Learning Network Size while Training with ShrinkNets

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Abstract

Let's write the abstract at the end

1. Introduction

As neural networks become increasingly widely deployed in a variety of applications – ranging from improving camera quality on mobile phones (**?**) to language translation (**?**) to text auto-completion (**?**) – and on diverse hardware architectures – from laptops to phones to embedded sensors – inference performance and model size are becoming as important in learning as metrics measures of prediction quality. However, these three aspect of model performance – quality, performance and size – are largely optimized separately today, often with suboptimal results.

Of course, the problem of finding compressed or small networks is not new. Existing techniques typically aim make a pre-trained neural network smaller (**??**) by taking one of two approaches: either by applying quantization (**?**) or code compilation (**?**) techniques that can be applied blindly to any network, or by analyzing the structure of the network and systematically pruning connections (Han et al., 2015; Cun et al., 1990) or neurons (**?**) based on some loss function. Although these techniques can substantially reduce model size, they have several drawbacks. First, they often negatively impact model quality. Second, they can (surprisingly) negatively impact inference time as they transform dense matrix operations into sparse ones, which can be substantially slower to execute on modern hardware (**?**), especially GPUs which do not efficiently support sparse linear algebra. Third, these techniques generally start by optimizing a particular architecture for prediction performance, and then, as a post- processing step, applying compression to generate a smaller model that meets the resource constraints of the deployment setting. Because the network architecture is essentially fixed during this post-processing, model architectures that work better in small settings may be missed – this is especially true in large networks like many-layered CNNs, where it is infeasible to try explore even a small fraction of possible network configurations.

In contrast, in this paper we present a new method to simultaneously optimize both network size and model performance. The key idea is to learn the right network size at the same time that we optimize for prediction performance. Our approach, called *ShrinkNets*, starts with an *oversized* network, and dynamically shrinks it by eliminating neurons during training time. Our approach has two main benefits. First, we explore the architecture of models that are both small and perform well, rather than starting with a high- performance model and making it small. This allows us to efficiently generate a family of smaller and accurate models without an exhaustive and expensive hyperparameter search over the number of neurons in each layer. Second, in contrast to existing neural network compression techniques (Aghasi et al., 2016; Han et al., 2015), our approach results in models that are not only small, but where the layers are dense, which means that inference time is also improved, e.g., on GPUs.

XXX people actually don't treat network size as a hyperparameter; we do!

In summary, our contributions are as follows:

1. We propose a novel technique based on dynamically switching on and off neurons, which allows us to optimize the network size as the network is trained.
2. **Sam:** xxx postprocessing techniques to strip out switch layers
3. We show that our technique is a relaxation of group LASSO (Yuan & Lin, 2006) and prove that our problem admits many global minima.
4. **Sam:** Some claim about model size vs performance
5. We demonstrate the efficacy of our technique on both convolutional and fully-connected neural nets, showing that ShrinkNets finds networks within +/-X% of best hand-crafted accuracy in XX% of training time compared to existing hyperparameter optimization methods. **Sam:** Mention the specific model/dataset where this is true.
6. We demonstrate that ShrinkNets can achieve this accuracy with only YY% of neurons. **Sam:** This claim should be something about how much smaller a network with say 99% accuracy is one some real dataset
7. **Sam:** Something about dense networks and then bene-

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Preliminary work. Under review by the International Conference on Machine Learning (ICML). Do not distribute.

fit over optimal brain damage
8. **Sam:** Some claim about inference time
9. **Sam:** Some claim about compatibility with existing compression techniques?

2. The ShrinkNets Approach

In this section we describe the ShrinkNets approach, which allows us to learn the network size as part of the training process. The key idea is the addition of *switch layers* added after each layer in the network; these layers allow us to disable certain neurons during training. Once we've introducd the basic method, we explain how to adapt the training procedure to support this new layer.

2.1. Overview

At a high-level, the ShrinkNets approach consists of two interconnected stages. The first stage identifies neurons that do not contribute to increasing the prediction accuracy of the network and deacivates them. The second stage is concerned with actually removing the neurons from the network and shrinking its size, therefore leading to faster inference times. We give an overview of both stages next:

Deactivating Neurons On-The-Fly: During the first stage, ShrinkNets applies an on/off switch to every neuron of an initially oversized network. We model the on/off switches by multiplying each input (or output) of each layer by a parameter $\boldsymbol{\theta}$, with values 0 or 1. A value of 0 will deactivate the neuron, while 1 will let the signal go through. These switches are part of a new layer, called **switch layer**, which can be applied to fully connected as well as convolutional layers.

We want to minimize the number of on switches to reduce the model size as much as we can, while preserving prediction accuracy. This can be achieved by jointly minimizing the objective of the network and a factor of the L0 norm of the vector containing all the on/off switches. Because finding an optimal binary assignement is an NP-Hard problem, we allow $\boldsymbol{\theta}$ to be a real number instead of a 0/1 value, thus constraining the L1 norm as opposed to the L0 norm.

Neuron Removal: During this stage, the neurons that are deactivated by the switch layers are actually removed from the network, effectively shrinking the network size. This is what leads to faster inference times, as we demonstrate in our evaluation. We choose to remove neurons at training time because we have observed that allows the active neurons to adapt to the new network architecture. Existing techniques focus on neuron removal after training, and require an extra fine-tuning process to compensate for the removal.

In the remainder of this section we describe in detail the switch layer as well as how to adapt the training process for ShrinkNets.

