Two Way ANOVA Calculator

Factorial ANOVA - Balanced design

Fixed effects, Mixed effects, Random effects and Mixed repeated measures

Video Information One way ANOVA Levene's test

Model: Fixed effect (A-Fixed, B-Fixed) - no repeats

Interaction: With interaction

Significance level (α): 0.05

Outliers: Included

Effect: Medium

Effect type: f²

Effect Size: 0.25

Digits: 4

Step by step

- Enter raw data directly
- Enter raw data from excel

Enter sample data directly

Balanced two Factor ANOVA with Replication - several values per cell. The data should be separated by Enter or , (comma).
ANOVA without Replication - one value per cell.
The tool ignores empty cells or non-numeric cells.

Var A \ Var B	Conventional	Assertiveness-based
Novice	6 6 6 8 8 8 19 19 19 18 18	18 19 19 17 17 18 20 20 1
Expert	17 17 17 16 9 9 17 18 18	20 19 20 20 19 17 18 20 2

Calculate Insert column Delete column Insert row Delete row Clear Load example

Enter sample data from excel

Calculate Clear Validate Load example Load last run

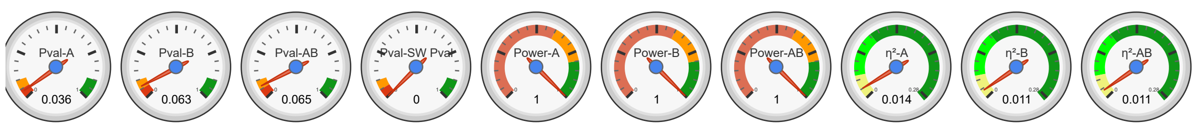
You may copy the data from Excel, Google sheets or any tool that separate the data with **Tab** and **Line Feed**. Copy the data, **one block of consecutive columns includes the header**, and paste. Click to see example:
Empty cells or non-numeric cells will be ignored

How to do with R?

ANOVA table

Hover over the cells for formulas and calculation.

Source	DF	Sum of Square (SS)	Mean Square (MS)	F Statistic (df_1,df_2)	P-value
Factor A - rows (A)	1	35.5776	35.5776	4.4538 (1,308)	0.03563
Factor B - columns (B)	1	27.7212	27.7212	3.4703 (1,308)	0.06343
Interaction AB	1	27.3336	27.3336	3.4218 (1,308)	0.0653
Error	308	2460.3388	7.9881		
Total	**311**	**2550.9712**	**8.2025**		



Two sample ANOVA - fixed test, using F distribution (right-tailed)

Factor - A

1. H_0 hypothesis

Since the p-value < α, H_0 is rejected.
Some of the groups' averages consider to be not equal.
In other words, the difference between the averages of some groups is big enough to be statistically significant.

2. P-value

The p-value equals 0.03563, ($P(x \le 4.4538) = 0.9644$). It means that the chance of type I error (rejecting a correct H_0) is small: 0.03563 (3.56%). The smaller the p-value the more it supports H_1.

3. Test statistic

The test statistic F_A equals 4.4538, which is not in the 95% region of acceptance: [-∞ : 3.8718].

4. Effect size

The observed effect size η^2 is **small**, **0.014**. This indicates that the magnitude of the difference between the average is small.

Factor - B

1. H_0 hypothesis

Since the p-value > α, H_0 can not be rejected.
The averages of all groups assume to be equal.
In other words, the difference between the averages of all groups is not big enough to be statistically significant.
A non-significance result can not prove that H_0 is correct, only that the null assumption can not be rejected.

2. P-value

The p-value equals 0.06343, ($P(x \le 3.4703) = 0.9366$). It means that the chance of type I error, rejecting a correct H_0, is too high: 0.06343 (6.34%). The larger the p-value the more it supports H_0.

3. Test statistic

The test statistic F_A equals 3.4703, which is in the 95% region of acceptance: [-∞ : 3.8718].

4. Effect size

The observed effect size η^2 is **small**, **0.011**. This indicates that the magnitude of the difference between the average is small.

Interaction AB

1. H_0 hypothesis

Since the p-value > α, H_0 can not be rejected.
The averages of all groups assume to be equal.
In other words, the difference between the averages of all groups is not big enough to be statistically significant.
A non-significance result can not prove that H_0 is correct, only that the null assumption can not be rejected.

