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In this work, uncertainty and disturbance estimation (UDE) technique is employed to robustify an input–output linearisation (IOL) controller. An IOL controller designed for a nominal system is augmented by the UDE estimated uncertainties to achieve robustness. In doing so, state dependent nonlinearities of the system are treated as a part of the uncertainties and thus, the controller does not require system states for its implementation. The resulting controller, however, needs derivatives of the output. To address the issue, a design of an observer that employs the UDE estimated uncertainties for robustness is proposed giving rise to the UDE based controller–observer structure. Closed loop stability of the overall system is established. The notable feature of the proposed design is that it neither requires accurate plant model nor system states or derivatives of output. Also the approach does not need any information about the uncertainty. To demonstrate the effectiveness, numerical simulation results of the proposed approach as applied to the wing-rock motion control problem are presented. Lastly, hardware implementation of the UDE based controller-observer structure for motion control of Quanser's DC servo motion control platform is carried out and it is shown that the proposed strategy offers a viable approach for designing implementable robust IOL controllers.

Keywords: uncertainty and disturbance estimation; input–output linearisation; nonlinear control; robust control; controller–observer structure

1. Introduction

Based on the concepts of differential geometry, the input–output linearisation (IOL) (Slotine and Li 1991; Vidyasagar 1993), also known as inverse dynamics in robotics and dynamic inversion in aerospace field, is one of the most prominent techniques used in the design of controller for nonlinear systems. While the IOL controller offers highly satisfactory performance in ideal scenario, it suffers from certain drawbacks when put into practice. First, the IOL controller requires availability of system state for its implementation. Next, the design requires exact cancellation of the system nonlinearities. It offers desired performance only when the models are exact and when the fed back states are available and are measured without any error. In reality, these requirements are hard to satisfy and in such situations, the IOL control law may not offer satisfactory performance.

To recover the performance in the presence of uncertainties, various approaches have been presented in the literature for robustification of the IOL controller (Spong 1987; Fernandez 1990; Chen and Chen 1991; Huang, Lin, Cloutier, Evers, and D'Souza 1992; Slotine and Hedrick 1993; Reiner, Balas, and Garrard 1995; Trebi-Ollennu and White 1996; Pollini, Innocenti, and Nasuti 1997; Leland 1997; Pfeiffer and Edgar 1999). In many of these approaches, knowledge of some characteristic of uncertainty, such as bounds, is required. Not having accurate information on the characteristics of the uncertainty results into degraded performance.

One approach for designing robust control law for uncertain systems is to estimate the effect of uncertainties and disturbances acting on the system and compensate it by augmenting the controller designed for nominal system. Techniques like disturbance observer (DO; Chen 2003), unknown input observer (UIO; Takahashi and Peres 1996) and perturbation observer (PO; Jiang and Wu 2002) have been in place for quite sometime, to estimate the effects of uncertainties and disturbances. A time delay control (TDC) is one such well-known strategy used for estimation of system uncertainties (Youcef-Toumi and Ito 1990). An application of TDC for robustification of IOL controller has also been reported in Youcef-Toumi and Wu (1992) wherein it is shown that the performance of the IOL controller in presence of uncertainty is indeed robust. Following the line of TDC, (Zhong and Rees 2004) have proposed a novel uncertainty and disturbance estimation (UDE) technique for

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estimation of uncertainties. Subsequently, in Talole and Phadke (2009), an application of the UDE in robustification of the input–output linearisation controller is presented wherein the UDE estimated uncertainties are used in robustifying an IOL controller. The robustification is achieved by estimating the uncertainties and external unmeasurable disturbances using the UDE and compensating the same by augmenting the IOL controller designed for nominal system. Further, the efficacy of the robustfication of IOL controller has been shown through application to wing-rock motion control problem. The robustified IOL control has been designated as the IOL+UDE controller. While the IOL+UDE controller achieves the objective of robustification of the IOL control quite effectively, the implementation of the same requires complete state vector as well as output derivatives. Both these requirements may not be easy to satisfy in practice and thus the issues need to be addressed.

In this article, the work presented in Talole and Phadke (2009) is extended with an objective of addressing the issues of requirement of system state as well as output derivatives for the implementation of the IOL+UDE controller. The IOL problem is formulated by considering the state dependent nonlinearities as a part of the uncertainties and thus the issue of requirement of state vector is addressed. The IOL controller is robustified by using the UDE estimated uncertainties. As the resulting controller requires derivatives of the output for implementation, a robust observer is proposed to provide estimates of the output derivatives. The observer design too employs the UDE estimated uncertainties to achieve robustness, thus giving rise to the UDE based controller–observer structure. Closed loop stability of the controller–observer structure is established. The significant feature of the proposed approach is that it does not need any information about the uncertainties. Also the design neither requires accurate plant model nor system states for its implementation. Effectiveness of the proposed approach is demonstrated through application to the wing-rock motion control problem considered in Talole and Phadke (2009). Finally, hardware implementation of the UDE based controller-observer structure for motion control of Quanser's DC servo motion control platform is carried out and the related results are presented.

The remaining article is organised as follows. In Section 2, a brief review of the I-O linearisation is presented. An overview of the UDE approach and its application for robustification of the IOL control is the subject of Section 3. In Section 4, the UDE based controller–observer structure is presented whereas closed loop stability of the overall system is presented in Section 5. Extension of the proposed strategy for multi-input multi-output (MIMO) system is discussed in Section 6. Application of the proposed control strategy to the problem of wing-rock motion control and related results are presented in Section 7 and in Section 8, the results of experimental validation of the proposed design as applied to Quanser's DC servo motion control platform are presented. Lastly, Section 9 concludes this work.

2. Input-output linearisation

An overview of the Input–output linearisation approach is presented in this section. While the treatment is presented for single-input single-output system, extension of the proposed design strategy for MIMO system is discussed in Section 6. To this end, consider a single-input single-output nonlinear dynamical system described by

$$\dot{x} = f(x) + g(x)u \tag{1}$$

$$y = c(x) \tag{2}$$

where $f \triangleq [f_1(x) \;\; f_2(x) \;\; \cdots \;\; f_n(x)]^T$; $g \triangleq [g_1(x) \;\; g_2(x) \;\; \cdots \;\; g_n(x)]^T$; and $x \in R^n$ is the state, $u(t) \in R^1$ represents the control input and $y(t) \in R^1$ is the output. The functions $f: R^n \to R^n$, $c: R^n \to R^1$ are assumed to be continuously differentiable nonlinear functions and $g: R^n \to R^n$ is a continuous function of x. Suppose that the desired output trajectory is specified by $y^\star(t)$, $0 \le t \le t_f$, which is an outcome of Equations (1), (2) for some feasible reference control $u^\star(t)$ for all $t \in [0, t_f]$. The objective is to design a controller such that $y(t)$ asymptotically tracks a reference output trajectory $y^\star(t)$.

Let λ be the relative degree of y. Following the IOL theory, the output is differentiated by λ times yielding

$$y^{(\lambda)} = a(x) + b(x)u \tag{3}$$

where $y^{(\lambda)}$ represents the λth order derivative of $y(t)$ with respect to time and $a(x) = L_f^\lambda c$ and $b(x) = L_g(L_f^{\lambda-1}(c))$. The quantities $L_f(\cdot)$ and $L_g(\cdot)$ denote the Lie derivatives of $(\cdot)$ with respect to $f(x)$ and $g(x)$, respectively. Assuming $b(x)$ to be non-singular, the IOL control is defined as

$$u(IOL) = \frac{1}{b(x)}(-a(x) + \nu) \tag{4}$$

where $\nu \in R^1$ is the outer loop control. Substituting (4) in (3) yields

$$y^{(\lambda)} = \nu \tag{5}$$

which is a linear relationship between the output and input. Now standard linear control techniques can be employed to design the outer loop control, ν, to achieve desired performance. For example, suppose that one desires the output $y(t)$ to asymptotically track a reference trajectory, $y^{\star}(t)$, then using the outer loop control

$$\nu = y^{\star(\lambda)}(t) + m_\lambda \tilde{y}^{(\lambda-1)}(t) + \cdots + m_2\dot{\tilde{y}}(t) + m_1\tilde{y}(t) \quad (6)$$

where $\tilde{y}(t) = y^{\star}(t) - y(t)$ results into a closed loop error dynamics as

$$\tilde{y}^{(\lambda)}(t) + m_\lambda \tilde{y}^{(\lambda-1)}(t) + \cdots + m_2\dot{\tilde{y}}(t) + m_1\tilde{y}(t) = 0 \quad (7)$$

The tracking error, $\tilde{y}(t)$, will asymptotically go to zero provided one chooses the controller gains, m_i, such that the polynomial (7) is Hurwitz. It may be noted that the IOL controller of (4) needs all state variables and up to $(\lambda-1)$ time derivatives of the plant output. Further the controller needs accurate plant model. Since in practice these requirements are hard to satisfy, it is necessary to address the issues. At this juncture, it is important to note that the IOL control will offer satisfactory results only if the associated zero dynamics (Slotine and Li 1991) of the system is stable and the same is assumed in this work.

