3T2: Fourier Transform properties (2 of 2)
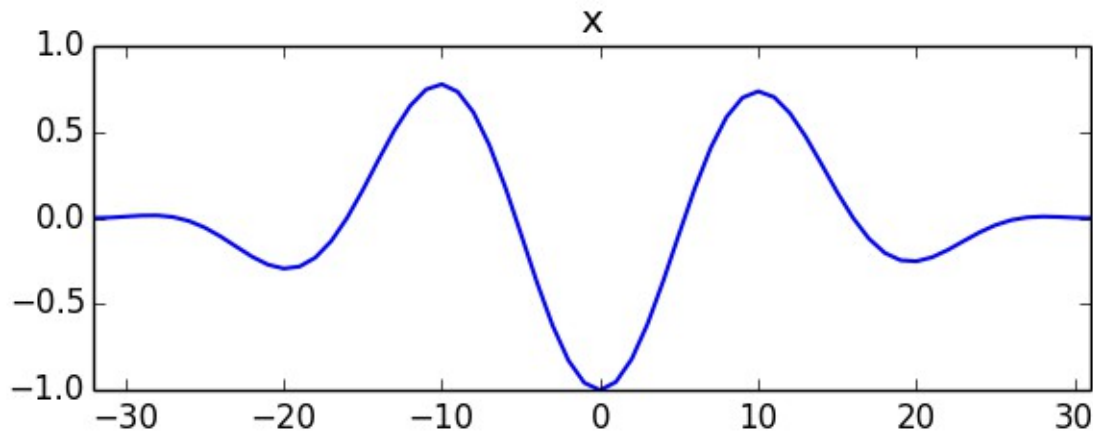
Xavier Serra
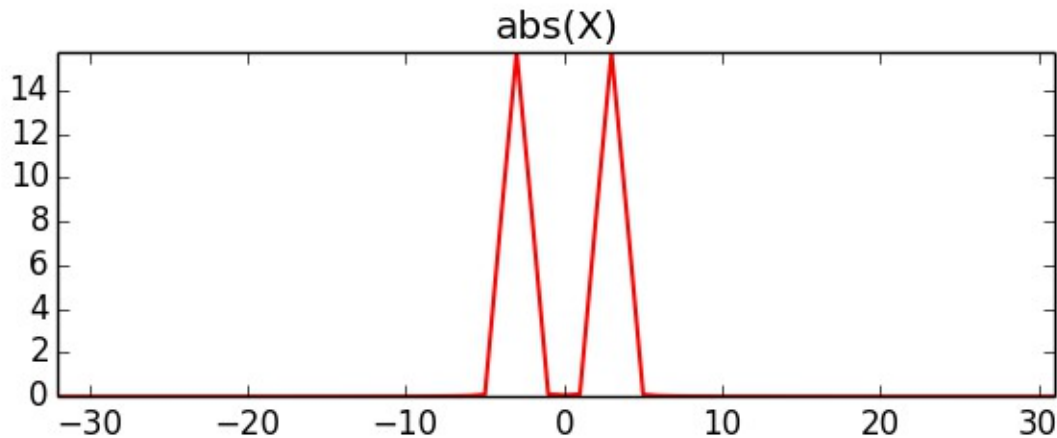
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Index

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- Phase unwrapping

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- Fast Fourier Transform (FFT)

