Model Following Sliding Mode Control Based on Uncertainty and Disturbance Estimator

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A new design of sliding mode control based on an uncertainty and disturbance estimator (UDE) is given. The control proposed does not require the knowledge of bounds of uncertainties and disturbances and is continuous. Thus, two main difficulties in the design of sliding mode control are overcome. Furthermore, the method of UDE is extended to plants having significant uncertainty in the control input matrix and subjected to disturbances that nonlinearly depend on states. [DOI: 10.1115/1.2909604]

Keywords: sliding mode control (SMC), uncertainty and disturbance estimator (UDE), uncertain systems

1 Introduction

Sliding mode control (SMC) is an effective strategy for controlling systems with significant uncertainties and unmeasurable disturbances. In the conventional SMC systems, insensitivity to uncertainties and disturbances is guaranteed by employing a discontinuous control based on the bounds of uncertainties and disturbances. In many situations, these bounds are hard to find, which result in an overestimation and consequently a large control. The discontinuous control is objectionable because it can cause excessive wear and tear of actuators and may excite unmodeled dynamics.

One method to overcome the need to know the bounds of the uncertainty explicitly is to find the bounds adaptively and then use them in the SMC. Slotine and Coetsee [1] proposed an SMC, in which the bounds on uncertain parameters are found adaptively. Simple control laws, in which a function bounding the uncertainty is found adaptively, are given in Refs. [2,3].

Youcef Toumi and Ito [4] proposed a nonadaptive approach called the time delay control (TDC) for uncertain systems. In this approach, the uncertainties and disturbances are estimated from past measurements under the assumption that the uncertainties and disturbances do not significantly change in a small interval of time. The estimate is then used in control so as to nullify the effect of uncertainty and disturbances on the system. Copious applications of this method including those to SMC are reported in the literature in the past 15 years [5–10]. In a similar approach, Chan [11] designed a control compensating the perturbation in the system. A common concern in all these methods is the need for the derivative of the state or the sliding surface variable.

Recently, Zhong and Rees [12] have proposed a very promising method called the uncertainty and disturbance estimator (UDE) for control of linear time invariant systems. This method, although based on an idea similar to the TDC, does not require the derivative of the system state and does not use time delayed signals. In this paper, the results of Ref. [12] are extended to SMC. The proposed control has several merits over both the conventional SMC and the UDE method.

In conventional SMC, the control is discontinuous, which results in undesirable chatter. Furthermore, the control can be designed if the bounds of uncertainty and disturbances are known. The chattering control is undesirable for several reasons and the bounds of uncertainty and disturbances are not always easy to find. The proposed control enforces sliding mode without using discontinuous control and without requiring the knowledge of uncertainties or their bounds.

Zhong and Rees [12] have proposed a control for linear systems, which have no uncertainties in the input matrix. In this paper, the method is extended to disturbances containing state dependent nonlinearities and to systems having significant uncertainties in the control input matrix. Furthermore, the choice of the sliding surface proposed in this paper avoids the difficulty of large initial amplitudes of the input seen with the UDE approach of Ref. [12]. This paper analyzes the accuracy of estimation and proposes a method to improve the accuracy. This paper is organized as follows: Section 2 states the problem. A model following control is designed in Sec. 3 and is illustrated by a numerical example in Sec. 4. The conclusions are stated in Sec. 5.

2 Statement of the Problem

Consider a single input single output plant

$$\dot{x} = Ax + bu + \Delta Ax + \Delta bu + d(x,t) \tag{1}$$

where x is the state vector, u is the control input, A and b are known constant matrices, ΔA and Δb are uncertainties, and $d(x,t)$ is an unknown disturbance.

