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Design and Implementation of Inertial Navigation System

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***Abstract*— Inertial Navigation combined with other navigation supports like GPS, has gained importance due to enhanced navigation and inertial reference performance. The INS individually can compute the position of the device without any help from the outside world. However, a huge number of errors are introduced by sensors giving rise to an unacceptable drift in the output. So, a GPS is used to help the INS, using a Kalman filter which helps in estimating the errors in the INS and thus updating position to improved accuracy.**

Keywords—Kalman filter, GPS, INS

I. INTRODUCTION

For automatic machines, be it robots, aircraft or other autonomous vehicles, navigation is of extreme importance. Various systems are used in navigation of aircraft, like inertial navigation systems (INS), global positioning systems (GPS), air-data dead reckoning systems, radio navigation systems, Doppler heading reference systems, are few among them. Our interest is in integrating both the GPS and the INS to provide the best possible estimate of the aircraft position in terms of the longitude, latitude and height above the surface of the earth. The INS gives us the position, velocity and attitude of the aircraft but it is included with errors due to the fact that any small drift error can grow the error with time. Hence, an update or position fix is considered from the GPS and using a Kalman filter we can estimate the errors in both the GPS and the INS, thus giving the better position information.

Applications are not limited to aircraft alone. Although these integrated systems find extensive usage in airborne vehicles, they have also been used in the navigation of cars, ships, satellites and many other vehicles.

There are many advantages in developing this kind of a navigation system as compared to the ones used earlier in terms of size and speed. GPS chips and Micro- gyroscopes can be integrated on a small board and can effectively give the position of the vehicle. With the advantage of MEMS technology, all this can be done at high levels of accuracy and at very lower costs.

Our objective is to develop the GPS-INS integrated system so that it can be implemented on real time hardware like a microcontroller, microprocessor or a digital signal processor. Even though high accuracy sensors like gyroscopes and accelerometers are available, their costs are higher. Usage of low cost and low accuracy sensors may find application where high accuracy is not compulsory. First the simulation of the whole navigation system would be done on a computer, where given the initial state of the vehicle and regular updates from the sensors (INS) and the GPS, the program will return the estimated position of the vehicle. Eventually this simulated model would be implemented on real hardware.

II. INS, GPS AND KALMAN FILTER

Current trend in navigation sees the increased use of integrated navigation systems, where the components (sensors) that are usually integrated are the Global Positioning System (GPS) and Inertial Navigation Systems (INS). The integration of two subsystems provides more accuracy than the individual subsystems.

A. *INS*

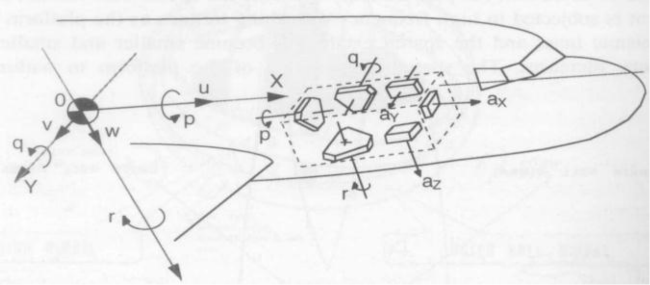

Fig. 1: Orientation of axes

The INS consists of 3-axis gyroscopes which gives the system computer the roll, pitch and yaw rates about the body axes as shown in figure 1 [6]. It also consists of 3-axis accelerometers which gives the accelerations along the axes. There are two basic inertial mechanisms which are used to derive the Euler angles from the rate gyros, viz. stable platform and strap-down INS. We would be interested with the strap-down INS where the gyros and accelerometers are `strapped-down' to the aircraft body frame. The values of acceleration from the accelerometers are then adjusted for rotation of the earth and gravity to give the velocity and position of the aircraft.

Equations of Motion

The aircraft orientation with respect to a fixed inertial frame of axes is given by three Euler angles. The aircraft is thought to be oriented parallel to the fixed reference frame of axes. A series of rotations bring it to the orientation about axes OX, OY and OZ.

1. A rotation in clockwise about the yaw axis, through the yaw (or heading) angle ψ followed by
2. A rotation in clockwise about the pitch axis, through the pitch angle θ followed by
3. A rotation in clockwise about the roll axis, through the back angle ϕ

The relationship between the angular rates of roll, pitch and yaw, p; q; r (measured by the body mounted gyros), the Euler angles, ψ, θ, ϕ and their rates, are given below.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{2.1}$$

By combining the above equations we can derive the Euler angles using initial conditions of a known attitude at a given time. But, for pitch angles around ±90.

