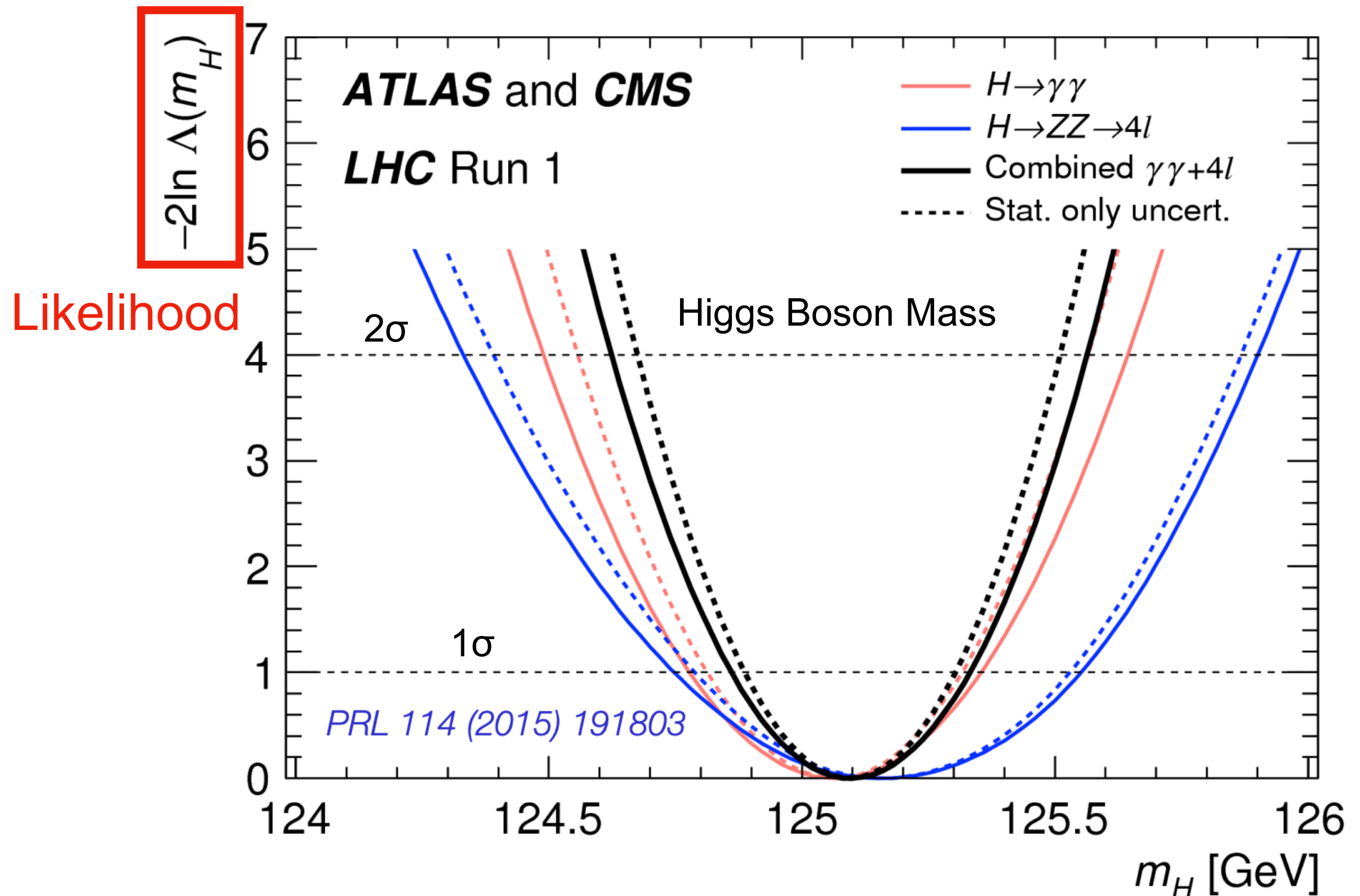




Lecture 5: Confidence

Recap

Making a Measurement



Important Properties

- Taylor expand in our floated parameter (μ in this case)

$$\chi^2(x_i, \mu) = \chi_{min}^2(x_i, \mu_0) + \frac{1}{2} \frac{\partial^2}{\partial \mu^2} \chi_{min}^2(x_i, \mu_0) (\mu - \mu_0)^2$$