ASSUMPTION 1. *The uncertainties* ΔA and Δb *and the disturbance* $d(x,t)$ *satisfy the matching conditions given by*

$$\Delta A = bD, \quad \Delta b = bE, \quad d(x,t) = bv(x,t) \tag{2}$$

where D and E are unknown matrices of appropriate dimensions and $v(x,t)$ *is an unknown function. The system of Eq. (1) can now be written as*

$$\dot{x} = Ax + bu + be(x,t) \tag{3}$$

where $e(x,t)=Dx+Eu+v(x,t)$. *Although* $e(x,t)$ *contains uncertainty and disturbances, in the sequel, it will be referred to as the lumped uncertainty for the sake of convenience. Next, let*

$$\dot{x}_m = A_m x_m + b_m u_m \tag{4}$$

be a stable model satisfying certain structural conditions stated by the following assumption:

ASSUMPTION 2. *The choice of the model is such that*

$$A - A_m = bL, \quad b_m = bM \tag{5}$$

where L and M are suitable known matrices.

The objective is to design a control u so as to force the plant of Eq. (3) to follow the model of Eq. (4) in spite of the uncertainties and disturbances represented by $e(x,t)$. Equations (2) and (5) are well-known matching conditions required to guarantee invariance and are an explicit statement of the structural constraint stated in Ref. [12].

3 Design of Control

In this section, a model following control is designed in the framework of SMC. The method is based on a sliding surface suggested in Ref. [13] and is different from the SMC based model following reported in literature.

Define a sliding surface,

Contributed by the Dynamic Systems, Measurement, and Control Division of ASME for publication in the JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL. Manuscript received January 11, 2005; final manuscript received July 11, 2007; published online April 24, 2008. Assoc. Editor: Prabhakar R. Pagilla.

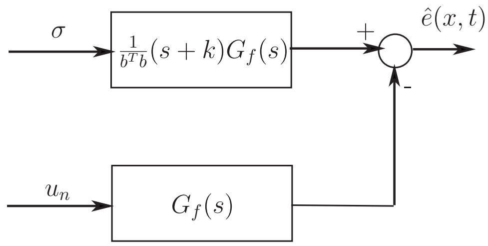

Fig. 1 Block diagram representation of Eq. (15)

$$\sigma = b^T x + z \tag{6}$$

where

$$\dot{z} = -b^T A_m x - b^T b_m u_m, \quad z(0) = -b^T x(0) \tag{7}$$

Equation (7) for the auxiliary variable z given here is different from that given in Ref. [13]. By virtue of the choice of the initial condition on z, $\sigma=0$ at $t=0$. If a control u can be designed ensuring sliding, then $\dot{\sigma}=0$ implies

$$\dot{x} = A_m x + b_m u_m \tag{8}$$

and thus fulfills the objective of model following. Differentiating Eq. (6) and using Eq. (3) with Eq. (7) give

$$\dot{\sigma} = b^T A x + b^T b u + b^T b e(x,t) - b^T A_m x - b^T b_m u_m \tag{9}$$

$$= b^T b L x - b^T b M u_m + b^T b u + b^T b e(x,t) \tag{10}$$

Let the required control be expressed as

$$u = u_{\mathrm{eq}} + u_n \tag{11}$$

Selecting

$$u_{\mathrm{eq}} = -Lx + M u_m - \frac{k}{b^T b}\sigma \tag{12}$$

where k is a positive constant, from Eqs. (11), (12), and (10), we get

$$\dot{\sigma} = b^T b u_n + b^T b e(x,t) - k\sigma \tag{13}$$

Next, the component u_n will be designed.

3.1 Compensation of Uncertainties and Disturbances. The lumped uncertainty $e(x,t)$ can be compensated by estimating it on the lines of Ref. [12]. Rewriting Eq. (13) as

$$e(x,t) = \frac{1}{b^T b}(\dot{\sigma} + k\sigma) - u_n \tag{14}$$

it can be seen that the lumped uncertainty $e(x,t)$ can be computed from the right hand side of Eq. (14). This, however, cannot be done directly. Let the estimate of the lumped uncertainty, denoted by $\hat{e}(x,t)$, be defined as

$$\hat{e}(x,t) = \frac{1}{b^T b}(s+k)G_f(s)\sigma - G_f(s)u_n \tag{15}$$

where $G_f(s)$ is a strictly proper low-pass filter with unity steady-state gain and broad enough bandwidth. It can be seen that Eq. (15) uses variables in both time and s-domain. While such a use is not uncommon in the literature, it may be stated that the equation can be interpreted as signals operating on hardware described by transfer functions, as illustrated in Fig. 1. Such an interpretation

