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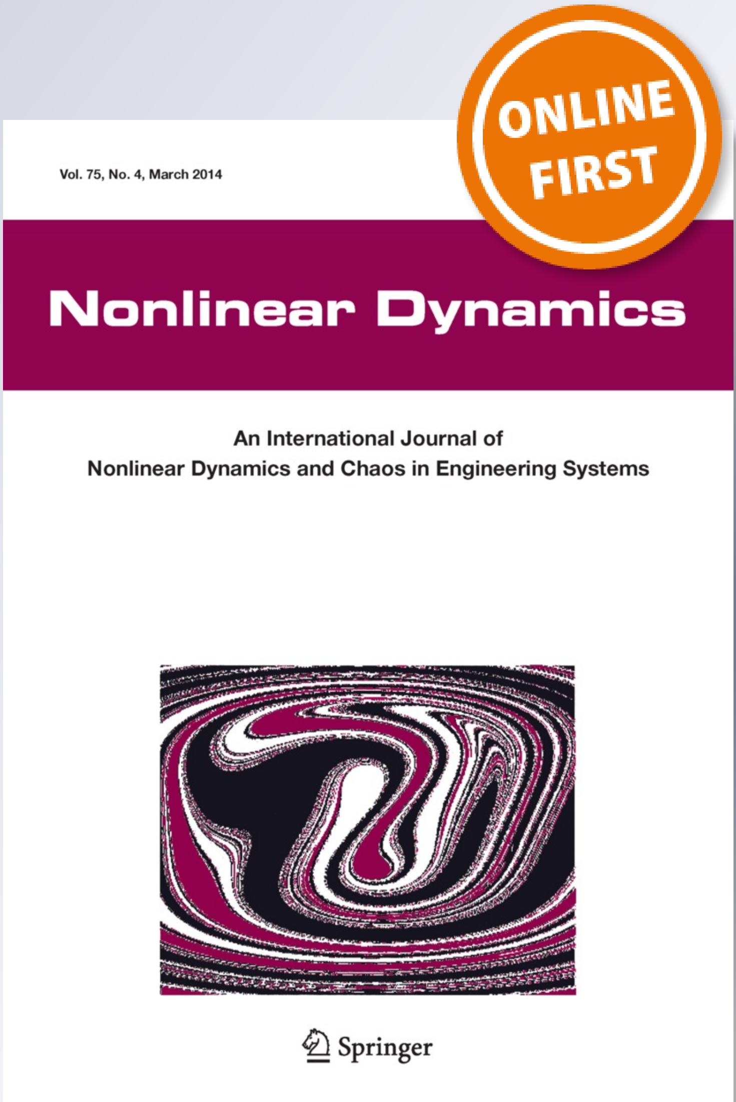
Nonlinear Dynamics
An International Journal of Nonlinear Dynamics and Chaos in Engineering Systems

ISSN 0924-090X

Nonlinear Dyn
DOI 10.1007/s11071-014-1337-x



Springer

Nonlinear Dyn
DOI 10.1007/s11071-014-1337-x

ORIGINAL PAPER

Improving the performance of UDE-based controller using a new filter design

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Received: 18 August 2012 / Accepted: 23 February 2014

Abstract Robust control of uncertain systems has been a field of active research and the technique of uncertainty and disturbance estimator (UDE) has proved itself as a viable tool in the design of a robust control strategy for systems having uncertainties and acted upon by disturbances. Though the technique is quite efficient for ensuring robustness under slow-varying disturbances, the presence of steady-state tracking and estimation errors cannot be ruled out for fast-varying or sinusoidal disturbances. To address this issue, a new form of filter in UDE-based controller is proposed in this work and it is shown that the errors can be kept within acceptable limits by means of appropriate choice of the design parameter α. Closed-loop stability and simulation results for wing-rock motion control problem employing an UDE-based controller using the new filter are carried out to demonstrate its efficacy against fast-varying uncertainties and disturbances.

Keywords Uncertainty and disturbance estimation · Input–output linearization · Nonlinear control · Robust control · Controller–observer structure · Wing-rock motion control

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1 Introduction

Owing to its practical importance, designing robust controllers for systems affected by significant uncertainties and unmeasurable external disturbances has been an area of active research; therefore, a large number of approaches have appeared in literature addressing the issue. One promising approach for the design of robust controller for such systems is based on the concept of uncertainty and disturbance estimation and compensation wherein the idea is to estimate the effect of uncertainties and disturbances acting on the system and compensate it by augmenting the controller designed for nominal system. To this end, various techniques such as unknown input observer (UIO) [1], disturbance observers (DO) [2,3], perturbation observer (PO) [4], equivalent input disturbance (EID) approach [5], time delay control (TDC) [6], etc., have been in place for quite sometime to estimate the effect of uncertainties and disturbances. Applications of these as well as uncertainty estimation based on some other approaches could also be found in literature. For example, the problem of attitude control of spacecrafts with flexible appendages is presented in [7] wherein the vibration from flexible appendages is modeled as a disturbance and a disturbance-observer-based control is formulated to compensate the same. In [8], the

Published online: 14 March 2014

authors have proposed to use fractional order disturbance observer (FO-DOB) for vibration suppression applications such as hard disk drive servo control. In [9], a new method for globally stabilizing nonlinear underactuated mechanical systems is presented. The system nonlinearities are treated as a disturbance and an EID- based robust controller is designed to globally stabilize the systems. A design of tracking controller for electrically driven flexible-joint robot manipulators can be found in [10] wherein the authors have introduced an interesting method of uncertainty estimation to obtain robust control law. Similarly, in [11], a novel robust decentralized control of electrically driven robot manipulators by adaptive fuzzy estimation and compensation of uncertainty is presented.

Following the similar line of thinking as that of TDC, a novel uncertainty and disturbance estimator (UDE) technique is proposed in [12] wherein the effect of system uncertainties and external disturbances is estimated by employing an appropriate filter. Since then, various theoretical and application results on UDE-based control have appeared in literature. In [13], the authors have brought out the two-degree-of-freedom nature of the UDE-based controllers and it is shown that, in addition to the known advantages over the time-delay control, the UDE-based control outperforms TDC under the same operational conditions. In [14], UDE is used in overcoming the issues of requirement of knowledge of uncertainty bound and chattering in sliding mode control. The UDE-based robust control designs for uncertain linear and nonlinear systems with state delays are presented in [15,16] and it is shown that the designs offer excellent tracking and disturbance rejection performance. Results on extension of the UDE technique for uncertain nonlinear systems can be found in [17]. Applications of the UDE technique have also been reported in literature. An application of UDE in robustifying a feedback linearizing control law for a robot having joint flexibility is presented in [18] wherein the effect of joint flexibility is treated as a disturbance. Design of robust flight path angle tracking controller in aircraft [19], trajectory tracking control of rigid link manipulators [20], control of VTOL aircraft [21,22], and design of a robust guidance law for tactical missiles [23] are some more examples to cite.

An application of the UDE in robustification of input–output linearization (IOL) controller is presented in [24] wherein the UDE estimated uncertainties are used in robustifying an IOL controller. The robustification is achieved by estimating the uncertainties and external disturbances using UDE and compensating the same by augmenting the IOL controller designed for nominal system. The robustified IOL control has been designated as IOL+UDE controller. While the IOL+UDE controller achieves the objective of robustification of IOL control quite effectively, implementation of the same requires complete state vector as well as output derivatives. Both of these requirements may not be easy to meet in practice. Addressing these issues, in [25] the IOL problem is formulated by considering the state-dependent nonlinearities as a part of the uncertainties and the IOL controller is robustified using the UDE estimated uncertainties. As the resulting controller requires derivatives of the output for implementation, a robust observer is proposed to provide estimate of the output derivatives. The observer design too employs the UDE estimated uncertainties to achieve robustness, thus giving rise to the UDE-based controller–observer structure. Further, to demonstrate the efficacy of the design, experimental results on Quanser's DC servo motion control platform are presented. The closed-loop stability analysis presented in [24,25] show that while the designs are quite effective against steady or slow-varying disturbances and uncertainties, steady-state tracking and estimation errors may exist for sinusoidal or fast-varying disturbances. The objective of this work is to address this issue and thus extend and improve upon the results presented in [24,25]. Here, to address the issue of fast-varying uncertainties, the use of a new filter in UDE is proposed. The resulting closed-loop stability analysis shows that by employing the proposed filter in the UDE formulation, the errors can be kept within acceptable limits by choosing the design parameter α appropriately. Simulation and comparison results of the proposed controller applied to wing-rock motion control problem in the presence of fast-varying uncertainties and disturbances are presented to demonstrate the efficacy of the design.