2.2. The Switch Layer

Let L be a layer in a neural network that takes an input tensor $\boldsymbol{x}$ and produces an output tensor y of shape $(c \times d_1 \times \cdots \times d_n)$ where c is the number of neurons in that layer. For instance, for fully connected layers, n=0 and the output is single dimensional of size c (ignoring batch size for now) while for a 2-D convolutional layer, n=2 and c is the number of output channels or feature maps.

We want to tune the size of L by applying a switch layer, S. The switch layer is parametrized by a vector $\theta \in \mathbb{R}^c$ such that the result of applying S to $L(\boldsymbol{x})$ is a tensor also of size $(c \times d_1 \times \cdots \times d_n)$ such that:

$$S_\theta(L(\boldsymbol{x}))_{i,\ldots} = \boldsymbol{\theta}_i L(\boldsymbol{x})_{i,\ldots} \tag{1}$$

Effectively, once passed through the switch layer, each output channel i produced by L is scaled by the corresponding θ_i. Note that when $\theta_i = 0$, the i^{th} channel is multiplied by zero and will not contribute to any computations after the switch layer. If this happens, we say the switch layer has *deactivated* the neuron of layer L corresponding to channel i.

We place switch layers after each layer whose size we wish to tune; these are typically fully connected and convolutional layers. We discuss next how to train ShrinkNets.

2.3. Training ShrinkNets

For training, we need to account for the effect of the swith layers in the loss function. The effect of switch layers can be expressed in terms of a sparsity constraint that pushes values in the θ vector to 0. Then, given a neural network parameterized by weights $\boldsymbol{W}$ and switch layer parameters $\boldsymbol{\theta}$, we optimize the following ShrinkNet loss:

$$L_{SN}(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{W}, \boldsymbol{\theta}) = L(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{W}) + \lambda \left\|\theta\right\|_1 + \lambda_2 \left\|\boldsymbol{W}\right\|_p \tag{2}$$

i.e., the expression augments the regular training loss with a regularization term for the switch parameters and another on the network weights.

We next analyze why such loss function works to train ShrinkNets and its connection to group sparsity:

Relation to Group Sparsity (LASSO): ShrinkNets removes neurons, i.e., inputs and outputs of layers. For a

fully connected layer defined as:

$$f_{\boldsymbol{A},\boldsymbol{b}}(\boldsymbol{x}) = a(\boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}) \tag{3}$$

Removing an input neuron j is equivalent to have $\left(\boldsymbol{A}^T\right)_j = \boldsymbol{0}$. Removing an output neuron i is the same as having $\boldsymbol{A}_i = \boldsymbol{0}$ and $\boldsymbol{b}_i = 0$. Solving optimization problems while trying to set entire group of parameters to zero is the goal of group sparsity regularization (). For any partitioning of the set of parameter defining a model in p groups: $\boldsymbol{\theta} = \bigcup_{i=1}^{P} \boldsymbol{\theta}_i$ we define it the following way:

$$\Omega_\lambda^{gp} = \lambda \sum_{i=1}^{p} \sqrt{\#\boldsymbol{\theta_i}} \left\|\boldsymbol{\theta_i}\right\|_2 \tag{4}$$

In fully-connected layers, the groups are either: columns of $\boldsymbol{A}$ if we want to remove inputs, or rows of $\boldsymbol{A}$ and the corresponding entry in $\boldsymbol{b}$ if we want to remove outputs. For simplicity, we focus our analysis in the simple one-layer case. In this case, filtering outputs does not make sense, this is why we will only consider the former case. The group sparsity regularization then becomes:

$$\Omega_\lambda^{gp} = \lambda \sum_{j=1}^{p} \left\|\left(\boldsymbol{A^T}\right)_{\boldsymbol{j}}\right\|_2 \tag{5}$$

Because $\forall i, \#\boldsymbol{\theta}_i = n$, **ra:** is # the general way of expressing cardinality? why not —x—?to make the notation simpler, we embedded $\sqrt{n}$ inside λ.

Group sparsity and ShrinkNets try to achieve the same goal. We discuss next how they are related to each other. First let's recall the two problems. The original ShrinkNet problem is:

$$\min_{\boldsymbol{A},\boldsymbol{\beta}} \left\|\boldsymbol{y} - \boldsymbol{A}\mathrm{diag}\left(\boldsymbol{\beta}\right)\boldsymbol{x}\right\|_2^2 + \lambda \left\|\boldsymbol{\beta}\right\|_1 \tag{6}$$

And the Group Sparsity problem is:

$$\min_{\boldsymbol{A}} \left\|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\right\|_2^2 + \Omega_\lambda^{gp} \tag{7}$$

We can prove that under the condition: $\forall j \in [\![1, p]\!], \left\|\left(\boldsymbol{A}^T\right)_j\right\|_2 = 1$ the two problems are equivalent (proposition A.1). However if we relax this constraint then shrinknet becomes non-convex and has no global minimum (propositions A.2 and A.3). Fortunately, by adding an extra term to the ShrinkNet regularization term we can proove that:

$$\min_{\boldsymbol{A},\boldsymbol{\beta}} \left\|\boldsymbol{y} - \boldsymbol{A}\mathrm{diag}\left(\boldsymbol{\beta}\right)\boldsymbol{x}\right\|_2^2 + \Omega_\lambda^s + \lambda_2 \left\|A\right\|_p^p \tag{8}$$

has global minimums for all $p > 0$ (proposition A.4). This is the reason we defined the *regularized ShrinkNet penalty* earlier this way:**ra:** the notation is slightly different, better to make it consistent

$$\Omega_{\lambda,\lambda_2,p}^{rs} = \lambda \left\|\boldsymbol{\beta}\right\|_1 + \lambda_2 \left\|\boldsymbol{\theta}\right\|_p^p \tag{9}$$

In practice we observed that $p = 2$ or $p = 1$ are good choice, while the latter will also introduce additional sparsity in the parameters.