2. P-value

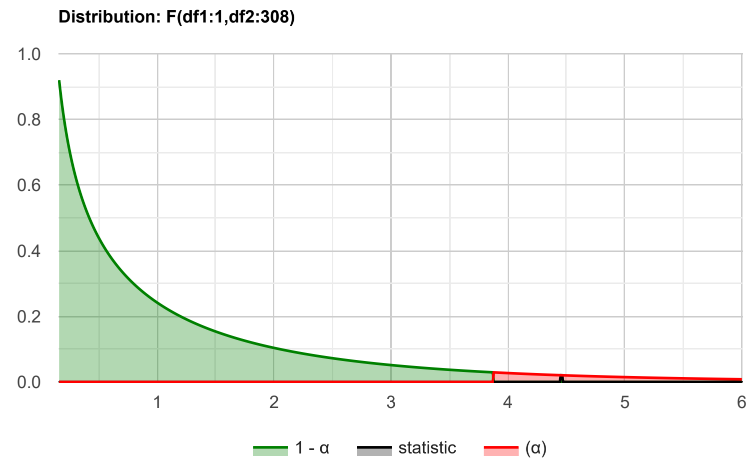
The p-value equals 0.0653, ($P(x \le 3.4218) = 0.9347$). It means that the chance of type I error, rejecting a correct H_0, is too high: 0.0653 (6.53%). The larger the p-value the more it supports H_0.

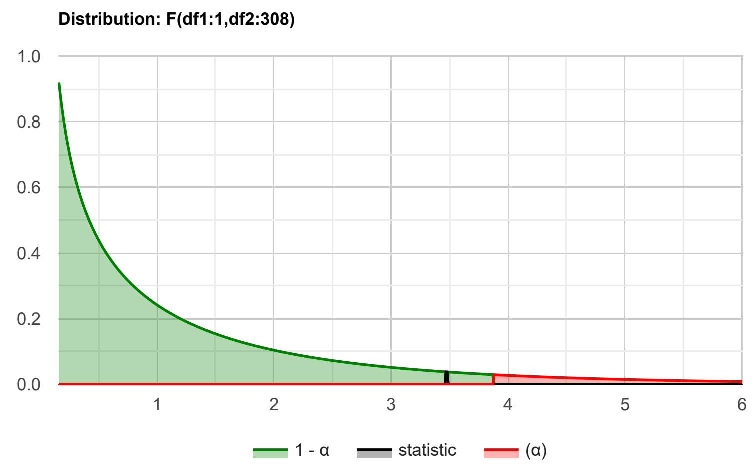
3. Test statistic

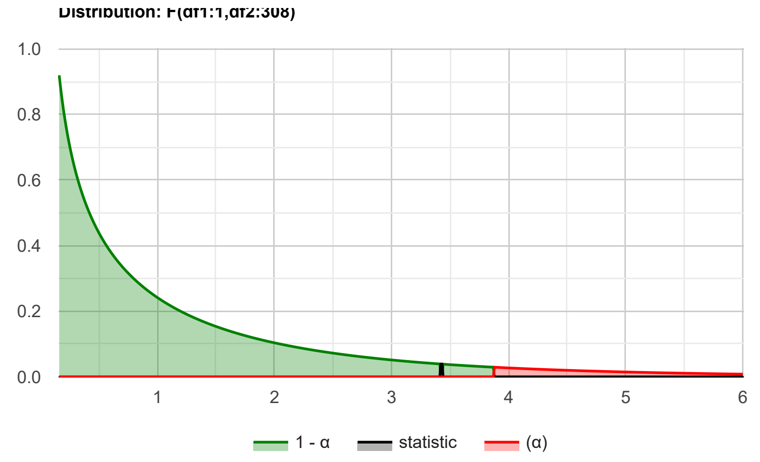
The test statistic F_A equals 3.4218, which is in the 95% region of acceptance: [-∞ : 3.8718].

