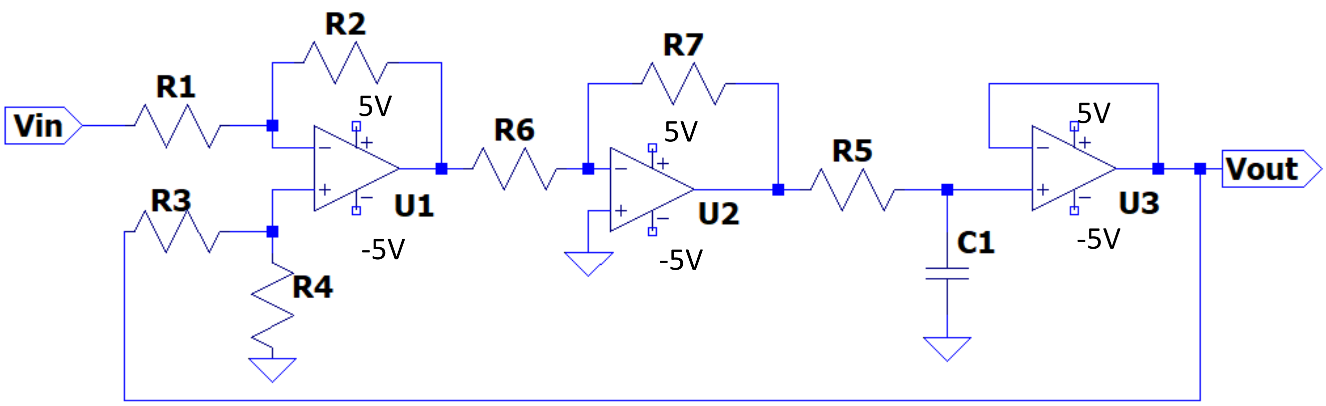
REPORT

Experiment 1: RC circuit with unit feedback



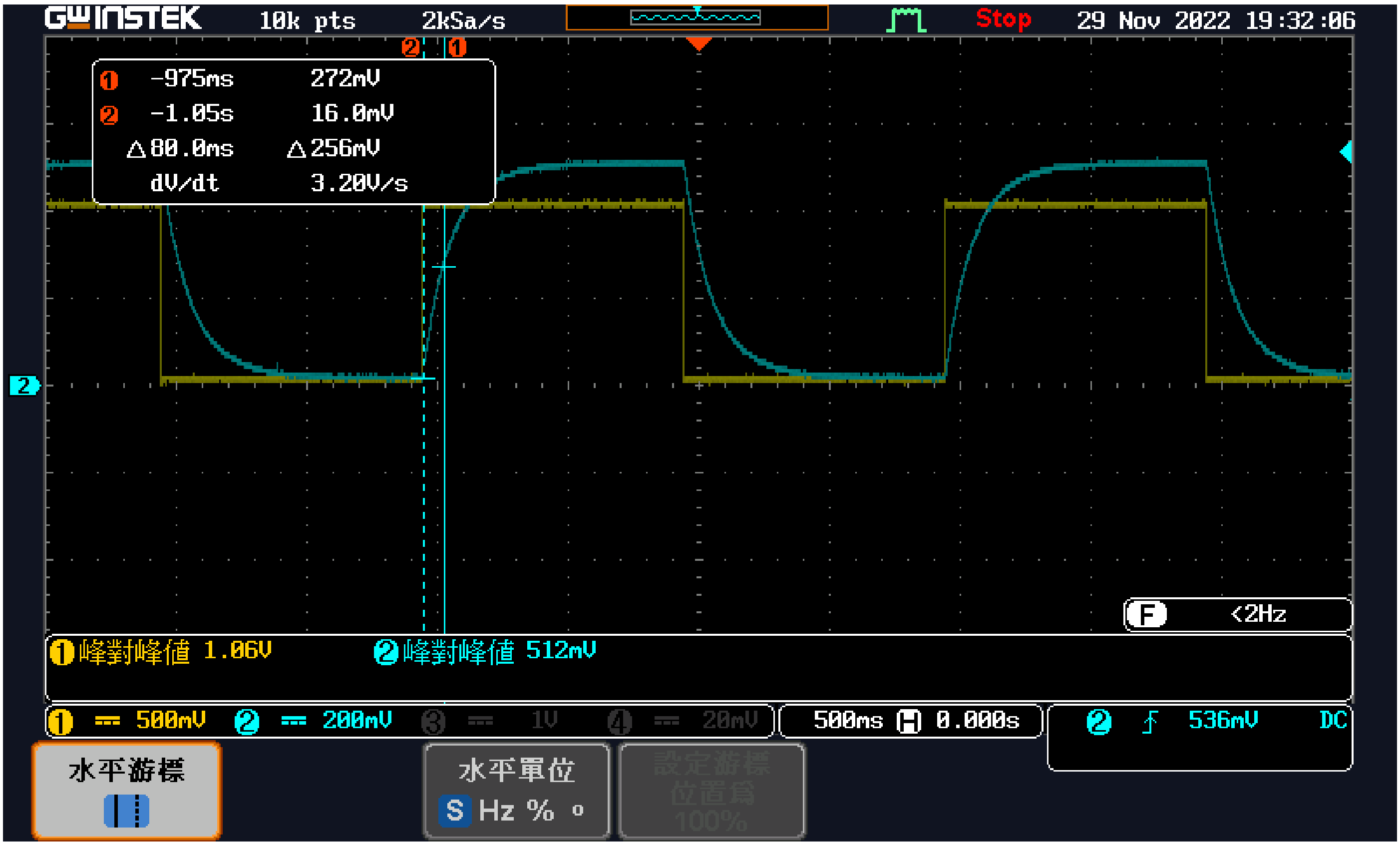
	rise time (s)	delay time (s)	steady-state error (V)
Theoretical Result			**0.5**
Experiment Result	**0.285**	**0.080**	**0.530**
Simulation Result	**0.2197**	**0.0737**	**0.5000**