The remaining paper is organized as follows. The design of UDE-based controller–observer structure using the proposed new filter and its closed-loop stability results are presented Sect. 2. Performance improvement analysis of the proposed filter is the subject of Sect. 3. Simulation results on performance comparison of the proposed controller as applied to the wing-rock motion control problem are presented in Sect. 4 and lastly, Sect. 5 concludes this work.

2 UDE-based controller–observer structure using new filter

IOL [26,27] is one of the most prominent techniques used in the design of controller for nonlinear systems. While the IOL controller offers satisfactory performance in ideal scenario, it suffers from certain drawbacks when put into practice. First, the IOL controller requires availability of system state for its implementation. Next, the design requires exact cancellation of the system nonlinearities. In reality, these requirements are hard to satisfy; therefore, the IOL control law may not offer satisfactory performance. To recover the performance in the presence of uncertainties, various approaches have been presented in literature for robustification of the IOL controller [28–36].

In [24], an application of the UDE technique in robustification of an IOL controller is presented and the efficacy of the design was demonstrated by applying it for wing-rock motion control in slender delta-winged aircraft. As mentioned before, though the UDE-based IOL controller could achieve the desired robustness, issues like need for complete state vector and output derivatives remained to be addressed. The same was tackled in [25] in an integrated manner, by the design of a UDE- based controller–observer structure. The authors have demonstrated the effectiveness of the approach through application to the wing-rock motion control problem considered in [24] followed by experimental validation.

From the closed-loop error dynamics presented in [24,25], it is seen that the UDE technique using a first-order low-pass filter is quite efficient in achieving robustness to IOL control when the uncertainties and disturbances are constant or slow-varying as the disturbance estimation error depends on the rate of change of the uncertainty/disturbance. It is, therefore, natural to examine the feasibility of extending the benefits of UDE technique to uncertain systems when acted upon by fast-varying disturbances. In this work, an attempt is made to address this issue and to this end a new form of filter, designated as α-filter, is proposed in obtaining estimate of the uncertainties and to contain the tracking and estimation errors in the presence of fast-varying uncertainties and disturbances. Following the treatment presented in [25], the derivation of the UDE-based controller–observer structure using the new filter design is presented in this section.

2.1 UDE-based controller

Consider a single-input single-output nonlinear dynamical system described by

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \\ y &= c(x). \end{aligned} \tag{1}$$

To account for uncertainties, the system dynamics is re-written as

$$\begin{aligned} \dot{x} &= \hat{f}(x) + \Delta f(x) + (\hat{g}(x) + \Delta g(x))u, \\ y &= c(x), \end{aligned} \tag{2}$$

where $\hat{f}(x)$ and $\hat{g}(x)$ represent the nominal part of the system whereas $\Delta f(x)$ and $\Delta g(x)$ represent their associated uncertainties. Following the IOL theory, the output is differentiated by λ times yielding

$$y^{(\lambda)} = a(x) + b(x)u, \tag{3}$$

where λ is the relative degree of the output and $y^{(\lambda)}$ represents the λth-order derivative of $y(t)$ with respect to time. Following [25], the dynamics of (3) is written as

$$y^{(\lambda)} = \bar{a}\bar{y} + (a(x) - \bar{a}\bar{y}) + (b(x) - \hat{b}_0)u + \hat{b}_0 u + d^{'}, \tag{4}$$

where $\bar{a} = [\hat{a}_1\ \hat{a}_2\ \dots\ \hat{a}_\lambda]$ and $\bar{y} = [y\ \dot{y}\ \ddot{y}\ \dots\ y^{(\lambda-1)}]^T$ with $\hat{a}_i$ being the nominal values of the coefficients and the quantity $d^{'}$ represents the effect of the external unmeasurable disturbances, if any. In writing (4), it is assumed that the nonlinear function, $a(x)$, may have components which are linear in output, y, and its higher-order derivatives up to the order of $(\lambda - 1)$. If such is not the case, the quantity $\bar{a}\bar{y}$ is taken as zero.

Defining $d \triangleq (a(x) - \bar{a}\bar{y}) + (b(x) - \hat{b}_0)u + d^{'}$, the dynamics of (4) is re-written as

$$y^{(\lambda)} = \bar{a}\bar{y} + \hat{b}_0 u + d, \tag{5}$$

where $\hat{b}_0$ is a chosen constant that approximates $b(x)$. It may be noted that the uncertainty, d, consists of all state-dependent nonlinearities and thus the requirement of availability of the plant states does not exist. The UDE robustified IOL control is designed as

$$u(\text{IOL} + \text{UDEM}) = \frac{1}{\hat{b}_0}(u_a - \hat{d} + \nu), \tag{6}$$

where

$$u_a = -\bar{a}\bar{y}, \tag{7}$$

and $\hat{d}$ is that part of the control which cancels the effect of uncertainties and disturbances and also the estimate of d. The controller is designated as IOL+UDEM control for being consistent with the terminology used in [25]. It may be noted that by considering $\bar{a}\bar{y}$ as a known part in the dynamics (5), the magnitude of the uncertainty that needs to be estimated reduces. This, in turn, results into reduced control effort as is evident from (6). Substituting (6) in (5) leads to

$$y^{(\lambda)} = -\hat{d} + \nu + d. \tag{8}$$

From (8) one gets

$$d = y^{(\lambda)} + \hat{d} - \nu. \tag{9}$$

Following [12,17,24,25], the lumped uncertainties (d) is estimated by passing it through the proposed filter $G_\alpha(s)$ having the form

$$G_\alpha(s) = \frac{(1-\alpha)\tau s + 1}{\tau s + 1} \tag{10}$$

as

$$\hat{d} = \frac{(1-\alpha)}{\alpha} y^{(\lambda)} - \frac{1}{\alpha}\nu + \nu + \frac{1}{\alpha\tau} y^{(\lambda-1)} - \frac{1}{\alpha\tau}\int \nu dt. \tag{11}$$

If it is desired that the output $y(t)$ tracks asymptotically a reference trajectory $y^*(t)$, then by defining $\tilde{y}(t) = y^*(t) - y(t)$, the outer loop control, ν, is chosen as

$$\nu = y^{\star(\lambda)}(t) + m_\lambda \tilde{y}^{(\lambda-1)}(t) + \cdots + m_2 \dot{\tilde{y}}(t) + m_1 \tilde{y}(t) \tag{12}$$

such that the choice of m_i ensures (12) is Hurwitz. Substitution of (7) and (11) in (6) gives the UDE-based robust IOL controller using $G_\alpha(s)$ as

$$u(\text{IOL}+\text{UDEM}) = \frac{1}{\hat{b}_0}\left[-\bar{a}\bar{y} - \frac{(1-\alpha)}{\alpha} y^\lambda - \frac{1}{\alpha\tau} y^{(\lambda-1)} + \frac{1}{\alpha}\nu + \frac{1}{\alpha\tau}\int \nu dt\right]. \tag{13}$$

Note that the controller does not require plant states; however, it needs up to λth-order time derivatives of y. The issue of obtaining the output derivatives is addressed by designing a UDE-based robust observer as discussed in the next section.

2.2 UDE-based observer

As is obvious from (13), the controller requires output derivatives up to the order of λ for its implementation. As this may not be practical in all situations,

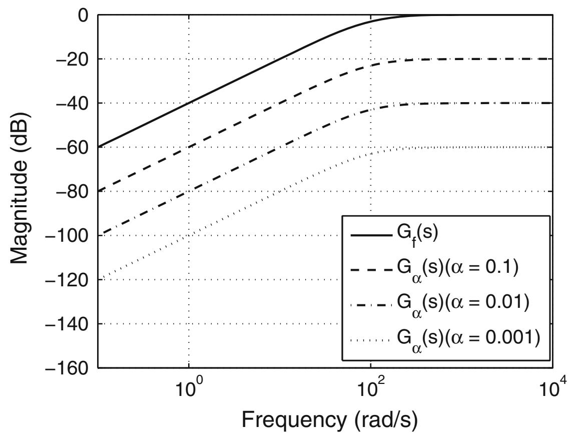

Fig. 1 Frequency responses of $H_f(s)$ with UDE filter ($G_f(s)$) and α-filter ($G_\alpha(s)$)

