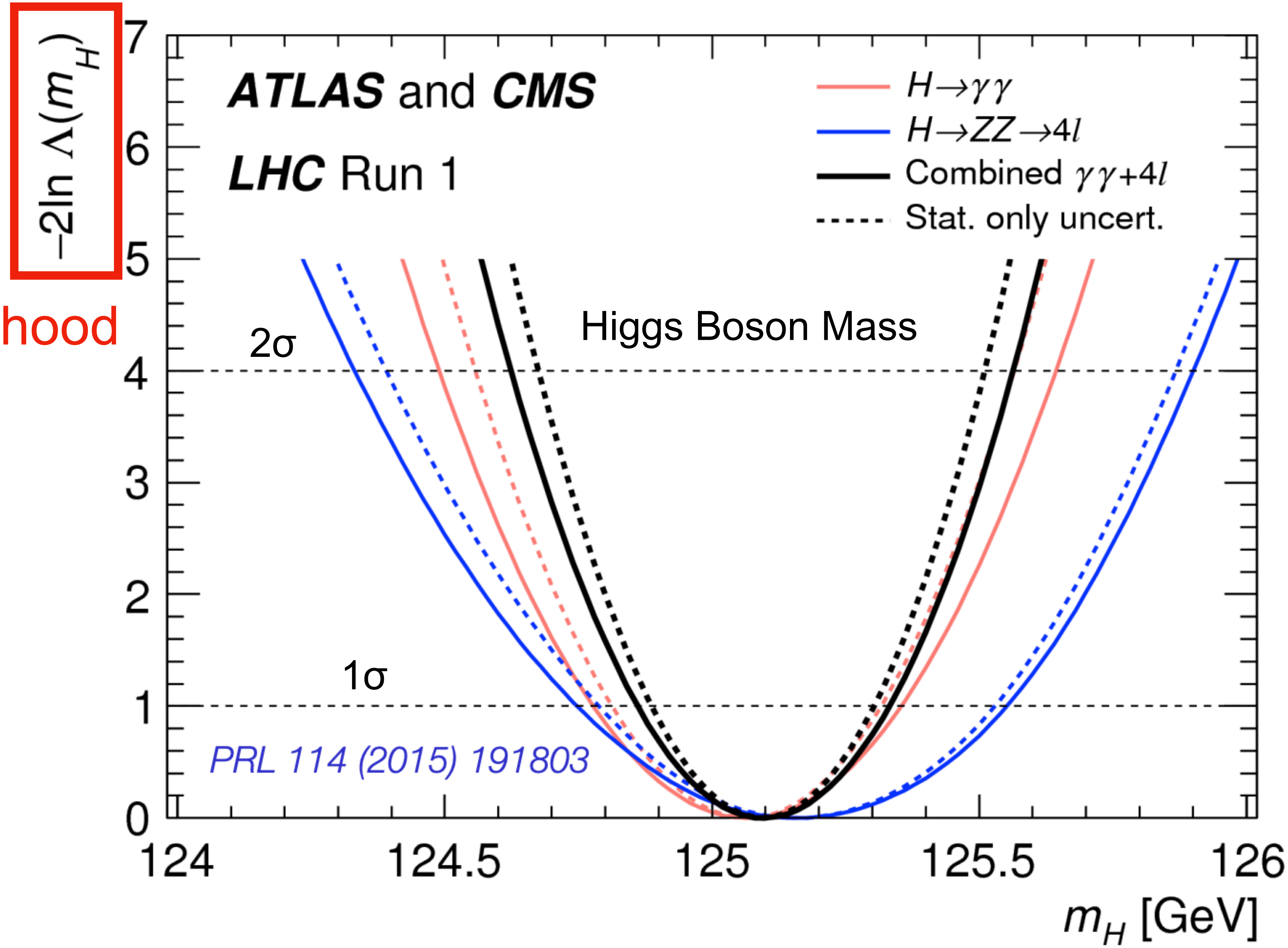
Lecture 5:Confidence

Recap

Making a Measurement



Likelihood

Important Properties

- Taylor expand in our floated parameter (μ in this case)

$$\chi^2(x_i,\mu) = \boxed{\chi^2_{min}(x_i,\mu_0) + \boxed{\frac{1}{2}\frac{\partial^2}{\partial\mu^2}\chi^2_{min}(x_i,\mu_0)}(\mu-\mu_0)^2}$$

$$\frac{1}{2}\frac{d^2\chi^2}{d\mu^2} \rightarrow \frac{1}{\sigma^2}$$

χ^2 distribution of 1 degree of freedom

V[$\chi^2(x)$]=1

$$\boxed{\frac{\partial^2\chi^2}{\partial\theta^2} = \frac{2}{\sigma^2_\theta}}$$

For any floated parameter uncertainty of that parameter is given by the 2nd derivative of χ^2