3. UDE based IOL controller

One of the approaches for designing robust control is to estimate the effect of the uncertainties and disturbances acting on the system and compensate for the same. As stated earlier, various techniques known as DO, UIO and PO have been presented in literature for estimation of the effects of the uncertainties and disturbances. The TDC is one such approach that has been widely used for estimation of uncertainties. To cite an example, an application of the TDC to estimate the target acceleration, which can be considered as a disturbance acting on the system, is presented in Talole, Ghosh, and Phadke (2006). While the TDC has been shown as a highly effective robust control strategy, there exists concerns such as the requirement of state and their derivatives for estimation of uncertainty, existence of oscillations in control signal and existence of time delay in the control law formulation. Addressing these issues, Zhong and Rees (2004) presented a formulation of a novel UDE technique and the authors have shown that while the UDE brings performance similar to that of the TDC, it addresses the stated concerns associated with it. The effectiveness of the formulation has been demonstrated by providing simulations of the UDE-based control with a comparison made with TDC. Recently in Zhong, Kuperman, and Stobart (2011), the authors have brought out the two-degree-of-freedom nature of the UDE based controllers and it is shown that, in addition to the known advantages over the time-delay control, the UDE-based control outperforms TDC under the same operational conditions. Application of UDE in various contexts have also been reported in the literature. An application of the UDE in robustifying a feedback linearising control law for a robot having joint flexibility is presented by Patel et al. (2006) wherein the effect of joint flexibility is treated as a disturbance. Conventional sliding mode control has two shortcomings, that is, the knowledge of bounds of uncertainties and disturbances is needed and the resulting control is discontinuous. With the application of UDE to sliding mode control, both these drawbacks can be removed as shown by Talole and Phadke (2008). In Kuperman and Zhong (2009, 2011) and Stobart, Kuperman, and Zhong (2011), UDE based robust control designs for uncertain linear and nonlinear systems with state delays are presented and the authors have shown that the designs offer excellent tracking and disturbance rejection performance.

As robustness is an important concern with the IOL control, application of the UDE technique for robustification of the IOL controller is presented in Talole and Phadke (2009) wherein the effect of system uncertainties are estimated by using the UDE and the IOL controller is augmented by using the estimated uncertainties. The resulting controller is designated as the IOL+UDE controller. While the IOL+UDE controller achieves the objective of robustification of the IOL control quite effectively, the implementation of the same requires complete state vector as well as time derivatives of output. Since these requirements may not be satisfied at all times in practical situations, the issues are required to be addressed. This article extends the work presented in Talole and Phadke (2009) with an objective of addressing these issues. To this end, a brief review of the IOL + UDE controller designed in Talole and Phadke (2009) is presented in this section for the sake of completeness and ready reference. Modifications of the controller are presented subsequently.

3.1 *IOL + UDE controller*

Consider the dynamics given in (1), (2). Since in practice the system model given by (1), (2) is rarely known exactly, it becomes necessary to account for the modelling errors and inaccuracies. To account for these uncertainties, the system dynamics is re-written as

$$\begin{aligned} \dot{x} &= \hat{f}(x) + \Delta f(x) + (\hat{g}(x) + \Delta g(x))u \\ y &= c(x) \end{aligned} \quad (8)$$

where $\hat{f}(x)$ and $\hat{g}(x)$ represent the nominal part of the system and $\Delta f(x)$ and $\Delta g(x)$ represent their associated uncertainties. In view of the uncertainties, (3) will get modified as

$$y^{(\lambda)} = \hat{a}(x) + \hat{b}(x)u + d \tag{9}$$

where d represents the effect of the uncertainties and external unmeasurable disturbances, if any. To address the issue of uncertainties, the IOL control of (4) is augmented as

$$u(\mathrm{IOL} + \mathrm{UDE}) = \frac{1}{\hat{b}(x)}(u_a + u_d + \nu) \tag{10}$$

where

$$u_a \triangleq -\hat{a}(x) \tag{11}$$

and u_d is that part of control which cancels the effect of uncertainties, external disturbances and non-linearities. The controller of (10) was designated as IOL + UDE controller in Talole and Phadke (2009). Substituting (10) in (9) leads to

$$y^{(\lambda)} = u_d + \nu + d \tag{12}$$

and from (12) one gets

$$d = y^{(\lambda)} - u_d - \nu \tag{13}$$

In view of (13) and following the procedure given in Zhong and Rees (2004) and Talole and Phadke (2009), the estimate of d is obtained as

$$\hat{d} = G_f(s)d = G_f(s)(y^{(\lambda)} - u_d - \nu) \tag{14}$$

where $\hat{d}$ is an estimate of d and $G_f(s)$ is the first-order low pass filter with unity steady-state gain and having a time constant of τ

$$G_f(s) = \frac{1}{1 + s\tau} \tag{15}$$

Selecting $u_d = -\hat{d}$ and using (14) gives

$$u_d = -G_f(s)(y^{(\lambda)} - u_d - \nu) \tag{16}$$

and solving for u_d leads to

$$u_d = -\frac{G_f(s)}{(1 - G_f(s))}(y^{(\lambda)} - \nu) \tag{17}$$

Substitution of (6), (11) and (17) in (10) gives the IOL + UDE controller as

$$u(\mathrm{IOL} + \mathrm{UDE}) = \frac{1}{\hat{b}(x)}\left(-\hat{a}(x) - \frac{1}{\tau}y^{(\lambda-1)} + \nu + \frac{1}{\tau}\int \nu \,\mathrm{d}t\right) \tag{18}$$

Clearly under the assumption of $\hat{d} \approx d$, application of this control to the dynamics of (9) results in the same error dynamics as given by (7) thus eliminating the effect of uncertainties and so robustifying the IOL controller. Similar to the IOL control of (4), the implementation of the IOL + UDE controller of (10) requires the states of the plant as well as up to $(\lambda - 1)$th order time derivatives of output. The issue of requirement of the system states is addressed by modifying the IOL + UDE control formulation as presented in the next section.

3.2 *Modified IOL + UDE controller*

In this section, the IOL+UDE controller formulation is modified to address the issue of requirement of plant states. To this end, the dynamics of (3) is re-written as

$$y^{(\lambda)} = \bar{a}\bar{y} + (a(x) - \bar{a}\bar{y}) + (b(x) - \hat{b}_0)u + \hat{b}_0 u + d' \tag{19}$$

where $\bar{a} = [\hat{a}_1\ \hat{a}_2 \cdots \hat{a}_\lambda]$ and $\bar{y} = [\,y\ \dot{y}\ \ddot{y}\ \cdots\ y^{(\lambda-1)}]^T$ with $\hat{a}_i$ being the nominal values of the coefficients and the quantity d' represents the effect of the external unmeasurable disturbances, if any. In writing (19), it is assumed that the nonlinear function, $a(x)$, may have components which are linear in output, y, and its higher order derivatives up to the order of $(\lambda - 1)$. If such is not the case, the quantity $\bar{a}\bar{y}$ is taken as zero. Re-writing (19) as

$$y^{(\lambda)} = \bar{a}\bar{y} + \hat{b}_0 u + d \tag{20}$$

where $d \triangleq (a(x) - \bar{a}\bar{y}) + (b(x) - \hat{b}_0)u + d'$ where $\hat{b}_0$ is a chosen constant that approximates $b(x)$. It may be noted that by considering $\bar{a}\bar{y}$ as a known dynamics, the magnitude of the to be estimated uncertainty, d, and thus the resulting control effort gets reduced. It may also be noted that the uncertainty, d, consists of all state dependent nonlinearities and thus the requirement of availability of the plant states does not exist. Following the procedure outlined in the earlier section, the modified IOL + UDE control is obtained as

$$u(\mathrm{IOL} + \mathrm{UDEM}) = \frac{1}{\hat{b}_0}(u_a + u_d + \nu) \tag{21}$$

where

$$u_a = -\bar{a}\bar{y} \tag{22}$$

and

$$u_d = -\hat{d} = -\frac{G_f(s)}{(1 - G_f(s))}(y^{(\lambda)} - \nu) \tag{23}$$

with the outer loop control, ν, is as defined in (6). Substitution of (6), (22), and (23) in (21) gives the modified IOL + UDE controller and is designated as IOL + UDEM controller. In time-domain form, the IOL + UDEM controller can be written as

$$u(\mathrm{IOL}+\mathrm{UDEM}) = \frac{1}{\hat{b}_0}\left(-\bar{a}\bar{y} - \frac{1}{\tau}y^{(\lambda-1)} + \nu + \frac{1}{\tau}\int \nu\,\mathrm{d}t\right) \tag{24}$$

The resulting controller does not require plant states, however needs up to $(\lambda-1)$th order time derivatives of y. The issue of obtaining the output derivatives is addressed by designing a robust observer presented in the next section.

4. UDE based controller-observer structure

As is obvious from (24), the IOL+UDEM controller requires output derivatives up to the order of $(\lambda-1)$ for its implementation. As this may not be practical in all situations, the derivatives need to be estimated from the available output. As a solution to this problem, design of a UDE based robust observer is proposed as follows.

