# Mean Shift clustering algorithm

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Firenze, 27 Aprile 2020

# Outline

# Introduction

- Mean Shift is a non-parametric clustering algorithm
- It is based on *Kernel Density estimation*
- The only parameter is the *bandwidth*
- $O(n^2)$ computational cost
- Common application in computer vision: image segmentation
- It is embarassingly parallel

# Algorithm

- At each step a kernel function is applied to each point to make it shift towards the local maxima
- Most used kernel: Gaussian kernel

$$K(x) = e^{-\frac{x^2}{2\sigma^2}} \tag{1}$$

- New position $x'$ where $x$ has to be shifted is computed as:

$$x' = \frac{\sum_{x_i \in N(x)} K(dist(x, x_i))x_i}{\sum_{x_i \in N(x)} K(dist(x, x_i))} \tag{2}$$

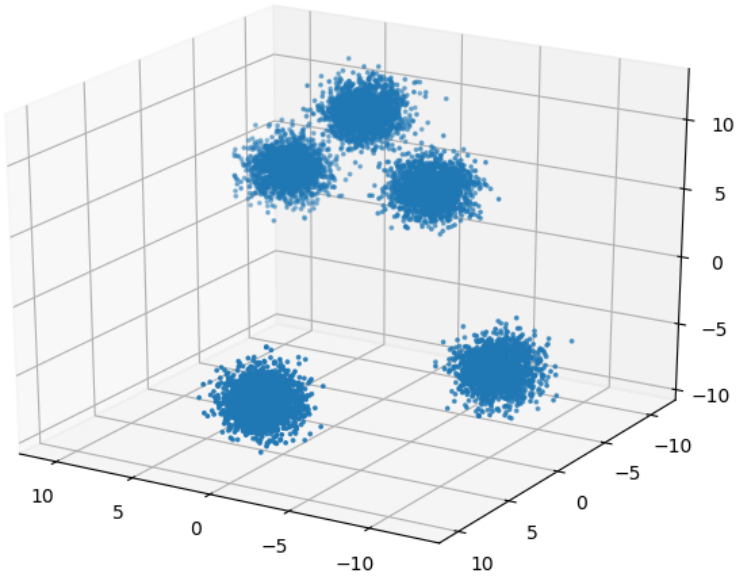
  $N(x)$ is the neighborhood of $x$, a set of points for which $K(x_i) \neq 0$

- Algorithm stops when all points have stopped shifting, that is they have reached the local maxima

# Sequential implementation

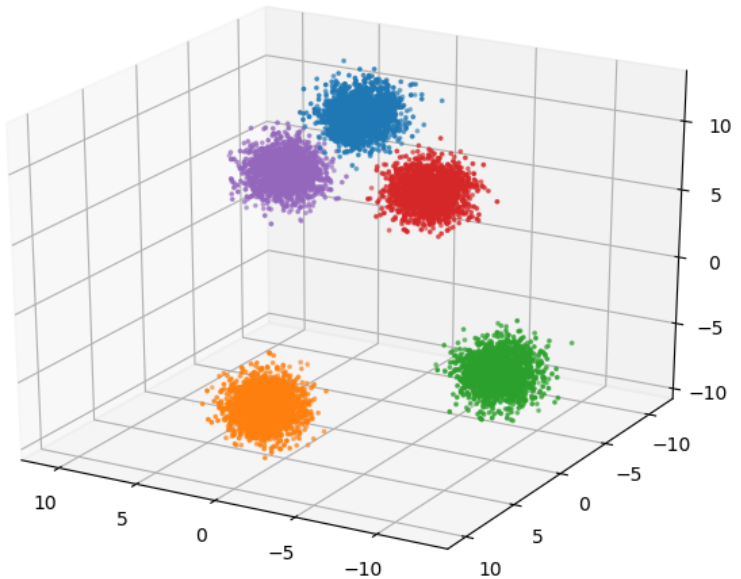
**Algorithm 1** Mean shift core

**function** MEANSHIFT(*originalPoints*)
  *shiftedPoints* ← *originalPoints*
  **while** iterationIndex < *MAX_ITERATIONS* **do**
    **for** each point *p* in *shiftedPoints* **do**
      *p* ← SHIFTPOINT(*p*, *originalPoints*)




