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A GENERALIZED MINMAX INFLUENCE COEFFICIENT METHOD FOR FLEXIBLE ROTOR BALANCING

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ABSTRACT

Rotor balancing is a requirement for the smooth operation of high-speed rotating machinery. In field balancing, minimization of the residual vibrations at important locations/speeds under practical constraints is usually a challenging task. In this paper, the generalized minmax coefficient influence method is formulated as an optimization problem with flexible objective functions and constraints. The optimization problem is cast in a Linear Matrix Inequality (LMI) form and a balancing code is developed to solve it. Two balancing examples are run to verify the efficiency and flexibility of the proposed method. Over the existing methods, current method is more flexible for the various requirements encountered in field balancing and can be solved accurate with current mathematical software.

INTRODUCTION

Rotating machineries can experience high amplitude vibrations during operation which can lead to machinery malfunction or even catastrophic failure of its components. One of the most common causes of these induced vibrations is the unbalance, which occurs when the principal inertia axis of the machinery does not overlap to its spin axis. Mechanical balancing tries to reduce the inherent rotor unbalance by addition and removal of small amount of mass along the rotor axis. Finding the optimal distribution of correction weights in the time down of the rotating machinery placed in service is a very challenging task. Therefore, field balancing has a great need for efficient and flexible balancing methods.

Once with increasing operating speeds and decreasing shaft stiffnesses, the rotors became more flexible and the traditional rigid balancing procedures became inadequate [1]. Therefore, various balancing techniques for the flexible rotors have been developed over past forty years. These methods can be divided mainly into two groups [1]: modal balancing methods and influence coefficient (IC) balancing methods. The principle of modal method is to balance rotors using their modal shapes, which are assumed to be planar. However, selecting of the trial masses for correcting specific modes need accurate analytical models of rotor system which make difficult field balancing [1]. The improvements in instrumentation and computation tools have made possible development of IC balancing method. This method uses only the assumption that the rotor system is a linear system in terms of its response to trial weights, which makes it very easy to apply in field balancing [1]. After the rotor run out and possible shaft bow effects are compensated [2], the balancing problem can be represented in the complex form as,

$$\{R_f\} = \{R_0\} + [H]\{B\} \tag{1}$$

where $\{R_0\}, \{R_f\} \in C^m \quad (m = k \times l)$ are the vibration measurements vectors representing the initial and residual response at k sensor locations and l rotation speeds. The influence coefficient matrix $[H] \in C^{m \times n}$ is usually obtained from the unbalance responses at different locations (sensor locations and speeds) using trial weights at the balancing planes. The objective of balancing is to find $\{B\} \in C^n$ (the vector variable representing the locations and amounts of the correction weights) which minimize the vector norm of the residual response according to some acceptance criteria ([2], [3]) while satisfying specifically constraints in terms of correction weights. The p- norm of a vector can be defined as

$$\|x\|_p := \begin{cases} \left(\sum_{i=1}^{p} |x_i|^p \right)^{\frac{1}{p}}, p \in N^* \\ \max |x_i|, p = \infty \end{cases} \quad (2)$$

The most common norms used in the IC balancing are: 2-norm -Least-Square (LS) balancing and infinity-norm- minmax balancing.

The first IC balancing procedure was proposed by Goodman [4], who obtained the analytical formula of the correction weight vector by minimization of the root-mean-square (RMS) values of residual vibrations. Due to its objective function, LS method in many problems can not reduce the residual vibrations at all locations, therefore an iterative weighted LS method was proposed [4] to correct it. However, this method depends on the chosen weighting matrix and there is no guarantee of its convergence to an optimal solution. Little and Pilkey [5] formulated the IC balancing problem as a constrained optimization problem. Numerical algorithms already developed in linear programming (LP) or quadratic programming (QP) were used ([5], [6]) to find approximate solutions of the IC balancing problems. However, casting the nonlinear problem of IC balancing in a LP or QP formulation can be made only approximately. Recent advance of the convex optimization theory, correlated with rapid increase of computational power, allowed the development of new algorithms which can solve more efficient and accurate the IC balancing problems. One of these new optimization methods is the semi-definite programming (SDP) techniques using the Linear Matrix Inequality (LMI) formulation. This technique was used by Kanki et al [7] to formulate several IC balancing problems. The correction weights were determined by minimization of the residual vibrations (infinity or 2-norm) or total amount of balancing weights, without constraints. Li et al [8] used the Second Order Cone Programming (SOCP) formulation (a SDP special case) in order solve the least-square and minmax balancing problems. A robust balancing approach was also defined considering the possible uncertainty in the measured data during the balancing process.