4.1 *UDE based observer*

To design an observer for estimation of the output derivatives, the system of (20) is re-written in a phase variable state space form as

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= y_3 \\ &\vdots \\ \dot{y}_\lambda &= \hat{a}_1 y_1 + \hat{a}_2 y_2 + \cdots + \hat{a}_\lambda y_\lambda + \hat{b}_0 u + d \\ y &= y_1 \end{aligned} \tag{25}$$

Defining the state vector as $x_p = [y_1\ y_2\ \cdots\ y_{\lambda-1}\ \ y_\lambda]^T = [y\ \dot{y}\ \ddot{y}\ \cdots\ y^{(\lambda-1)}]^T$, the system of (25) is written as

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_p u + B_d d \\ y_p &= C_p x_p \end{aligned} \tag{26}$$

where

$$A_p = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & \ddots & & \\ 0 & 0 & 0 & \cdots & 1 \\ \hat{a}_1 & \hat{a}_2 & \hat{a}_3 & \cdots & \hat{a}_\lambda \end{bmatrix};\quad B_p = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \hat{b}_0 \end{bmatrix};\quad B_d = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

and $C_p = [1\ \ 0\ \ \cdots\ \ 0\ \ 0]$. It may be noted that a conventional Luenberger observer will not be able to provide accurate state estimation for the plant of (26) owing to the presence of the uncertainty. In view of this, an Luenberger like observer of the following form is proposed as

$$\begin{aligned} \dot{\hat{x}}_p &= A_p \hat{x}_p + B_p u + B_d \hat{d} + L(y_p - \hat{y}_p) \\ \hat{y}_p &= C_p \hat{x}_p \end{aligned} \tag{27}$$

where $L = [\beta_1\ \beta_2\ \cdots\ \beta_\lambda]^T$ is the observer gain vector. The observer, however, needs the estimate of the uncertainty, i.e. $\hat{d}$. As the uncertainty is the same as present in (20), the UDE estimated uncertainty is used in the observer (27) too giving rise to the UDE based controller-observer structure. It may be noted that the proposed observer does not require an accurate plant model and is robust.

Noting that $\hat{x}_p = [\hat{y}_1\ \hat{y}_2\ \cdots\ \hat{y}_{\lambda-1}\ \hat{y}_\lambda]^T = [\hat{y}\ \dot{\hat{y}}\ \ddot{\hat{y}}\ \cdots\ \hat{y}^{(\lambda-1)}]^T$, the IOL+UDEM control of (21) can be computed by using the UDE based observer estimated output derivatives and thus the issue of requirement of output derivatives is addressed. A functional block diagram of the observer based IOL+UDEM control is shown in Figure 1.

5. Closed loop stability

In this section, results on the closed loop stability of the overall system are presented. The IOL+UDEM control (21) using u_a of (22) and ν of (6) evaluated using the observer estimated output derivatives and using $u_d = -\hat{d}$ can be written as

$$\begin{aligned} u = \frac{1}{\hat{b}_0}\big[&-\bar{a}\hat{\bar{y}} + \dot{y}^\star_\lambda + m_1(y^\star_1 - \hat{y}_1) + m_2(y^\star_2 - \hat{y}_2) \\ &+ \cdots + m_{\lambda-1}(y^\star_{\lambda-1} - \hat{y}_{\lambda-1}) + m_\lambda(y^\star_\lambda - \hat{y}_\lambda) - \hat{d}\big] \end{aligned} \tag{28}$$

Denoting the reference state vector $R = [y^\star_1\ \ y^\star_2\ \cdots\ \ y^\star_{\lambda-1}\ \ y^\star_\lambda]^T$ and defining the state feedback gain vector, K_p as $K_p = [k_1\ k_2\ \cdots\ k_{\lambda-1}\ k_\lambda]$ with the elements as $k_1 = \frac{m_1+\hat{a}_1}{\hat{b}_0}$, $k_2 = \frac{m_2+\hat{a}_2}{\hat{b}_0}, \ldots, k_{\lambda-1} = \frac{m_{\lambda-1}+\hat{a}_{\lambda-1}}{\hat{b}_0}$, $k_\lambda = \frac{m_\lambda+\hat{a}_\lambda}{\hat{b}_0}$, the controller (28) can be re-written in compact form as

$$u = -K_p\hat{x}_p + K_p R + K_R R - \frac{1}{\hat{b}_0}\hat{d} + \frac{1}{\hat{b}_0}\dot{y}^\star_\lambda \tag{29}$$

where $K_R = [-\frac{\hat{a}_1}{\hat{b}_0}\ \ -\frac{\hat{a}_2}{\hat{b}_0}\ \cdots\ -\frac{\hat{a}_{\lambda-1}}{\hat{b}_0}\ \ -\frac{\hat{a}_\lambda}{\hat{b}_0}]$. It is straightforward to show that the dynamics of reference state vector, R, can be written as

$$\dot{R} = A_p R + A_o R + B_d \dot{y}^\star_\lambda \tag{30}$$

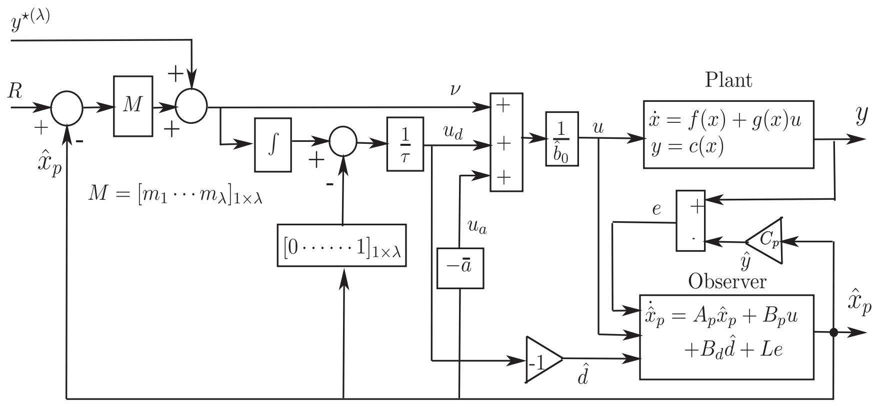

Figure 1. Functional block diagram of observer based IOL+UDEM controller.

where A_o is defined as

$$A_0 = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ & & \ddots & & \\ 0 & 0 & 0 & \cdots & 0 \\ -\hat{a}_1 & -\hat{a}_2 & -\hat{a}_3 & \cdots & -\hat{a}_\lambda \end{bmatrix}$$

Defining the state tracking error, $e_c = R - x_p$ and using (26), (29) and (30), and carrying out some simplifications leads to the following state tracking error dynamics:

$$\dot{e}_c = (A_p - B_pK_p)e_c - (B_pK_p)e_o - B_d\tilde{d} \tag{31}$$

where $\tilde{d} = d - \hat{d}$ is the uncertainty estimation error and $e_o = x_p - \hat{x}_p$ is the observer state estimation error vector.

Next, the observer error dynamics can be obtained by subtracting (27) from (26) as

$$\dot{e}_o = (A_p - LC_p)e_o + B_d\tilde{d} \tag{32}$$

Finally, the uncertainty estimation error dynamics is obtained. From (14), the estimate of the uncertainty, d, is given as

$$\hat{d} = G_f(s)d \tag{33}$$

where $\hat{d}$ is an estimate of d and $G_f(s)$ is a first-order filter as given in (15). From (15) and (33) one has

$$d = (1 + s\tau)\hat{d} \tag{34}$$

With the uncertainty estimation error defined as $\tilde{d} = d - \hat{d}$ and carrying out some simplifications gives

$$\dot{\tilde{d}} = -\frac{1}{\tau}\tilde{d} + \dot{d} \tag{35}$$

Combining (31), (32) and (35) yields the following error dynamics for the Controller–Observer combination

$$\begin{bmatrix} \dot{e}_c \\ \dot{e}_o \\ \dot{\tilde{d}} \end{bmatrix} = \begin{bmatrix} (A_p - B_pK_p) & -(B_pK_p) & -B_d \\ 0 & (A_p - LC_p) & B_d \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} e_c \\ e_o \\ \tilde{d} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{d} \tag{36}$$

From (36), the system matrix being in a block triangular form, it can be easily verified that the eigenvalues of the system matrix are given by

$$|sI - (A_p - B_pK_p)||sI - (A_p - LC_p)|\left|s - \left(-\frac{1}{\tau}\right)\right| = 0 \tag{37}$$

Noting that the pair (A_p, B_p) is controllable and the pair (A_p, C_p) is observable, the controller gain, K_p, and the observer gain, L, can be chosen appropriately along with $\tau > 0$ to ensure stability for the error dynamics. As the error dynamics is driven by $\dot{d}$, it is obvious that for bounded $|\dot{d}|$, bounded input-bounded output stability is assured. Also, if the rate of change of uncertainty is negligible, i.e. if $\dot{d} \approx 0$, then the error dynamics is asymptotically stable. As has been stated, the error dynamics (36) is asymptotically stable if $\dot{d} \approx 0$. However, asymptotic stability for the error dynamics can always be assured if some higher derivative of the uncertainty is equal to zero. For example, if $\dot{d} \neq 0$ but some higher derivative of d is zero, then the asymptotic stability of the error dynamics can be guaranteed by choosing an appropriate higher order filter in place of the one chosen in (15) as shown in Talole and Phadke (2009). In general, if kth derivative of the disturbance, d, is zero, then it can be easily verified that asymptotic stability for the error dynamics is guaranteed if one chooses the filter as

$$G_f = \frac{(1+\tau s)^k - (\tau s)^k}{(1+\tau s)^k} \tag{38}$$

Now some comments on the choice of observer poles and filter time constant are in order. From (35) it can be observed that choice of the value of filter time constant, τ, affects the uncertainty estimation error accuracy, i.e. smaller value leads to a smaller estimation error. However, in the present formulation, the uncertainty estimation is obtained by use of estimated states and thus, it is important to select the filter time constant and the observer poles appropriately to achieve the desired performance for the observer based IOL+UDEM controller. As is obvious from (17), the estimate of the uncertainty, $\hat{d}$, is given by

$$\hat{d} = -u_d = -\frac{1}{\tau}\left(\hat{y}^{(\lambda-1)} - \int \nu \, \mathrm{d}t\right) \tag{39}$$

Noting the fact that ν employs estimates of output and its derivatives, it is obvious from (39) that the accuracy of the uncertainty estimation depends upon the accuracy of the estimates of the output derivatives. Thus the estimation of the uncertainties is only accurate when the output derivatives are estimated well in time. The natural step in such situations would be to design an observer with a bandwidth higher or comparable to the bandwidth of the filter. As a guideline, it can be stated that the observer bandwidth should be equal or greater than the filter bandwidth for satisfactory uncertainty estimation.

