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Robust control of uncertain nonlinear systems with state delays based on an uncertainty and disturbance estimator[‡]

Alon Kuperman[1,2,*,†] and Qing-Chang Zhong[1,3]

[1] *Department of Electrical Engineering and Electronics, The University of Liverpool, U.K.*
[2] *Department of Electrical Engineering and Electronics, Ariel University Center of Samaria, Israel*
[3] *School of Electrical and Information Engineering, Changsha University of Science and Technology, People's Republic of China*

SUMMARY

In this article, one linear and one nonlinear robust control strategies are proposed for uncertain nonlinear continuous-time systems with disturbances and state delays. The approaches are based on the uncertainty and disturbance estimator (UDE) introduced in 2004. In the case of a linear controller, the terms containing the nonlinear functions and time delays are treated as additional disturbances to the system. In the case of a nonlinear controller, both known and unknown delay scenarios are considered. In the case of an unknown time delay, the terms containing the delay are treated as additional disturbances to the system. The algorithms provide excellent tracking and disturbance rejection performance. Simulations are given to show the effectiveness of the strategies, first via a simple example and second via an application to a continuous stirred tank reactor system. Copyright © 2010 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Various systems, such as nuclear and chemical reactors, electrical networks, hydraulic systems, etc. can be modelled as nonlinear systems with time delay. Some of the existing time-delay systems have delays in the state. During the recent decades there has been a substantial increase in attempting to solve the stabilization and control problem of time delay systems [1, 2]. In many practical applications, either the parameters of the system are poorly known or large unpredictable parameter variations and unexpected disturbances exist. For these systems, the common fixed-gain controller is inadequate to achieve satisfactory performance in the entire range over which the characteristics of the system may vary and robust control strategies are needed.

*Correspondence to: Alon Kuperman, Department of Electrical Engineering and Electronics, The University of Liverpool, U.K.
†E-mail: alonk@liv.ac.uk

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Some literature relevant to this topic are briefly outlined here. Different methods to robust stabilization of nonlinear time-delay systems were proposed in [3, 4], whereas global stabilization of such systems was discussed in [5]. Robust control [6, 7] has been proved to be a good candidate to solve the control problem of time-delay systems. Optimal control of a class of nonlinear time-delay systems and related problems were successfully described in [8]. Fuzzy [9], adaptive [10], variable structure [11] and sliding mode approaches [12–14] have gained popularity during the last decade. Neural network-based predictive control [15] and LMI approach [16, 17] were also considered. Recently, state feedback controllers were constructed via backstepping method for a class of nonlinear systems with delay [18–20].

A technique called uncertainty and disturbance estimator (UDE), proposed in [21] as a replacement for time-delay control [22], is able to quickly estimate uncertainties and disturbances and thus provides excellent robust performance. The UDE technique is based on the assumption that a continuous signal can be approximated as it is appropriately filtered. The UDE has been successfully applied to input–output liberation [23, 24] and combined with sliding mode control [25–27]. Recently, the UDE-based control strategy has been extended to uncertain linear systems with a state delay in [28]. In this article, the UDE-based control strategy is extended to uncertain nonlinear systems with state delays. Both nonlinear and linear controllers are proposed. The terms containing time delay (when the information about the delay is absent) and the uncertainties and the nonlinear terms (in the case of a linear controller) are considered as disturbances. The developed strategies are then applied to a first-order example and a continuous stirred tank reactor (CSTR) system to show the effectiveness.

The remainder of this article is organized as follows: In Section 2, the general linear and nonlinear UDE-based control laws for uncertain nonlinear systems with state delays and disturbances are derived. A simple example is given in Section 3, followed by the application to a CSTR system in Section 4. Conclusions are made in Section 5.

2. UDE-BASED CONTROL LAWS

Consider the nonlinear system with uncertainties and disturbances

$$\begin{aligned}\dot{\mathbf{x}}(t) = {} & \mathbf{g_1}(\mathbf{x}(t),t)+\mathbf{g_2}(\mathbf{x}(t-\tau),t)+\mathbf{g_3}(\mathbf{x}(t),\mathbf{x}(t-\tau),t)+\mathbf{b}(\mathbf{x}(t),t)\mathbf{u}(t)\\ & +\mathbf{f}(\mathbf{x}(t),\mathbf{x}(t-\tau),\mathbf{u}(t),\mathbf{d}(t),t).\end{aligned} \tag{1}$$

Here $\mathbf{x}=(x_1,\cdots,x_n)^{\mathrm{T}}$ is the state vector, $\mathbf{u}(t)=(u_1(t),\cdots,u_r(t))^{\mathrm{T}}$ is the control input vector, $\mathbf{d}(t)$ the unpredictable disturbances vector, $\mathbf{g_1}(\mathbf{x}(t),t)$, $\mathbf{g_2}(\mathbf{x}(t-\tau),t)$ and $\mathbf{g_3}(\mathbf{x}(t),\mathbf{x}(t-\tau),t)$ the known smooth nonlinear vector functions of the state vector, $\mathbf{f}(\mathbf{x}(t),\mathbf{x}(t-\tau),\mathbf{u}(t),\mathbf{d}(t),t)$ the unknown smooth nonlinear vector function of the state vector, the control input and the unpredictable disturbances, and τ is a delay. $\mathbf{b}=\mathbf{B_1}+\mathbf{B_2}((\mathbf{x}(t),t))$ is a known nonzero control matrix function of the state vector, where $\mathbf{B_1}$ is a constant matrix and $\mathbf{B_2}$ a matrix function of the state vector.

A reference model is chosen according to the desired specifications as

$$\dot{\mathbf{x}}_m(t)=\mathbf{A}_m\mathbf{x}_m(t)+\mathbf{B}_m\mathbf{c}(t), \tag{2}$$

which is linear and does not involve any delay. The control objective is to force the error $\mathbf{e}$ between the states of the reference model and the states of the system

$$\mathbf{e}(t)=\mathbf{x}_m(t)-\mathbf{x}(t) \tag{3}$$

to be stable and satisfy the error dynamic equation

$$\dot{\mathbf{e}}(t)=(\mathbf{A}_m+\mathbf{K})\mathbf{e}(t), \tag{4}$$

where $\mathbf{K}$ is an error feedback gain matrix with appropriate dimensions, $\mathbf{c}(t)=(c_1(t),\cdots,c_r(t))^{\mathrm{T}}$ is a piecewise continuous and uniformly bounded command to the system. If the reference model is chosen to be stable, $\mathbf{K}$ may be chosen as $\mathbf{0}$. If different error dynamics are desired or required

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to guarantee stability, then common control strategies, e.g. pole placement, can be used to choose $\mathbf{K}$. It is worth noting that the dimension of $\mathbf{c}(t)$ does not have to be the same as that of $\mathbf{u}(t)$. This provides more freedom for the choice of $\mathbf{B}_m$.

