

Typechecking Cool and PA3

Agenda

- About PA3 & Starter Code
- Typechecking the AST (simplified)
- Typechecking the AST (with rules in detail)

PA#3

- It's big. More code than PA1 and PA2.
- Some tricky concepts.
- Many corner cases.
- If you haven't started, start ASAP.

PA#3 Observations

- You have a lot of flexibility
 - Choices of data structures
 - Error messages
 - Code organization
 - Algorithmic choices (how many passes, etc.)
 -
- The starter code is minimal
 - You'll need to write some boilerplate
 - You should (must?) change starter code

We provide
'classes' object

`Classes_class`

is-list-of

`Class__class`

“Phylum” for Cool classes

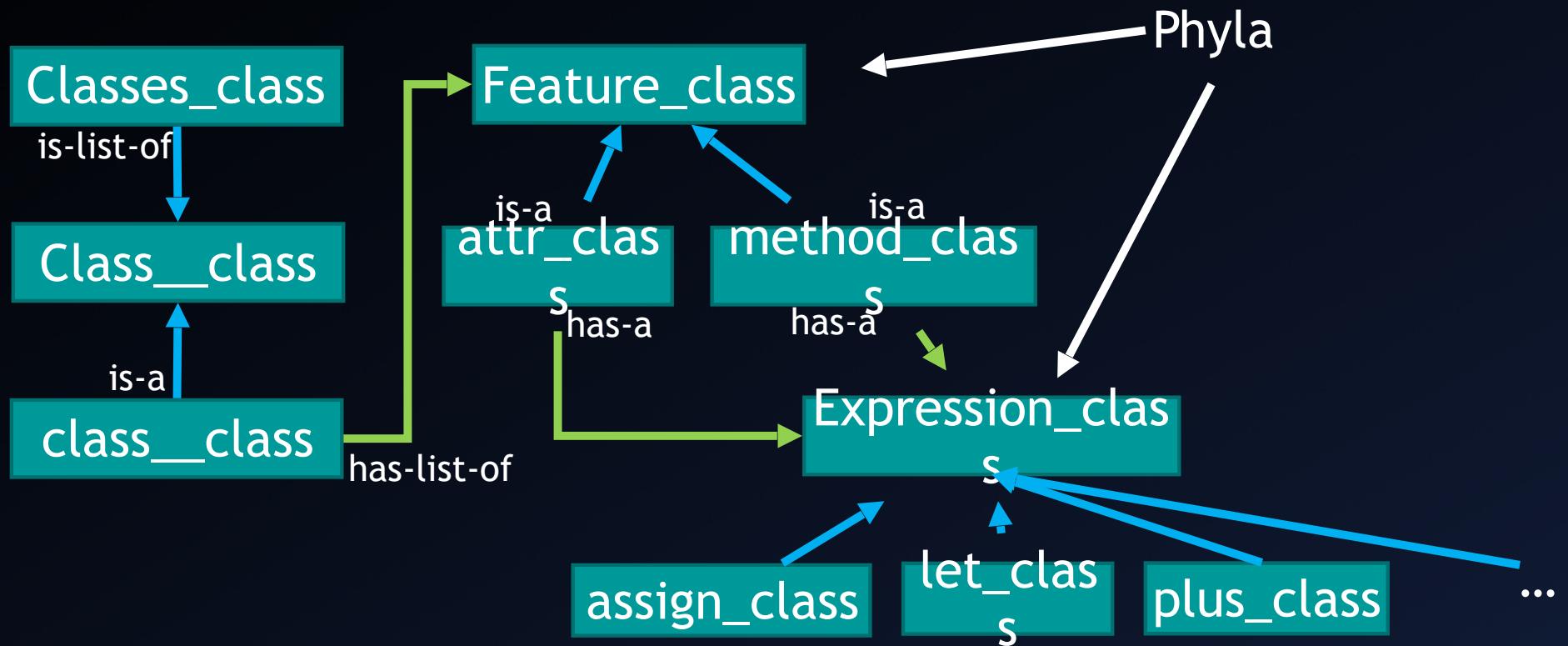
is-a

`class__class`

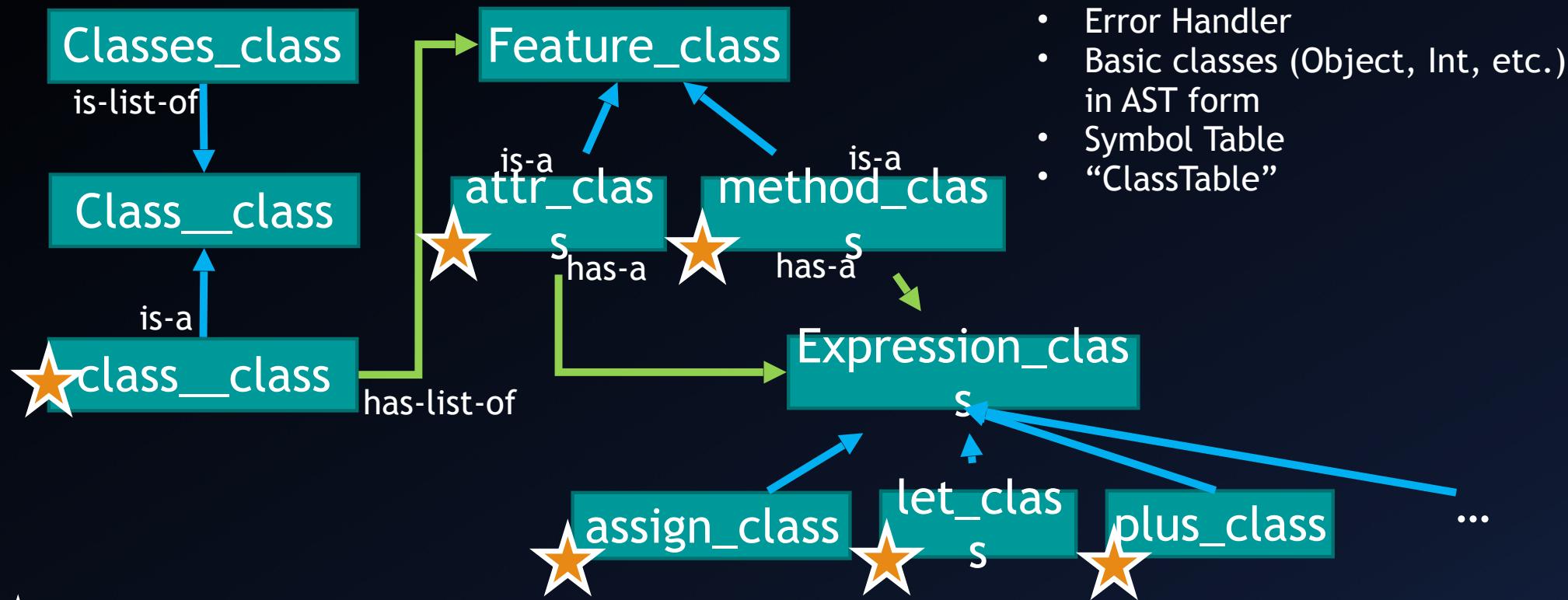
“Constructor” for Cool classes

PA#3 starter code

PA#3 starter code

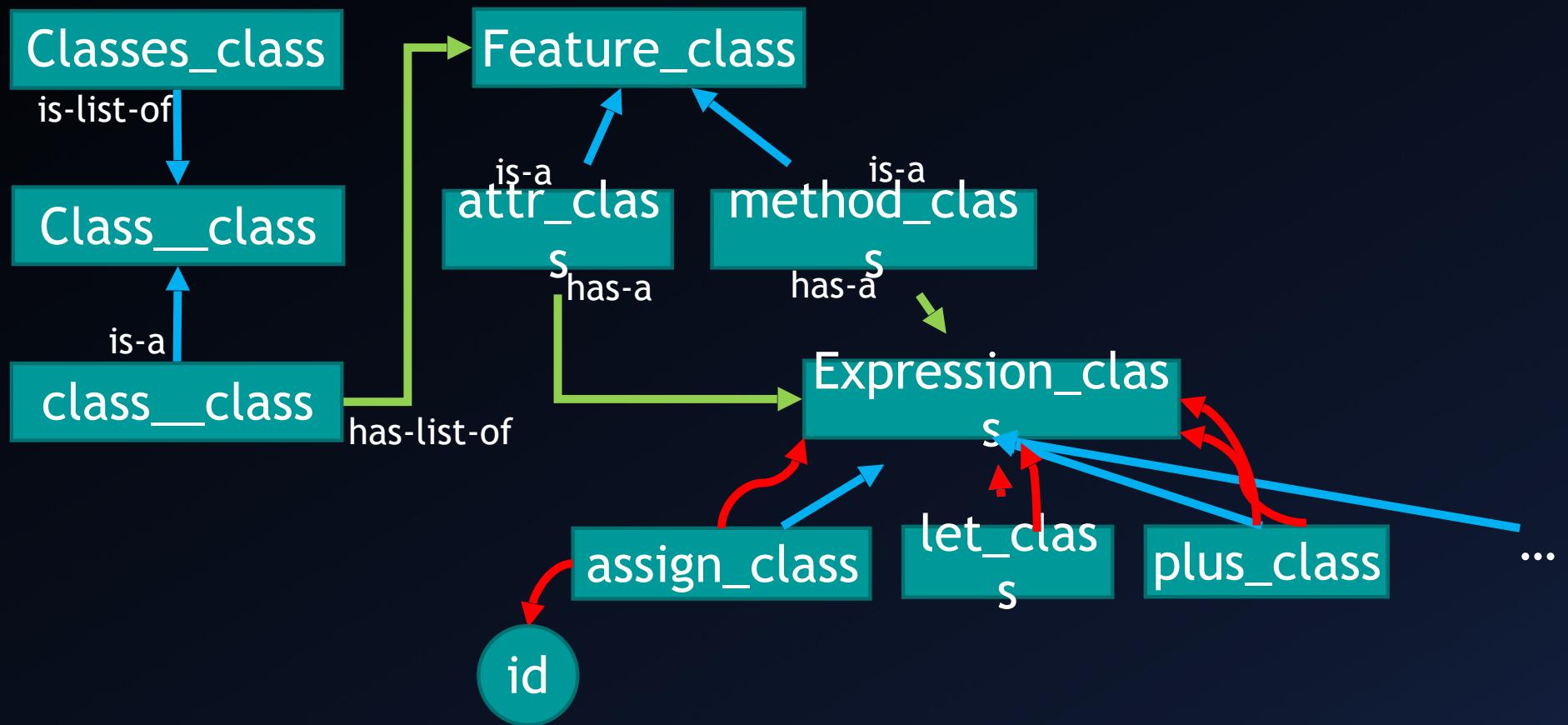


PA#3 starter code



★ constructor: has members you need to write accessors for

PA#3 starter code



PA#3 Tips

- Don't write phases independently (to combine later)
 - The first phase(s) are much simpler than the typechecking phase.
 - Typechecking has many dependencies on first phase.
- Understand the type checking rules!