6. Extension to MIMO systems

The UDE based controller-observer structure designed for robustification of the IOL controller can be easily extended for MIMO systems as well. For example, consider a MIMO nonlinear dynamic system described by

$$\dot{x} = f(x) + G(x)u \tag{40}$$

$$y = c(x) \tag{41}$$

where $f \triangleq [f_1\ f_2\ \cdots\ f_n]^T$; $G \triangleq [g_1\ g_2\ \cdots\ g_m]$; $c \triangleq [c_1\ c_2\ \cdots\ c_m]^T$ and $x \in R^n$ is the state, $u(t) \in R^m$ represents the control and $y(t) \in R^m$ is the output vector. The functions $f: R^n \to R^n$, $c: R^n \to R^m$ are assumed to be continuously differentiable nonlinear functions and $G: R^n \to R^{n\times m}$ is a continuous function of x. Suppose that the desired output trajectory is specified by $y^\star(t)$, $0 \le t \le t_f$, which is an outcome of Equations (40), (41) for some feasible reference control $u^\star(t)$ for all $t \in [0, t_f]$. The objective is to design a controller such that $y(t)$ asymptotically tracks a reference output trajectory $y^\star(t)$. Let λ_i be the relative degree of y_i, the ith component of output y. Following the IOL theory, one can obtain the output dynamics as

$$Y_\lambda = A(x) + B(x)u \tag{42}$$

where $Y_\lambda \triangleq [y_1^{(\lambda_1)}, y_2^{(\lambda_2)}, \ldots, y_m^{(\lambda_m)}]^T$,

$$A(x) \triangleq [\alpha_1\ \alpha_2 \ldots \alpha_m]^T; \quad \alpha_i(x) = L_f^{\lambda_i} c_i, \quad i = 1, \ldots, m \tag{43}$$

and $B(x) \in R^{m\times m}$ has each of its rows in the form of

$$b_i = \left[L_{g_1}(L_f^{\lambda_i - 1}(c_i)), \ldots, L_{g_m}(L_f^{\lambda_i - 1}(c_i))\right], \quad i = 1, \ldots, m \tag{44}$$

Assuming that $B(x)$ can be written as

$$B(x) = \hat{B}_0 + \Delta B(x) \tag{45}$$

with $\hat{B}_0$ is a non-singular diagonal matrix with diagonal elements as $\hat{b}_{ii}$; $i = 1, 2, \ldots, m$ and $\Delta B(x)$ being the associated uncertainty. In view of (45), the dynamics of (42) is written as

$$Y_\lambda = \bar{A}\bar{Y} + (A(x) - \bar{A}\bar{Y}) + (B(x) - \hat{B}_0)u + \hat{B}_0 u + D' \tag{46}$$

where $\bar{A}$ and $\bar{Y}$ are appropriately defined matrices and D' represents the effect of the external unmeasurable disturbances, if any. Re-writing (46) as

$$Y_\lambda = \bar{A}\bar{Y} + \hat{B}_0 u + D \tag{47}$$

where $D \triangleq (A(x) - \bar{A}\bar{Y}) + (B(x) - \hat{B}_0)u + D'$. From (47), it is obvious that the dynamics of each output are decoupled and hence represents as a single-input single-output systems for which the UDE based controller observer structure can be designed independently as outlined earlier.

7. Illustrative example: control of wing-rock motion

To demonstrate the effectiveness, results of the control of wing-rock motion using the proposed approach is presented in this section. The wing-rock motion (Nayfeh, Elzebda, and Mook 1989; Monahemi and Krstic 1996) represents a self induced limit cycle oscillation, i.e. oscillatory rolling motion in the presence of some initial disturbance in slender delta wings that needs to be prevented. The problem has gained considerable attention owing to its practical importance and to this end, various approaches have been presented in the literature for the control of the wing-rock motion (Luo and Lan 1993; Singh, Yim, and Wells 1995; Monahemi and Krstic 1996; Araujo and Singh 1998). In general, the controllers designed on the assumption of knowledge of the aerodynamic parameters will not offer satisfactory performance in the presence of uncertainty and modelling inaccuracies. In fact although different models of wing-rock motion have been proposed in literature, these models are only approximate and thus controllers are required to cater for these inaccuracies. Here the observer based IOL+UDEM controller is employed for the control of wing-rock motion. The effectiveness of this approach is presented for wing-rock motion suppression (stabilisation) as well as tracking of the desired roll command. To this end, consider the dynamics of the wing-rock motion as taken from Monahemi and Krstic (1996):

$$\begin{aligned} \dot{\phi} &= p \\ \dot{p} &= c_1 + c_2\phi + c_3 p + c_4|\phi|p + c_5|p|p + c_6 u \\ y &= \phi \end{aligned} \tag{48}$$

The co-efficients c_1 till c_6 are the aerodynamic co-efficients describing the roll dynamics and ϕ and p are the roll angle (rad) and roll rate (rad/s), respectively. In stabilisation, the objective is to suppress the motion by regulating the output, i.e. ϕ to 0 deg. With ϕ as output, the system has relative degree of two and to this end, differentiating y twice gives

$$\ddot{y} = c_1 + c_2\phi + c_3 p + c_4|\phi|p + c_5|p|p + c_6 u \tag{49}$$

Let $c_2 = \hat{c}_2 + \Delta c_2$, $c_3 = \hat{c}_3 + \Delta c_3$ and $c_6 = \hat{c}_6 + \Delta c_6$ where $\hat{c}_2$, $\hat{c}_3$ and $\hat{c}_6$ are the nominal values of the respective parameters and Δc_2, Δc_3 and Δc_6 being their associated uncertainties. To this end, the dynamics of (49) can be represented in the same form as that of (20) as

$$\ddot{y} = \hat{c}_2\phi + \hat{c}_3 p + \hat{c}_6 u + d \tag{50}$$

where $\bar{a} = [\hat{c}_2\ \hat{c}_3]$, $\bar{y} = [\phi\ p]^T$, $\hat{b}_0 = \hat{c}_6$ and $d = c_1 + \Delta c_2\phi + \Delta c_3 p + c_4|\phi|p + c_5|p|p + \Delta c_6 u + d'$. Note that the uncertainty, d, consists of the term involving state dependent nonlinearities. An observer of the form of (27) for the present problem is obtained as

$$\begin{aligned} \dot{\hat{\phi}} &= \hat{p} + \beta_1 e_{o1} \\ \dot{\hat{p}} &= \hat{c}_2\hat{\phi} + \hat{c}_3\hat{p} + \hat{c}_6 u + \hat{d} + \beta_2 e_{o1} \end{aligned} \tag{51}$$

where $e_{o1} \triangleq \phi - \hat{\phi}$, and $L = [\beta_1\ \beta_2]^T$ is the observer gain vector. Using the observer estimated states in evaluation of u_a, u_d and ν in (21) as

$$\begin{aligned} u_a &= -(\hat{c}_2\hat{\phi} + \hat{c}_3\hat{p}) \\ u_d &= -(\ddot{y} - \nu)\left(\frac{G_f}{1 - G_f}\right) \\ \nu &= \ddot{y}^{\star} + m_2\dot{\tilde{y}} + m_1\tilde{y} \end{aligned}$$

where $\tilde{y} = \phi^{\star} - \hat{\phi}$. Using $G_f(s)$ as given in (15), the controller (21) for the present problem is obtained as

$$u(\text{IOL} + \text{UDEM}) = \frac{1}{\hat{c}_6}\left[-(\hat{c}_2\hat{\phi} + \hat{c}_3\hat{p}) - \frac{\hat{p}}{\tau} + \frac{(1+\tau s)}{\tau s}\nu\right] \tag{52}$$

Equation (52) can be written in time-domain form as

$$u(\text{IOL} + \text{UDEM}) = \frac{1}{\hat{c}_6}\left[-(\hat{c}_2\hat{\phi} + \hat{c}_3\hat{p}) - \frac{\hat{p}}{\tau} + \nu + \frac{1}{\tau}\int \nu\, \mathrm{d}t\right] \tag{53}$$

For simulations, we desire a settling time of 2 s and damping ratio of 0.8 for the error dynamics and accordingly the feedback gains m_1 and m_2 are chosen. The time constant, τ, of the filter of (15) is taken as 0.01. Following the guideline that the observer bandwidth should be greater or comparable to the filter bandwidth as discussed earlier, the observer gains, β_i's are obtained by placing the observer poles at -100. The nominal values of the parameters c_1 till c_6 appearing in (48) for an angle of attack of 30° (Monahemi and Krstic 1996) are given as

$$\begin{aligned} &\hat{c}_1 = 5, \quad \hat{c}_2 = -26.7, \quad \hat{c}_3 = 0.765 \\ &\hat{c}_4 = -2.9, \quad \hat{c}_5 = -2.5, \quad \hat{c}_6 = 0.75 \end{aligned} \tag{54}$$

First simulations are carried out assuming there are no uncertainties in the parameters c_2, c_3 and c_6 and the results are presented in Figure 2. In Figure 2(a), the roll angle history is presented and it can be seen that the wing-rock motion is suppressed effectively. It may be noted that the dynamics exhibits limit cycle behaviour when simulated in open loop manner with initial conditions (Talole and Phadke 2009). The actual and estimated roll rate histories are given in Figure 2(b) whereas the control input, i.e. aileron angle history is presented in Figure 2(c). In Figure 2(d), the profile of actual and estimated uncertainty is presented and it can be observed that the UDE has estimated the uncertainty accurately. For this purpose the actual uncertainty, d, is calculated from (50) and its estimate is given by $\hat{d} = -u_d$. Note that the uncertainty is present due to the presence of the state dependent nonlinearities in d.