注意事項:請參考投影片第 5 頁的定義

1. Calculate the **closed loop transfer function**.

$$V_{out} = -1 \times (V_{out} - V_{in}) \times \frac{1}{0.2s+1}$$
$$\Rightarrow (0.2s+1)V_{out} = -V_{out} + V_{in}$$
$$\Rightarrow (0.2s+2)V_{out} = V_{in}$$
$$\Rightarrow \underline{\frac{V_{out}}{V_{in}} = \frac{1}{0.2s+2}}_{\#}$$

2. V_{out} and Vin waveform (1 pic):

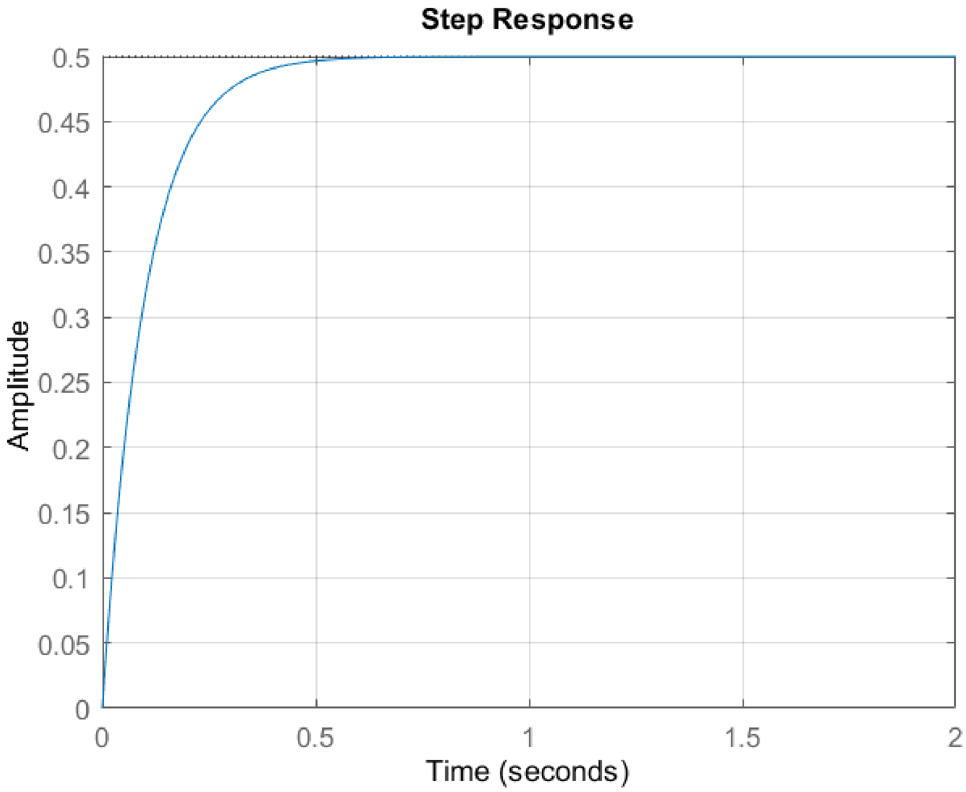

GW INSTEK 10k pts 2kSa/s Stop 29 Nov 2022 19:32:06
1 -975ms 272mV
2 -1.05s 16.0mV
△80.0ms △256mV
dV/dt 3.20V/s
F <2Hz
1 峰對峰值 1.06V
2 峰對峰值 512mV
1 500mV 2 200mV 3 1V 4 20mV
500ms 0.000s
2 536mV DC
水平游標
水平單位
S Hz % °
設定游標
位置爲
100%

3. Simulation

Closed loop transfer function from command window:

```
sys =

      1
  ---------
  0.2 s + 2
```

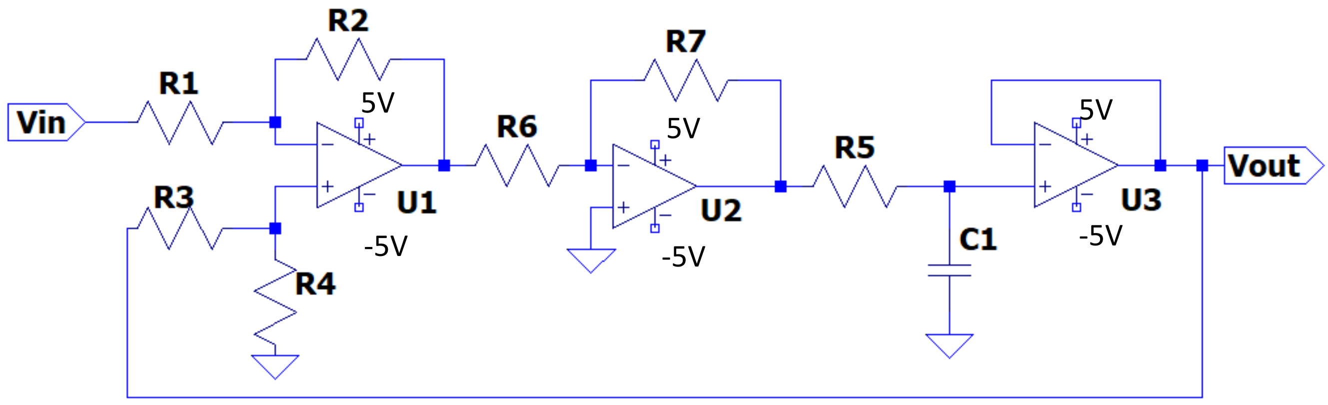
The unit step response(1 pic):




```
ans =

  struct with fields:

        RiseTime: 0.2197
    SettlingTime: 0.3912
     SettlingMin: 0.4523
     SettlingMax: 0.5000
       Overshoot: 0
      Undershoot: 0
            Peak: 0.5000
        PeakTime: 1.0546
```

```
delayTime =

    0.0737


sserror =

    0.5000
```