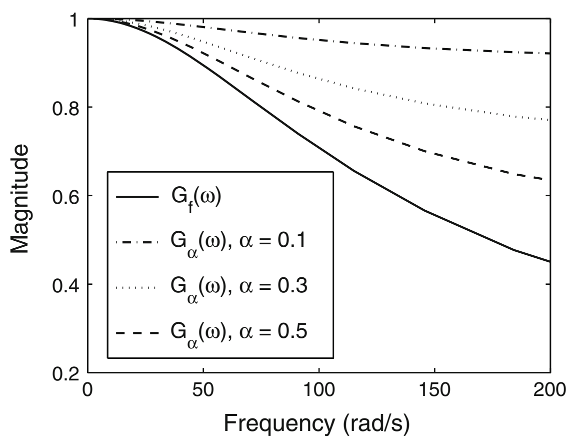

Fig. 2 Frequency response of $G_\alpha(s)$ as a lag filter

the derivatives need to be estimated from the available output. As a solution to this problem, design of a UDE-based robust observer is proposed. To this end, defining the state vector as $x_p = [y_1\ y_2\ \cdots\ y_{\lambda-1}\ y_\lambda]^T = [y\ \dot{y}\ \ddot{y}\ \cdots\ y^{(\lambda-1)}]^T$, the dynamics of (5) can be written in a phase variable state space form as

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_p u + B_d d, \\ y_p &= C_p x_p, \end{aligned} \tag{14}$$

where

$$A_p = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & \ddots & & \\ 0 & 0 & 0 & \cdots & 1 \\ \hat{a}_1 & \hat{a}_2 & \hat{a}_3 & \cdots & \hat{a}_\lambda \end{bmatrix}; \quad B_p = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \hat{b}_0 \end{bmatrix};$$

$$B_d = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

and $C_p = [1\ 0\ \cdots\ 0\ 0]$. An Luenberger-like observer of the form

$$\dot{\hat{x}}_p = A_p \hat{x}_p + B_p u + B_d \hat{d} + L(y_p - \hat{y}_p),$$
$$\hat{y}_p = C_p \hat{x}_p. \tag{15}$$

is proposed for the system of (14), where $L = [\beta_1\ \beta_2\ \ldots\ \beta_\lambda]^T$ is the observer gain vector. The observer, however, needs the estimate of the uncertainty, i.e., $\hat{d}$. As the uncertainty is the same as present in (5), the UDE estimated uncertainty is used in the observer (15) too giving rise to the UDE-based controller–observer structure. It may be noted that the proposed observer does not require an accurate plant model and is robust. Noting that $\hat{x}_p = [\hat{y}_1\ \hat{y}_2\ \cdots\ \hat{y}_{\lambda-1}\ \hat{y}_\lambda]^T = [\hat{y}\ \dot{\hat{y}}\ \ddot{\hat{y}}\ \cdots\ \hat{y}^{(\lambda-1)}]^T$, the IOL+UDEM control of (13) can be computed using the UDE-based observer estimated output derivatives up to $\lambda - 1$th order. Further, the last state equation in (15) gives the estimate of the λth-order time derivative of the output y and thus the issue of requirement of output derivatives is addressed completely.

2.3 Closed-loop stability

Following the development presented in [25], it can be shown that the closed-loop error dynamics for the linearized system of (14) with observer of (15) controller–observer combination using $G_\alpha(s)$ can be shown as

$$\begin{bmatrix} \dot{e}_c \\ \dot{e}_o \\ \dot{\tilde{d}} \end{bmatrix} = \begin{bmatrix} (A_p - B_p K_p) & -(B_p K_p) & -B_d \\ 0 & (A_p - LC_p) & B_d \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \times \begin{bmatrix} e_c \\ e_o \\ \tilde{d} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix} \dot{d}, \tag{16}$$

where $e_c = R - x_p$ is the state tracking error vector, $R = [y_1^\star\ y_2^\star \cdots\ y_{\lambda-1}^\star\ y_\lambda^\star]^T$ is the reference state vector, $e_o = x_p - \hat{x}_p$ is the observer state estimation error