Next, uncertainties are introduced in all parameters by taking their values as

$$\begin{aligned} &c_1 = 5\sin(15t), \quad c_2 = -20, \quad c_3 = 1 \\ &c_4 = 3\cos(5t), \quad c_5 = 10\sin(10t), \quad c_6 = 0.5 \end{aligned} \tag{55}$$

It may be noted that parameters such as c_4 and c_5 are time-varying, thus introducing large time-varying uncertainty in the system. In Talole and Phadke (2009) it has been shown that with these values, the IOL control results into unstable closed loop system

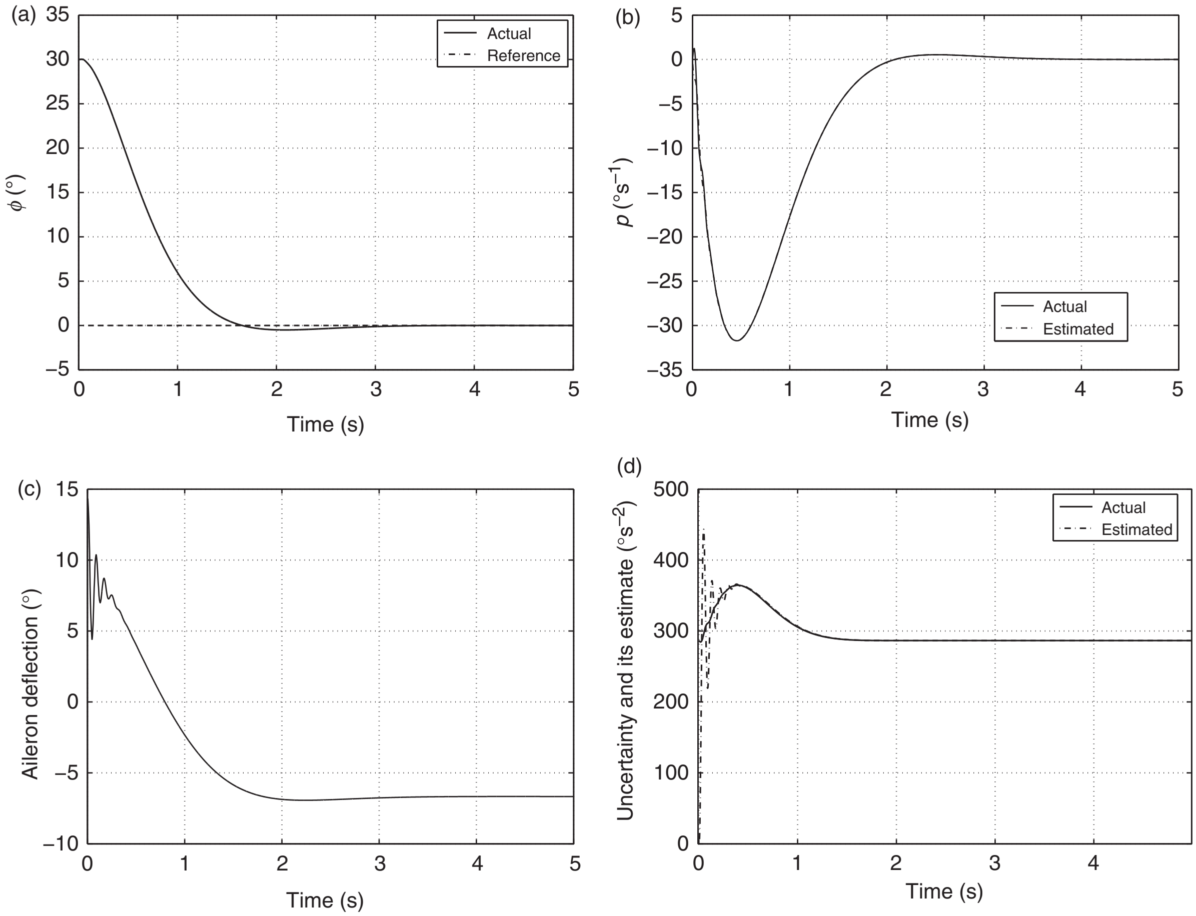

Figure 2. Performance of observer based IOL + UDEM controller in absence of uncertainties: (a) roll angle, (b) roll rate, (c) control history and (d) uncertainty estimation.

while the IOL+UDE controller ensures closed loop stability. Here, to verify the performance of the proposed IOL+UDEM controller, simulations are carried out and the results are presented in Figure 3. From Figure 3(a), it can be noted that the controller has indeed offered the desired robustness notwithstanding the significant uncertainties introduced in the plant. The efficacy of UDE in estimation of uncertainties can be seen from Figure 3(d) from where it can be seen that the UDE has estimated the uncertainty quite accurately in spite of its large and rapid variation. Figure 3(b) and (c) gives the corresponding roll rate and control input histories, respectively. Next, we show the effectiveness of the IOL+UDEM controller for tracking problem wherein the objective is together with suppressing the undesirable wing-rock motion, a pre-specified reference trajectory is to be followed. To this end, it is desired that roll angle to follow a commanded roll angle trajectory of $\phi^{\star}(t) = 20\sin(0.4\pi t)$. Simulations are carried out by introducing uncertainty by taking the plant parameter values as given in (55) and the results are presented in Figure 4. In Figure 4(a) the reference and actual roll angle profiles are presented and it can be observed that the controller offers satisfactory tracking in spite of the large uncertainties introduced into the parameters. The corresponding control input, i.e. aileron angle profile is given in Figure 4(c). The profile of actual and estimated uncertainty is presented in Figure 4(d).

The simulation results presented so far for the wing-rock problem use filter time constant of 0.01 with observer poles placed at -100. This choice of the observer poles satisfy the guideline that observer bandwidth should be higher or comparable with the bandwidth of the filter. To verify the effect of observer poles on the performance, simulations have also been carried out by considering observer poles as -50 and -200. When the observer poles are taken as -50, the simulation results of the performance of the observer based IOL+UDEM controller in the absence of uncertainty are shown in Figure 5 and comparing the results with the one presented in Figure 2 clearly brings out the fact that slower observer has resulted in the deterioration in the performance. On the other hand,