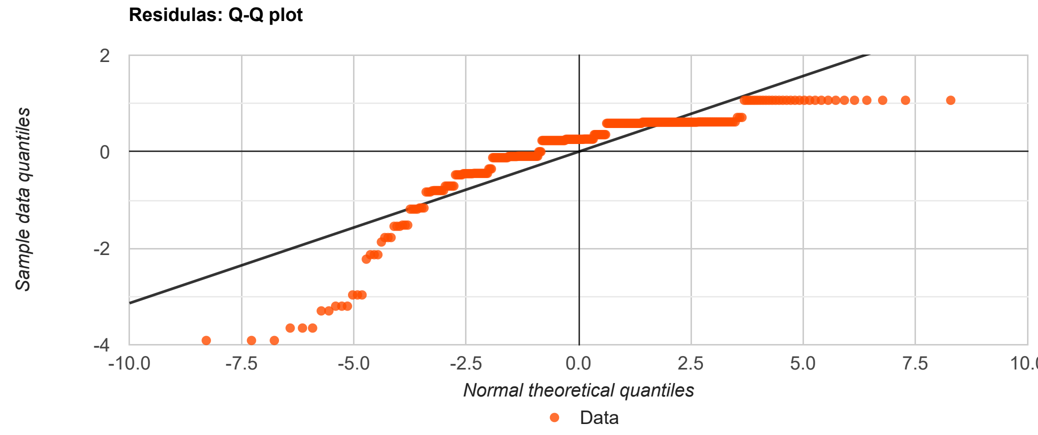
4. Effect size

The observed effect size η^2 is **small**, **0.011**. This indicates that the magnitude of the difference between the average is small.

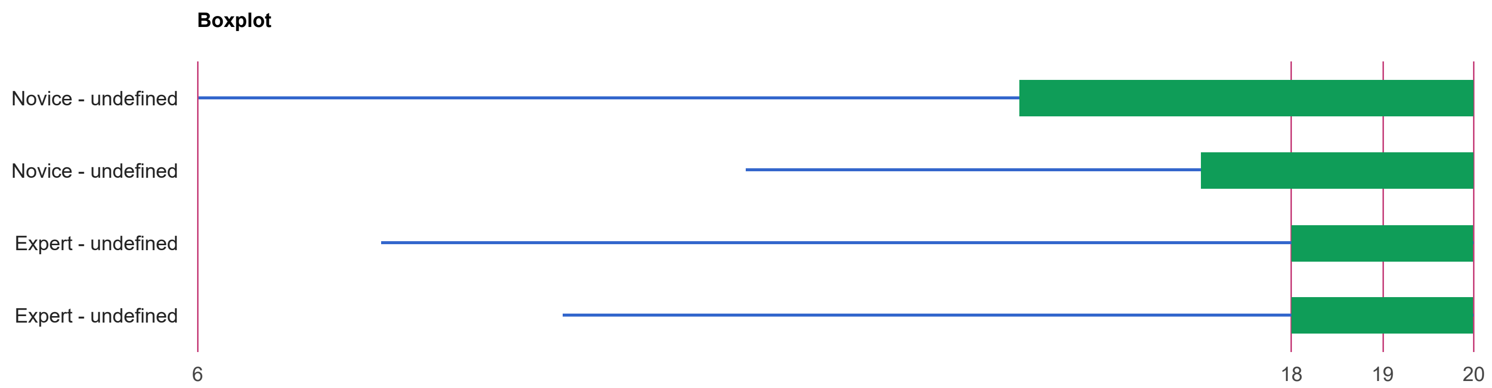








Right click on: Save image, (please use 'save link as...' or 'open link in new tab').



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Outliers

Outliers' detection method: Tukey Fence, k=1.5.
Residuals doesn't contain outliers. The **two way ANOVA** test is robust to the presence of outliers.

Normality

The assumption was checked based on the Shapiro-Wilk Test. (α=0.05)
It is assumed that **the residuals does not** follow a normal distribution (p-value is 0).
The test is considered robust for moderate violation of the normality assumption.

Test power: Factor - A

The test priori power is strong **1**

Test power: Factor - B

The test priori power is strong **1**

Test power: Interaction

The test priori power is strong **1**

Balanced design

undefined

Tukey HSD / Tukey Kramer

Count

Var A \ Var B	undefined	undefined	Total
Novice	69	69	**138**

Expert	87	87	**174**
Total	**156**	**156**	**312**

Average

Var A \ Var B	undefined	undefined	Total
Novice	17	18.2609	**17.6304**
Expert	18.2759	18.3448	**18.3103**
Total	**17.7115**	**18.3077**	**18.0096**

