2T1: The Discrete Fourier Transform (1 of 2)

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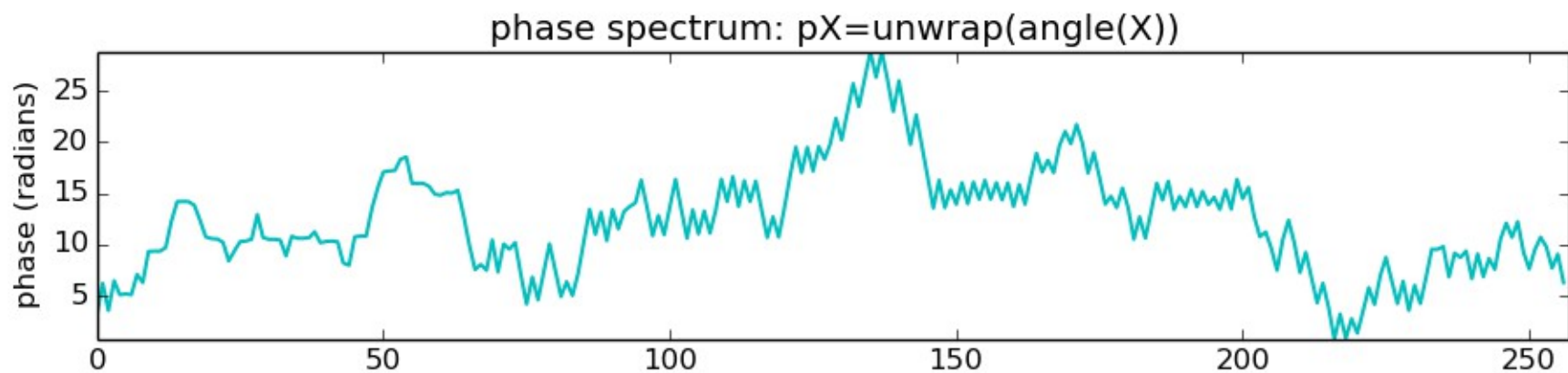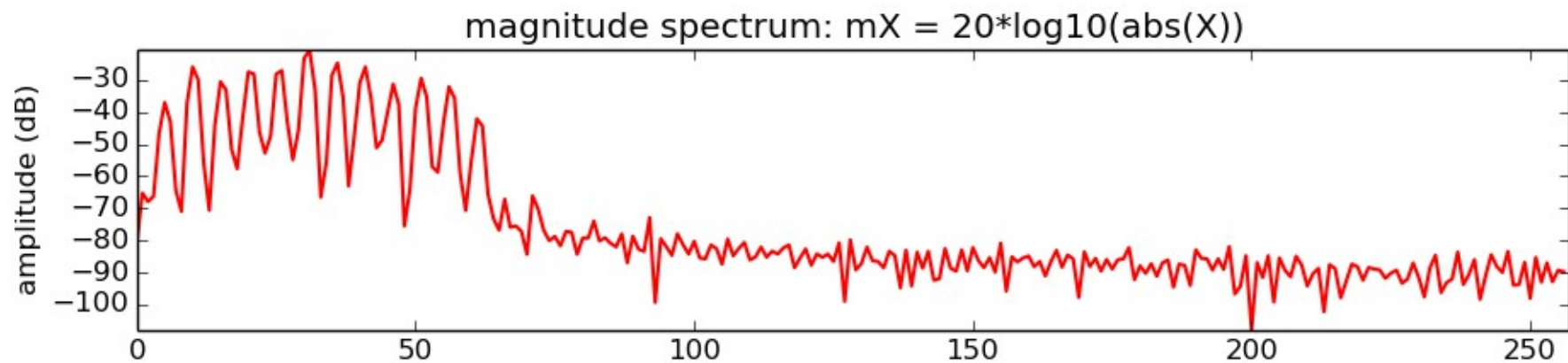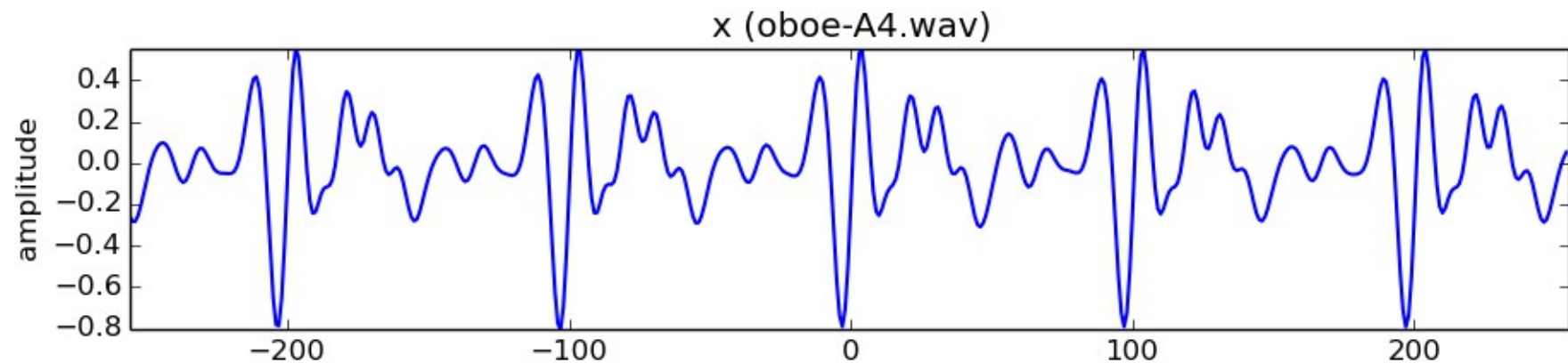
Discrete Fourier Transform