**Algorithm 2** Shift a single point

**function** SHIFTPOINT(*p*, *originalPoints*)
  *shiftedP* ← 0
  *weight* ← 0
  **for** each point *x* in *originalPoints* **do**
    *dist* ← *dist*(*p*, *x*)
    *w* ← *GKernel*(*dist*, *BW*)
    *shiftedP* ← *shiftedP* + *w* ∗ *x*
    *weight* ← *weight* + *w*
  **return** *shiftedP* / *weight*



# OpenMP implementation

**Algorithm 3** OpenMP Mean shift core

```
function OpenMPMeanShift(originalPoints)
    shiftedPoints ← originalPoints
    while iterationIndex < MAX_ITERATIONS do
        #pragma parallel for schedule(static)
        for each point p in shiftedPoints do
            p ← ShiftPoint(p, originalPoints)
```

**Algorithm 4** Shift a single point

```
function ShiftPoint(p, originalPoints)
    shiftedP ← 0
    weight ← 0
    for each point x in originalPoints do
        dist ← dist(p, x)
        w ← GKernel(dist, BW)
        shiftedP ← shiftedP + w ∗ x
        weight ← weight + w
    return shiftedP / weight
```

- just a pragma directive
- static scheduling: loop divided statically in chunks of equal size

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Taking advantage of GPUs:

- One thread for each point to shift
- Coalescing of accesses to memory:
  - Points stored as a *Structure of Arrays*

$$[x_1, \ldots, x_n, y_1, \ldots, y_n, z_1, \ldots, z_n] \tag{3}$$

  - Access to the array in the form of:

$$blockDim.x * blockIdx.x + threadIdx.x \tag{4}$$

# Naive CUDA implementation

**Algorithm 5** CUDA Naive version Mean Shift core

**function** NAIVECUDAMS(*originalPoints*)
  *shiftedPoints* $\leftarrow$ *originalPoints*
  **while** iterationIndex $<$ *MAX_ITERATIONS* **do**
    NAIVEKERNEL(*shiftedPoints*, *originalPoints*)

**Algorithm 6** CUDA Naive version Kernel

**function** NAIVEKERNEL(*shiftedPts*, *originalPts*)
  $tx \leftarrow threadIdx.x$
  $bx \leftarrow blockIdx.x$
  $idx \leftarrow bx * blockDim.x + tx$
  **if** $idx < |originalPts|$ **then**
    $shiftedP \leftarrow 0$
    $weight \leftarrow 0$
    $p \leftarrow shiftedPts[idx]$
    **for** each point $x$ in *originalPts* **do**
      $dist \leftarrow dist(p, x)$
      $w \leftarrow GKernel(dist, BW)$
      $shiftedP \leftarrow shiftedP + w * x$
      $weight \leftarrow weight + w$
    $shiftedPts[idx] \leftarrow shiftedP / weight$

# Tiling CUDA

A further optimization is possible:

- Each thread reads *O(n)* points from global memory to compute the shift
- Shared Memory can be exploited with the Tiling pattern
- *TILE_WIDTH = BLOCK_DIM*
- *O(n)* accesses reduced to *O(n/TILE_WIDTH)*

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**Algorithm 7** CUDA Tiling version Mean Shift core

```
function TilingCUDAMS(originalPoints)
    shiftedPoints ← originalPoints
    while iterationIndex < MAX_ITERATIONS do
        TilingKernel(shiftedPoints, originalPoints)
```

**Algorithm 8** CUDA Tiling version Kernel

```
function TilingKernel(shiftedPts, originalPts)
    tx ← threadIdx.x
    bx ← blockIdx.x
    idx ← bx ∗ blockDim.x + tx
    tile ← SharedMemArray[TILE_WIDTH]
    shiftedP ← 0
    weight ← 0
    for tileIter < numTiles do
        tileIdx ← tileIter ∗ TILE_WIDTH + tx
        if tileIdx < |originalPts| then
            tile[tx] ← originalPts[tileIdx]
        else
            tile[tx] ← nullPoint
        __synchthreads()                          ▷ End of loading
        if idx < |originalPts| then
            p ← shiftedPts[idx]
            for i with i < TILE_WIDTH do
                x ← tile[i]
                if x! = nullPoint then
                    dist ← dist(p, x)
                    w ← GKernel(dist, BW)
                    shiftedP ← shiftedP + w ∗ x
                    weight ← weight + w
        __synchthreads()                          ▷ End of computing
    if idx < |originalPts| then
        shiftedPts[idx] ← shiftedP / weight
```