In the current paper, a general optimization problem of the IC balancing method is formulated, based on the practical requirements of field balancing. The different objective functions and constraints in the magnitudes of residual vibrations and correction weights are cast in a LMI formulation. This formulation makes easier programming and solving the balancing problem using current mathematical software packages (e.g. Matlab [9]). The efficiency and flexibility of new method is tested in two balancing examples from the literature.

METHODOLOGY

Generalized MinMax Balancing Problem

In field balancing, balancers encounter hard constraints which must be solved in short time. The maximum residual vibrations at the sensor locations after balancing should be less than a maximum acceptable value corresponding to the rotor [2]. However, at some of the sensor locations due to the critical clearance or force requirement (e.g. seal locations), the residual vibration should be minimized as much as possible, in order to increase the service life of the rotating machinery. Sometimes, certain holes of balance planes may be filled up or even damaged, which makes the addition of the correction weights obtained by optimization impossible. Therefore, the magnitude of correction weights should be limited by the maximum weight which can be added to every balance plane. Based on these practical requirements, the optimization problem of IC balancing can be formulated.

If we consider the first *p* terms, as critical locations, and the other *m-p*, as non-critical locations, the equation (1) can be cast in the form:

$$\begin{Bmatrix} R_f^c \\ R_f^{nc} \end{Bmatrix} = \begin{Bmatrix} R_0^c \\ R_0^{nc} \end{Bmatrix} + \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \{B\} \quad (3)$$

where $\{R_0^c\}, \{R_f^c\} \in C^p$ and $\{R_0^{nc}\}, \{R_f^{nc}\} \in C^{m-p}$

The optimization problem of the generalized minmax balancing method can be formulated as:

$$minimize \ v_c \quad (4)$$

with the following constraints:

$$\left|R_{fi}^c\right| \le v_c \quad i = \overline{1, p} \quad (5)$$

$$\left|R_{fj}^{nc}\right| \le v_{\max} \quad j = \overline{1, m-p} \quad (6)$$

$$\left|B_k\right| \le w_k \quad k = \overline{1, n} \quad (7)$$

where v_c is the infinity-norm of residual vibration vector for critical sensor locations $\{R_f^c\}$, $v_{\max}$ is the maximum acceptance level of the residual vibration at non-critical locations, and w_k is the maximum weight which can be used at the balancing plane *k*. The optimization variables, which should be determined by optimization, are the components of correction weight vector *{B}* and the infinity-norm of critical locations v_c. The classical minmax method is a special case, when all locations are considered as critical ($p = m$).

SDP Technique- For several decades, linear programming (LP) was the most common used optimization method. However, many engineering problems, as for example the IC balancing problem, are nonlinear and can not be solved without some simplifying assumptions. Over last decade, semidefinite programming (SDP) has been one of the most exciting and active research areas in optimization. This remarkable research activity conducted to the development of efficient interior-point algorithms for solving SDP problems, and important applications in a wide variety of problems in many domains (system and control theory, signal processing, VLSI etc.).

A convex optimization problem in SDP has the following form:

$$minimize \ b^T x \quad (8)$$

$$\text{subject to } A_j(x) := A_{j0} + x_1 A_{j1} + ... + x_k A_{jk} > 0 \,,\ j = \overline{1, t} \quad (9)$$

where $A_{j0}, A_{j1}, ..., A_{jk}$ are given symmetric matrices and $x = (x_1, ..., x_k)$ represents the optimization variables. The constraints (9) are called linear matrix inequalities (LMIs) and means that the matrices $A_j(x)$ are positive definite. In other words, SDP can be regarded as an extension of LP (has a linear objective function), where the inequalities between vectors are replaced by matrix inequalities. Although SDP problems are much more general than linear problems, they are not much harder to solve. Most interior-point methods for linear programming have been generalized to semi-definite programs, and perform very well in practice [10]. However, in order to be solved with SDP the engineering problems should be cast in the LMI forms. Two lemmas ([7], [9]) presented below are very useful in casting the balancing problem in a LMI form.