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N} \qquad k = 0, \dots, N-1$$

n : discrete time index $\left(\text{normalized time, } \ T=1 \right)$

k : discrete frequency index

$\omega_k = 2\pi k / N$: frequency in radians per second

$f_k = f_s k / N$: frequency in Hz $\left(f_s : \text{sampling rate} \right)$

x (oboe-A4.wav)

magnitude spectrum: mX = 20*log10(abs(X))

phase spectrum: pX=unwrap(angle(X))

DFT: complex exponentials

$$s_k^* = e^{-j2\pi kn/N} = \cos(2\pi kn/N) - j\sin(2\pi kn/N)$$
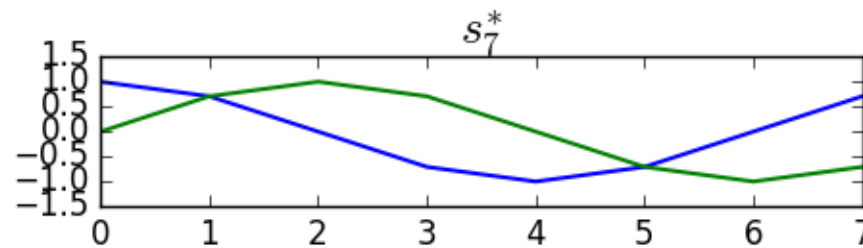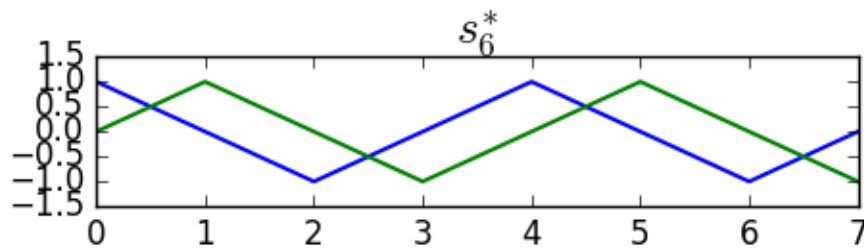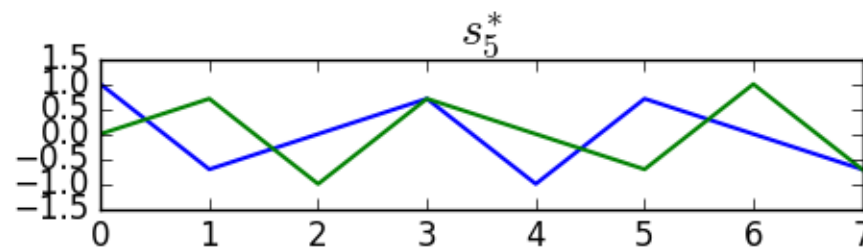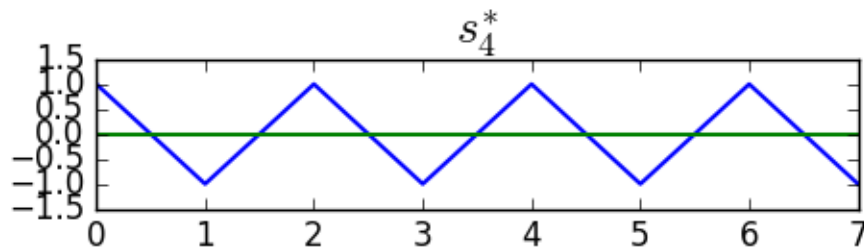
for $N=4$, thus for $n=0,1,2,3$; $k=0,1,2,3$