Experiment 2: RC circuit with P-controller and unit feedback



	rise time (s)	delay time (s)	steady-state error (V)
Theoretical Result			**0.0909**
Experiment Result	**0.050**	**0.014**	**0.1**
Simulation Result	**0.0399**	**0.0134**	**0.0909**

注意事項:請參考投影片第 5 頁的定義

1. Calculate K_P and the **closed loop transfer function**.

Assume V_2 is the output of U1. The relation of input and output of difference amplifier:

$$V_2 = V_{out} - V_{in}$$

Assume V_3 is the output of U2. The relation of input and output of inverting amplifier:

$$\frac{0 - V_2}{R_6} + \frac{0 - V_3}{R_7} = 0$$

$$\Rightarrow V_3 = -\frac{R_7}{R_6} V_2$$

Substitute $V_2 = V_{out} - V_{in}$:

$$= \frac{100}{10}(V_{in} - V_{out})$$

$$= 10(V_{in} - V_{out})$$

The relation of input and output of plant:

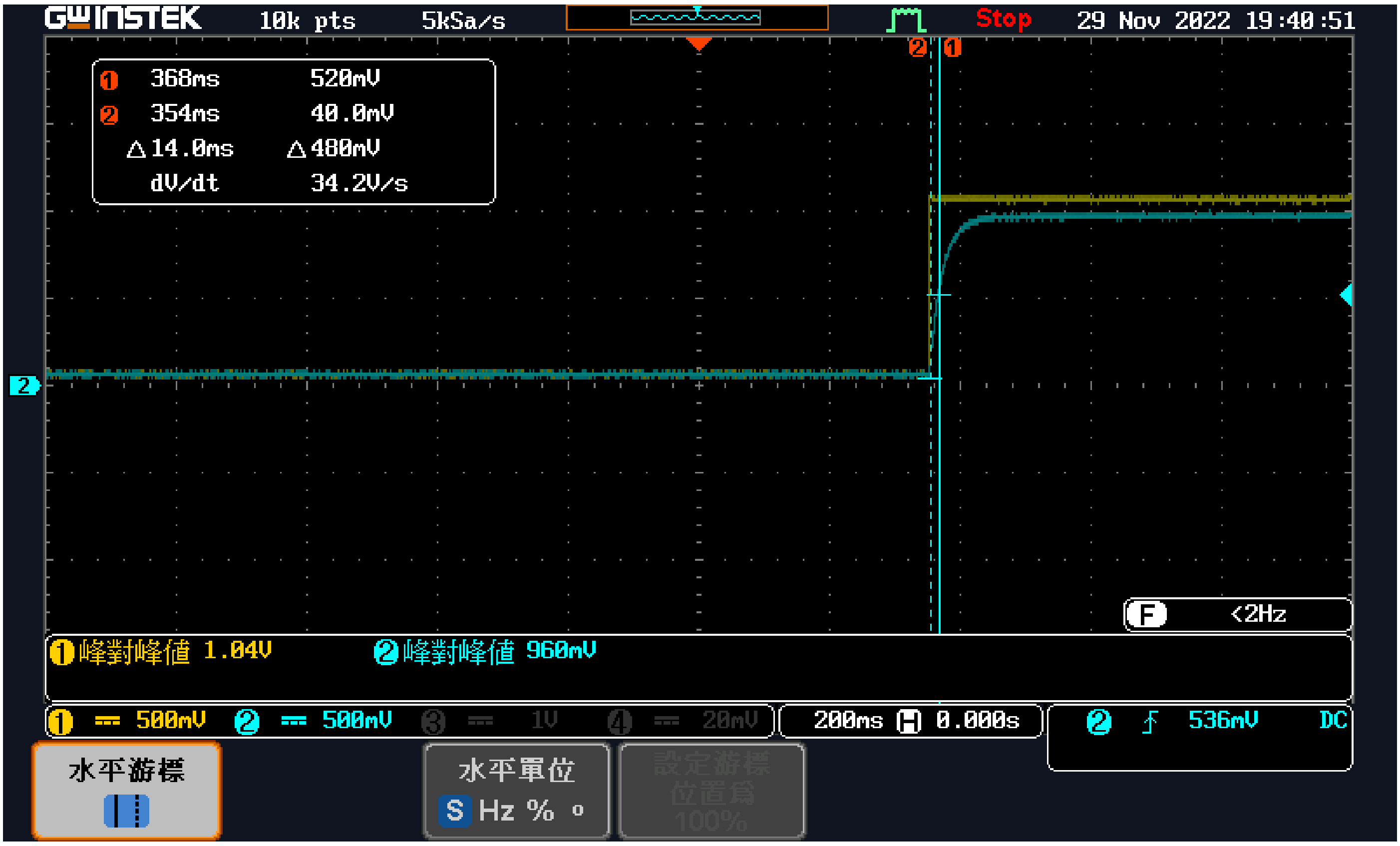
$$10V_{in} - 10V_{out} = (sR_5C_1 + 1)V_{out}$$

