Elementary parameter estimation

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Prior 6.14

Suppose we draw N balls from a Bernoulli distribution of probability p (i.e. each ball is red with probebility p and white with probability $1-p$). The probability we get R reds is $\binom{N}{R}p^R(1-p)^{(N-R)}$. Now, if our prior probability for p is uniform, then the probability of getting R reds is $\int_0^1 \binom{N}{R}p^R(1-p)^{(N-R)}$. Bayes 1763 paper tells us this is $\frac{1}{N+1}$ (see "Bayes' billiards", for example Story 8.3.2 in Blitzstein-Hwang, "Introduction to Probability"). So indeed, 6.14 is an uniformed prior in this sense as well.

Remark: This works in multicolor setting as well: Suppose N balls drawn from a large vat of balls in which there are balls of K colors in total, with unknown fractions $p_1, \ldots, p_K$ of balls of each color. Then the probability of getting N_1 balls of the first color, N_2 balls of the second color etc. - averaged over all possible tuples $p_1, \ldots, p_K$ - is always the same, no matter what the numbers N_i are. Since there are $\binom{N+K-1}{K-1}$ such tuples, each one has probability $\binom{N+K-1}{K-1}^{-1}$; for $K=2$ this is $\frac{1}{N+1}$ as before.

Summation formula 6.16

To choose $n+1$ balls from $N+1$: first choose the number $R+1$ of the $r+1$st chosen ball; then choose r balls from the first R; and, finally, $n-r$ from the last $N-R$.

Remark: This also follows from the more obvious Vandermonde identity $\sum_r \binom{R}{r}\binom{N-R}{n-r} = \binom{N}{n}$ by "upper index negation", see here.

Most probable value 6.21

This is the same as derivation of formulas 3.26 and 3.27 in Chapter 3 (see notes for that chapter).

6.25 and 6.29

$$E\left[\frac{R-r}{N-n}\right] = \frac{E[R+1]-(r+1)}{N-n} =$$

$$\frac{\frac{(N+2)(r+1)}{n+2} - (r+1)}{N-n} = \frac{r+1}{n+2}$$

In light of our remarks about the Prior 6.14 this **is** equivalent the fact that uniform is the $\beta(1,1)$, conjugate prior to Bernoulli with the two parameters $\alpha = 1$ and $\beta = 1$ equal to the number of prior imaginary successes and failures.