3. ShrinkNets in Practice

In this section we discuss practical aspects of ShrinkNets.

3.1. On-The-Fly Neuron Removal

Switch layers are initialized with weights sampled from $\mathcal{N}(0, 1)$, and their values change as part of the training process so as to swith *on* or off neurons. When neurons are deactivated by switch layers, we want to remove them to shrink the network size. Because after removing a neuron it will no longer contribute to improving prediction accuracy, we must devise strategies that remove neurons while being robust to changing values. We observe three strategies:

Threshold strategy: Under this strategy neurons are removed based on a fixed threshold expressed in terms of its absolute value. This is the method used by deep compression (). We found this method is not robust to noise in the gradients—which occurs commonly when training networks with stochastic gradient descent—because the threshold depends on the scale of each weight.

Sign change strategy: Under this strategy neurons are removed when the weight value changes its sign. This strategy works well in practice but it is also sensible to gradient noise. **ra:** explain what's the consequence of that If we sample a gradient that is not representative of the dataset when performing gradient descent then we might wrongly kill a neuron.**ra:** last sentence is unclear

Sign variance strategy: We measure the exponential moving variance of the sign $(-1, 1)$ of each parameter in the *switch layer*. When the value of the exponential moving variances goes over a predefined threshold, we consider the parameter is deactivating the neuron, and therefore we remove it. This strategy introduces two extra parameters, one to control the behavior of the inertia of the variance, and the threshold, but we have found it is the best performing in practice.

The last two strategies perform the best in practice; they reduce the network parameters by an order of magnitude with only a minimal impact on the final error obtained by the model.

3.2. Additional Optimizations for ShrinkNets

Preparing for Inference: With ShrinkNets we obtain reduced-sized networks while training, which is the first benefit towards faster inference. This networks are readily available for inference. However, because they include switch layers—and therefore more parameters—they introduce unnecessary overhead at inference time. To avoid this overhead, we reduce the network parameters by combining each switch layer with its respective network layer by multiplying the respective parameters. **ra:** can't we just simply prevent the following problem at network design time?In the unlikely case that a switch layer is sandwiched between two non-linearly scalable layers, we leave the switch layer as is.

Neural Garbage Collection: ShrinkNets decide on-the-fly which neurons to remove. Since ShrinkNets remove a large fraction of neurons, we must dynamically resize the network at runtime to not carry on the unnecessary overhead. We implement a neural garbage collection method as part of our library which takes care of updating the necessary network layers as well as updating optimizer state to reflect the neuron removal.

4. Evaluation

The goal of our evaluation is to explore:

- Whether, by varying λ, *ShrinkNets* can efficiently explore (in terms of number of training runs) the spectrum of high-accuracy models from small to large, on both CNNs and fully connected networks. Our results show that, for each network size, we obtain models that perform as well or better than *Static Networks*, trained via traditional hyperparameter optimization.
- Whether, because these smaller networks are dense, they result in improved inference times on both CPUs and GPUs.
- Whether the ShrinkNets approach results in network architectures that are substantially different than the best network architecutres (in terms of relative number of neurons per layered) identified in the literature.
- **Sam:** something else?

ra: add here the impl. thingy from s3, so we give the context of how we use shrinknets

4.1. Performance vs. Traditional Methods

We evaluate ShrinkNets on two datasets: `CIFAR10` and `COVERTYPE`. In this section, we report how we employed ShrinkNets on these datasets and compare our results to training without ShrinkNets.

4.1.1. `CIFAR10`

ra: this is related workPrevious research (Scardapane et al., 2017) that attempts to optimize network size during training focused on simple datasets and simple (i.e., fully connected) architectures. We believe that self-resizing network are most important for problems where the architecture involves many layers, such that that it is impractical to explore the space of all possible layer-size configurations.**ra:** until here For that reason, we picked `CIFAR10` (Krizhevsky, 2009). It is an image classification dataset containing 60000 color images ($3\times32\times32$), belonging to 10 different classes, running on the `VGG16` network (Srivastava et al., 2014). VGG16 consists of alternating convolutional layers and *MaxPool* layers interleaved by *BatchNorm* (Ioffe & Szegedy, 2015) and *ReLU* (Nair & Hinton, 2010) layers. The two last layers are fully connected layers separated by a *ReLU* activation function.

Sam: keep this? Although ShrinkNets supports larger networks and datasets as well (i.e., ImageNet (Russakovsky et al., 2015)), the time required to such train models and get statistically significant result was prohibitive – especially when comparing against classical hyperparameter search over network architecture.

We applied ShrinkNets to the VGG16 network by adding *Switch Layers* after each *BatchNorm* layer and each fully connected layer (except the last). Recall that ShrinkNets assume that the starting size of the network is an upper bound on the optimal size. For this reason, we simply started with a network with double the recommended size for each layer as an upper bound (this is larger than what ImageNet, for example uses). Thus, for the classification layers we use 5000 neurons as a starting limit (ImageNet uses 4096 **Gl:** This is on the top of my head, need to be double checked).

