Calibration of 3-axis Magnetometers

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Abstract: Calibration of all 3 axes of a 3D magnetometer in cases where the true field strength is unknown involves an elaborate test set-up and rotation of the unit about all three axes so as to cover a sphere/ellipsoid. This paper presents an alternate method for calibrating all three axis of a magnetometer, i.e. estimating bias and scale factors for all axes, using a simpler 2 step procedure. The proposed technique exploits the relation between the body axis rates and the Euler angles to define a limited rotation and an associated optimization problem to determine the calibration parameters.

Keywords: Magnetometer, Calibration, Scale factor, Bias, Optimization.

1. INTRODUCTION

Magnetometers are an important aiding sensor for navigation systems. They enable attitude (magnetic heading) computation by comparing the measured earth's magnetic field with geomagnetic charts. The outputs of magnetometers are corrupted by wide band measurement noise, stochastic biases due to sensor imperfections, installation errors and magnetic interference in the vicinity of the sensors. The unwanted or interfering magnetic fields can be classified into two distinct groups. The first group consists of constant or slowly time-varying fields generated by ferromagnetic structural materials in the proximity of the magnetometers. The field measurement errors resulting from such interferences are referred to as hard iron biases. The second group of interfering magnetic fields result from materials that generate their own magnetic field in response to an externally applied field. This generated field is affected by both the magnitude and direction of the externally applied magnetic field. Such materials are called soft irons and the error they generate is referred to as a soft iron bias, which is time varying [Demoz, 2006].

The magnetometer output with errors are modeled as

$$h^2 = \left(\frac{h_x - B_x}{\gamma_x}\right)^2 + \left(\frac{h_y - B_y}{\gamma_y}\right)^2 + \left(\frac{h_z - B_z}{\gamma_z}\right)^2 \tag{1}$$

where

B_x, B_y, B_z are the hard iron parameters.

$\gamma_x, \gamma_y, \gamma_z$ are the scale factor and soft iron error parameters.

h_x, h_y, h_z are the actual magnetic field strength.

The calibration procedures then involve computing the above parameters, and there are well known techniques for doing this, e.g. [Nathaniel Bowditch (1995), Demoz (2006), Vasconcelos et. al. (2008)].

The classic compass swinging calibration technique [Nathaniel Bowditch (1995)] is a heading calibration algorithm that computes scalar parameters using a least squares algorithm. The major shortcoming of this approach is the necessity of external heading information, which is a strong requirement in many applications. We adopt the technique from [Demoz, 2006] (which is not only conceptually similar to other standard techniques, but also provides a good theoretical justification); and we show that while 2D calibration is simple and straightforward; none of the standard techniques are easily implementable in the 3D case. We propose to show a new technique for 3D calibration which overcomes these issues.

2. EXPERIMENTAL SET UP AND INITIAL VALUE ESTIMATION

In this section we outline the experimental set up for the calibration procedure and the truth reference. We also study existing calibration routines and their limitations.

2.1 The experimental set up

The following experimental set up has been designed in our lab. As a truth reference sensor for angular positions we use Ideal Aerosmith's Position Table [Ideal, 2010], which has angular position accuracies better than 0.1°. The following are the general specifications of our set up:

Sensors
Micromag 3 – 3-axis Magnetometer, [PNI 2010]
ADIS16365 – 3-axis Inertial Measurement Unit, [Analog 2010]
Communication Board
Fox board with processor ETRAX100lx

Figure (1): Experimental set up with 3axis magnetometer, IMU and FoxBoard embedded system

Both the sensors are communicating through the Software SPI device driver code. Initially both sensors with communication board are mounted on the position table as shown in Figure (1). A data set of a 3-axes magnetometer is collected for one complete heading rotation of 360.

2.2 A 2D calibration procedure

The plot of the X vs. Y axes values of the magnetometer under a full heading rotation is as shown in Figure (2). The plot looks like an off centred scaled ellipse. The main consideration in the calibration procedure is to obtain a circle from the ellipse by finding the correction factors for hard iron and soft iron with scale factor error.