Combining equations (1)–(4), then[§]

$$\mathbf{A_m x} + \mathbf{B_m c} - \mathbf{g_1} - \mathbf{g_2} - \mathbf{g_3} - \mathbf{bu} - \mathbf{f} = \mathbf{Ke}. \tag{5}$$

Hence, the control signal $\mathbf{u}$ needs to satisfy

$$\mathbf{bu} = \mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c} - \mathbf{Ke} - \mathbf{g_1} - \mathbf{g_2} - \mathbf{g_3} - \mathbf{f}. \tag{6}$$

According to the availability of τ, define

$$\varepsilon = \begin{cases} 1 & \text{if } \tau \text{ is known,} \\ 0 & \text{if } \tau \text{ is unknown.} \end{cases}$$

When τ is known, the first six terms on the right-hand side of (6) are known and $\varepsilon = 1$; otherwise only the first four are known and $\varepsilon = 0$.

2.1. *A nonlinear control law*

The unknown terms in (6), including the uncertainties and the external disturbance, are denoted hereafter by $\mathbf{u}_d$. According to the system dynamics (1), $\mathbf{u}_d$ can be represented as

$$\mathbf{u}_d = -(\mathbf{1} - \varepsilon)(\mathbf{g_2} + \mathbf{g_3}) - \mathbf{f} = -\dot{\mathbf{x}} + \mathbf{g_1} + \varepsilon(\mathbf{g_2} + \mathbf{g_3}) + \mathbf{bu}. \tag{7}$$

Hence, the unknown dynamics and disturbances can be obtained from the known dynamics of the system and the control signal. However, it cannot be directly used to formulate a control law. The UDE control strategy proposed in [21] adopts an estimation of this signal to construct control laws. Assume that $g_f(t)$ is the impulse response of a strictly proper filter $G_f(s)$, whose passband contains the frequency content of $\mathbf{u}_d$. Then $\mathbf{u}_d$ can be accurately estimated from the output of the UDE as

$$\mathbf{ude} = \mathbf{u}_d \star g_f, \tag{8}$$

where '$\star$' is the convolution operator. Going back to (6), the control action satisfies

$$\begin{aligned} \mathbf{bu} &= \mathbf{A}_m\mathbf{x} + \mathbf{B}_m - \mathbf{cKe} - \mathbf{g}_1 - \varepsilon(\mathbf{g_2} + \mathbf{g_3}) + \mathbf{ude} \\ &= \mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c} - \mathbf{Keg}_1 - \varepsilon(\mathbf{g_2} + \mathbf{g_3}) + (-\dot{\mathbf{x}} + \mathbf{g_1} + \varepsilon(\mathbf{g_2} + \mathbf{g_3}) + \mathbf{bu}) \star g_f. \end{aligned}$$

This brings the nonlinear UDE-based control law

$$\mathbf{u} = \mathbf{b}^+ \left[-\mathbf{g_1} - \varepsilon(\mathbf{g_2} + \mathbf{g_3}) + \mathscr{L}^{-1}\left\{ \frac{1}{1 - G_f(s)} \right\} \star (\mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c} - \mathbf{Ke}) - \mathscr{L}^{-1}\left\{ \frac{sG_f(s)}{1 - G_f(s)} \right\} \star \mathbf{x} \right] \tag{9}$$

where $\mathbf{b}^+ = (\mathbf{b}^\mathrm{T}\mathbf{b})^{-1}\mathbf{b}^\mathrm{T}$ is the pseudo-inverse of $\mathbf{b}$, $G_f(s) = \mathscr{L}\{g_f(t)\}$ and $\mathscr{L}\{\cdot\}$ is the Laplace transform operator. The control signal has nothing to do with the unknown dynamics and disturbances. Since $\mathbf{u}$ is an approximate solution of (5), Equations (4) and (5) are not always met and, when choosing the control parameters, the following structure constraint needs to be met:

$$(\mathbf{I} - \mathbf{bb}^+)(\mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c} - \mathbf{g}_1 - \mathbf{g_2} - \mathbf{g_3} - \mathbf{f} - \mathbf{Ke}) = 0. \tag{10}$$

Obviously, if $\mathbf{b}$ is a square matrix and invertible, the above structural constraint is always met. If not, the choice of the reference model and the error feedback gain matrix are restricted. Moreover, the unknown dynamics and disturbances play a role in the above constraint. As shown in [22], a system in the canonical form always satisfies this constraint.

[§]In order to simplify the exposition, the arguments of functions in the time-domain are omitted hereafter.

2.2. A linear control law

It is possible to construct a linear control law for the nonlinear system to achieve the desired performance given by the reference model. In this case, it is necessary that $\mathbf{B_1}$ is not zero. Moreover, the information of the delay is not used even when available. Denote the terms in (6) that include the uncertainties, the external disturbance and the nonlinear dynamics as

$$\mathbf{u}_d = -\mathbf{g_1} - \mathbf{g_2} - \mathbf{g_3} - \mathbf{f} - \mathbf{B_2}\mathbf{u} = -\dot{\mathbf{x}} + \mathbf{B_1}\mathbf{u}. \tag{11}$$

With the UDE defined according to (8), there is

$$\mathbf{B_1}\mathbf{u} = \mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c} - \mathbf{Ke} + \mathbf{ude} = \mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c} - \mathbf{Ke} + (-\dot{\mathbf{x}} + \mathbf{B_1}\mathbf{u}) \star g_f.$$

This results in the following linear UDE control law, after using the laplace transform:

$$\mathbf{U}(s) = \frac{1}{1-G_f(s)}\mathbf{B_1^+}(\mathbf{A}_m\mathbf{X}(s) + \mathbf{B}_m\mathbf{C}(s) - \mathbf{KE}(s) - s\mathbf{X}(s)G_f(s)), \tag{12}$$

where $\mathbf{B_1^+} = (\mathbf{B_1^T}\mathbf{B_1})^{-1}\mathbf{B_1}^{\mathrm{T}}$ is the pseudo-inverse of $\mathbf{B_1}$. Note that only if the filter $G_f(s)$ is strictly proper, $sG_f(s)$ is physically implementable and there is no need of measuring the derivative of states in both linear and nonlinear control laws.

