Spotlight on *Schubert2*: A package for intersection theory in *Macaulay2*

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A brief history of *Schubert2*

Schubert

Schubert was a Maple-based package written by Sheldon Katz and Stein-Arild Stromme in the early 1990's.

- A Maple package for "doing" intersection theory of smooth projective varieties and enumerative geometry (e.g. how many lines are on a hypersurface?)
- Computing chern classes of bundles, Riemann-Roch formulas, solving enumerative geometry problems
- Early use: doing counts of rational curves of different degrees on a general quintic hypersurface in $\mathbb{P}^4$.
- Matched numbers predicted by physicists using string theory and mirror symmetry (after bug fix!).
- Problem: With changes to Maple, eventually Schubert would not even compile any longer, and can no longer easily be used.

Solution? Write a version for *Macaulay2*!

Key ideas of Schubert

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- Represent a smooth projective variety as **abstract**: not by equations, instead by its intersection ring $A(X)$ of algebraic cycles modulo numerical equivalence. (A graded ring, graded by codimension, where multiplication is intersection, and work over $\mathbb{Q}$ too).
- Represent a vector bundle (or sheaf) by its total Chern class
- Represent a map $f : X \to Y$ of smooth projective varieties by a pair of maps

$$f^* : A(Y) \to A(X), \text{ a ring map}$$

and

$$f_* : A(X) \to A(Y), \text{ a map of } A(Y)\text{-modules.}$$

Example

If $X = \mathbb{P}^1 \times \mathbb{P}^2$, then $A(X) = \mathbb{Q}[s,t]/(s^3, t^2)$.
s represents $\mathbb{P}^1 \times L$, $L = \textit{line}$, and t represents $p \times \mathbb{P}^2$. The element $s^2 t$ represents a point in X.

Schubert2

Some basic Schubert2 constructions

- You can create hand crafted abstract varieties, bundles and maps
- Projective spaces, products
- Grassmannians, Flag varieties
- Projective bundles, Grassmann bundles, Flag bbundles Uses Gröbner bases to compute in all of these intersection rings.
- Toric varieties
- Zero sections in an abstract variety
- Degeneracy loci of a map of vector bundles
- Blowups along smooth subvarieties
- Hirzebruch-Riemann-Roch and Grothendieck-Riemann-Roch

Examples to do in emacs and Macaulay2

- 27 lines on a cubic surface
- Riemann-Roch for a divisor on a 3-fold
- A Calabi-Yau section of a toric 4-fold