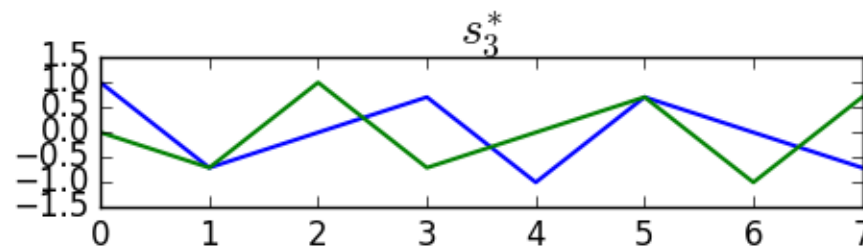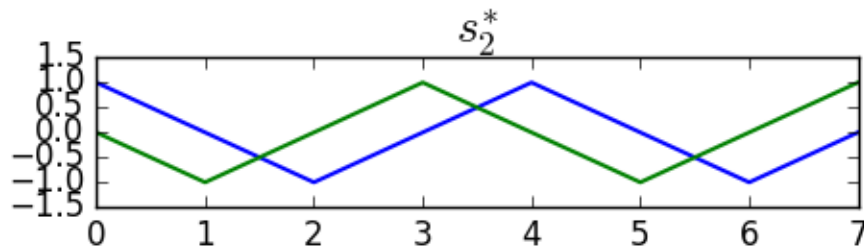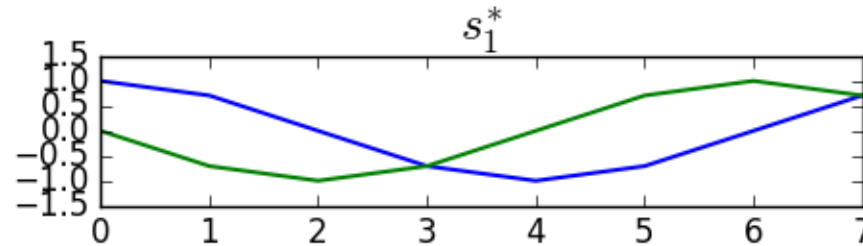$$s_0^* = \cos(2\pi \times 0 \times n/4) - j\sin(2\pi \times 0 \times n/4) = [1,1,1,1]$$
$$s_1^* = \cos(2\pi \times 1 \times n/4) - j\sin(2\pi \times 1 \times n/4) = [1,-j,-1,j]$$
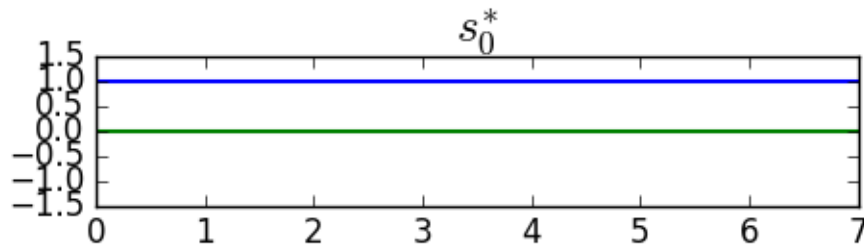$$s_2^* = \cos(2\pi \times 2 \times n/4) - j\sin(2\pi \times 2 \times n/4) = [1,-1,1,-1]$$
$$s_3^* = \cos(2\pi \times 3 \times n/4) - j\sin(2\pi \times 3 \times n/4) = [1,j,-1,-j]$$

DFT: complex exponentials

DFT: scalar product

$$\langle x, s_k \rangle = \sum_{n=0}^{N-1} x[n] s_k^*[n] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}$$

Example:

$$x[n] = [1, -1, 1, -1]; \ N = 4$$

$$\langle x, s_0 \rangle = 1 \times 1 + (-1) \times 1 + 1 \times 1 + (-1) \times 1 = 0$$
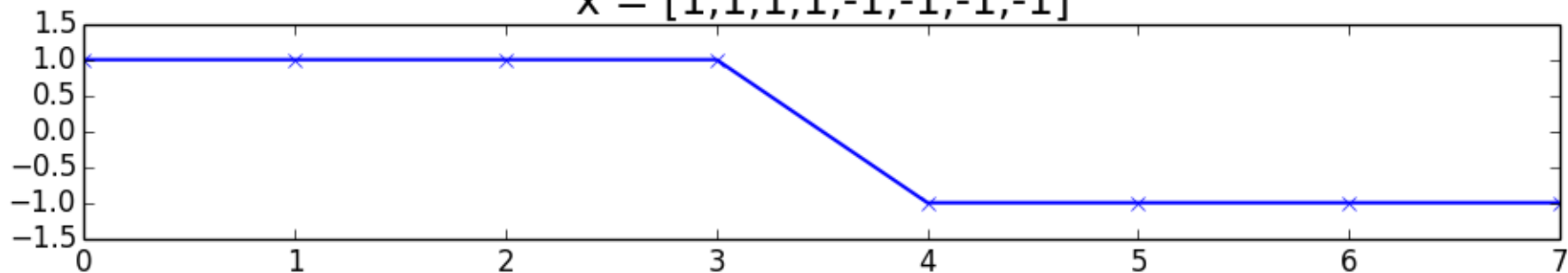
$$\langle x, s_1 \rangle = 1 \times 1 + (-1) \times (-j) + 1 \times (-1) + (-1) \times j = 0$$