2.3. Simplification of the control laws

Assume that the frequency range of the system dynamics and the external disturbance is limited by ω_f. A practical low-pass filter can be chosen as having the impulse response of

$$g_f(t) = \frac{1}{T}e^{-\frac{t}{T}}1(t), \tag{13}$$

where $T = 1/\omega_f$ and $1(t)$ is the unit step signal. Note that

$$\frac{1}{1-G_f(s)} = 1 + \frac{1}{Ts} \quad \text{and} \quad \frac{sG_f(s)}{1-G_f(s)} = \frac{1}{T}.$$

Hence, an integral action is included in the controller when the low-pass filter (13) is used. Moreover,

$$\mathscr{L}^{-1}\left\{\frac{1}{1-G_f(s)}\right\} \star (\mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c} - \mathbf{Ke}) = \mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c} - \mathbf{Ke} + \frac{1}{T}\int_0^t (\mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c} - \mathbf{Ke})\,\mathrm{d}t$$

and

$$\mathscr{L}^{-1}\left\{\frac{sG_f(s)}{1-G_f(s)}\right\} \star \mathbf{x} = \frac{1}{T}\mathbf{x}(t).$$

The nonlinear and linear control laws (9) and (12) can then be simplified, respectively, as

$$\begin{aligned}
\mathbf{u}(t) &= \mathbf{b}^+\left(-(\mathbf{g_1}(t) + \varepsilon(\mathbf{g_2}(t) + \mathbf{g_3}(t))) + \mathbf{A}_m\mathbf{x}(t) + \mathbf{B}_m\mathbf{c}(t) - \mathbf{Ke}(t) - \frac{1}{T}\mathbf{x}(t)\right) \\
&\quad + \mathbf{b}^+\frac{1}{T}\int_0^t (\mathbf{A}_m\mathbf{x}(t) + \mathbf{B}_m\mathbf{c}(t) - \mathbf{Ke}(t))\,\mathrm{d}t \\
&= \mathbf{b}^+\left(-(\mathbf{g_1}(t) + \varepsilon(\mathbf{g_2}(t) + \mathbf{g_3}(t))) + \mathbf{A}_m\mathbf{x}_m(t) - \mathbf{A}_m\mathbf{e}(t) + \mathbf{B}_m\mathbf{c}(t) - \mathbf{Ke}(t) - \frac{1}{T}\mathbf{x}(t)\right) \\
&\quad + \mathbf{b}^+\frac{1}{T}\int_0^t (\mathbf{A}_m\mathbf{x}(t) - \mathbf{A}_m\mathbf{x}_m(t) + \dot{\mathbf{x}}_m(t) - \mathbf{Ke}(t))\,\mathrm{d}t
\end{aligned}$$

$$
\begin{aligned}
&= \mathbf{b}^{+}(-(\mathbf{g_1}(t)+\varepsilon(\mathbf{g_2}(t)+\mathbf{g_3}(t)))+\mathbf{A}_m\mathbf{x}_m(t)+\mathbf{B}_m\mathbf{c}(t)-(\mathbf{A}_m+\mathbf{K})\mathbf{e}(t))\\
&\quad +\mathbf{b}^{+}\frac{1}{T}\left(\mathbf{x}_m(t)-\mathbf{x}(t)-(\mathbf{A}_m+\mathbf{K})\int_0^t \mathbf{e}(t)\,\mathrm{d}t\right)\\
&= \mathbf{b}^{+}(-(\mathbf{g_1}(t)+\varepsilon(\mathbf{g_2}(t)+\mathbf{g_3}(t)))+\mathbf{A}_m\mathbf{x}_m(t)+\mathbf{B}_m\mathbf{c}(t))\\
&\quad +\mathbf{b}^{+}\frac{1}{T}\left((\mathbf{I}-(\mathbf{A}_m+\mathbf{K})T)\mathbf{e}(t)-(\mathbf{A}_m+\mathbf{K})\int_0^t \mathbf{e}(t)\,\mathrm{d}t\right)
\end{aligned}
$$

and

$$
\mathbf{U}(s)=\mathbf{B_1^{+}}\left((\mathbf{A}_m\mathbf{X}_m(s)+\mathbf{B}_m\mathbf{C}(s))+\frac{1}{T}\left(\mathbf{I}-(\mathbf{A}_m+\mathbf{K})T-\frac{1}{s}(\mathbf{A}_m+\mathbf{K})\right)\mathbf{E}(s)\right).
$$

The simplified nonlinear control law consists of three terms. The first term cancels all the known system dynamics, whereas the second term introduces the desired dynamics given by the reference model and the last term performs a PI control action. The simplified linear control law only has the two last terms because the system dynamics is regarded as unknown. These control laws can also be re-written, respectively, as

$$
\begin{aligned}
\mathbf{u}(t) &= \mathbf{b}^{+}(-(\mathbf{g_1}(t)+\varepsilon(\mathbf{g_2}(t)+\mathbf{g_3}(t)))+\mathbf{A}_m\mathbf{x}(t)+\mathbf{B}_m\mathbf{c}(t))\\
&\quad +\mathbf{b}^{+}\frac{1}{T}\left((\mathbf{I}-\mathbf{K}T)\mathbf{e}(t)-(\mathbf{A}_m+\mathbf{K})\int_0^t \mathbf{e}(t)\,\mathrm{d}t\right)
\end{aligned}
\tag{14}
$$

and

$$
\mathbf{U}(s)=\mathbf{B_1^{+}}\left((\mathbf{A}_m\mathbf{X}(s)+\mathbf{B}_m\mathbf{C}(s))+\frac{1}{T}\left(\mathbf{I}-\mathbf{K}T-\frac{1}{s}(\mathbf{A}_m+\mathbf{K})\right)\mathbf{E}(s)\right).
\tag{15}
$$

