SCUTTLE Robot Kinematics Guide

revised 2021.11.17

SCUTTLE Kinematics

This guide covers:

- Robot geometry, r, and L
- Key variables: phi, x, y, and theta
- Kinematic equation: convert wheel speeds to chassis speeds
- Inverse Kinematic equation: convert chassis speeds to wheel speeds
- Time-derivatives of the wheel and chassis displacements
- Rotation matrix to convert body-fixed coordinates to global coordinates

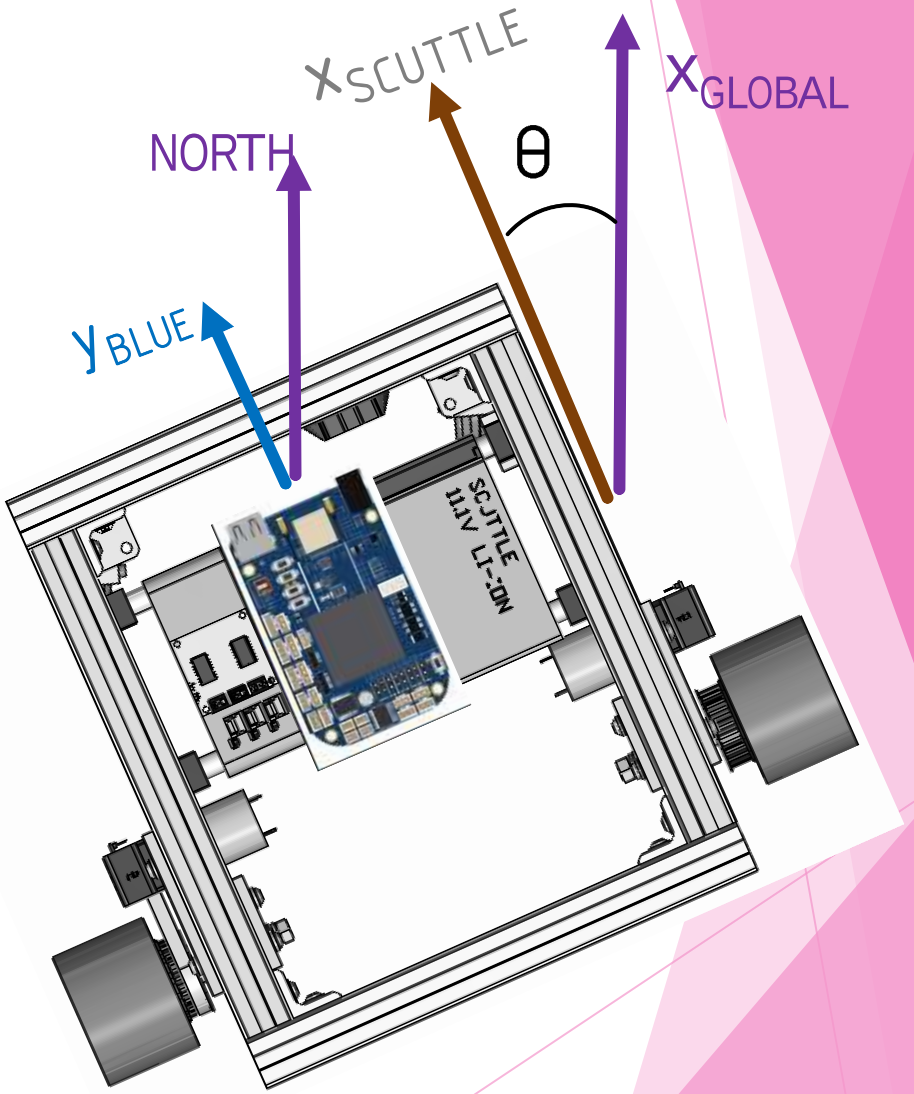
SCUTTLE Coordinates

This slide is for reference.

The beagle circuit board has an IMU with y-axis pointing along the USB port.

The global x- is often decided to be aligned with magnetic North.

The SCUTTLE body-fixed X vector is aligned with the forward direction.