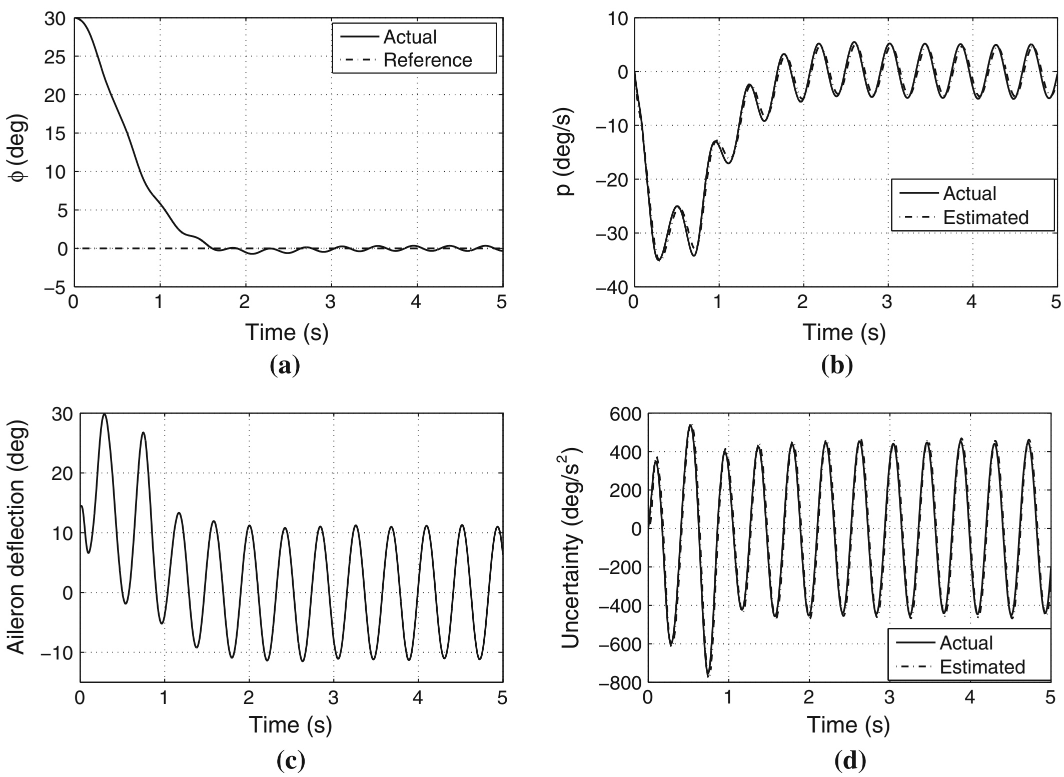

Fig. 3 Performance of IOL+UDEM1 controller in stabilization with uncertainties

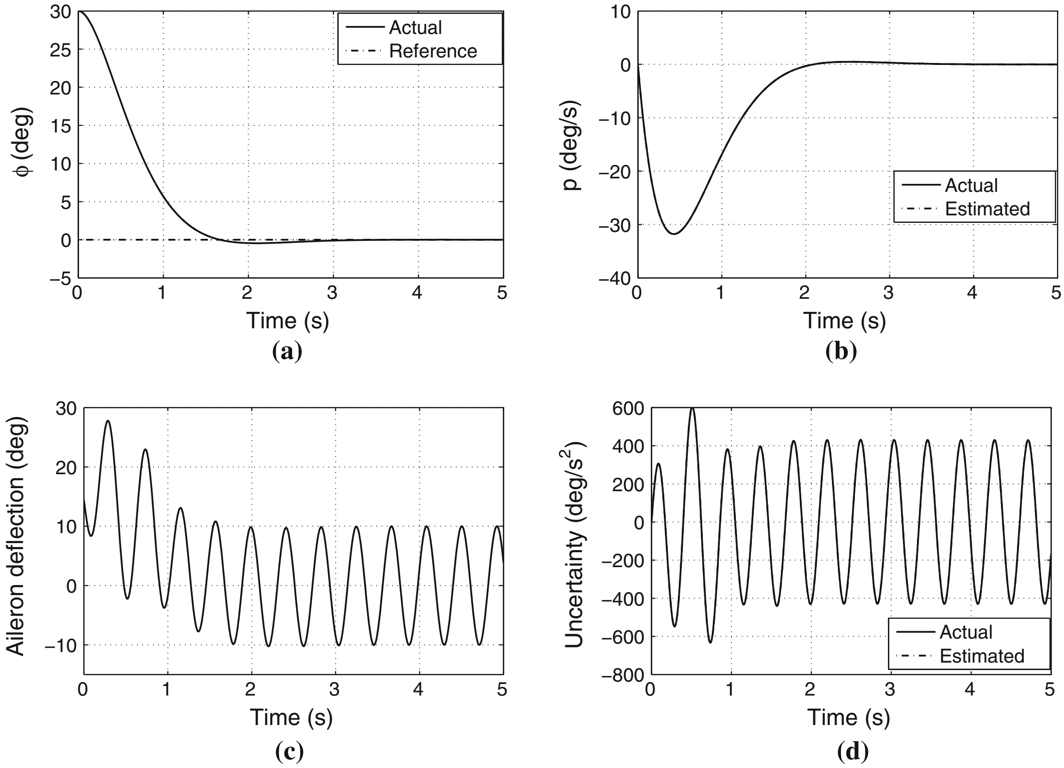

Fig. 4 Performance of IOL+UDEM2 controller in stabilization with uncertainties

vector, $\tilde{d} = d - \hat{d}$ is the uncertainty estimation error, and $K_p = [k_1 \;\; k_2 \;\; \cdots \;\; k_{\lambda-1} \;\; k_\lambda]$ is the state feedback gain vector where its elements are $k_1 = \frac{m_1+\hat{a}_1}{b_0}$, $k_2 = \frac{m_2+\hat{a}_2}{b_0}, \ldots, k_{\lambda-1} = \frac{m_{\lambda-1}+\hat{a}_{\lambda-1}}{b_0}$, and $k_\lambda = \frac{m_\lambda+\hat{a}_\lambda}{b_0}$. As the system matrix in (16) is in block triangular form, it can be easily verified that its eigenvalues are given by

$$|sI - (A_p - B_pK_p)||sI - (A_p - LC_p)||s - \left(-\frac{1}{\tau}\right) | = 0. \tag{17}$$

Noting that the pair (A_p, B_p) is controllable and the pair (A_p, C_p) is observable; the controller gain, K_p, and the observer gain, L, can be chosen appropriately along with $\tau > 0$ to ensure stability for the error dynamics. As the dynamics is driven by $\dot{d}$, it is obvious that for bounded $|\dot{d}|$, bounded input-bounded output stability is assured. Also, if the rate of change of uncertainty is negligible, i.e., if $\dot{d} \approx 0$, then the error dynamics is asymptotically stable. However, if $\dot{d} \neq 0$, then proper choice of α ensures in keeping the steady-state errors within desired limits.

2.4 Some comments on the controller

It is worth mentioning that the IOL+UDEM controller of (13) needs $y^{(\lambda)}$ in addition to $(\lambda - 1)$th-order time derivatives of y. While the estimate of the latter part is given by the UDE-based observer (15), $y^{(\lambda)}$ can be obtained using the last state equation of the observer, i.e.,

$$\dot{\hat{y}}_\lambda = \hat{a}_1 y_1 + \hat{a}_2 y_2 + \cdots + \hat{a}_\lambda y_\lambda + \hat{d} + \beta_\lambda(y_1 - \hat{y}_1). \tag{18}$$

Next, it is interesting to note that for the proposed filter $G_\alpha(s)$ as in (10), $\frac{G_\alpha(s)}{(1-G_\alpha(s))} = \frac{((1-\alpha)\tau s+1)}{\alpha\tau s}$ which further can be expressed as $\frac{(1-\alpha)}{\alpha} + \frac{1}{(\alpha\tau s)}$ which is similar to a PI Control. Further, for $\alpha = 0$, $G_\alpha(s) = \frac{\tau s+1}{\tau s+1}$ and the expression $\frac{G_\alpha(s)}{1-G_\alpha(s)}$ becomes ∞. Therefore, $\alpha = 0$ cannot be used in the proposed filter. Meanwhile, for $\alpha = 1$, $G_\alpha(s) = \frac{1}{\tau s+1}$ which is same as $G_f(s)$ given in [12,25]. Thus, the value of α needs to be taken between 0 and 1 as $0 < \alpha \leq 1$ depending on the requirements of the tracking errors.

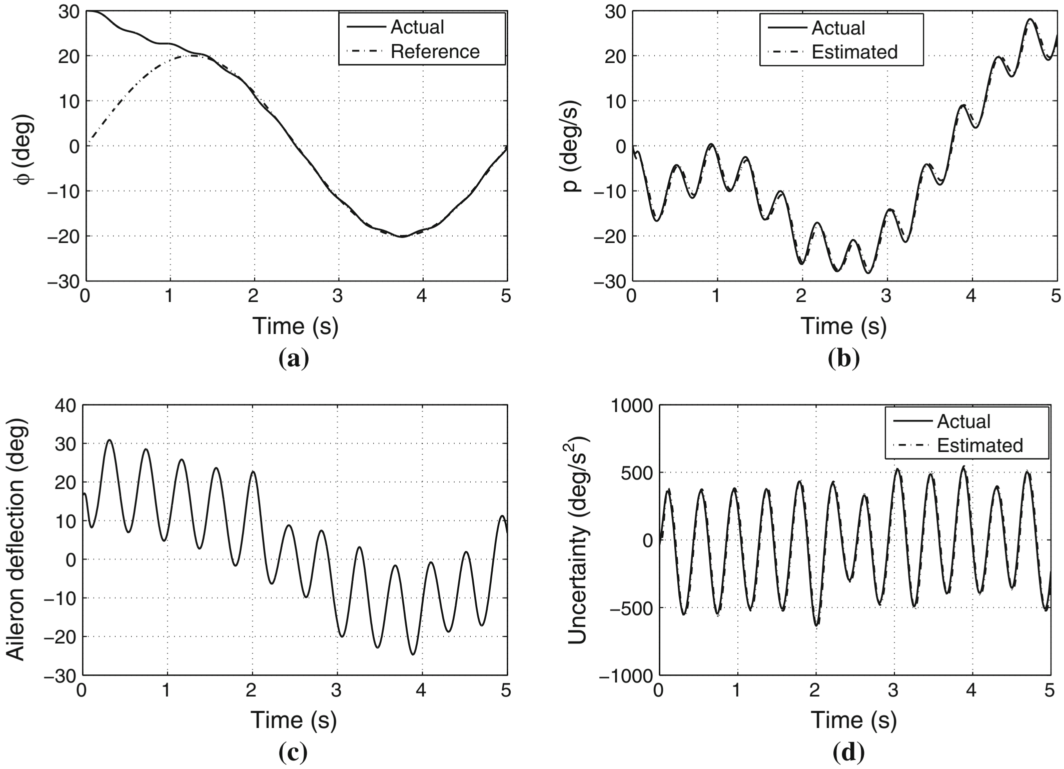

Fig. 5 Performance of IOL+UDEM1 controller in tracking with uncertainties

3 Performance improvement analysis

3.1 Fast-varying uncertainties

In [25], the UDE-based controller–observer structure is presented using the filter as

$$G_f(s) = \frac{1}{1+\tau s}, \tag{19}$$

whereas in the present work a new filter as given in (10) is proposed. To distinguish, the controller designed using (19) will be referred to as the IOL+UDEM1 controller whereas the controller designed using the proposed filter of (10) will be designated as IOL+UDEM2 controller. The IOL+UDEM1 controller and its closed-loop error dynamics as given in [25] is

$$u(\text{IOL}+\text{UDEM1}) = \frac{1}{\hat{b}_0}\left(-\bar{a}\bar{y} - \frac{1}{\tau}y^{(\lambda-1)} + \nu + \frac{1}{\tau}\int \nu dt\right), \tag{20}$$

$$\begin{bmatrix} \dot{e}_c \\ \dot{e}_o \\ \dot{\tilde{d}} \end{bmatrix} = \begin{bmatrix} (A_p - B_pK_p) & -(B_pK_p) & -B_d \\ 0 & (A_p - LC_p) & B_d \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \times \begin{bmatrix} e_c \\ e_o \\ \tilde{d} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{d}. \tag{21}$$

Similarly, the IOL+UDEM2 controller and its closed-loop error dynamics are given as

$$u(\text{IOL}+\text{UDEM2}) = \frac{1}{\hat{b}_0}\left[-\bar{a}\bar{y} - \frac{(1-\alpha)}{\alpha}y^{\lambda} - \frac{1}{\alpha\tau}y^{(\lambda-1)} + \frac{1}{\alpha}\nu + \frac{1}{\alpha\tau}\int \nu dt\right], \tag{22}$$