$$\frac{1}{2} \frac{d^2 \chi^2}{d\mu^2} \rightarrow \frac{1}{\sigma^2}$$

χ^2 distribution of 1 degree of freedom
 $V[\chi^2(x)] = 1$

$$\frac{\partial^2 \chi^2}{\partial \theta^2} = \frac{2}{\sigma_\theta^2}$$

$$\Delta \chi^2 = 2 \Delta \log L = 1$$

For one degree of freedom

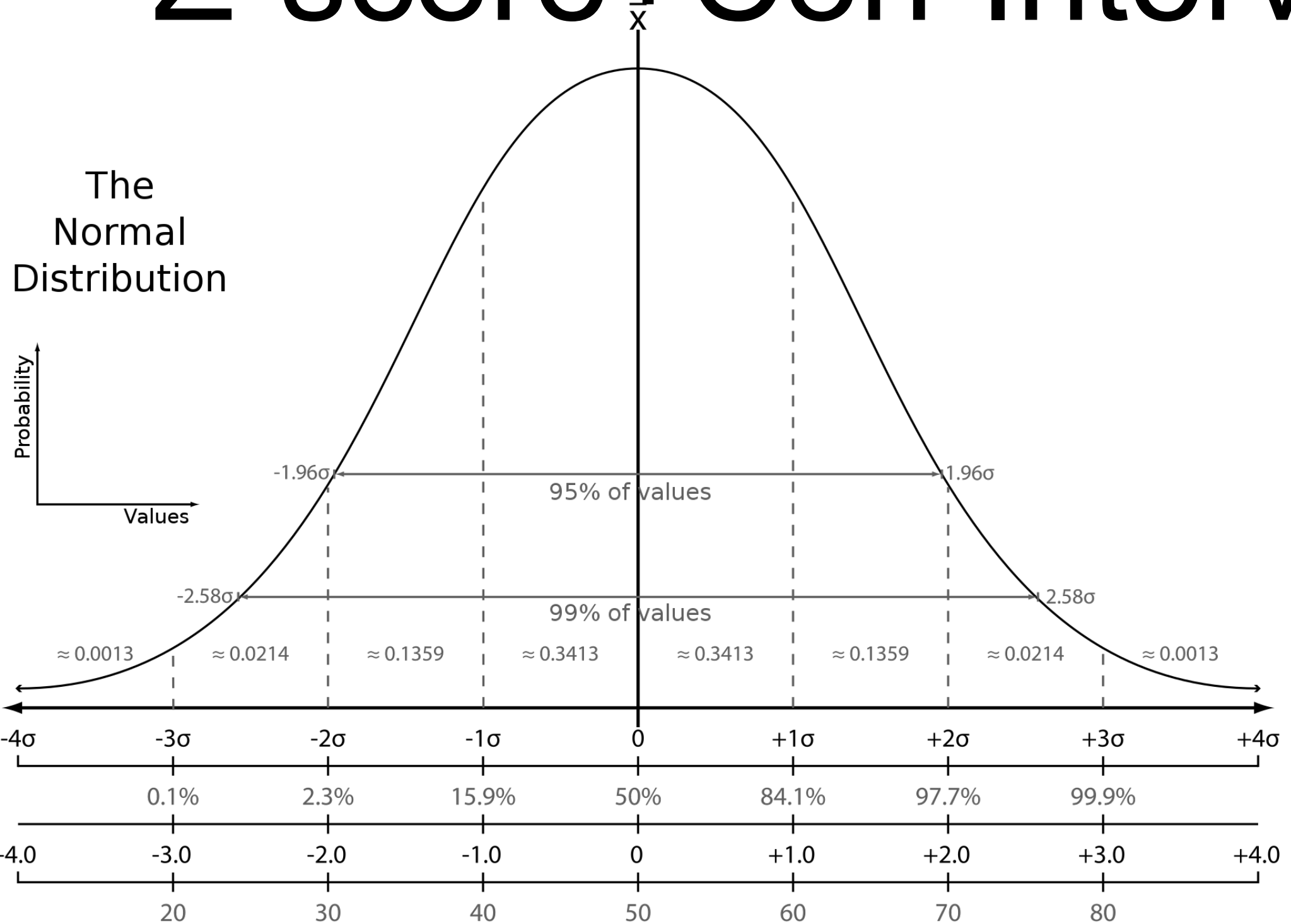
For any floated parameter
 uncertainty of that parameter is
 given by the 2nd derivative of χ^2