$$\Rightarrow V_{out} = \frac{10V_{in}}{sR_5C_1 + 11}$$

$$\Rightarrow \underline{K_p = 10}_{\#}$$

$$\Rightarrow \underline{V_{out} = \frac{10}{0.2s + 11} V_{in}}_{\#}$$

2. V_{out} and Vin waveform (1 pic):

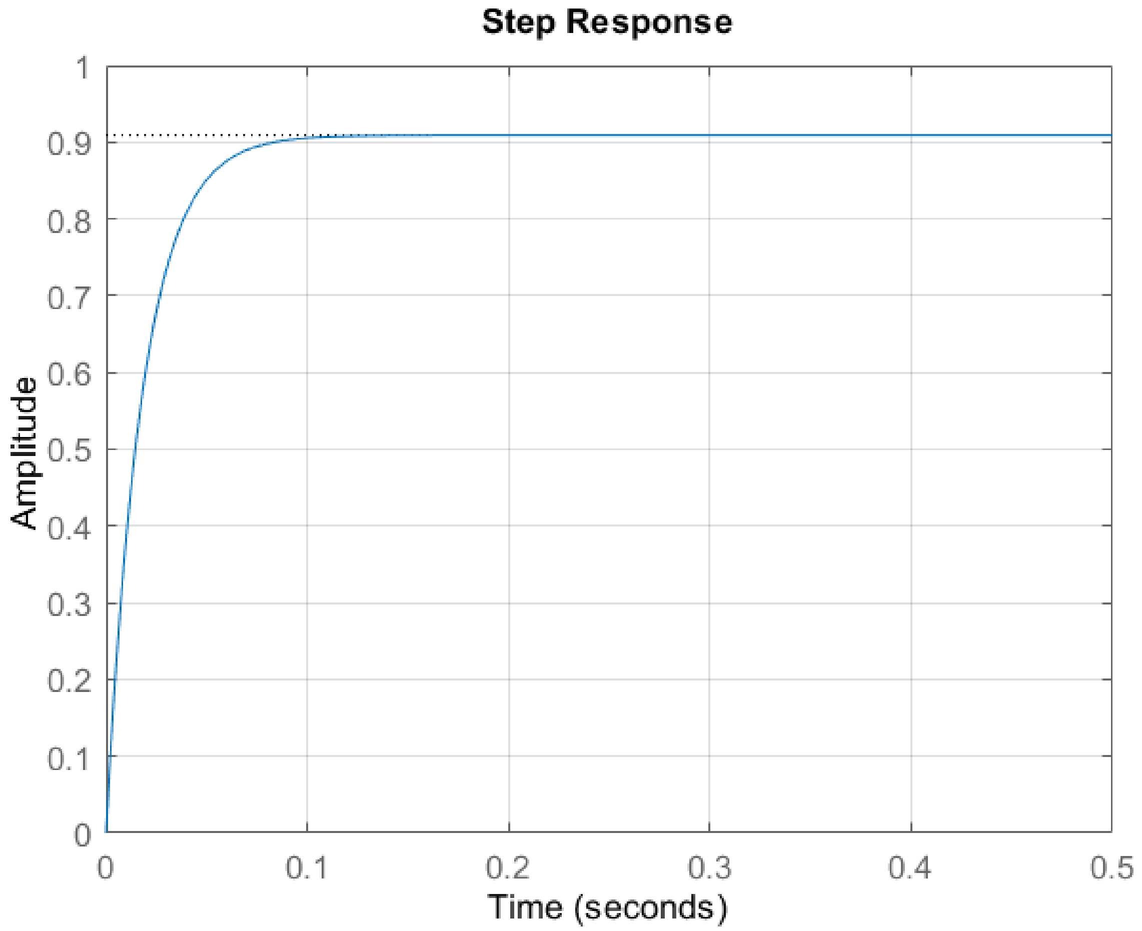

GW INSTEK
10k pts
5kSa/s
Stop
29 Nov 2022 19:40:51
1 368ms 520mV
2 354ms 40.0mV
△14.0ms △480mV
dV/dt 34.2V/s
<2Hz
1 峰對峰值 1.04V
2 峰對峰值 960mV
1 500mV
2 500mV
3 1V
4 20mV
200ms 0.000s
2 536mV DC
水平游標
水平單位
S Hz % °
設定游標
位置爲
100%

3. Simulation

Closed loop transfer function from command window:

```
sys =

        10
    ----------
    0.2 s + 11
```

The unit step response(1 pic):




```
ans =

  struct with fields:

        RiseTime: 0.0399
    SettlingTime: 0.0711
     SettlingMin: 0.8223
     SettlingMax: 0.9091
       Overshoot: 0
      Undershoot: 0
```

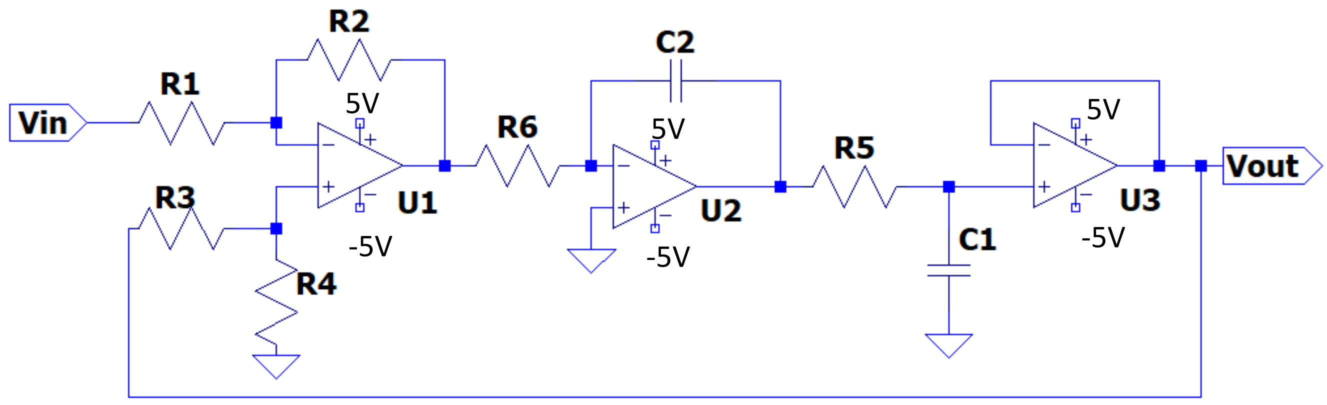
```
           Peak: 0.9091
       PeakTime: 0.1917


  delayTime =

      0.0134


  sserror =

      0.0909
```

Discuss:

Try to explain the effect of the gain K_P on the overall system.