PA#3 Tips

- Before submitting, read chap 1-13 of the cool manual again.
- Look for corner cases mentioned in manual.
- Testing:
 - (common case) make sure your implementation does what you think it does!
 - (corner case) think of examples that will break your code

Example

```
class Foo {  
    bar(id1 : Type1, id2 : Type2) : Type3 { ..... }  
};
```

Other than typechecking, what must we verify about bar?

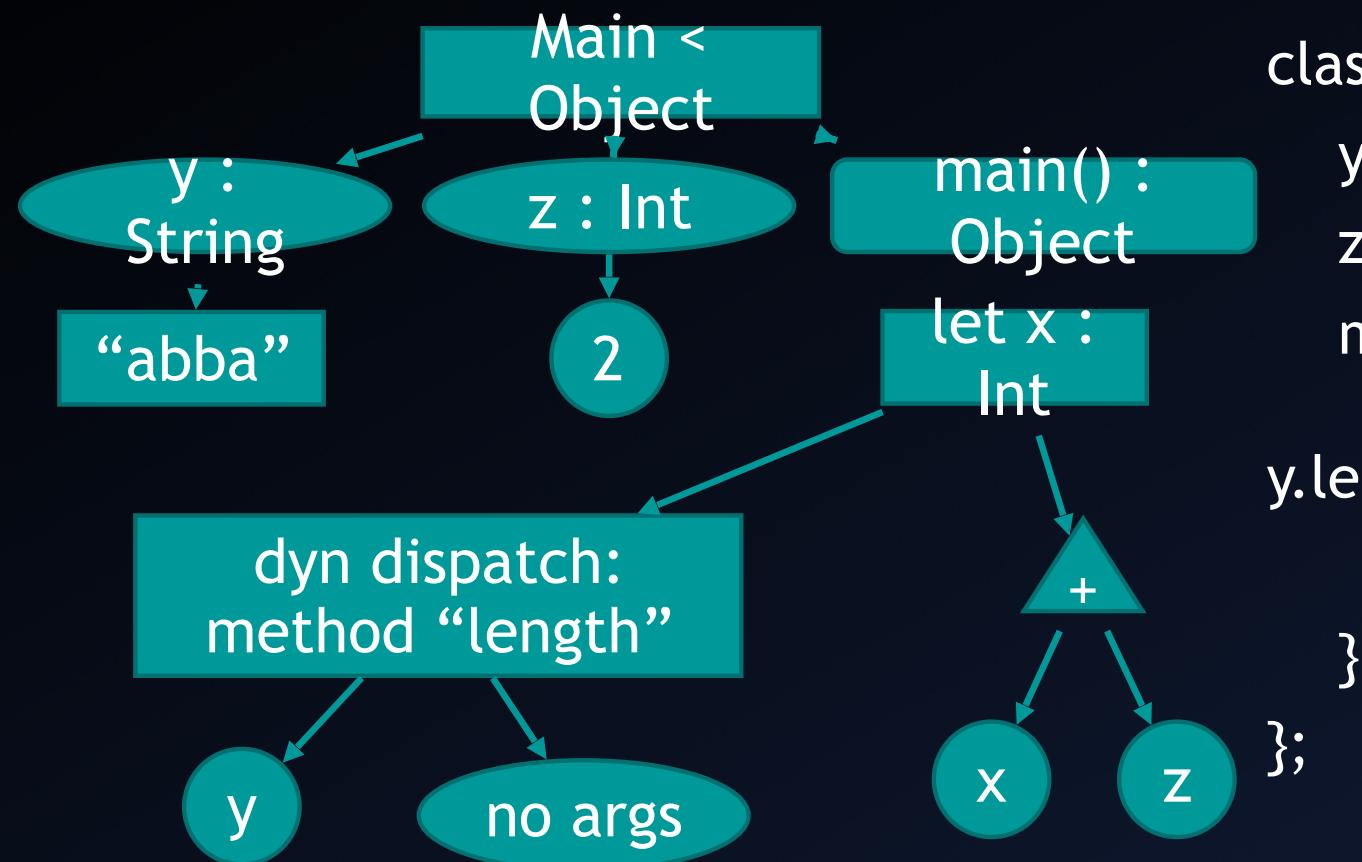
Questions on PA3 or the starter code?

Typechecking Examples

Running Example

```
class Main {  
    y : String <- "abba";  
    z : Int <- 2;  
    main() : Object {  
        let x : Int <- y.length() in x+z  
    };  
};
```

Running Example



```
class Main {  
    y : String <- "abba";  
    z : Int <- 2;  
    main() : Object {  
        let x : Int <-  
            y.length() in  
            x+z  
    };  
};
```

Before Typechecking

- Compute “Method Environment” (M)
- Any data structure you like!
- Need to be able to lookup methods by type and name

Main.main() : Object	String.concat(String) : String
Object.cool_abort() : Object	String.length() : Int
Object.type_name() : String	String.substr(Int, Int) : String
Object.copy() : SELF_TYPE	(and all other functions...)

Before Typechecking

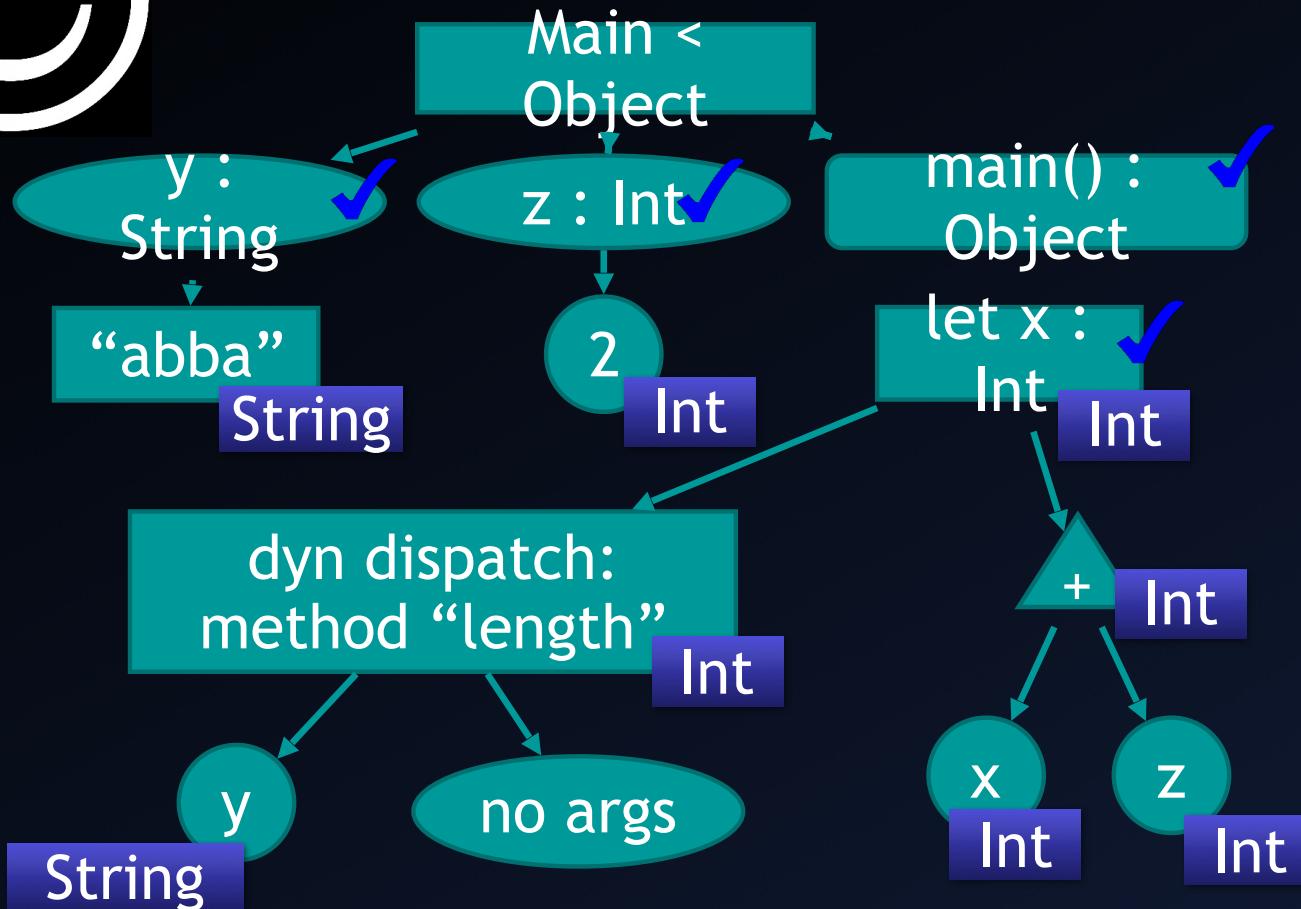
- Compute initial “Object Environment” for each class C . (O_C)
- Good idea to use symbol table (but not required!)
- Lists the attributes of the class
- Example for our Main class

O_{MAIN}

Symbol	Type
y	String
z	Int



Typechecking Example



Id	Type
y	String
z	Int
self	SELF_TYPE _{Main}

Id	Type
x	Int

but what about all those
rules?

Rule for Typechecking an Attribute

$$\frac{\begin{array}{c} O_C(x) = T_0 \\ O_C[\text{SELF_TYPE}_C/\text{self}], M, C \models e_1 : \\ T_1 \end{array}}{O_C, M, C \models x : T_0 \leftarrow e_1}$$

- What are O , M and C ?

Rule for Attributes

- O is the object environment.

Id	Type
y	String
z	Int
self	SELF_TYPE _{Main}

- M is the method environment.

- C is the class environment.

Main.main() : Object	String.concat(String) : String
Object.cool_abort() : Object	String.length() : Int
Object.type_name() : String	String.substr(Int, Int) : String
Object.copy() : SELF_TYPE	(and all other functions...)

$O, M, C \vdash e_1 : T_1$

$T_1 \leq T_0$

$O[T_0/x], M, C \vdash e_2 : T_2$

$O, M, C \vdash \text{let } x : T_0 \leftarrow e_1 \text{ in } e_2 : T_2$

To typecheck an expression, you need to know O , M and C .

- The method environment (M) is constant.
- The object environment (O) starts with O_C when typechecking C .
 - As you traverse top-to-bottom scopes are added.
- C is just the class that contains the expression.

Rule for Typechecking an Attribute

$$\frac{\begin{array}{c} O_C(x) = T_0 \\ O_C[\text{SELF_TYPE}_C/\text{self}], M, C \models e_1 : \\ T_1 \\ \hline T_1 \leq T_0 \end{array}}{O_C, M, C \models x : T_0 \leftarrow e_1}$$

- What are x , e_1 , T_0 , T_1 ?