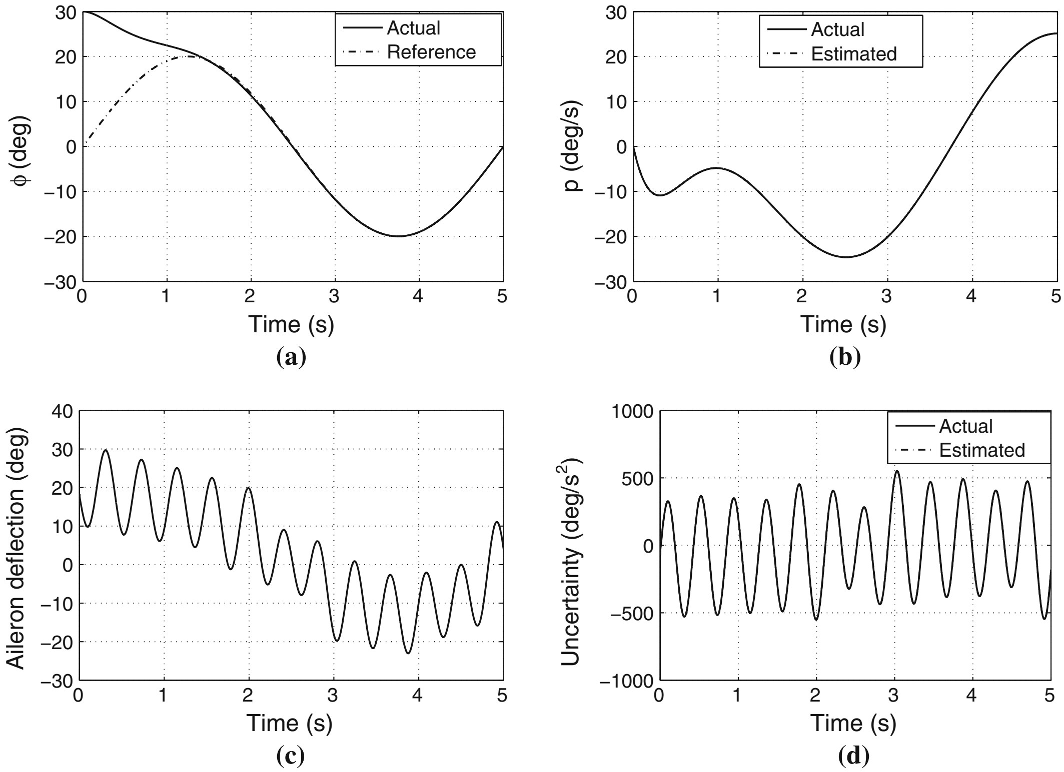

Fig. 6 Performance of IOL+UDEM2 controller in tracking with uncertainties

$$\begin{bmatrix} \dot{e}_c \\ \dot{e}_o \\ \dot{\tilde{d}} \end{bmatrix} = \begin{bmatrix} (A_p - B_p K_p) & -(B_p K_p) & -B_d \\ 0 & (A_p - LC_p) & B_d \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \times \begin{bmatrix} e_c \\ e_o \\ \tilde{d} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix} \dot{d}. \tag{23}$$

Comparing (21) and (23), it can be noticed that the presence of α in (23) permits one to contain the tracking and estimation errors, especially when the disturbance is fast-varying. If $\dot{d}_i \approx 0$, then the choice of α would not make much difference and asymptotic stability for the error dynamics can be assured. However, when $\dot{d} \neq 0$, i.e., when uncertainties and disturbances are fast-varying, use of (19) would give rise to steady-state errors. On the contrary, the use of the proposed α-filter helps in containing the tracking and estimation errors as is obvious from (23). It has been observed during simulations that keeping α sufficiently small improves stabilization/tracking performance of the plant when subjected to fast-varying uncertainties and disturbances. The guidelines on choice of the observer poles in relation to the filter time constant are the same as discussed in [25].

3.2 Analysis of α-filter

In [13], model following design of UDE-based controller for an uncertain linear time-varying system is presented wherein the authors have shown that the design of controllers using the UDE technique offers two-degree-of-freedom nature. The set-point response is determined by the reference model and the disturbance response is determined by the error feedback gain and the filter used in the uncertainty estimation. Further, it is shown that designing the error feedback gain and the filter can be decoupled in the frequency domain; the error feedback gain can be chosen to attenuate the high-frequency components of the uncertainty and disturbance signal, while the filter can be chosen to attenuate the low-frequency components of the uncertainty and disturbance signal. In presenting the results on two-degree-of-freedom nature of the UDE-based controller, the authors have shown that the signal, $u_d(t)$,

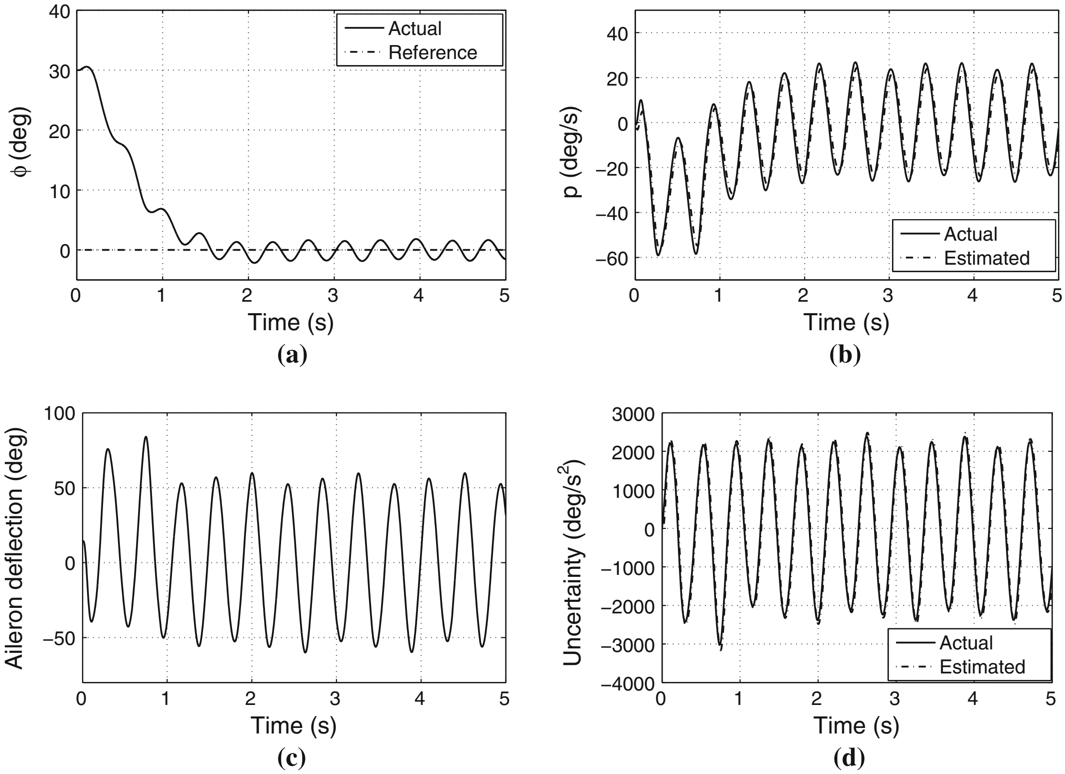

Fig. 7 Performance of IOL+UDEM1 controller in stabilization with uncertainties and varying disturbance

which represents the effect of uncertainty and disturbances in the system is attenuated twice—first by a low-pass filter (LPF) $H_k(s) = (sI - (A_m + K))^{-1}$ and then by a frequency-selective high-pass filter (HPF) $H_f(s) = 1 - G_f(s)$, where A_m is the system matrix of the reference model, K is the error feedback gain matrix, and $G_f(s)$ is the filter used in the estimation of uncertainty. It is interesting to note that if the filter $G_f(s)$ of (19) is replaced with the proposed filter $G_\alpha(s)$ of (10), the function of $H_f(s)$ continues to be HPF with improved performance. To appreciate the extent of the improvement, the magnitude–frequency response plots of $H_f(s)$ with both filters, i.e., $G_f(s)$ and $G_\alpha(s)$ using $\tau = 0.01$ are shown in Fig. 1. For instance, if we take a case with $\alpha = 0.1$, from the figure it can be observed that with the chosen values of τ and α, there is an appreciable improvement in the attenuation performance by a factor of 20 dB over the whole frequency range from 0 to 100π rad/s. It is this attenuation property of the α-filter that helps improving the steady-state error in the presence of fast-varying uncertainties and disturbances. Further, the attenuation improves in steps of 20 dB if α is chosen as 0.01, 0.001, etc., as seen in the figure.