Make the response faster and improve the steady state error.

Experiment 3: RC circuit with I-controller and unit feedback



	Maximum overshoot (%)	peak time (s)	rise time (s)	delay time (s)	settling time for 5% (s)	steady-state error (V)
Theoretical Result						**0**
Experiment Result	**52.83**	**0.150**	**0.092**	**0.055**	**1.31**	**0.010**
Simulation Result	**70.2118**	**0.1405**	**0.0509**	**0.2817**	**1.5630**	**0.0064**

注意事項:請參考投影片第 7 頁的定義

1. Calculate K_I and the **closed loop transfer function**.

The output of U1 V_1 is equal to

$$V_1 = V_{out} - V_{in}$$

The relationship between V_1 and V_2 around U2 is

$$\frac{0 - V_1}{R_6} + \frac{0 - V_2}{\frac{1}{sC_2}} = 0$$

Substitute $V_1 = V_{out} - V_{in}$:

$$\Rightarrow V_2 = \frac{1}{sC_2R_6}(V_{in} - V_{out})$$

$$= \frac{1}{0.01s}(V_{in} - V_{out})$$

$$\Rightarrow \underline{K_I = 100_{\#}}$$

The relationship between V_2 and V_{out}:

$$\frac{V_{out} - V_2}{R_5} + \frac{V_{out}}{\frac{1}{sC_1}} = 0$$

$$\Rightarrow V_{out} - V_2 = -sC_1R_5V_{out}$$

Substitute V_2:

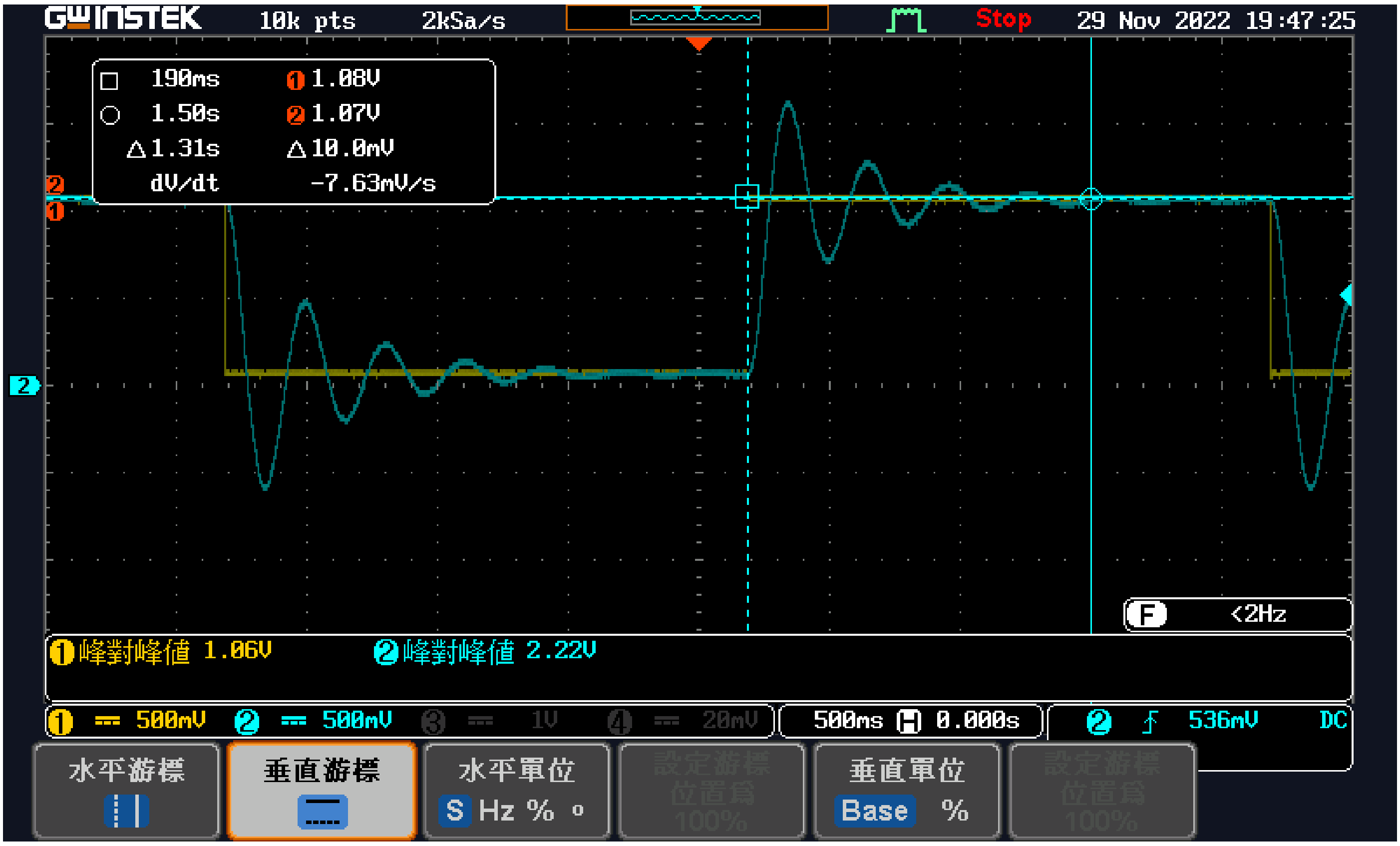
$$\Rightarrow (sC_1R_5 + 1)V_{out} = V_2 = \frac{1}{0.01s}(V_{in} - V_{out})$$