We assume no prior knowledge on the optimal batch size, learning rate, λ or weight decay (λ_2). Instead, we trained a number of models, randomly and independently selecting the values of these parameters from a range of reasonable values (**Gl:** should we make them explicit ? **Sam:** yes). We trained using our modified `PyTorch` (Paszke et al., 2017) library that supports dynamically resized layers (our code is available at **Sam:** add anonymized link). Training is done using gradient descent and the *Adam* optimizer (Kingma & Ba, 2014). Specifically, we start with the learning rate sampled randomly; for every 5 epochs of non-improvement in validation accuracy we divide the learning rate by 10. We stop training after 400 epochs or when the learning rate is under 10^{-7}, whichever comes first. **Gl:** Should I also give the details about the removal strategy we used and the γ and threshold I used ? because this is clearly getting boring **Sam:** probably not necessary For each of the models we trained, we pick the epoch with the best validation accu-

racy and report the corresponding testing accuracy. Because of the nature of our method, it can happen that for networks that are aggressively compressed, the best validation accuracy is obtained early in training, before the size has converged. To be sure that accuracy measured corresponds to the final shape and not the starting shape, we only consider the second half of the training when picking the best epoch. For each model, we also measure the total size, in terms of number of floating point parameters, excluding the *Switch Layers* because as described in section 3.1, these are eliminated after training.

We compare against classical (*Static*) networks. In such networks, the number of parameters that control the size is large: 13 parameters for the convolutional layers and 2 for the fully connected layers. ShrinkNets effectively fuse all these parameters in a single λ, but in conventional architectures where all of these parameters are free, it is infeasible to obtain a reasonable sample of a search space of this size. For this reason, we rely on the conventional heuristic that the original VGG architecture (and many CNNs) **Gl:** try to find the paper that introduces this heuristic user, where **Sam:** describe what the heuristic is – doubling of neurons up to apoint? For *Static Networks* we sample the size between 0.1 and 2 times the size **Sam:** size of what? optimized for ImageNet. **Sam:** Why are we using ImageNet and not CIFAR10 as the comparison point? We report the same numbers as we did for *ShrinkNets* and we compare the two distributions. **Sam:** I worry that this method of selecting network sizes for Static will of course constrain it to larger sizes and worse perforamance. What if we just randomly sample networks? Would it look terrible?

The results are shown in the top figure of fig. 1. Here, the blue dots show the models produced by ShrinkNets, and the orange dots show the models produced for the static networks. For each model, we plot its accuracy and model size. The lines show the Pareto frontier of models in each of the two optimization settings. From the figure, it's apparent that ShrinkNets much more effectively explores the trade-off between model size and accuracy. **Sam:** Would be good to show Lambda values on the figure somehow. **Sam:** I thought we were going to summarize with distributional plots instead of Pareto frontiers? **Sam:** Summarize more quantitatively – best model performance, model performance drop off for a factor of 10 reduction in seize, etc. so that we can repeat a quantitative claim in the intro, i.e., "Note that the best performing ShrinkNets models has 9x.x% accuracy while the best static model has 9x.x% accuracy, while the ShrinkNets model is x times smaller. In addition, if we give up just 1% error, ShrinkNets finds a model that is z times smaller." (If we do this, would be good to highlight these points in the figure somehow)

4.1.2. COVERTYPE DATASET

The second dataset we use for evaluation is the COVERTYPE (Blackard, 1998) dataset. This dataset contains 581012 descriptions of geographical area (elevation, inclination, etc...) and the goal is to predict the type of forest growing in each area. We picked this dataset for several reasons. First it is simple, such that we can reach good accuracy with only a few fully-connected layers. This important because we want to show that *ShrinkNets* find sizes as good as *Static Networks*, even if we are sampling the entire space of possible network sizes. The second reason is that Scardapane et al (Scardapane et al., 2017) also perform their evaluation on this dataset, which allow us to compare the results obtained by our method with the method in (Scardapane et al., 2017).

The experimental setup on this dataset is similar to CIFAR10. We use the same architecture as (Scardapane et al., 2017), i.e., three fully-connected layers network with no *Dropout* (Srivastava et al., 2014) or *BatchNorm*. **Gl:** Should we say here that we don't expect Dropout to work here ? I could write an entire paragraph about it if needed. In this case, for the *Static Networks*, we independently samples the sizes of the three different layers to explore all possible architectures.

The results are shown in the top figure of fig. 2, with the two optimization methods plotted as before. Here, *Static* method finds models that perform well at a variety of sizes, because it is able to explore the entire parameter space. This is as expected; the fact that ShrinkNets performs as well as the Static indicates that ShrinkNets is doing an effective job of exploring the parameter space using just the single λ parameter. **Sam:** Summarize more quantitatively – best model performance, model performance drop off for a factor of 10 reduction in seize, etc. so that we can repeat a quantitative claim in the intro, i.e., "Here, the best performing ShrinkNets models has 9x.x% accuracy while the best static model has 9x.x% accuracy, while the ShrinkNets model is x times smaller. Further, note that ShrinkNets finds a model with 9x.x% accuracy that is y times than the best performing model. Also, as on CIFAR10, if we give up just 1% error, ShrinkNets finds a model that is z times smaller." (If we do this, would be good to highlight these points in the figure somehow)

4.1.3. DISCUSSION

Sam: we may not need to keep this unless we can think of something more to say

The Pareto frontier in the two figures shows that that *ShrinkNets* is able find models that both as accurate and smaller than *Static Networks*. when we sample the entire size space (in the COVERTYPE case), *ShrinkNets* are bet-

ter or equally as good as *Static Networks*. This has two implications. First it reinforces our observation **Sam:** our observation? in [ref] that showed that the models do not suffer from the non-convexity introduced by the multiplication by the *Switch Layer*. Secondly it could let us conjecture that the parametrization from a λ (real value) to the size of the size of the network (vector) close to optimal. Indeed, if it was not the case then eventually a randomly sampled size would have beaten *ShrinkNets* models. **Gl:** Too strong ? **Sam:** I think this is too strong – we have no evidence this is true, do we?