- FFT and zero-phase windowing

- Analysis/synthesis

Energy conservation

$$\sum_{n=-N/2}^{N/2-1} |x[n]|^2 = \frac{1}{N} \sum_{k=-N/2}^{N/2-1} |X[k]|^2$$
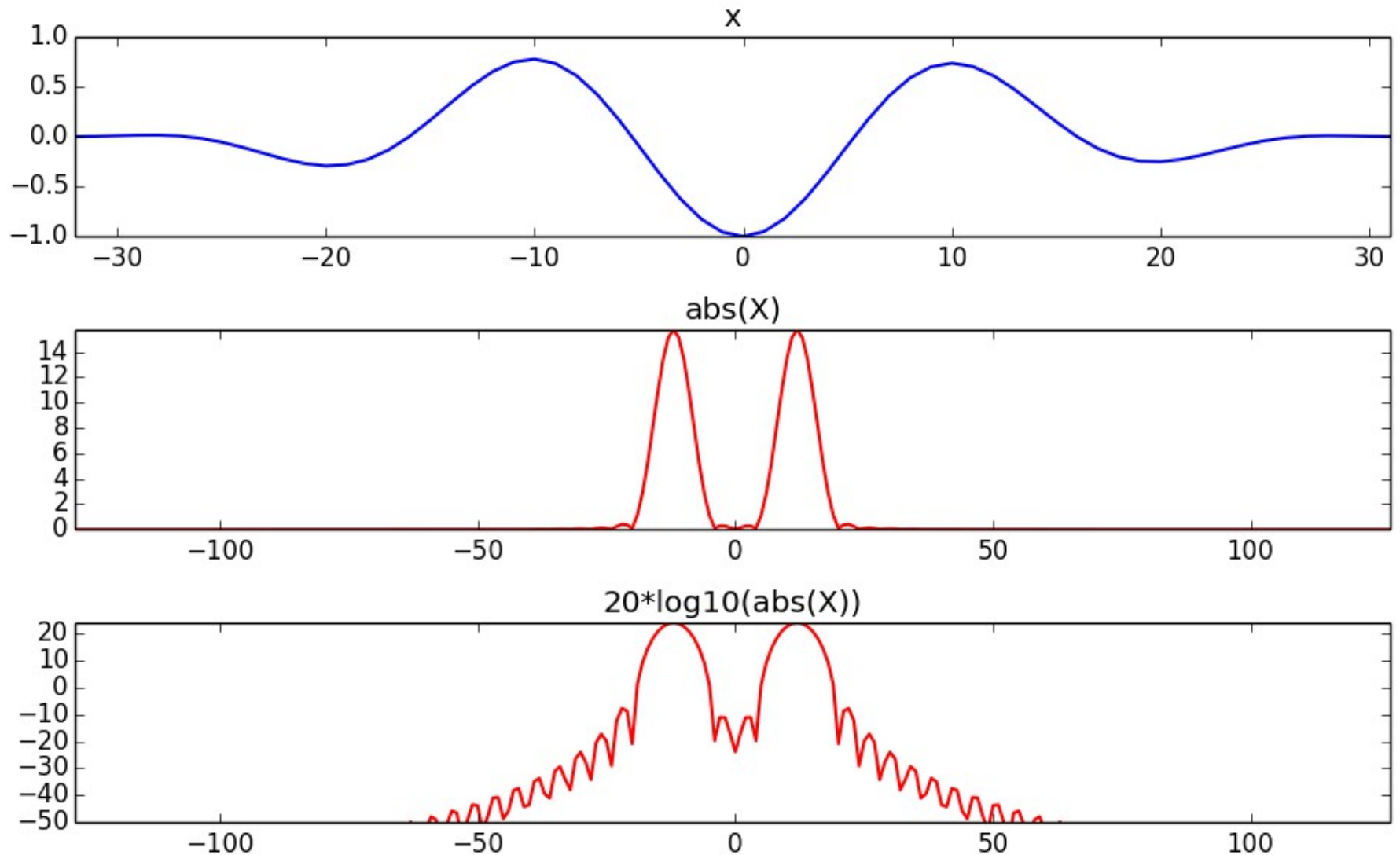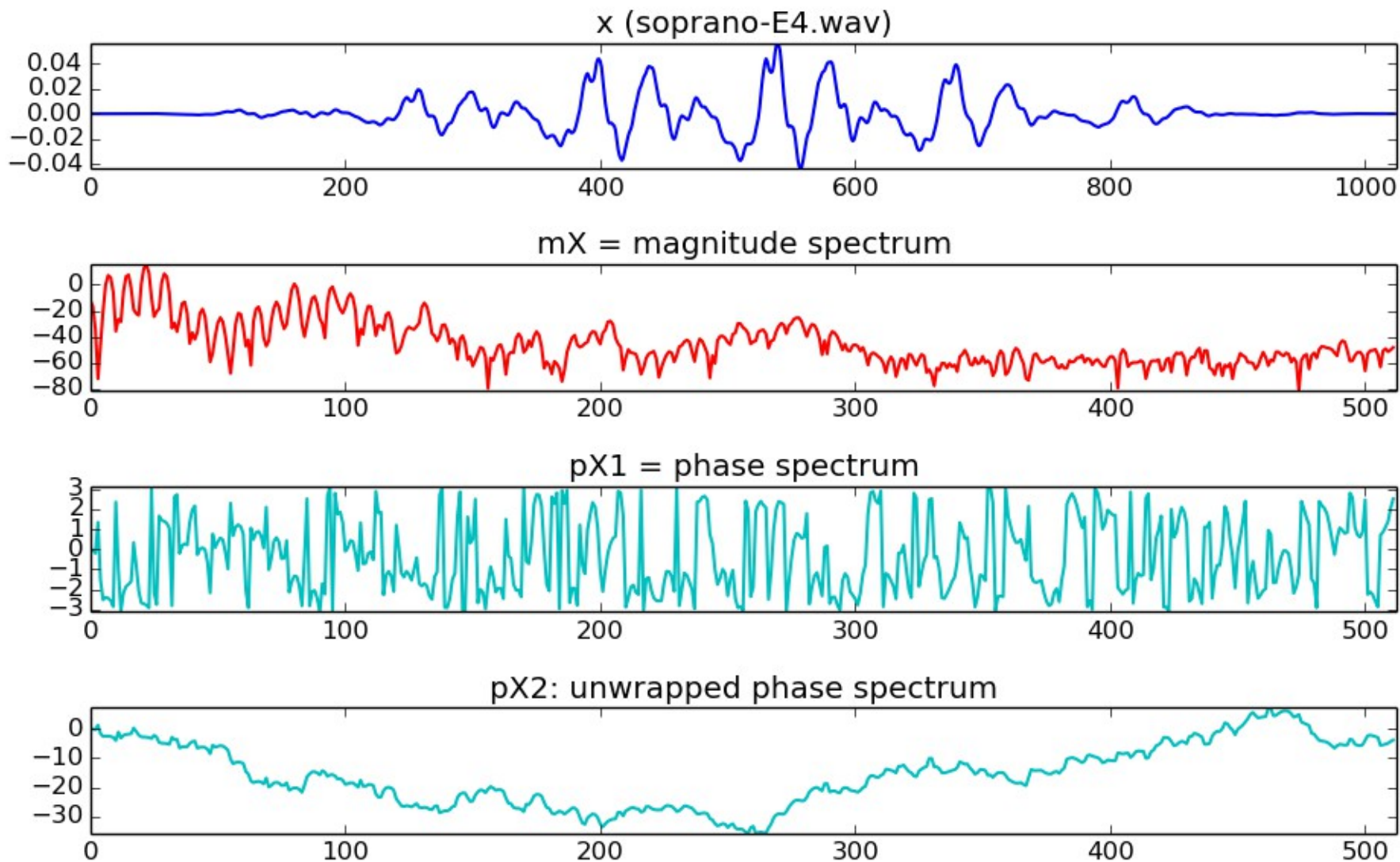