Variance

Var A \ Var B	undefined	undefined	Total
Novice	16.3235	2.7251	**9.8551**
Expert	8.4579	5.089	**6.7355**
Total	**12.2582**	**4.0208**	**8.2025**

Mean confidence interval (CL:0.95)

Var A \ Var B	undefined	undefined	Total
Novice	[9.1389,24.8611]	[15.0489,21.4728]	**[11.4999,23.761]**
Expert	[12.6087,23.9431]	[13.9489,22.7408]	**[13.2383,23.3824]**
Total	**[10.8714,24.5517]**	**[14.3902,22.2252]**	**[12.4053,23.6139]**

Differential effects

Var A \ Var B	undefined	undefined	Total
Novice	-0.3324	0.3324	**-0.3792**
Expert	0.2636	-0.2636	**0.3007**
Total	**-0.2981**	**0.2981**	**0**

Cells - the differential effects of the interactions.
For example, the effect of the interactions of the categories **undefined** and **Novice** is **-0.3324**.
Totals - the differential effects of factor A (right column) and factor B (bottom row).
For example, the effect of factor B category **undefined** is **-0.2981**.

Residuals

A	B	$Y_{i,j,k}$	Formula: $Y_{i,j,k}$ - $\bar{Y}_{i,j}$	Residual
Novice	undefined	6	6 - 17	-11
Novice	undefined	6	6 - 17	-11
Novice	undefined	6	6 - 17	-11
Novice	undefined	8	8 - 17	-9
Novice	undefined	8	8 - 17	-9
Novice	undefined	8	8 - 17	-9
Novice	undefined	19	19 - 17	2
Novice	undefined	19	19 - 17	2
Novice	undefined	19	19 - 17	2
Novice	undefined	18	18 - 17	1
Novice	undefined	18	18 - 17	1
Novice	undefined	18	18 - 17	1
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	12	12 - 17	-5
Novice	undefined	12	12 - 17	-5
Novice	undefined	12	12 - 17	-5
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	11	11 - 17	-6
Novice	undefined	11	11 - 17	-6
Novice	undefined	11	11 - 17	-6
Novice	undefined	18	18 - 17	1
Novice	undefined	18	18 - 17	1
Novice	undefined	18	18 - 17	1
Novice	undefined	17	17 - 17	0
Novice	undefined	17	17 - 17	0
Novice	undefined	17	17 - 17	0
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	16	16 - 17	-1

Novice	undefined	16	16 - 17	-1
Novice	undefined	16	16 - 17	-1
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	18	18 - 17	1
Novice	undefined	18	18 - 17	1
Novice	undefined	18	18 - 17	1
Novice	undefined	15	15 - 17	-2
Novice	undefined	15	15 - 17	-2
Novice	undefined	15	15 - 17	-2
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	18	18 - 17	1
Novice	undefined	18	18 - 17	1
Novice	undefined	18	18 - 17	1
Novice	undefined	15	15 - 17	-2
Novice	undefined	15	15 - 17	-2
Novice	undefined	15	15 - 17	-2
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
Novice	undefined	20	20 - 17	3
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Novice	undefined	19	19 - 18.261	0.7391
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Novice	undefined	17	17 - 18.261	-1.2609
Novice	undefined	18	18 - 18.261	-0.2609
Novice	undefined	20	20 - 18.261	1.7391
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Novice	undefined	19	19 - 18.261	0.7391
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Novice	undefined	17	17 - 18.261	-1.2609
Novice	undefined	17	17 - 18.261	-1.2609

Novice	undefined	16	16 - 18.261	-2.2609
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Novice	undefined	20	20 - 18.261	1.7391
Novice	undefined	20	20 - 18.261	1.7391
Novice	undefined	19	19 - 18.261	0.7391
Novice	undefined	19	19 - 18.261	0.7391
Novice	undefined	19	19 - 18.261	0.7391
Novice	undefined	16	16 - 18.261	-2.2609
Novice	undefined	16	16 - 18.261	-2.2609
Novice	undefined	16	16 - 18.261	-2.2609
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Novice	undefined	18	18 - 18.261	-0.2609
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Expert	undefined	10	10 - 18.345	-8.3448
Expert	undefined	10	10 - 18.345	-8.3448
Expert	undefined	10	10 - 18.345	-8.3448
Expert	undefined	14	14 - 18.345	-4.3448
Expert	undefined	14	14 - 18.345	-4.3448
Expert	undefined	14	14 - 18.345	-4.3448
Expert	undefined	16	16 - 18.345	-2.3448
Expert	undefined	16	16 - 18.345	-2.3448
Expert	undefined	16	16 - 18.345	-2.3448
Expert	undefined	19	19 - 18.345	0.6552
Expert	undefined	19	19 - 18.345	0.6552
Expert	undefined	19	19 - 18.345	0.6552
Expert	undefined	15	15 - 18.345	-3.3448
Expert	undefined	15	15 - 18.345	-3.3448
Expert	undefined	15	15 - 18.345	-3.3448