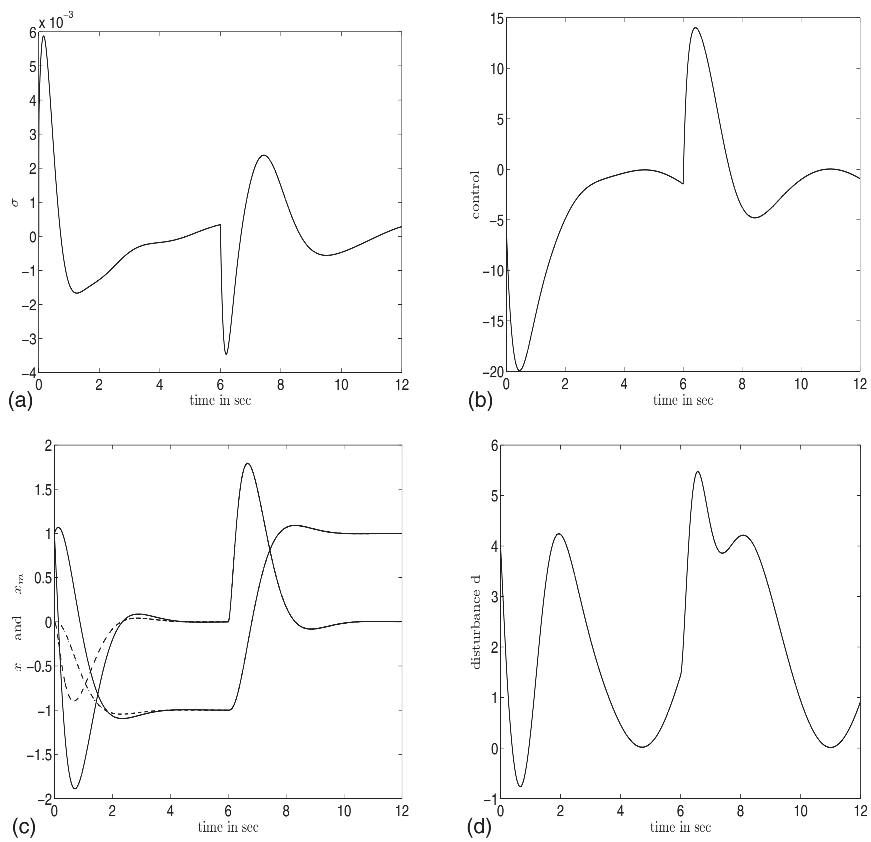

Fig. 2 Case 1—plots of the model following: (a) σ, (b) control input, (c) states of plant and model, (d) disturbance