If e_0, e_1, e_2, e_3 were the four parameters then in terms of angular rates, we have the following

$$\begin{aligned} \dot{e}_0 &= -\frac{1}{2}(e_1 p + e_2 q + e_3 r) \\ \dot{e}_1 &= \frac{1}{2}(e_0 p + e_2 r - e_3 q) \\ \dot{e}_2 &= \frac{1}{2}(e_0 q + e_3 p - e_1 r) \\ \dot{e}_3 &= \frac{1}{2}(e_0 r + e_1 q - e_2 p) \end{aligned} \tag{2.2}$$

with the parameters satisfying the following equation at all points of time.

$$e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1 \tag{2.3}$$

The above equations can be used to generate the time history of the four parameters e_0, e_1, e_2, and e_3. The initial values of the Euler angles are given which are used to calculate the initial values of the four parameters using the following equations

$$\begin{aligned} e_0 &= \cos\frac{\psi}{2}\cos\frac{\theta}{2}\cos\frac{\phi}{2} + \sin\frac{\psi}{2}\sin\frac{\theta}{2}\sin\frac{\phi}{2} \\ e_1 &= \cos\frac{\psi}{2}\cos\frac{\theta}{2}\sin\frac{\phi}{2} - \sin\frac{\psi}{2}\sin\frac{\theta}{2}\cos\frac{\phi}{2} \\ e_2 &= \cos\frac{\psi}{2}\sin\frac{\theta}{2}\cos\frac{\phi}{2} + \sin\frac{\psi}{2}\cos\frac{\theta}{2}\sin\frac{\phi}{2} \\ e_3 &= -\cos\frac{\psi}{2}\sin\frac{\theta}{2}\sin\frac{\phi}{2} + \sin\frac{\psi}{2}\cos\frac{\theta}{2}\cos\frac{\phi}{2} \end{aligned} \tag{2.4}$$

We now have the attitude of the aircraft. To compute the position we use the accelerations taken by the accelerometers. The accelerations (a_x, a_y and a_z) of the aircraft along the three body axes, as read by the accelerometers, are given by the equations (2.5) U, V, W and p, q, r are all available as states. If the acceleration due to gravity (g) model is supplied as a function of location around the earth, then $\dot{u}$, $\dot{v}$, $\dot{w}$ can be computed.

$$\begin{aligned} \dot{U} &= a_X + Vr - Wq + g\sin\theta \\ \dot{V} &= a_Y - Ur + Wp - g\cos\theta\sin\phi \\ \dot{W} &= a_Z + Uq - Vp - g\cos\theta\cos\phi \end{aligned} \tag{2.5}$$

Errors in the INS

Most errors in INS are associated with the inertial sensors (instrument errors). These are the residual errors exposed by the installed gyros and accelerometers following INS calibration. The major error sources are shown in table 1[3, 7].

TABLE 1 SENSOR GENERATED ERRORS IN THE INS

Alignment errors	roll, pitch and heading errors
Accelerometer bias or offset	a constant offset in the accelerometer output that changes randomly after each turn-on.
Accelerometer scale factor error	results in an acceleration error proportional to sensed acceleration.
Nonorthogonality of gyros and accelerometers	the axes of accelerometer and gyro uncertainty and misalignment.
Gyro drift or bias (due to temperature changes)	a constant gyro output without angular rate presence.
Gyro scale factor error	results in an angular rate error proportional to the sensed angular rate
Random noise	Random noise in measurement

B. Global positioning system

GPS uses a one-way ranging technique from the GPS satellites that are also broadcasting their estimated positions. Signals from four satellites are used with the user generated replica signal and the relative phase is calculated. Using triangulation the location of the receiver is fixed. Four unknowns can be determined using the four satellites and appropriate geometry: latitude, longitude, altitude and a rectification to the user's clock. The GPS receiver combined with the receiver computer gives elevation angle between the user and satellite, azimuth angle between the user and satellite, measured clockwise positive from the true north, geodetic latitude and longitude of the user.