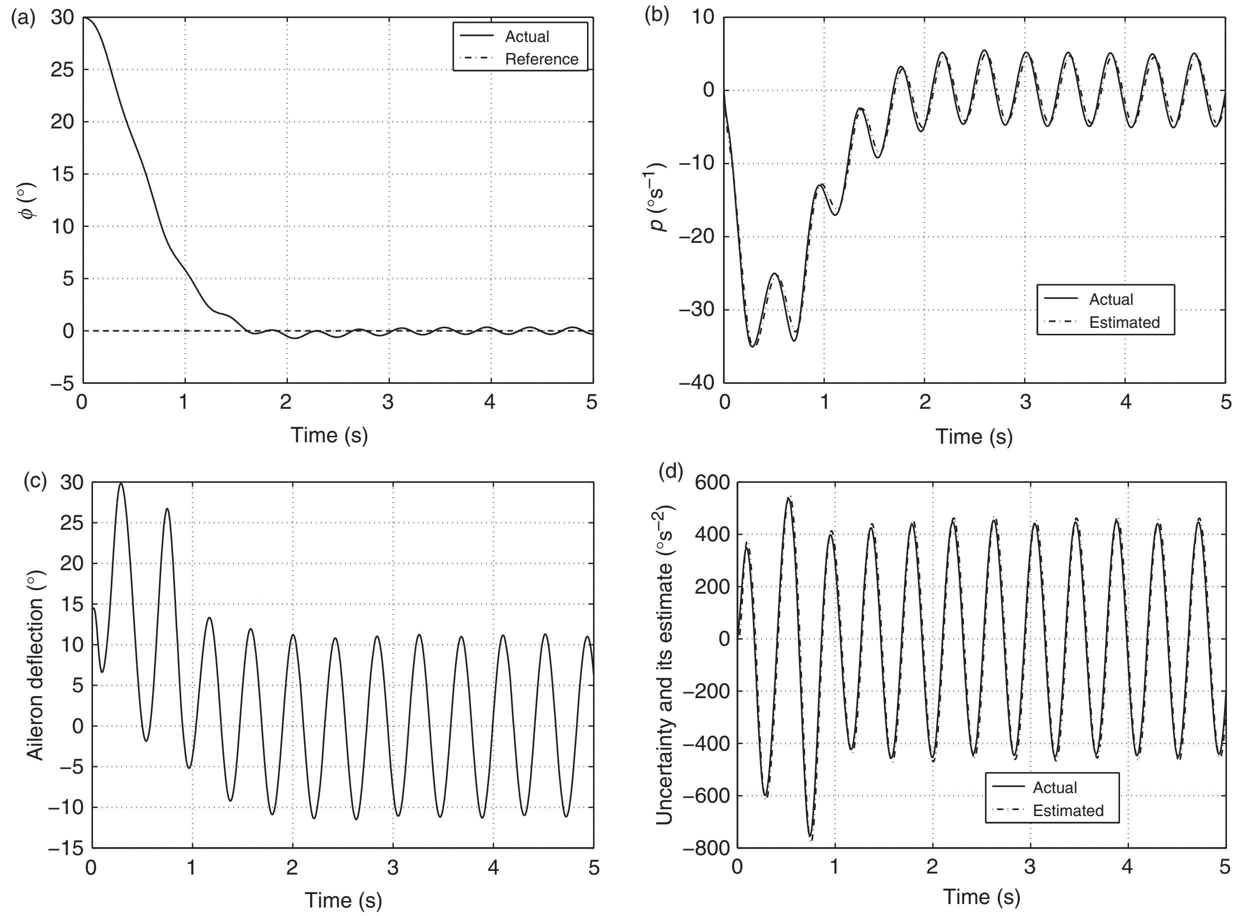

Figure 3. Performance of observer based IOL+UDEM controller in stabilisation: (a) roll angle, (b) roll rate, (c) control history and (d) uncertainty estimation.

the simulation results with observer poles placed at -200 are found to be quite similar to that of the one with observer poles placed at -100.

8. Experimental validation

In this section, the efficacy of the proposed UDE based controller–observer structure is demonstrated experimentally on the Quanser's DC servo motor motion control module.

8.1 *Mathematical model*

In the Quanser's DC servo motor motion control platform, the transfer function for the linearised dynamics of the Quanser's module as given in Quanser Inc. (2008a) is

$$\frac{\theta_l(s)}{V_m(s)} = \frac{\eta_g \eta_m K_t K_g}{J_{eq} R_m s^2 + (B_{eq} R_m + \eta_g \eta_m K_m K_t K_g^2)s} \quad (56)$$

The quantities appearing in these expressions are: θ_l motor load angle, η_m the motor efficiency, η_g the gearbox efficiency, K_t the motor torque constant, K_m the back EMF constant, K_g the gearbox ratio, B_{eq} the viscous damping coefficient, R_m the armature resistance, J_{eq} the equivalent moment of inertia at the load and V_m the motor control voltage. The aim is to design a controller that will force the load angle, θ_l, to track its desired trajectory. Here the IOL+UDEM controller is designed to achieve this objective. To this end, defining $y = \theta_l$, the transfer function (56) is written as

$$\ddot{y} = -\frac{1}{J_{eq} R_m}(B_{eq} R_m + \eta_g \eta_m K_m K_t K_g^2)\dot{y} + \frac{\eta_g \eta_m K_t K_g}{J_{eq} R_m} V_m \quad (57)$$

Now to demonstrate the efficacy of the proposed approach, it is assumed that the dynamics given by $\left[-\frac{1}{J_{eq} R_m}(B_{eq} R_m + \eta_g \eta_m K_m K_t K_g^2)\dot{y}\right]$ in (57) is unknown and thus represents the uncertainty. To this end,

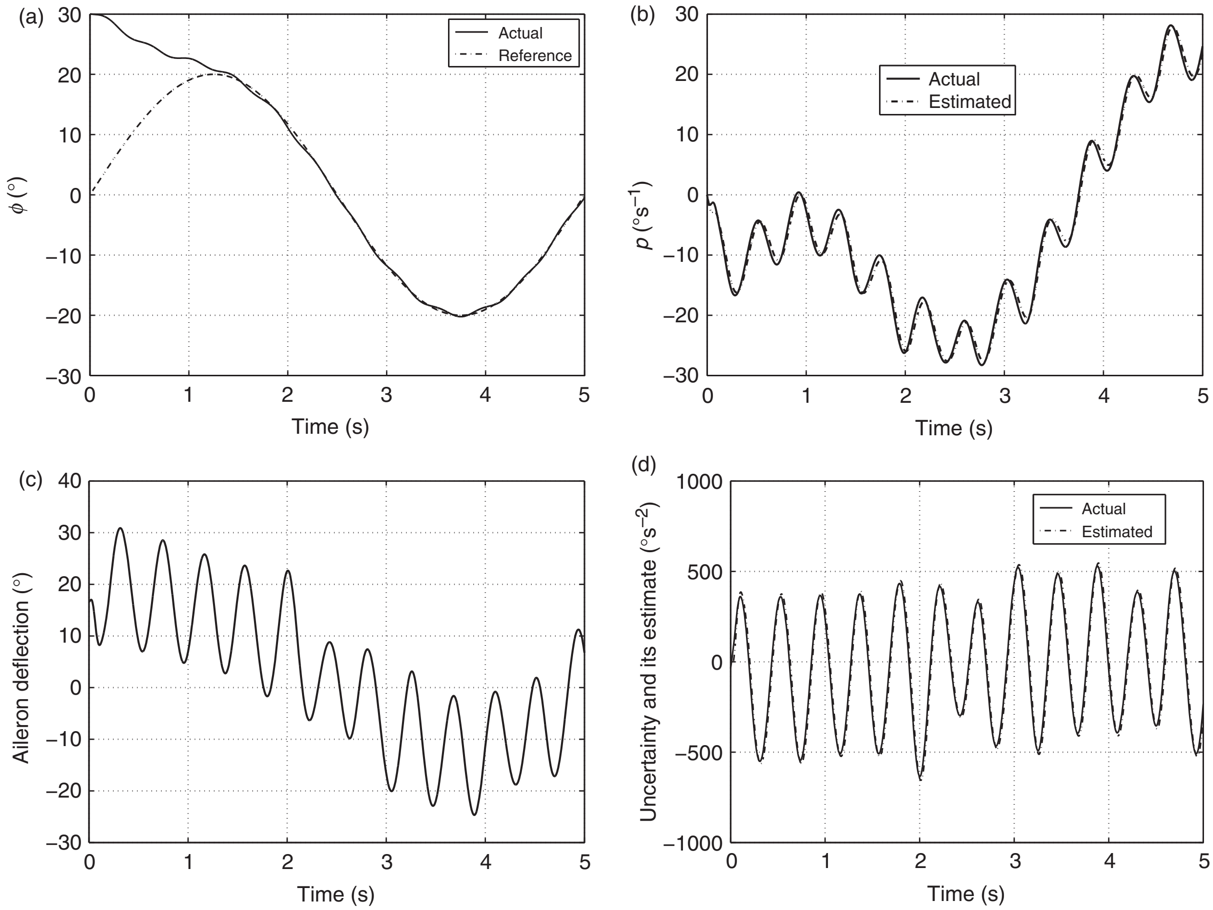

Figure 4. Performance of observer based IOL+UDEM controller in tracking: (a) roll angle, (b) roll rate, (c) control history and (d) uncertainty estimation.

defining the quantities, d, and $\hat{b}_0$ as

$$d \triangleq \frac{-(B_{eq}R_m + \eta_g\eta_m K_m K_t K_g^2)}{J_{eq}R_m}\dot{y} \quad \text{and} \tag{58}$$

$$\hat{b}_0 \triangleq \frac{\eta_g\eta_m K_t K_g}{J_{eq}R_m} \tag{59}$$

the dynamics of (57) reduces to the form of (20) as

$$\ddot{y} = d + \hat{b}_0 V_m \tag{60}$$

where $\bar{a}\bar{y}$ has been assumed to be zero.

