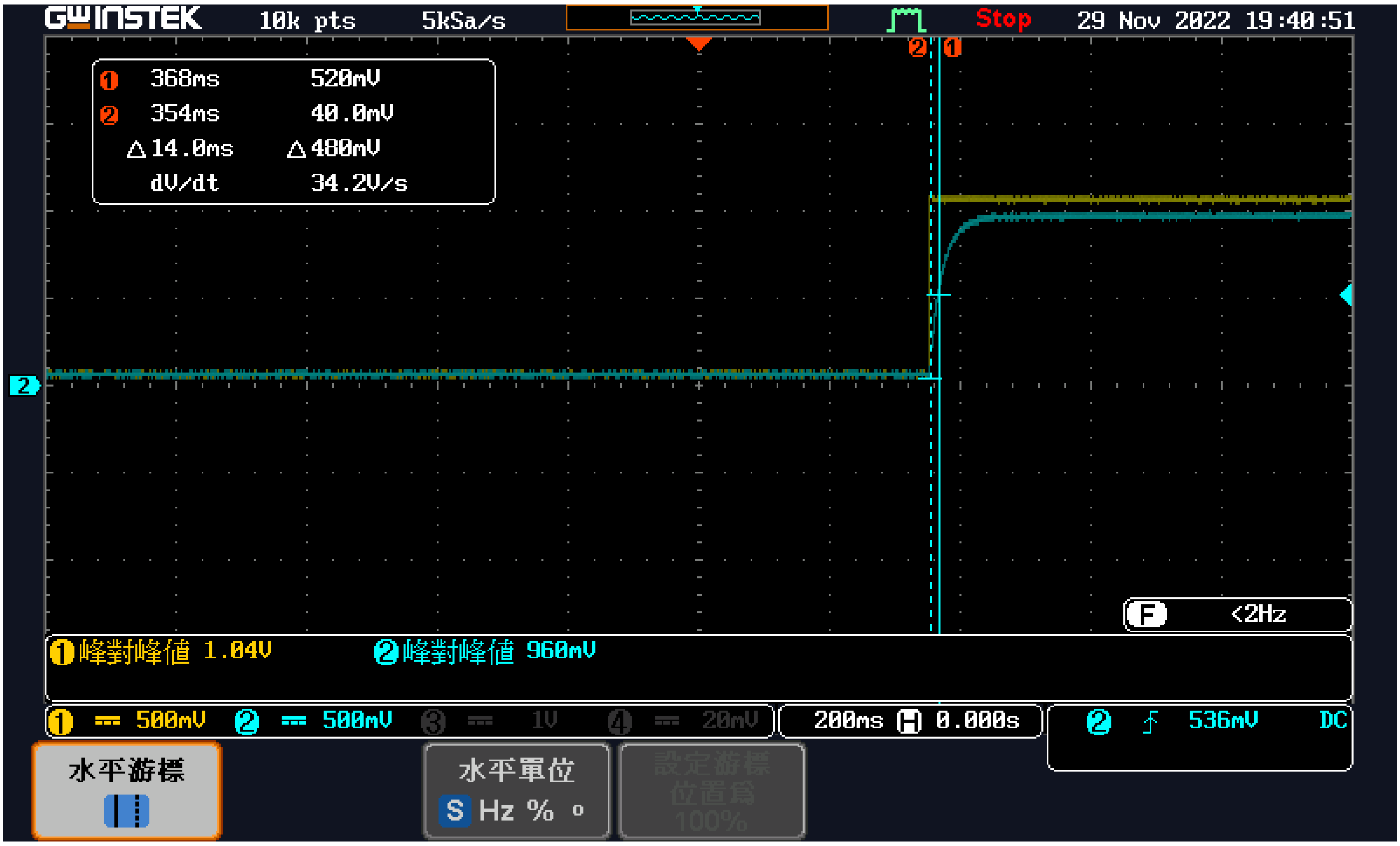
$$10V_{¿} - 10V_{out} = \left(sR_5C_1 + 1\right)V_{out}$$