This is known as Wilk's Theorem

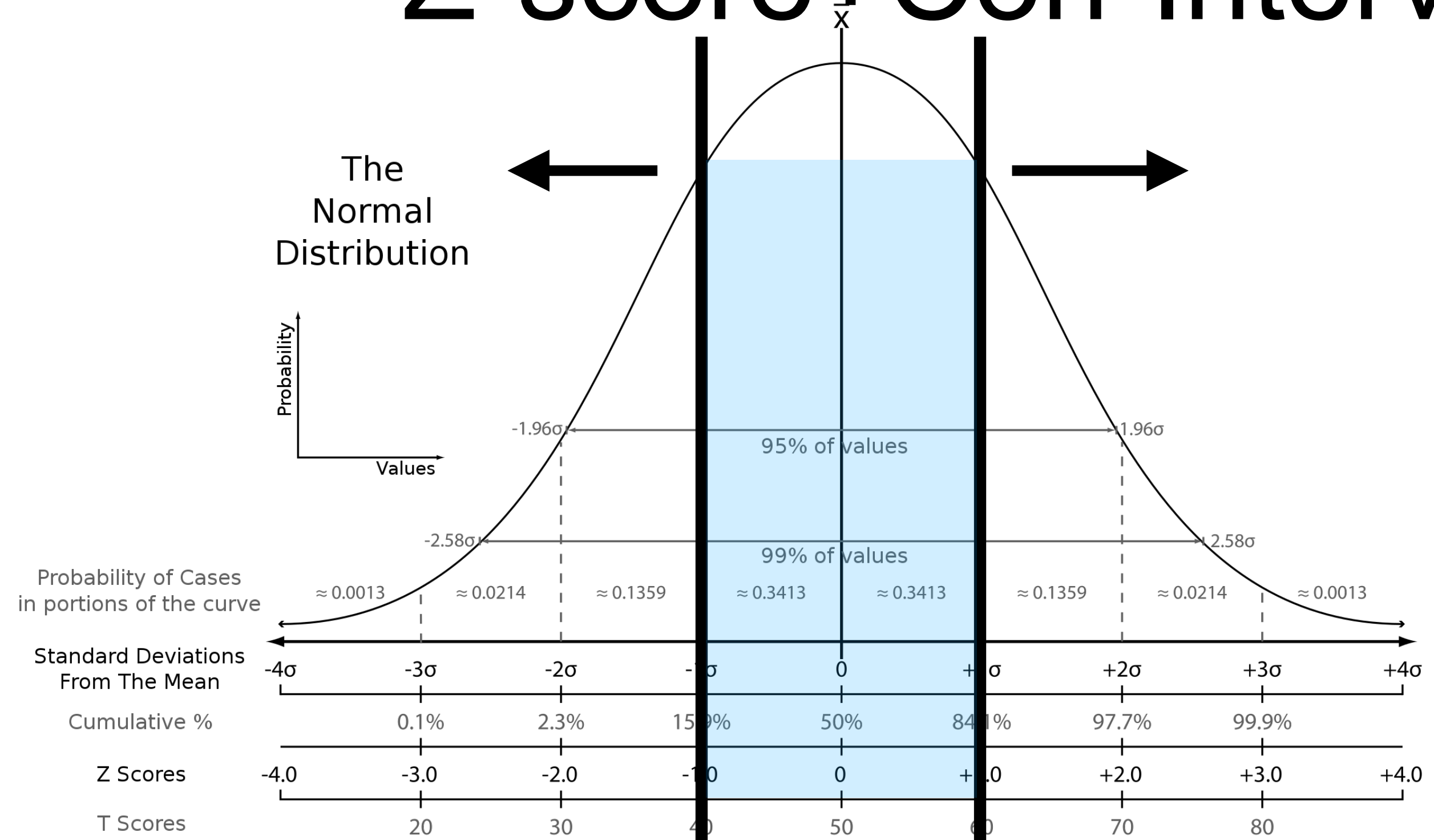
$$\sigma_\theta^2 = \left(\frac{\partial^2 \log L}{\partial \theta^2} \right)^{-1}$$