Lemma1: (The Schur Complement Lemma)

Let complex matrix A, B and C be given. The following statements are equivalent

$$\text{a)} \begin{bmatrix} A & B \\ B^* & C \end{bmatrix} > 0 \tag{10}$$

$$\text{b)} \begin{cases} A > 0 \\ C - B^* A^{-1} B > 0 \end{cases} \tag{11}$$

$$\text{c)} \begin{cases} C > 0 \\ A - B C^{-1} B^* > 0 \end{cases} \tag{12}$$

where * means complex conjugate

Lemma2:

Let the complex matrix X and if we denote $\Re X$ and $\Im X$ the real and imaginary parts of X, the following statements are equivalents.

$$\text{a)}\ X = \Re X + j\Im X > 0 \tag{13}$$

$$\text{b)} \begin{bmatrix} \Re X & \Im X \\ -\Im X & \Re X \end{bmatrix} > 0 \tag{14}$$

Using the lemmas mentioned above the following theorem can be formulated and proved.

Theorem:

Let consider a complex vector $X = \{X_i\} \in C^n$ and a real vector $w = \{w_i^2\} \in R^n$ with $w_i \geq 0$. If $X_i^* X_i = |X_i|^2 < w_i^2$ for $i = \overline{1,n}$ then

$$\begin{bmatrix} d(w) & \Re d(X) & 0 & -\Im d(X) \\ \Re d(X) & I_n & \Im d(X) & 0 \\ 0 & \Im d(X) & d(w) & \Re d(X) \\ -\Im d(X) & 0 & \Re d(X) & I_n \end{bmatrix} > 0 \tag{15}$$

where $d(X)$ express a diagonal matrix which has on the diagonal all components X_i of the vector X.

Proof: The conditions

$$X_i^* X_i = |X_i|^2 < w_i^2 \text{ for } i = \overline{1,n} \tag{16}$$

can be put in the following form:

$$d(w) - d(X)^* d(X) > 0 \tag{17}$$

Using Schur Complement formula the following matrix inequality can be obtained:

$$\begin{bmatrix} d(w) & d(X)^* \\ d(X) & I_n \end{bmatrix} > 0 \tag{18}$$

Applying Lemma 2, the equation (18) becomes equation (15) and the theorem is proved. It is important to mention that the components of vector *w* can be either optimization variables or constants.

Gneralized MinMax Balancing Problem - LMI Formulation

Using the theorem formulated above the generalized IC balancing problem expressed by equations (3-6) can be cast in a LMI form as follows:

$$\textit{minimize } v_c^2 \tag{19}$$

subject to the following LMIs resulted from the constrains (5-7).

$$\begin{bmatrix} v_c^2 I_p & \Re d(R_f^c) & 0 & -\Im d(R_f^c) \\ \Re d(R_f^c) & I_p & \Im d(R_f^c) & 0 \\ 0 & \Im d(R_f^c) & v_c^2 I_p & \Re d(R_f^c) \\ -\Im d(R_f^c) & 0 & \Re d(R_f^c) & I_p \end{bmatrix} > 0 \tag{20}$$

$$\begin{bmatrix} d(v_{\max}^2) & \Re d(R_f^{nc}) & 0 & -\Im d(R_f^{nc}) \\ \Re d(R_f^{nc}) & I_{m-p} & \Im d(R_f^{nc}) & 0 \\ 0 & \Im d(R_f^{nc}) & d(v_{\max}^2) & \Re d(R_f^{nc}) \\ -\Im d(R_f^{nc}) & 0 & \Re d(R_f^{nc}) & I_{m-p} \end{bmatrix} > 0 \tag{21}$$

$$\begin{bmatrix} w_k^2 & \Re B_k & 0 & -\Im B_k \\ \Re B_k & 1 & \Im B_k & 0 \\ 0 & \Im B_k & w_k^2 & \Re B_k \\ -\Im B_k & 0 & \Re B_k & 1 \end{bmatrix} > 0 \quad \overline{k = 1.n} \tag{22}$$

The elements of diagonal matrices with the components of residual vibration vector can be obtained from its decomposition in real and complex parts:

$$R_{fk} = \Re R_{fk} + j\Im R_{fk} = R_{0k} + H_k B \tag{23}$$

where $H_k = \{H_{k1} \quad H_{k2} \quad . \quad . \quad H_{kn}\}$

After mathematical manipulations, the following real and imaginary components will be:

$$\Re R_{fk} = \Re R_{0k} + \Re H_k \cdot \Re B - \Im H_k \cdot \Im B \tag{24}$$
$$\Im R_{fk} = \Im R_{0k} + \Im H_k \cdot \Re B + \Re H_k \cdot \Im B$$

After replacing these terms in equations (20-22), the IC balancing problem is formulated as LMI optimization problem with the correction mass vector - *{B}* and the infinity-norm of residual vibration vector for critical sensor locations - v_c, as optimization variables.

Application: IC Balancing examples.