The GPS ranging signal is broadcasted at two frequencies: a primary signal at 1575.42 MHz (L1) and a secondary broadcast at 1227.6 MHz (L2). Civilians use L1 frequency that has two modulations, C/A or Clear Acquisition (or Coarse Acquistion) Code and P or Precise or Protected Code. C/A is unencrypted signal broadcast at a higher bandwidth and is available only on L1. P code is more accurate because it is broadcasted at a higher bandwidth and is restricted for military applications.

The primary use of GPS is to provide more accurate position and velocity worldwide, depending on range and range-rate measurements. GPS can be implemented in navigation as a fixing aid by being a part of an integrated navigation system, for example INS/GPS.

1. Errors in GPS

Ephemeris errors occur when the GPS message does not transmit the correct satellite location and this affects the ranging accuracy. These grow with time from the last update from the control station. The errors from Satellite clock affect both C/A and P code users and leads to an error of 1-2m over 12hr updates [8]. Measurement noise affects the position accuracy of GPS pseudo range absolute positioning by a few meters. The spreading of these errors into the position solution can be characterized by a quantity called Dilution of Precision (DOP) which expresses the geometry between the satellite and the receiver and is typically between 1 and 100. If the DOP is greater than 6, then the satellite geometry is bad. Ionospheric and tropospheric delays are introduced due to the atmosphere and this tends to a phase lag in computation of the pseudo range. These can be rectified with dual-frequency P-code receivers. Multipath errors are caused by reacted signals entering the front end of the receiver and masking the correlation peak. These effects tend to be more prominent due to the presence of reflective surfaces, where 15m or more in ranging error can be found in some cases.

C. Kalman filtering

The Kalman Filter (KF) is a very effective stochastic estimator for a huge number of problems, be it in computer graphics or in navigation. It is an optimal combination, in terms of minimization of variance, between the prediction of parameters from a previous time instant and external observations at a present time instant.

1. Discrete Kalman Filter

The KF deals with the general problem of trying to estimate the state x €$\Re^n$of a discrete-time controlled process that is controlled by the linear stochastic difference equation [1, 2, 9]

$$\mathbf{x}_k = A\mathbf{x}_{k-1} + B\mathbf{u}_k + \mathbf{w}_{k-1} \tag{2.6}$$

With a measurement z €$\Re^m$ that is

$$\mathbf{z}_k = H\mathbf{x}_k + \mathbf{v}_k. \tag{2.7}$$

The variables wk and vk denotes the process and measurement noise respectively. They are assumed to be independent of each other, white, and with normal probability distributions

$$p(\mathbf{w}) \sim N(0, \mathbf{Q}),$$

$$p(\mathbf{v}) \sim N(0, \mathbf{R}). \tag{2.8}$$

Q is the process noise covariance and R is the measurement noise covariance. Equation 2.6 is same as the standard state differential equation

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \tag{2.9}$$

Where x denotes the state vector and u denotes the input or driving function, the only difference being that equation 2.6 is a system whose state vector is sampled for discrete time state, whereas equation 2.9 is sampled for continuous time state.

The n x n matrix A in the difference equation 2.6, relates the state at the previous time step k x1 to the state at the present time step k, in the absence of a driving function or a process noise. The n x1 matrix B relates the optional control input u $\in \Re^n$ to the state x. The m x n matrix H in equation 2.7 links the state x to the measurement z_k.

An initial estimate of the process at some point tk is assumed, and this estimate is based on our knowledge of the process before tk. Let this a priori estimate be denoted by $\hat{x}_k^-$, where the “hat" denotes estimate, and the “super minus" indicates us that this is the best estimate we have prior to assimilating the measurement at tk. Assuming that the error covariance matrix associated with $\hat{x}_k$ is also known, then the estimation error is defined as

$$\hat{\mathbf{e}}_k^- = \mathbf{x}_k - \hat{\mathbf{x}}_k^-, \tag{2.9}$$

and the associated covariance matrix as

$$\mathbf{P}_k{}^- = E[\mathbf{e}_k^- \mathbf{e}_k^-{}^T] = E[(\mathbf{x}_k - \hat{\mathbf{x}}^-)(\mathbf{x}_k - \hat{\mathbf{x}}^-)^T] \tag{2.10}$$

Since we have assumed a prior estimate $\hat{x}_k$, we use z_k to improve the prior estimate, by the following equation.