Typechecking an Attribute

- E.g. how do we typecheck $y : \text{String} \leftarrow \text{"abba"}$ in class Main?
- Trick: read the rule backward!

$$\frac{\begin{array}{c} O_C(x) = T_0 \\ O_C[\text{SELF_TYPE}_C/\text{self}], M, C \models e_1 : \\ T_1 \end{array}}{T_1 \leq T_0} \quad O_C, M, C \models x : T_0 \leftarrow e_1$$

- We want $x \rightarrow y$, $T_0 \rightarrow \text{String}$, $e_1 \rightarrow \text{"abba"}$. Don't know T_1 yet.

$y : \text{String} \leftarrow \text{"abba"} \quad (\text{in class Main})$

- We instantiate the rule with
 $C = \text{Main}$, $x = y$, $T_0 = \text{String}$, $e_1 = \text{"abba"}$

- There are three things we need to check!

$$\frac{\begin{array}{c} O_C(x) = T_0 \\ O_C[\text{SELF_TYPE}_C/\text{self}], M, C \models e_1 : \\ T_1 \end{array}}{T_1 \leq T_0} \quad O_C, M, C \models x : T_0 \leftarrow e_1$$

$$\frac{\begin{array}{c} O_{\text{Main}}(y) = \text{String} \\ O_{\text{Main}}[\text{SELF_TYPE}_{\text{Main}}/\text{self}], M, \text{Main} \models \text{"abba"} : T_1 \\ T_1 \leq \text{String} \end{array}}{O_C, M, \text{Main} \models y : \text{String} \leftarrow \text{"abba"}}$$

$y : \text{String} \leftarrow \text{"abba"} \quad (\text{in class Main})$

$$\frac{\begin{array}{c} O_{\text{Main}}(y) = \text{String} \\ O_{\text{Main}}[\text{SELF_TYPE}_{\text{Main}}/\text{self}], M, \text{Main} \models \text{"abba"} : T_1 \\ T_1 \leq \text{String} \end{array}}{O_C, M, \text{Main} \models y : \text{String} \leftarrow \text{"abba"}}$$

- $O_{\text{Main}}(y) = \text{String} \rightarrow$ Look at the table we made earlier!

Symbol	Type
y	String
z	Int

$y : \text{String} \leftarrow \text{"abba"} \quad (\text{in class Main})$

$$\frac{\begin{array}{c} O_{\text{Main}}(y) = \text{String} \checkmark \\ O_{\text{Main}}[\text{SELF_TYPE}_{\text{Main}}/\text{self}], M, \text{Main} \models \text{"abba"} : T_1 ? \\ T_1 \leq \text{String} \end{array}}{O_{\text{Main}}, M, \text{Main} \models y : \text{String} \leftarrow \text{"abba"}}$$

- $O_{\text{Main}}[\text{SELF_TYPE}_{\text{Main}}/\text{self}], M, \text{Main} \models \text{"abba"} : T_1$
 - We need to prove that “abba” has type T_1 in extended environment
 - We need not know what T_1 is (yet).
 - This is where the typechecker makes a recursive call.

Typechecking “abba”

- We need to prove “abba” has type T_0
- with this augmented object environment,
- Only one rule applies to a string constant:

Symbol	Type
y	String
z	Int
self	SELF_TYPE _{MAIN}

$$\frac{x \text{ is string constant}}{\mathcal{O}, M, C \models x : \text{String}}$$

- We can return “String” - no further checks required.

Now,
replace T_1
by String

$y : \text{String} \leftarrow \text{"abba"} \quad (\text{in class Main})$

$$O_{\text{Main}}(y) = \text{String} \checkmark$$

$$O_{\text{Main}}[\text{SELF_TYPE}_{\text{Main}}/\text{self}], M, \text{Main} \models \text{"abba"} : T_1$$

$$T_1 \leq \text{String}$$

$$O_{\text{Main}}, M, \text{Main} \models y : \text{String} \leftarrow \text{"abba"}$$

T_1
replaced
by String

$y : \text{String} \leftarrow \text{"abba"} \quad (\text{in class Main})$

$$O_{\text{Main}}(y) = \text{String} \checkmark$$

$$O_{\text{Main}}[\text{SELF_TYPE}_{\text{Main}}/\text{self}], M, \text{Main} \models \text{"abba"} : \text{String}$$

$$\text{String} \leq \text{String} \checkmark$$

$$O_{\text{Main}}, M, \text{Main} \models y : \text{String} \leftarrow \text{"abba"}$$

- $O_{\text{Main}}[\text{SELF_TYPE}_{\text{Main}}/\text{self}], M, \text{Main} \models \text{"abba"} : T_1$
 - We just proved this with the recursive call.
 - $\text{String} \leq \text{String}$ is true since \leq is reflexive.

A complete proof

“abba” is a string constant

 $O_{\text{Main}} [\text{SELF_TYPE}_{\text{Main}}/\text{self}], M, \text{Main} \vdash \text{“abba”} : \text{String}$

$O_{\text{Main}} (y) = \text{String}$
 $O_{\text{Main}} [\text{SELF_TYPE}_{\text{Main}}/\text{self}], M, \text{Main} \vdash \text{“abba”} :$

 $\text{String} \leq \text{String}$
 $O_{\text{Main}}, M, \text{Main} \vdash y : \text{String} \leftarrow \text{“abba”}$

Last example: typechecking
main()

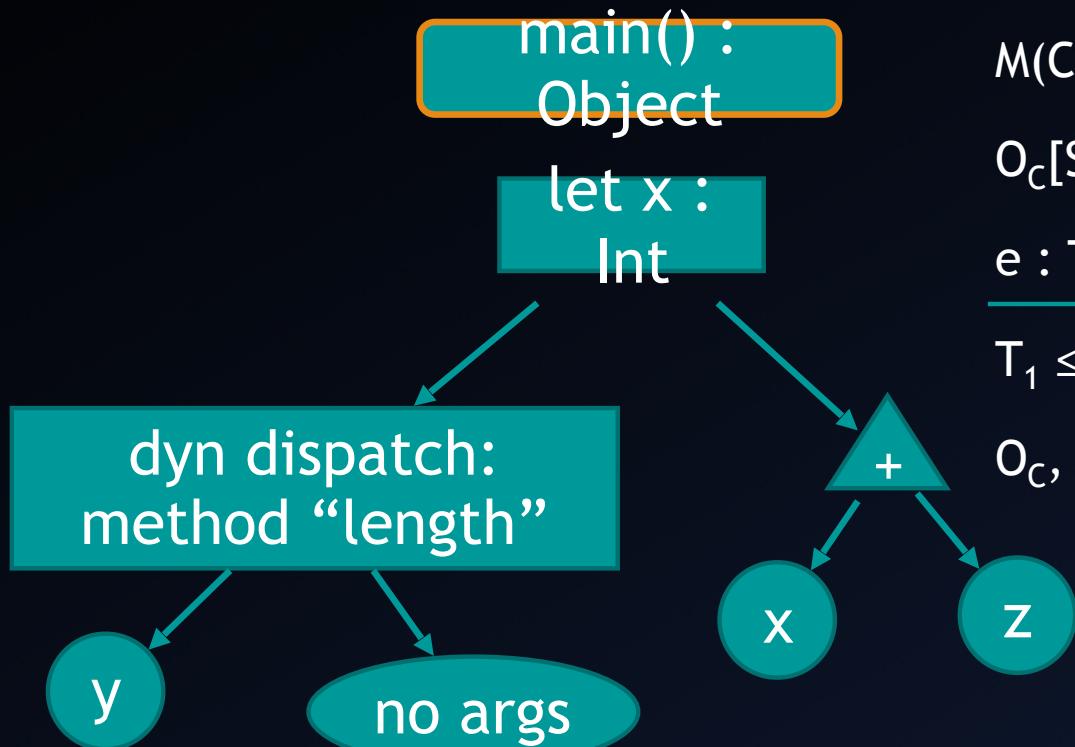
Last example

- Type check main()
- Going to simplify the rules slightly
(will only consider some of the SELF_TYPE cases)

```
class Main {  
    y : String <- "abba";  
    z : Int <- 2;  
    main() : Object {  
        let x : Int <-  
            y.length() in  
            x+z  
    };  
};
```

Id	Type
y	String
z	Int

Typechecking Example



[Method]

$$M(C, f) = (T_1, T_2, \dots, T_n, T_0)$$

$$O_C[\text{SELF_TYPE}_C/\text{self}][T_1/x_1] \dots [T_n/x_n], M, C \vdash e : T_1$$

$$T_1 \leq T_0$$

$$O_C, M, C \vdash f(x_1 : T_1, \dots, x_n : T_n) : T_0 \{ e \}$$

$C \rightarrow \text{Main}$

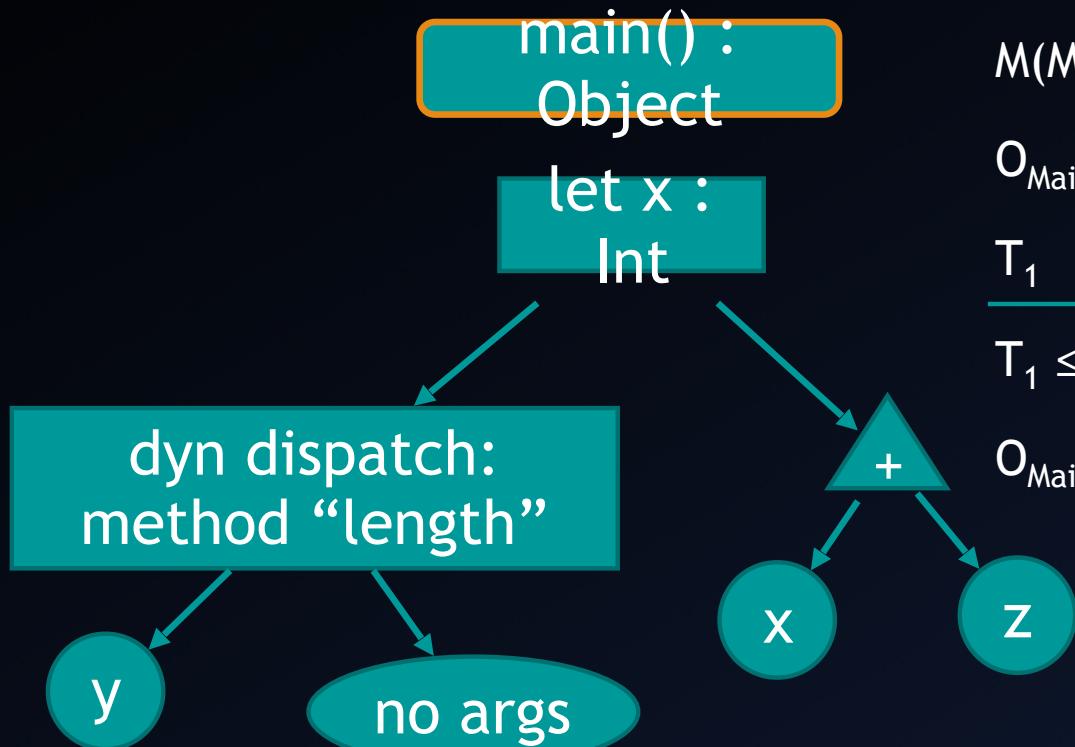
$f \rightarrow \text{main}$

$n \rightarrow 0$

$e \rightarrow \text{let } x : \text{Int} \dots$

Id	Type
y	String
z	Int

Typechecking Example



[method]

$$M(\text{Main}, \text{main}) = (T_0)$$

$$O_{\text{Main}}[\text{SELF_TYPE}_C/\text{self}], M, \text{Main} \vdash \{ \text{let } x \dots \} :$$