# Experimental results

- Performances compared with the speedup metric, computed as:

$$S = \frac{t_S}{t_P} \tag{5}$$

- Tests executed on a machine with:
  - OS: Ubuntu 18.04 LTS
  - CPU: Intel Core i7-8565U 1.8GHz up to 4.6GHz with Turbo Boost, 4 cores/8 threads
  - RAM: 16 GB DDR4
  - GPU: NVidia GeForce MX250 2GB with CUDA 10.1
- Each time is the average of 5 experiments for the sequential and OpenMP versions and of 15 experiments for both CUDA implementations

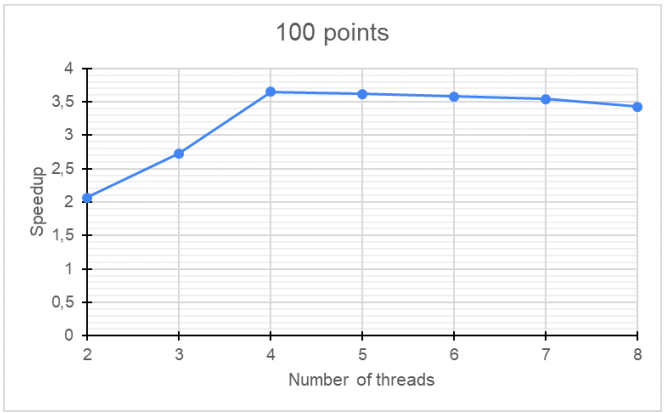
# Experimental results (cont.)

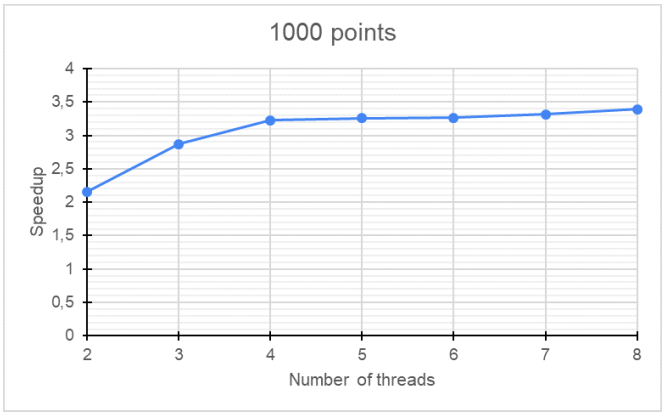


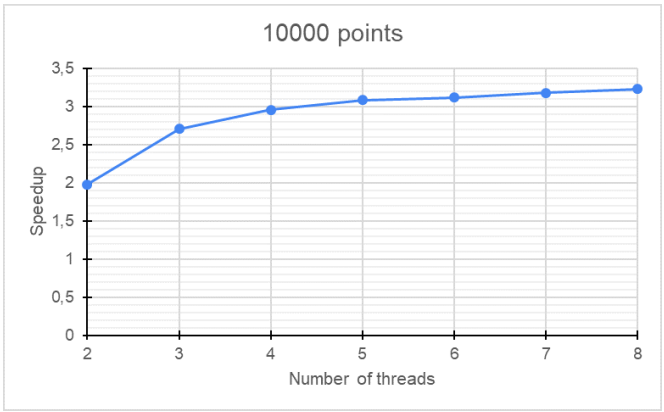
- The implementations have been evaluated on gaussian distributions composed by respectively 100, 1000, 10000, 100000 and 250000 3D points
- bandwidth set to 2
- *MAX_ITERATIONS* constant set to 10, which has been empirically estimated to be enough to make all points converge to the local maxima