$$\Longrightarrow V_{out} = \frac{10V_{¿}}{sR_5C_1 + 11}$$

$$\Longrightarrow \underline{K_p = 10}_{\#}$$

$$\Longrightarrow \underline{V_{out} = \frac{10}{0.2s + 11} V_{¿}}_{\#}$$

2. V_{out} and Vin waveform (1 pic):

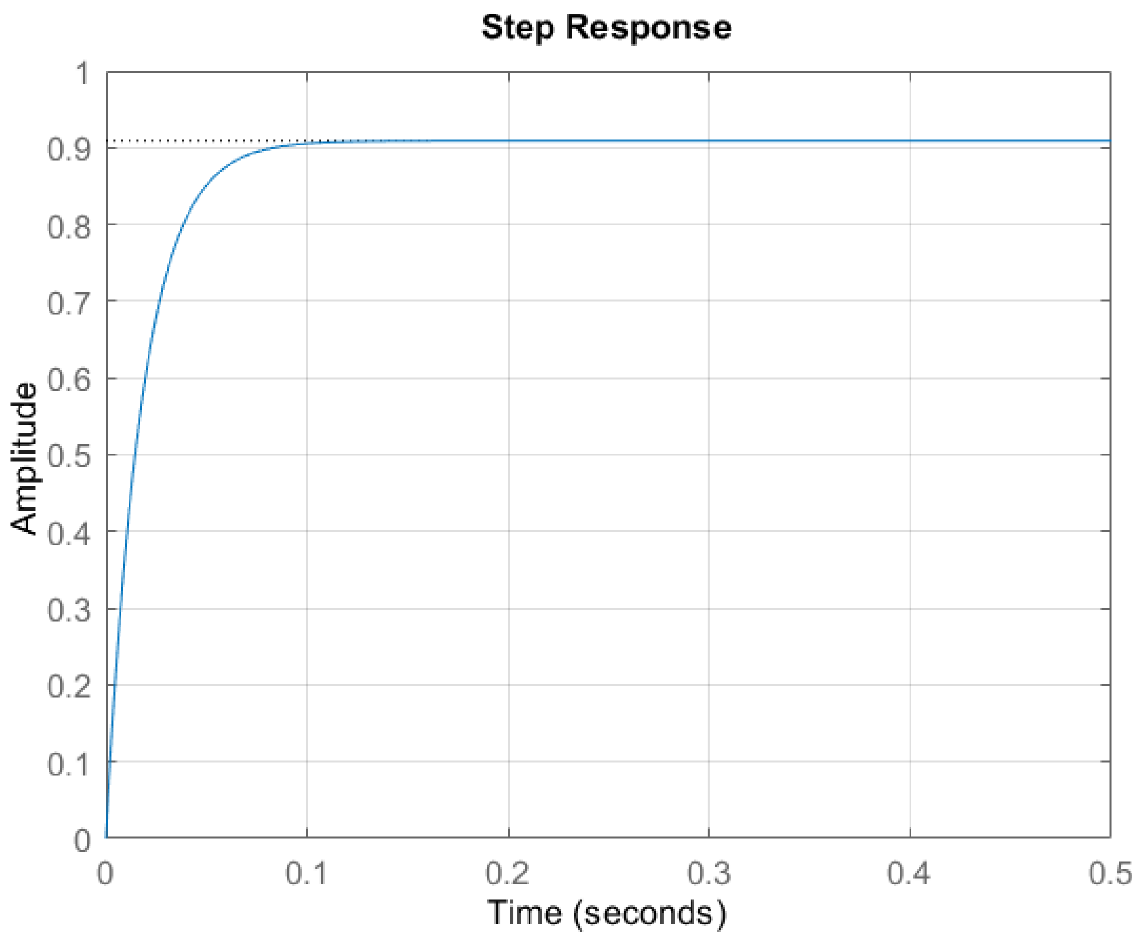

GW INSTEK
10k pts
5kSa/s
Stop
29 Nov 2022 19:40:51
368ms 520mV
354ms 40.0mV
△14.0ms △480mV
dV/dt 34.2V/s
<2Hz
峰對峰值 1.04V
峰對峰值 960mV
500mV
500mV
1V
20mV
200ms 0.000s
536mV DC
水平游標
水平單位
S Hz % °
設定游標
位置爲
100%

3. Simulation

Closed loop transfer function from command window:

```
sys =

      10
  ----------
  0.2 s + 11
```