4.2. Benefits of smaller sizes

We showed in the previous experiments that the *ShrinkNets* approach is able to find networks with accuracy equal to or better than *Static Networks*. In this experiment we want to determine benefits of the smaller size we get.

For some applications, the absolute best accuracy is not something desirable if it implies having enormous models. This observation motivates our methodology: for a given target accuracy that you want your model to achieve, we pick the smallest that satisfy the constraint. For both *ShrinkNets* and *Static Networks* we measure multiple metrics and compute the ratio between the two. The metrics are interested in are: the final model size (in number of parameters) and inference speed on CPU and GPU for small batch sizes (1 input) and large batch sizes (depend on the dataset and hardware). Results are shown on fig. 1 and fig. 2 after the distribution of sizes and accuracy. We limit our plot to the $(80 - 100\%$ accuracy range because we consider that models with lower accuracy have less practical use).

Regarding size, the key observation we can make is that size improvements are significant for the `CIFAR10`. In the range of accuracies we are interested in, improvements in size go from 4x to 40x. On the `COVERTYPE` dataset even if the compression ratio is always above 1, it hardly exceeds 3x, except for very high accuracy where *ShrinkNets* obtained some particularly good solutions.

We observe that unlike local sparsity compression based method, our method translates directly improvement in size to higher bandwidth at inference time. In situations where inference is already incredibly fast (`COVERTYPE` with a batch size of 1 and `CIFAR10` with a batch size of 1 on GPU), improvements are dominated by noise. However, on machine learning frameworks more focused on latency it should be possible to measure them.

4.3. Architectures obtained after convergence

The previous experiments showed that *ShrinkNets* finds architectures the effectively explore the frontier of model size and accuracy. Further, for a given accuracy, the size needed is significantly smaller than when we use the **Sam:** name it heuristic commonly used to size convolutional neural networks. This suggests that this conventional heuristic may not in fact be optimal, especially when looking for smaller models. Empirically we observed this to often be the case. For example, during our experimentations on the `MNIST` (LeCun et al., 2001) and `FashionMNIST` (Xiao et al., 2017) datasets (not reported here due to space constraints), we observed that even though these datasets have the same number of classes, input features, and output distributions, for a fixed λ *ShrinkNets* converged to considerably bigger networks in the case of `FashionMNIST`. This evidence shows that optimal architecture not only depends on the output distribution or shape of the data but actually reflects the dataset. This makes sense, as `MNIST` is a much easier problem than `FashionMNIST`.

To illustrate this point on a larger dataset, we show two examples of architectures learned by *ShrinkNets* in Figure 3. The top plot shows the model with the best test accuracy, with **Sam:** identical performance to the best static network?; the bottom shows a network that slightly under-performs the best in terms of accuracy but is significantly smaller that the best equivalent *Static Network* **Sam:** how much smaller?. In the plot, the dashed line shows the number of neurons in each layer of the original VGG net, and the shaded regions show the size of the ShrinkNet as it converges (with the darkest region representing the fully converged network). Observe that the final network that is trained looks quite different in the two cases, with the optimal performing network appearing similar to the original VGG net, whereas the shrunken network allocates many fewer neurons to the middle layers, and then additional neurons to the final fewer layers.

5. Related Work

- Other techniques for choosing network size (e.g., hyperparmeter optimization) – manual effort vs optimal techniques – people actually don't treat network size as a hyperparameter; we do! Also brute force techniques (**?**)
- distillation techniques – train a big net, then train a smaller net from it
- post-training compression techniques – brain damage ,
- group sparsity e.g., (Scardapane et al., 2017) and non-parametric neural networks –
- training dynamics paper: first overfitting and then randomization?, **Gl:** Here is the ref, if you can introduce it in the flow (Shwartz-Ziv & Tishby, 2017)

Given the importance of network structure, many techniques have been proposed to find the best network structure for a