Questions: Info@scuttlerobot.org

SCUTTLE: a Non-Holonomic System

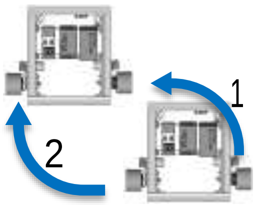
A holonomic robot has the same number (or more) of controllable degrees of freedom as the number of degrees of freedom.

SCUTTLE DOF: (x, y, theta)
CONTROLLABLE DOF: (left motor, right motor)
Mecanum Robot DOF: (x,y,theta)

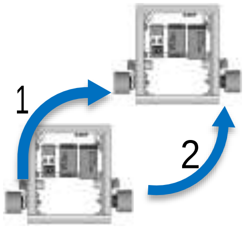
In a non-holonomic system, the final position of the robot depends on the path taken to achieve the movement.

Example:

Move right wheel, then left

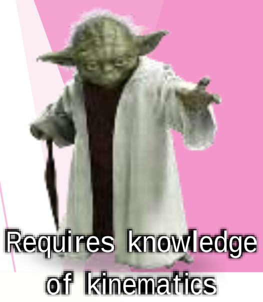


Move left wheel, then Right



Holonomic:

Easier navigation
More parts
Less robust



Non-Holonomic:

Harder navigation
Fewer parts
More robust

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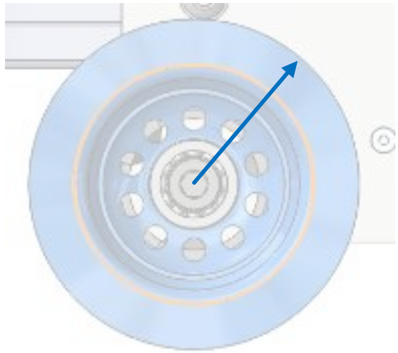
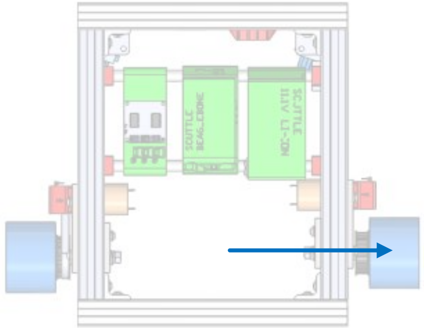
SCUTTLE Kinematics

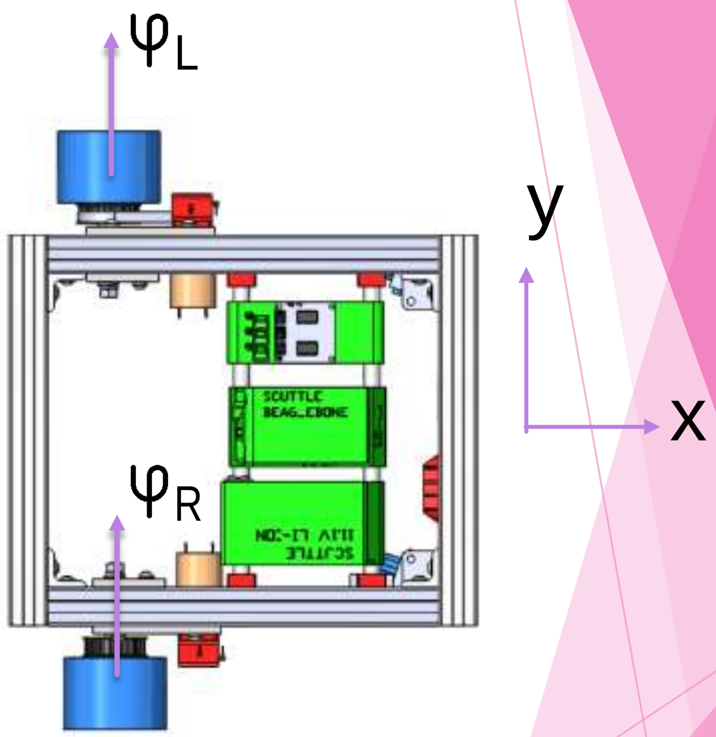
- The **Chassis Geometry** determines the equations for kinematics.
- The **radius**, r, is the radius of the driven wheel
- The **half-wheelbase**[1], L, is the space from wheel center to center divided in two