$$\Delta\chi^2 = 2\Delta\log \mathrm{L} = 1$$

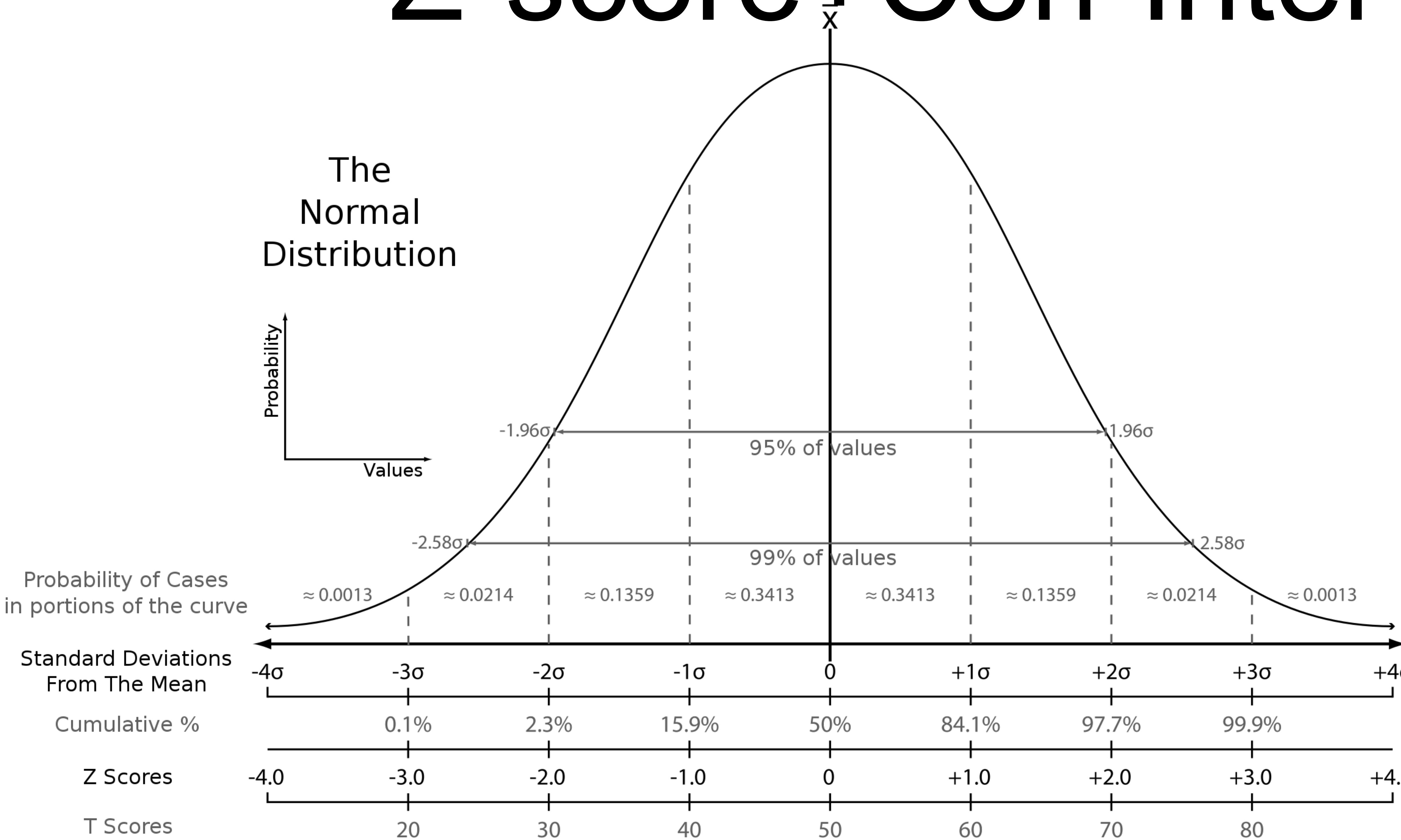
For one degree of freedom

This is known as Wilk's Theorem

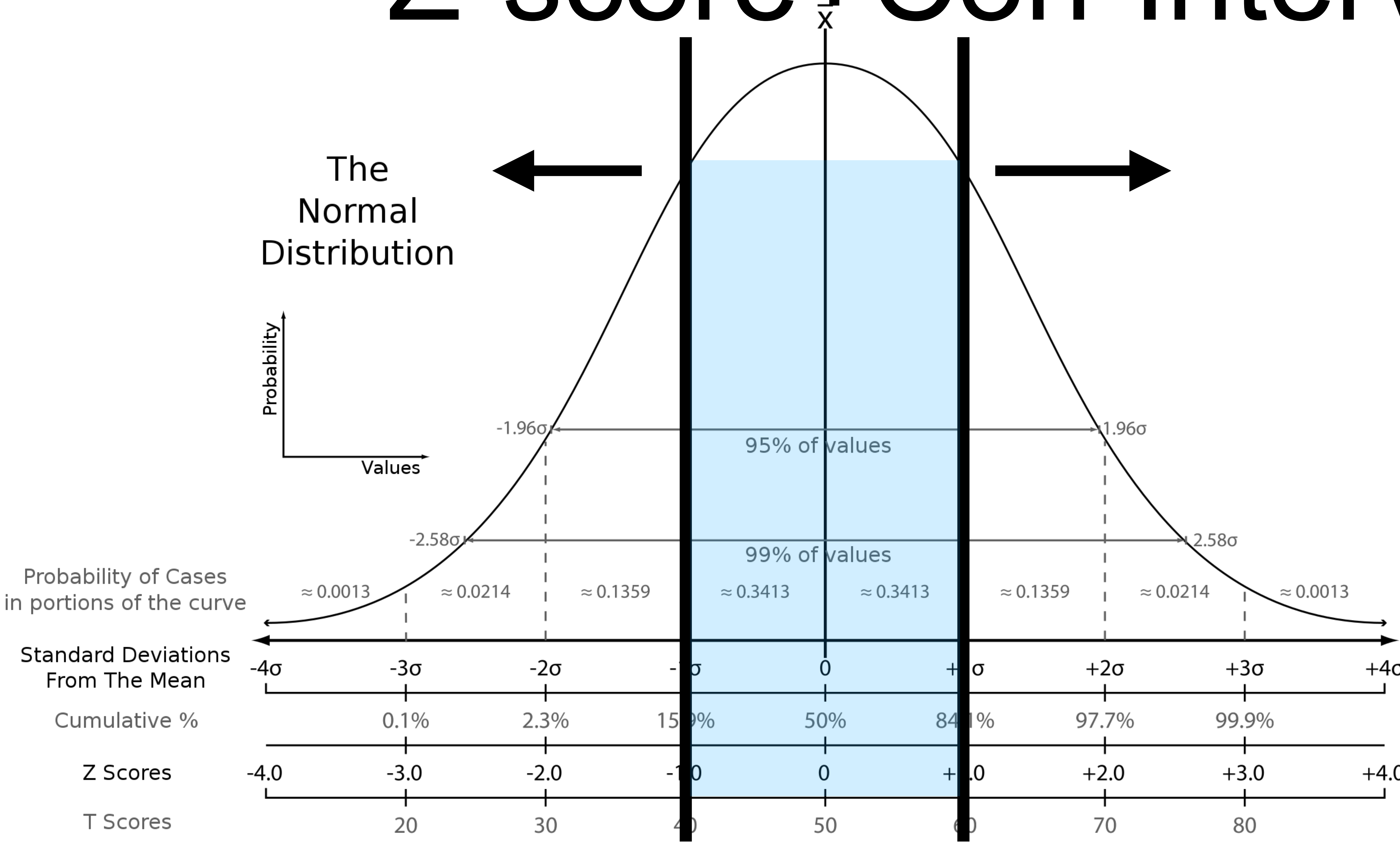
$$\sigma^2_\theta = \left(\frac{\partial^2 \log L}{\partial\theta^2}\right)^{-1}$$