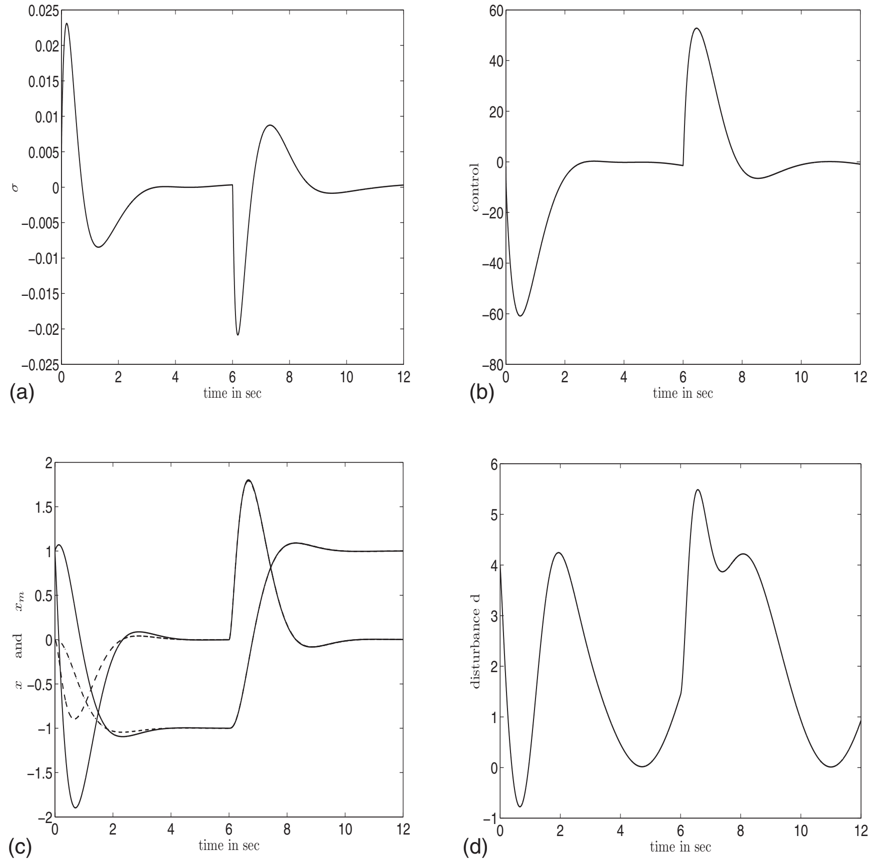

Fig. 3 Case 2–increased uncertainty $\Delta b=[0-0.8]^T$: (a) σ, (b)control input, (c) states of plant and model, (d) disturbance

affords simplification of equations on the lines of block diagram simplification. With such a filter,

$$\hat{e}(x,t) \simeq e(x,t) \tag{16}$$

which enable the design of u_n as

$$u_n = -\hat{e}(x,t) \tag{17}$$

$$= -\frac{1}{b^T b}(s+k)G_f(s)\sigma + G_f(s)u_n \tag{18}$$

Solving for u_n gives

$$u_n = -\frac{1}{b^T b(1-G_f(s))}(s+k)G_f(s)\sigma \tag{19}$$

Clearly, since $G_f(s)$ is strictly proper, the control signal u_n in Eq. (19) is implementable.

3.2 Existence of Sliding Mode. The existence of the sliding mode can be proved easily. We define the error in estimation as

$$\tilde{e}(x,t) = e(x,t) - \hat{e}(x,t) \tag{20}$$

Using Eq. (17) in Eq. (13), we get

$$\dot{\sigma} = -k\sigma + b^T b\tilde{e}(x,t) \tag{21}$$

which, in view of Eq. (16), leads to

$$\sigma\dot{\sigma} = -k\sigma^2 \tag{22}$$

Since $\sigma=0$ at $t=0$ by virtue of the choice of σ in Eqs. (6) and (7), satisfaction of Eq. (22) ensures $\sigma=0$ for all $t \geqslant 0$. This makes the uncertain plant follow the stable model chosen by the designer for all $t \geqslant 0$.

3.3 Accuracy of Estimation. The above result is based on the premise that Eq. (16) holds. In this section, we take a closer look at the error in estimation.

We consider a practical low-pass filter

$$G_f(s) = \frac{1}{Ts+1} \tag{23}$$

where T is a small positive constant. With the above $G_f(s)$ and in view of Eqs. (14), (15), and (20),

$$\tilde{e}(x,t) = (1-G_f(s))\left[\frac{1}{b^T b}(\dot{\sigma}+k\sigma) - u_n\right] \tag{24}$$

$$= T\dot{e}(x,t)G_f(s) \tag{25}$$

Therefore, Eq. (16) will hold, if the term $T\dot{e}(x,t)$ is sufficiently small. Interestingly, this is similar to the usual assumption $e(x,t) \simeq e(x,t-L)$, where L is a small interval of time, found in the TDC approach.

It is worth noting that for $G_f(s)$ in Eq. (23), the control u_n works out to

$$u_n = -\frac{1}{b^T bT}\left[\sigma + \frac{k}{s}\sigma\right] \tag{26}$$

and has a simple time domain interpretation. From Eqs. (26) and (25), it is clear that smaller T implies a smaller estimation error but a larger magnitude of control if sigma is not small. The choice of σ as given in Eq. (6) and (7) enables the designer to strike a favorable compromise in this respect.