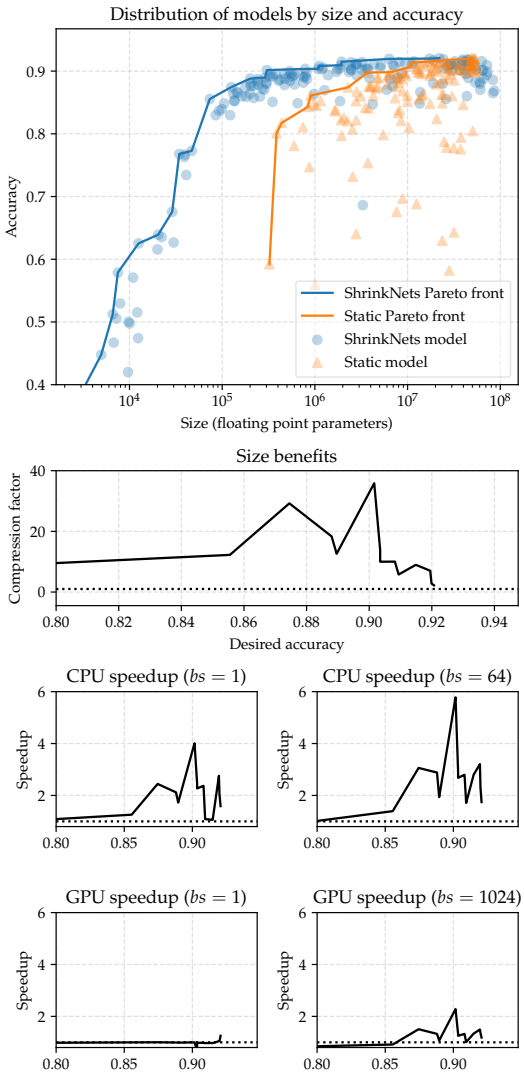

Figure 1. Summary of the result of random search over the hyper-parameters the `CIFAR10` dataset

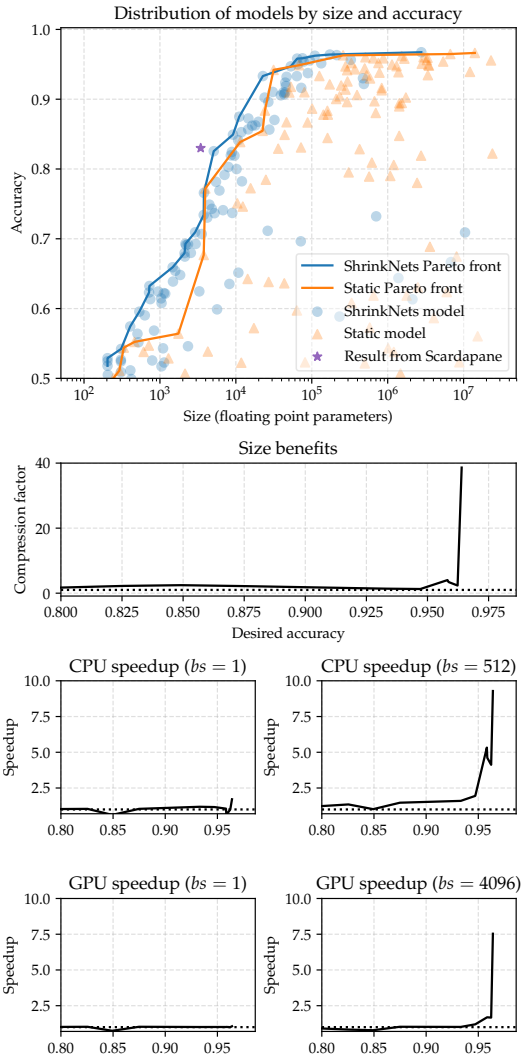

Figure 2. Summary of the result of random search over the hyper-parameters the `COVERTYPE` dataset **Sam:** label the x axes on the bottom plots

given learning task. These techniques broadly fall into four categories: hyperparameter optimization strategies, post-training model compression for inference as well as model simplification, techniques to resize models during training, and automated architecture search methods.

The most popular techniques for hyperparameter optimization include simple methods such as random search (Bergstra & Yoshua, 2012) which have been shown to work suprisingly well compared to more complex methods such as those based on Bayesian optimization (Snoek et al., 2012). Techniques such as (Snoek et al., 2012) model generalization performance via Gaussian Processes (Rasmussen & Williams, 2006) and select hyperparameter combinations that come from uncertain areas of the hyperparameter space. Recently, methods based on bandit algorithms (e.g. (Li et al., 2016; Jamieson & Talwalkar, 2016)) have also become a popular way to tune hyperparameters. As noted before, all of the above techniques require many tens to hundreds of models to be trained, making this process computationally inefficient and slow.

In contrast with hyperparameter tuning methods, some methods such as DeepCompression (Han et al., 2015) seek to compress the network to make inference more efficient. This is accomplished by pruning connections and quantizing weights. On similar lines, multiple techniques such as (Romero et al., 2014; Hinton et al., 2015) have been proposed for distilling a network into a simpler network or a different model. Unlike our technique which works during training, these techiques are used after training and it would be interesting to apply them to ShrinkNets as well. (Abadi et al., 2016) share the common goal of removing entire blocks of parameter to maintain dense matrices, however

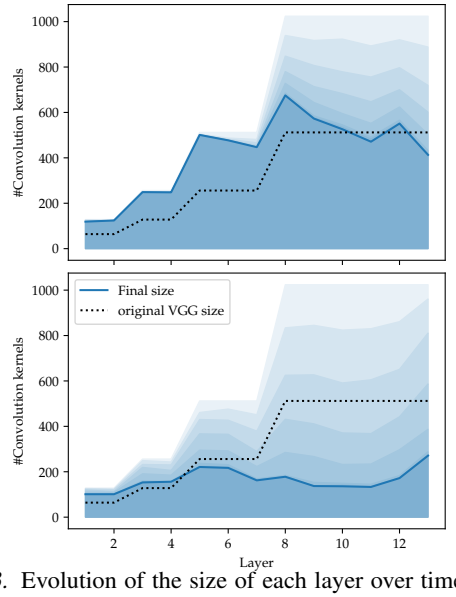

Figure 3. Evolution of the size of each layer over time (lighter: beginning, darker: end). On top a very large network performing 92.07%, at the bottom a simpler model with 90.5% accuracy.

their method only applies to convolutional layers.