R = 41 mm



L = 201 mm [1]



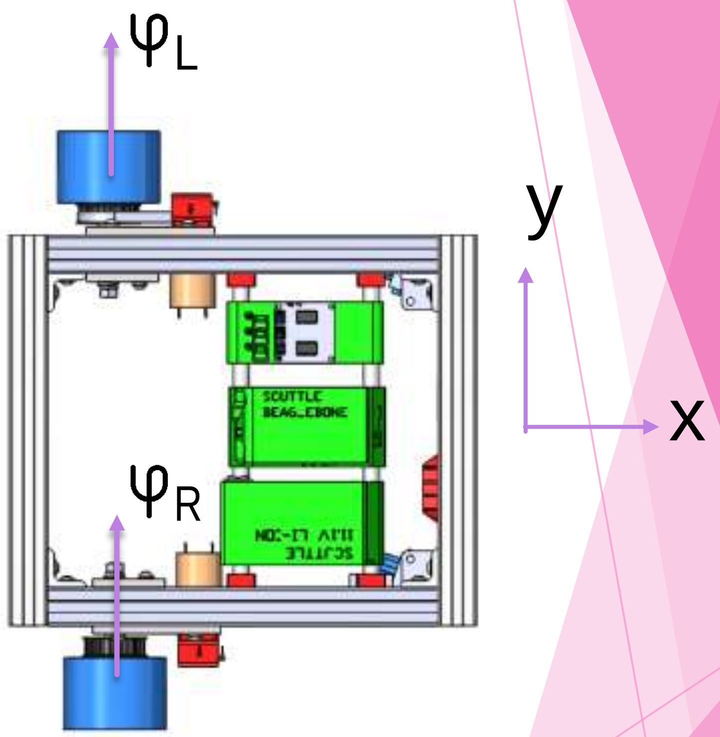
SCUTTLE Kinematics

- **Phi dot** is the derivative of phi with respect to time.