Information

Models

There are many possible models, this calculator deal currently only with the following balanced models:

- **Fixed effect model (A-Fixed, B-Fixed), no repeats** - both factors are fixed.
- **Mixed effect model (A-Random, B-Fixed), no repeats** - factor A is random, factor B is fixed, each subject is measured only once.
- **Mixed effect model (A-Fixed, B-Random), no repeats** - factor A is fixed, factor B is random, each subject is measured only once.
- **Random effect model (A-Random, B-Random), no repeats**
- **Mixed repeated measures (A-Fixed, B-Repeated)** - factor A is fixed, factor B uses the same subject for all the categories.

You may use data with replications, or data without replications.

What is balanced model?

The balanced design has the same number of observations in each cell - each combination of factor.
Currently this calculator supports only the balanced design.
When the model is **unbalanced**, it causes correlation between the factors and the interaction if it is proportional, and also between the factors if it is unbalance but not proportional.
hence you don't know how to divide the shared sum of squares between the two factors.
There are several methods how to deal with the shared sum of squares.
Type I - sequenceial, the first some of squares (SS) you calculate get the shared some of squares, in this case the order is matter!
Type II - conservative, it assumes there is no interaction between the factors, it ignores the shared SS between the factors. Type III - it assumes there is interaction between the factors, it ignores all the shared SS between the factors and between the factors and the intercation.

Targets

The two way ANOVA test checks the following targets using sample data.

- Checks if the difference between **Factor A** averages of two or more categories is significant
- Checks if the difference between **Factor B** averages of two or more categories is significant
- Checks if there is an interaction between **Factor A** and **Factor B**

When performing ANOVA test, we try to determine if the difference between the averages reflects a real difference between the groups, or is due to the random noise inside each group.
The F statistic represents the ratio of the variance between the groups and the variance inside the groups. Unlike many other statistic tests, the smaller the F statistic the more likely the averages are equal.

Right-tailed F test, for ANOVA test you can use only the right tail. Why?

Hypotheses

Factor A: H_0: μ_1 = .. = μ_a
There is no difference in the means of variable A categories.

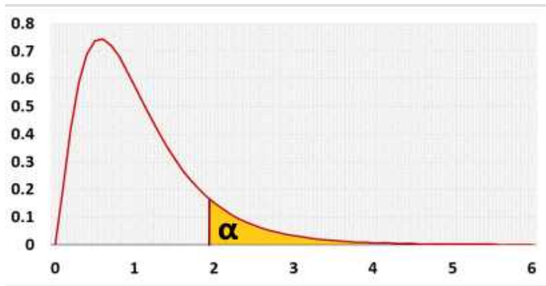
Factor B: H_0: μ_1 = .. = μ_b
There is no difference in the means of variable B categories.

H_0: Interaction(A_iB_j) = 0 (∀ i = 1 to a, j = 1 to b)
There is no interaction between variable A and variable B, i.e., for all the cells, the effect of variable A on the cells' means is not depend on the effect of variable B, and vice versa.

Test statistic

Fixed Model	Mixed Model	Random Model	Mixed Repeated
$F_A=\frac{MS_A}{MS_E}$	$F_A=\frac{MS_A}{MS_{AB}}$	$F_A=\frac{MS_A}{MS_{AB}}$	$F_A=\frac{MS_A}{MS_{SWA}}$
$F_B=\frac{MS_B}{MS_E}$	$F_B=\frac{MS_B}{MS_E}$	$F_B=\frac{MS_B}{MS_{AB}}$	$F_B=\frac{MS_B}{MS_{BSWA}}$
$F_{AB}=\frac{MS_{AB}}{MS_E}$	$F_{AB}=\frac{MS_{AB}}{MS_E}$	$F_{AB}=\frac{MS_{AB}}{MS_E}$	$F_{AB}=\frac{MS_{AB}}{MS_{BSWA}}$