The techniques closed to our work are those based on group sparsity such as (Scardapane et al., 2017), and those like (Philipp & Carbonell, 2017) who grow and shrink dynamically the size of the network during training.

Finally, there has also been recent work in automatically learning model architecture through the use of genetic algorithms and reinforcement learning techniques (Zoph & Le, 2016; Zoph et al., 2017). These techniques are focused more on learning higher-level architectures (e.g. building blocks for neural network architectures) as opposed to learning network size.

6. Discussion

Gl: Did not have time to write it but added some pointers

- Where does the method shine: We shine where there are too many layers To explore the network size and we have to rely on heuristics. It shines by it simplicity compared to other methods **Gl:** Do we want to talk about the number of lines of codes ?. Prunning while we train, from the compression results seem to be benificial to prune while training to achieve very good compression.
- Side-effects of method: smaller networks, time to train. There is not significant difference in training time, mostly because the filter layer is not optimized. However we can say the convergence speed is improved. For example for `CIFAR10` the mean number of epoch to reach the best is 62.5 for *ShrinkNets* vs 68.4 for normal networks. for `COVERTYPE` the numbers are respectively 117.9 and 130.
- Potential extensions/limitations: We could add a filter that learn the size of convolutions for convolution layers. Supporting Residual Network would allow us to support more architecture. We could also try to mix try to learn the number of layers such is described in (Meier)

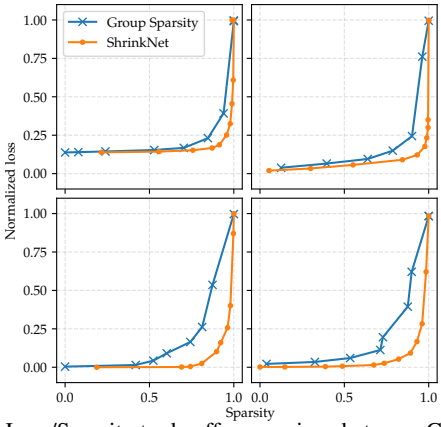

Figure 4. Loss/Sparsity trade off comparison between Group Sparsity and Shrinknet on linear and logistic regression. From top to bottom and left to right we show the results for `scm1d`, `oes97`, `gina_prior2` and `gsadd`.

7. Conclusion

A. Appendix

A.1. Proofs

Unless stated explicitely, all the propositions consider the Multi-Target linear regression problem.

Proposition A.1. $\forall (n,p) \in \mathbb{N}_+^2, \boldsymbol{y} \in \mathbb{R}^n, \boldsymbol{x} \in \mathbb{R}^p \lambda \in \mathbb{R}$

$$\min_{\mathbf{A}} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda \sum_{j=1}^{p} \left\| \left(A^T\right)_j \right\|_2$$
$$= \min_{\mathbf{A}',\boldsymbol{\beta}} \|\mathbf{y} - \mathbf{A}' diag\left(\boldsymbol{\beta}\right) \mathbf{x}\|_2^2 + \lambda \|\boldsymbol{\beta}\|_1$$
$$s.t. \forall j, 1 \leq j \leq p, \left\| \left(A'^T\right)_j \right\|_2^2 = 1$$

Proof. we will split this proof in two different sections. the first one will demonstrate that there is at least one global minimum. and the second will show how to construct 2^k distinct solutions from a single global minimum. In order to prove this statement we will show that for any solution $\boldsymbol{A}$ in the first problem, there exists a solution in the second with the exact same value, and vice-versa.
Part 1: We assume we have a potential solution $\boldsymbol{A}$ for the

first problem and we define $\boldsymbol{\beta}$ such that $\boldsymbol{\beta}_j = \left\| \left(\boldsymbol{A}^T\right)_j \right\|_2^2$, and $\boldsymbol{A'} = \boldsymbol{A}\left(\text{diag}\left(\boldsymbol{\beta}\right)\right)^{-1}$. It is easy to see that the constraint on $\boldsymbol{A'}$ is statisfied by construction. Now:

$$\begin{aligned}
&\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda \sum_{j=1}^{p} \left\| \left(A^T\right)_j \right\|_2 \\
&= \|\mathbf{y} - \mathbf{A'}\text{diag}\left(\boldsymbol{\beta}\right)\mathbf{x}\|_2^2 + \lambda \sum_{j=1}^{p} \left\| \left(A'^T\right)_j \beta_j \right\|_2 \\
&= \|\mathbf{y} - \mathbf{A'}\text{diag}\left(\boldsymbol{\beta}\right)\mathbf{x}\|_2^2 + \lambda \sum_{j=1}^{p} |\beta_j| \cdot 1 \\
&= \|\mathbf{y} - \mathbf{A'}\text{diag}\left(\boldsymbol{\beta}\right)\mathbf{x}\|_2^2 + \lambda \|\boldsymbol{\beta}\|_1
\end{aligned}$$

Which concludes the first part.

Part 2: Assuming we take an $\boldsymbol{A'}$ that satisfy the constraint and a $\boldsymbol{\beta}$, we can define $\boldsymbol{A} = \boldsymbol{A'}\text{diag}\left(\boldsymbol{\beta}\right)$. We can apply the same operations in reverse order and obtain an instance of the first problem with the same value.