The unit step response(1 pic):




```
ans =

  struct with fields:

        RiseTime: 0.0399
    SettlingTime: 0.0711
     SettlingMin: 0.8223
     SettlingMax: 0.9091
       Overshoot: 0
      Undershoot: 0
```

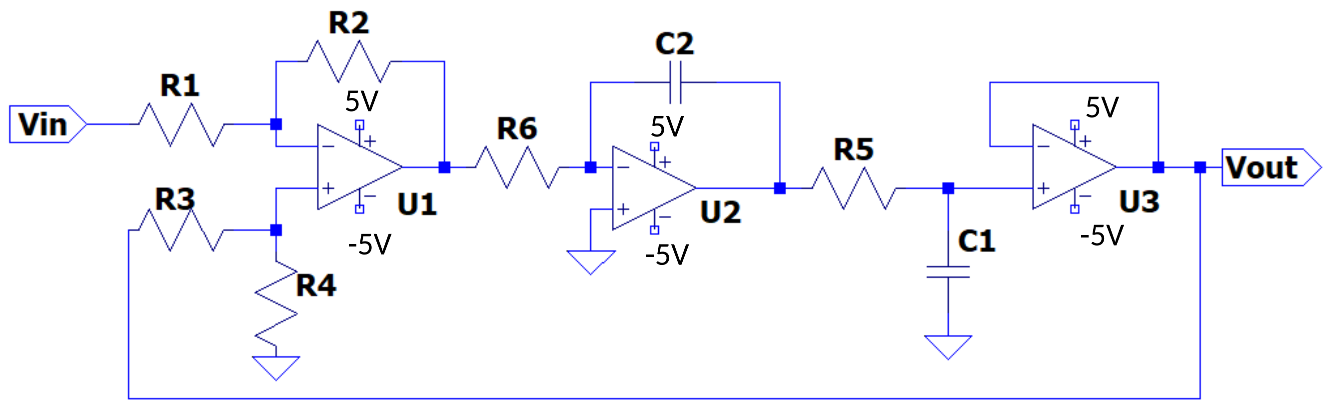
```
          Peak: 0.9091
      PeakTime: 0.1917


delayTime =

    0.0134


sserror =

    0.0909
```

Discuss:

Try to explain the effect of the gain K_P on the overall system.

Make the response faster and improve the steady state error.

Experiment 3: RC circuit with I-controller and unit feedback



	Maximum overshoot (%)	peak time (s)	rise time (s)	delay time (s)	settling time for 5% (s)	steady-state error (V)
Theoretical Result						**0**
Experiment Result	**52.83**	**0.150**	**0.092**	**0.055**	**1.31**	**0.010**
Simulation Result	**70.2118**	**0.1405**	**0.0509**	**0.2817**	**1.5630**	**0.0064**

注意事項:請參考投影片第７頁的定義

1. Calculate K_I and the **closed loop transfer function**.

The output of U1 V_1 is equal to

$V_1 = V_{out} - V_{¿}$

The relationship between V_1 and V_2 around U2 is

$$\frac{0-V_1}{R_6} + \frac{0-V_2}{\frac{1}{sC_2}} = 0$$

Substitute $V_1 = V_{out} - V_{¿}$:

$$\Longrightarrow V_2 = \frac{1}{sC_2R_6}\left(V_{¿} - V_{out}\right)$$

$$¿\frac{1}{0.01s}\left(V_{¿} - V_{out}\right)$$

$$\Longrightarrow \underline{K_I = 100}_{\#}$$

The relationship between V_2 and V_{out}:

$$\frac{V_{out}-V_2}{R_5} + \frac{V_{out}}{\frac{1}{sC_1}} = 0$$

$$\Longrightarrow V_{out} - V_2 = -sC_1R_5V_{out}$$

Substitute V_2:

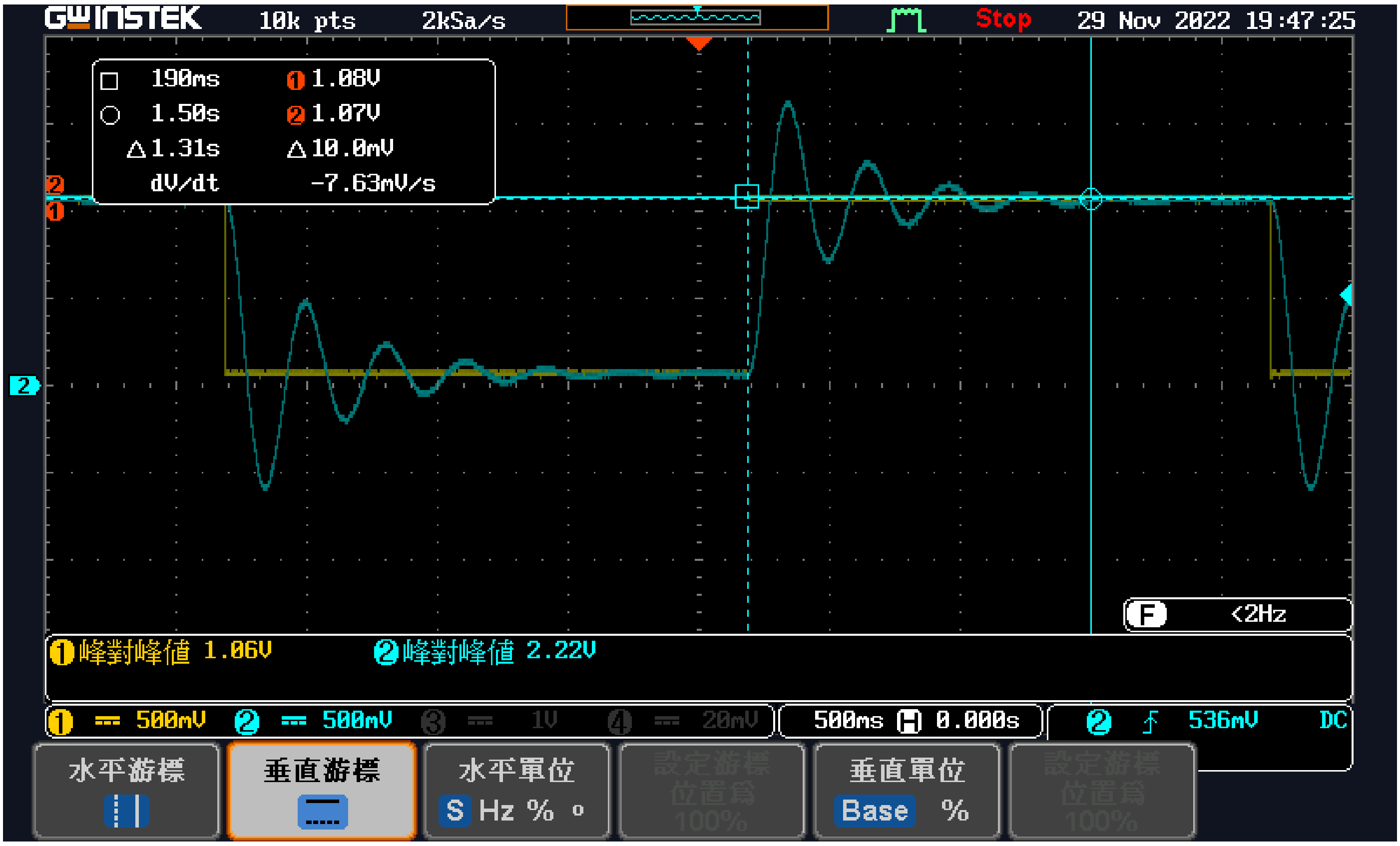
$$\Longrightarrow \left(sC_1R_5+1\right)V_{out} = V_2 = \frac{1}{0.01s}\left(V_{¿} - V_{out}\right)$$