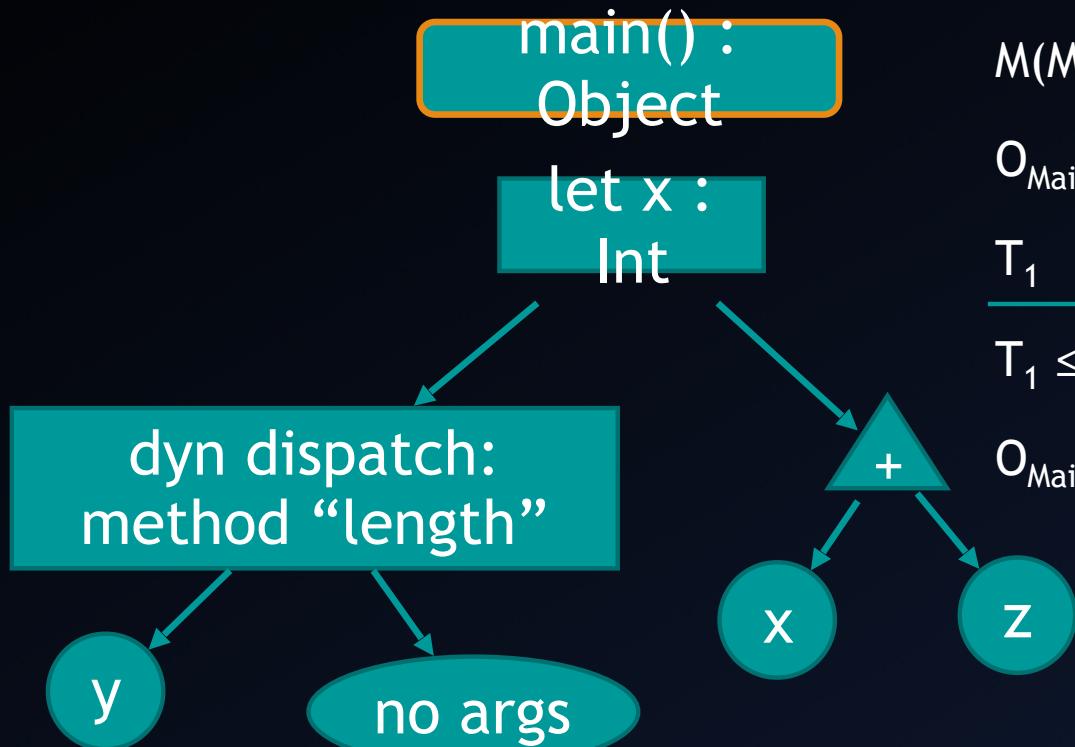
$$T_1$$

$$T_1 \leq \text{Object}$$

$$O_{\text{Main}}, M, \text{Main} \vdash \text{main}() : T_0 \{ \text{let } x \dots \}$$

Id	Type
y	String
z	Int

Typechecking Example



[Method]

$$M(\text{Main}, \text{main}) = (T_0)$$

$$O_{\text{Main}}[\text{SELF_TYPE}_C/\text{self}], M, \text{Main} \models \{ \text{let } x \dots \} :$$

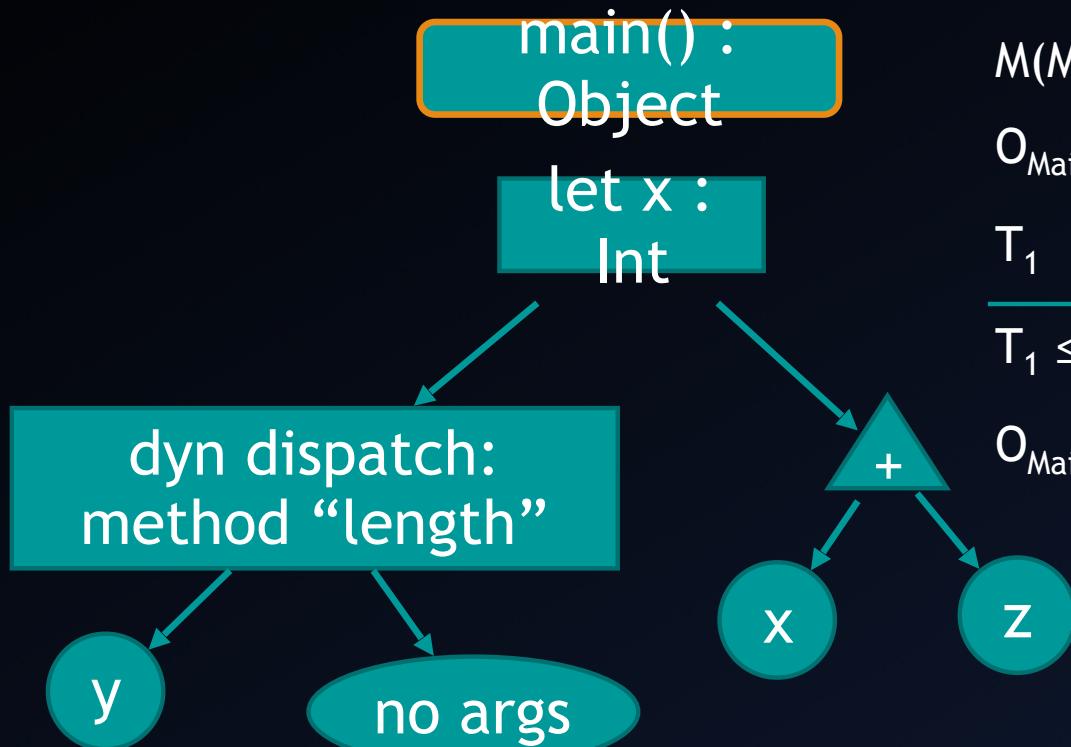
$$T_1$$

$$T_1 \leq \text{Object}$$

$$O_{\text{Main}}, M, \text{Main} \models \text{main}() : T_0 \{ \text{let } x \dots \}$$

Id	Type
y	String
z	Int

Typechecking Example



[Method]

$M(\text{Main}, \text{main}) = (\text{Object})$

$O_{\text{Main}}[\text{SELF_TYPE}_C/\text{self}], M, \text{Main} \vdash \{ \text{let } x \dots \} :$

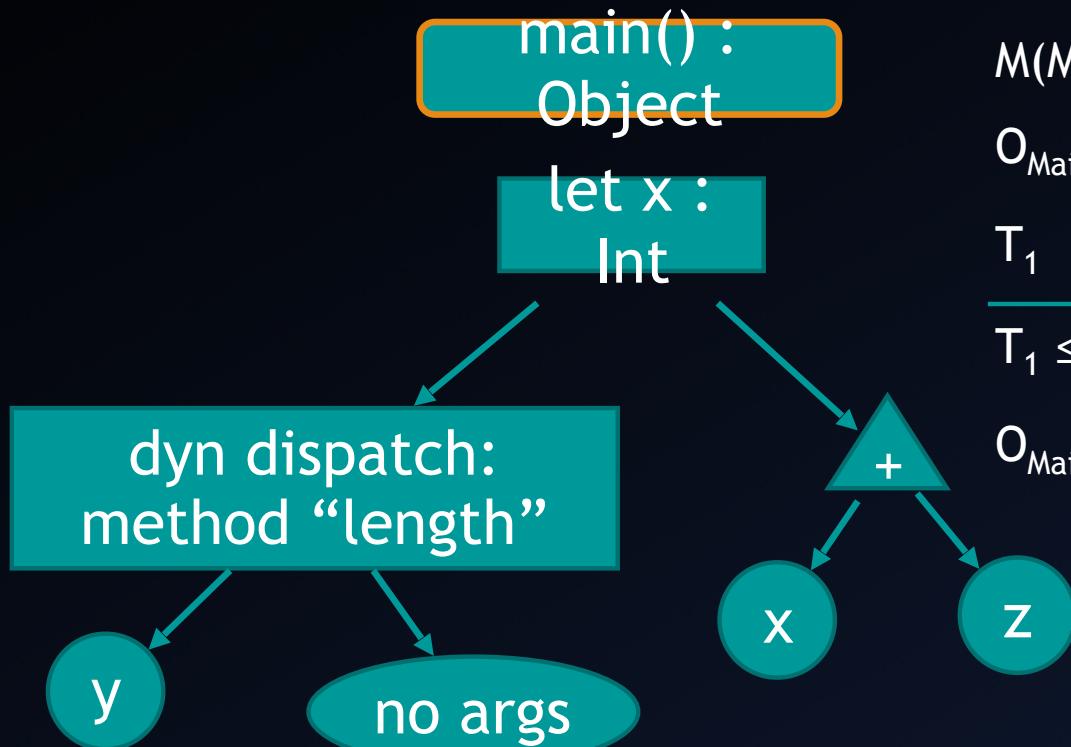
T_1

$T_1 \leq \text{Object}$

$O_{\text{Main}}, M, \text{Main} \vdash \text{main}() : \text{Object} \{ \text{let } x \dots \}$

Id	Type
y	String
z	Int

Typechecking Example



[Method]

$M(\text{Main}, \text{main}) = (\text{Object})$ ✓

$O_{\text{Main}}[\text{SELF_TYPE}_C/\text{self}], M, \text{Main} \vdash \{ \text{let } x \dots \} :$

