

# Package ‘bifactor’

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**Type** Package

**Title** Exploratory factor and bi-factor modeling with multiple general factors

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**Description** Fit exploratory factor models and bi-factor models with multiple general factors.

**License** GPL-3

**LazyData** TRUE

**Depends** R (>= 4.0)

**Imports** Rcpp, Rcsdp

**URL** <https://github.com/Marcosjnez/bifactor>

**BugReports** <https://github.com/Marcosjnez/bifactor/issues>

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**LinkingTo** Rcpp, RcppArmadillo

**NeedsCompilation** yes

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asyp_cov	<i>Asymptotic standard errors for correlation matrices.</i>
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**Description**

Get the asymptotic standard errors of correlation matrices of normal or arbitrary random deviates.

**Usage**

```
asyp_cov(R, X = NULL, eta = 1, type = "normal")
```

**Arguments**

R	Correlation matrix.
X	Optional raw data matrix.
eta	Skewness parameter for elliptical data distributions.
type	Type of random deviates: "normal", "elliptical" or "general".

**Details**

If type = "normal", the calculation assumes that the raw data follows a multivariate normal distribution. If type = "elliptical", the calculation assumes that the raw data follows an elliptical distribution with skewness parameter eta. If type = "general", no assumption is made but need to provide the raw data via the X argument.

**Value**

The asymptotic covariance matrix of R.

References

M.W. Browne and A. Shapiro (1986). The asymptotic covariance matrix of sample correlation coefficients under general conditions. *Linear Algebra and its Applications*, 82, 169-176. [https://doi.org/10.1016/0024-3795\(86\)90150-3](https://doi.org/10.1016/0024-3795(86)90150-3)

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bifactor	<i>Fit an exploratory bi-factor model with one or multiple general factors.</i>
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Description

Jiménez, M., Abad, F. J., Garcia-Garzon, E., & Garrido, L. E. (2023). Exploratory Bi-factor Analysis with Multiple General Factors. *Multivariate behavioral research*, 1–18. Advance online publication. <https://doi.org/10.1080/00273171.2023.2189571>

Usage

```
bifactor(X, n_generals, n_groups, method = "GSLiD", cor = "pearson",
estimator = "uls", projection = "oblq", nobs = NULL, PhiTarget = NULL,
PhiWeight = NULL, blocks = NULL, block_weights = NULL,
oblq_factors = NULL, init_Target = NULL, maxit = 20L, cutoff = 0,
normalization = "none", w = 1, random_starts = 1L, cores = 1L,
init = NULL, efa_control = NULL, rot_control = NULL, first_efa = NULL,
second_efa = NULL, verbose = TRUE)
```

Arguments

X	Raw data matrix or correlation matrix.
n_generals	Number of general factors to extract.
n_groups	Number of group factors to extract.
method	"GSLiD", "SL" (Schmid-Leiman), and "botmin" (bifactor-oblimin-target minimization). Defaults to "GSLiD".
cor	Correlation method. Available correlations: c("pearson", "poly"). Defaults to "pearson".
estimator	EFA fitting estimator: "ml" (maximum likelihood for multivariate normal variable), "uls" (minimum residuals), "pa" (principal axis) or "minrank" (minimum rank). Defaults to "uls".
projection	Projection method. Available projections: "orth" (orthogonal), "oblq" (oblique) and "poblq" (partially oblique). Defaults to "oblq".
nobs	Sample size. Defaults to NULL.
PhiTarget	Target matrix for the factor correlations. Defaults to NULL.
PhiWeight	Weight matrix for the factor correlations. Defaults to NULL.
blocks	Vector with the number of factors for which separately applying the rotation criterion. Defaults to NULL.