Confidence Intervals

Z-score+Con-Interval

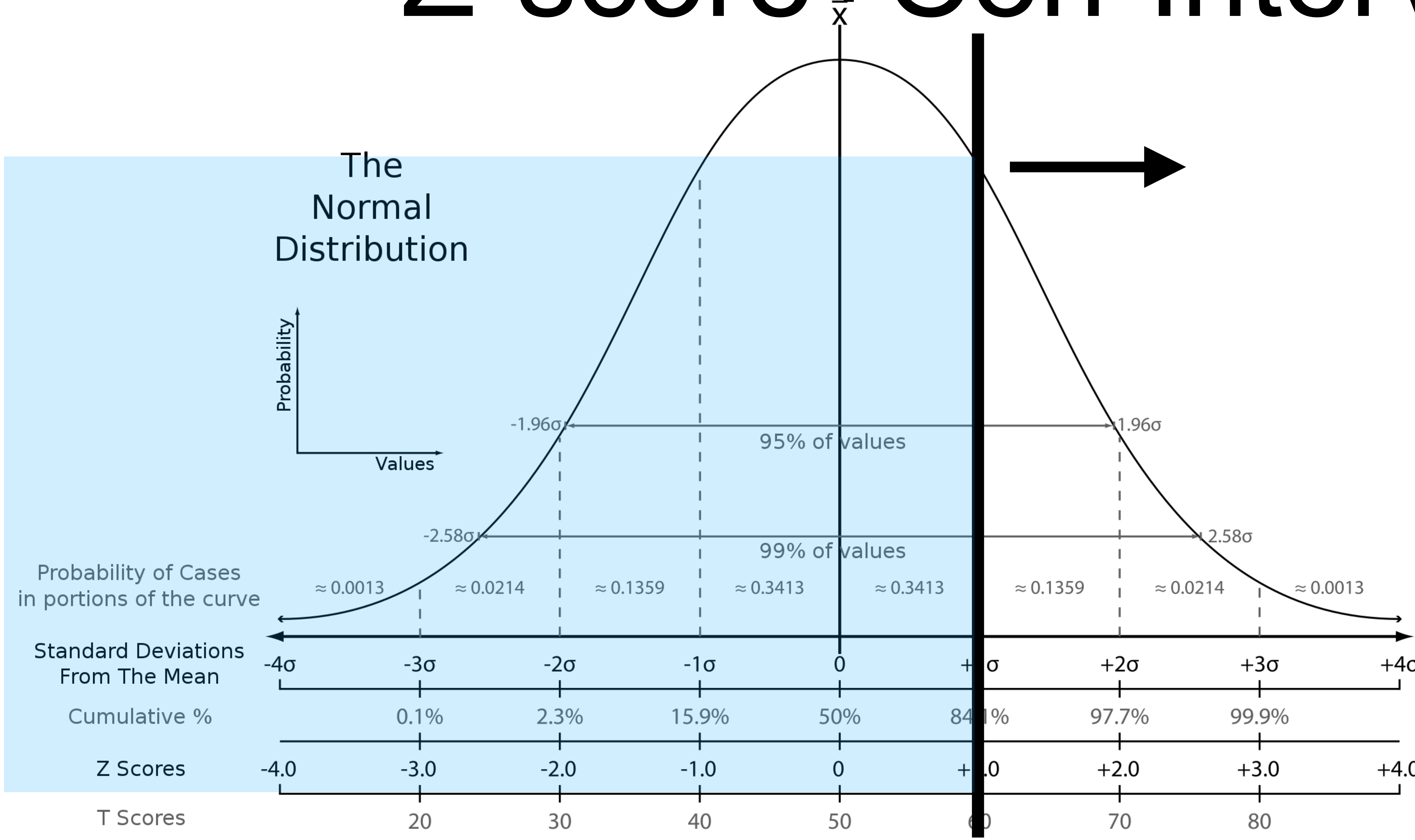


Z-score+Con-Interval



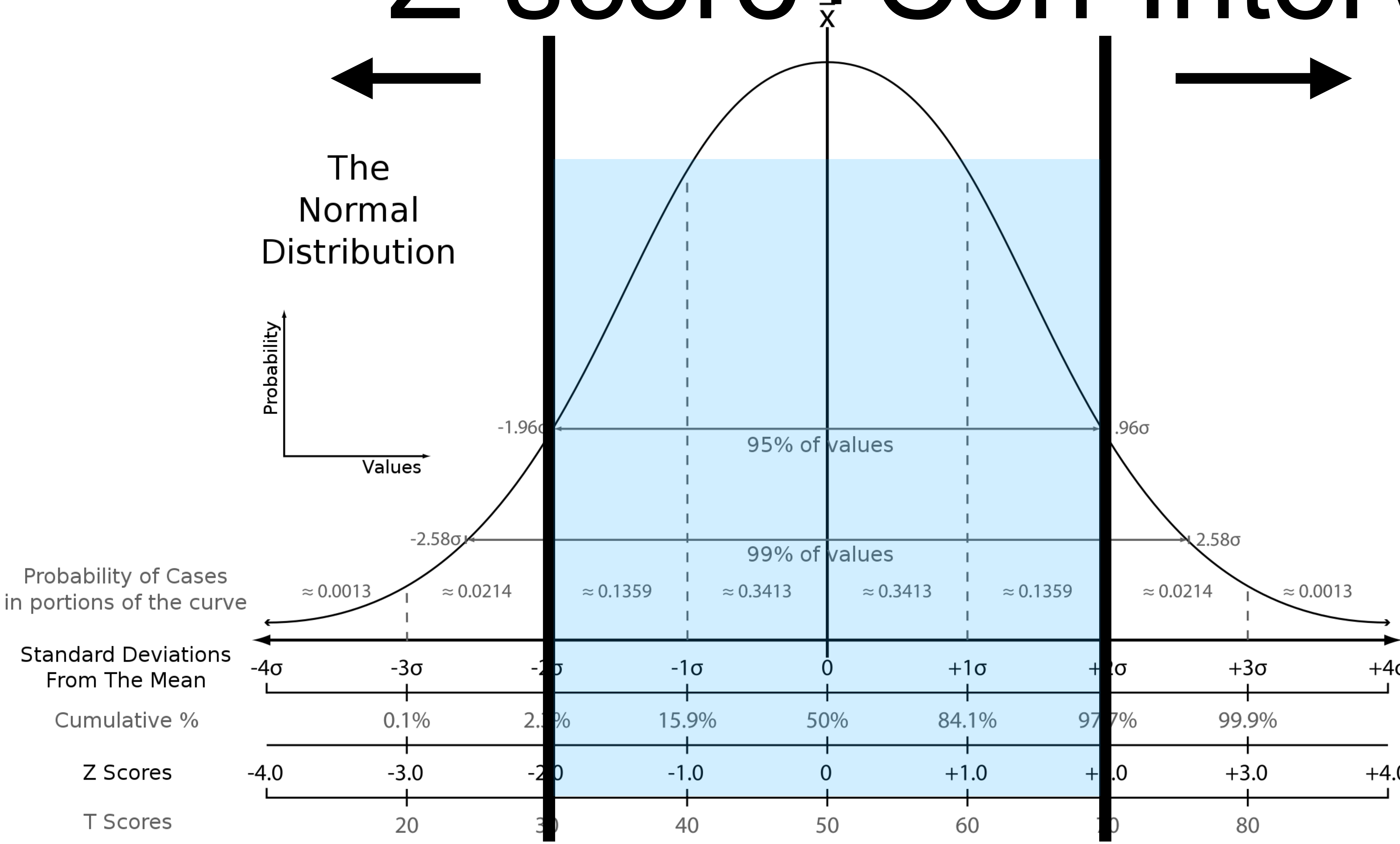
Z-score of 1: 67% chance of being within 1 standard deivation