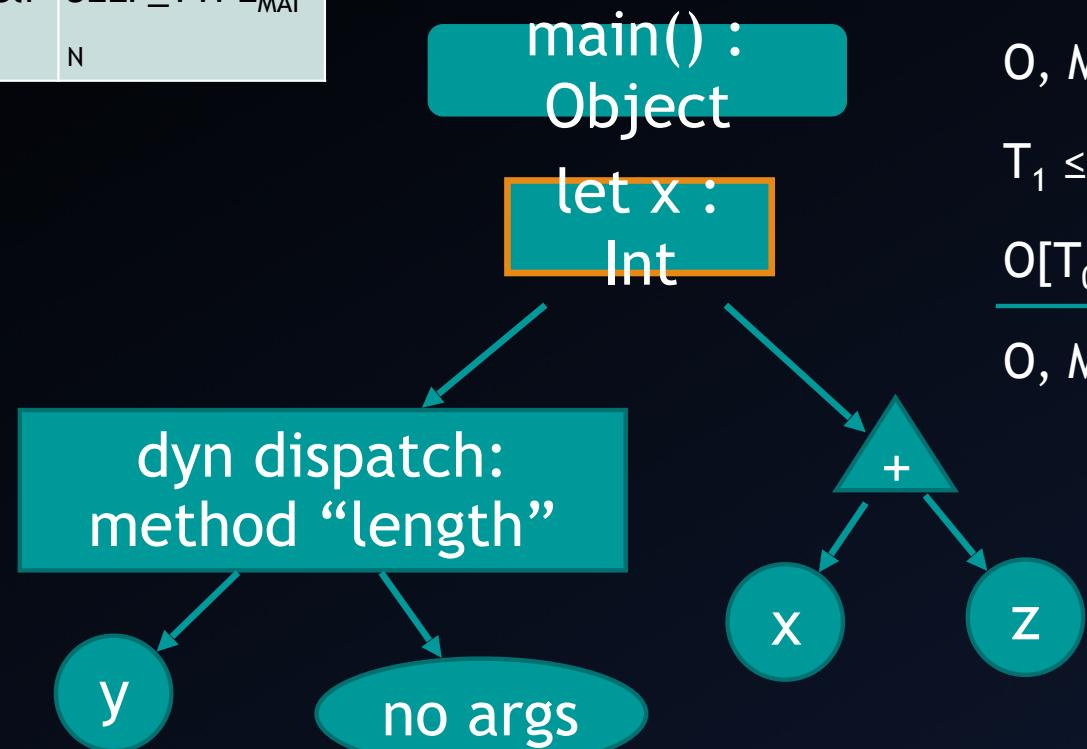
T_1

$T_1 \leq \text{Object}$

$O_{\text{Main}}, M, \text{Main} \vdash \text{main}() : \text{Object} \{ \text{let } x \dots \}$

Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
N	

Typechecking Example



[Let-Init]

$$O, M, C \models e_1 : T_1$$

$$T_1 \leq T_0$$

$$O[T_0/x], M, C \models e_2 : T_2$$

$$\frac{}{O, M, C \models \text{let } x : T_0 \leftarrow e_1 \text{ in } e_2 : T_2}$$

$e_1 \rightarrow y.\text{length}()$

$e_2 \rightarrow x + z$

$T_0 \rightarrow \text{Int}$

Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
N	

Typechecking Example

main() :
Object
let x :
Int

[Let-Init]

$$O, M, C \models y.length() : T_1$$

$$T_1 \leq \text{Int}$$

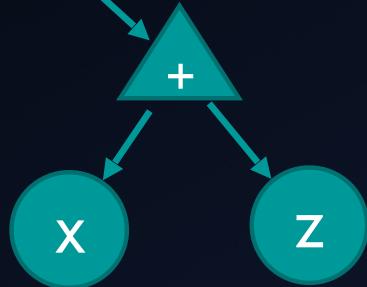
$$O[\text{Int}/x], M, C \models x+z : T_2$$

$$O, M, C \models \text{let } x : \text{Int} \leftarrow y.length() \text{ in } x+z : T_2$$

dyn dispatch:
method “length”

y

no args



Id	Type
y	String
z	Int
self	$\text{SELF_TYPE}_{\text{MAI}}$
N	

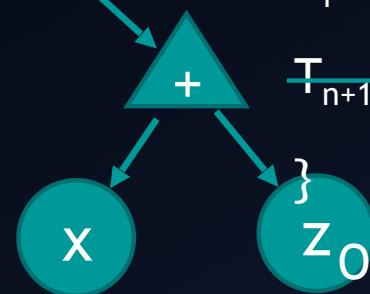
Typechecking Example

main() :
Object
let x :
Int

dyn dispatch:
method “length”



no args



[Dispatch]

$O, M, C \vdash e_0 : T_0 \dots O, M, C \vdash e_n : T_n$

$T'_0 = \{ C \text{ if } T_0 = \text{SELF_TYPE}_C; T_0 \text{ otherwise } \}$

$M(T'_0, f) = (T'_1, \dots, T'_{n+1})$

$T_i \leq T'_i \text{ for } 1 \leq i \leq n$

~~$T'_{n+1} = \{ T_0 \text{ if } T'_{n+1} = \text{SELF_TYPE}; T'_{n+1} \text{ otherwise } \}$~~

$O, M, \mathcal{A} \vdash e_0 @ (e_1 \text{ there are no arguments})$
 $f \rightarrow \text{“length”}$

$e_0 \rightarrow y$

$C \rightarrow \text{Main}$

Id	Type
y	String
z	Int
self	$\text{SELF_TYPE}_{\text{Main}}$
N	

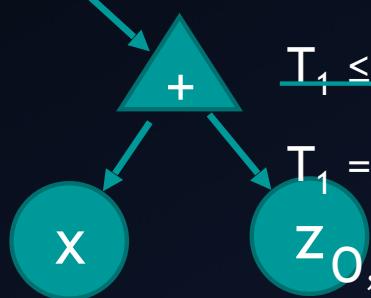
Typechecking Example

main() :
Object
let x :
Int

dyn dispatch:
method “length”

y

no args



[Dispatch]

$O, M, \text{Main} \vdash e_0 : T_0$

$T_0' = \{ \text{Main if } T_0 = \text{SELF_TYPE}_{\text{Main}}; T_0 \text{ otherwise} \}$

$M(T_0', \text{“length”}) = (T_1')$

$T_1 \leq T_1'$

$T_1 = \{ T_0 \text{ if } T_1' = \text{SELF_TYPE}; T_1' \text{ otherwise} \}$

$O, M, \text{Main} \vdash y.\text{length}() : T_1$

Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
N	

Typechecking Example

main() :
Object
let x :
Int

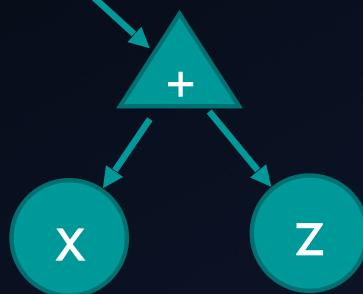
[Var]

$$\frac{O(\text{Id}) = T}{O, M, C \models \text{Id} : T}$$

dyn dispatch:
method “length”

y

no args



Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
N	

Typechecking Example

main() :
Object
let x :
Int

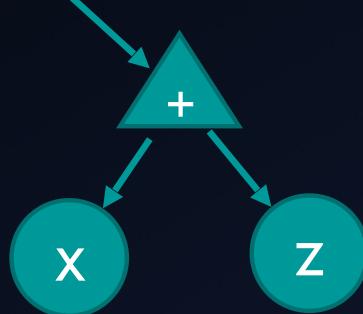
dyn dispatch:
method “length”

y

no args

[Var]

$$\frac{O(y) = \text{String} \checkmark}{O, M, C \models y : \text{String} \checkmark}$$



Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
N	

Typechecking Example

main() :
Object
let x :
Int

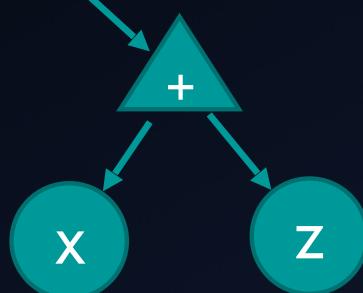
[Var]

$$\frac{O(y) = \text{String} \checkmark}{O, M, C \models y : \text{String}} \checkmark$$

dyn dispatch:
method “length”

y
String

no args



Id	Type
y	String
z	Int
self	$\text{SELF_TYPE}_{\text{Main}}$
N	

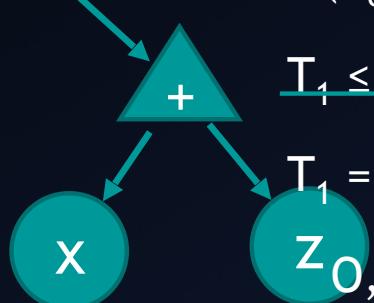
Typechecking Example

main() :
Object
let x :
Int

dyn dispatch:
method “length”

y
String

no args



[Dispatch]

$O, M, C \vdash y : T_0$

$T_0' = \begin{cases} \text{Main} & \text{if } T_0 = \text{SELF_TYPE}_{\text{Main}}; \\ T_0 & \text{otherwise} \end{cases}$

$M(T_0', "length") = (T_1')$

$T_1 \leq T_1'$

$T_1 = \begin{cases} T_0 & \text{if } T_1' = \text{SELF_TYPE}; \\ T_1' & \text{otherwise} \end{cases}$

$O, M, C \vdash y.\text{length}() : T_1$
 $T_0 \rightarrow \text{String}$

Id	Type
y	String
z	Int
self	$\text{SELF_TYPE}_{\text{Main}}$
N	

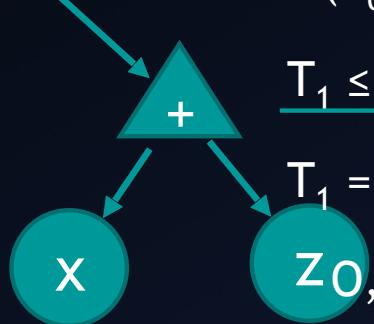
Typechecking Example

main() :
Object
let x :
Int

dyn dispatch:
method “length”

y
String

no args



[Dispatch]

$O, M, C \vdash y : \text{String}$ ✓

$T_0' = \{ \text{Main if } \text{Str} = \text{SELF_TYPE}_{\text{Main}}; \text{ Str otherwise} \}$

$M(T_0', \text{"length"}) = (T_1')$

$T_1 \leq T_1'$

$T_1 = \{ \text{Str if } T_0' = \text{SELF_TYPE}; T_0' \text{ otherwise} \}$

Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
N	

Typechecking Example

main() :
Object
let x :
Int

[Dispatch]

O, M, C ⊢ y : String ✓

T₀' = String ✓

M(String, "length") = (T₁')

T₁ ≤ T₁'

T₁ = { Str if T'₁ = SELF_TYPE; T'₁ otherwise }

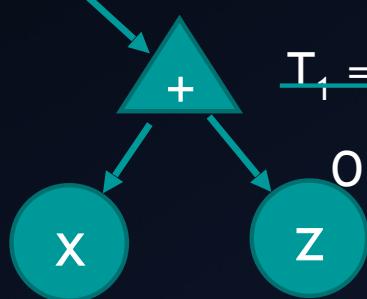
O, M, C ⊢ y.length() : T₁

Now, look up String.length in the method environment. Does it really have no parameters?

dyn dispatch:
method "length"

y
String

no args



Id	Type
y	String
z	Int
self	SELF_TYPE _{MAIN}
N	

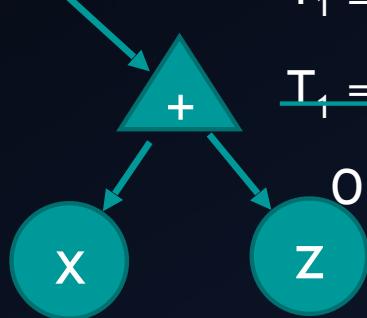
Typechecking Example

main() :
Object
let x :
Int

dyn dispatch:
method “length”

y
String

no args



O, M, C ⊢ y : String ✓

T_{0'} = String ✓

M(String, “length”) = (T_{1'}) = (Int)

T₁ ≤ T_{1'}

T₁ = { Str if T_{0'} = SELF_TYPE; T_{0'} otherwise }

O, M, C ⊢ y.length() : T₁

Yes! It returns an Int.
Now T₁ = T_{1'} = Int.