Based on the mathematical formulation shown before, a balancing program was realized in MatLab using LMI Control Toolbox [9]. In order to compare the generalized minmax balancing method proposed in this paper to other balancing methods or formulations, two numerical examples from the literature ([1], [11]) were run.

Example 1. Ill Conditioned Influence Coefficient Matrix

As was shown in [1], the LS balancing method is prone to certain numerical difficulties when the balancing planes are almost non-independent. In these cases, the correction weights obtained by LS balancing method are very large and can not be applied to the balancing planes. In order to avoid this problem, Darlow[1] developed a method which identify and eliminate the "least independent" of the balancing planes using a Gram-Schmidt orthogonalization procedure.

In this section a balancing problem [1] with ill conditioned influence matrix, as shown in Tab. 1, is presented. The problem is solved by three methods: LS balancing [3], LS balancing with elimination (LSE) of non-independent balancing planes[1], and the minmax balancing method. In the minmax problem, the maximum correction weight is limited to be less than one at all balancing planes and the optimization objective is minimization of the maximum residual vibration.

Plane 1	Plane 2	Plane 3
$1.41<45^0$	$3.61<34^0$	$3.61<34^0$
$3.16<72^0$	$2.24<27^0$	$2.24<27^0$
$2.83<45^0$	$5.0<37^0$	$5.0<37^0$
$3.16<18^0$	$3.61<34^0$	$4.47<27^0$

Tab. 1: The coefficient influence matrix –ill conditioned case

Example 2. Turbine-Generator Balancing Case

In this section, an 1150 MW turbine-generator balancing problem presented in [11] is solved. This 11-bearing nuclear turbine system has five balancing planes. The vibration measurements were collected at all bearings locations and the coefficient influence matrix obtained from measurement is presented in Tab. 2.

Plane 1	Plane 2	Plane 3	Plane 4	Plane 5
$9.8<177^0$	$17<124^0$	$7.2<114^0$	$20.6<29^0$	$38.5<77^0$
$2.7<43^0$	$14.3<317^0$	$4.5<213^0$	$16.1<229^0$	$14.3<270^0$
$12.5<323^0$	$25<261^0$	$15.2<158^0$	$70<130^0$	$30<238^0$
$22.4<92^0$	$32.6<45^0$	$23.3<315^0$	$15.2<17^0$	$27.8<210^0$
$26<94^0$	$40.3<9^0$	$25<330^0$	$5.4<295^0$	$34<213^0$
$40.3<355^0$	$43<144^0$	$29.6<61^0$	$33.2<94^0$	$65.4<322^0$
$20.6<339^0$	$32.3<152^0$	$36.7<41^0$	$35.8<110^0$	$61.8<322^0$
$12.6<226^0$	$37.6<52^0$	$18.8<153^0$	$37.6<54^0$	$26<176^0$
$13.4<209^0$	$26.9<76^0$	$47.5<98^0$	$60<94^0$	$71.7<312^0$
$13.4<154^0$	$22.4<307^0$	$52<299^0$	$65.4<308^0$	$102<165^0$
$5.4<24^0$	$7.2<199^0$	$22.4<2^0$	$25.1<187^0$	$27.8<99^0$

Tab. 2: The coefficient influence matrix ($\mu m / kg <^0$)

In practice, the balancing plane 4 is not available; therefore it is not used in the balancing process. The balancing problem is solved using minmax method with three different formulations. In the first formulation the maximum residual vibrations are minimized without constraints (minmax method). The results obtained using LMI technique are compared to the results obtained by SOCP technique in [8] for the same problem. In the second formulation, the minmax method with weight constraints (maximum 120 oz = 3.402 kg per balancing plane) is solved. In the last formulation, the generalized minmax method is used. The location 2 and 10 are considered as critical, and their maximum value is minimized. For other locations an acceptable vibration level of 3 mils (76 μm) is considered. The maximum correction weight per balance plane is set as 120 oz (3.402 kg) for all balancing planes.

RESULTS AND DISCUSSION

The solutions (balance weights) of the first balancing example, obtained using the balancing methods mentioned before, are illustrated in Tab. 3.

Balance Plane No.	Balance Weights		
	LS	LSE	Minmax & weight constrain
1	$0.88<99^0$	$0.51<46^0$	$0.41<339^0$
2	$4.78<98^0$	-	$1<122^0$
3	$5.14<271^0$	$1.13<205^0$	$1<239^0$

Tab. 3: Balance Weights

The initial and the residual vibrations at the sensor locations are shown in Tab. 4.