$$\Rightarrow (0.2s + 1)(0.01s)V_{out} + V_{out} = V_{in}$$

$$\Rightarrow \left(\frac{1}{500}s^2 + \frac{1}{100}s + 1\right)V_{out} = V_{in}$$

$$\Rightarrow V_{out} = \frac{1}{0.002s^2 + 0.01s + 1}V_{in} \#$$

2. V_{out} and Vin waveform (1 pic):

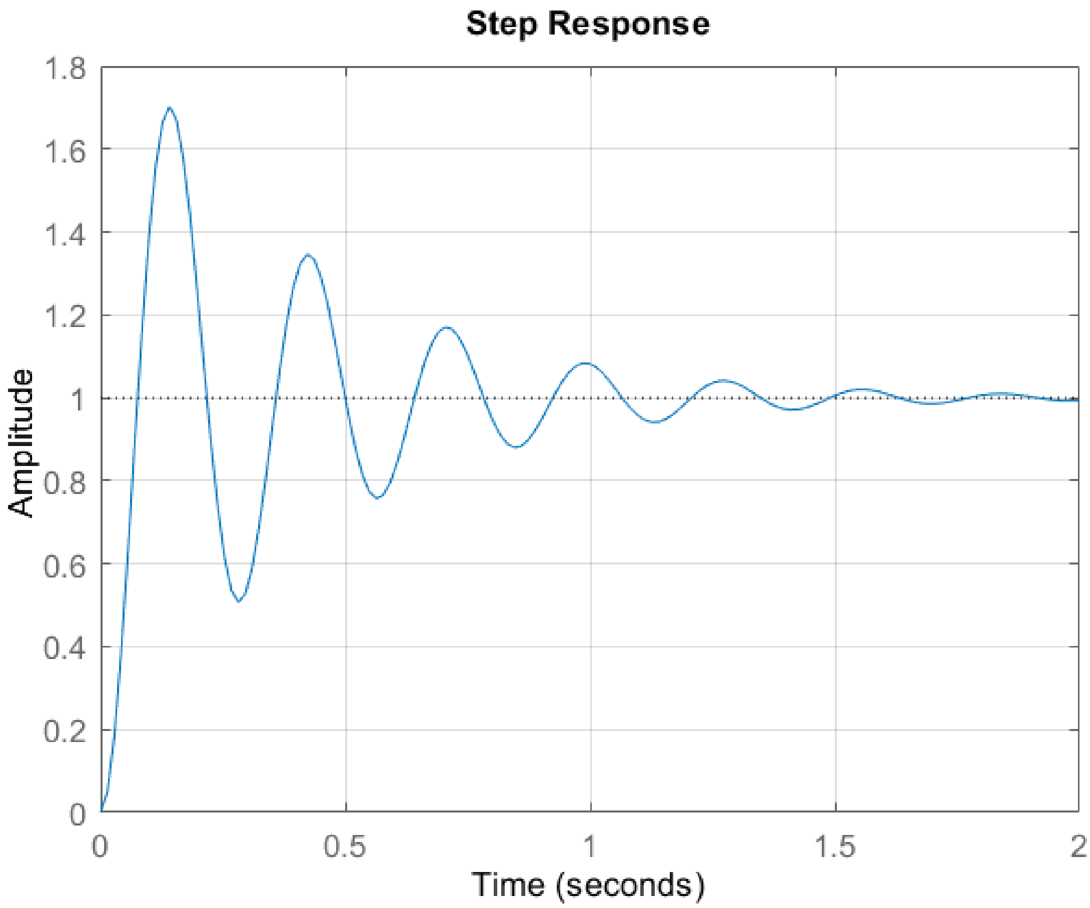


3. Simulation

Closed loop transfer function from command window:

```
sys =

           1
  ----------------------
  0.002 s^2 + 0.01 s + 1
```

The unit step response(1 pic):




```
ans =

  struct with fields:

        RiseTime: 0.0509
    SettlingTime: 1.5630
     SettlingMin: 0.5072
     SettlingMax: 1.7021
       Overshoot: 70.2118
      Undershoot: 0
```

```
          Peak: 1.7021
      PeakTime: 0.1405


delayTime =

    0.2817


sserror =

    0.0064
```

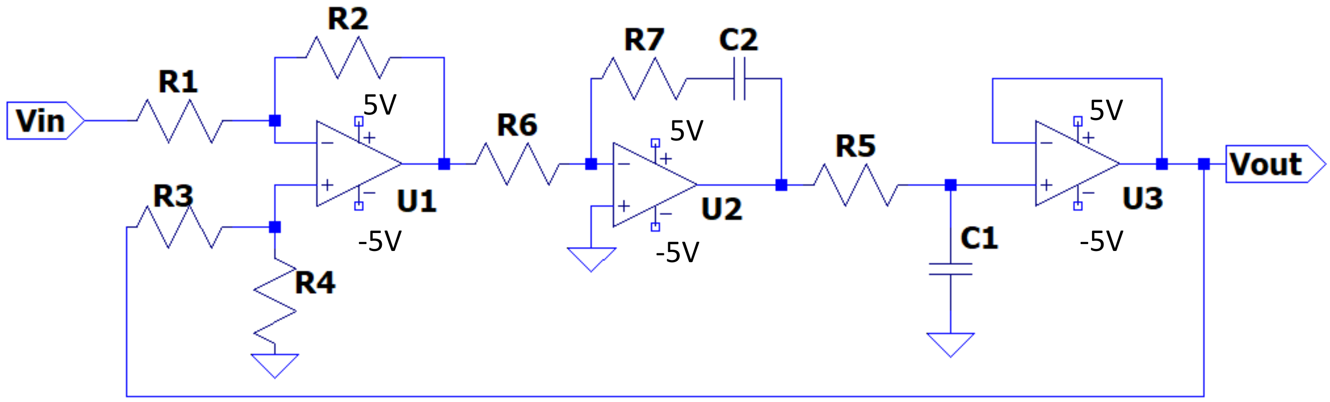
Discuss:

Try to explain why the steady-state error is close to zero and the effect of the gain K_I on the overall system.

