$$\mathcal{L}_B = \sum_{s_i \in \{0,1\}} \left[\alpha_\ell(s_i) \cdot \log p_{a_i,\theta^\ell}(x_i^\ell, s_i) - \alpha_\ell(s_i) \cdot \log \alpha_\ell(s_i) \right]$$

maximize $\mathcal{L}_B$

subject to $\sum_{s_i \in \{0,1\}} \alpha_\ell(s_i) = 1$

$$\mathcal{L}_B' = \lambda \cdot \left[1 - \sum_{s_i \in \{0,1\}} \alpha_\ell(s_i) \right] + \mathcal{L}_B$$

$$\frac{\partial \mathcal{L}_B'}{\partial \lambda} = 1 - \sum_{s_i \in \{0,1\}} \alpha_\ell(s_i) = 0 \qquad \frac{\partial \mathcal{L}_B'}{\partial \alpha_\ell(s_i)} = -\lambda + \log p_{a_i,\theta^\ell}(x_i^\ell, s_i) - \left[\log \alpha_\ell(s_i) + 1 \right] = 0$$

$$\sum_{s_i \in \{0,1\}} \alpha_\ell(s_i) = 1$$

$$-\lambda - 1 + \log \frac{p_{a_i,\theta^\ell}(x_i^\ell, s_i)}{\alpha_\ell(s_i)} = 0$$

$$\alpha_\ell(s=0) = 1 - \alpha_\ell(s=1)$$

$$\log \frac{p_{a_i,\theta^\ell}(x_i^\ell, s_i)}{\alpha_\ell(s_i)} = 1 + \lambda$$

$$\frac{p_{a_i,\theta^\ell}(x_i^\ell, s_i)}{\alpha_\ell(s_i)} = e^{1+\lambda}$$

$$\alpha_\ell(s_i) = \frac{p_{a_i,\theta^\ell}(x_i^\ell, s_i)}{e^{1+\lambda}} \quad *$$

$$\frac{p_{a_i,\theta^\ell}(x_i^\ell, s=0)}{e^{1+\lambda}} = 1 - \frac{p_{a_i,\theta^\ell}(x_i^\ell, s=0)}{e^{1+\lambda}}$$

$$e^{1+\lambda} = p_{a_i,\theta^\ell}(x_i^\ell, s=0) + p_{a_i,\theta^\ell}(x_i^\ell, s=1)$$

$$\lambda = \log\left[p_{a_i,\theta^\ell}(x_i^\ell, s=0) + p_{a_i,\theta^\ell}(x_i^\ell, s=1) \right] - 1$$

$$* \quad \alpha_\ell(s_i) = \frac{p_{u_i,\theta^\ell}(x_i^\ell, s_i)}{e^{1+\lambda}}$$

$$\lambda = \log\left[p_{u_i,\theta^\ell}(x_i^\ell, s=0) + p_{u_i,\theta^\ell}(x_i^\ell, s=1)\right] - 1$$

$$\Rightarrow \alpha_\ell(s_i) = \frac{p_{u_i,\theta^\ell}(x_i^\ell, s_i)}{e^{1+\log[p_{u_i,\theta^\ell}(x_i^\ell, s=0) + p_{u_i,\theta^\ell}(x_i^\ell, s=0)] - 1}}$$

$$= \frac{p_{u_i,\theta^\ell}(x_i^\ell, s_i)}{\sum_{\bar{s}\in\{0,1\}} p_{u_i,\theta^\ell}(x_i^\ell, \bar{s})}$$

$$\alpha_\ell(s_i) = p_{u_i,\theta^\ell}(s_i \mid x_i^\ell)$$