Z-score+Con-Interval



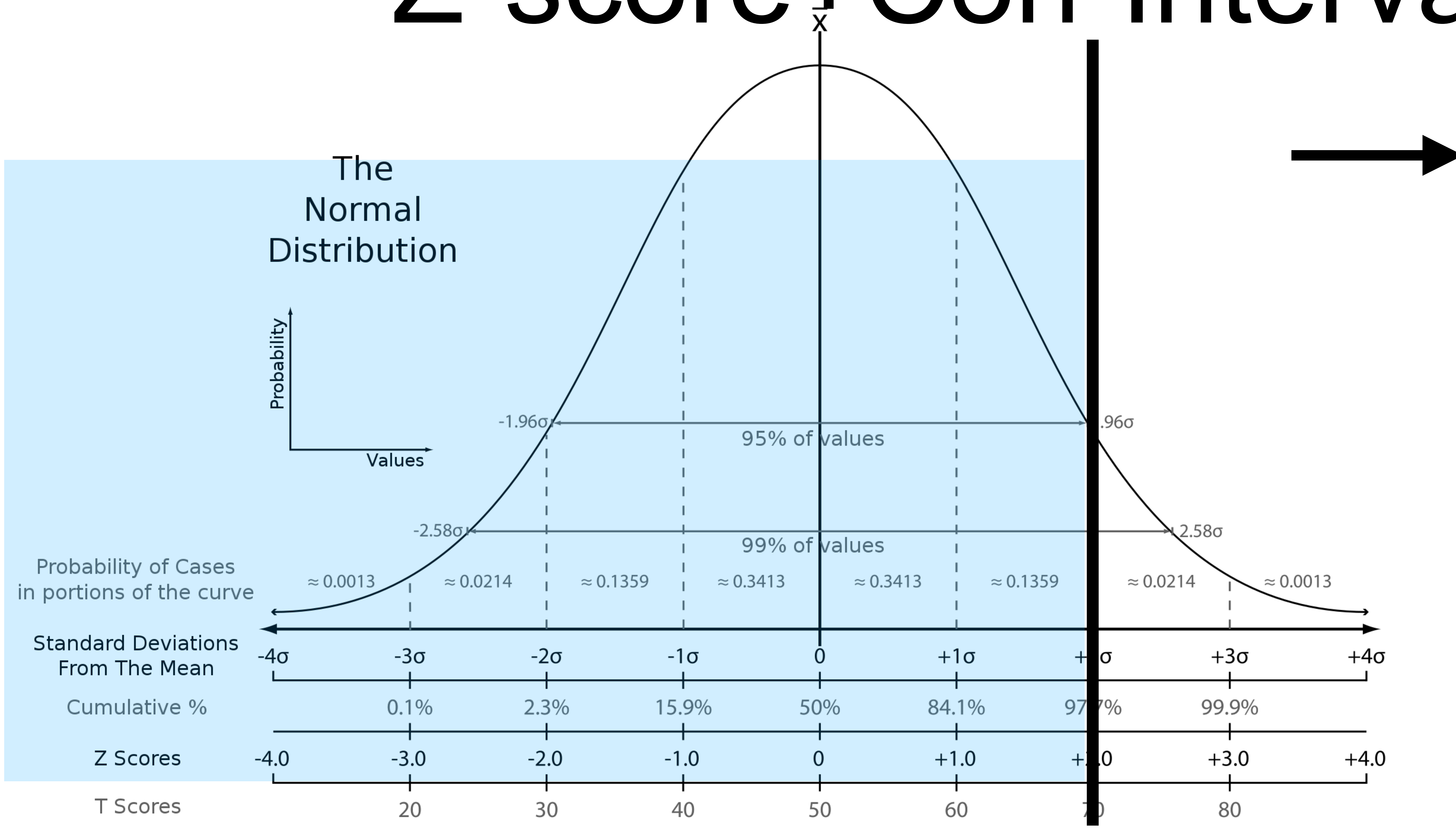
Single sided Z-score of 1: 84% chance of being above 1 standard deviation

Z-score+Con-Interval



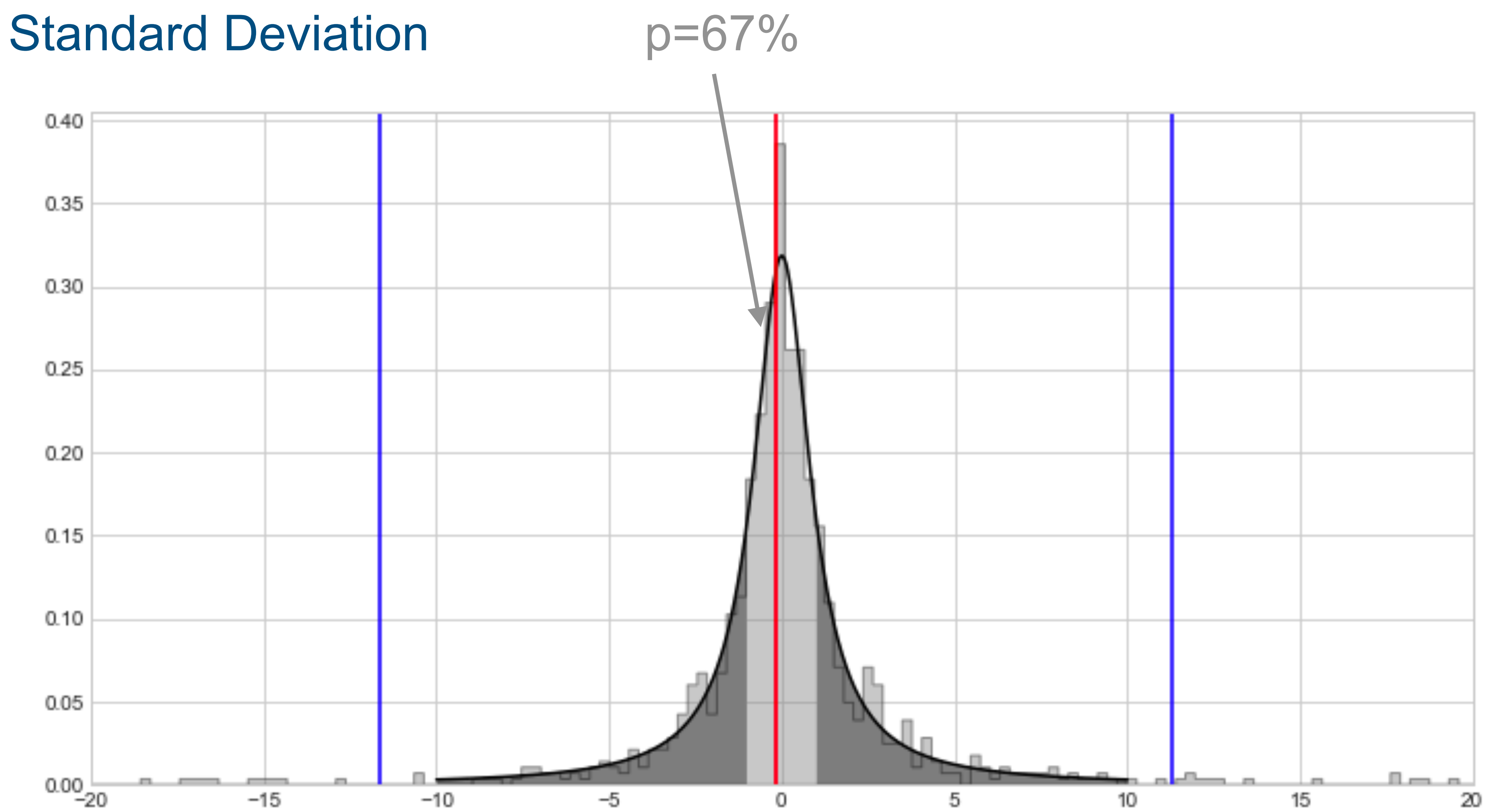
Z-score of 2: 95% chance of being within 1 standard deivation