Confidence Intervals

Z-score+Con-Interval

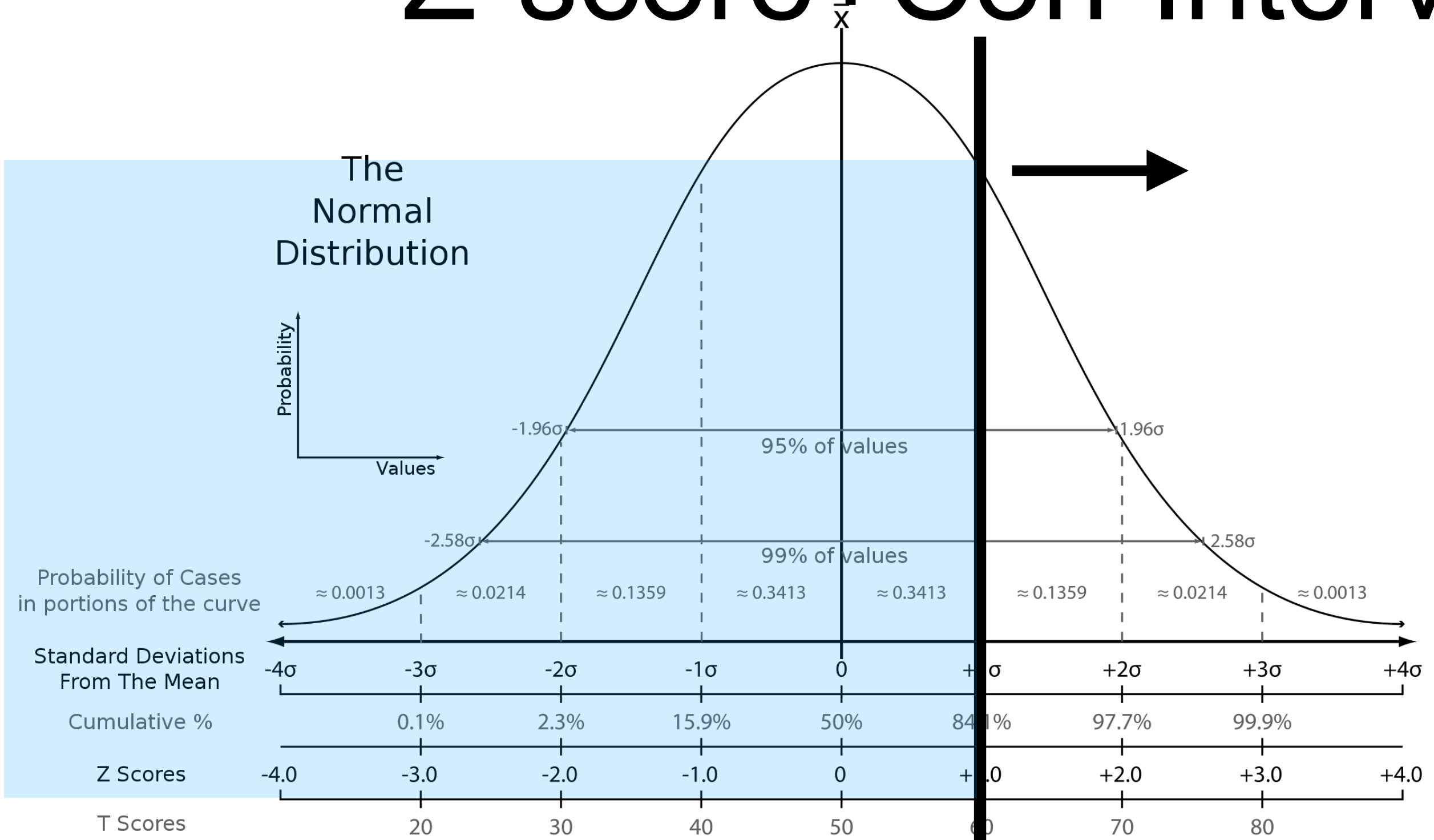


Z-score+Con-Interval



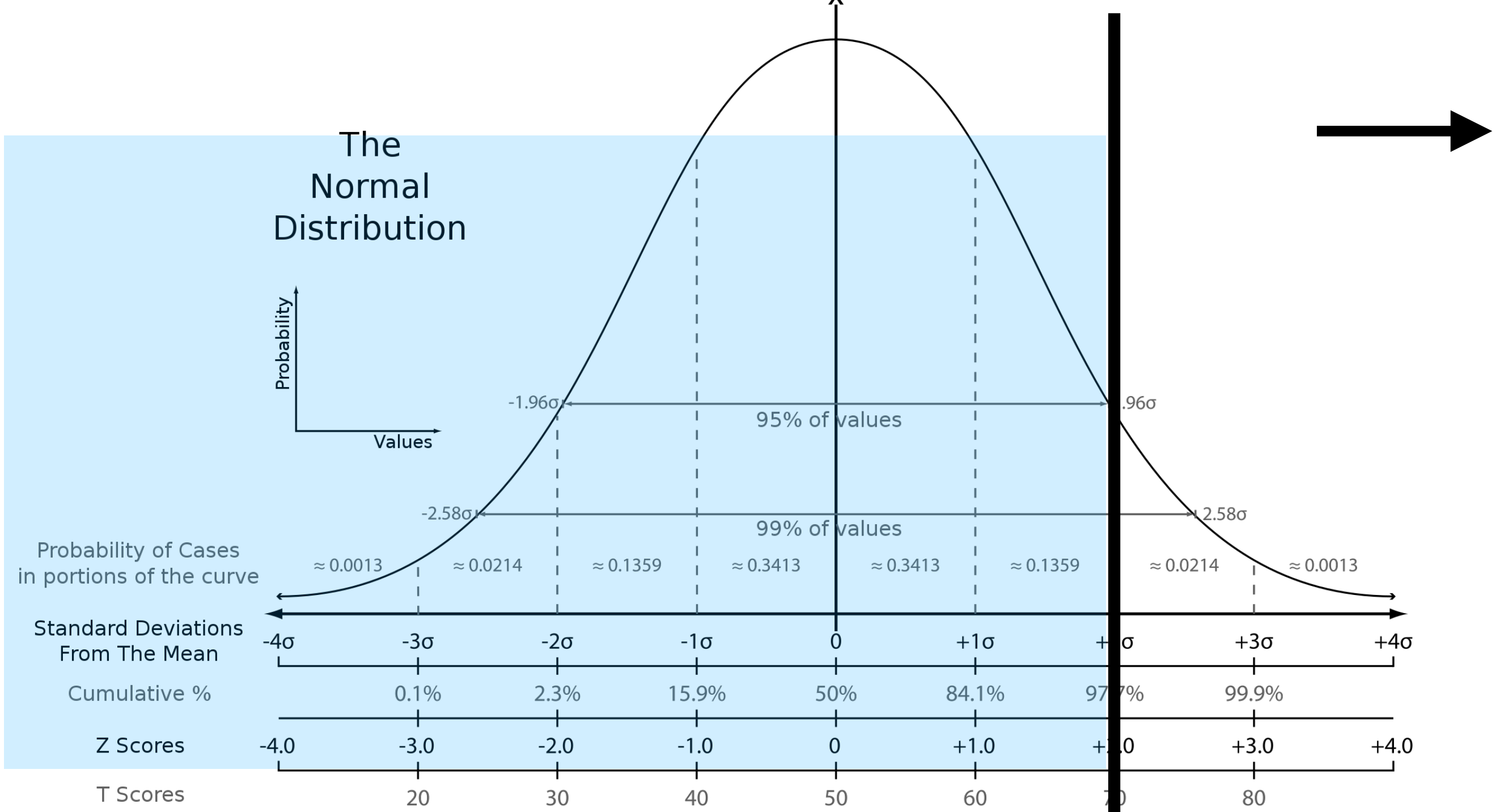
Z-score of 1: 67% chance of being within 1 standard deviation