$$\sum_{n=N/2}^{N/2-1} |x[n]|^2 = 11.81182$$

$$\frac{1}{N} \sum_{k=N/2}^{N/2-1} |X[k]|^2 = 11.81182$$
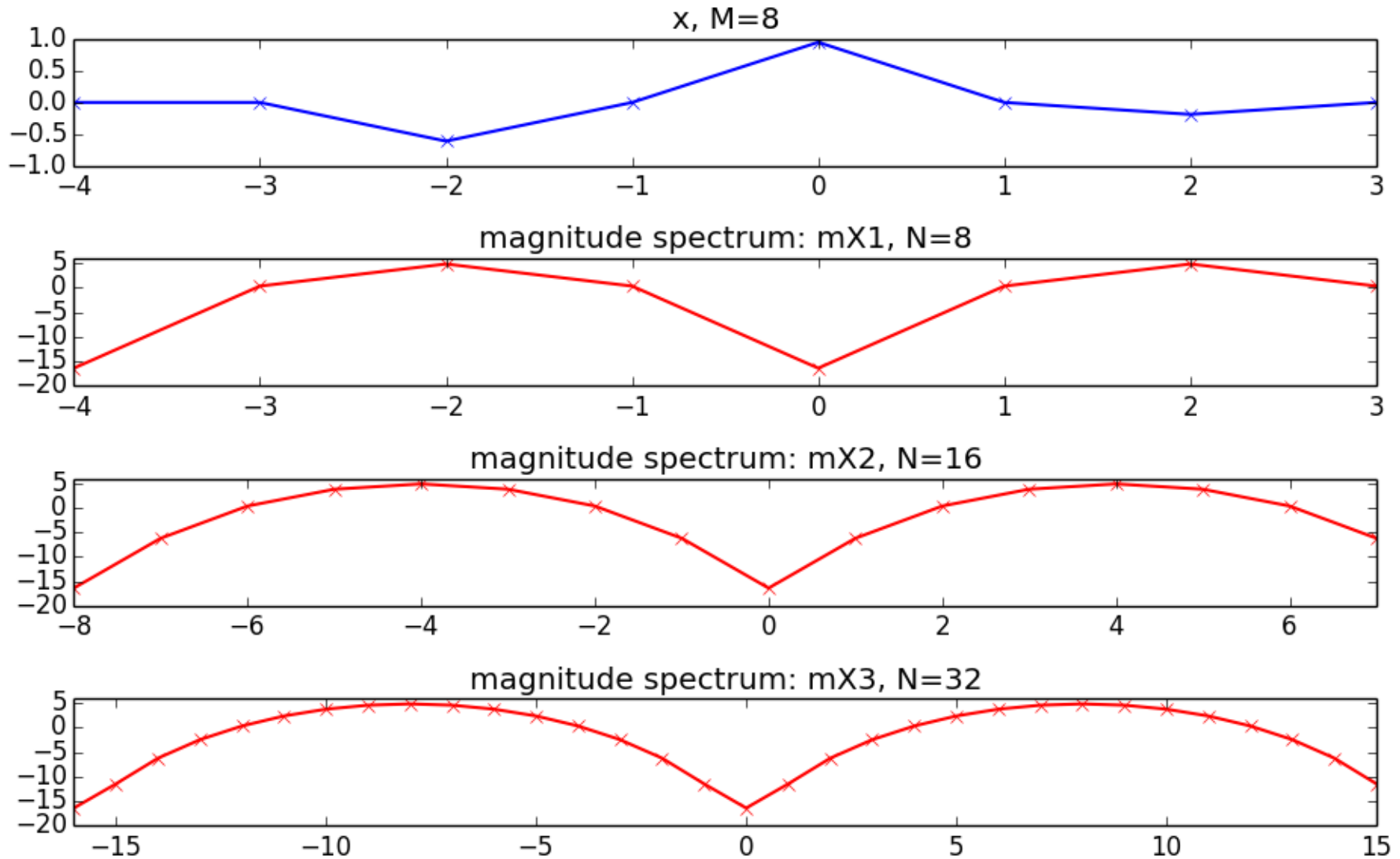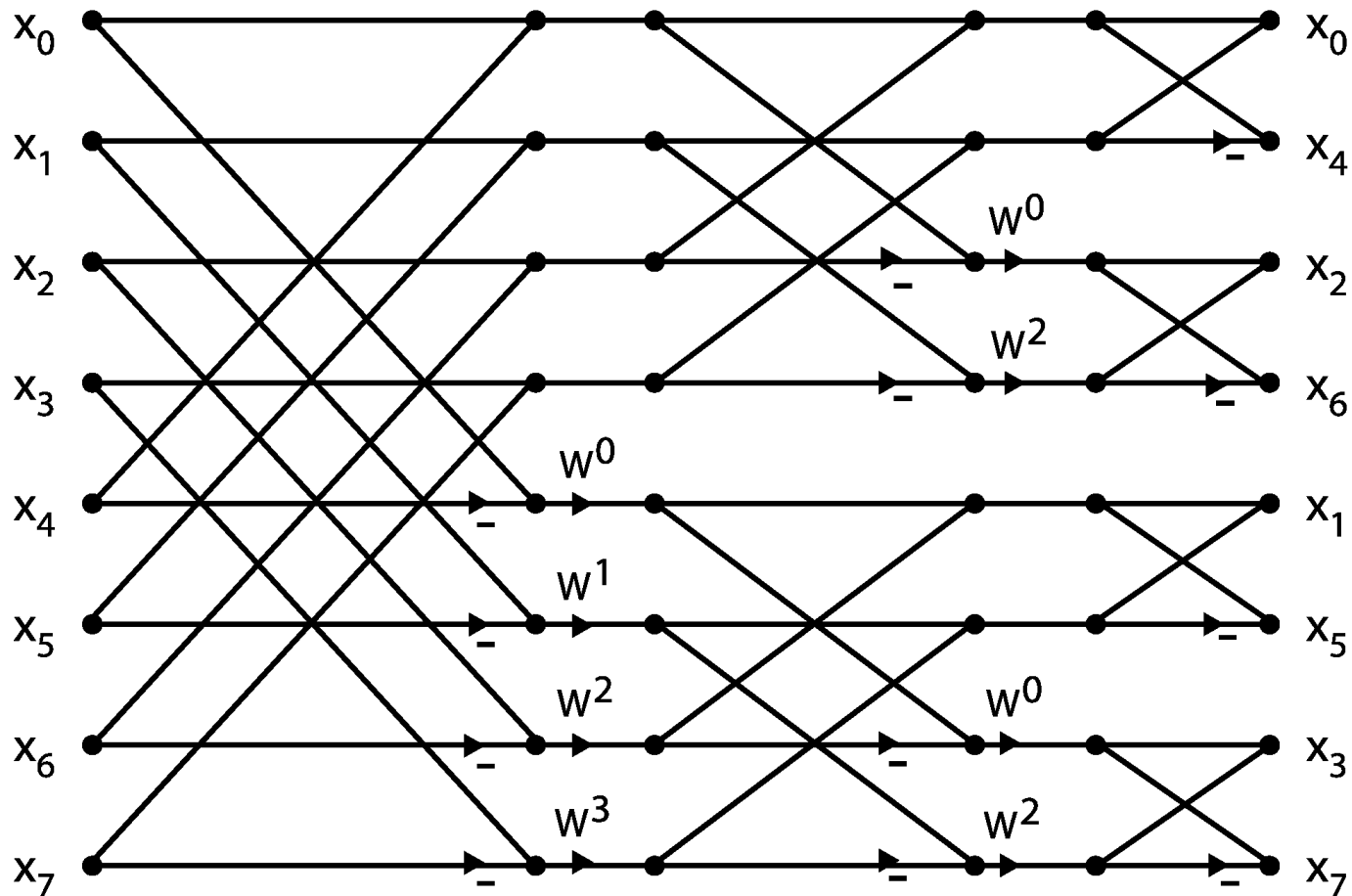
Amplitude in decibels (dB)

