Queer uses for probability theory

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Exercise 5.3

From 5.31 we want $x > y$ but

$$\log \frac{x}{1-x} + \log \frac{a_x}{b_x} < \log \frac{y}{1-y} + \log \frac{a_y}{b_y}$$

i.e.

$$\log \frac{x}{1-x} - \log \frac{y}{1-y} < \log \frac{a_y}{b_y} - \log \frac{a_x}{b_x}$$

or, multiplicatively

$$\frac{x(1-y)}{y(1-x)} < \frac{a_y b_x}{a_x b_y}.$$

This is of course very possible. It means simply that the level of trust of N is sufficiently different between X and Y that the the difference of evidence perceived by X and Y from the announcement is bigger than their prior difference of evidences.

Exercise 5.5

Everything in this exercise is conditional on I.

If the "graphical model" assumption $C \to B \to A$, is true (meaning that A and C are independent conditional on B), then 5.43 becomes

$$P(A|CI) = qP(A|BI) + (1-q)P(A|\overline{B}I)$$

and since $P(A|\overline{B}I)$ is bounded by 1, as $q \to 1$ we do have $P(A|CI) \to P(A|BI)$.

In general, however, even $q = 1$ does not imply closeness of $P(A|BI)$ and $P(A|CI)$. Suppose we have a fair 4 sided die. Let C be the event "4 is rolled", B the event "the result is even", A the event "the result is 1 or 2". Then $q = P(B|C) = 1$, but $P(A|C) = 0$ while $P(A|B) = 1/2$.

By taking more-sided dice we can even make $P(A|B)$ arbitrarily small while keeping the other implications.

An example “from life”: $B =$“I’m in San Francisco” implies that the probability that $A|B =$“It is snowing around me” is very low. However, if I know $C =$“I’m in San Francisco and the date is February 5, 1976” then I am certain of $B|C$ (so $q = 1$), but also certain of $A|C$.