Z-score+Con-Interval



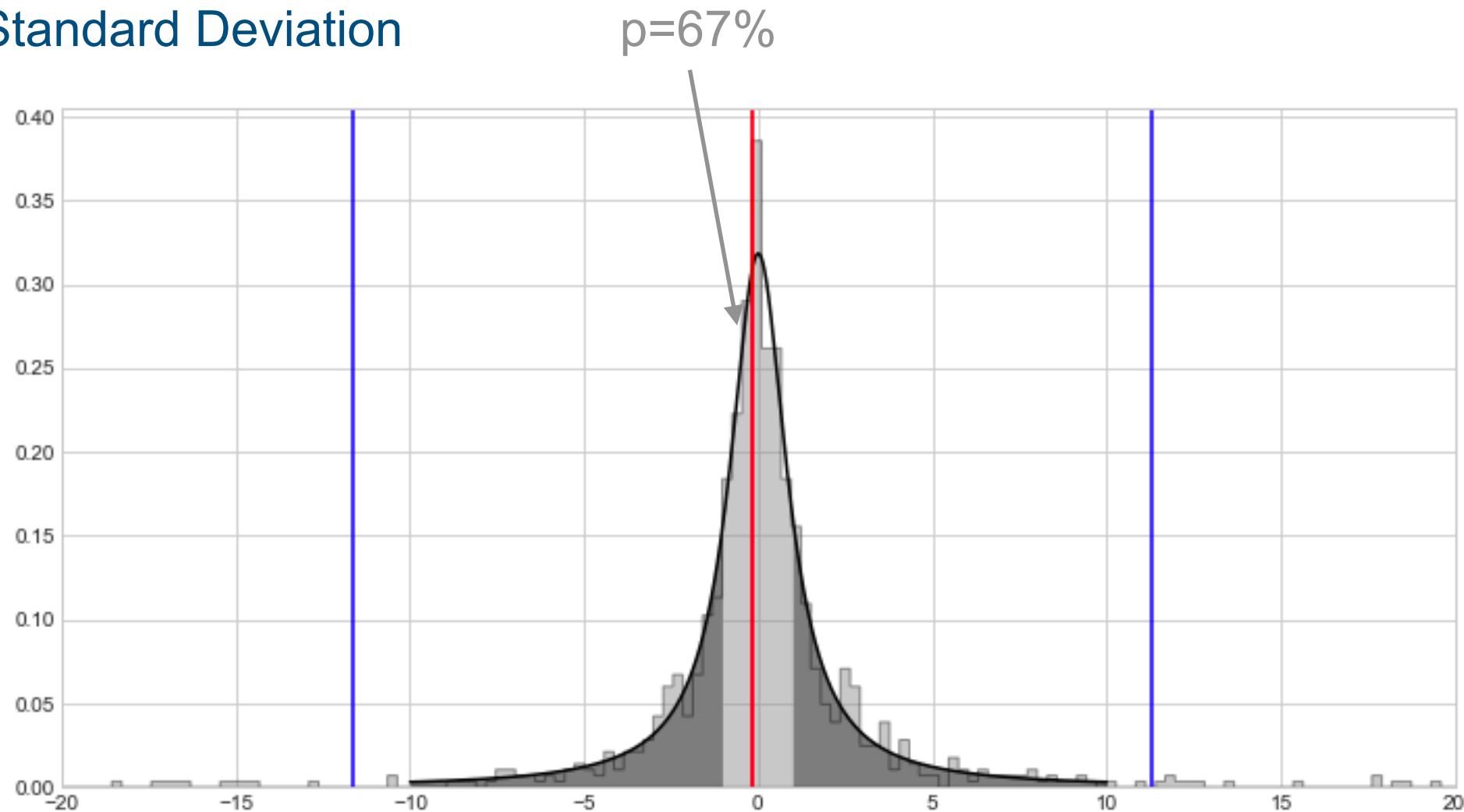


Z-score+Con-Interval



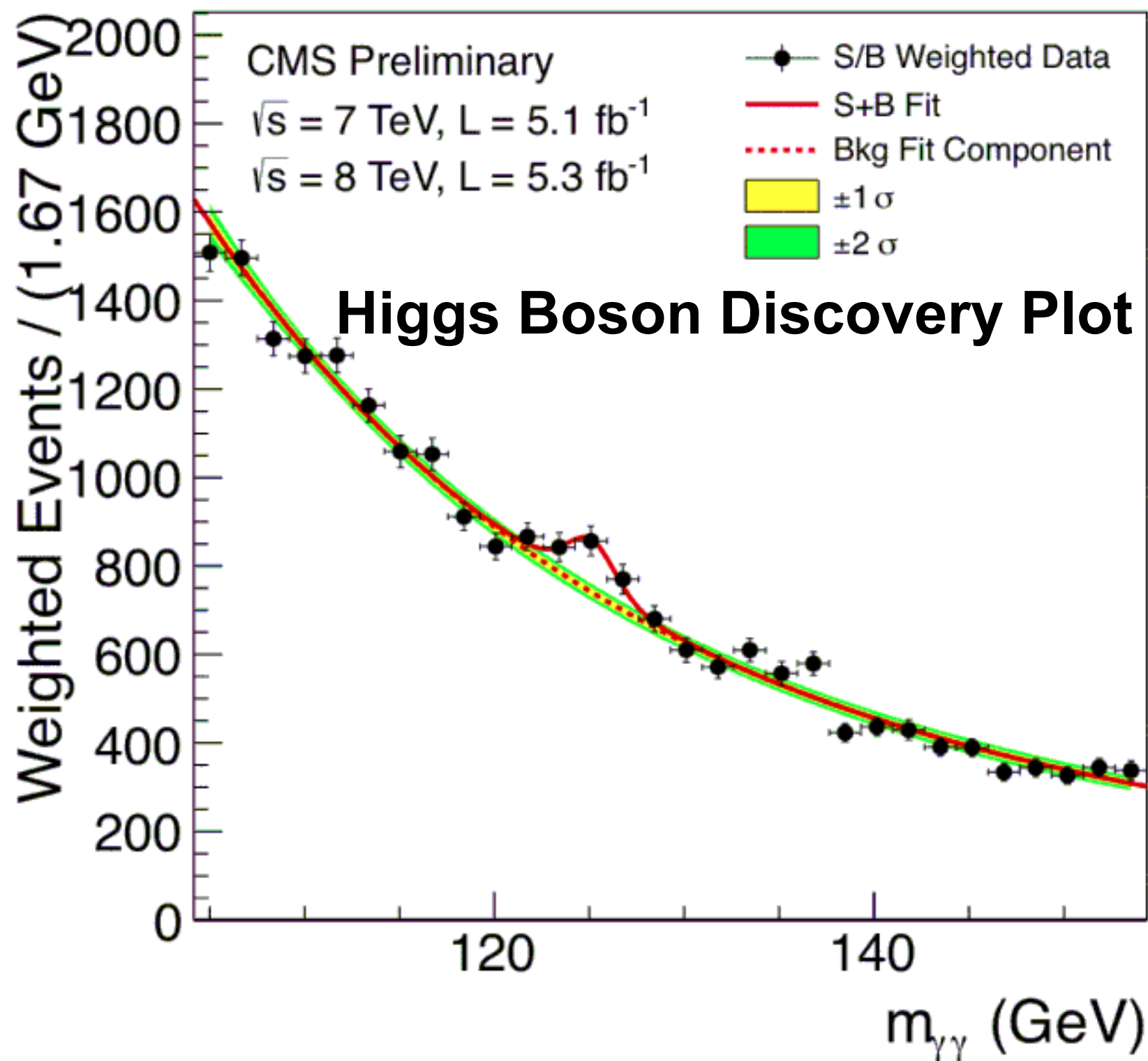
Z-score+Con-Interval

Standard Deviation



- Z score works on any system
- However standard deviation does not necessarily reflect the z-score

Confidence Plots



The rules of significance

- How significant is our measurement? (High Energy physics rules)
- 3 sigma is considered “Evidence”
- 5 sigma is considered “Discovery”

<https://understandinguncertainty.org/explaining-5-sigma-higgs-how-well-did-they-do>



The screenshot shows the homepage of the 'Understanding Uncertainty' website. The header features a yellow 'uu' logo and a navigation bar with links: Home, Blog, Articles, Videos, Animations, Guest Articles, Links, and About Us. The main content area displays a blog post titled 'Explaining 5-sigma for the Higgs: how well did they do?' by david, dated 08/07/2012. A warning note states 'Warning, this is for statistical pedants only.' The post begins with 'To recap, the results on the Higgs are communicated in terms of the numbers of'. On the right side, there is a search bar, a dropdown menu for 'Featured Content', and a 'Main menu' link.

uu

Understanding Uncertainty

Home Blog Articles Videos Animations Guest Articles Links About Us

Home » Blogs » david's blog

Explaining 5-sigma for the Higgs: how well did they do?

Submitted by david on Sun, 08/07/2012 - 1:17pm

Warning, this is for statistical pedants only.