Probe No.	Initial vibration	Residual Vibration		
		LS	LSE	Minmax & weight constrain
1	$3.16<72^0$	$1.64<124^0$	$1.16<169^0$	$1.95<146^0$
2	$3.16<18^0$	$0.46<180^0$	$0.8<30^0$	$1.95<27^0$
3	$4.12<14^0$	$1.29<315^0$	$2.87<297^0$	$1.95<297^0$
4	$5.39<68^0$	$0<90^0$	$2.5<99^0$	$1.95<99^0$

Tab. 4: Initial Vibration and Residual Vibrations

As it can be observed, the residual vibrations obtained using LS are smallest, but the correction weight needed to obtain these values are too large. The LSE method proposed in [1] use only two balancing plane, which reduce the correction weights, but the maximum residual vibration levels increase too much. The minmax method with a constraint in the magnitude of correction weights reduces both maximum residual vibrations and correction masses under the maximum values obtained using LSE method.

The solutions obtained by the formulations of the minmax method in the second example are presented in Tab. 5. The initial and residual vibrations at the sensor locations are shown in Tab. 6. The minmax solution without constraints using LMI technique gives a solution similar with that obtained by SCOP technique in [8]. However, SCOP algorithms are not so well developed and validated as LMI algorithms, which were used and verified for many years in control problems. Also LMI algorithms are implemented in the current mathematical software (e.g. Matlab [9]) and can be easily applied in an IC balancing code.

Balance Plane No.	Balance Weight		
	Minmax	Minmax & weight constrain	Minmax generalized
1	$4.423<88^0$	$3.402<91^0$	$3.402<96^0$
2	$2.92<352^0$	$2.325<354^0$	$2.041<359^0$
3	$1.588<322^0$	$1.361<318^0$	$1.304<336^0$
5	$1.928<304^0$	$1.786<35^0$	$1.616<315^0$

Tab. 5: Balance Weights (kg $< ^0$)

Probe No.	Initial vibration	Residual Vibration		
		Minmax	Minmax & weight constrain	Minmax generalized
1	$55<259^0$	$39<337^0$	$36<339^0$	$30<327^0$
2	$45<118^0$	$33<178^0$	$32<169^0$	$30<165^0$
3	$124<21^0$	$70<31^0$	$73<28^0$	$76<26^0$
4	$138<349^0$	$70<14^0$	$73<2^0$	$76<354^0$
5	$107<349^0$	$70<331^0$	$73<330^0$	$76<329^0$
6	$90<280^0$	$70<104^0$	$13<124^0$	$24<123^0$
7	$58<354^0$	$70<14^0$	$73<352^0$	$76<7^0$
8	$108<201^0$	$70<107^0$	$73<129^0$	$76<144^0$
9	$88<190^0$	$70<249^0$	$73<250^0$	$66<238^0$
10	$56<48^0$	$35<122^0$	$57<124^0$	$36<110^0$
11	$73<158^0$	$70<117^0$	$69<123^0$	$76<122^0$

Tab. 6: Initial Vibration and Residual Vibrations (μm $< ^0$)

Using the minmax method with constraint on the maximum correction weight, the maximum residual vibration of all locations increased slightly (about 5 %) than the previous formulation. However, the maximum vibration level on the critical locations has increased with about 62% (sensor 10). Applying the generalized minmax method, the maximum vibration level at critical locations remained almost at the same level as in the formulation without constraint and in the same time the residual vibrations and correction weights are maintained under the acceptance levels.

Previous examples illustrate several advantages of the generalized minmax IC balancing method over existing balancing methods and formulations. The method is currently tested in industry and it is believed that its application will prove better its efficiency and flexibility in solving the complex real world balancing problems.

CONCLUSION

The field balancing of modern high-speed machinery is very costly process due to down time of the rotating machinery. Therefore it is crucial to apply effective and flexible balancing methods which could find solutions satisfying the complex constraints encountered in real world balancing problems. In order to answer at these practical requirements, a generalized minmax method is proposed in this paper. In this method, the sensor locations are divided in critical and non-critical based on their importance in the reliability of rotating machinery. The levels of the vibrations on the non-critical locations are limited at the acceptance level corresponding to the rotating machinery using mathematical constraints. This relaxed approach on non-critical locations makes possible obtaining better results in the critical locations which could increase the service life of the rotating machinery. Constraints on the magnitude of correction weights can be applied individual or global on the balancing planes based on the practical requirements. The generalized minmax problem is cast as a LMI problem. Based on this formulation, a balancing code is developed and two balancing examples from the literature are solved. The results show that the proposed balancing method is very efficient and flexible.

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