[Dispatch]

Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
N	

Typechecking Example

main() :
Object
let x :
Int

[Dispatch]

$O, M, C \vdash y : \text{String}$ ✓

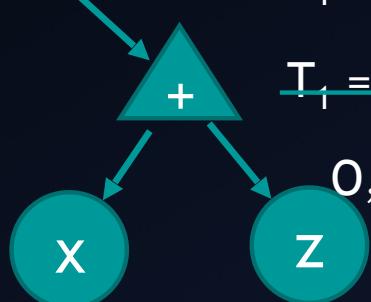
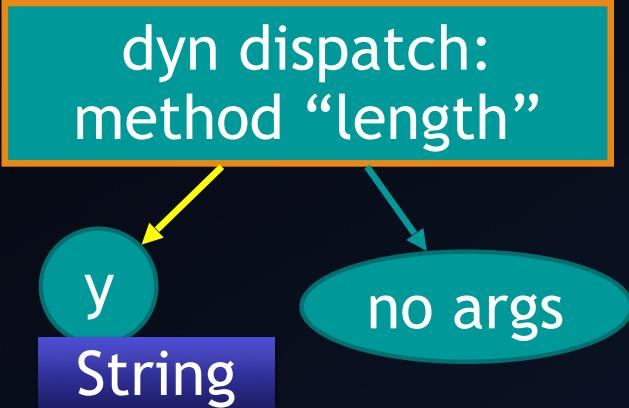
$T_0' = \text{String}$ ✓

$M(\text{String}, \text{"length"}) = (T_1') = (\text{Int})$ ✓

$T_1 \leq T_1' \Leftrightarrow \text{Int} \leq \text{Int}$ ✓

$T_1 = \text{Int}$ ✓

$O, M, C \vdash y.\text{length}() : \text{Int}$



Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
N	

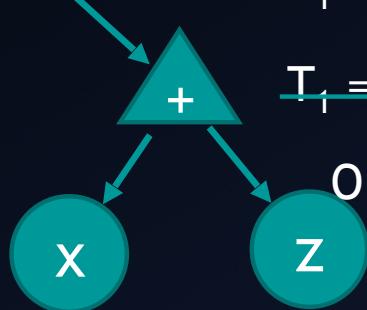
Typechecking Example

main() :
Object
let x :
Int

dyn dispatch:
method “length”

String

Int
no args



[Dispatch]

$O, M, C \vdash y : \text{String}$ ✓

$T_0' = \text{String}$ ✓

$M(\text{String}, \text{“length”}) = (T_1') = (\text{Int})$ ✓

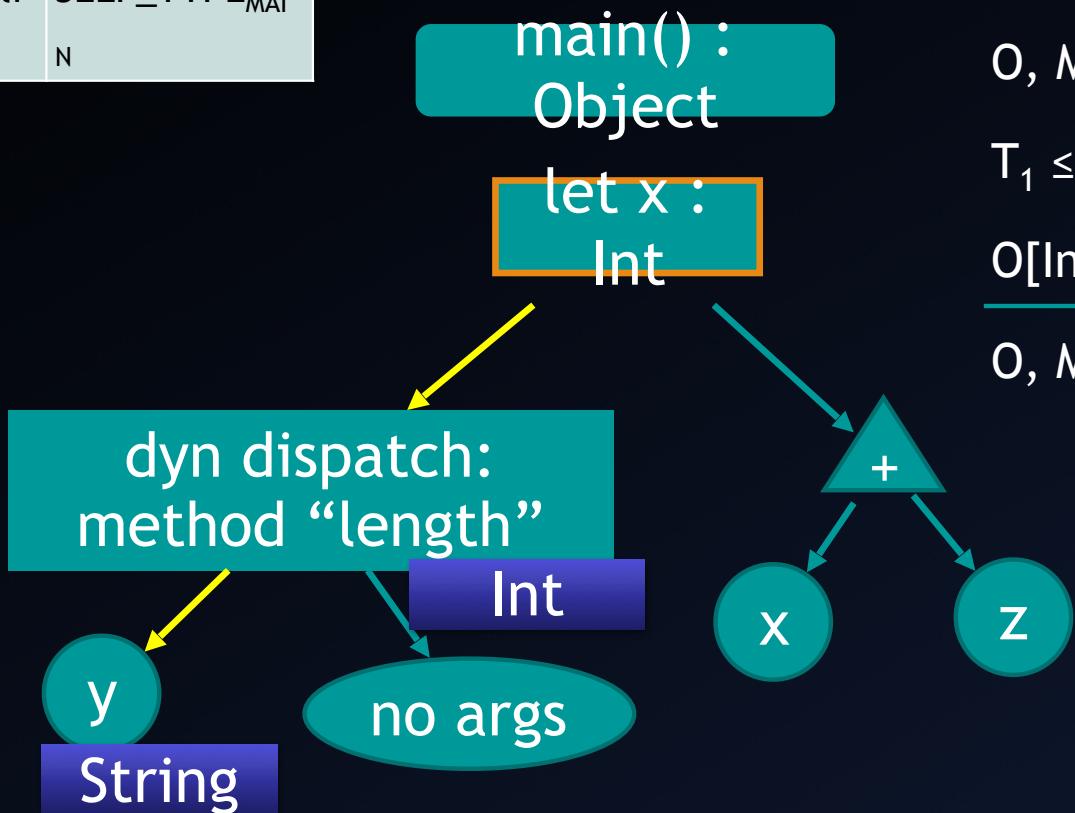
$T_1 \leq T_1' \Leftrightarrow \text{Int} \leq \text{Int}$ ✓

$T_1 = \text{Int}$ ✓

$O, M, C \vdash y.\text{length}() : \text{Int}$ ✓

Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
N	

Typechecking Example



[Let-Init]

$$O, M, C \models y.length() : T_1$$

$$T_1 \leq \text{Int}$$

$$O[\text{Int}/x], M, C \models x+z : T_2$$

$$\frac{}{O, M, C \models \text{let } x : \text{Int} \leftarrow y.length() \text{ in } x+z : T_2}$$

Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
N	

Typechecking Example

main() :
Object
let x :
Int

[Let-Init]

$O, M, C \vdash y.length() : Int$ ✓

$\text{Int} \leq \text{Int}$ ✓

$O[\text{Int}/x], M, C \vdash x+z : T_2$

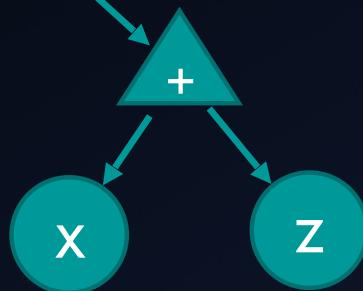
$O, M, C \vdash \text{let } x : \text{Int} \leftarrow y.length() \text{ in } x+z : T_2$

dyn dispatch:
method “length”

Int

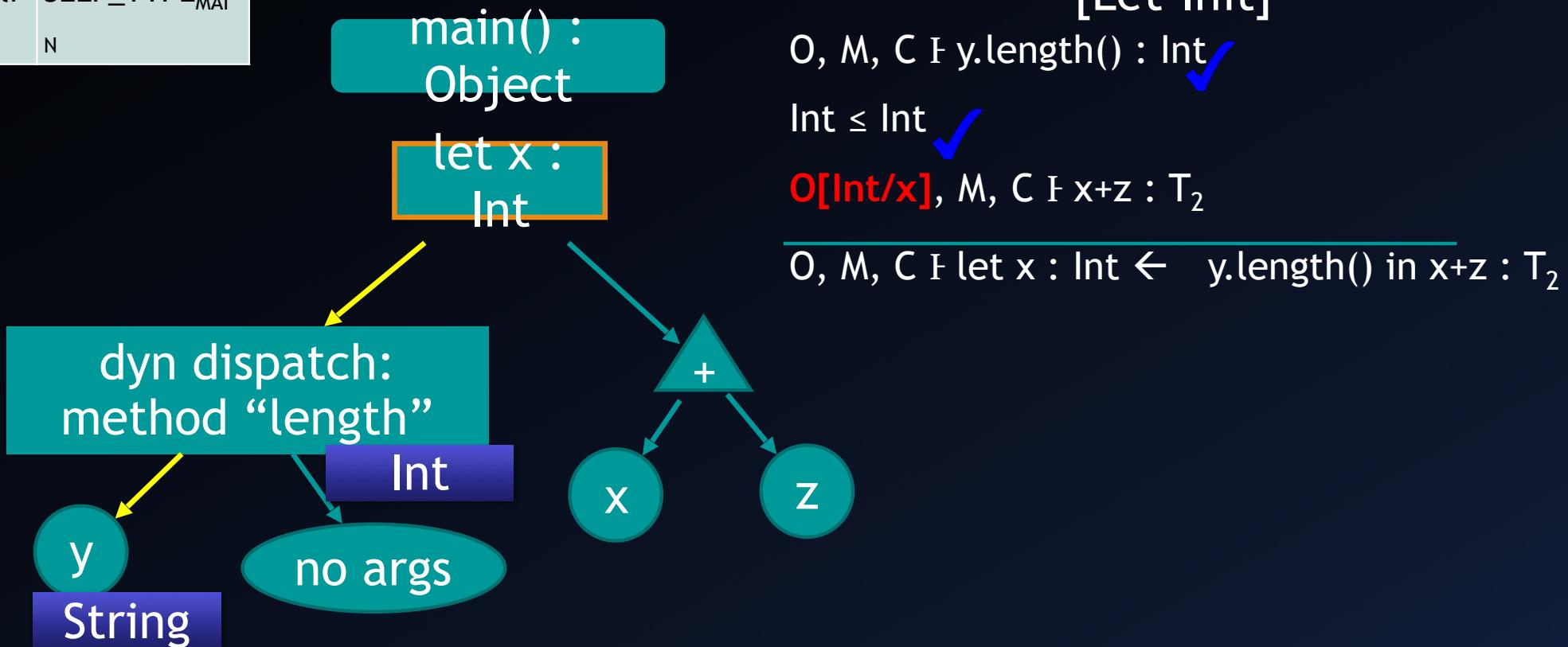
no args

y
String



Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
N	

Typechecking Example



Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
x	Int

Typechecking Example

main() :
Object
let x :
Int

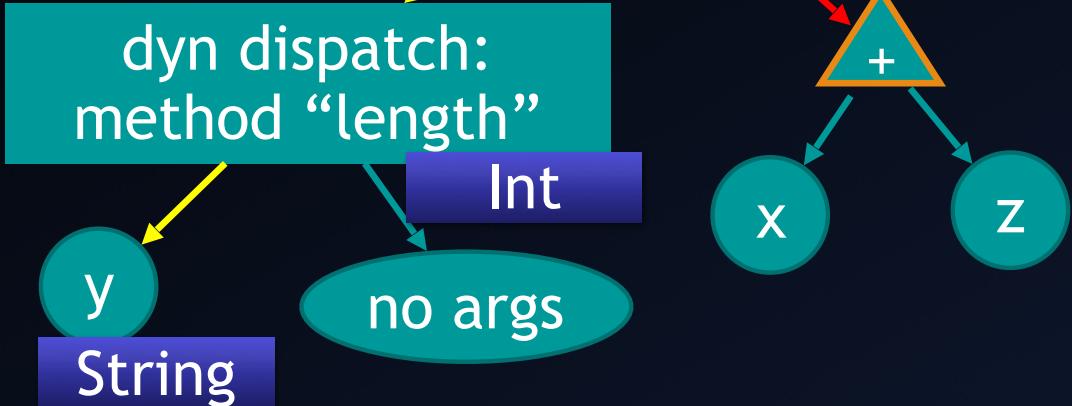
[Arith]