3.3 Implementation issues of α-filter

The technique of UDE employing a first-order filter has distinct advantages in terms of simplicity, bandwidth, ease of implementation, etc. However, owing to the presence of steady-state errors for fast-varying uncertainties and disturbances that may not be acceptable in precision applications, the issue needs to be addressed. In this work, the issue is addressed by carrying out suitable modifications to the first-order filter. It may be noted that the proposed filter $G_\alpha(s)$ with $0 < \alpha \leq 1$ is indeed a lag filter that has the typical characteristic of reducing the steady-state errors. The lag characteristics of the filter are obvious from the frequency response of the $G_\alpha(s)$ for $\alpha = 0.1, 0.3, 0.5$, and 1 with $\tau = 0.01$ shown in Fig. 2. It may be noted that choice of α as 1 results in $G_f(s)$. Since the objective of this work is to prove a concept of improving the performance of

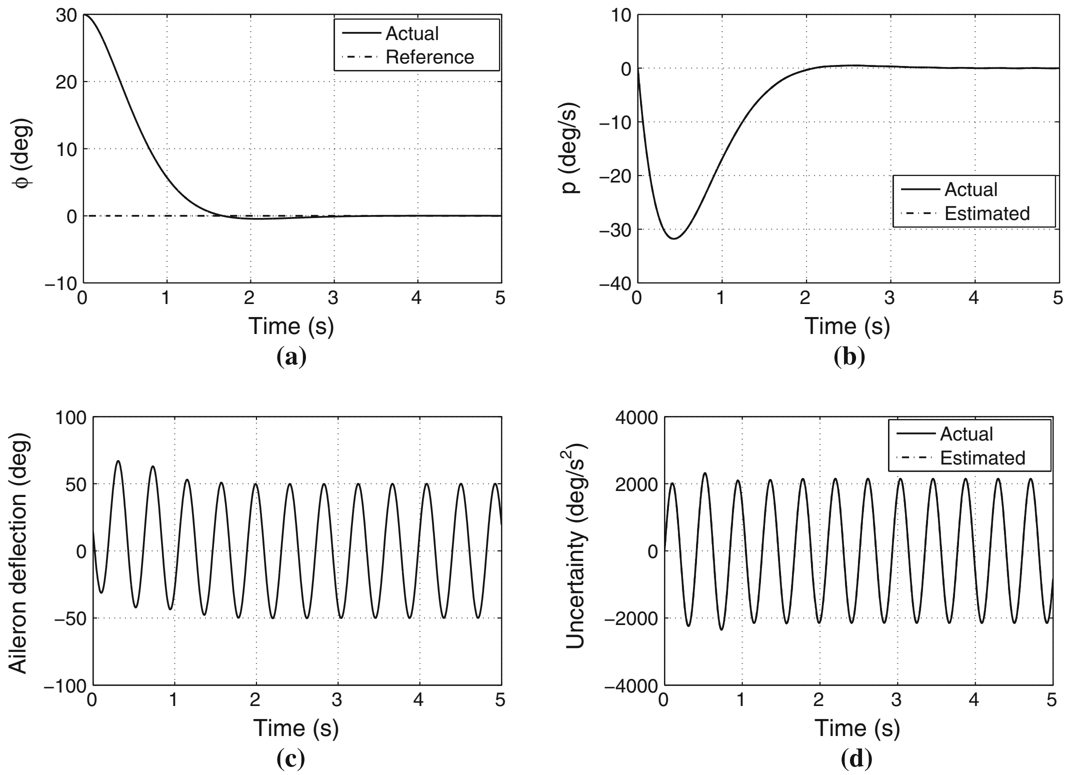

Fig. 8 Performance of IOL+UDEM2 controller in stabilization with uncertainties and varying disturbance

UDE by proposing a new filter, hardware implementation and experimental validation issues are not considered and the same can form the scope for further research.

4 Performance comparison control of wing-rock motion

The phenomenon of wing-rock motion [37–45] represents a self-induced limit cycle oscillation, i.e., oscillatory rolling motion in the presence of some initial disturbance in slender delta-wings that needs to be prevented. The problem gained considerable attention due to its practical importance. In general, the controllers designed based on the assumption of accurate knowledge of the aerodynamic parameters may not offer satisfactory performance in the presence of uncertainty and modeling inaccuracies. In fact, although different models of wing-rock motion have been proposed in literature, they are only approximate and thus controllers are required to cater for the modeling inaccuracies. Here, the dynamics of the wing-rock motion as given in [38] is considered and the same is given by

$$\begin{aligned} \dot{\phi} &= p, \\ \dot{p} &= c_1 + c_2\phi + c_3 p + c_4 |\phi| p + c_5 |p| p + c_6 u, \\ y &= \phi, \end{aligned} \tag{24}$$

wherein c_1 till c_6 are the aerodynamic coefficients describing the roll dynamics and ϕ and p are the roll angle (rad) and roll rate (rad/s), respectively. In stabilization, the objective is to suppress the motion by regulating the output, i.e, ϕ to 0.

4.1 Performance comparison of UDE-based controllers

Now re-writing the dynamics of (24) as

$$\ddot{y} = \hat{c_2}\phi + \hat{c_3} p + \hat{c_6} u + d, \tag{25}$$

where

$$\begin{aligned} d \triangleq\ & c_1 + \Delta c_2\phi + \Delta c_3 p + c_4|\phi|p + c_5|p|p \\ & + \Delta c_6 u + d', \end{aligned} \tag{26}$$

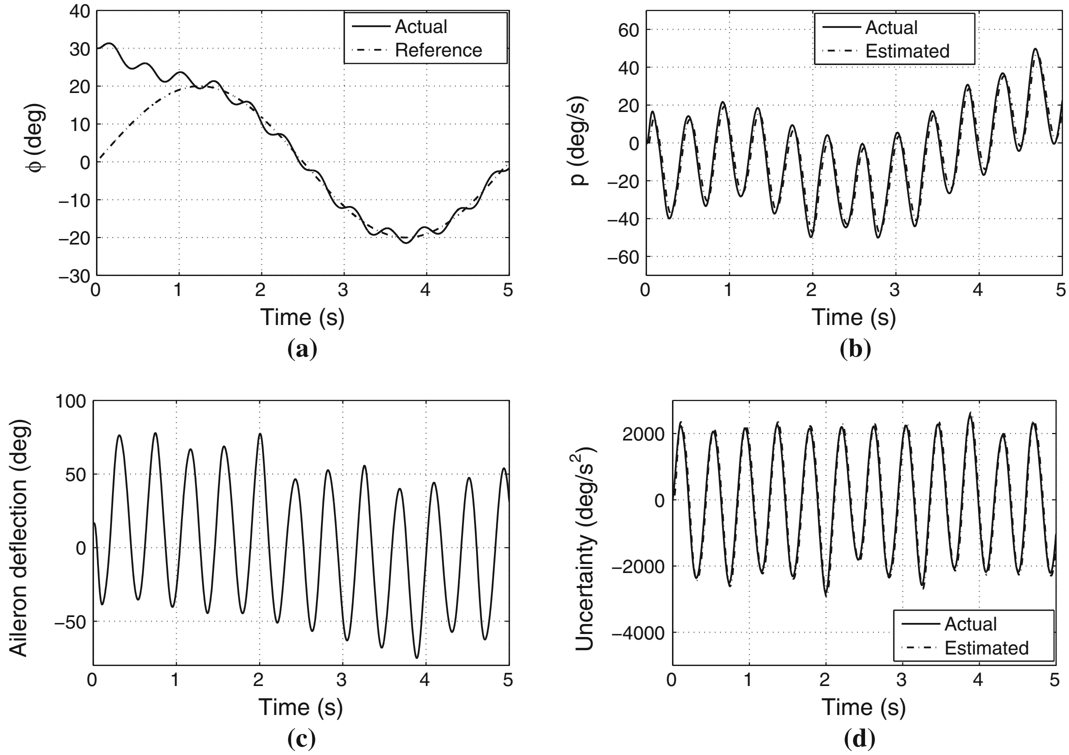

Fig. 9 Performance of IOL+UDEM1 controller in tracking with uncertainties and varying disturbance

and d' denotes unmeasurable external disturbance, if any. The quantities, $\hat{c_2}$, $\hat{c_3}$, and $\hat{c_6}$ are the nominal values of the respective parameters; whereas Δc_2, Δc_3, and Δc_6 are their associated uncertainties. Comparing (25) with (4), one have $\bar{a} = [\hat{c_2}\ \hat{c_3}]$, $\bar{y} = [\phi\ p]^T$, and $\bar{b}_0 = \hat{c_6}$. An UDE-based observer of the form of (15) for the present problem is designed as

$$\begin{aligned} &\dot{\hat{\phi}} = \hat{p} + \beta_1 e_{o1}, \\ &\dot{\hat{p}} = \hat{c_2}\hat{\phi} + \hat{c_3}\hat{p} + \hat{c_6}u + \hat{d} + \beta_2 e_{o1}, \end{aligned} \tag{27}$$

where $e_{o1} \triangleq \phi - \hat{\phi}$ and $L = [\beta_1\ \ \beta_2]^T$ is the observer gain vector. The IOL+UDEM1 controller can be obtained using the observer estimated states in u_a, u_d, and ν in (6) with the UDE filter of (19) as

$$u(\mathrm{IOL} + \mathrm{UDEM1}) = \frac{1}{\hat{c_6}}\left[-(\hat{c_2}\hat{\phi} + \hat{c_3}\hat{p}) - \frac{\hat{p}}{\tau} + \nu + \frac{1}{\tau}\int \nu dt\right]. \tag{28}$$