$$\Rightarrow (0.2s+1)(0.01s)V_{out}+V_{out}=V_{¿}$$

$$\Longrightarrow \left(\frac{1}{500}s^2+\frac{1}{100}s+1\right)V_{out}=V_{¿}$$

$$\Longrightarrow \underline{V_{out}=\frac{1}{0.002s^2+0.01s+1}V_{¿}}_{\#}$$

2. V_{out} and Vin waveform (1 pic):

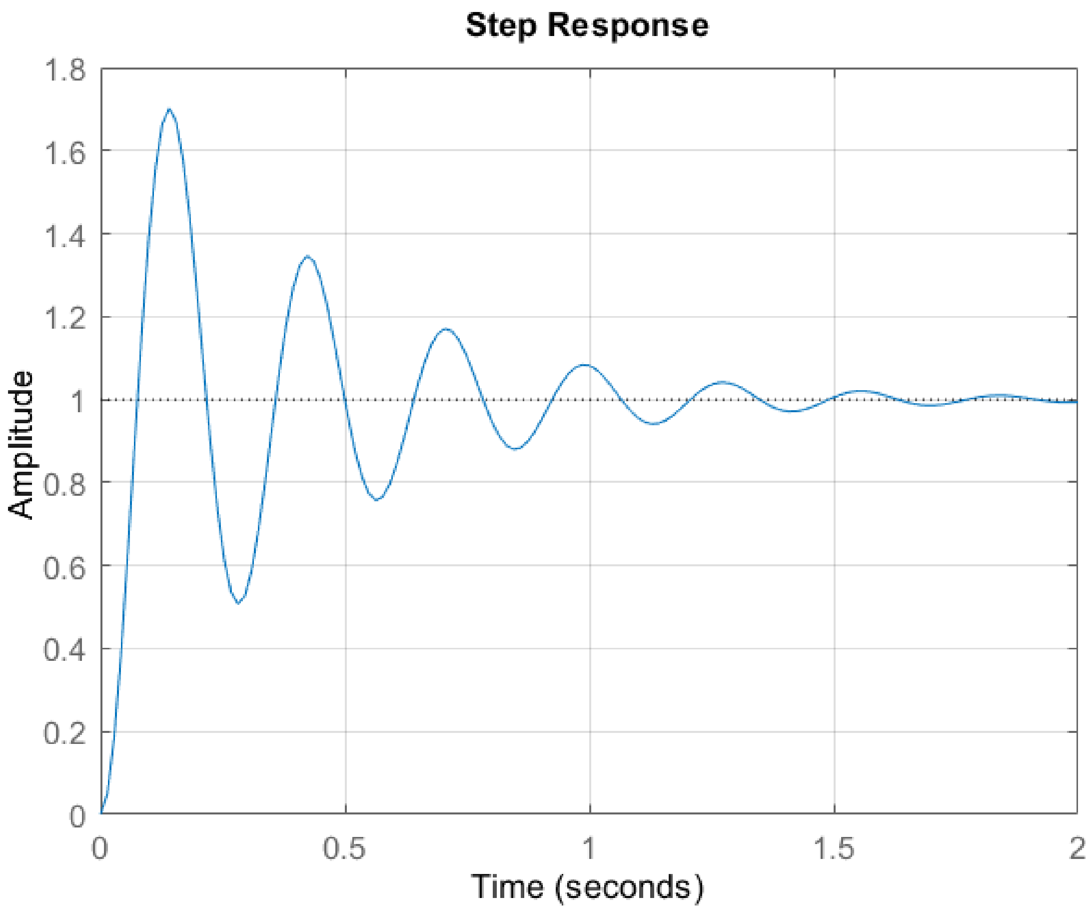


3. Simulation

Closed loop transfer function from command window:

```
sys =

          1
  ----------------------
  0.002 s^2 + 0.01 s + 1
```

The unit step response(1 pic):




```
ans =

  struct with fields:

        RiseTime: 0.0509
    SettlingTime: 1.5630
     SettlingMin: 0.5072
     SettlingMax: 1.7021
       Overshoot: 70.2118
      Undershoot: 0
```

```
              Peak: 1.7021
          PeakTime: 0.1405


  delayTime =

      0.2817


  sserror =

      0.0064
```