0, M, C ⊢ e₁ : Int

0, M, C ⊢ e₂ : Int

op ∈ { *, +, -, / }

0, M, C ⊢ e₁ op e₂ : Int



e₁ → x

e₂ → z

op → +

Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
x	Int

Typechecking Example

main() :
Object
let x :
Int

[Arith]

0, M, C ⊢ x : Int

0, M, C ⊢ z : Int

+ ∈ { *, +, -, / }



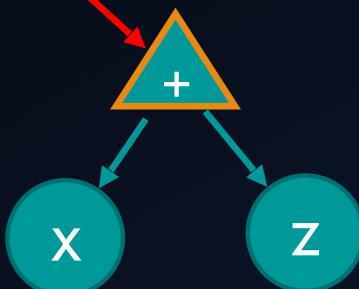
0, M, C ⊢ x + z : Int

dyn dispatch:
method “length”

Int

y
String

no args



Id	Type
y	String
z	Int
self	SELF_TYPE _{MAIN}
x	Int

Typechecking Example

main() :
Object
let x :
Int

[Var]

$$\frac{O(x) = \text{Int} \quad \checkmark}{O, M, C \models y : \text{Int}} \quad \checkmark$$



Id	Type
y	String
z	Int
self	SELF_TYPE _{MAIN}
x	Int

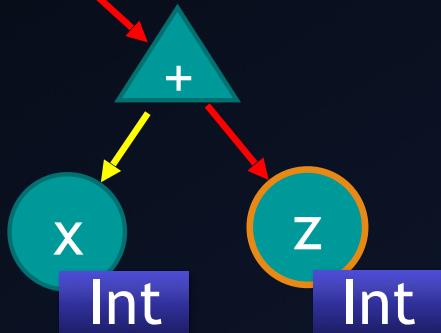
Typechecking Example

main() :
Object
let x :
Int

[Var]

$$\frac{O(z) = \text{Int} \quad \checkmark}{O, M, C \models z : \text{Int}} \checkmark$$

dyn dispatch:
method “length”
Int
no args



Id	Type
y	String
z	Int
self	SELF_TYPE _{MAIN}
x	Int

Typechecking Example

main() :
Object
let x :
Int

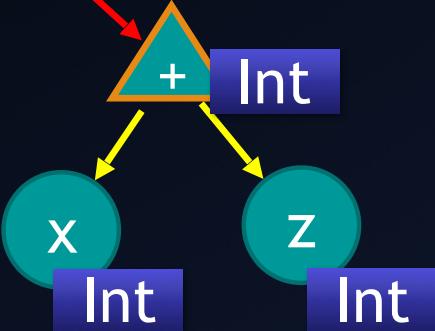
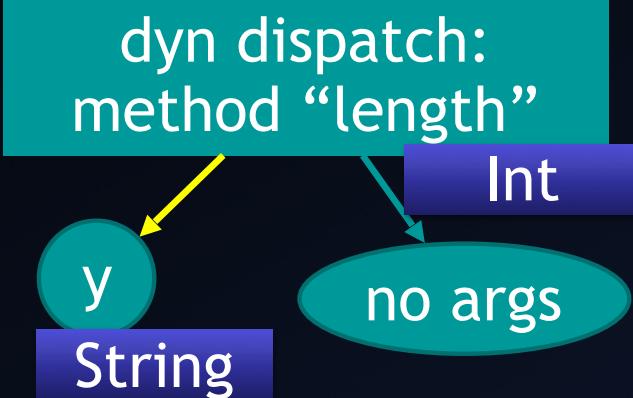
[Arith]

0, M, C ⊢ x : Int ✓

0, M, C ⊢ z : Int ✓

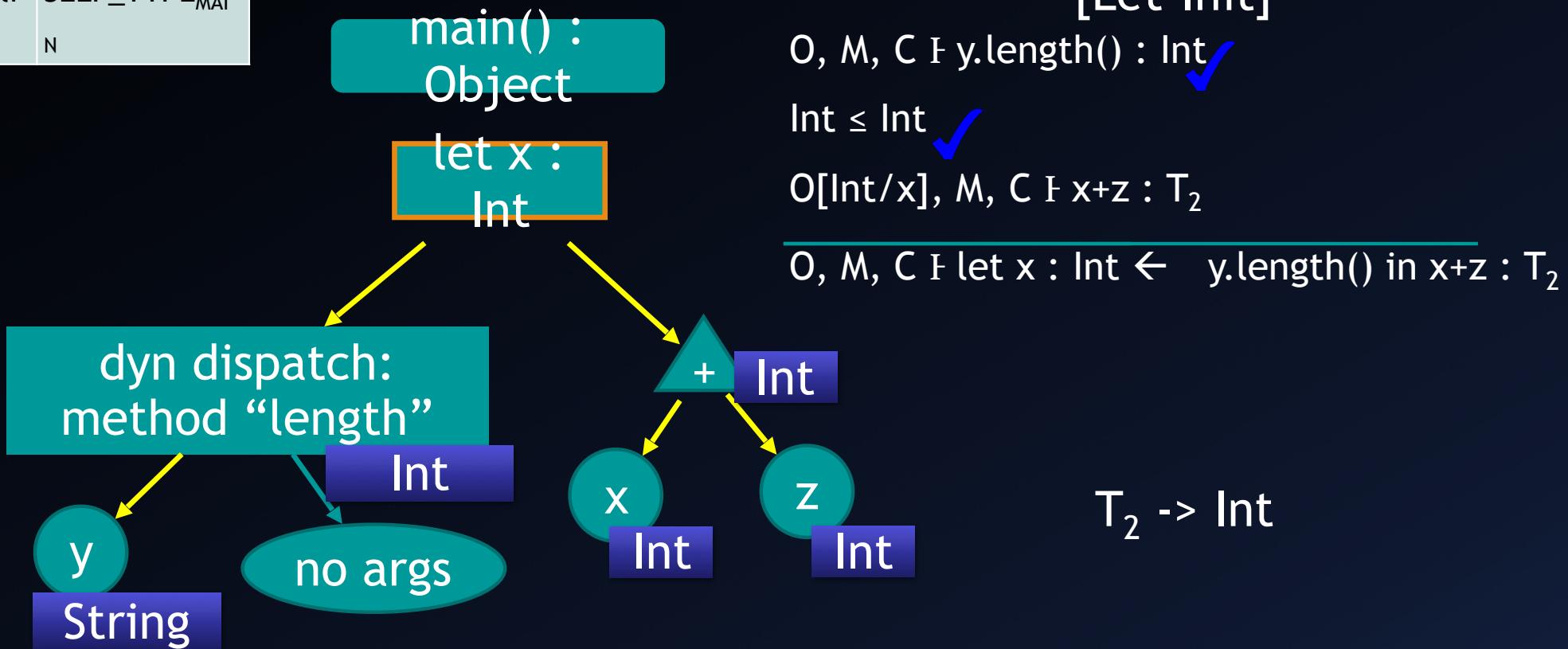
+ ∈ { *, +, -, / } ✓

0, M, C ⊢ x + z : Int ✓



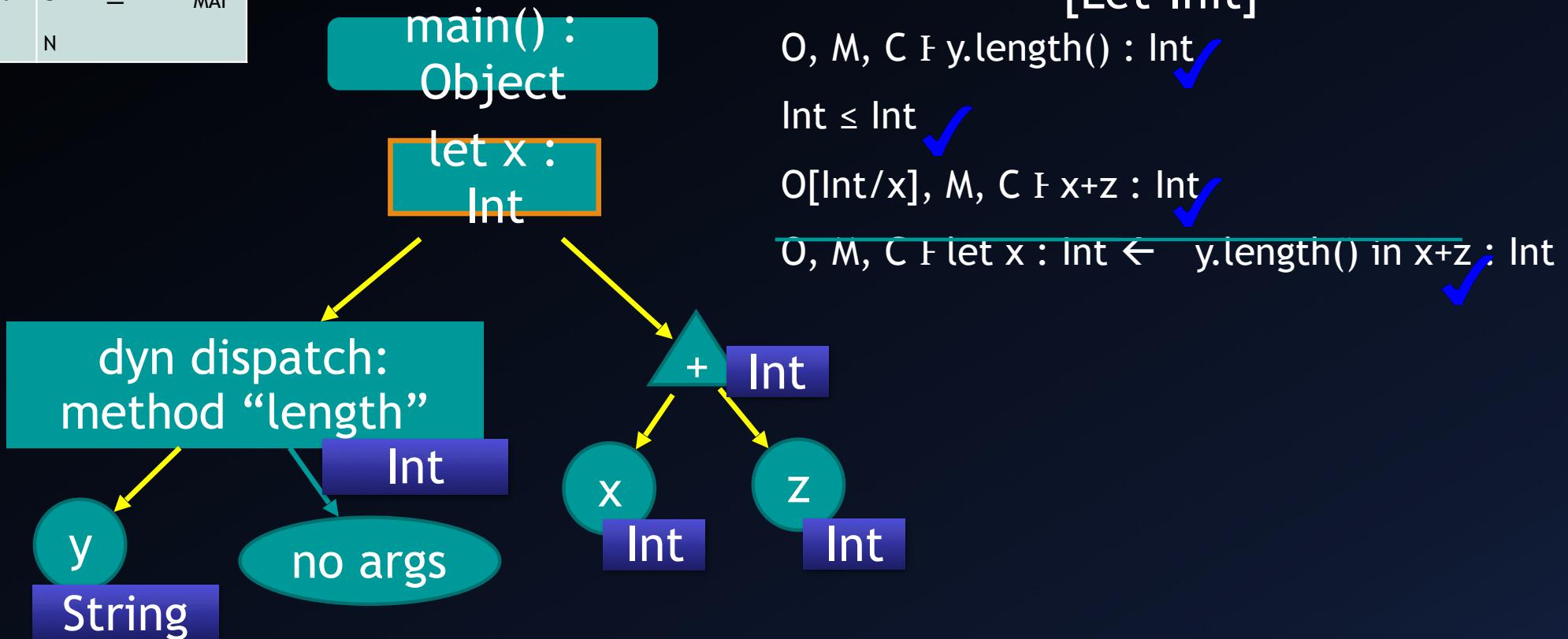
Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
N	