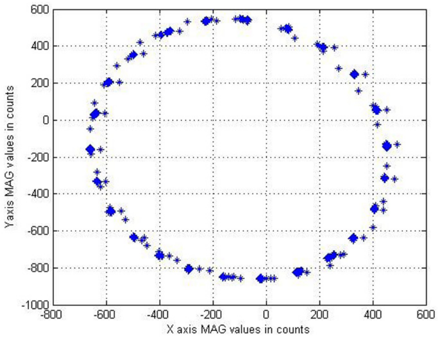

Figure (2): Magnetometer output under a full 360 rotation.

2.3 Initial value estimation from the data set to find the hard iron and soft iron with scale factor errors: The following method is adopted from [Demoz, 2006]. The basic form of the magnetometer equation as discussed before in Section 1 is:

$$h^2 = \left(\frac{h_x - B_x}{\gamma_x}\right)^2 + \left(\frac{h_y - B_y}{\gamma_y}\right)^2 \qquad (2)$$

Expanding the above expression

$$h^2 = \frac{h_x^2 - 2\cdot h_x \cdot B_x + B_x^2}{\gamma_x^2} + \frac{h_y^2 - 2\cdot h_y \cdot B_y + B_y^2}{\gamma_y^2} \qquad (3)$$

In matrix form the standard equation will be written as $\vec{z} = H\vec{x} + \vec{v}$ where $\vec{v}$ is the noise vector, $\vec{z}$ is residual or the error vector and $\vec{x}$ is the change in the parameters (magnetic field values from the magnetometers). Assuming $\vec{v}$, the noise vector, is zero or too less comparatively with the gain factor of magnetometer, then by algebraic combination of the expanded equations we get the H matrix as

$$H = \begin{bmatrix} -2h_x(t_1) & -2h_y(t_1) & (h_y(t_1))^2 & 1 \\ -2h_x(t_2) & -2h_y(t_2) & (h_y(t_2))^2 & 1 \\ .. & .. & .. & 1 \\ .. & .. & .. & .. \\ -2h_x(t_k) & -2h_y(t_k) & (h_y(t_k))^2 & 1 \end{bmatrix} \qquad (4)$$

Since $\vec{v}$ is assumed to be sufficiently small, an estimate of $\vec{x}$ can be obtained by the least squares solution as

$$\hat{\vec{x}} = (H^T H)^{-1} H^T \vec{z} \qquad (5)$$

The change in the parameters vector can be expressed as

$$\vec{x} = \begin{bmatrix} \mu_1 \\ \mu_2 \cdot B_y \\ \mu_2 \\ \mu_3 \end{bmatrix} \qquad (6)$$

where

$$\mu_1 = B_x$$

$$\mu_2 = \frac{\gamma_x^2}{\gamma_y^2}$$

$$\mu_3 = \mu_2 B_y^2 + B_x^2 - h^2\gamma_x^2$$

By algebraic manipulation we can compute the estimate of the initial conditions as

$$B_x = \mu_1$$

$$B_y = \frac{\vec{x}(2)}{\vec{x}(3)}$$

$$\gamma_x = \sqrt{\frac{\mu_2 B_y^2 + B_x^2 - \mu_3}{h^2}}$$

$$\gamma_y = \sqrt{\frac{\gamma_x^2}{\mu_2}}$$

To find the optimal values from the initial values

Since the ellipse equation is non linear, we use the *fsolve* command in MATLAB to find the exact roots of the equation to fit the magnetometer data to a circle with the initial conditions as calculated in Section 2.3. As the non linear equation has numerous solutions, computing good initial values is important for convergence to the correct solution using the initialization technique described in Section 2.3. After calculating the exact roots the correction factors are offset from the magnetometer data with respect to corresponding axis and the Figure 3 below shows the circle fit after calibration.