3. A SIMPLE EXAMPLE

Consider the following first-order nonlinear system,

$$
\begin{aligned}
\dot{x}(t) &= (a_1+\Delta a_1)\cos(x(t))+(a_2+\Delta a_2)\sin(x(t-\tau))\\
&\quad +(a_3+\Delta a_3)x(t)x(t-\tau)+(b+\triangle b)(2+x^2(t))u(t)+\mathrm{d}(t),
\end{aligned}
$$

where $a_1=1$, $a_2=1$, $a_3=0.5$ and $b=1$ are known parameters; $\Delta a_1=0.8\sin(t)$, $\Delta a_2=0.5\sin(2t-\frac{1}{4}\pi)$, $\Delta a_3=0.3\sin(0.5t-\frac{1}{8}\pi)$ and $\Delta b=\sum_{k=0}^{\infty}0.5[1(t-2k)-1(t-1-2k)]$ are uncertain time-varying parameters. The external disturbance is $\mathrm{d}(t)=-2(1(t-3.5)-1(t-7))$ and the delay $\tau=1+0.5\sin(0.4\pi t)$ s is slowly varying. This system can be re-formulated into the form of (1) with $\mathbf{g_1}=a_1\cos(x(t))$, $\mathbf{g_2}=a_2\sin(x(t-\tau))$, $\mathbf{g_3}=a_3x(t)x(t-\tau)$, $\mathbf{b}=2b+bx^2(t)$ ($\mathbf{B_1}=2b$, $\mathbf{B_2}=bx^2(t)$) and $\mathbf{f}=\Delta a_1\cos(x(t))+\Delta a_2\sin(x(t-\tau))+\Delta a_3x(t)x(t-\tau)+\triangle b(2+x^2(t))u(t)+d(t)$. It is worthy noting, although the above control laws were developed when the delay is constant, this example shows that it also works for some cases with time-varying delays. The condition on the allowable time-varying bound will not be discussed here but is left for the future investigation.

The reference model is chosen as $\dot{x}_m=-2x_m+2c$ and the error feedback gain is taken as $\mathbf{K}=-2$. The time constant of the low-pass filter is chosen as $T=0.001$ s and the reference input is set to $c(t)=1(t)-1(t-5)$. According to (14) and (15), the nonlinear and linear control laws are obtained, respectively, as

$$
\begin{aligned}
u(t) &= \frac{1}{2+x^2(t)}\Bigg(-\cos(x(t))-\varepsilon(\sin(x(t-\tau))+0.5x(t)x(t-\tau))\\
&\quad -2x(t)+2c(t)+1002e(t)+4000\int_0^t e(t)\,\mathrm{d}t\Bigg)
\end{aligned}
$$

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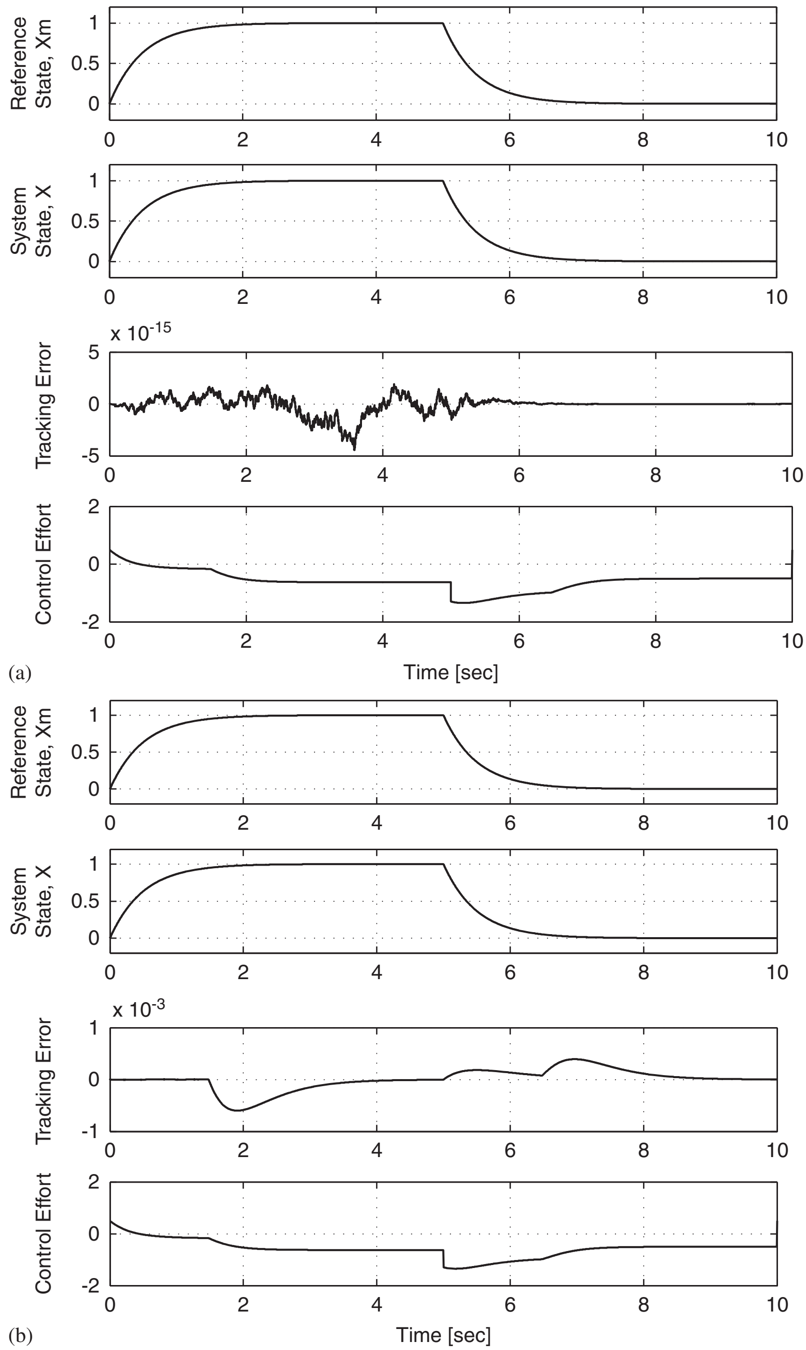

Figure 1. Nominal responses with the nonlinear controller: (a) The time delay is known and (b) the time delay is unknown.

