a) Itemset I of size m has $2^m - 1$ subsets
where support is greater than or equal to $sup(I)$
If I is among the top-K most frequent patterns:-

$$2^m - 1 \le K$$
$$2^m \le K + 1$$
$$m \le \log_2 (K+1) \le \lceil \log_2 (K+1) \rceil$$

b.1) let x = a particular subset in the stream
$C(x)$ = counter of x
$f(x)$ = frequency of x in the stream
$a(x)$ = occurance of x in the stream without incrementing $C(x)$
$b(x)$ = # of times $C(x)$ was decremented
hence $f(x) = C(x) + a(x) + b(x)$
$a(x) + b(x) = f(x) - C(x)$

counters maintained = C
total subsets = L
$\hat{f}_s \in [f_s - \frac{L}{C+1}, f_s] \rightarrow$ claim

proof:-
consider this alternate algorithm:-
1. Initialize S to be an empty-set
2. for $i = 1$ to L do:
3. if $x_i \in S$, then add (a copy) of x_i to S
4. else if there are less than C distinct elements, add x_i to S
5. else add x_i, then delete it as well as one copy of each of the C elements in S
6. End for

Subsets deleted are in groups of $C+1$, hence total deletions for a subset is at most $L/(C+1)$
therefore $a(x) + b(x) \le L/(C+1)$
hence $f(x) - C(x) \le L/(C+1)$
proof completed

b.2) $\hat{f}_K \leq f_{s^K}$ (trivial)

using the same approach as b.1:-

deletions of a K-th most frequent pattern is at most $L/(C+1)$

hence $f_{s^K} - \hat{f}_K \leq L/(C+1)$

therefore, $\hat{f}_K \in [f_{s^K} - L/(C+1), f_{s^K}]$

b.3) ① $\frac{L}{C+1} \rightarrow$ a constant

let f_{s^K} be the K-th largest support:-

and $f_{s^K} - \frac{L}{C+1} \leq \hat{f}_K \leq f_{s^K}$

if a pattern x is frequent:-

$$f_x \geq f_{s^K} \quad \text{and} \quad f_x - \frac{L}{C+1} \leq \hat{f}_x \leq f_x$$

therefore $\hat{f}_x \geq f_{s^K} - \frac{L}{C+1}$

if $\hat{f}_x \geq f_{s^K} - \frac{L}{C+1}$ then $\hat{f}_x \geq \hat{f}_K - \frac{L}{C+1}$

because $\hat{f}_K \leq f_{s^K}$

hence recall would be 100%

② let $A = \{S_1, S_2, \cdots, S_K\}$

$minsup(A) = \min\{sup(S_1), sup(S_2), \cdots, sup(S_K)\}$

let $f_{s'}$ be $minsup(A)$:-

$$\hat{f}_{s'} \geq \hat{f}^K - \frac{L}{C+1}, \quad \hat{f}_{s'} \geq f_{s'} + \frac{L}{C+1}, \quad \hat{f}^K \geq f_{s^K} - \frac{L}{C+1}$$

$$\hat{f}_{s'} + \frac{L}{C+1} \geq \hat{f}^K \rightarrow \hat{f}_{s'} + \frac{L}{C+1} \geq f_{s^K} - \frac{L}{C+1} \rightarrow \hat{f}_{s'} \geq f_{s^K} - \frac{2L}{C+1}$$

$$\hat{f}_{s'} \leq f_{s'} \rightarrow f_{s'} \geq f_{s^K} - \frac{2L}{C+1}$$