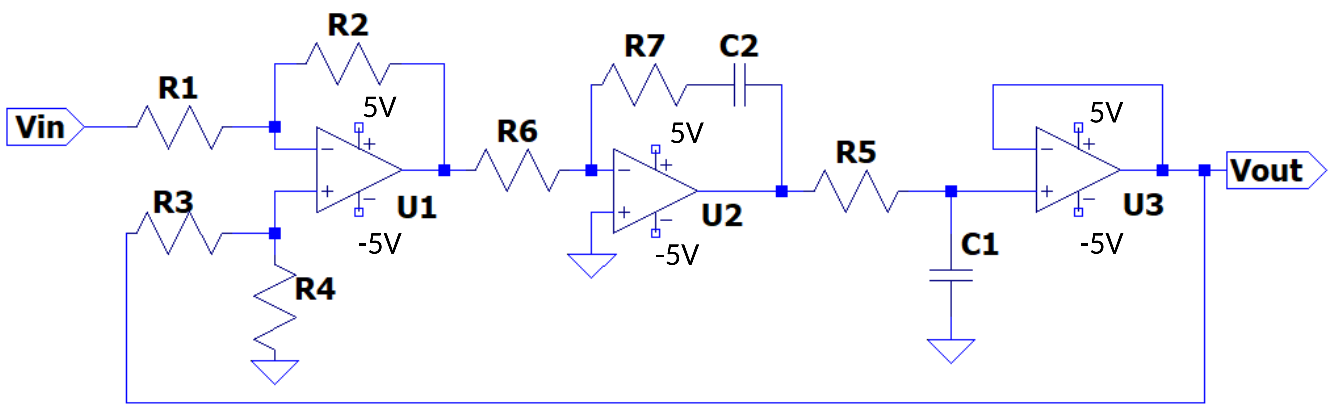
Discuss:

Try to explain why the steady-state error is close to zero and the effect of the gain K_I on the overall system.

When the output reach steady state, capacitor C2 will be open circuit because the current through it $\frac{V_{out} - V_{¿} - 0}{R_6}$ is approximately 0. Therefore, the steady state error is close to zero.

The effect of the gain K_I is to eliminate the steady state error.

Experiment 4: RC circuit with PI-controller and unit feedback



	Maximum overshoot (%)	peak time (s)	rise time (s)	delay time (s)	settling time for 5% (s)	steady-state error (V)
Theoretical Result						**0**
Experiment Result	**3.774**	**0.085**	**0.070**	**0.019**	**0.198**	**0.004**
Simulation Result	**4.8838**	**0.0974**	**0.0358**	**0.0127**	**0.2007**	**0.00064794**

注意事項:請參考投影片第7頁的定義

1. Calculate K_P , K_I and the **closed loop transfer function**.

The difference amplifier:

$$V_1 = V_{out} - V_{¿}$$

The PI controller:

$$\frac{0-V_1}{R_6} + \frac{0-V_2}{R_7 + \frac{1}{sC_2}} = 0$$

$$\Longrightarrow \frac{V_2}{\frac{sC_2R_7+1}{sC_2}} = \frac{-V_1}{R_6}$$

$$\Longrightarrow V_2 = \frac{-1}{R_6} \times \frac{sC_2R_7+1}{sC_2} V_1$$

$$¿ - \frac{C_2R_7s+1}{sC_2R_6} V_1$$

$$¿ \frac{\frac{s}{10}+1}{\frac{s}{100}} \left(V_{¿} - V_{out}\right)$$

$$¿ \frac{10s+100}{s} \left(V_{¿} - V_{out}\right)$$

$$\Longrightarrow \underline{K_p = 10, K_I = 100}_{\#}$$

The plant:

