



SCUTTLE Robot Kinematics Guide

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This guide covers:

- Robot geometry, r, and L
- Key variables: phi, x, y, and theta
- Kinematic equation: convert wheel speeds to chassis speeds
- Inverse Kinematic equation: convert chassis speeds to wheel speeds
- Time-derivatives of the wheel and chassis displacements
- Rotation matrix to convert body-fixed coordinates to global coordinates

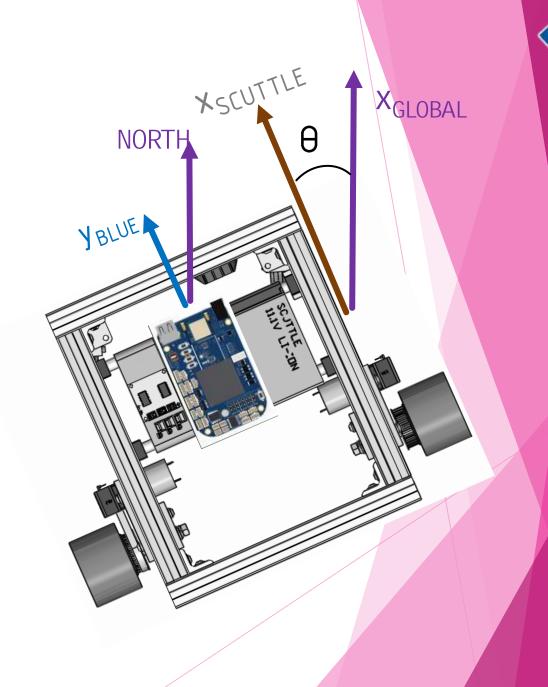
SCUTTLE Coordinates

This slide is for reference.

The beagle circuit board has an IMU with y-axis pointing along the USB port.

The global x- is often decided to be aligned with magnetic North.

The SCUTTLE body-fixed X vector is aligned with the forward direction.



SCUTTLE: a Non-Holonomic System



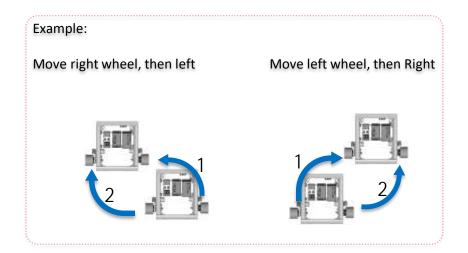
A holonomic robot has the same number (or more) of controllable degrees of freedom as the number of degrees of freedom.

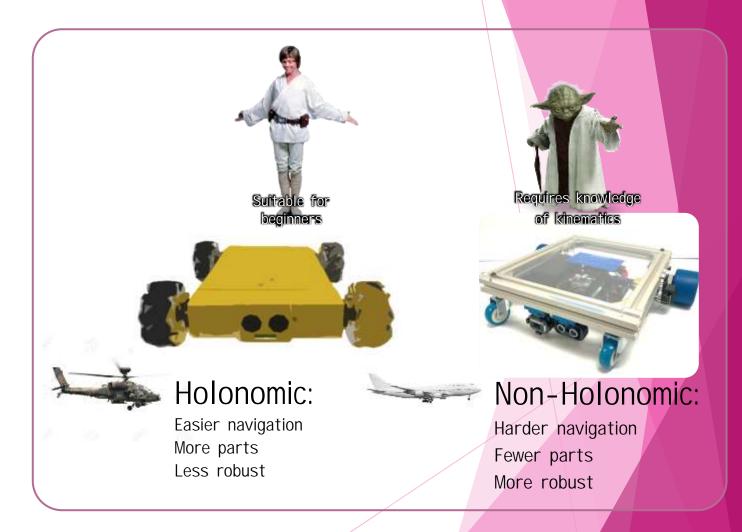
SCUTTLE DOF: (x, y, theta)

CONTROLLABLE DOF: (left motor, right motor)

Mecanum Robot DOF: (x,y,theta)

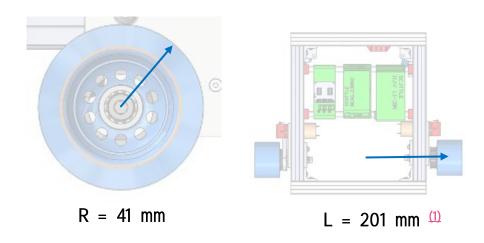
In a non-holonomic system, the final position of the robot depends on the path taken to achieve the movement.