<code>block_weights</code>	Vector of weights for each block of factors.
<code>oblq_factors</code>	Vector with the number of factors for each oblique block. E.g.: <code>c(2, 4)</code> means that there are two blocks of oblique factors: one block with 2 factors and another block with 4 factors. Everything else is orthogonal. Defaults to NULL.
<code>init_Target</code>	Initial target matrix for the loadings. Defaults to NULL.
<code>maxit</code>	Maximum number of iterations for the GSLiD algorithm. Defaults to 20L.
<code>cutoff</code>	Cut-off used to update the target matrix upon each iteration. Defaults to 0.
<code>normalization</code>	Available normalizations: "kaiser". Defaults to "none".
<code>w</code>	$w$ parameter for the extended target criterion ("xtarget"). Defaults to 1L.
<code>random_starts</code>	Number of rotations with different random starting values. The rotation with the smallest cost function value is returned. Defaults to 1L.
<code>cores</code>	Number of cores for parallel execution of multiple rotations. Defaults to 1L.
<code>init</code>	Initial uniquenesses values for exploratory factor analysis estimation. Defaults to NULL.
<code>efa_control</code>	List of control parameters for efa fitting. Defaults to NULL.
<code>rot_control</code>	List of control parameters for the rotation algorithm. Defaults to NULL.
<code>first_efa</code>	List of arguments to pass to <code>efast</code> to perform the first-order solution for the Schmid-Leiman method. Defaults to NULL.
<code>second_efa</code>	List of arguments to pass to <code>efast</code> to perform the second-order solution for the Schmid-Leiman method. Defaults to NULL.
<code>verbose</code>	Print the convergence progress information. Defaults to TRUE.

## Details

If `efa.control` = NULL, then `list(maxit = 1e4)` is passed to `efa.control`. If `rot_control` = NULL, then `list(maxit = 1000, eps = 1e-05)` is passed to `rot_control`, where `eps` is the absolute tolerance. When the objective function does not make a larger improvement than `eps`, the algorithm is assumed to converge.

If `Target` is provided but not `Weight`, then `Weight = 1 - Target` by default, which means a partially specified target rotation is performed. The same applies for `PhiTarget` and `PhiWeight`.

If `init` = NULL, then the squared multiple correlations of each item with the remaining ones are used as initial values (These are known to be upper bounds).

If `init_Target` is provided, then an initial target by means of the Schmid-Leiman transformation is not necessary.

If `cutoff` is not 0, loadings smaller than such a cut-off are fixed to 0. When `cutoff` = 0, an empirical cut-off is used for each column of the loading matrix. They are the mean of the one-lagged differences of the sorted squared normalized loadings. Then, the target is determined by fixing to 0 the squared normalized loadings smaller than such cut-offs.

## Value

List of class `bifactor`.

efa	List containing objects related to the exploratory factor analysis estimation. See efast.
bifactor	List with the following components: <ul style="list-style-type: none"> <li>• loadings - Rotated loading matrix.</li> <li>• Phi - Factor correlation matrix.</li> <li>• T - Transformation matrix.</li> <li>• f - Objective value at the minimum.</li> <li>• iterations - Number of iterations performed by the rotation algorithm.</li> <li>• convergence - Convergence of the rotation algorithm.</li> <li>• uniquenesses - Vector of uniquenesses.</li> <li>• Rhat - Correlation matrix predicted by the model.</li> <li>• Target - Updated target matrix.</li> <li>• Weight - Weight matrix. It is the complement of the updated target.</li> <li>• GSLiD_iterations - Number of iterations performed by the GSLiD algorithm.</li> <li>• GSLiD_convergence - Convergence of the GSLiD algorithm.</li> <li>• min_congruences - Vector containing, for each iteration, the minimum Tucker's congruence between the current loading matrix and the previous loading matrix.</li> <li>• max_abs_diffs - Vector containing, for each iteration, the maximum absolute difference between the current loading matrix and the previous loading matrix.</li> </ul>
elapsed	Total amount of time spent for execution (in nanoseconds).