Typechecking Example



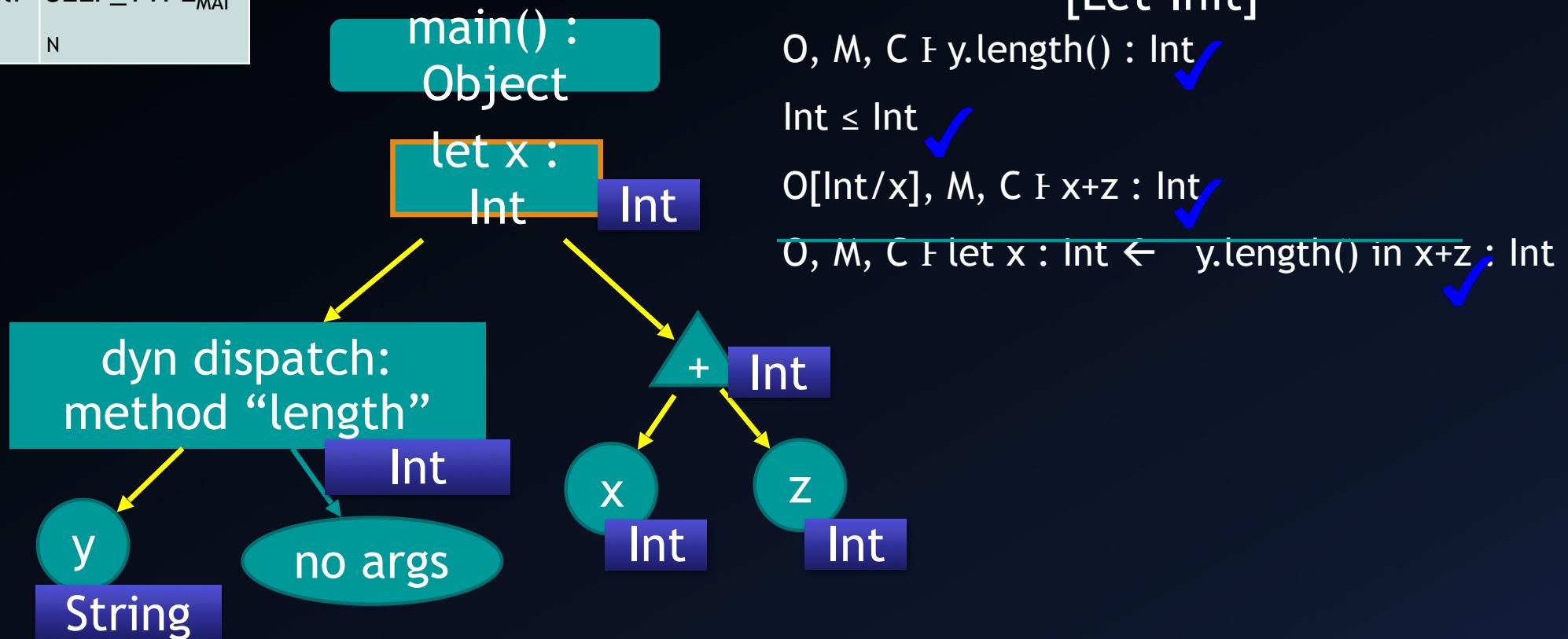
Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
N	

Typechecking Example



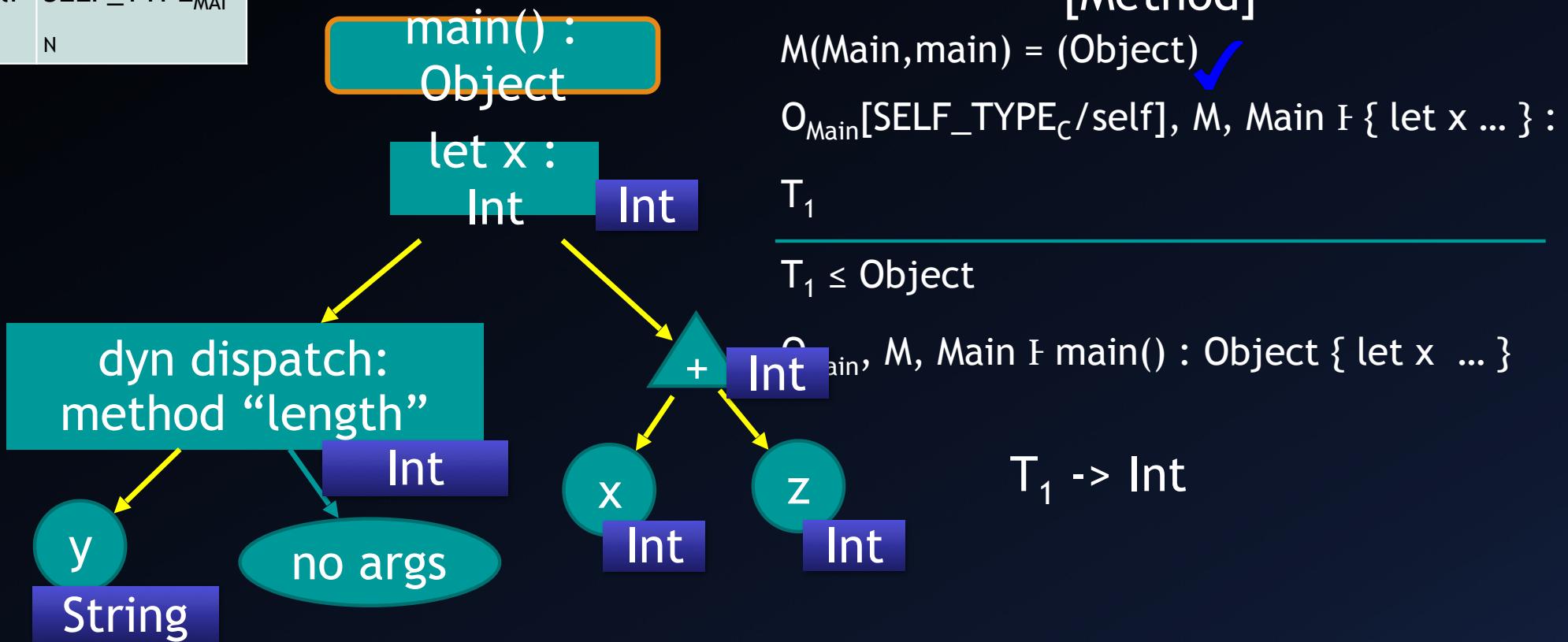
Id	Type
y	String
z	Int
self	SELF_TYPE _{MAI}
N	

Typechecking Example



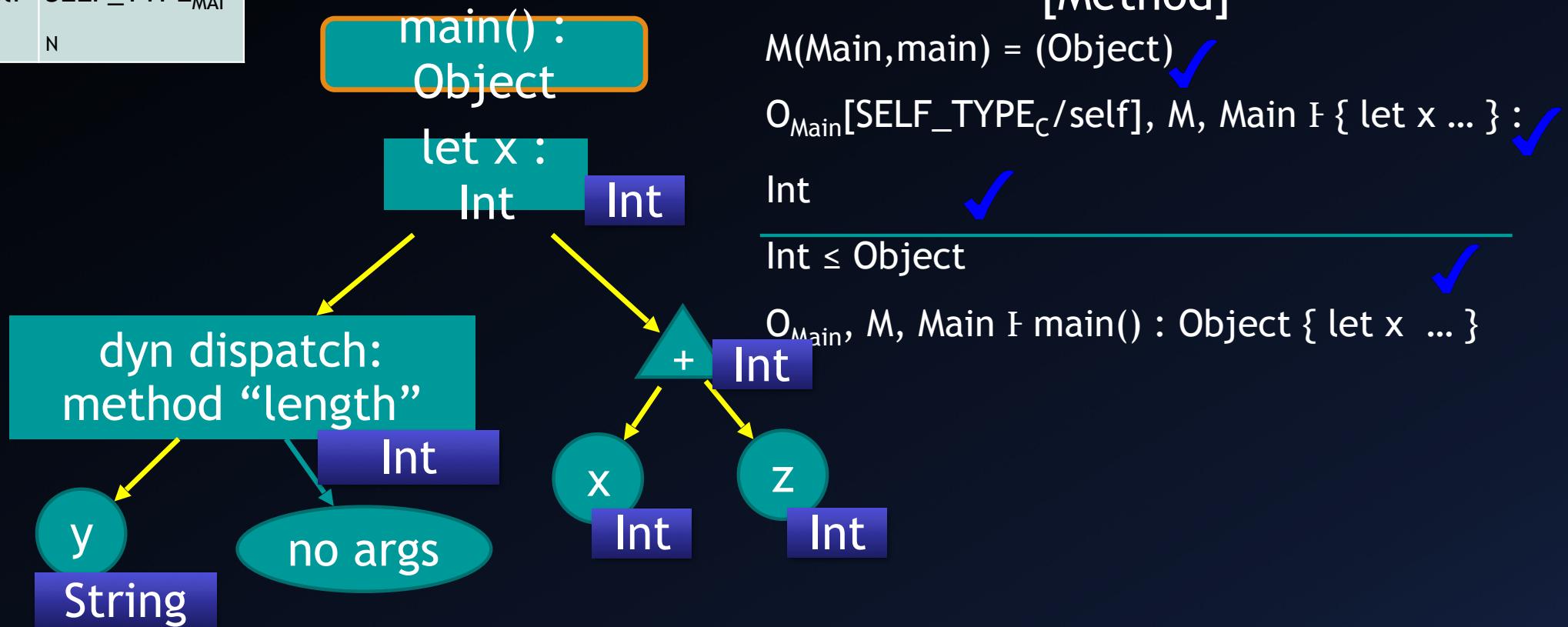
Id	Type
y	String
z	Int
self	$\text{SELF_TYPE}_{\text{MAI}}$
N	

Typechecking Example



Id	Type
y	String
z	Int
self	$\text{SELF_TYPE}_{\text{MAI}}$
N	

Typechecking Example



Id	Type
y	String
z	Int
self	$\text{SELF_TYPE}_{\text{MAI}}$
N	

Typechecking Example