and

$$U(s) = -X(s) + C(s) + \left(501 + \frac{2000}{s}\right) E(s).$$

3.1. Nominal performance

The nominal response (when the uncertain parameters and the disturbance are zero) using the nonlinear control law for the case of known τ ($\varepsilon = 1$) is shown in Figure 1(a) and the nominal

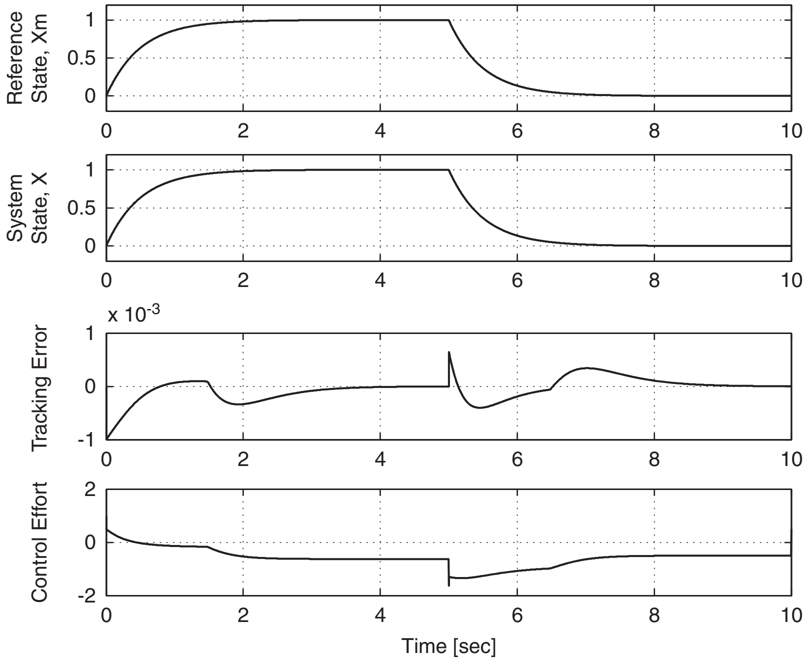

Figure 2. Nominal responses with the linear controller.

response for the case of unknown τ ($\varepsilon = 0$) is shown in Figure 1(b). The system state x tracks the desired state x_m very well in both scenarios. In the case of known τ, the tracking error is practically zero at all times, whereas in the case of unknown τ there is a tracking error change around the moment of the time delay because the terms containing the time delay act like an unknown input to the system. The nominal response using the linear control law is shown in Figure 2. Excellent tracking performance is obtained.

3.2. Robust performance

The robust response using the nonlinear control law for the case of known τ is shown in Figure 3(a) and the robust response for the case of unknown τ is shown in Figure 3(b). When the uncertain parameters are nonzero, the performance does not degrade significantly because the controller can well estimate the uncertainty. There are visible jumps in the control signal u when the disturbance is applied at $t = 3.5$ s and removed at t=7 s, which indicates that the controller estimates the disturbance very quickly. The robust response using the linear control law is shown in Figure 4. The performance is quite good as well.

3.3. Effect of the bandwidth of G_f

The bandwidth (time constant) of the low-pass filter has an influence on the system performance. The tracking errors for three cases with different time constants (when the delay is assumed not known and the controller is nonlinear) are shown in Figure 5. The smaller the time constant (the broader the bandwidth), the smaller the tracking error (i.e. the better the performance). However, in practise, the time constant is limited by the computation capability and the measurement noise. This is normally not a problem nowadays for many applications, as the technology in microcontrollers and digital signal processors has advanced considerably.

3.4. Effect of the error feedback gain

The error dynamics can be changed via changing the error feedback gain **K**. The tracking errors for three cases with different **K** (when the delay is assumed not known and the controller is nonlinear)

