Circuit analysis

The circuit for the grounded electrode is the simplest and is represented by the circuit drawn in figure 1. The capacitor Cgnd represent the self-capacitance of the grounded electrode and the resistor Rgnd represents the resistance of the connection towards GND (including resistance of the wire, resistors, etc.). The voltage across Cgnd will be determined by the influence of the discharge in the gap. And will evolve in time as:

$$Vgnd(t + \Delta t) = Vgnd(t) \exp(-\frac{\Delta t}{RgndCgnd}) \tag{1}$$

The current can be calculated from this newly calculated potential as:

$$Ignd(t + \Delta t) = \frac{Vgnd(t + \Delta t)}{Rgnd} \tag{2}$$

The powered electrode will have two different possible circuits: Small lab (SL) circuit, and Marx generator (MG) circuit. The SL circuit is shown in figure 2. The powered electrode is represented by the capacitor C2, which signifies the self-capacitance of the powered electrode. The voltage V2 across the capacitor C2 will be determined by the influence of the discharge on the potential of the powered electrode and the response of the circuit on this change of potential.

In the SL circuit we can have V1(t=0) = Vs to signify that the capacitor had time to charge before it got connected to the electrode C2. We can setup equations using Kirchhoff's laws for the potentials V1 and V2. The currents and voltage loops chosen for the setup of these equations are shown in figure 3.

The current equation will be:

$$I1 = I2 + I3 \tag{3}$$

with $I2 = C1\frac{dV1}{dt}$, and $I3 = C2\frac{dV2}{dt}$ we get:

$$I1 = C1\frac{dV1}{dt} + C2\frac{dV2}{dt} \tag{4}$$

Applying the voltage laws in loop 1 gives us:

$$Vs - I1R1 - V1 = 0 \tag{5}$$

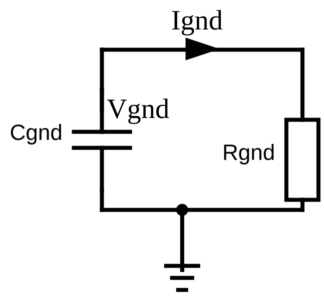

Figure 1. The circuit of the grounded electrode. The ground electrode is represented by the capacitor Cgnd (self-capacitance). The potential across the capacitor Cgnd is determined by the influence of the discharge gap

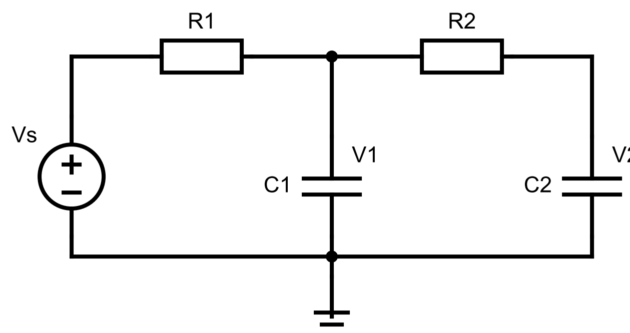

Figure 2. The circuit of the powered electrode in the SL circuit. The powered electrode is represented by the capacitor C2 (self-capacitance). The potential across capacitor C2 is determined by the influence of the discharge gap and the influence of the rest of the circuit.

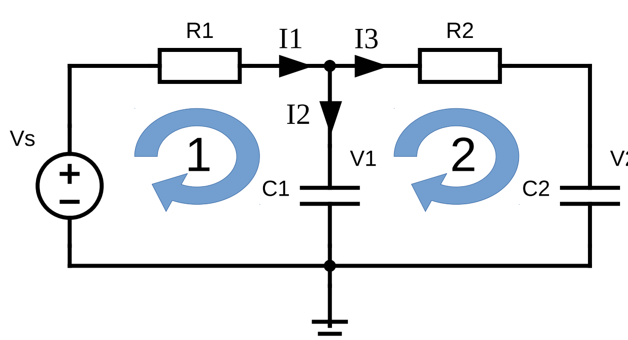

Figure 3. The circuit for the powered electrode in the SL circuit annotated for the application of Kirchhoff's laws.

Loop 2 gives us:

$$-I3R2 - V2 + V1 = 0 \tag{6}$$

The set of equations for this circuit are then:

$$I1 = C1\frac{dV1}{dt} + C2\frac{dV2}{dt} \tag{7}$$
$$Vs = I1R1 + V1 \tag{8}$$
$$V1 = I3R2 + V2 \tag{9}$$

Substituting eq.(7) into eq.(8) and using $I3 = C2\frac{dV2}{dt}$ in eq.(9) we get:

$$Vs = \left(C1\frac{dV1}{dt} + C2\frac{dV2}{dt}\right)R1 + V1 \tag{10}$$
$$V1 = C2\frac{dV2}{dt}R2 + V2 \tag{11}$$

After some rewriting we obtain the coupled differential equations for V1 and V2:

$$\frac{dV2}{dt} = \frac{1}{C2R2}V1 - \frac{1}{C2R2}V2 \tag{12}$$
$$\frac{dV1}{dt} = \frac{1}{C1R1}Vs - \frac{R2+R1}{C1R1R2}V1 + \frac{1}{C1R2}V2 \tag{13}$$

Using these coupled set of equations we can adjust V2 (the powered electrode potential) depending on the rest of the circuit (including energy stored in capacitor C1 due to the change in V1).

We can solve eq.(12) and eq.(13) numerically using first order euler method:

$$V2(t+\Delta t) = \left(\frac{1}{C2R2}V1(t) - \frac{1}{C2R2}V2(t)\right)\Delta t + V2(t) \tag{14}$$
$$V1(t+\Delta t) = \left(\frac{1}{C1R1}Vs - \frac{R2+R1}{C1R1R2}V1(t) + \frac{1}{C1R2}V2(t)\right)\Delta t + V1(t) \tag{15}$$

The current that is usually measured in experiments the current going through R2 due to the potential difference of the powered electrode and the rest of the circuit. This should be I3 given by the following equation:

$$I3 = C2\frac{dV2}{dt} \tag{16}$$

and can be written in discretized form as:

$$I3(t + \Delta t) = C2\frac{V2(t + \Delta t) - V2(t)}{\Delta t} \tag{17}$$

but it is possible that because we are using self-capacitance and we have an "open" circuit that the current I3 using the self-capacitance is not correct. The current through R2 can be written as:

$$Ir = \frac{V1 - V2}{R2} \tag{18}$$

discretized this can be written as:

$$Ir(t) = \frac{V1(t) - V2(t)}{R2} \tag{19}$$