Z-score+Con-Interval



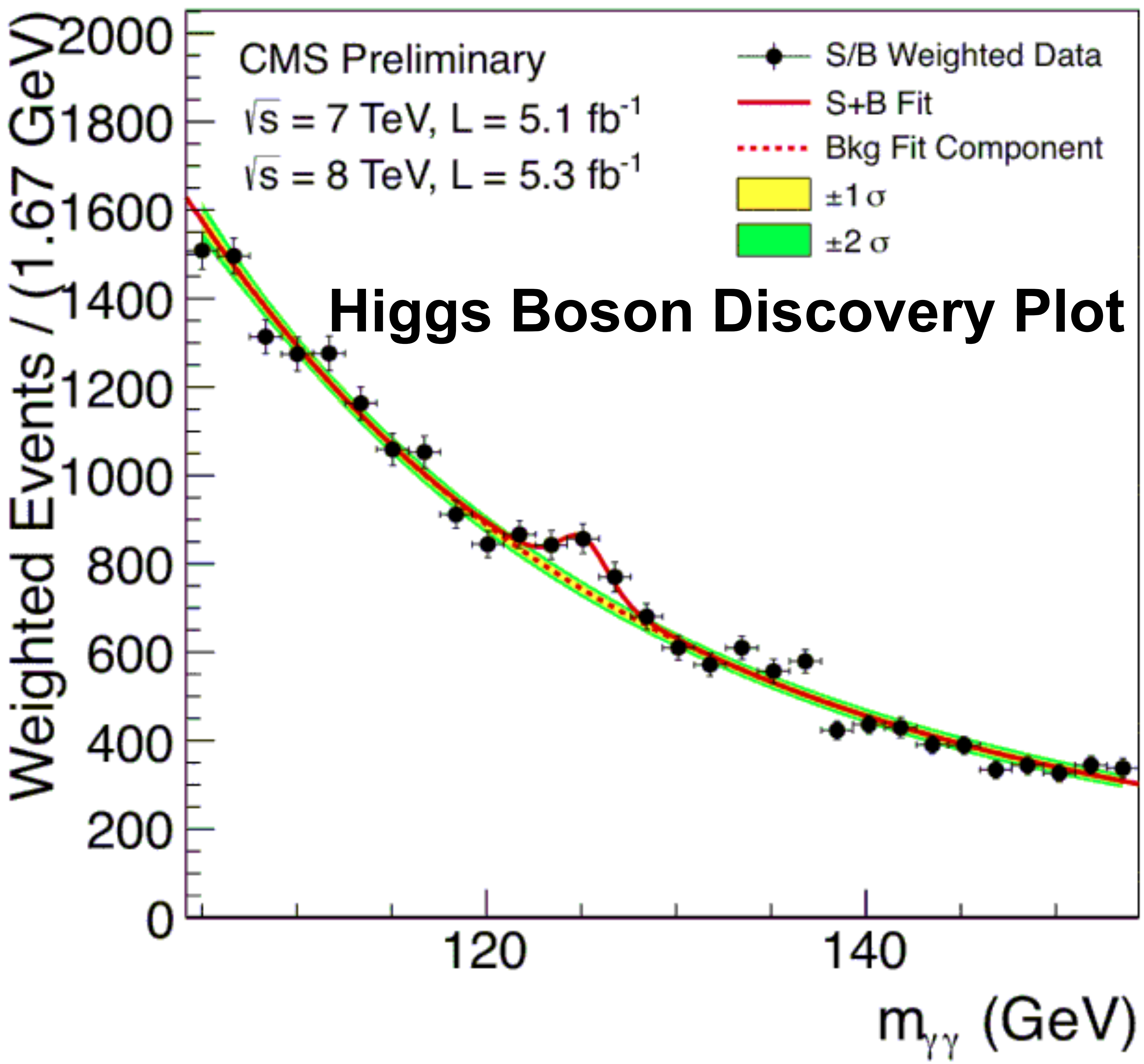
Single sided Z-score of 2: 97% chance of being above 1 standard deviation

Z-score+Con-Interval



- Z score works on any system
- However standard deviation does not necessarily reflect the z-score