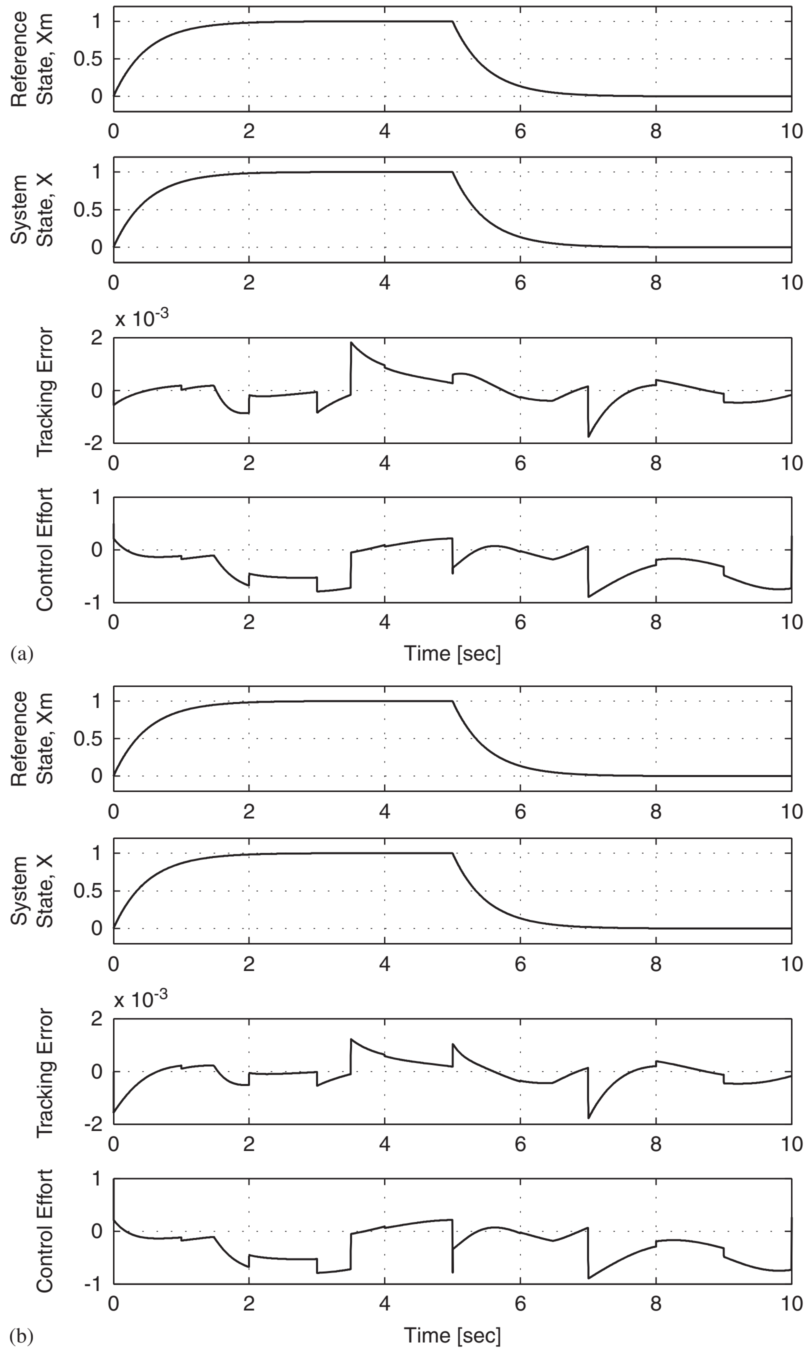

Figure 3. Robust responses with the nonlinear controller. (a) The time delay is known and (b) the time delay is unknown.

are shown in Figure 6. In order to see the effect easily, the time constant of G_f is chosen as $T=0.01$ s. A bigger (absolute) value of $\mathbf{K}$ results in faster error dynamics.

4. APPLICATION TO A CONTINUOUS STIRRED TANK REACTOR SYSTEM

CSTR is a common ideal type of reactors in chemical engineering. A first-order irreversible exothermic reaction occurs in a CSTR, where the fresh-fed and recycled streams of reactants are

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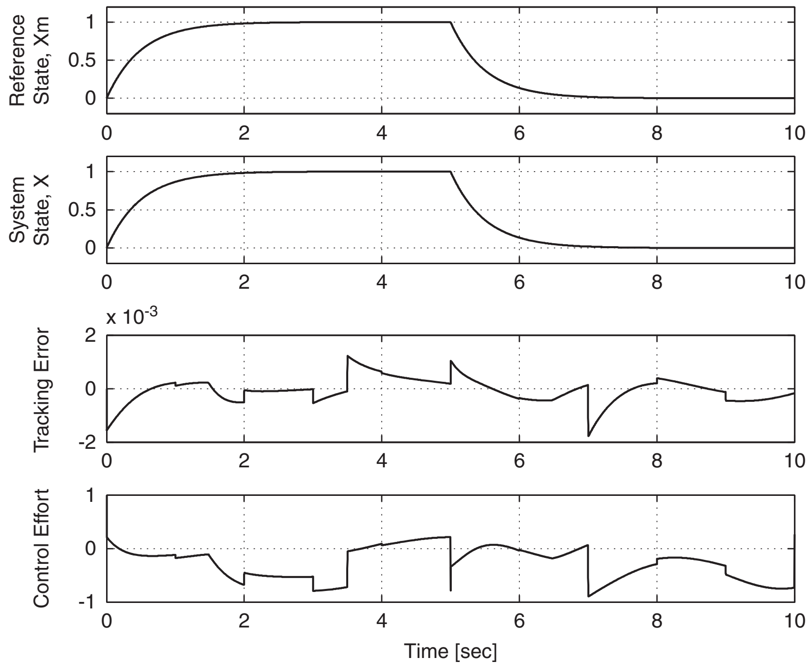

Figure 4. Robust responses with the linear controller.

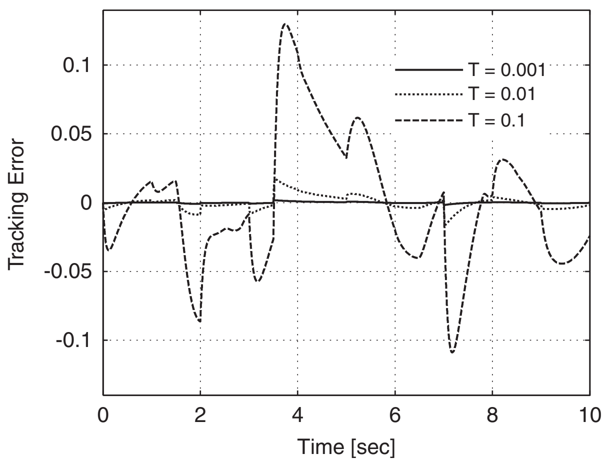

Figure 5. Tracking errors for different values of T with the nonlinear controller.

mixed. CSTR is a highly nonlinear system that exhibits stability in certain regions and instability in some regions [9]. It can be described by the following equations:

$$\dot{x}_1(t) = -\frac{1}{\lambda}x_1(t) + D_a(1-x_1(t))\exp\left(\frac{x_2(t)}{1+\dfrac{x_2(t)}{\gamma_0}}\right) + \left(\frac{1}{\lambda}-1\right)x_1(t-\tau),$$

$$\dot{x}_2(t) = -\left(\frac{1}{\lambda}+\beta\right)x_2(t) + HD_a(1-x_1(t))\exp\left(\frac{x_2(t)}{1+\dfrac{x_2(t)}{\gamma_0}}\right) + \left(\frac{1}{\lambda}-1\right)x_2(t-\tau) + \beta u(t),$$

where $x_1(t)$ is the reactor conversion rate and $x_2(t)$ is the dimensionless temperature. See [9] for more details about the model.

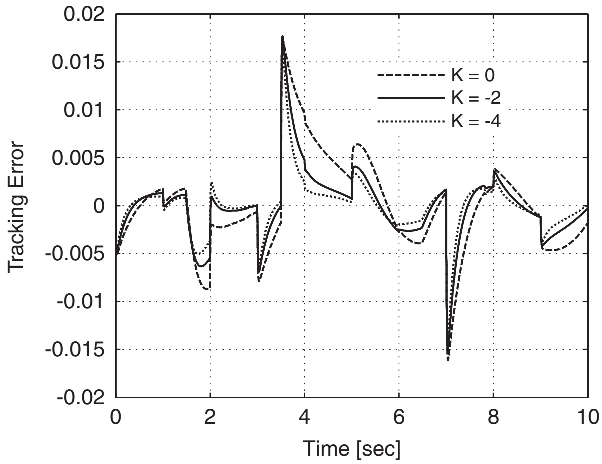

Figure 6. Tracking errors for different **K** with the nonlinear controller.