$\dot{\varphi}_L$ = pdl, as in phi_dot_l

$\dot{\varphi}_R$ = pdr, as in phi_dot_r

phiDots = np.array([pdl, pdr]) # python syntax

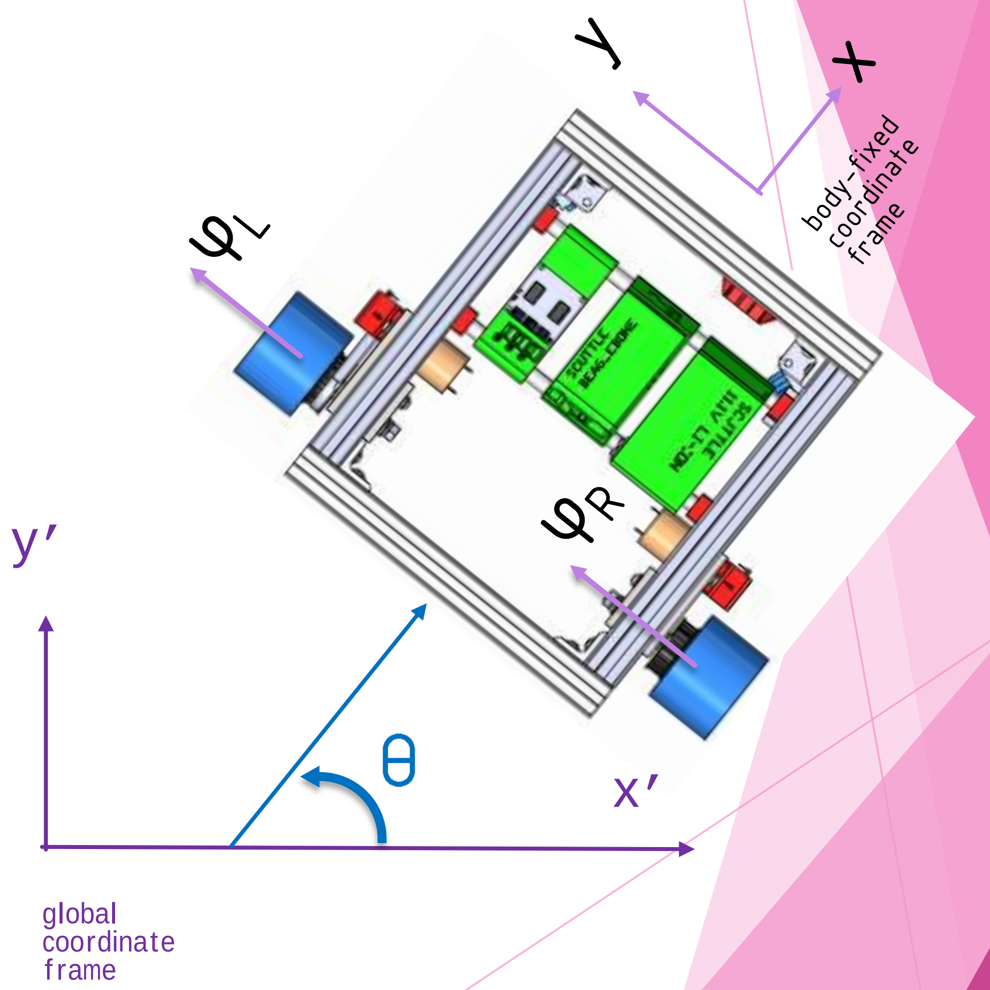


SCUTTLE Kinematics

- **Theta** describes the difference between the body-fixed frame and the global frame.

- The **rotation matrix** converts body-fixed coordinates to the global coordinates

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x_{bf} \\ y_{bf} \end{bmatrix}$$



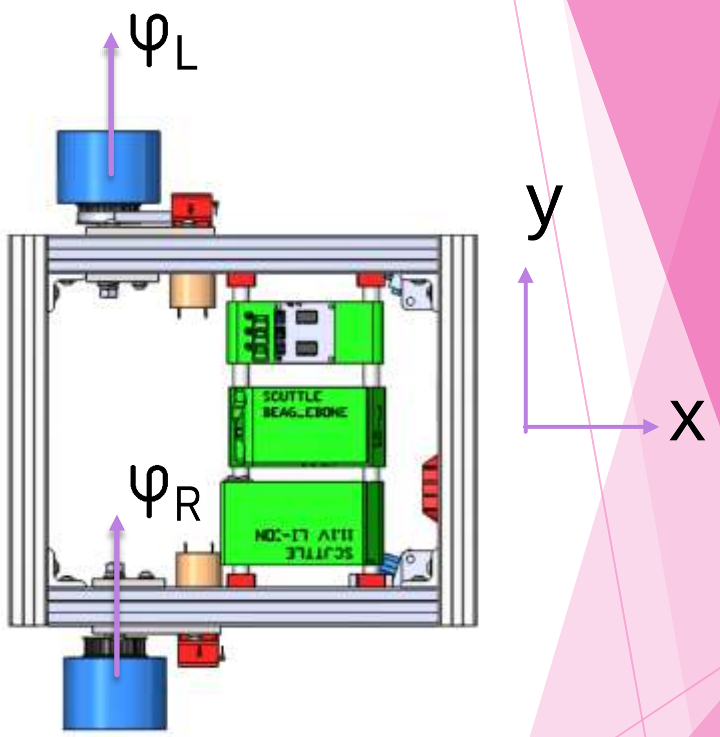
SCUTTLE Kinematics

- Phi is the angle of the wheel.
  - It is used to define incremental changes in wheel position and to calculate wheel speeds

- The x,y coordinate system has x pointing forward on the bot.
  - Positive movement of both phi's result in positive movement of the robot along the x-direction

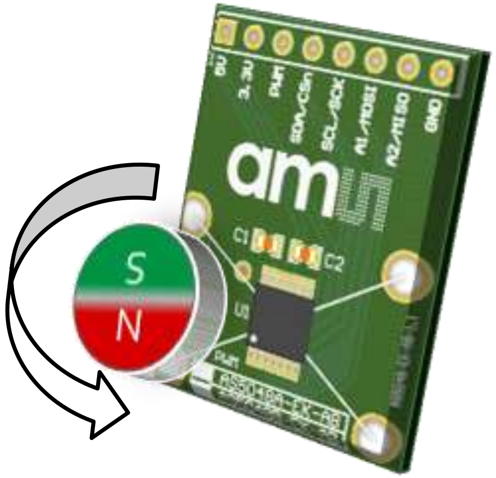
- The Kinematic Equation generates chassis motion information.
  - input the wheel speeds and output the (translational and rotational) chassis speeds