Conclusion There is no way these two problems have different minima, because we are able to construct a solution to a problem from the solution of the other while preserving the value of the objective. □

Proposition A.2.

$$\|\mathbf{y} - \mathbf{A}\textit{diag}\left(\boldsymbol{\beta}\right)\mathbf{x}\|_2^2$$

is not convex in $\boldsymbol{A}$ and $\boldsymbol{\beta}$.

Proof. To prove this we will take the simplest instance of the problem: where everything is a scalar. We have $f(a, \beta) = (y - a\beta x)^2$. For simplicty let's take $y = 0$ and $x > 0$. If we consider two candidates $s_1 = (0, 2)$ and $s_2 = (2, 0)$, we have $f(s_1) = f(s_2) = 0$. However $f(\frac{2}{2}, \frac{2}{2}) = x > \frac{1}{2}f(0, 2) + \frac{1}{2}f(2, 0)$, which break the convexity property. Since we showed that a particular case of the problem is non-convex then necessarily the general case cannot be convex. □

Proposition A.3.

$$\min_{\mathbf{A},\boldsymbol{\beta}} \|\mathbf{y} - \mathbf{A}\textit{diag}\left(\boldsymbol{\beta}\right)\mathbf{x}\|_2^2 + \lambda \|\boldsymbol{\beta}\|_1$$

has no solution if $\lambda > 0$.

Proof. Let's assume this problem has a minimum $\boldsymbol{A}^*, \boldsymbol{\beta}^*$. Let's consider $2\boldsymbol{A}^*, \frac{1}{2}\boldsymbol{\beta}^*$. Trivially the first component of the sum is identical for the two solutions, however $\lambda \left\|\frac{1}{2}\boldsymbol{\beta}\right\| < \lambda \|\boldsymbol{\beta}\|$. Therefore $\boldsymbol{A}^*, \boldsymbol{\beta}^*$ cannot be the minimum. We conclude that this problem has no solution. □

Proposition A.4. *For this proposition we will not restrict ourselves to single layer but the composition of an arbitrary large (n) layers as defined individually as $f_{\boldsymbol{A}_i,\boldsymbol{\beta}_i,\boldsymbol{b}_i}(x) = a(\boldsymbol{A_i}\textit{diag}\left(\boldsymbol{\beta_i}\right)\boldsymbol{x} + \boldsymbol{b_i})$. The entire network follows as: $N(\boldsymbol{x}) = \left(\bigcirc_{i=1}^{n} f_{\boldsymbol{A_i},\boldsymbol{\beta_i},\boldsymbol{b_i}}\right)(\boldsymbol{x})$. For $\lambda > 0$, $\lambda_2 > 0$ and $p > 0$ we have:*

$$\min \|\boldsymbol{y} - N(\boldsymbol{x})\|_2^2 + \Omega^{rs}_{\lambda,\lambda_2,p}$$

has at least 2^k global minimum where $k = \sum_{i=1}^{n} \#\boldsymbol{\beta_i}$

Proof. We will split this proof in two parts. First we will show that there is at least one global minimum, then we will show how to construct $2^n - 1$ other distinct solutions with the same objective.

Part 1: The two components of the expression are always positive so we know that this problem is bounded by below by 0. $\Omega^{rs}_{\lambda,\lambda_2,p}$ is trivially coercive. Since we have a sum of terms, all bounded by below by 0 and one of them is coercive, then the entire function admit at least one global minimum.

Part 2: Let's consider one global minimum. For each component k of $\boldsymbol{\beta_i}$ for some i. Negating it and negating the k^{th} column of $\boldsymbol{A_i}$ do not change the the first part of the objetive because the two factors cancel each other. The two norms do not change either because by definition the norm is independant of the sign. As a result these two sets of parameter have the same value and by extension also global minimuma. It is easy to see that going from this global minimum we can decide to negate or not each element in each $\boldsymbol{\beta_i}$. We have a binary choice for each parameter, there are $k = \sum_{i=1}^{n} \#\boldsymbol{\beta_i}$ parameters, so we have at least 2^k global minima.

□

A.2. Multi-Target Linear and Multi-Class Logistic regressions experiments

As we showed, Group sparsity share similarities with our method, and we claim that ShrinkNets are a relaxation of group sparsity. In this experiment we want to compare the two aproaches. We decided to focus on multi-target linear regression because in the single target case, groups in the Group Sparsity problem would have a size of one ($\boldsymbol{A}$ would be a vector in this case).

The evaluation will be done on two datasets `scm1d` and `oes97` (Spyromitros-Xioufis et al., 2016) for linear regressions and we will use `gina_prior2` (Guyon et al., 2007) and the *Gas Sensor Array Drift Dataset* (Vergara et al., 2012) (that we shorten in `gsadd`) for logistic regressions.

For each dataset we fit with different regularization parameters and measure the error and sparsity obtained after convergence. In this context we define sparsity as the ratio of columns that have all their weight under 10^{-3} in absolute value. Regularization parameters were choosed in order to

obtain the widest sparsity spectrum. Loss is normalized depending on the problem to be in the $[0, 1]$ range. We summarized the results in fig. 4. From our experiments it is clear that ShrinkNets can fit the data closer than Group Sparsity for the same amount of sparsity. The fact that we are able to reach very low loss demonstrate that even if our objective function is non convex, in practice it works as good or better as convex alternatives.

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