## References

Jiménez, M., Abad, F. J., Garcia-Garzon, E., & Garrido, L. E. (2023). Exploratory Bi-factor Analysis with Multiple General Factors. *Multivariate behavioral research*, 1–18. Advance online publication. <https://doi.org/10.1080/00273171.2023.2189571>

## Examples

```
## Not run: # Simulate data:
sim <- sim_factor(n_generals = 3, groups_per_general = 5, items_per_group = 6,
  generals_rho = 0.3)
scores <- MASS::mvrnorm(1e4, rep(0, nrow(sim$R)), Sigma = sim$R)
s <- cor(scores)

# Fit an exploratory bi-factor model with GSLiD:
fit <- bifactor(s, n_generals = 3, n_groups = 15, method = "GSLiD",
  estimator = "uls", projection = "poblq", nobs = NULL, oblq_factors = 3,
  random_starts = 10, cores = 8, w = 1, maxit = 20, verbose = TRUE, normalization = "none")

## End(Not run)
```

check\_deriv

*Check the derivatives and differentials of rotation criteria.***Description**

Check the derivatives and differentials of rotation criteria.

**Usage**

```
check_deriv(L, Phi, dL, dP, rotation = "oblimin", projection = "oblq",
  Target = NULL, Weight = NULL, PhiTarget = NULL, PhiWeight = NULL,
  blocks = NULL, block_weights = NULL,
  oblq_factors = NULL, gamma = 0,
  epsilon = 0.01, k = 0L, w = 1)
```

**Arguments**

L	Loading matrix.
Phi	Factor correlation matrix.
dL	Perturbation for L.
dP	Perturbation for Phi.
rotation	Rotation criterion. Available rotations: "varimax", "cf" (Crawford-Ferguson), "oblimin", "geomin", "target", "xtarget" (extended target) and "none". Defaults to "oblimin".
projection	Projection method. Available projections: "orth" (orthogonal), "oblq" (oblique), "poblq" (partially oblique). Defaults to "oblq".
Target	Target matrix for the loadings. Defaults to NULL.
Weight	Weight matrix for the loadings. Defaults to NULL.
PhiTarget	Target matrix for the factor correlations. Defaults to NULL.
PhiWeight	Weight matrix for the factor correlations. Defaults to NULL.
blocks	Vector with the number of factors for which separately applying the rotation criterion. Defaults to NULL.
block_weights	Vector of weights for each block of factors.
oblq_factors	Vector with the number of factors for each oblique block. E.g.: c(2, 4) means that there are two blocks of oblique factors: one block with 2 factors and another block with 4 factors. Everything else is orthogonal. Defaults to NULL.
gamma	$\gamma$ parameter for the oblimin criterion. Defaults to 0 (quartimin).
epsilon	$\epsilon$ parameter for the geomin criterion. Defaults to 0.01.
k	$k$ parameter for the Crawford-Ferguson family of rotation criteria. Defaults to 0.
w	$w$ parameter for the extended target criterion ("xtarget"). Defaults to 1L.

**Details**

None yet.

**Value**

A list with the objective value and the gradients and differentials of L and Phi.

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cv_eigen	<i>Cross-validated eigenvalues.</i>
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**Description**

Estimate cross-validated eigenvalues and the dimensionality using the Kaiser's rule.

**Usage**

```
cv_eigen(X, N = 100L, hierarchical = FALSE, efa = NULL, cores = 1L)
```

**Arguments**

X	Raw data matrix.
N	Number of cross-validated samples.
hierarchical	Logical indicating whether a second cross-validated eigenvalues estimation should be performed from the factor scores obtained after a first factor analysis analysis.
efa	A list of arguments to pass to efast when hierarchical = TRUE.
cores	Number of cores to perform parallel computations.

**Details**

None yet.

**Value**

A list with the cross-validated eigenvalues and the estimated dimensionality.

**References**

Chen F., Roch S., Rohe K., Yu S (2021). Estimating Graph Dimension with Cross-validated Eigenvalues, arXiv. <https://arxiv.org/abs/2108.03336>

---

efast

*Fast exploratory factor analysis.*


---

## Description

Fast exploratory factor analysis.