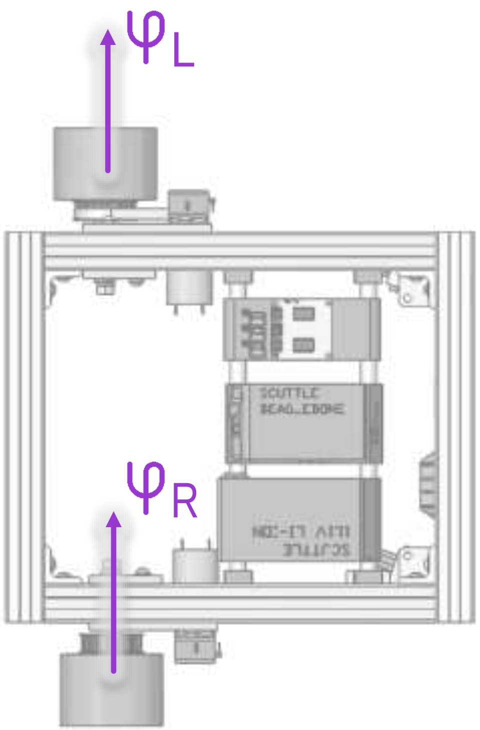
$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} R/2 & R/2 \\ -R/2L & R/2L \end{bmatrix} \begin{bmatrix} \dot{\varphi}_L \\ \dot{\varphi}_R \end{bmatrix}$$



SCUTTLE Kinematics - Encoder

- Phi is the angle of the wheel.
  - Let ‘a’ be the encoder reading (from 0 to 360)
  - Let ‘N’ be the small:large pulley ratio
  - Then,
    - $\varphi_R = a_R * N$
    - $\varphi_L = (360 - a_L) * N$