Phase unwrapping

x (soprano-E4.wav)

mX = magnitude spectrum

pX1 = phase spectrum
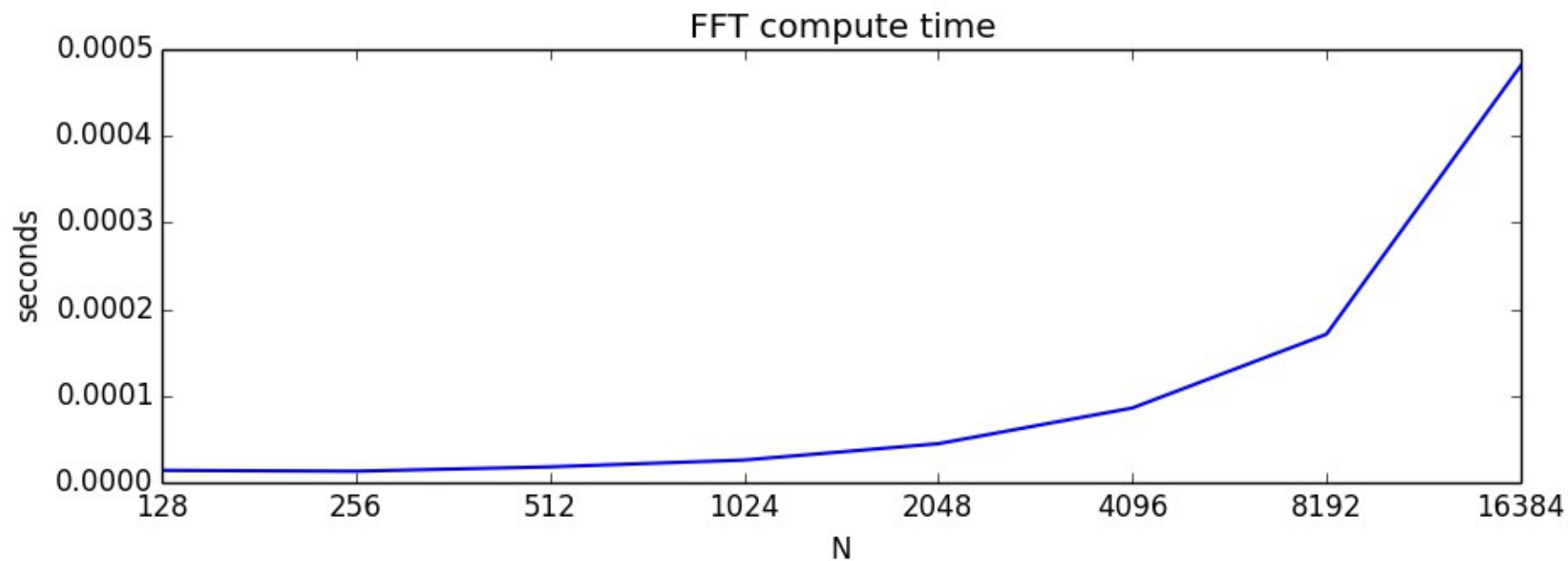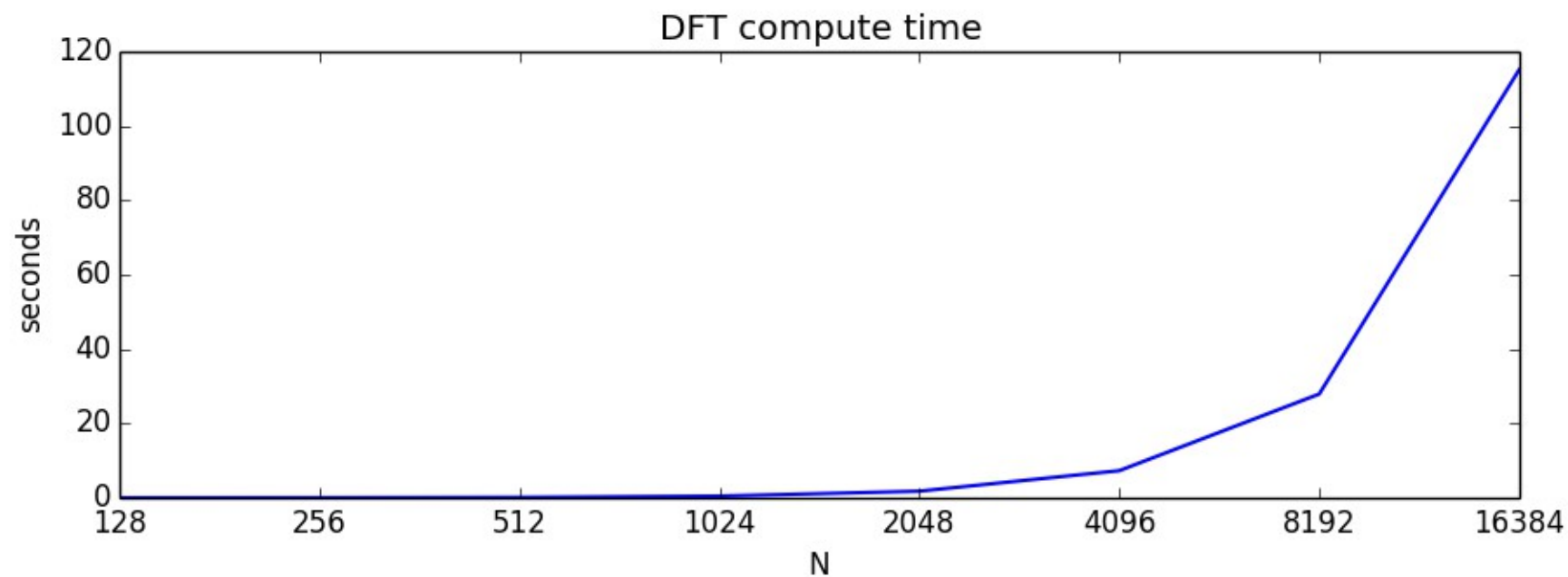
pX2: unwrapped phase spectrum

Zero-padding

zero padding $\leftrightarrow$ interpolation
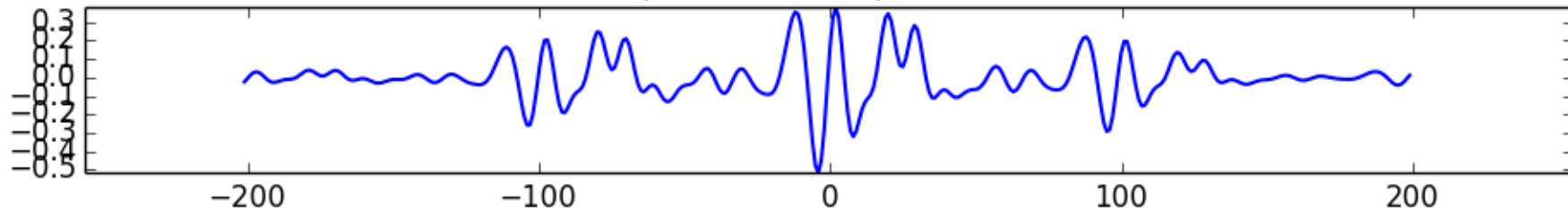
Fast Fourier Transform

