Problem 1.2

(a) ITHINKTHATISHALLNEVERSEEABILLBOARDLOVELYASATREE

(b) LOVEISNOTLOVEWHICHALTERSWHENITALTERATIONFINDS

(c) INBAITINGAMOUSETRAPWITHCHEESEALWAYSLEAVEROOMFORTHEMOUSE

Problem 1.4

Problem 1.5

For simplicity's sake, I'll be using A, B, C, D as the alphabet.

(a) $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ possible substitution ciphers

(b)　(i) For no fixed letters, A can map to B, C, or D, so...

$3 \cdot 2 \cdot 1 = 6$ possible substitution ciphers that have no letters fixed

(ii) For at least one fixed letter, we have 4 choices for the fixed letter, and then we simply deal with a 3-letter alphabet.

$4 \cdot 3 \cdot 2 \cdot 1 = 24$ possible substitution ciphers that have at least one letter fixed

(iii) For only one fixed letter, we have 4 choices for the fixed letter, and then we simply deal with a 3-letter alphabet. without any fixed letters, so...

$4 \cdot 2 \cdot 1 = 8$ possible substitution ciphers that have exactly one letter fixed

(iv) For exactly two fixed letters, we first have $4 \cdot 3$ choices for the fixed letters, and then we simply deal with a 2-letter alphabet.

$4 \cdot 3 \cdot 1 = 12$ possible substitution ciphers that have exactly two letters fixed

Problem 1.9

(a) gcd(291, 252)

$$291 = 252(1) + 39$$
$$252 = 39(6) + 18$$
$$39 = 6(6) + \mathbf{3}$$
$$6 = 6(1) + 0$$
$$\rightarrow 3$$

(b) $\gcd(16261,\,86562)$

$$86562 = 16261(5) + 5257$$
$$16261 = 5257(3) + 490$$
$$5257 = 490(10) + 357$$
$$490 = 357(1) + 133$$
$$357 = 133(2) + 91$$
$$133 = 91(1) + 42$$
$$91 = 42(2) + \mathbf{7}$$
$$42 = 7(7) + 0$$
$$\rightarrow 7$$

Problem 1.17

(a) $347 + 513 = 860 \equiv 97 \pmod{763}$

(b) $3264 + 1238 + 7231 + 6437 = 18170 \equiv 8916 \pmod{9254}$

(c) $153 \cdot 287 = 43851 \equiv 79 \pmod{353}$

(d) $357 \cdot 862 \cdot 193 = 59392662 \equiv 1545 \pmod{8157}$

(e) $5327 \cdot 6135 \cdot 7139 \cdot 2187 \cdot 5219 \cdot 1873 = 4.06854 \times 10^{23} \equiv 603 \pmod{8157}$

(f) $137^2 = 18769 \equiv 137 \pmod{327}$

(g) $373^6 = 2693103168443689 \equiv 463 \pmod{581}$

(h) $23^3 \cdot 19^5 \cdot 11^4 = 441084963939653 \equiv 93$

Problem 1.26