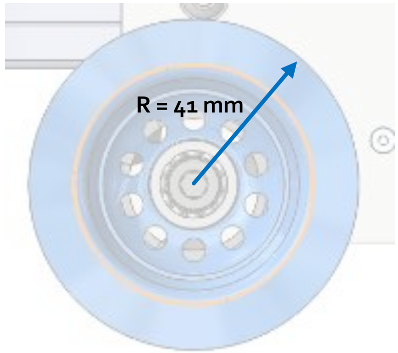


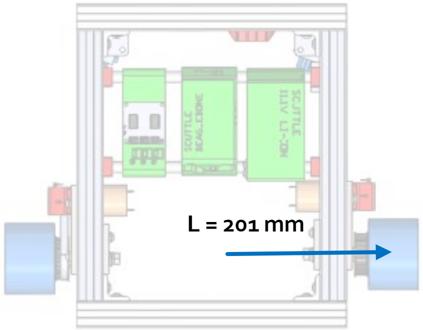
SCUTTLE Kinematics

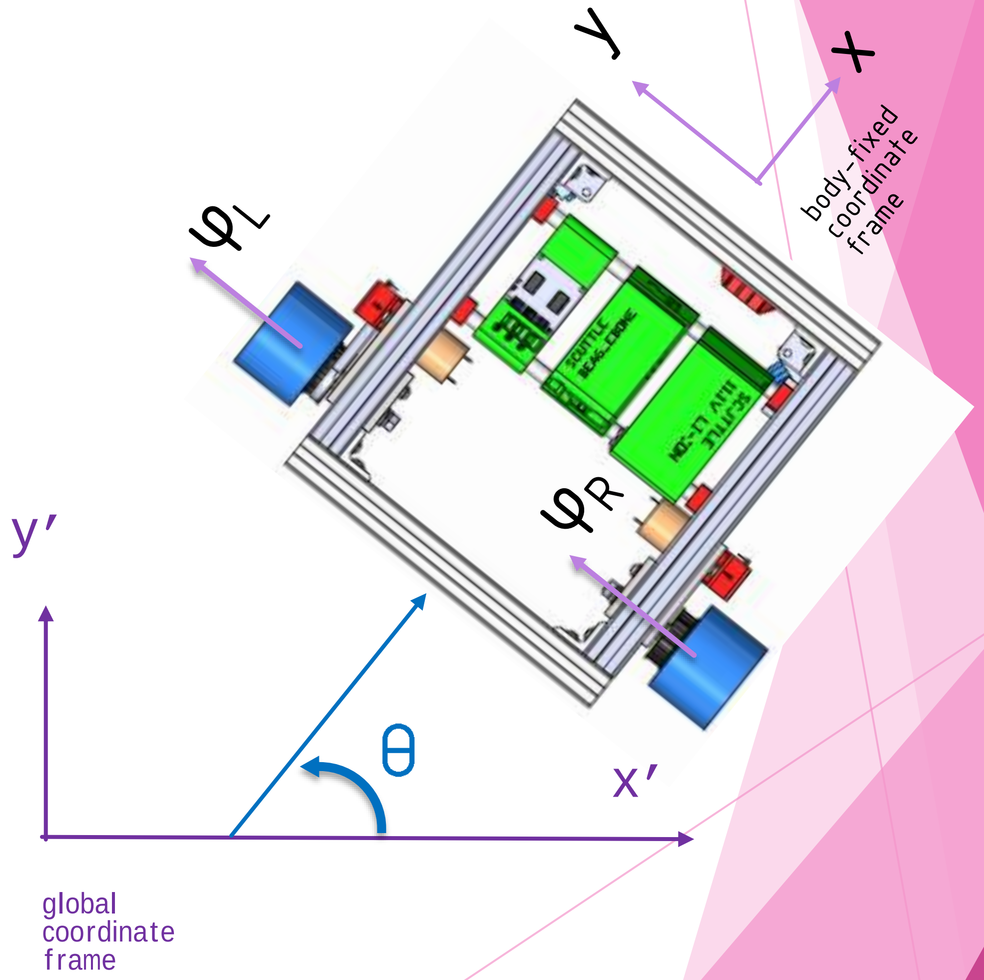
- Inverse Kinematic equation:
  - Input the desired speed and angular speed, and output the left and right wheel speeds.
  - These equations are written in the robot-fixed frame

$$\begin{bmatrix}\dot{\varphi}_L \\ \dot{\varphi}_R\end{bmatrix} = \begin{bmatrix}1/R & -L/R \\ 1/R & L/R\end{bmatrix}\begin{bmatrix}\dot{x} \\ \dot{\theta}\end{bmatrix}$$

matrix multiplication: $[C] = [A][B]$







SCUTTLE Kinematics

Forward and inverse kinematics <u>in the robot frame</u>:

"Kinematics"

Use the **wheel** speeds to obtain the **chassis** speeds

$$\begin{bmatrix}\dot{x}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}R/2 & R/2\\ -R/2L & R/2L\end{bmatrix}\begin{bmatrix}\dot{\varphi}_L\\ \dot{\varphi}_R\end{bmatrix}$$

Solve for

"Inverse Kinematics"

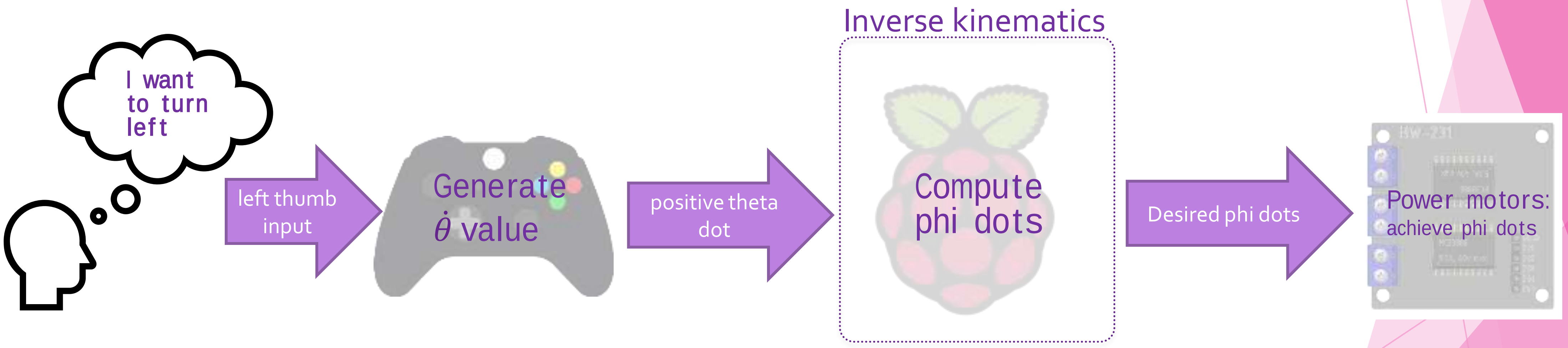
Use the **chassis** speeds to obtain the **wheel** speeds