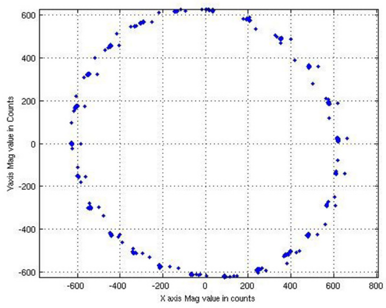

Figure (3): Magnetometer output after 2D calibration

2.4 Calibration does not require location information

In standard calibration routines where the ellipse is fit to a circle in order to compute the scale factor and bias – a key assumption is that the radius of the circle is a known quantity. Scale factors are the soft iron disturbances and the bias is the hard iron disturbances. The radius of the circle is in fact the total strength of the true magnetic field. The true magnetic field is location dependent, i.e. it can be computed only by knowing the (latitude, longitude, altitude). We propose a simple procedure to compute scale factor and bias without any knowledge of the true magnetic field – and hence the proposed procedure does not require location information! The procedure is described now.

In a 2D calibration there are 4 unknowns: 2 scale factors (s_x, s_y) and 2 bias values (b_x, b_y). If the true magnetic field is unknown then there are 5 parameters to be estimated: $\Omega \quad s_x, s_y, b_x, b_y, r)$ where r is the radius (or true magnetic field) and Ω is the set of parameters to be estimated.
Since the radius of the circle affects only the computation of the scale factors (this can be easily visualized geometrically), and has no effect in estimating the bias values, the above parameter set can be rewritten as:

$$\Omega \quad \frac{s_x}{\sqrt{r}}, \frac{s_y}{\sqrt{r}}, b_x, b_y) := \hat{s}_x, \hat{s}_y, b_x, b_y) \tag{7}$$

Now there are only 4 parameters to be estimated. Then we can pose a standard least squares problem, similar to what was done in Section 2.3, with these 4 parameters as the unknowns.
This implies that the true magnetic field and hence the actual vehicle location is not required. This was the procedure followed in the previous section.

2.5 Problems with standard techniques when extending the calibration procedure to the 3D case

A common method to calibrate a magnetometer in 3 dimensions is to collect the data of all 3 axes by rotating in a sphere. This is not feasible, as it requires an infinite number of circles to form a sphere. Hence for 3D calibration, see [Demoz 2006, Vasconcelos et. al. (2008)], a minimum 20˚strip of the sphere data is collected and calibration procedure in carried out. This would involve a failr elaborate set up for the calibration procedure. The least squares technique does not converge when data collection is less than 20˚ strip, and it often fails to converge even when a 20˚ strip is chosen. Finally, [Demoz, 2006] concludes that "for relatively low cost magnetometers with relatively large magnitude output noise, this algorithm is not suitable unless a large portion of the ellipsoid is available".

Hence for any practical application, there is a need for a simpler and accurate 3D calibration procedure. We illustrate one such procedure below. Firstly, we calibrate the magnetometer in the horizontal plane. Next, we make use of a unique relationship between the Roll (ϕ) and Yaw (ψ) Euler angles. Namely, that when the magnetometer is level, and it undergoes a rotation about the forward direction (i.e. roll rotation) there must be no change in the ψ . This relationship is used to construct a cost function with the Z axis calibration parameters being a solution to the cost function minima.

3. A 2-STEP PROCEDURE FOR 3-AXIS MAGNETOMETER CALIBRATION

This section presents a new technique to calibrate magnetometers in 3axes without requiring to cover a (minimum area of a) sphere as discussed previously in Section 2.4.

3.1 Euler angle relationships with body rates

- The formulation of the Euler angles results in certain unique relations between them. We make use of one such relationship in order to formulate a new cost function, the solution of which gives optimal estimates for the vertical axis errors. The relationship is the following: *whenever the magnetometer rotates about its forward axis (i.e. roll rotation), then there must be no change in the yaw angle.* Let us see what happens to this behaviour in our experimental set up, when only the horizontal axes of the magnetometer has been calibrated (as described before). From Figure 4 we can observe that as ϕ varies from -90 to +90°, there is a significant variation in the heading angle, ψ , as well. Here we see that φ varies by almost 40°. Note that the truth reference is the angular position given by the position table.

