Sp 516 - Télécommunications Spatiales: Project de cursus

Stefano Fortunati

Institut Polytechnique des Sciences Avancées (IPSA) &
Université Paris-Saclay, CNRS, CentraleSupélec, L2S.

Matlab implementation and analysis of a digital communication system

Main steps of the project

- During the course and in the TD, we studied the main building blocks of a digital communication system.
- We have three fundamental blocks:
  - The transmitter,
  - The channel,
  - The receiver.
- During this project, we will simulate all of them in Matlab.
- For each step, we will perform a time domain and a frequency domain analysis of the various signals.
- This allows us to better understand the issues that a practitioner working in communications field has to face.

Transmitter

- Let's start with the transmission step.
- These are the main tasks that a transmitter of a digital communication system has to perform:
  - Generation of the (complex) symbols $a[m] \triangleq I[m] + jQ[m]$,
  - Generation of the pulse $p(t)$,
  - Generation of the In-phase and Quadrature information signals:

$$x_I(t) = \sum_{k=-\infty}^{+\infty} I[k]p(t - kT_s), \quad x_Q(t) = \sum_{k=-\infty}^{+\infty} Q[k]p(t - kT_s),$$

  - Generation of the transmitted signal:

$$x_T(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t).$$

Transmitter: QPSK modulation

- We consider a QPSK modulation such that:

$$a[m] = I[m] + jQ[m] \in \{1+j, 1-j, -1+j, -1-j\},$$

where:

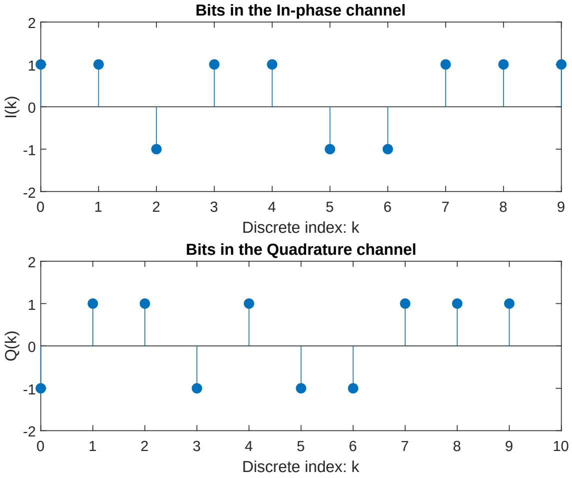
$$I[m] \in \{1, -1\},\ \Pr\{I[m]=1\} = \Pr\{I[m]=-1\} = 1/2,$$

and

$$Q[m] \in \{1, -1\},\ \Pr\{Q[m]=1\} = \Pr\{Q[m]=-1\} = 1/2.$$

- To create this two sequence in Matlab, you can use the function "randi".

Transmitter: $I[m]$ and $Q[m]$ sequences



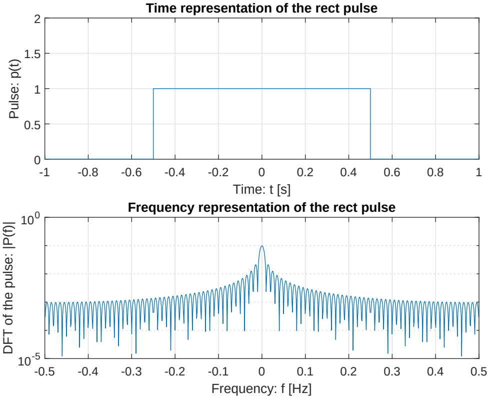
- Here you can see an example of the sequences $I[m]$ and $Q[m]$, where $m = 0, \dots, M-1$ and $M = 10$.

Transmitter: the pulse $p(t)$

- Let us start by generating the simplest pulse

$$p(t) = A_c \mathrm{rect}\left(\frac{t}{T_s}\right),$$

where T_s is the symbol rate (symbol/second) and A_c is used to choose the transmitted power.



Transmitter: the pulse $p(t)$

- In Matlab, the time variable is discrete. This means that you need to choose a given number of samples N to define the support of $p(t)$.
- Since $p(t)$ is intrinsically discrete in Matlab, the frequency analysis has to be done using the *Discrete Fourier transform*:

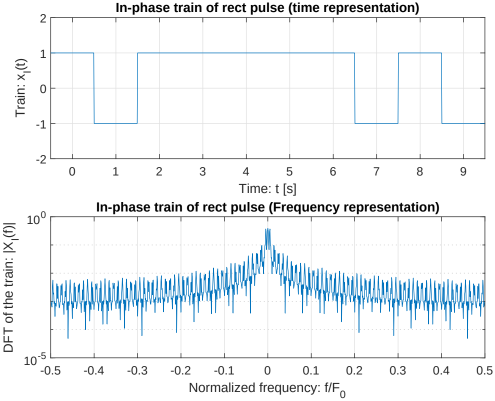
$$P(f_k) = \sum_{n=0}^{N-1} p(n\Delta T) e^{j2\pi f_k n\Delta T} = \sum_{n=0}^{N-1} p(n\Delta T) e^{j2\pi \frac{k}{\Delta T} n},$$

  where $\Delta T = T_s/N$ is the sampling interval and $F_0 = 1/\Delta T = N/T_s$ is the sampling frequency.
- In Matlab, this is obtained using the functions "fft" and "fftshift".