For the dynamics of (60), the controller and the observer design is then carried out in a similar manner as outlined in Sections 3 and 4. The resulting IOL+UDEM controller using the observer estimated states is obtained as

$$V_m = \frac{1}{\hat{b}_0}[\ddot{y}^\star + m_1(y^\star - \hat{y}) + m_2(\dot{y}^\star - \dot{\hat{y}}) - \hat{d}] \tag{61}$$

8.2 *Simulation results*

Simulations are carried out for the position control problem, and the results are validated through experimentation. The nominal values of the Quanser's DC servo parameters as taken from Quanser Inc. (2008b) are: $\eta_g = 0.9$, $\eta_m = 0.69$, $K_t = 0.00767\,\text{Nm}$, $K_m = 0.00767\,\text{V/(rad s}^{-1})$, $K_g = 70$, $R_m = 2.6\Omega$, $B_{eq} = 0.004\,\text{Nm/(rad s}^{-1})$ and $J_{eq} = 0.0021\,\text{kg} - \text{m}^2$. The time constant, τ, of the first-order filter is taken as 0.01. The controller gains, m_i, are obtained by placing the poles at $(1 + \frac{\tau_c}{2}s)^2$ with a time constant of $\tau_c = 0.1$. The observer gains, β_i's are obtained by placing the observer poles at $(1 + \frac{\tau_o}{2}s)^2$ with a time constant of $\tau_o = \frac{\tau_c}{5}$. Note that the observer time constant is one-fifth that of the controller. The initial conditions are assumed to be zero for the plant as well as the observer. The desired position trajectory, $y^\star$, is taken as square wave with a magnitude 30° and frequency $2\,\text{rad s}^{-1}$. Simulations are carried out using the observer based IOL+UDEM controller and also IOL+UDE controller wherein the IOL+UDE

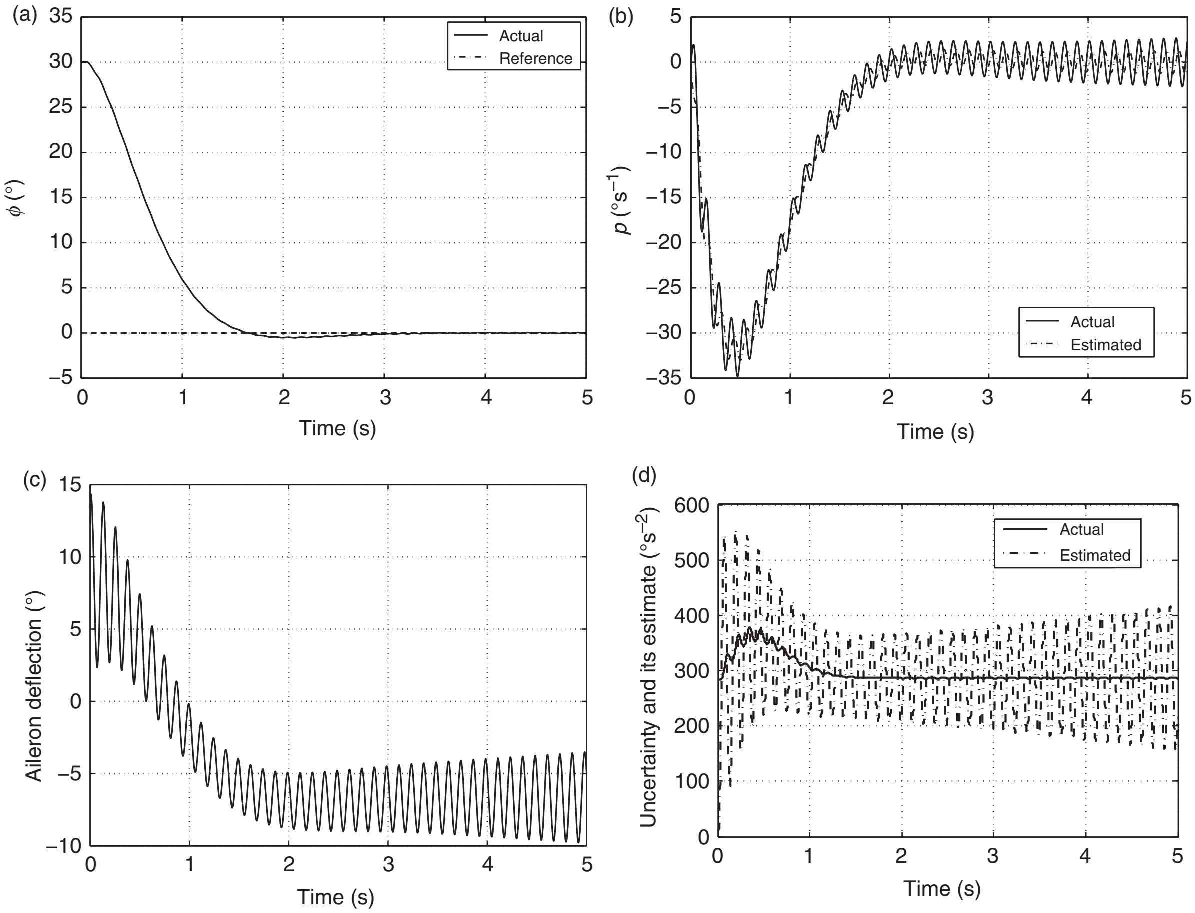

Figure 5. Performance of observer based IOL+UDEM controller in stabilisation: observer poles at −50: (a) roll angle, (b) roll rate, (c) control history and (d) uncertainty estimation.

controller is designed for the dynamics of (60) by following the procedure discussed in Section 3.1. As stated earlier, the IOL+UDE controller uses actual states of the plant for feedback purpose. The results are presented in Figure 6. In Figure 6(a), the tracking performance is presented from where one can observe that the motor shaft has tracked the reference trajectory quite accurately. The shaft velocity and the control input histories are as shown in Figures 6(b) and (c), respectively. The actual and estimated uncertainty are shown in Figure 6(d) from where it can be observed that the UDE has estimated the uncertainty quite accurately. In the simulation results presented in Figure 6, the derivatives of reference signal required in the controller are taken zero as the considered reference is a square wave signal. When the reference is a smooth signal, it is possible to use the higher derivatives of the reference in the controller. For example, taking the desired position trajectory as $y^{\star} = 30\sin(2t)^{\circ}$, simulations are carried out and the results are presented in Figure 7. Once again, it can be observed from Figure 7(a) that the tracking performance is quite satisfactory. The corresponding shaft velocity history is as shown in Figure 7(b) and the control input history is presented in Figure 7(c). The actual and estimated uncertainty history is given in Figure 7(d).

8.3 *Experimental validation*

The Quanser's DC servo motion control experimental setup is as shown in Figure 8 and more details about the same can be found in Quanser Inc. (2008a, b). To evaluate the performance of the observer based IOL+UDEM controller experiments are carried out on the Quanser's DC servo motion control module. The nominal values of the various DC servo module are taken same as taken in simulation. The experimental setup consists of Quanser UPM 1503 module, Quanser Q4 data acquisition and control board, Quaser SRVO2 plant, and a PC equipped with necessary softwares including the Quanser WinCon. The Q4 is an hardware in loop data acquisition and

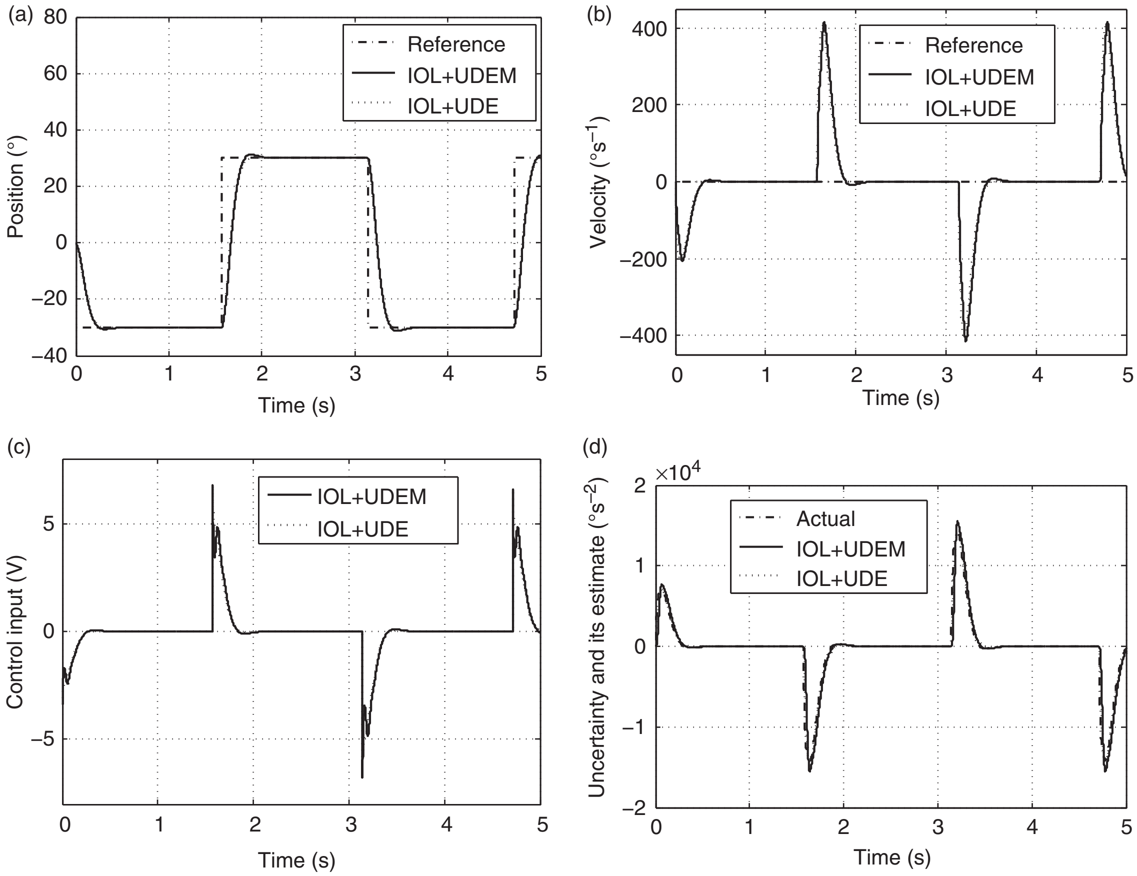

Figure 6. Square wave trajectory: tracking performance.

