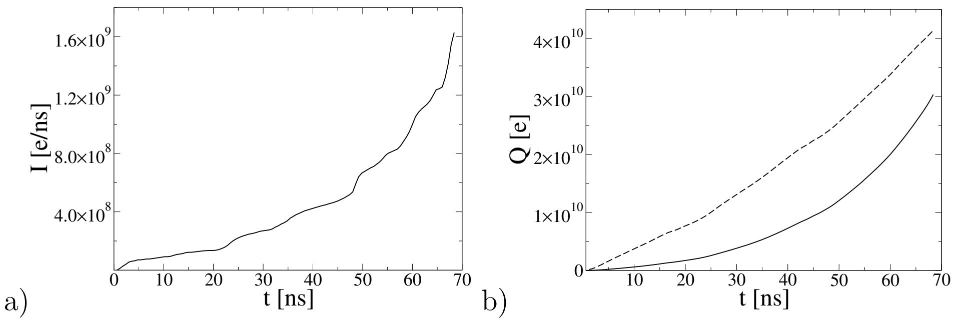

Figure 6. a) Current $I(t)$ in the external circuit as a function of time t, calculated with Eq. (14), and expressed in multiples of e/ns (10^{10} e/ns = 1.602 A). b) Solid line: The temporal integral $Q_I(t) = \int_0^t I(\tau)\, d\tau$ from panel a. Dashed line: The total charge inside the streamer $Q_{streamer}(t) = \int Q(z,t)\, dz$ from Fig. 4c as a function of time t. Both charges are expressed in multiples of the elementary charge e.

Defining the field enhancement at a given time t more accurately as the ratio between the maximal electric field $E_{max}(t)$ and the background field E_{back} at the location of the field maximum in the absence of the streamer, but with the needle electrode present, one finds that the field enhancement exceeds a factor of 10 after about 20 ns, and then stays approximately constant until effects of electrode proximity lead to a field increase during the final stages.

Fig. 5b shows velocity $v(t)$ and radius $R(t)$ as a function of time, defined as earlier in [20]. For comparison, the electron drift velocity in the maximal electric field at the streamer head is included. The figure shows two interesting facts, namely (i) that velocity and radius increase in a similar manner — in agreement with the relation (1) for fixed $E_{\max}$ —, and (ii) that the velocity is first smaller and later larger than the electron drift velocity in the streamer head.

For positive streamers slower than the electron drift velocity, some earlier theories [50, 16] based on "ultrafast" streamer propagation (i.e., on streamer velocities much larger than the electron drift velocity) are clearly not applicable.

3.3. Electric current and charge content

The current $I(t)$ in the external circuit is calculated as follows. Charge conservation $\partial_t q + \nabla\cdot\mathbf{j} = 0$ together with Gauss's law $\nabla\cdot\mathbf{E} = q/\epsilon_0$ can be expressed as the conservation of the total current density $\mathbf{J}$,

$$\nabla\cdot\mathbf{J} = 0, \qquad \mathbf{J} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}, \tag{12}$$

where $\epsilon_0 \partial_t \mathbf{E}$ is the displacement current density and $\mathbf{j}$ is the particle current density. (The equation can be derived as well by taking the divergence over the Maxwell equation that contains the current.) Therefore the integral of the total current density over an arbitrary closed surface S vanishes

$$\oint_S \mathbf{J}\cdot\mathbf{dS} = 0. \tag{13}$$

If the closed surface is laid through the conductor in the external circuit and over the cathode plane that the streamer approaches, then the circuit current is (before the streamer reaches the cathode)

$$I(t) = \int \mathbf{J}(t) \cdot \mathbf{dS} = \int_A \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{S} = -\epsilon_0 \frac{\partial}{\partial t} \int_0^{L_r} E_z(r, 0, t)\; 2\pi r\, dr, \qquad (14)$$

where A is the surface of the planar cathode.

Panel a of Fig. 6 shows the electric current $I(t)$ as a function of time that the streamer creates in the external electric circuit; it increases up to 1.6 10^9 e/ns = 260 mA when the streamer crosses the gap. The somewhat noisy structure of $I(t)$ is a numerical artifact: far from the streamer, we used a rather coarse grid, therefore there is some numerical inaccuracy in the local displacement current.

The total charge $Q(t)$ in the discharge gap is plotted in panel b of Fig. 6; more precisely, the solid line in panel b is the integral over the current in panel a, $Q_I(t) = \int_0^t I(\tau)\; d\tau$, while the dashed line is the total charge in the streamer $Q_{streamer}(t) = \int Q(z,t)\; dz$ with the data for Q(z,t) as in Fig. 4c. Panel b shows that the streamer charge is actually larger than the charge that has been transported into the system, $Q_I < Q_{streamer}$. This is because the net positive charge on the electrode needle actually decreases while the positive charge in the streamer increases: while initially the voltage on the electrode needle was created by charges on the electrode itself, later the streamer charge largely creates the electrode potential, and the charge on the electrode decreases.

4. Evaluation of the simulation results for a single streamer — UPDATE AND CUT A LOT

4.1. Streamer diameter, velocity and current

4.1.1. Comparison with experiments

We focus below on the time span between 8 and 65 ns, when the streamer propagates due to the externally applied field and to the current supplied by the electrode. The earlier and later stages lack a sufficiently accurate model of the electrodes; therefore they do not show the initial formation of an inception cloud around the electrode needle and its destabilization into a streamer [TANJA, SANDER], and they do not model e.g., photo-induced electron emissions from the planar cathode when the streamer comes close.

After the transients of the first 8 ns, the streamer radius increases approximately linearly from ≈500 to ≈940 μm. This radius characterizes the radius of the space charge layer around the streamer head, i.e., it is the electrodynamic radius [REF Starikovskii, Pancheshnyi ON THAT DEF]. Assuming that the optical radius is about half the electrodynamic radius [REFS], the optical diameter ranges from ≈500 to ≈940 μm. The velocity increases from ≈0.35 to ≈0.9 mm/ns; first the increase is linear as well, and then after 50 ns it accelerates. The electric field stays at an approximately constant