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k(\mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_k^-) \tag{2.11}$$

Where $\hat{x}_k$ is the updates estimate and K_k is the blending factor or Kalman gain that minimizes the a posteriori error covariance equation 3.33.

$$\mathbf{P}_k = E[\mathbf{e}_k\mathbf{e}_k^T] = E[(\mathbf{x}_k - \hat{\mathbf{x}})(\mathbf{x}_k - \hat{\mathbf{x}})^T] \tag{2.12}$$

Substituting equation 2.7 into 2.12 and then substituting the resulting expression into 3.33 we get

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_k^- \tag{2.13}$$

where the Kalman gain which minimizes the mean-square estimation error is given by

$$\mathbf{K}_k = \mathbf{P}_k^-\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_k^-\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \tag{2.14}$$

We then estimate the next step measurement $\hat{x}_{k+1}$, the error covariance P_{k+1} and repeat the process

$$\hat{\mathbf{x}}_{k+1}^- = A_k\hat{\mathbf{x}}_k + B_k\mathbf{u}_k$$

$$\mathbf{P}_{k+1}^- = A_k\mathbf{P}_kA_k^T + \mathbf{Q}_k \tag{2.15}$$

We reject the contribution of wk because it is a zero mean function and not correlated with the earlier w's.

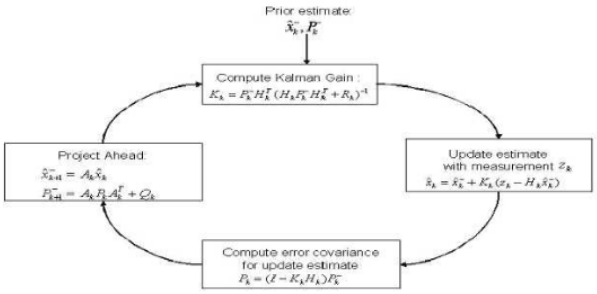
Prior estimate:
$\hat{x}_k^-, P_k^-$
Compute Kalman Gain :
$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}$
Update estimate with measurement z_k
$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H_k \hat{x}_k^-)$
Compute error covariance for update estimate
$P_k = (I - K_k H_k) P_k^-$
Project Ahead:
$\hat{x}_{k+1}^- = A_k \hat{x}_k$
$P_{k+1}^- = A_k P_k A_k^T + Q_k$

Fig 2. Kalman filtr loop

Kalman filter and navigation

KF is a very effective and versatile procedure for combining noisy sensor outputs to estimate the state of a system with unsure dynamics. Noisy sensor outputs include outputs from the INS and GPS; state of the system might include position, velocity, attitude and attitude rate of a vehicle or an aircraft; and uncertain dynamics includes unpredictable disturbances in the sensor parameters or disturbances caused by a human operator or a medium (like wind).

The KF is used to calculate the errors introduced into the unaided INS system due to the gyros and accelerometers as discussed in table 1. These errors form the state vector $\hat{x}_k$ and the measured values of the state vector from the GPS forms the measurement vector z. After modeling the errors, the KF loop, as shown in figure 2, is implemented after giving the initial estimates of the state vector and its covariance matrix at time t = 0. This makes the GPS-aided INS system configuration, and the errors are either compensated by the feed forward or the feedback mechanism as shown in figures 3 and 4 respectively.

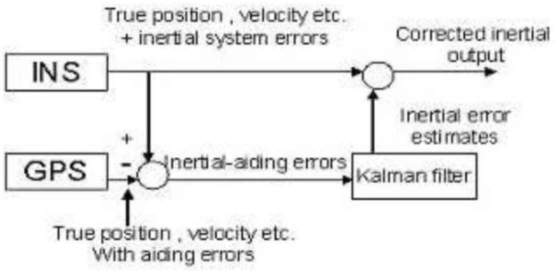
True position , velocity etc.
+ inertial system errors
Corrected inertial output
INS
Inertial error estimates
+
GPS
-
Inertial-aiding errors
Kalman filter
True position , velocity etc.
With aiding errors

Fig 3. Feedforward aided INS

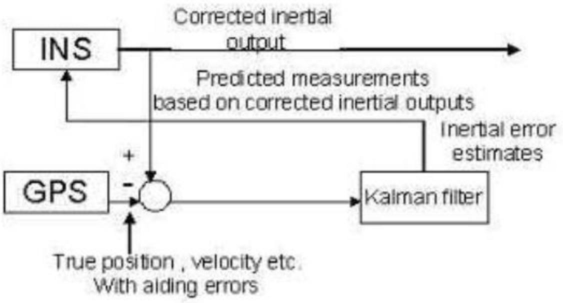
Corrected inertial output
INS
Predicted measurements based on corrected inertial outputs
Inertial error estimates
+
GPS
-
Kalman filter
True position , velocity etc.
With aiding errors

Fig 4. Feedback aided INS

III. SIMULATION

The simulation of the integration of INS and GPS using a Kalman filter has been done. Separate programs are written for modeling the INS, creating the errors in the sensors and modeling the output of the GPS using both MATLAB and C.