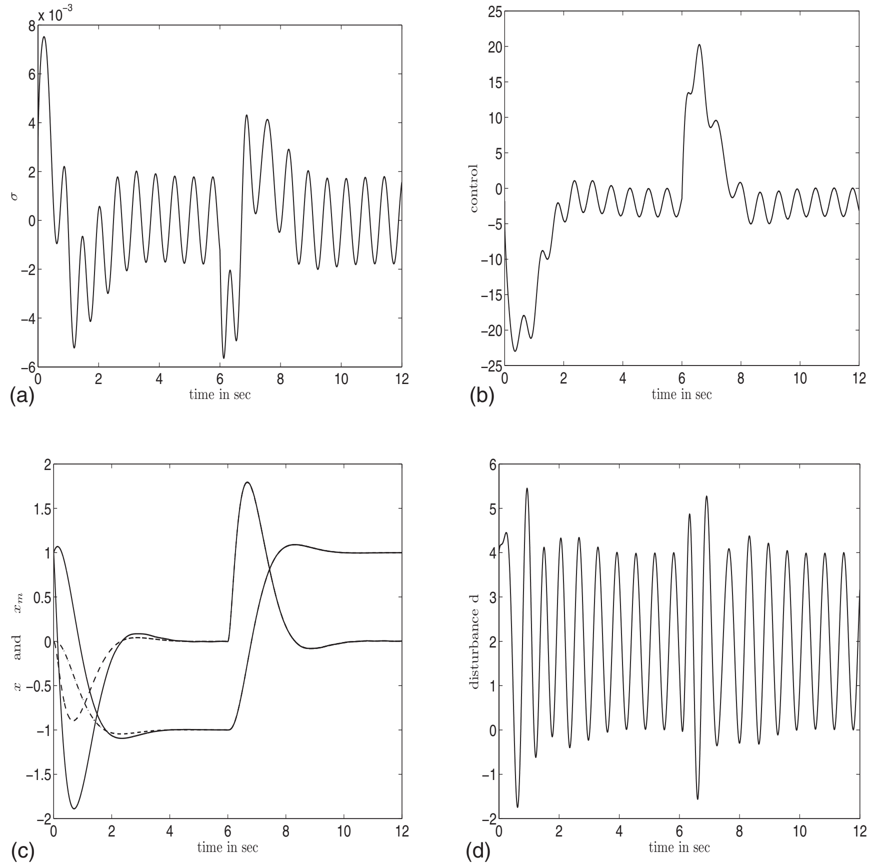

Fig. 4 Case 3—a more rapidly changing disturbance: (a) σ, (b) control input, (c) states of plant and model, (d) disturbance

3.3.1 Improvement of Accuracy. The accuracy of estimation can be improved as much as desired by an appropriate choice of $G_f(s)$. For example, if $T\dot{e}(x,t)$ is not sufficiently small, it can be accounted for by the following filter:

$$G_f(s) = \frac{1}{T^2s^2 + 2Ts + 1} \tag{27}$$

The lumped uncertainties and disturbances can be written as

$$\begin{aligned} e(x,t) &= e(x,t)G_f(s) + e(x,t)(1 - G_f(s)) \\ &= e(x,t)G_f(s) + e(x,t)(2Ts + T^2s^2)G_f(s) \\ &= e(x,t)(1 + 2Ts)G_f(s) + T^2G_f(s)\ddot{e}(x,t) \end{aligned}$$

Now, defining

$$\begin{aligned} \hat{e}(x,t) &= e(x,t)(1 + 2Ts)G_f(s) \\ &= (1 + 2Ts)G_f(s)\left(\frac{1}{b^Tb}(\dot{\sigma} + k\sigma) - u_n\right) \\ &= (1 + 2Ts)G_f(s)\left(\frac{s+k}{b^Tb}\sigma - u_n\right) \end{aligned} \tag{28}$$

the control u_n based on the above $\hat{e}(x,t)$ is implementable since $(1+2Ts)(s+k)G_f(s)$ is proper for the choice of $G_f(s)$ in Eq. (27). Now, Eq. (16) will hold if $T^2\ddot{e}(x,t)$ is sufficiently small. The result can be easily generalized.