## Usage

```
efast(X, nfactors, cor = "pearson", estimator = "uls",
      rotation = "oblimin", projection = "oblq", nobs = NULL,
      Target = NULL, Weight = NULL, PhiTarget = NULL, PhiWeight = NULL,
      blocks = NULL, block_weights = NULL,
      oblq_factors = NULL, gamma = 0,
      epsilon = 1e-02, k = 0, w = 1,
      random_starts = 1L, cores = 1L,
      init = NULL, efa_control = NULL, rot_control = NULL)
```

## Arguments

X	Raw data matrix or correlation matrix.
nfactors	Number of common factors to extract.
cor	Correlation method. Available correlations: c("pearson", "poly"). Defaults to "pearson".
estimator	EFA fitting estimator: "ml" (maximum likelihood for multivariate normal variables), "uls" (minimum residuals), "pa" (principal axis) and "minrank" (minimum rank). Defaults to "uls".
rotation	Rotation criterion. Available rotations: "varimax", "cf" (Crawford-Ferguson), "oblimin", "geomin", "target", "xtarget" (extended target) and "none". Defaults to "oblimin".
projection	Projection method. Available projections: "orth" (orthogonal), "oblq" (oblique), "poblq" (partially oblique). Defaults to "oblq".
nobs	Sample size. Defaults to NULL.
Target	Target matrix for the loadings. Defaults to NULL.
Weight	Weight matrix for the loadings. Defaults to NULL.
PhiTarget	Target matrix for the factor correlations. Defaults to NULL.
PhiWeight	Weight matrix for the factor correlations. Defaults to NULL.
blocks	List of vectors with the indexes of the factors for which separately applying the rotation criterion. Defaults to NULL.
block_weights	Vector of weights for each block of factors.
oblq_factors	Vector with the number of factors for each oblique block. E.g.: c(2, 4) means that there are two blocks of oblique factors: one block with 2 factors and another block with 4 factors. Everything else is orthogonal. Defaults to NULL.



gamma	$\gamma$ parameter for the oblimin criterion. Defaults to 0 (quartimin).
epsilon	$\epsilon$ parameter for the geomin criterion. Defaults to 0.01.
k	$k$ parameter for the Crawford-Ferguson family of rotation criteria. Defaults to 0.
w	$w$ parameter for the extended target criterion ("xtarget"). Defaults to 1L.
random_starts	Number of rotations with different random starting values. The rotation with the smallest cost function value is returned. Defaults to 1L.
cores	Number of cores for parallel execution of random starts. Defaults to 1L.
init	Initial uniquenesses values for exploratory factor analysis estimation. Defaults to NULL.
efa_control	List of control parameters for efa fitting. Defaults to NULL.
rot_control	List of control parameters for the rotation algorithm. Defaults to NULL.

### Details

If `efa.control = NULL`, then `list(maxit = 1e4)` is passed to `efa.control`. If `rot_control = NULL`, then `list(maxit = 1000, eps = 1e-05)` is passed to `rot_control`, where `eps` is the absolute tolerance. When the objective function does not make a larger improvement than `eps`, the algorithm is assumed to converge.

If `Target` is provided but not `Weight`, then `Weight = 1 - Target` by default, which means a partially specified target rotation is performed. The same applies for `PhiTarget` and `PhiWeight`.

If `init = NULL`, then the squared multiple correlations of each item with the remaining ones are used as initial values (These are known to be upper bounds).

If a Heywood case is encountered, then `estimator = "minrank"` is automatically applied to ensure positive uniquenesses.

### Value

List of class `efast` with the following components:

efa	List containing the following objects: <ul style="list-style-type: none"> <li>• loadings - Unrotated loadings.</li> <li>• uniquenesses - Vector of uniquenesses.</li> <li>• Rhat - Correlation matrix predicted by the model.</li> <li>• residuals - Residual correlation matrix.</li> <li>• f - Objective value at the minimum.</li> <li>• Heywood - TRUE if any Heywood case is encountered and FALSE otherwise.</li> <li>• iterations - Number of iterations for the L-BFGS-B algorithm to converge.</li> <li>• convergence - TRUE if the L-BFGS-B algorithm converged and FALSE otherwise.</li> <li>• estimator - Method used to fit the exploratory factor analysis.</li> </ul>
rotation	List of class <code>rotation</code> . Only if the argument <code>rotation</code> is not "none". See <code>rotate</code> for the components.
elapsed	Total amount spent for execution (in nanoseconds).