Similarly, the IOL+UDEM2 controller using the proposed filter of (10) is obtained as

$$u(\mathrm{IOL} + \mathrm{UDEM2}) = \frac{1}{\hat{c_6}}\left[-(\hat{c_2}\hat{\phi} + \hat{c_3}\hat{p}) - \left(\frac{1-\alpha}{\alpha}\right)\dot{\hat{p}} - \frac{1}{\alpha\tau}\hat{p} + \frac{1}{\alpha}\nu + \frac{1}{\alpha\tau}\int \nu dt\right]. \tag{29}$$

It is desired to have a settling time of 2 s and damping ratio of 0.8 and accordingly, the gains m_1 and m_2 are selected. The time constant, τ, of the filters is taken as 0.01, whereas the value of α is chosen as 0.001. The nominal values of the various parameters appearing in (24) for an angle of attack of 30^0 as taken from [38] are

$$\hat{c_2} = -26.7, \quad \hat{c_3} = 0.765, \text{ and } \hat{c_6} = 0.75. \tag{30}$$

Uncertainties are introduced by taking the values of the various parameters in plant equations (24) as

$$\begin{aligned} &c_1 = 5\sin(15t), \quad c_2 = -20, \quad c_3 = 1, \\ &c_4 = 3\cos(5t), \quad c_5 = 10\sin(10t), \quad c_6 = 0.5. \end{aligned} \tag{31}$$

Following the guideline that the observer bandwidth should be greater or comparable to the filter bandwidth,

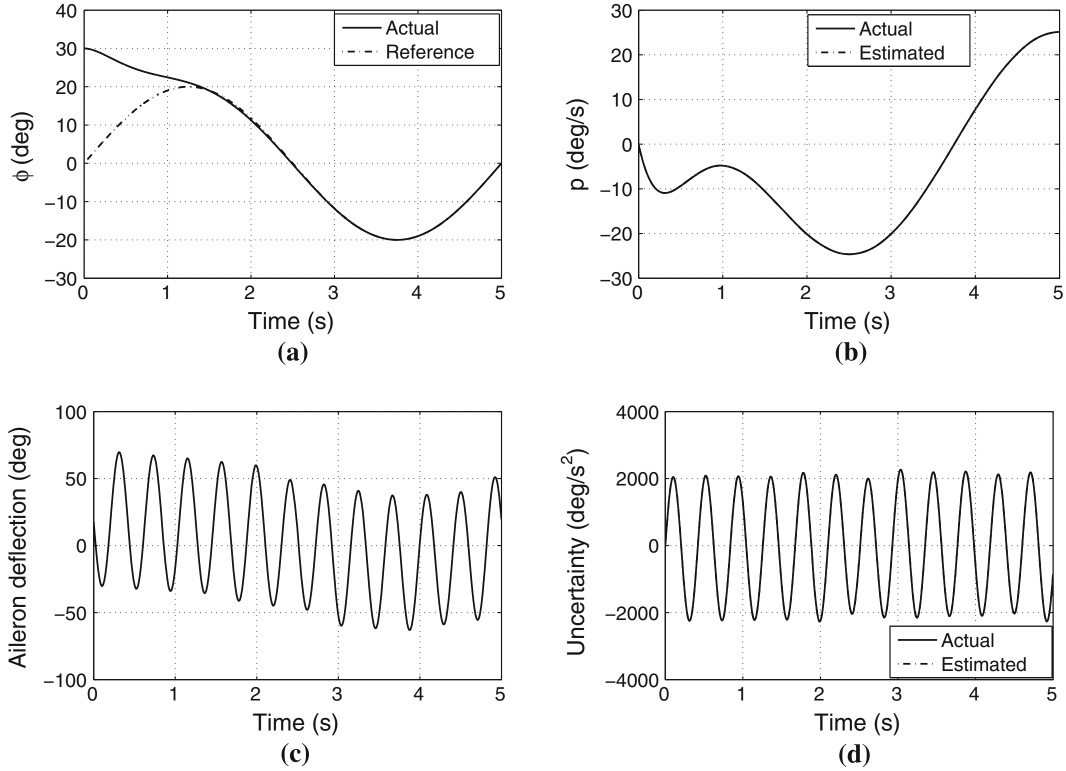

Fig. 10 Performance of IOL+UDEM2 controller in tracking with uncertainties and disturbance

the observer gains, β_i, are chosen to place the observer poles at -100. The initial conditions in plant are chosen as $x_1(0) = 30$ deg and $x_2(0) = 0$ deg/s, whereas the observer initial conditions are taken as zero.

First, the case of parametric uncertainties alone is considered. It may be noticed that the values of the various aerodynamic parameters are significantly different from their nominal values and also some of them are time-varying, thus introducing large time-varying uncertainty in the system. The simulation results using IOL+UDEM1 controller are shown in Fig. 3, whereas the results corresponding to the IOL+UDEM2 controller for the same scenario are as shown in Fig. 4.

From the figures, it can be observed that the performance of the IOL+UDEM2 controller i.e., the controller using the proposed α-filter is quite superior in comparison with the IOL+UDEM1 controller. Next, simulations are carried out for tracking a desired roll angle trajectory of $\phi^{\star}(t) = 20\sin(0.4\pi t)$ and the results are presented in Figs. 5 and 6 for the IOL+UDEM1 and IOL+UDEM2 controllers, respectively. Once again, it can be observed that the IOL+UDEM2 controller offers better performance.

Next, the case with parametric uncertainties as well as fast-varying disturbances acting on the plant is considered. To this end, the system is subjected to a varying disturbance of the form of $20\ \sin(15t)$. The simulation results in stabilization of the roll attitude for IOL+UDEM1 and IOL+UDEM2 controllers are presented in Figs. 7 and 8, respectively, and it can be observed that the IOL+UDEM2 controller outperforms IOL+UDEM1 in containing the errors and ensuring the desired stabilization performance.

Finally, simulations are carried out to assess the tracking performance of a reference command of $\phi^{\star}(t) = 20\ \sin(0.4\pi t)$ in the presence of parametric uncertainties as well as the disturbance and the results are presented in Figs. 9 and 10 for the for IOL+UDEM1 and IOL+UDEM2 controllers, respectively. From the figures it can be observed that the IOL+UDEM2 controller offers superior performance in comparison to that of the IOL+UDEM1 controller.

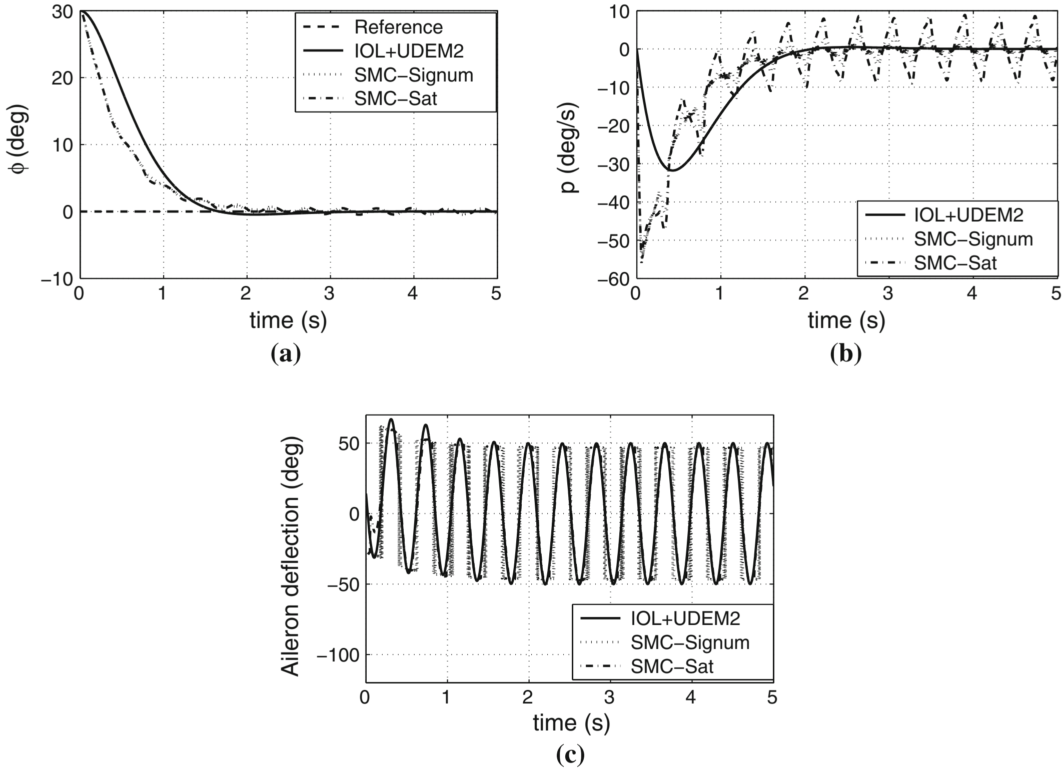

Fig. 11 Performance of IOL+UDEM2 and SMC controller in stabilization with uncertainties and varying disturbances