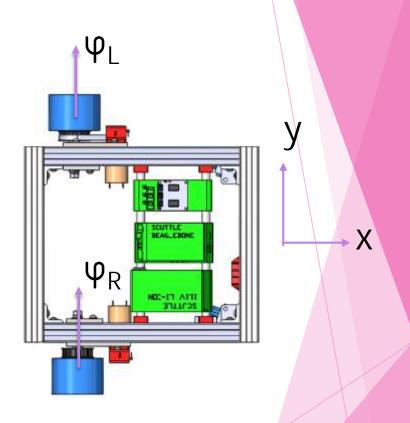




Questions: Info@scuttlerobotorg

- The Chassis Geometry determines the equations for kinematics.
- The radius, r, is the radius of the driven wheel
- The half-wheelbase⁽¹⁾, L, is the space from wheel center to center divided in two

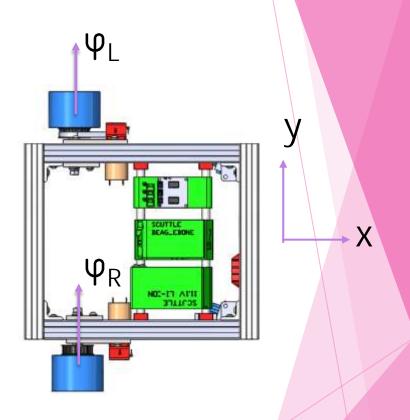




• **Phi dot** is the derivative of phi with respect to time.

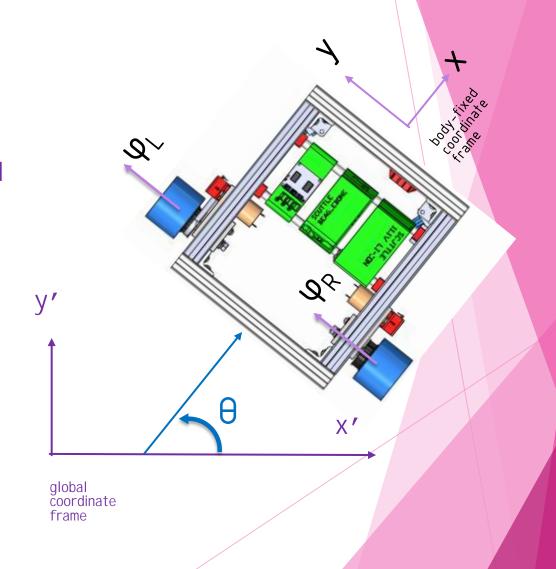
```
\dot{\phi}_L = pdI, as in phi_dot_I
\dot{\phi}_R = pdr, as in phi_dot_r
```

phiDots = np.array([pdl, pdr]) # python
syntax



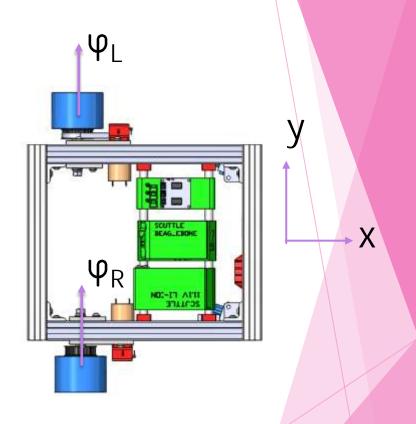
- Theta describes the difference between the body-fixed frame and the global frame.
- The **rotation matrix** converts body-fixed coordinates to the global coordinates

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x_{bf} \\ y_{bf} \end{bmatrix}$$



- Phi is the angle of the wheel.
 - It is used to define incremental changes in wheel position and to calculate wheel speeds
- The x,y coordinate system has x pointing forward on the bot.
 - Positive movement of both phi's result in positive movement of the robot along the x-direction
- The Kinematic Equation generates chassis motion information.
 - input the wheel speeds and output the (translational and rotational) chassis speeds

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} R/2 & R/2 \\ -R/2L & R/2L \end{bmatrix} \begin{bmatrix} \dot{\varphi}_L \\ \dot{\varphi}_R \end{bmatrix}$$