## References

Jiménez, M., Abad, F. J., Garcia-Garzon, E., & Garrido, L. E. (2023). Exploratory Bi-factor Analysis with Multiple General Factors. *Multivariate behavioral research*, 1–18. Advance online publication. <https://doi.org/10.1080/00273171.2023.2189571>

## Examples

```
## Not run:
# Simulate data:
sim <- sim_factor(n_generals = 0, groups_per_general = 5, items_per_group = 6)
scores <- MASS::mvrnorm(1e3, rep(0, nrow(sim$R)), Sigma = sim$R)
s <- cor(scores)

# Fit efa:
fit <- efast(s, nfactors = 5, estimator = "uls", rotation = "oblimin",
projection = "oblq", gamma = 0, random_starts = 10L, cores = 1L)

## End(Not run)
```

---

fit

---

*Compute fit measures for exploratory factor models.*


---

## Description

Compute fit measures for exploratory factor models.

## Usage

```
fit(model, nobs = NULL)
```

## Arguments

nobs	Sample size. Defaults to NULL.
model	Object of class efa or cfa.

## Details

fit... to be explained

## Value

Vector of fit measures.

## Author(s)

Vithor R. Franco & Marcos Jiménez

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fscores	<i>Compute factor scores</i>
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**Description**

Compute the factor scores from an exploratory factor model

**Usage**

```
fscores(fit, scores = NULL, method = "regression")
```

**Arguments**

fit	object of class ‘efa’ or ‘bifactor’.
scores	Matrix of raw scores.
method	Method to compute the factor scores.

**Details**

...

**Value**

List...

**Author(s)**

Marcos Jiménez & Vithor R. Franco

---

get_target	<i>Get a target from a loading matrix.</i>
------------	--

---

**Description**

Get a target for the loading matrix using a custom or empirical cut-off.

**Usage**

```
get_target(loadings, Phi = NULL, cutoff = 0)
```

**Arguments**

loadings	A matrix of loadings.
Phi	A correlation matrix among the factors. Defaults to NULL.
cutoff	The cut-off used to create the target matrix. Defaults to 0.

Details

If cutoff is not 0, loadings smaller than such a cut-off are fixed to 0. When cutoff = 0, an empirical cut-off is used for each column of the loading matrix. They are the mean of the one-lagged differences of the sorted squared normalized loadings. Then, the target is determined by fixing to 0 the squared normalized loadings smaller than such cut-offs.

Value

A target matrix.

References

Garcia-Garzon, E., Abad, F. J., & Garrido, L. E. (2019). Improving bi-factor exploratory modeling: Empirical target rotation based on loading differences. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*, 15(2), 45–55. <https://doi.org/10.1027/1614-2241/a000163>

Jiménez, M., Abad, F. J., Garcia-Garzon, E., & Garrido, L. E. (2023). Exploratory Bi-factor Analysis with Multiple General Factors. *Multivariate behavioral research*, 1–18. Advance online publication. <https://doi.org/10.1080/00273171.2023.2189571>

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parallel	<i>Hierarchical parallel analysis using either principal components (PCA) or principal axis factoring (PAF).</i>
----------	--

---

Description

Perform hierarchical parallel analysis to detect dimensionality using either principal components or principal axis factoring.

Usage

```
parallel(X, n_boot = 100L, type = "pearson", quant = NULL, mean = TRUE, replace = FALSE,
PA = NULL, hierarchical = FALSE, efa = NULL, cores = 1L)
```

Arguments

X	Raw data matrix.
n_boot	Number of bootstrap samples.
type	Type of correlations: "pearson" or "poly".
quant	Vector of quantiles of the distribution of bootstrap eigenvalues to which the compare the sample eigenvalues.
mean	Logical. Compare the sample eigenvalues to the mean of the bootstrap eigenvalues. Defaults to TRUE.
replace	Logical indicating whether the columns of X should be permuted with replacement.