4.2 Performance comparison with SMC-based controller

Having compared the performances of the IOL+UDEM1 and IOL+UDEM2 controllers in the preceding section, it is also important to compare the performance of the proposed IOL+UDEM2 controller with some other robust control strategy. To this end, the performance of the IOL+UDEM2 controller is compared with the well-known sliding mode control (SMC)-based design and the results are discussed in this section.

Consider the wing-rock motion dynamics of (24) which is re-written as (25), where the total uncertainty, d, is given by (26). Following [20], the SMC control, u, is defined as

$$u = \frac{1}{\hat{c}_6}\left[u_a + u_d + \nu\right], \tag{32}$$

where u_a is the nominal control, u_d is the part of the SMC control that caters for the effect of d, and ν is the outer loop control. The nominal control is designed as

$$u_a = -\hat{c}_2\phi - \hat{c}_3 p, \tag{33}$$

whereas the outer loop control is considered as

$$\nu = \ddot{\phi}^* + m_2(\dot{\phi}^* - \dot{\phi}) + m_1(\phi^* - \phi), \tag{34}$$

where ϕ^* is the desired roll angle, and m_1 and m_2 are the gains chosen to meet the specifications of settling time of 2 s and a damping of 0.8 for the closed-loop response. Now defining $\tilde{y} = \phi^* - \phi$ and the sliding surface by $s = \dot{\tilde{y}} + K_s\tilde{y}$, the control u_d is designed as

$$u_d = D\mathrm{sgn}(s), \tag{35}$$

where D is the assumed bound of uncertainties and disturbances. Substitution of (33), (34), and (35) in (32) gives the SMC-based controller. As is well known in the SMC theory, the signum function in (35) may result into chattering and to address the issue frequently the function is replaced by saturation function of the form

$$\begin{aligned}\mathrm{sat}\left(\frac{s}{\varepsilon}\right) &= \frac{s}{|s|} \quad \text{if } |s| > \varepsilon, \\ &= \frac{s}{\varepsilon} \quad \text{if } |s| \le \varepsilon,\end{aligned} \tag{36}$$

where ε is width of the boundary layer.

Simulations are carried out using the SMC-based design for both the cases, i.e., using signum as well

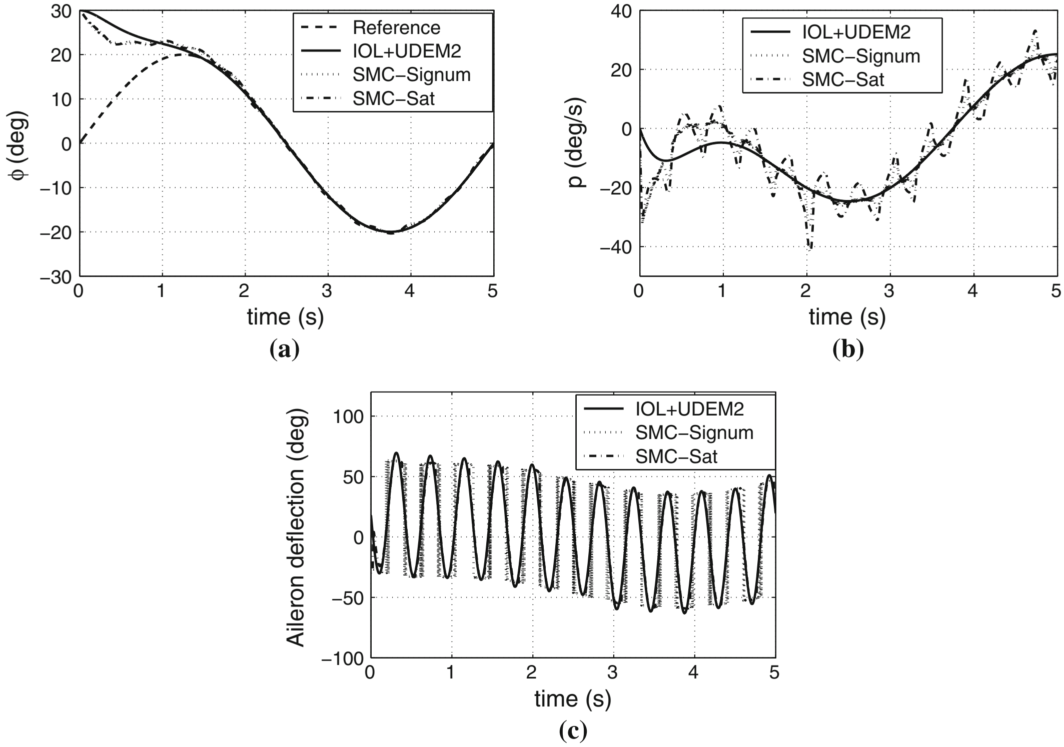

Fig. 12 Performance of IOL+UDEM2 and SMC controller in tracking with uncertainties and varying disturbances

as saturation functions for the stabilization as well as tracking scenarios considered in the previous section. In simulations, the bound on uncertainty, D, is taken as 500 for the case of parametric uncertainties alone and 2, 000 when parametric uncertainties as well as disturbances are present. The values are chosen by observing the simulation results on estimated and actual uncertainties using the IOL+UDEM2 controller. The value of $K_s = 2$ is chosen to meet the specifications. For valid comparison, the parametric uncertainties given by (31) used for IOL+UDEM2 controller are considered. For the case of saturation function, the width of the boundary layer, ϵ, is taken as 0.1. The results of the simulations with IOL+UDEM2 and SMC controllers for stabilization as well as tracking a reference trajectory given by $20\sin(0.4\pi t)$ under the conditions of parametric uncertainties and varying disturbance of the form $20\sin(15t)$ are given in Figs. 11 and 12.

From the figures it can be observed that due to signum function, the SMC controller has not resulted in a smooth control effort (aileron deflection) while the same has been smooth with IOL+UDEM2 controller. It can also be observed that the performance of the IOL+UDEM2 controller is better than the SMC controller for stabilization as well as tracking albeit the use of the saturation function in the SMC controller to some extent managed to improve its performance. It is important to note that while the SMC controller employed actual states, estimated states are utilized in implementing the IOL+UDEM2 controller. In spite of using the estimated states, the IOL+UDEM2 controller has been able to perform better in comparison with the SMC controller. Needless to mention that if the SMC controller had used estimated states (by designing a separate observer), the performance of the SMC controller is likely to degrade. Also, successful implementation of the SMC controller depends upon a priori knowledge on the bound of uncertainties and disturbance. In contrast, the proposed IOL+UDEM2 controller does not require the same. In the simulations, the values of the bound used are quite accurate since they are taken from the UDE estimated uncertainties. However, in general, the value of the bound may not be always known accurately. This situation is critical while designing controllers for plants with poor or insufficient knowledge of uncertainties, where design based on the IOL+UDEM2 strategy would be ideally suited.

5 Conclusions

In this work, a new filter for UDE-based controller is proposed to achieve reduced tracking and estimation errors in the presence of fast-varying uncertainties. Closed-loop stability results are presented and it is shown that it is possible to keep the error within acceptable limits by choosing filter parameters appropriately. Simulation results for wing-rock motion control problem employing the UDE-based controller–observer structure using the proposed filter and its comparison with similar controller appeared in literature are presented to demonstrate the efficacy and performance benefits offered by the proposed design in the presence of fast-varying uncertainties and disturbances.

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