When the output reach steady state, capacitor C2 will be open circuit because the current through it $\frac{V_{out}-V_{in}-0}{R_6}$ is approximately 0. Therefore, the steady state error is close to zero.

The effect of the gain K_I is to eliminate the steady state error.

Experiment 4: RC circuit with PI-controller and unit feedback



	Maximum overshoot (%)	peak time (s)	rise time (s)	delay time (s)	settling time for 5% (s)	steady-state error (V)
Theoretical Result						**0**
Experiment Result	**3.774**	**0.085**	**0.070**	**0.019**	**0.198**	**0.004**
Simulation Result	**4.8838**	**0.0974**	**0.0358**	**0.0127**	**0.2007**	**0.00064794**

注意事項:請參考投影片第 7 頁的定義

1. Calculate K_P , K_I and the **closed loop transfer function**.

The difference amplifier:

$$V_1 = V_{out} - V_{in}$$

The PI controller:

$$\frac{0 - V_1}{R_6} + \frac{0 - V_2}{R_7 + \frac{1}{sC_2}} = 0$$

$$\Rightarrow \frac{\mathrm{V_2}}{\frac{\mathrm{sC_2R_7 + 1}}{\mathrm{sC_2}}} = -\frac{\mathrm{V_1}}{\mathrm{R_6}}$$

$$\Rightarrow V_2 = -\frac{1}{R_6} \times \frac{sC_2R_7 + 1}{sC_2} V_1$$

$$= -\frac{C_2R_7s + 1}{sC_2R_6} V_1$$

$$= \frac{\frac{s}{10} + 1}{\frac{s}{100}} (V_{in} - V_{out})$$

$$= \frac{10s + 100}{s} (V_{in} - V_{out})$$

$$\Rightarrow \underline{K_p = 10, K_I = 100_{\#}}$$

The plant:

$$\frac{V_{out}}{\frac{1}{sC_1}} + \frac{V_{out} - V_2}{R_5} = 0$$

$$\Longrightarrow (sC_1R_5 + 1)V_{out} = V_2$$

$$\Longrightarrow V_{out} = \frac{1}{sC_1R_5 + 1}V_2$$

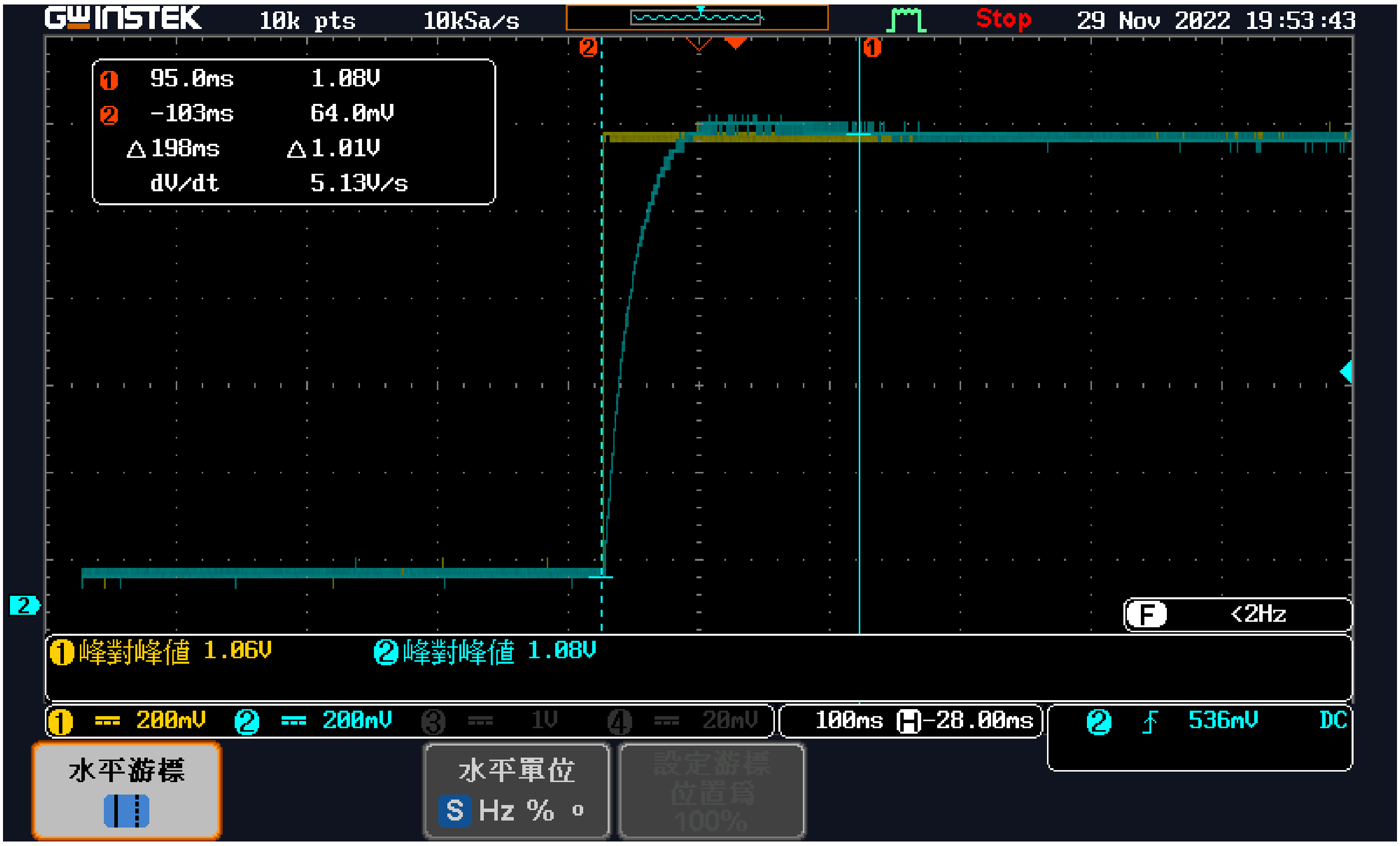
$$= \frac{1}{0.2s + 1}\left(\frac{10s + 100}{s}\right)(V_{in} - V_{out})$$