Cooley-Tukey algorithm: breaks down recursively the DFT of a power of 2 size into two pieces of size N/2.
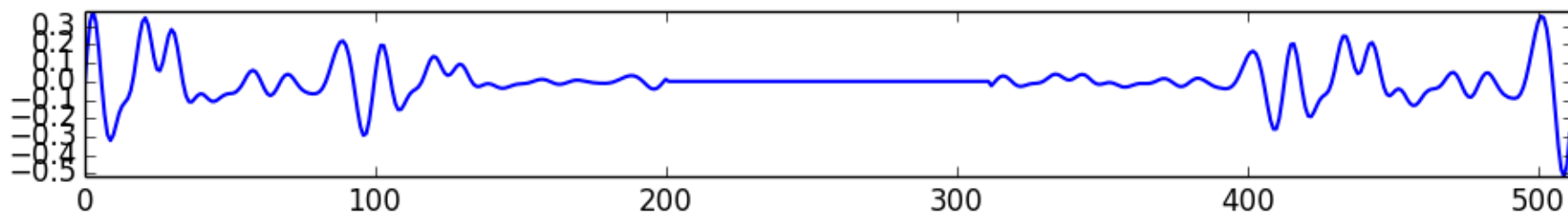
DFT compute time

FFT compute time
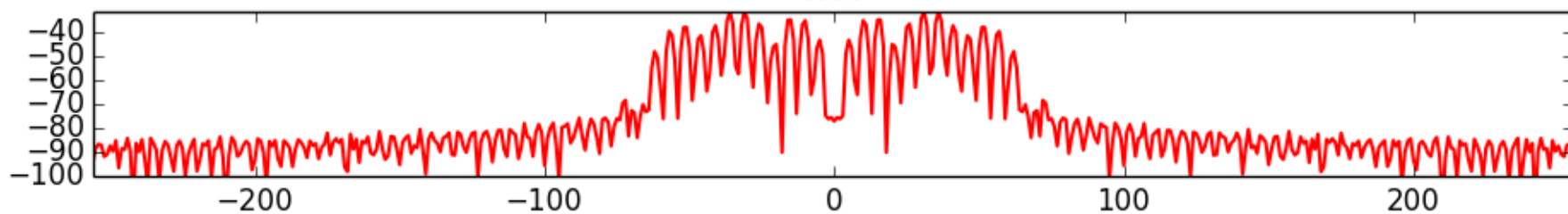
FFT and zero-phase windowing
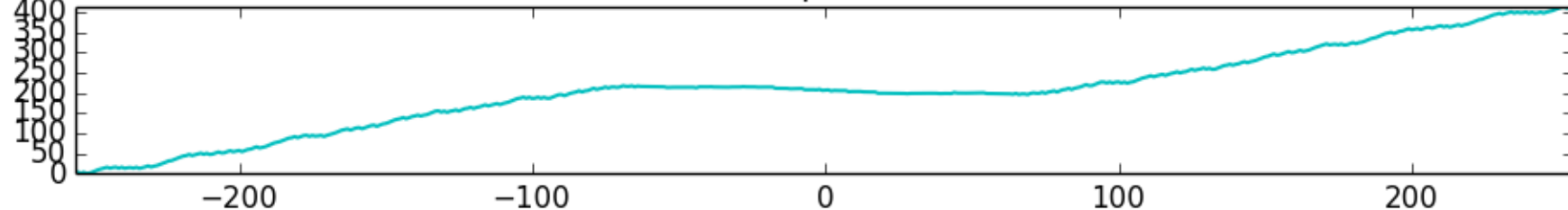
Analysis/synthesis

$$x[n] \xrightarrow{\quad\quad} \boxed{FFT} \xrightarrow{\ |X[k]|\ } \boxed{IFFT} \xrightarrow{\ y[n]\ }$$