F distribution



Assumptions

- The dependent variable is continuous (ratio or interval)
- Two categorical independent variables
- Independent observations (no repeated measure)
- The residuals distribution is normal
- Homogeneity of variances, a similar variance for each cell

Required Sample Data

Sample data from all compared groups

Parameters

a - the number of categories in variable A, number of rows.
b - the number of categories in variable B, number of columns.
$\mathbf{n_i}$ - sample side of category i of variable A (row i).
$\mathbf{n_j}$ - sample side of category j of variable B (column j).
$\mathbf{n_{i,j}}$ - sample side of cell i,j (row i, column j). In the balance $n_{i,j}$=n/(a*b)
n - overall sample side, includes all the groups ($\Sigma n_{i,j}$, i=1 to a, j=1 to b).
$\mathbf{\bar{Y}_i}$ - average of all the observations of category i of variable A (row i).
$\mathbf{\bar{Y}_j}$ - average of all the observations of category j of variable B (column j).
$\mathbf{\bar{Y}}$ - overall average ($\Sigma Y_{i,j,k}$ / n, i=1 to a, j=1 to b, k=1 to $n_{i,j}$).

Repeated measures ANOVA

s - represent the order of subject in category i (subject 1 in category 1 is different than subject 1 in category 2)
sub - number of subjects per cell, cell is one combination of variable A and variable B. For the balance design: N=a*b*sub.
$\mathbf{\bar{Y}_{i,s}}$ - subject's average, $\Sigma Y_{i,j,s}$ for subject i,s ,the average of all the observations of subject s of category j of variable B (column j).
$\mathbf{\bar{Y}}$ - overall average ($\Sigma Y_{i,j,s}$ / n

Results calculations

Sum of squares

The sum of squares accumulates the squared differences related to the effect we try to estimate.
$\mathbf{SS_A}$ - the squared differences related to the effect of variable A. You compare the average of every category to the total average. The same value as the sum of squares between groups in one way ANOVA.
$\mathbf{SS_B}$ - the same as SS_A, for variable B.
$\mathbf{SS_{AB}}$ - the squared differences related to the effect of the combination of variable A and variable B in each cell, Since we try to understand the influence of the interaction AB, the interaction of the specific value of variable A and the specific value of variable B, we take the average of each cell, remove the influence of variable A and variable B, and compare to the total average.
A effect = $\bar{Y}_i$ - $\bar{Y}$
B effect = $\bar{Y}_j$ - $\bar{Y}$
AB effect = Cell average - A effect - B effect - Total average.
= $\bar{Y}_{i,j}$ - ($\bar{Y}_i$ - $\bar{Y}$) - ($\bar{Y}_j$ - $\bar{Y}$) - $\bar{Y}$.
= $\bar{Y}_{i,j}$ - $\bar{Y}_i$ - $\bar{Y}_j$ + $\bar{Y}$.
Take the square of each difference
$\bar{Y}_{i,j}$ - $\bar{Y}_i$ - $\bar{Y}_j$ + $\bar{Y})^2$.
Count the square differences of each value in the cell, hence multiply by the sample size of each cell ($n_{i,j}$).

$SS_{AB}=\Sigma_i^a\Sigma_j^b n_{i,j}(\bar{Y}_{i,j} - \bar{Y}_i - \bar{Y}_j + \bar{Y})^2$

Fixed and Random Effects

The fixed and random effects are related to the independent variables ().

Fixed Effect

The effect is constant across individuals.

- The categories of the variable contains the entire categories' list
- The effect of this variable is interesting. The difference between the categories is important
- There is no know pattern on the difference between the categories

Random Effect

The effect vary across individuals, the individuals may be people, products.

- The categories' list is only a sample from the entire categories' list
- The effect of this variable is not interesting by itself. The difference between the categories is not important.
- There is no know pattern on the difference between the categories

Example: collecting data from several schools.
A sample from the entire groups' population.
There is no pattern about the difference between the schools, and if there will be a pattern, it will be another factor, like school's size.
Each school is not important by itself.