control board with an extensive range of input and output support. The UPM-1503 is a power amplifier to drive the motor. The WinCon software provides an hardware-in-the-loop simulation environment, i.e. a Simulink based user designed controller can be run in real-time using the WinCon environment. The DC servo motor load angle is measured using an optical encoder. The experimental set permits one to validate a controller designed for the DC servo motion control module to track a desired load angle position. The control signal generated is applied to the module through Quanser's interfacing hardware board. Experiments are carried out by implementing the observer based IOL+UDEM as well as IOL+UDE controllers. For the experimental implementation of the IOL+UDE controller, the motor shaft angle is measured by optical encoder while the shaft velocity is measured by using tacho-generator. The experimental results for both the controllers are plotted together to show the comparative performance. First, experiment is carried out by taking the desired position trajectory, $y^\star$, as a square wave with a magnitude 30° and frequency 2 rad s^{-1} and the results are presented in Figure 9(a), (b). In Figure 9(a), the tracking performance is presented from where one can observe that the shaft has tracked the reference quite accurately. The corresponding control input history is as shown in Figure 9(b). Next, desired position trajectory is taken as $y^\star = 30\sin(2t)^\circ$ and the experimental results are presented in Figures 10(a), (b). From Figure 10(a), it can be observed that the tracking performance is quite satisfactory. The corresponding control input history is given in Figure 10(b). It can also be seen that the tracking performance given in Figures 9 and 10 are quite similar to the simulation results given in Figures 6 and 7, respectively. From the figures, it can be observed that the performance of the observer based IOL+UDEM controller in simulation as well as in experimentation case is quite similar to that of the IOL+UDE controller. It may be noted that the IOL+UDE controller uses actual states of the system. As the implementation of IOL+UDE controller employs motor velocity measured using tacho-generator mounted on motor shaft, the control input are noisy in case of IOL+UDE controller.

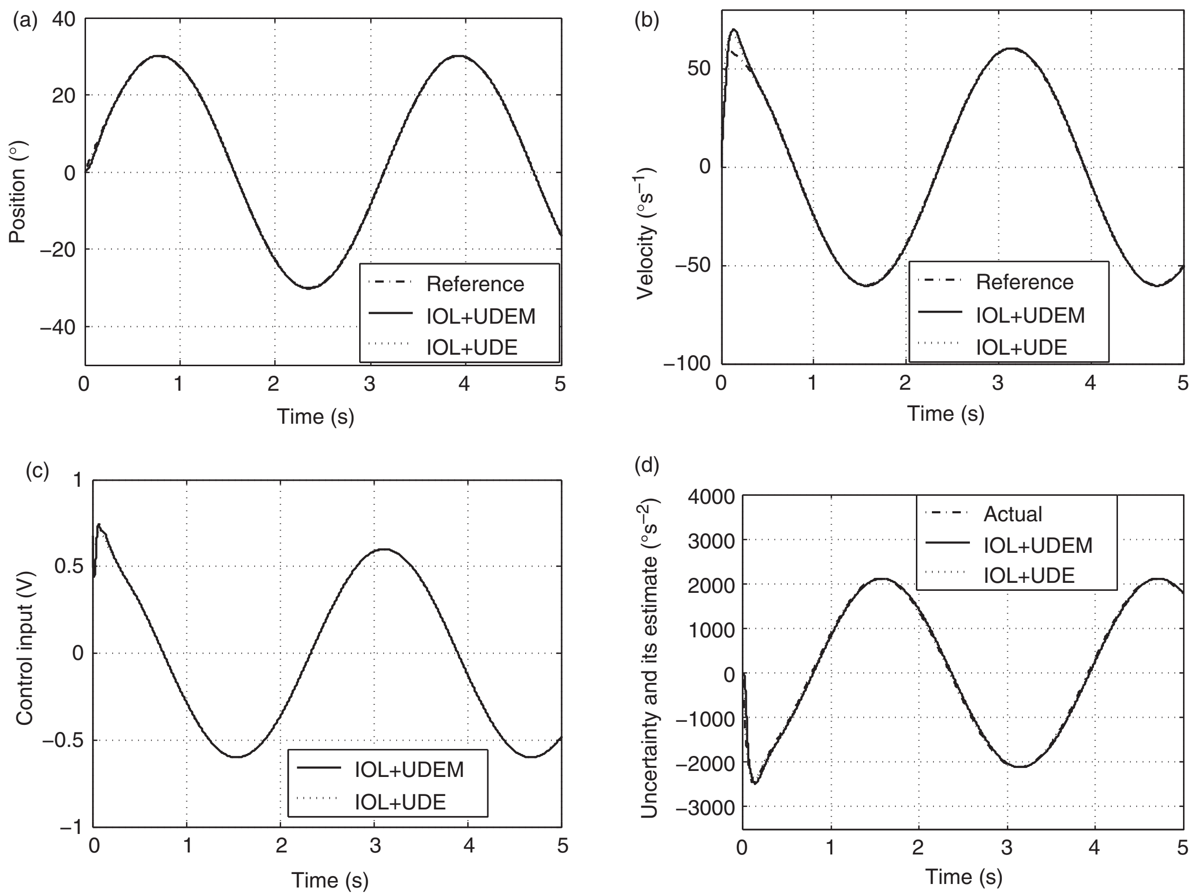

Figure 7. Sinusoidal trajectory: tracking performance.

Figure 8. Quanser DC Servo Motion Control Module.

9. Conclusion

In this article, a new approach based on the UDE technique is proposed to robustify an input–outout linearisation controller. The IOL controller is robustified by augmenting it with the UDE estimated uncertainties. The design does not need system states for feedback as the state dependent nonlinearities have been considered as a part of the estimated uncertainty. As the controller thus designed needs output derivatives, the same are obtained by designing a robust observer which too employs the UDE estimated uncertainties to achieve robustness. The notable

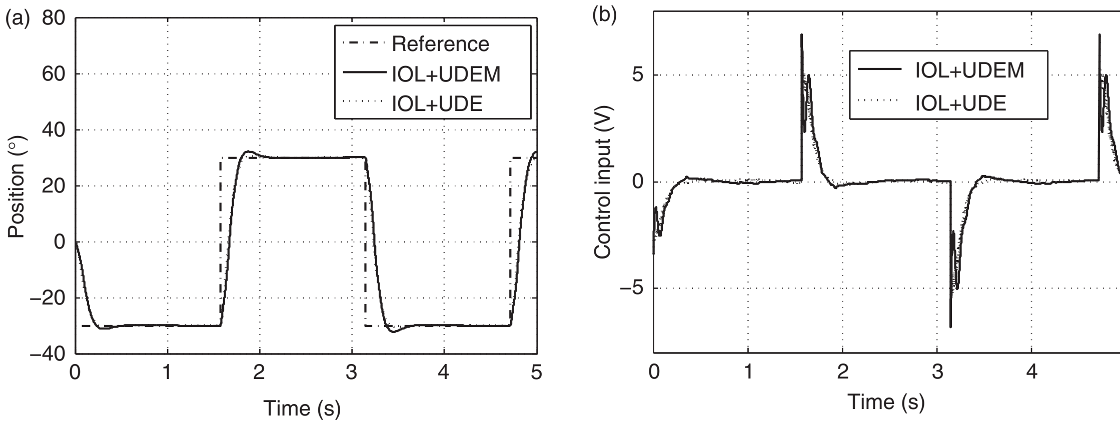

Figure 9. Square wave trajectory: experimental tracking performance.

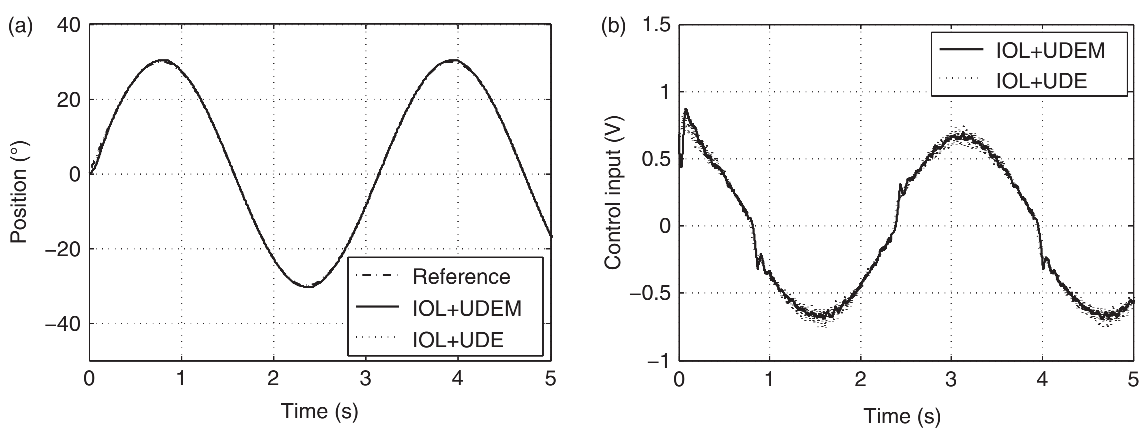

Figure 10. Sinusoidal trajectory: experimental tracking performance.

features of the proposed strategy is that it does not need knowledge of any characteristic of the uncertainty. Also, the design does not require accurate plant model. The closed stability of the overall system is established. Numerical simulation results of application of the proposed strategy to wing-rock motion control problem are presented and it is shown that proposed strategy offers desired performance notwithstanding significant plant uncertainties. Finally, efficacy of the proposed approach is established through experimental validation as applied to the Quanser's DC servo motion control platform.

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