$$\frac{V_{out}}{\frac{1}{sC_1}}+\frac{V_{out}-V_2}{R_5}=0$$

$$\Longrightarrow \left(sC_1R_5+1\right)V_{out}=V_2$$

$$\Longrightarrow V_{out}=\frac{1}{sC_1R_5+1}V_2$$

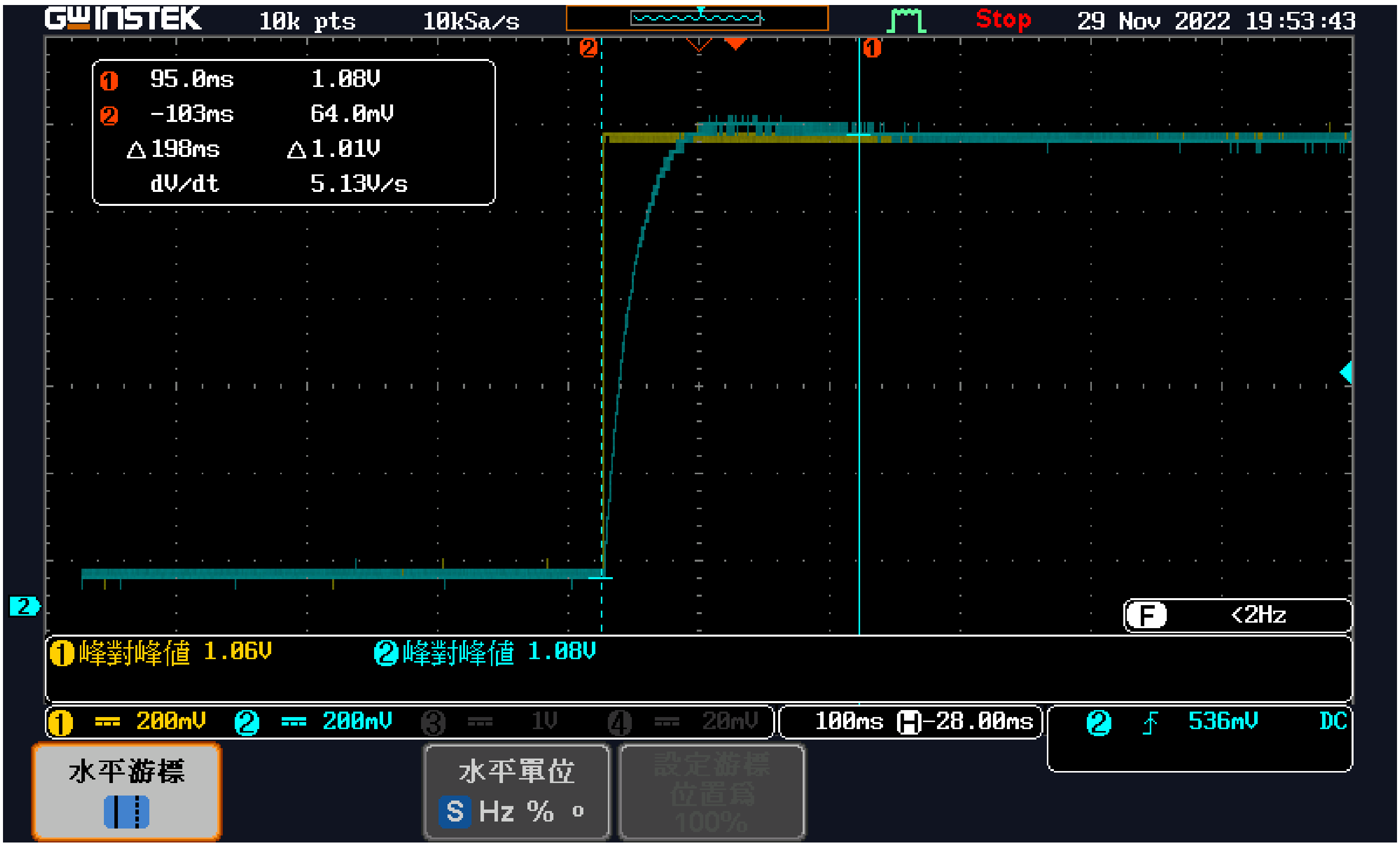
$$¿\ \frac{1}{0.2s+1}\left(\frac{10s+100}{s}\right)\left(V_{¿}-V_{out}\right)$$

$$¿\ \frac{10s+100}{0.2s^2+s}\left(V_{¿}-V_{out}\right)$$

$$\Longrightarrow V_{out}=\frac{\frac{10s+100}{0.2s^2+s}}{1+\frac{10s+100}{0.2s^2+s}}V_{¿}$$