Confidence Plots

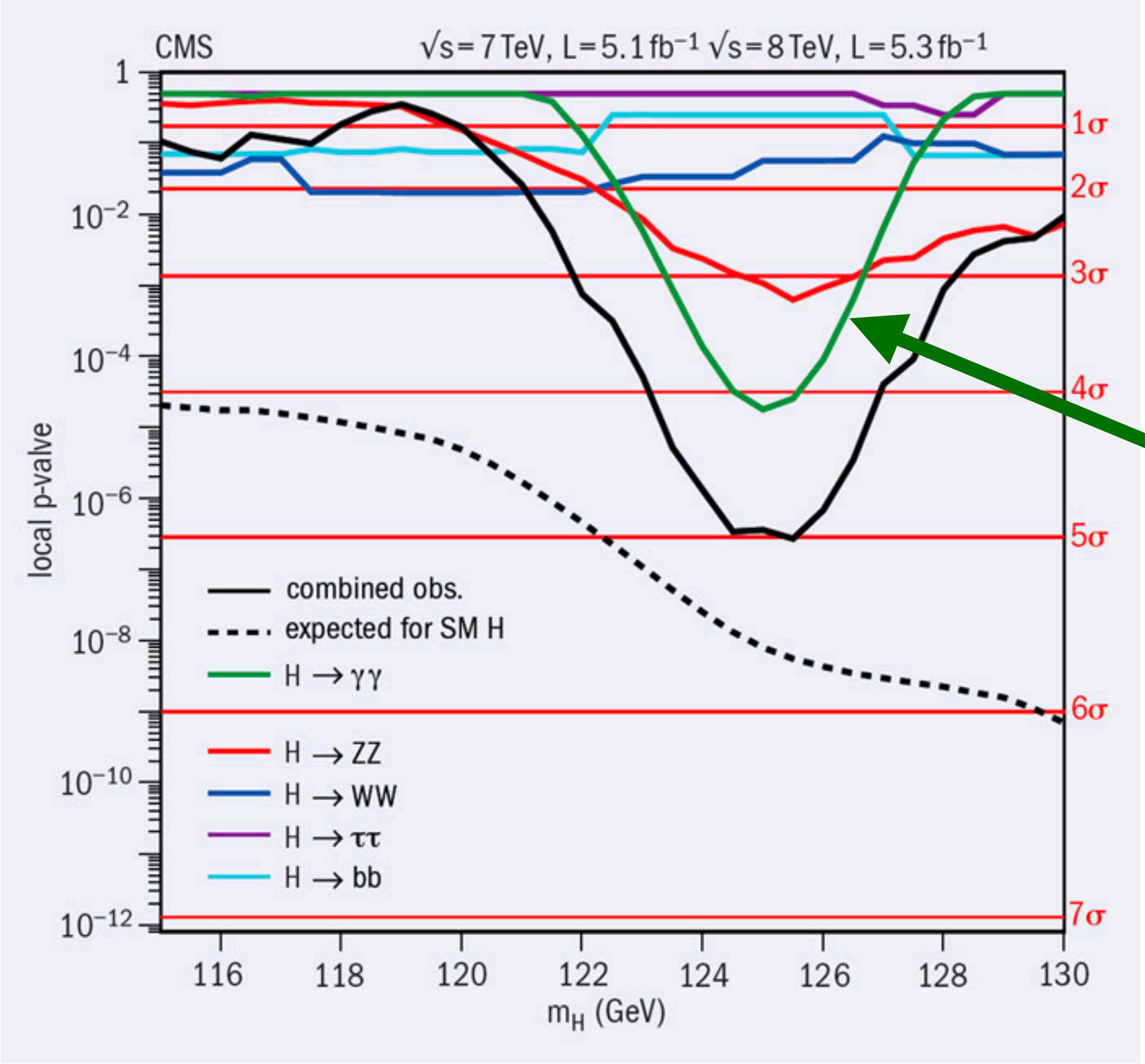


The rules of significance

- How significant is our measurement? (High Energy physics rules)
- 3 sigma is considered “Evidence”
- 5 sigma is considered “Discovery”

https://understandinguncertainty.org/explaining-5-sigma-higgs-how-well-did-they-do