$$\begin{bmatrix}\dot{\varphi}_L\\ \dot{\varphi}_R\end{bmatrix} = \begin{bmatrix}1/R & -L/R\\ 1/R & L/R\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{\theta}\end{bmatrix}$$

Solve for

SCUTTLE Kinematics

How do we use inverse kinematics?



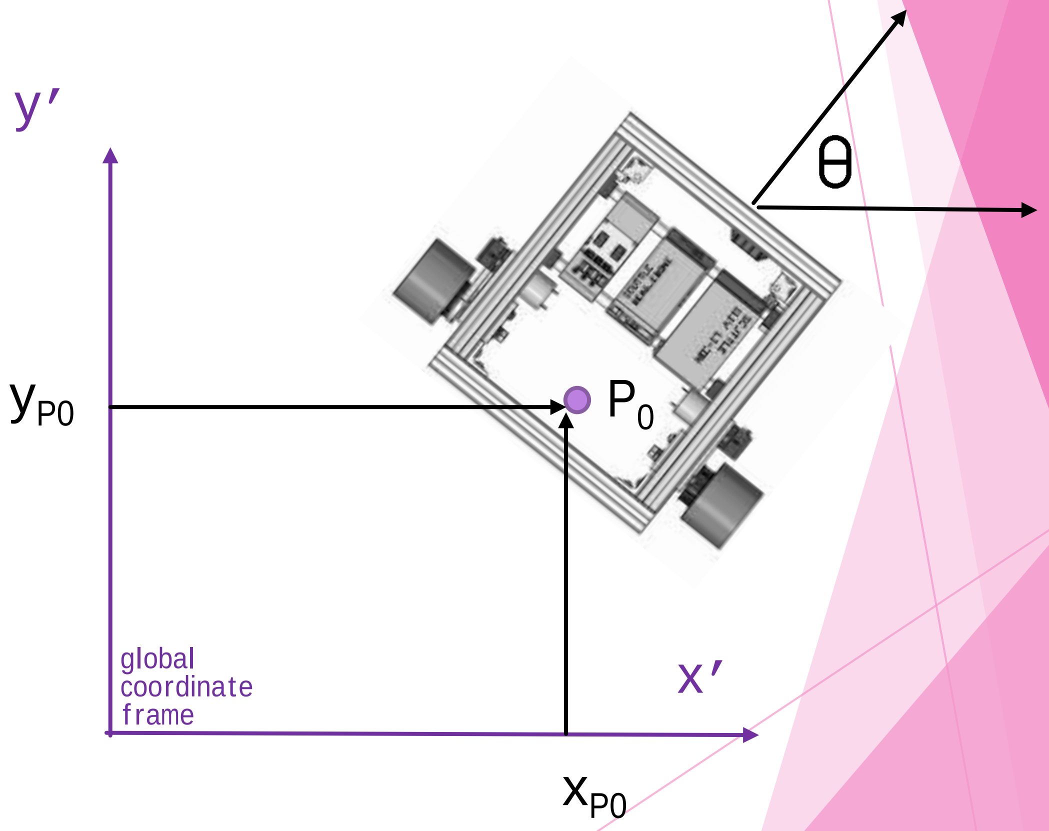
SCUTTLE Kinematics (prt 2)

This section will be expanded to discuss navigation & inertial frame

Describing the robot in the inertial (global) frame:

- P0 describes the location of the robot and is defined by the center of the wheelbase.