Transmitter: In-phase information signal

- After having generated the sequence $I[m]$ and the pulse $p(t)$, we need to generate the In-phase information signal:

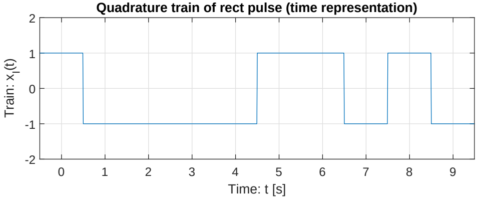
$$s_I(t) = \sum_{k=-\infty}^{+\infty} I[k]p(t - kT_s).$$

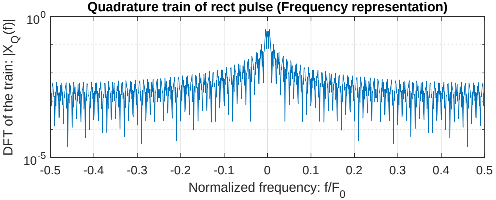


Transmitter: Quadrature information signal

- After having generated the sequence $Q[m]$ and the pulse $p(t)$, we need to generate the Quadrature information signal:

$$s_Q(t) = \sum_{k=-\infty}^{+\infty} Q[k]p(t - kT_s).$$





Transmitter: transmitted signal

- Transmitter last task: generate the transmitted signal:

$$x_T(t) = \sum_{k=-\infty}^{+\infty} I[k]p(t-kT_s)\cos(2\pi f_c t) - \\ - \sum_{k=-\infty}^{+\infty} Q[k]p(t-kT_s)\sin(2\pi f_c t),$$

- Be careful here in choosing the carrier frequency f_c!

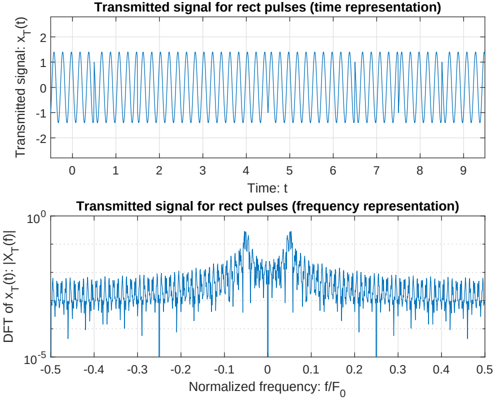
- In Matlab, all the signal are intrinsically discrete, so we need to satisfy the Nyquist's condition:

$$F_0 > 2f_{max},$$

where f_{max} is the maximum frequency of the transmitted signal (assumed to be *bandlimited* with bandwidth $B = f_{max} - (-f_{max}) = 2f_{max}$).

Transmitter: transmitted signal

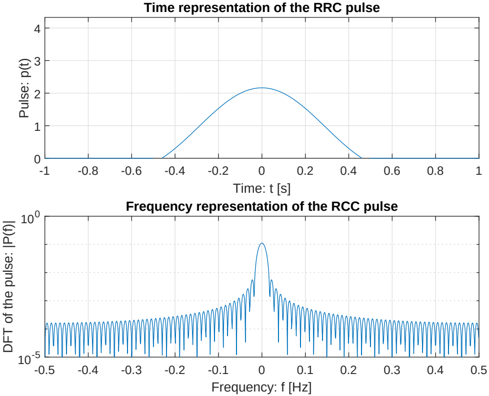
- Which is the resulting constraint on f_c?
- In the following example, I used $T_s = 1s$, $N = 100$, $f_c = 5$.



Transmitter: Root-Raised Cosine pulse

- Before moving to the receiver side, let's re-do the previous steps for a different pulse: the root-raised cosine (RCC) pulse:

$$p(t) = \frac{\sin(2\pi t(1-\alpha)/T_s) + \frac{8\alpha t}{T_s}\cos(2\pi t(1+\alpha)/T_s)}{\frac{2\pi t}{T_s}(1-(8\alpha t/T_s)^2)}.$$



Channel: adding the random noise

- As already done in the course, in this project we assume an ideal “distortionless” channel.
- This assumption implies the following channel impulse response:

$$h_C(t) = \delta(t).$$

- However, the Gaussian random disturbance is always present!
- The received signal is as follow:

$$x_R(t) = x_T(t) + \sigma_n n(t),$$

where $n(t)$ is a white Gaussian random process (use “randn” in Matlab) and σ_n is a coefficient that controls his power.

Receiver: Demodulation

- We know from the course that the first task to perform at the receiver is the demodulation.