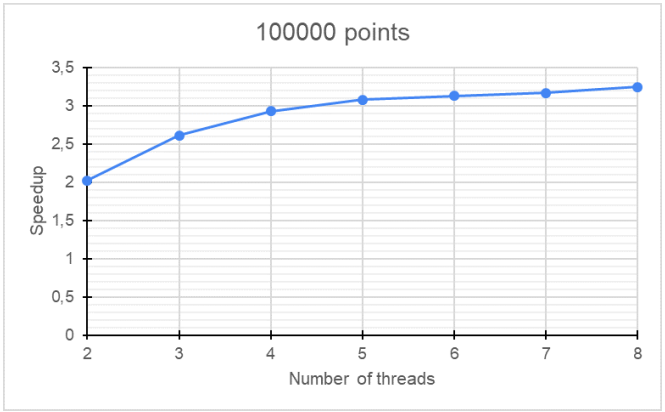
# OpenMP: Increasing threads

Number of threads gradually increased for each dataset



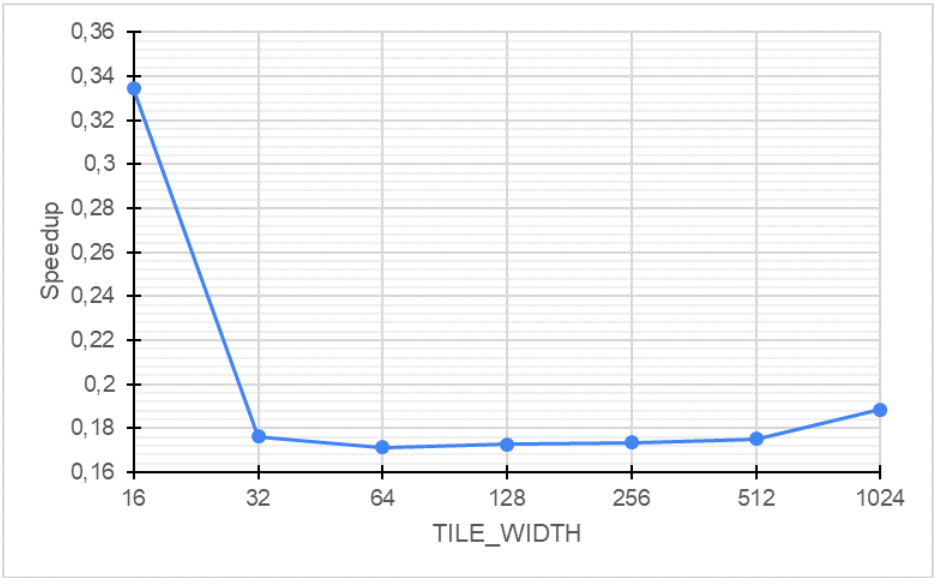




Not tested on 250k points dataset: too long execution time

# Tiling CUDA: *TILE_WIDTH*

Execution time for 10000 points for the CUDA tiling implementation varying *TILE_WIDTH*




| TILE_WIDTH | CUDA Tiling |
|---|---|
| 16 | 0.334405 s |
| 32 | 0.176171 s |
| **64** | **0.171354 s** |
| 128 | 0.172666 s |
| 256 | 0.173306 s |
| 512 | 0.175091 s |
| 1024 | 0.18845 s |

# Naive vs Tiling CUDA

| Dim | CUDA Naive | CUDA Tiling | Speedup |
|---|---|---|---|
| 100 | 0.000401 s | 0.000437 s | 1.09 |
| 1000 | 0.003044 s | 0.003359 s | 1.11 |
| 10000 | 0.171354 s | 0.191216 s | 1.12 |
| 100000 | 15.79 s | 18.29 s | 1.16 |
| 250000 | 100.38 s | 121.10 s | 1.20 |

# Tiling CUDA speedup

| Dim | Sequential | CUDA Tiling | Speedup |
|---|---|---|---|
| 100 | 0.005384 s | 0.000401 s | 13.43 |
| 1000 | 0.475485 s | 0.003044 s | 156.21 |
| 10000 | 49.29 s | 0.171354 s | 287.64 |
| 100000 | 4560.37 s | 15.79 s | 288.80 |
| 250000 | † 27768 s | 100.38 s | † 276.61 |

- † time has been estimated with a quadratic regression (due to the $O(n^2)$ computational cost)

# Global comparison

Global comparison between sequential, OpenMP and Tiling CUDA best results:

- Greatest speedups with CUDA, at the expense of a more complicated implementation
- OpenMP lets to reach noticeable speedups with a simple implementation (just a directive)

| Dim | Sequential | OpenMP | OpenMP Speedup | CUDA Tiling | CUDA Speedup |
|---|---|---|---|---|---|
| 100 | 0.005384 s | 0.001473 s | 3.66 | 0.000401 s | 13.43 |
| 1000 | 0.475485 s | 0.140161 s | 3.39 | 0.003044 s | 156.21 |
| 10000 | 49.29 s | 15.25 s | 3.24 | 0.171354 s | 287.64 |
| 100000 | 4560.37 s | 1403.78 s | 3.25 | 15.79 s | 288.80 |
| 250000 | † 27768 s | † 8720 s | † 3.18 | 100.38 s | † 276.61 |

Table: † times have been estimated with a quadratic regression

# Conclusions

- The embarassingly parallel structure of Mean Shift makes it suitable for parallel implementations
- OpenMP has an excellent speedup and development cost ratio
- CUDA makes Mean Shift applicable to datasets intractable with a CPU