$$\langle x, s_2 \rangle = 1 \times 1 + (-1) \times (-1) + 1 \times 1 + (-1) \times (-1) = 4$$
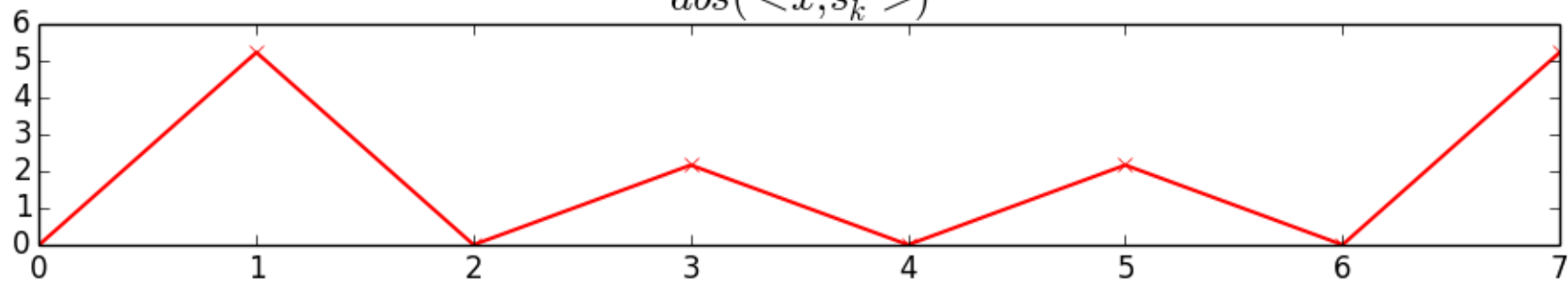
$$\langle x, s_3 \rangle = 1 \times 1 + (-1) \times j + 1 \times (-1) + (-1) \times (-j) = 0$$
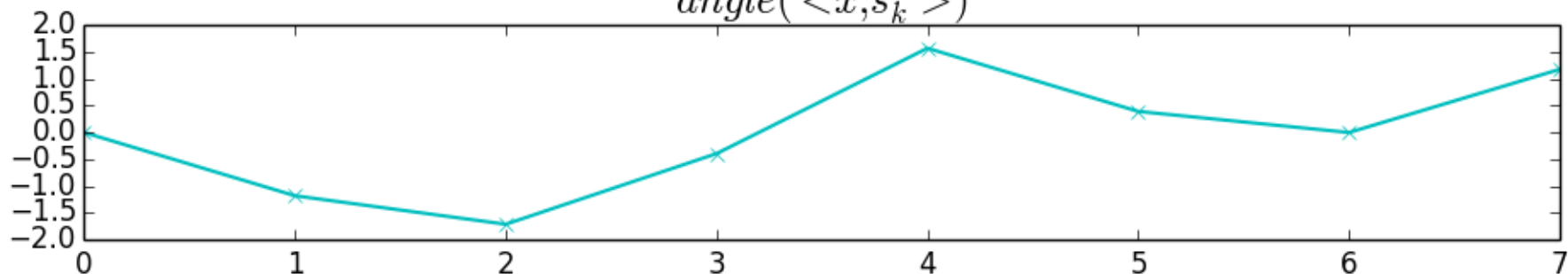
DFT: scalar product

$x = [1,1,1,1,-1,-1,-1,-1]$

$abs(<x,s_k>)$

$angle(<x,s_k>)$

References and credits

- More information in: https://en.wikipedia.org/wiki/Discrete_Fourier_transform

- Reference on the mathematics of the DFT from Julius O. Smith: https://ccrma.stanford.edu/~jos/mdft/

- Sounds from: http://www.freesound.org/people/xserra/packs/13038

- Slides released under CC Attribution-Noncommercial-Share Alike license and code under Affero GPL license; available from https://github.com/MTG/sms-tools

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