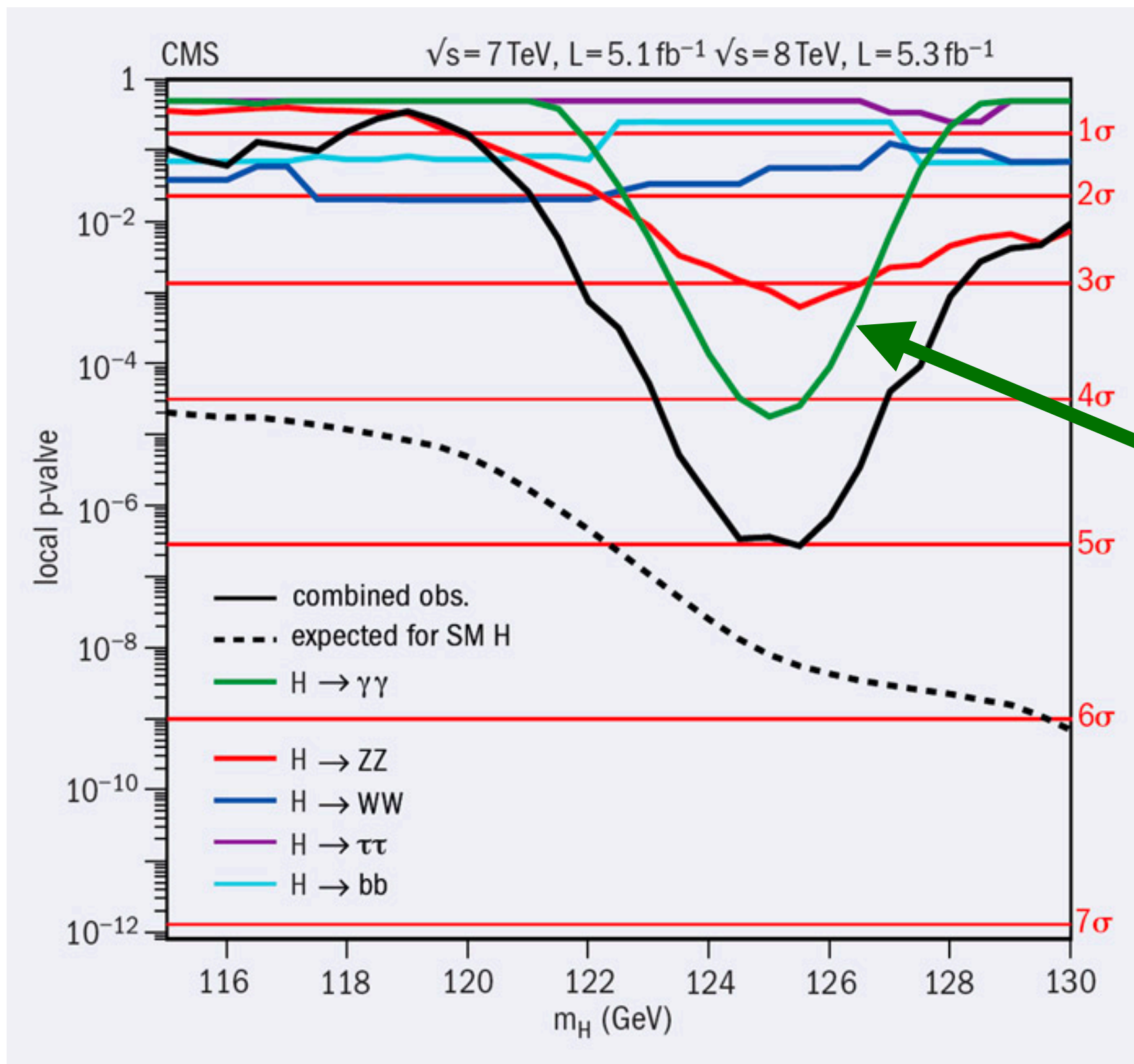
To recap, the results on the Higgs are communicated in terms of the numbers of

Search

- Featured Content -

Main menu

Confidence Plots



Significance

Moments

$$\mu_n = m^n(x) = E[x^n p(x)] = \int_{-\infty}^{\infty} x^n p(x) dx$$

Skewness

The coefficient of Skewness is a measure for the degree of symmetry in the variable distribution.



Negatively skewed distribution
or Skewed to the left
Skewness < 0



Normal distribution
Symmetrical
Skewness = 0



Positively skewed distribution
or Skewed to the right
Skewness > 0

- Moments are a way to characterize the function
- n=1 is mean
- n=2 is variance
- n=3 is Skew
- n=4 is kurtosis

Kurtosis

The coefficient of Kurtosis is a measure for the degree of peakedness/flatness in the variable distribution.