A. Implementation

A program called Flight Dynamics and Controls (FDC) toolbox, when given the initial conditions of the aircraft thrust and aerodynamics, gave as its output the time history of the aircraft in the form of a state vector X, where

$$\mathbf{X} = \begin{bmatrix} \phi & \theta & \psi & p & q & r & a_x & a_y & a_z & X & Y & Z & V_T & \alpha & \beta \end{bmatrix}^T \tag{3.1}$$

- θ; ϕ;ψ are the Euler angles in radians,
- p ; q; r are the roll, pitch and yaw rates from the gyroscopes in radians per second,
- a_x; a_y; a_z are the accelerations from the accelerometers in m/s2
- X; Y; Z are the distances along the three axes in the navigation frame in meters,
- V_T;α ; β are the velocity of the aircraft in m/s, the angle of attack in radians and the sideslip angle in radians, respectively.

B. Results

In this section the results obtained from the simulation of integrated systems i.e. the INS and GPS are discussed.

1. Integrated system

A nine state model Kalman filter is implemented as described above. Figures 5-7, show the output of the simulation as well as the GPS output simulated for a period of 200s. The standard deviation chosen for the accelerometers here was 10mGal. The standard deviations of the accelerometers were varied and we have got two sets of outputs. As given in the works by Ronnback [4] and Shin [10], the standard deviations of the accelerometers were increased to give an output with a much better accuracy as seen in figures 5-7. The standard deviations of the accelerometers chosen were 30mGal.

The update from the accelerometers and gyroscopes was taken every 0.01s, the GPS update was taken every 1s and the Kalman filter is run every 0.1s [5] to achieve better accuracy. Every alternate 0.1s instant, when the GPS update is present, equation 2.11 is used to predict the error state $\hat{x}_k$, using the newest GPS update as the measurement, i.e. the GPS update is taken constant for that whole period of time (0.2 sec). This also comes in use when there are GPS outages. Whenever the GPS update is taken, $\hat{x}_k$ is made zero, and whenever the GPS update is not taken (every alternate 0.1s or when there are GPS outages), $\hat{x}_k$ is left as it is and updated using the equations 2.11-2.15.

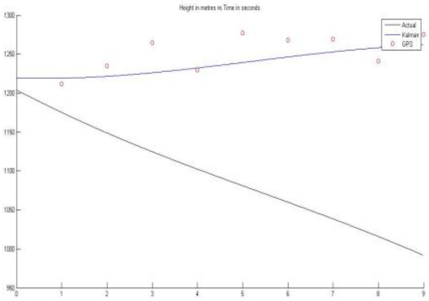

Fig 5. Hieght vs time

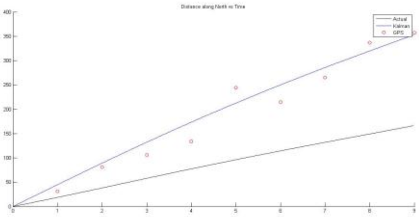

Fig 6.Distance along north vs time

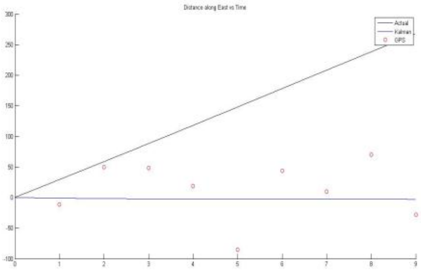

Fig 7. Distance alond east vs time

CONCLUSION

The INS system is modeled as given by the specification sheets. A nine state Kalman filter is designed and implemented using the perturbation theory model for position, velocity and attitude. The accuracy of the results obtained was better than the accuracy given by the GPS and INS as individual systems. The accuracy can be further improved if we increase the states of the filter and model for the scale factors, biases and nonorthogonality of the sensors.

The program of integration of the INS and GPS using Kalman filtering was run on the DSP simulator software and the computation time was well within the requirements of 10ms.

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