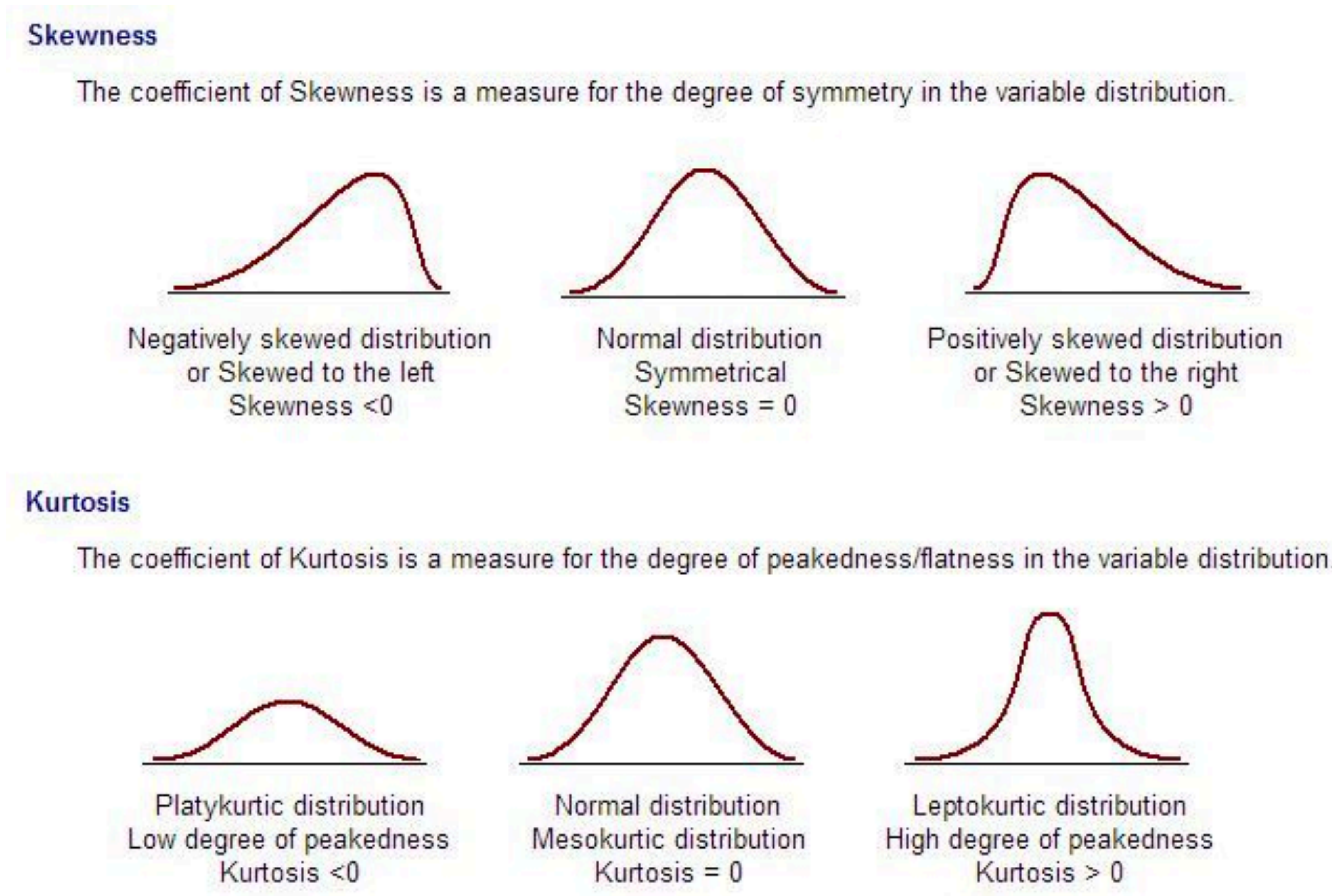
Confidence Plots



Significance

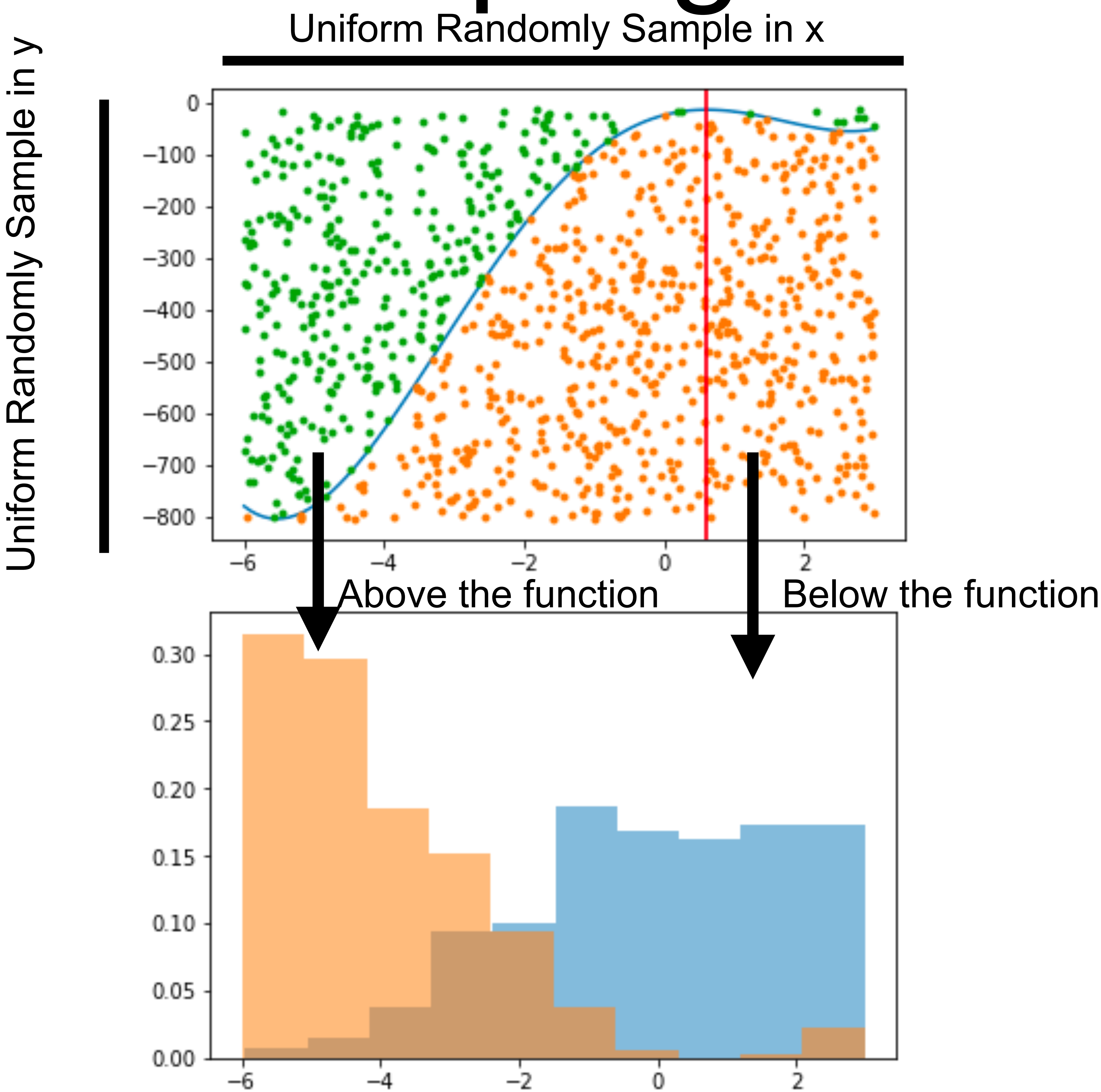
Moments

$$\mu_n = m^n(x) = E[x^n p(x)] = \int_{-\infty}^{\infty} x^n p(x) dx$$



- Moments are a way to characterize the function
- n=1 is mean
- n=2 is variance
- n=3 is Skew
- n=4 is kurtosis

Sampling a Function



Our final expansion Plot