3.2 Formulation of a cost function

First note that standard transformations have been applied to convert magnetometer readings in the body frame to the inertial frame. The cost function is defined as the difference between two computed heading values: one heading is computed when the platform is stationary and leveled; the other when the platform is experiencing a roll rotation. The cost function is:

$$J = \psi \; L) - \psi \; R) \tag{8}$$

Where
$\psi \; L)$: is the Euler heading angle when the magnetometer is stationary and levelled,
$\psi \; R)$: is the Euler heading angle when the magnetometer is undergoing a rolling motion.

Ideally J should be 0 when the magnetometer is undergoing a roll. Note that $\psi \; = \tan^{-1}\left(\frac{M_y}{M_x}\right)$, and this in turn is related to the parameters (B_z, γ_{-}) to be estimated. Here M_y, M_x are the magnetic field measurements in the horizontal plane.

By using the *fminsearch* optimization routine in MATLAB, the solution to the minimum of the cost function can be computed, and the solution gives the optimal values of B_z and γ_z .

After computing the optimal values of the Z axis parameters the heading values are again plotted with the roll manoeuvre. The heading error is restricted to ±7˚ as shown in Figure (4), except where the roll rotation comes close to +90˚. Note that the error is computed with respect to the position table.

Finally note that the initial conditions for the optimization routine have not been chosen carefully, and we could use the procedure described in Section 2.3 to compute accurate initial values which could further reduce the above heading error.

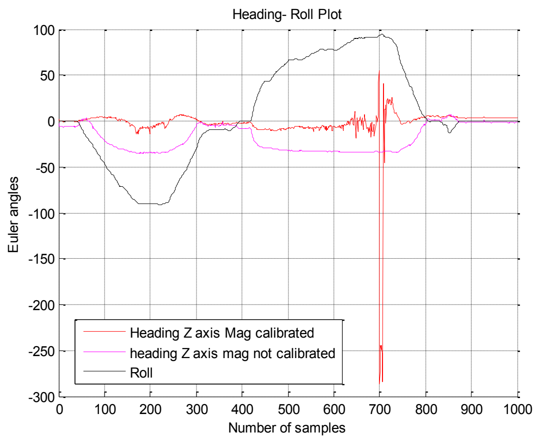

Fig (4): Comparison of Heading calibrated and not calibrated with Roll Plot

4. FUTURE WORK

In this paper we have presented a detailed result on calibrating a 3-axis magnetometer in the presence of a pure roll manoeuvre. In many cases, e.g. for ground vehicles the magnetometer needs to be calibrated in the presence of pitch motion as well. Thus calibration is required when both pitch and roll motions are present. The technique outlined in this paper can be extended to this case as follows. There is an analytical relationship in a change in the heading angle when a vehicle is undergoing a pure pitch motion. Consider the standard transformation between body rates and Euler angles:

$$\begin{bmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi \tan\theta & \cos\phi \tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{9}$$

Then we see

$$\dot\psi = \frac{\sin\phi}{\cos\theta} q \tag{10}$$

If the Euler angles ϕ, θ can be measured (e.g. using an accelerometer under a constant acceleration assumption), along with pitch rate q using a gyro, then the measured ψ can be compared to the true change in heading computed analytically; and a simple cost function can be defined and solved for obtaining the magnetometers Z-axis parameters.
In a more general case when the vehicle experiences both pitch and roll motion the following relationship between the heading angle and body rates

$$\dot\psi = 0 \cdot p + \frac{\sin\phi}{\cos\theta} \cdot q \tag{11}$$

can be used to define an appropriate cost function and solve for the magnetometer Z-axis parameters.

5. CONCLUSIONS

We have presented a simple technique for performing 3D magnetometer calibration.

The resulting heading error after our calibration has been reduced to 7° - further reduction in the error can be obtained by controlling measurement errors (e.g. modelled as white noise), misalignment errors, and by improving our initial condition estimates for the Z-axis calibration parameters.
The complicated procedure of having to gather a large amount of data by rotating the magnetometer in a sphere is avoided. Instead, the problem is simplified to only having to rotate the sensor only about its roll axis.
Further refinement of this procedure would involve obtaining good initial conditions for the Z-calibration parameters; extracting a much larger dataset with the sensor in roll motion, and estimating measurement noise and incorporating these into the optimization procedure described in this work.

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