$$\xrightarrow{\ <X[k]\ }$$
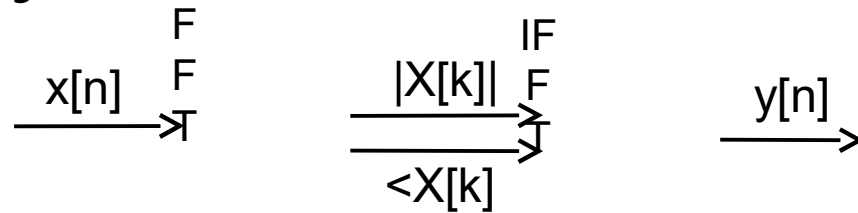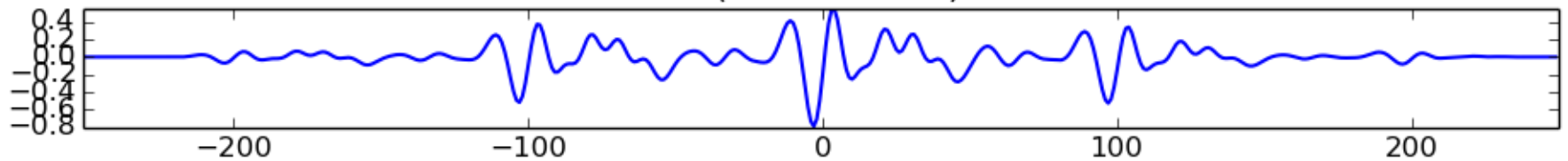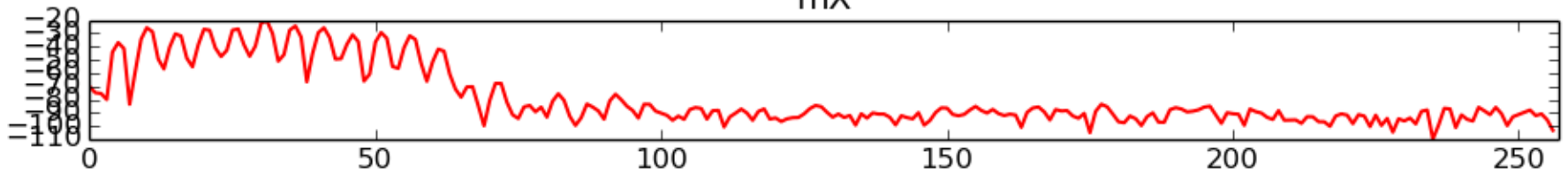
References and credits

- More information on:
  https://en.wikipedia.org/wiki/Discrete_Fourier_transform
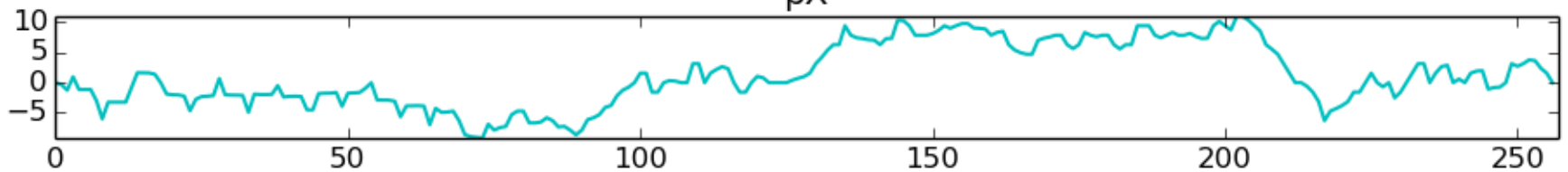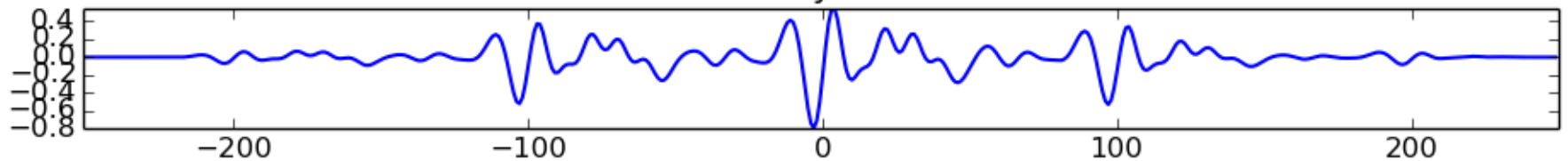 https://en.wikipedia.org/wiki/Fast_Fourier_transform

- Sounds from:
  http://www.freesound.org/people/xserra/packs/13038/

- Reference for the DFT by Julius O. Smith:
  https://ccrma.stanford.edu/~jos/mdft/

- Slides released under CC Attribution-Non Commercial-Share Alike license and code under Affero GPL license; available from https://github.com/MTG/sms-tools

3T2: Fourier Transform properties (2 of 2)

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