PA	Parallel analysis method. It can be either principal components ("PCA"), principal axis ("PAF") or both ("PCA" and "PAF"). Defaults to NULL, which sets c("PCA", "PAF").
hierarchical	Logical indicating whether a second parallel analysis should be performed from the factor scores obtained after a first factor analysis analysis.
efa	A list of arguments to pass to efast when hierarchical = TRUE.
cores	Number of cores to perform the parallel bootstrapping.

### Details

Not yet.

### Value

A list with the bootstrapped eigenvalues and the estimated dimensionality.

### References

Horn, J. L. (1965). A Rationale and Test For the Number of Factors in Factor Analysis, *Psychometrika*, 30, 179-85. <https://doi.org/10.1007/BF02289447>

---

polyfast	<i>Fast polychoric correlations.</i>
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---

### Description

Compute huge polychoric correlation matrices very fast.

### Usage

```
polyfast(X, acov = "none", smooth = "pd", min_eigval = 0.001, nboot = 1000L, fit = FALSE, cores = 1L)
```

### Arguments

X	Matrix of categorical scores. The lowest score must start at 0.
acov	Use acov = 'cov' to obtain the asymptotic covariance matrix and acov = 'var' to simply obtain the asymptotic variances. Use "bootstrap" for estimating the asymptotic covariance matrix by resampling. Defaults to "none".
smooth	Smooth the matrix to be positive definite ("pd"), positive semi-definite ("psd"), or estimate the maximum likely polychoric correlation matrix under the positive semi-definite constraint ("analytical"). Defaults to "none".
min_eigval	Minimum eigenvalue when smooth = "pd". Defaults to 0.001.
nboot	Number of bootstrap samples to compute the standard errors. It only works if acov = "bootstrap". Defaults to 1000L.
fit	Should the fit value be calculated? Defaults to FALSE.
cores	Number of parallel cores to compute the polychoric correlations.

**Details**

None yet.

**Value**

A list with the polychoric correlations, the thresholds, and the elapsed time in nanoseconds.

---

prints

*S3Methods for Printing*

---

**Description**

Prints for bifactor objects

**Usage**

```
## S3 method for class 'efa'  
print(x, nobs=NULL, ...)  
  
## S3 method for class 'bifactor'  
print(x, nobs=NULL, ...)
```

**Arguments**

x	Object from bifactor package.
nobs	Optional number of observations. If not provided, Chi-squared-based statistics will not be computed.
...	Additional arguments

**Value**

Prints bifactor object

**Author(s)**

Marcos Jimenez <marcosjnezhquez@gmail.com> and Víthor R. Franco <vithorfranco@gmail.com>

---

random_oblq	<i>Generate random oblique matrices.</i>
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---

**Description**

Generate random oblique matrices from a standard normal distribution.

**Usage**

```
random_oblq(p, q)
```

**Arguments**

p	Number of rows.
q	Number of columns. Should not be greater than p.

**Value**

An oblique matrix with normally distributed data.

**References**

Jiménez, M., Abad, F. J., Garcia-Garzon, E., & Garrido, L. E. (2023). Exploratory Bi-factor Analysis with Multiple General Factors. *Multivariate behavioral research*, 1–18. Advance online publication. <https://doi.org/10.1080/00273171.2023.2189571>

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random_orth	<i>Generate random orthogonal matrices.</i>
-------------	---

---

**Description**

Generate random orthogonal matrices from a standard normal distribution. First, a matrix of random standard normal variables is simulated and then, the Q factor from the QR decomposition is returned.

**Usage**

```
random_orth(p, q)
```

**Arguments**

p	Number of rows.
q	Number of columns. Should not be greater than p.

**Value**

An orthogonal matrix with normally distributed data.

**References**

Jiménez, M., Abad, F. J., Garcia-Garzon, E., & Garrido, L. E. (2023). Exploratory Bi-factor Analysis with Multiple General Factors. *Multivariate behavioral research*, 1–18. Advance online publication. <https://doi.org/10.1080/00273171.2023.2189571>

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random_poblq	<i>Generate a random partially oblique matrix.</i>
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---

**Description**

First, a matrix is simulated from a standard normal distribution. Second, the matrix is normalized and the Gram-Schmidt process is performed between the oblique blocks. Finally, the orthogonal blocks correspond to those columns of the Q matrix from the QR decomposition.