4 Illustrative Example

The plant is of the form of Eqs. (1) and (2) with

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \Delta A = \begin{bmatrix} 0 & 0 \\ 2 & -3 \end{bmatrix},$$

$$\Delta b = \begin{bmatrix} 0 \\ -0.4 \end{bmatrix}, \quad v(x,t) = 2(\sin(t)x_1^2 + \cos(t)x_2 + 1)$$

The model to be followed is

$$A_m = \begin{bmatrix} 0 & 1 \\ -4 & -2.8 \end{bmatrix}, \quad b_m = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

There is an initial condition mismatch between the plant and the model given by $x(0)=[1\ 1]^T$ and $x_m(0)=[0\ 0]^T$. The low-pass filter is chosen as $1/(1+Ts)$ with $T=0.001$ and the control gain $k=5$. The reference input u_m is a square wave switching at 6 s. The simulation results in Fig. 2 show that in spite of significant uncertainties and disturbances, the system follows the model very closely. In the illustration, these results are referred to as Case 1.

Next, the uncertainty in the control input matrix was increased to 80% by taking $\Delta b=[0 -0.8]^T$, while the uncertainty ΔA was kept the same as in Case 1. The simulation results shown in Fig. 3 show that the control effort has increased but the model following accuracy is almost the same as that of the case with $\Delta b=[0 -0.4]^T$. This is Case 2.

Finally, the effectiveness of the controller was assessed for a disturbance that changed significantly faster than in Cases 1 and 2. The disturbance term for this case is given by $v(x,t)=2(\sin(10t)x_1^2+\cos(10t)x_2+1)$. The results shown in Fig. 4 confirm the following good model. Without going into a detailed comparison, it appears that the method of UDE copes with fast varying disturbances and large control input matrix uncertainties better than the TDC [14,15]. This is referred to as Case 3. Simulation with pure SMC was also conducted for the plant described above. The results (not shown here) confirmed the drawbacks of pure SMC, as mentioned in Sec. 1.

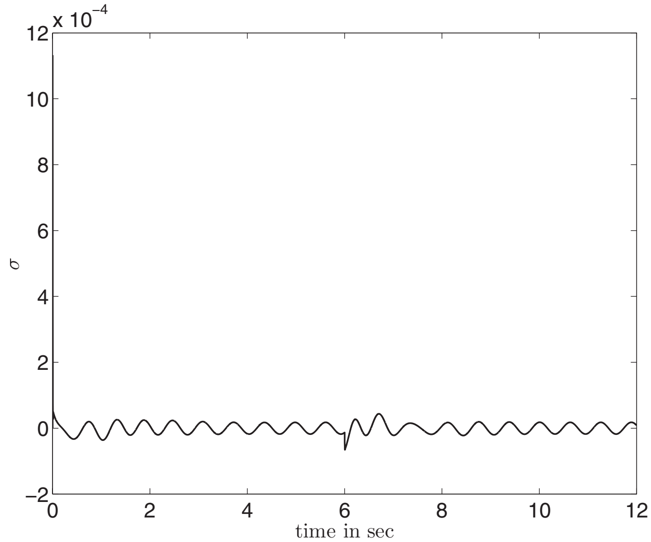

Fig. 5 Improvement of accuracy with second order filter

As indicated in Sec. 3.3.1, the accuracy of estimation can be improved by employing a filter like that in Eq. (27). The reduction of σ with such a modification for the plant with the same rapidly changing disturbance as in Case 3 is shown in Fig. 5. The reduction is significant.

5 Conclusion

In this paper, uncertainty and disturbance estimator (UDE) is extended and applied to SMC of uncertain plants overcoming two main problems in the design of SMC. The effectiveness of the proposed controller is verified by simulation. The controller tolerates significant uncertainties, including those in control input matrix, and disturbances. This paper also suggests how the accuracy of the estimator can be improved.

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