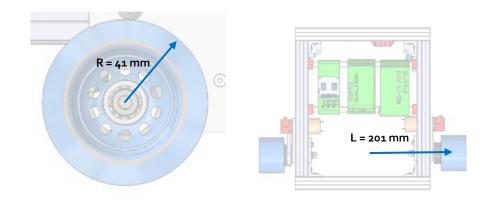


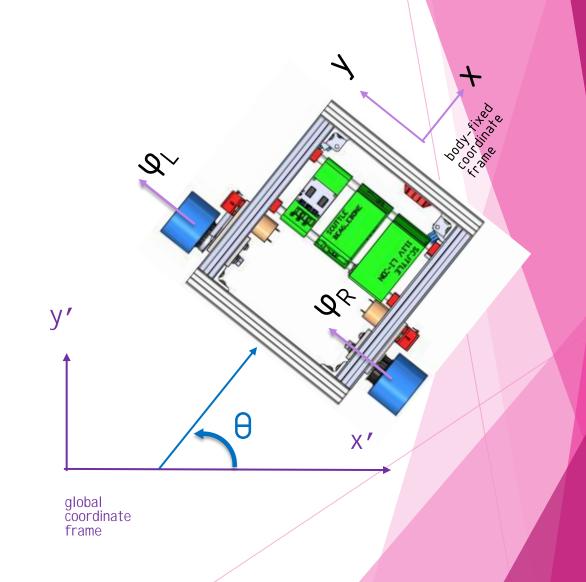


- Input the desired speed and angular speed, and output the left and right wheel speeds.
- These equations are written in the robotfixed frame

$$\begin{bmatrix} \dot{\varphi}_L \\ \dot{\varphi}_R \end{bmatrix} = \begin{bmatrix} 1/R & -L/R \\ 1/R & L/R \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix}$$

matrix multiplication: [C] = [A][B]







Comparing forward and inverse kinematics:

"Kinematics"
Use the **wheel** speeds to obtain the **chassis** speeds

"Inverse Kinematics"
Use the **chassis** speeds to obtain the **wheel** speeds

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} R/2 & R/2 \\ -R/2L & R/2L \end{bmatrix} \begin{bmatrix} \dot{\varphi}_L \\ \dot{\varphi}_R \end{bmatrix}$$

$$\begin{bmatrix} \dot{\varphi}_L \\ \dot{\varphi}_R \end{bmatrix} = \begin{bmatrix} 1/R & -L/R \\ 1/R & L/R \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix}$$



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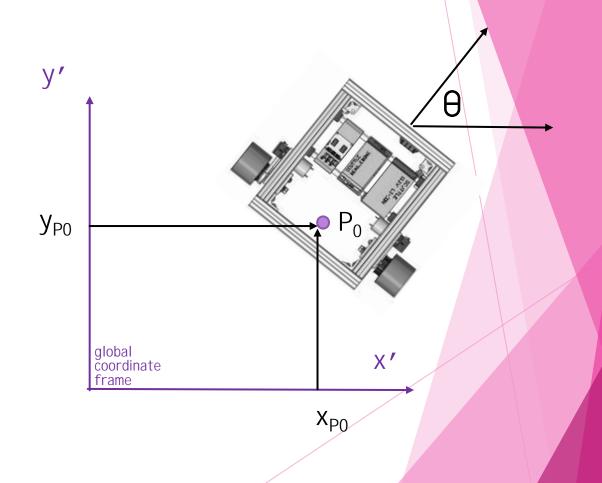
SCUTTLE Kinematics (prt 2)

This section will be expanded to discuss navigation

Describing the robot in the inertial (global) frame:

 P0 describes the location of the robot and is defined by the center of the wheelbase.

$$q^{I} = \begin{bmatrix} x_{a} \\ y_{a} \\ \theta \end{bmatrix}$$



Kinematics Footnotes

- 1) The true wheelbase is determined by the contact patch of the wheels on the ground.
 - Most wheels have a draw angle from the manufacturing die, resulting in a larger inner diameter.
 - Then, the contact patch may lie closer to the robot center, especially with no payload.
 - The inner-most contact patch yields a 355mm wheelbase.
 - The wheelbase may change when a heavy load compresses the urethane.

Wheel spec: 83mm diam