**Usage**

```
random_poblq(p, q, oblq_factors)
```

**Arguments**

- |              |   |
|--------------|---|
| p            | Number of rows.   |
| q            | Number of columns. Should not be greater than p.  |
| oblq_factors | A vector with the number of factors for each oblique block. E.g.: c(2, 4) means that there are two blocks of oblique factors: one with 2 factors and another with 4 factors. Everything else is orthogonal. |

**Value**

A partially oblique matrix.

**References**

Jiménez, M., Abad, F. J., Garcia-Garzon, E., & Garrido, L. E. (2023). Exploratory Bi-factor Analysis with Multiple General Factors. *Multivariate behavioral research*, 1–18. Advance online publication. <https://doi.org/10.1080/00273171.2023.2189571>

**Examples**

```
random_poblq(p = 7, q = 7, oblq_factors = c(3, 2))
```



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retr_oblq	<i>Retraction of a matrix onto the oblique manifold.</i>
-----------	--

---

**Description**

Transform a matrix into an oblique matrix.

**Usage**

```
retr_oblq(X)
```

**Arguments**

X	A matrix.
---	-----------

**Value**

An oblique matrix.

**References**

Jiménez, M., Abad, F. J., Garcia-Garzon, E., & Garrido, L. E. (2023). Exploratory Bi-factor Analysis with Multiple General Factors. *Multivariate behavioral research*, 1–18. Advance online publication. <https://doi.org/10.1080/00273171.2023.2189571>

---

retr_orth	<i>Retraction of a matrix onto the orthogonal manifold.</i>
-----------	---

---

**Description**

Transform a matrix into an orthogonal matrix.

**Usage**

```
retr_orth(X)
```

**Arguments**

X	A matrix.
---	-----------

**Value**

An orthogonal matrix.

## References

Jiménez, M., Abad, F. J., Garcia-Garzon, E., & Garrido, L. E. (2023). Exploratory Bi-factor Analysis with Multiple General Factors. *Multivariate behavioral research*, 1–18. Advance online publication. <https://doi.org/10.1080/00273171.2023.2189571>

---

retr\_poblq

*Retraction of a matrix onto the partially oblique manifold.*


---

## Description

Transform a matrix into a partially oblique matrix.

## Usage

```
retr_poblq(X, oblq_factors)
```

## Arguments

X	A matrix.
oblq_factors	A vector with the number of factors for each oblique block. E.g.: c(2, 4) means that there are two blocks of oblique factors: one with 2 factors and another with 4 factors. Everything else is orthogonal.

## Value

A partially oblique matrix.

## References

Jiménez, M., Abad, F. J., Garcia-Garzon, E., & Garrido, L. E. (2023). Exploratory Bi-factor Analysis with Multiple General Factors. *Multivariate behavioral research*, 1–18. Advance online publication. <https://doi.org/10.1080/00273171.2023.2189571>

## Examples

```
X <- replicate(8, rnorm(8))
retr_poblq(X, c(2, 3, 3))
```

---

rotate	<i>Fast rotation algorithm for factor analysis.</i>
--------	---

---

### Description

Riemannian Newton Trust-Region algorithm to quickly perform (parallel) rotations with different random starting values.

### Usage

```
rotate(loadings, rotation = "oblimin", projection = "oblq",
gamma = 0, epsilon = 0.01, k = 0, w = 1,
Target = NULL, Weight = NULL, PhiTarget = NULL, PhiWeight = NULL,
blocks = NULL, block_weights = NULL, oblq_factors = NULL,
normalization = "none",
rot_control = NULL, random_starts = 1L, cores = 1L)
```