Consider a CSTR with the following nominal values [9]: $\gamma_0=20$, $H=8$, $D_a=0.072$, $\lambda=0.8$, $\beta=0.3$ and $\tau=2\,\text{s}$. This system can be reformulated into the form of system (1) with

$$\mathbf{g_1}=\begin{pmatrix}-\dfrac{1}{\lambda}x_1(t)\\ -\left(\dfrac{1}{\lambda}+\beta\right)x_2(t)\end{pmatrix},\quad \mathbf{g_2}=\left(\frac{1}{\lambda}-1\right)\begin{pmatrix}x_1(t-\tau)\\ x_2(t-\tau)\end{pmatrix},$$

$$\mathbf{g_3}=D_a(1-x_1(t))\exp\left(\frac{x_2(t)}{1+\dfrac{x_2(t)}{\gamma_0}}\right)\begin{pmatrix}1\\ H\end{pmatrix},\quad \mathbf{b}=\begin{pmatrix}0\\ \beta\end{pmatrix}$$

and

$$\mathbf{f}=\begin{pmatrix}0\\ 0\end{pmatrix}.$$

Uncertainties will be added to γ_0, H and β at a later stage when investigating the system robust performance.

The reference model is chosen as the second-order system

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix}=\begin{pmatrix}-1 & 0\\ 0 & -4\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}+\begin{pmatrix}1 & 0\\ 0 & 4\end{pmatrix}\mathbf{c}(t),$$

where $\mathbf{c}(t)=\begin{pmatrix}c_1(t)\\ c_2(t)\end{pmatrix}$ is the desired (steady-state) operating point for the system states $\begin{pmatrix}x_1\\ x_2\end{pmatrix}$. Note that the dimension of $\mathbf{c}(t)$ is not the same as that of $u(t)$. The error feedback gain is set to $\mathbf{K}=\mathbf{0}$ and the time constant of the low-pass filter is chosen as $T=0.005\,\text{s}$. A linear control law is obtained, according to (12), as

$$U(s)=-\frac{40}{3}X_2(s)+\frac{10}{3}C_2(s)+\left(\frac{2000}{3}+\frac{8000}{3s}\right)E_2(s),$$

where $E_2(s)=X_{m2}(s)-X_2(s)$.

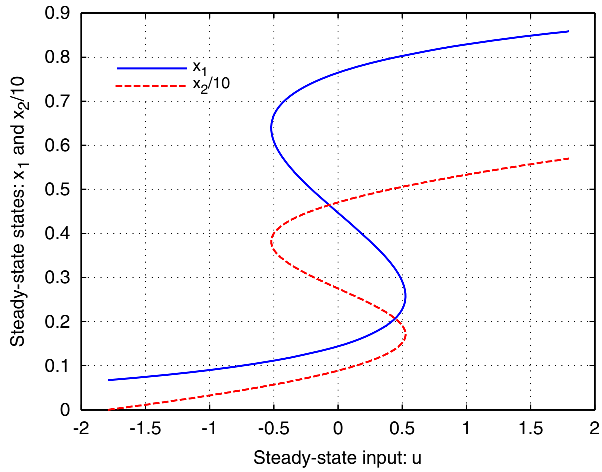

Figure 7. The steady-state CSTR curve.

4.1. Nominal performance

The open-loop system may converge to different steady-state operating points under different initial conditions and control effort u. These steady-state operating points, as shown in Figure 7, are characterized by

$$x_1 = D_a(1-x_1)\exp\left(\frac{x_2}{1+\frac{x_2}{\gamma_0}}\right),$$

$$x_2 = \frac{H}{\beta+1}D_a(1-x_1)\exp\left(\frac{x_2}{1+\frac{x_2}{\gamma_0}}\right)+\frac{\beta}{\beta+1}u.$$

When $u=0$, there are two open-loop stable steady-state operating points $\mathbf{x_s^1}=(0.1440, 0.8862)^{\mathrm{T}}$ and $\mathbf{x_s^2}=(0.7646, 4.7052)^{\mathrm{T}}$ and one open-loop unstable steady-state operating points $\mathbf{x_s^3}=(0.4472, 2.752)^{\mathrm{T}}$. Figure 8 demonstrates the convergence of the open-loop state responses when $u=0$. For the initial condition $\mathbf{x_0}=(0,0)^{\mathrm{T}}$, the system states converge to $\mathbf{x_s^1}$; for the initial condition $\mathbf{x_0}=(0.8,6)^{\mathrm{T}}$, the system states converge to $\mathbf{x_s^2}$. When the UDE-based control law is used, the system states can converge to the open-loop unstable operating point $\mathbf{x_s^3}=(0.4472, 2.752)^{\mathrm{T}}$ regardless of $\mathbf{x_0}$, as shown in Figure 9.

In order to show the ability of the controller to track the change of operating points, the desired operating point $\mathbf{c}(t)$ was set to $(0.4472, 2.752)^{\mathrm{T}}$ initially at $t=0$ s and then changed to $(0.7, 4.2107)^{\mathrm{T}}$ at $t=10$ s (both are on the steady-state curve). The responses are shown in Figure 10. It can be seen from Figure 10 that a quite satisfactory results are obtained even though only a linear controller is used.

4.2. Robust performance

In order to demonstrate the robust performance of the controller, the following parameter drifts and uncertainties were added to the nominal system values: $\Delta\gamma_0=5$, $\Delta H=-2$, and $\Delta\beta=-0.1\sum_{k=0}^{\infty}(1(t-2k)-1(t-1-2k))$. The robust responses for the change of operating points from $(0.4472, 2.752)^{\mathrm{T}}$ to $(0.7, 4.2107)^{\mathrm{T}}$ at $t=10$ s using the linear control law is shown in Figure 11. As expected, the responses remain almost unaffected despite the uncertainties. The fluctuations in the control effort reflect the fast estimation of uncertainties and disturbances.