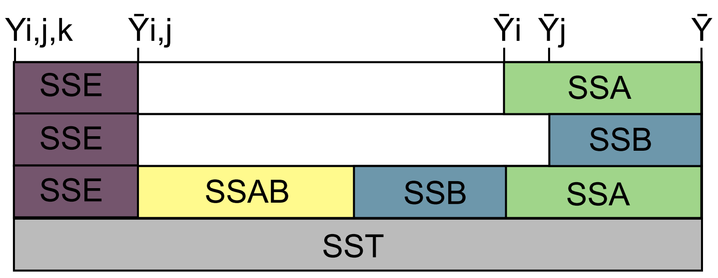
When you change the **interaction field** or the **model**, the following ANOVA table and diagram will be adjusted!

ANOVA table - with interaction

Source	Degrees of Freedom (DF)	Sum of Squares (SS)	Mean Square (MS)	F statistic	p-value
Factor A (rows) Between the categories of factor A	$DF_A = a - 1$	$SS_A = \Sigma_i^a n_i(\bar{Y}_i-\bar{Y})^2$	$MS_A = SS_A / DF_A$	$F_A = MS_A / MS_E$	$P(x > F_A)$
Factor B (Columns) Between the categories of factor B	$DF_B = b - 1$	$SS_B = \Sigma_j^b n_j(\bar{Y}_j-\bar{Y})^2$	$MS_B = SS_B / DF_B$	$F_B = MS_B / MS_E$	$P(x > F_B)$
Interaction AB Between the cells after reducing factor A and factor B effects	$DF_{AB} = (a - 1)(b - 1)$	$SS_{AB}=\Sigma_i^a\Sigma_j^b n_{i,j}(\bar{Y}_{i,j} - \bar{Y}_i - \bar{Y}_j + \bar{Y})^2$	$MS_{AB} = SS_{AB} / DF_{AB}$	$F_{AB} = MS_{AB} / MS_E$	$P(x > F_{AB})$
Error Within the cells	$DF_E = n - a*b$	$SS_E=\Sigma_i^a\Sigma_j^b\Sigma_k^{n_{i,j}}(Y_{i,j,k} - \bar{Y}_{i,j})^2$	$MS_E = SS_E / DF_E$		
Total All the deviations from the average	$DF_T = n - 1$	$SS_T=\Sigma_i^a\Sigma_j^b\Sigma_k^{n_{i,j}}(Y_{i,j,k} - \bar{Y})^2$ SS_T=Sample Variance*(n-1) $SS_T=SS_A+SS_B+SS_{AB}+SS_E$	$MS_E = S^2 = SS_T / (n - 1)$		

Sum of squares diagram - with interaction

In the following diagram you may see the differences per each observation $Y_{i,j,k}$ that used to calculate the sum of squares.
A effect: $\bar{Y}_i - \bar{Y}$.
B effect: $\bar{Y}_j - \bar{Y}$.
Interaction effect (AB): $Y_{i,j} - \bar{Y}_i - \bar{Y}_j + \bar{Y}$.
Error: $Y_{i,j,k} - \bar{Y}_{i,j}$.
Total effect: $Y_{i,j,k} - \bar{Y}$.



R Code

The following R code should produce the same results

```
A0 =
c("Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice","Novice
A1 = c("Expert","Expert","Expert","Expert","Expert","Expert","Expert","Expert","Expert","Expert","Expert","Expert")
A = c(A0,A1)
B0 =
c("undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefi
B1 = c("undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined","undefined")
B = c(B0,B1)
DV0 =
c(6,6,6,8,8,8,19,19,19,18,18,18,20,20,20,12,12,12,20,20,20,20,20,20,11,11,11,18,18,18,17,17,17,20,20,20,20,20,20,16,16,16,20,20,20,18,18,18,15,15,15,20,20,20,18,18,18,15,15,15,20,20,20,20,20,20,20,20,20,20,17,17,17,16,9,9,17,18,18
DV1 = c(14,14,14,16,16,16,19,19,19,15,15,15)
DV = c(DV0,DV1)
ID0 =
c(0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,7
ID1 = c(144,145,146,147,148,149,150,151,152,153,154,155)
ID = c(ID0,ID1)
df1 <- data.frame(A,B,DV,ID)
Model1 <- aov(DV ~ A + B + A:B, data=df1)
summary(Model1)
res=residuals(object = Model1)
```