Platykurtic distribution
Low degree of peakedness
Kurtosis < 0



Normal distribution
Mesokurtic distribution
Kurtosis = 0

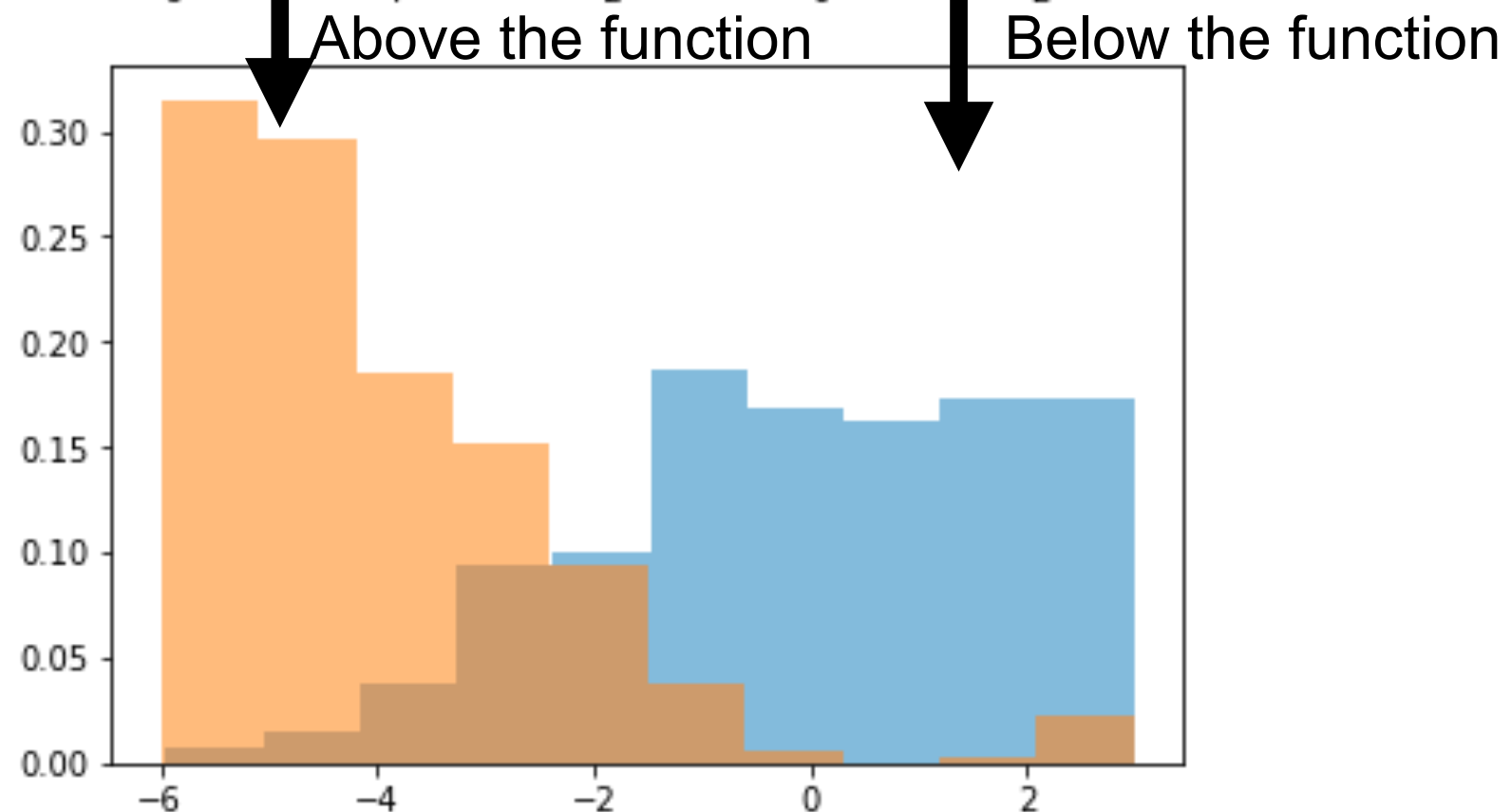
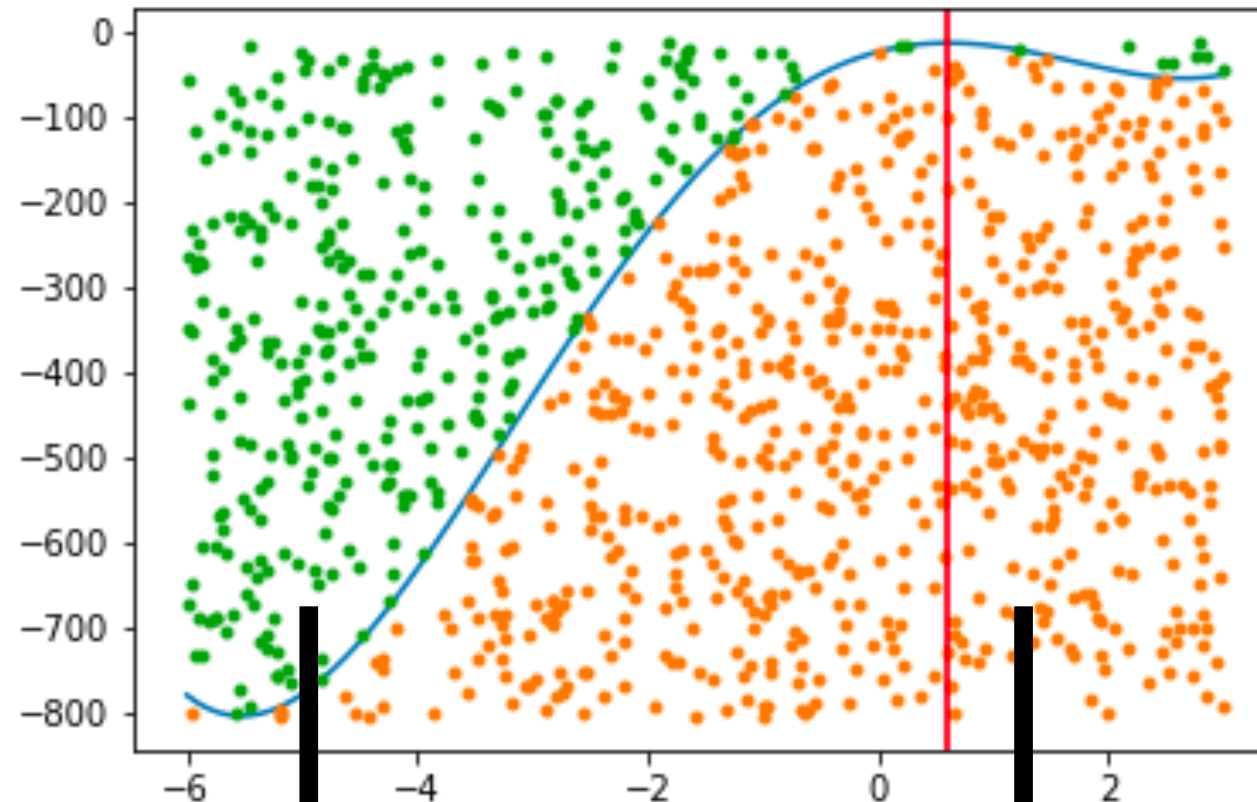


Leptokurtic distribution
High degree of peakedness
Kurtosis > 0

Sampling a Function¹⁶

Uniform Randomly Sample in x

Uniform Randomly Sample in y



Our final expansion Plot