- Specifically, we need to do the following two operations:

  - Retrieval of $x_I(t)$:

$$v_I(t) = h(t) * [x_R(t)\,(\cos(2\pi f_c t))] = \frac{1}{2}s_I(t) + n_I(t),$$

  - Retrieval of $x_Q(t)$:

$$v_Q(t) = h(t) * [x_R(t)\,(-\sin(2\pi f_c t))] = \frac{1}{2}s_Q(t) + n_Q(t).$$

- The frequency response of the low-pass filter $h(t)$ is:

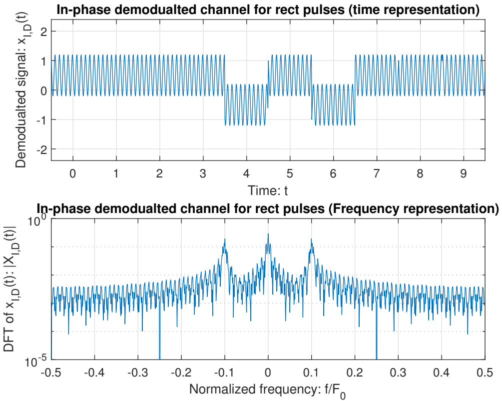
$$H(f) \triangleq \begin{cases} 1 & |f| \leqslant B_x/2 \\ 0 & |f| > B_x/2 \end{cases} = \Pi\left(\frac{f}{B_x}\right),$$

where B_x is the bandwidth of the information signal.

Receiver: $v_I(t)$ before the filter $h(t)$

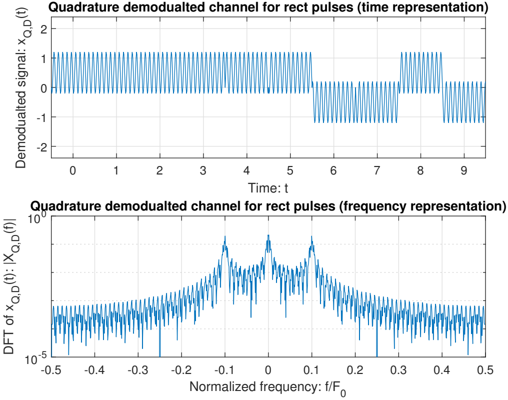
- In order to reconstruct $s_I(t)$, the first step is the multiplication of the received signal by $\cos(2\pi f_c t)$.

- In the frequency representation, we can clearly see the "useful low pass" portion of the signal and the two "useless" replica at $2f_c$.



Receiver: $v_Q(t)$ before the filter $h(t)$

- In order to reconstruct $s_Q(t)$, the first step is the multiplication of the received signal by $-\sin(2\pi f_c t)$.
- As expected, in the frequency representation, we can see the "low pass" portion of the signal and the two replica at $2f_c$.



How to choose the filter $h(t)$

- Our two information signals, $s_I(t)$ and $s_Q(t)$, are not band limited signals.

- As a consequence, the filter

$$H(f) \triangleq \begin{cases} 1 & |f| \leqslant B_x/2 \\ 0 & |f| > B_x/2 \end{cases} = \Pi\left(\frac{f}{B_x}\right),$$

  will cut some useful frequency component.

- We should choose the filter bandwidth as a good compromise between the lost of useful information and the rejection of the two replica.

- As empirical rule, we may choose

$$B_x/2 \backsimeq f_c.$$

Matched filter $h_{MF}(t)$

- After the demodulation, we need to apply the matched filter $h_{MF}(t)$ to maximize the Signal-to-Noise Ratio.

- Let us focus on the In-Phase channel, for the Quadrature one, we will follow exactly the same procedure.

- Let's define $x_{RI}(t) = x_R(t)\,(\cos(2\pi f_c t))$. Then the signal after the matched filter can be indicated as:

$$y_I(t) = h_{MF}(t) * h(t) * x_{RI}(t), \quad h_{MF}(t) \triangleq p(-t).$$

- By using the properties of the Fourier Transform, we have:

$$y_I(t) = \mathcal{F}^{-1}\left\{H_{MF}(f)H(f)X_{RI}(f)\right\}.$$

where $H_{MF}(f) = \left(\mathcal{F}\left\{p(t)\right\}\right)^*$ and $X_{RI}(f) = \mathcal{F}\left\{x_{RI}(t)\right\}$.

Matched filter $h_{MF}(t)$

- Find the expression of $y_I(t)$ in Matlab in two different ways:
  - Frequency domain using the command "fft",
  - Time domain using the command "conv".
- Repeat exactly the same procedure for the Quadrature channel.
- Provide some consideration on the impact of the Gaussian noise for different noise power σ_n^2.
- Repeat the exercise for the Root Raised Cosine pulse.

Sampling and sequence reconstruction

- The last step is the sampling of the continuous time signals $y_I(t)$ and $y_Q(t)$ as:
  - $y_I[m] = y_I(t)|_{t=mT_s}$,
  - $y_Q[m] = y_Q(t)|_{t=mT_s}$.
- Apply the sampling (with the appropriate modification) to both the frequency and time domain-obtained signals $y_I(t)$ and $y_Q(t)$.
- Provide some consideration on the impact of the Gaussian noise on the sequence reconstruction for different noise power σ_n^2.
- Repeat the exercise for the Root Raised Cosine pulse.

Sampling and sequence reconstruction

- Here an example of reconstructed sequence for $\sigma_n = 1$.
- The used pulse is the rectangular one.