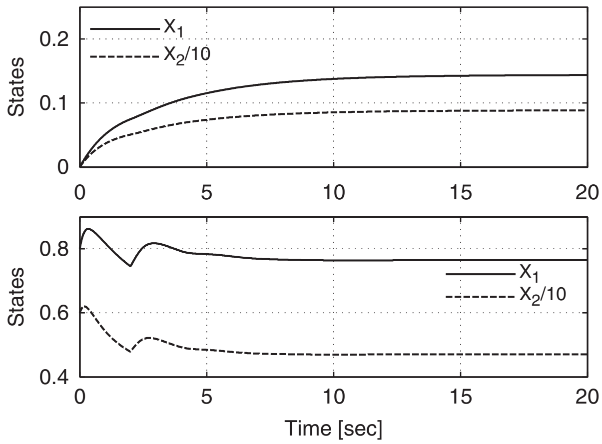

Figure 8. Zero-input open-loop system responses with different initial conditions: upper for $\mathbf{x_0}=(0,0)^{\mathrm{T}}$ and lower for $\mathbf{x_0}=(0.8,6)^{\mathrm{T}}$.

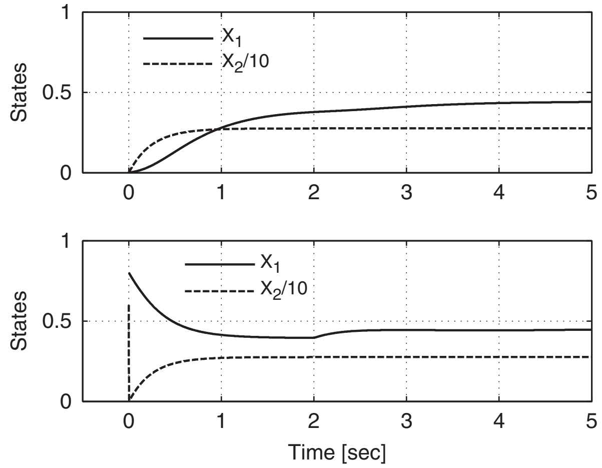

Figure 9. Closed-loop system responses with different initial conditions for the set operating point $\mathbf{x_s^3}$: upper for $\mathbf{x_0}=(0,0)^{\mathrm{T}}$ and lower for $\mathbf{x_0}=(0.8,6)^{\mathrm{T}}$.

5. CONCLUSIONS

In this article, the UDE-based control has been extended to uncertain nonlinear continuous systems with state delays and disturbances. Both linear and nonlinear control strategies have been derived. The proposed algorithms have shown excellent tracking and disturbance rejection capabilities almost independently on the availability of the information about the time delay and nonlinear dynamics. In the case of unknown time delay and nonlinear dynamics, the terms containing the delay and nonlinear dynamics are treated as additional disturbances to the system and can be quickly estimated by the controller. The proposed strategies have been successfully applied via simulations to a simple first-order nonlinear uncertain system with disturbances and state delays as well as to a CSTR system. Although the approach was developed for the case with constant delays, simulations of the first example show that it also works for the case with time-varying delays. The conditions on the bound of the time-varying delays are not discussed here and are left for the future study.

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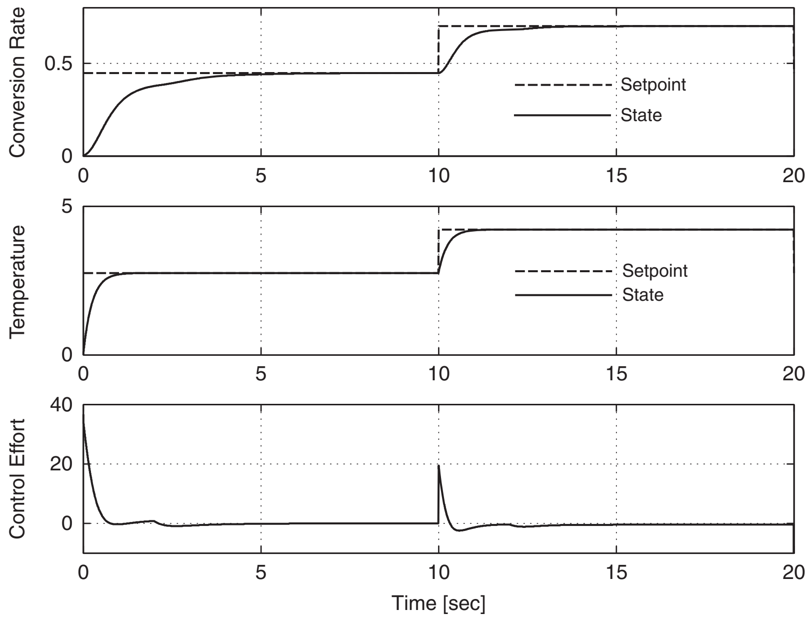

Figure 10. Responses to the change of operating points: nominal case.

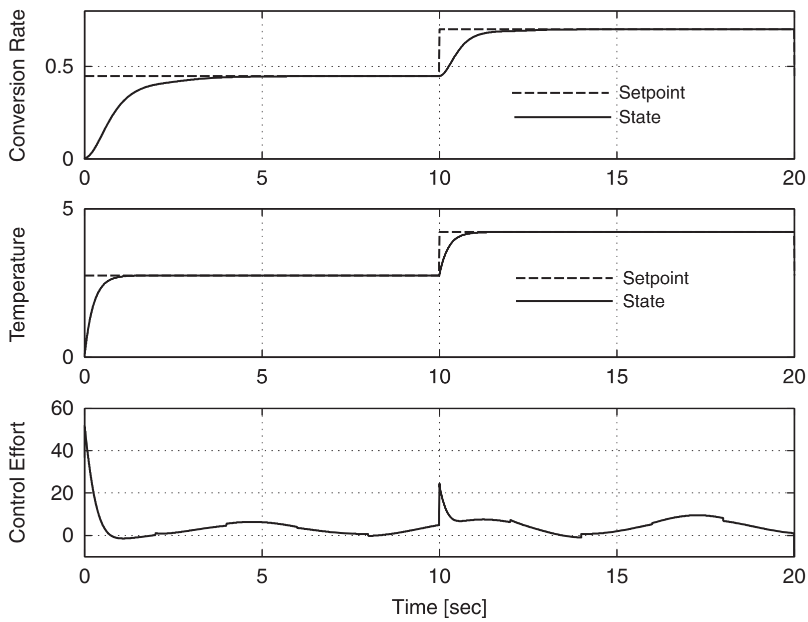

Figure 11. Responses to the change of operating points: robust case.

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