$$\Longrightarrow \underline{V_{out}=\frac{10s+100}{0.2s^2+11s+100}V_{¿}}_{\#}$$

2. V_{out} and Vin waveform (1 pic):

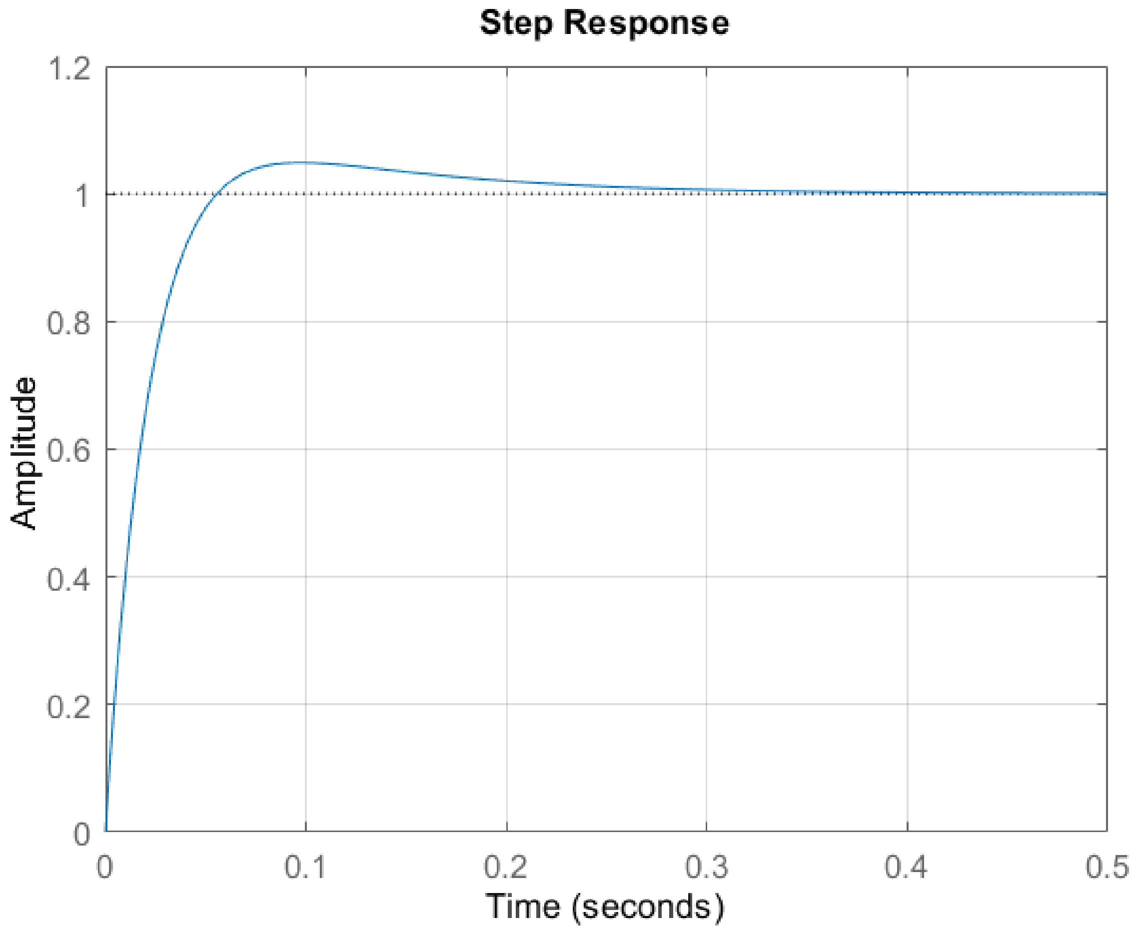


3. Simulation

Closed loop transfer function from command window:

```
sys =

        0.1 s + 1
  ----------------------
  0.002 s^2 + 0.11 s + 1
```

The unit step response(1 pic):




```
ans =

  struct with fields:

        RiseTime: 0.0358
    SettlingTime: 0.2007
     SettlingMin: 0.9017
     SettlingMax: 1.0488
       Overshoot: 4.8838
      Undershoot: 0
```

```
             Peak: 1.0488
         PeakTime: 0.0974


delayTime =

    0.0127


sserror =

   6.4794e-04
```

Discuss:

Try to explain the effect of PI-controller on the overall system and how to design $K_P$and K_I.

The PI-controller both the rise time and the steady state errors of the system.

Increasing K_p results in a faster response of the control system. However, an increase in K_p above a certain value can make the system unstable.

Increasing K_i helps in eliminating steady-state error, but increases oscillations and overshoot.