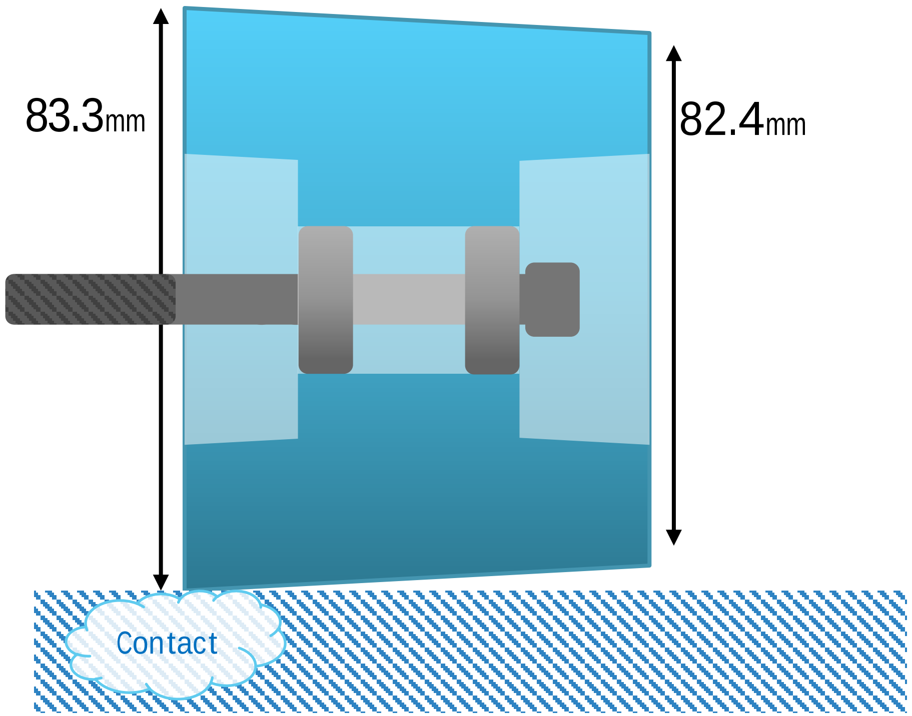
$$q^I = \begin{bmatrix} x_a \\ y_a \\ \theta \end{bmatrix}$$

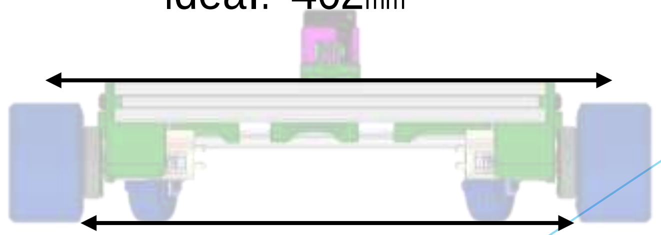


Kinematics Footnotes

1) The true wheelbase is determined by the contact patch of the wheels on the ground.
   - Most wheels have a draw angle from the manufacturing mold, resulting in a larger inner diameter.
   - Then, the contact patch may lie closer to the robot center, especially with no payload.
   - The inner wheel edge gives a 355mm wheelbase.
   - The wheelbase may change when a heavy load compresses the urethane.

Wheel spec: 83mm diam





Recommended reading:

1. Dhaouadi, Rached, and A. Abu Hatab. "Dynamic modelling of differential-drive mobile robots using lagrange and newton-euler methodologies: A unified framework." *Advances in Robotics & Automation* 2.2 (2013): 1-7.
2. Hur, Byul, et al. "Open-source Embedded Linux Mobile Robot Platform for Mechatronics Engineering and IoT Education." *Journal of Management & Engineering Integration* 13.2 (2020): 34-44.
3. Achmad, MS Hendriyawan, et al. "ROS-based 2-D Mapping Using Non-holonomic Differential Mobile Robot." *JURNAL INFOTEL* 10.2 (2018): 75-82.