$$= \frac{10s + 100}{0.2s^2 + s}(V_{in} - V_{out})$$

$$\Longrightarrow V_{out} = \frac{\frac{10s + 100}{0.2s^2 + s}}{1 + \frac{10s + 100}{0.2s^2 + s}}V_{in}$$

$$\Longrightarrow V_{out} = \frac{10s + 100}{0.2s^2 + 11s + 100}V_{in}\#$$

2. V_{out} and Vin waveform (1 pic):

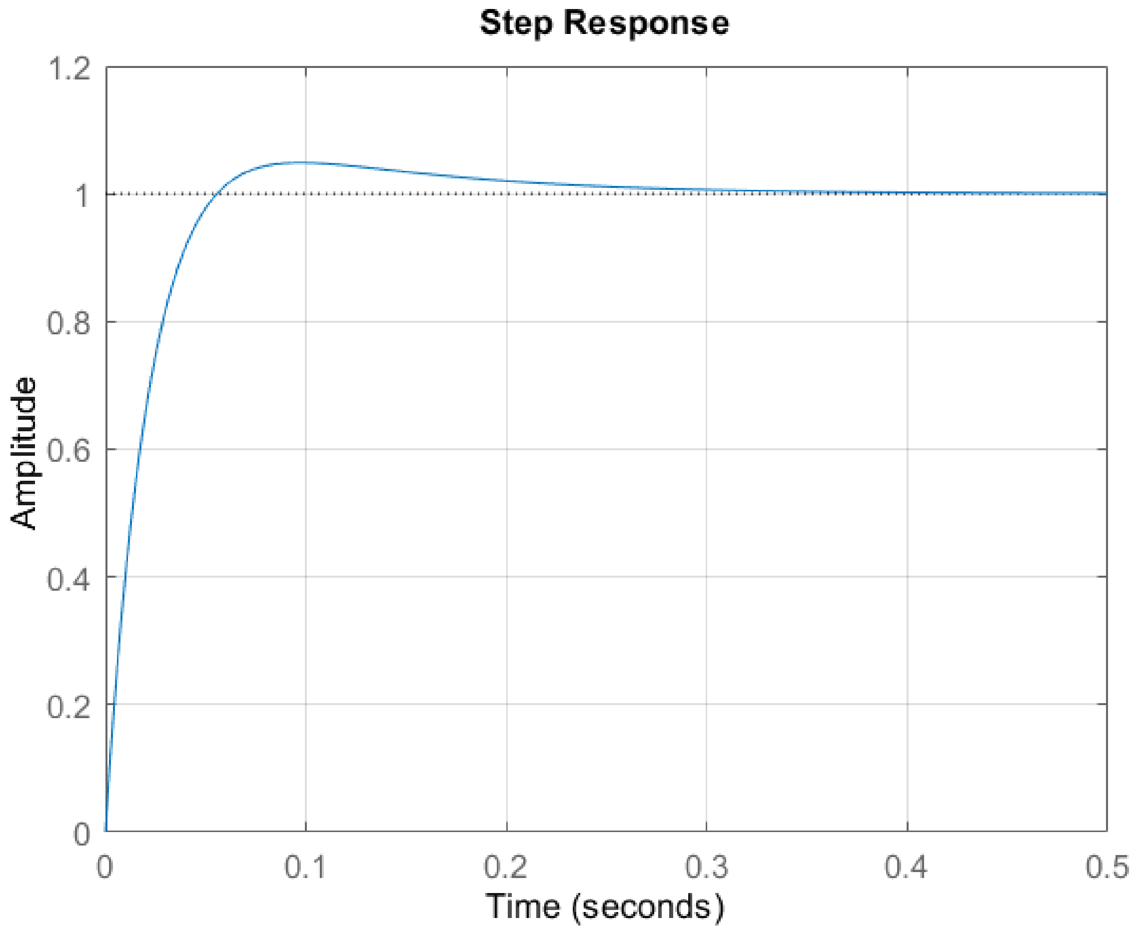


3. Simulation

Closed loop transfer function from command window:

```
sys =

        0.1 s + 1
  ---------------------
  0.002 s^2 + 0.11 s + 1
```

The unit step response(1 pic):




```
ans =

  struct with fields:

        RiseTime: 0.0358
    SettlingTime: 0.2007
     SettlingMin: 0.9017
     SettlingMax: 1.0488
       Overshoot: 4.8838
      Undershoot: 0
```

```
            Peak: 1.0488
        PeakTime: 0.0974


delayTime =

    0.0127


sserror =

   6.4794e-04
```

Discuss:

Try to explain the effect of PI-controller on the overall system and how to design K_P and K_I.

The PI-controller both the rise time and the steady state errors of the system.
Increasing K_p results in a faster response of the control system. However, an increase in K_p above a certain value can make the system unstable.
Increasing K_i helps in eliminating steady-state error, but increases oscillations and overshoot.