### Arguments

loadings	Unrotated loading matrix.
rotation	Rotation criterion. Available rotations: "varimax", "cf" (Crawford-Ferguson), "oblimin", "geomin", "target", "xtarget" (extended target) and "none". Defaults to "oblimin".
projection	Projection method. Available projections: "orth" (orthogonal), "oblq" (oblique), "poblq" (partially oblique). Defaults to "oblq".
gamma	$\gamma$ parameter for the oblimin criterion. Defaults to 0 (quartimin).
epsilon	$\epsilon$ parameter for the geomin criterion. Defaults to 0.01.
k	$k$ parameter for the Crawford-Ferguson family of rotation criteria. Defaults to 0.
w	$w$ parameter for the extended target criterion ("xtarget"). Defaults to 1.
Target	Target matrix for the loadings. Defaults to NULL.
Weight	Weight matrix for the loadings. Defaults to NULL.
PhiTarget	Target matrix for the factor correlations. Defaults to NULL.
PhiWeight	Weight matrix for the factor correlations. Defaults to NULL.
blocks	Vector with the number of factors for which separately applying the rotation criterion. Defaults to NULL.
block_weights	Vector of weights for each block of factors.
oblq_factors	Vector with the number of factors for each oblique block. E.g.: c(2, 4) means that there are two blocks of oblique factors: one block with 2 factors and another block with 4 factors. Everything else is orthogonal. Defaults to NULL.
normalization	Available normalizations: "kaiser". Defaults to "none".
rot_control	List of control parameters for the rotation algorithm. Defaults to NULL.
random_starts	Number of rotations with different random starting values. The rotation with the smallest cost function value is returned. Defaults to 1L.
cores	Number of cores for parallel execution of random starts. Defaults to 1L.

Details

If `rot_control = NULL`, then `list(maxit = 1000, eps = 1e-05)` is passed to `rot_control`, where `eps` is the absolute tolerance. When the objective function does not make a larger improvement than `eps`, the algorithm is assumed to converge. If `Target` is provided but not `Weight`, then `Weight = 1 - Target` by default, which means a partially specified target rotation is performed. The same applies for `PhiTarget` and `PhiWeight`.

Value

List of class rotation with the following components:

<code>loadings</code>	Rotated loading matrix.
<code>Phi</code>	Correlation matrix among the factors.
<code>T</code>	Rotation matrix.
<code>f</code>	Objective value at the minimum.
<code>iterations</code>	Number of iterations for the rotation algorithm to converge.
<code>convergence</code>	TRUE if the algorithm converged and FALSE otherwise.
<code>elapsed</code>	Total amount of time spent for execution (in nanoseconds).

References

Jiménez, M., Abad, F. J., Garcia-Garzon, E., & Garrido, L. E. (2023). Exploratory Bi-factor Analysis with Multiple General Factors. *Multivariate behavioral research*, 1–18. Advance online publication. <https://doi.org/10.1080/00273171.2023.2189571>

Zhang, G., Hattori, M., Trichtinger, L. A., & Wang, X. (2019). Target rotation with both factor loadings and factor correlations. *Psychological Methods*, 24(3), 390–402. <https://doi.org/10.1037/met0000198>

---

<code>se</code>	<i>Standard errors for rotated factor loadings, factor correlations and uniquenesses.</i>
-----------------	---

---

Description

Compute the sandwich standard errors of factor loadings, factor correlations and uniquenesses.

Usage

```
se(fit = NULL, n = NULL, X = NULL, type = "normal", eta = 1)
```

Arguments

<code>fit</code>	Optional efast model.
<code>X</code>	Raw data matrix.
<code>type</code>	Type of random deviates: "normal", "elliptical" or "general".
<code>eta</code>	Skewness parameter for elliptical data distributions.
<code>n</code>	Sample size.

Details

Currently, only available for estimator = uls.

Value

A list with the standard errors of the rotated factor loadings, factor correlations and uniquenesses.

References

Zhang G, Preacher KJ, Hattori M, Jiang G, Trichtinger LA (2019). A sandwich standard error estimator for exploratory factor analysis with nonnormal data and imperfect models. *Applied Psychological Measurement*, 43, 360–373. <https://doi.org/10.1177/0146621618798669>

---

sim_factor	<i>Simulate a bi-factor structure with either one or multiple general factors.</i>
------------	--

---

Description

Simulate bifactor structures with multiple general factors, cross-loadings, pure items, correlated factors, and more.

Usage

```
sim_factor(n_generals = 0, groups_per_general = 5,
items_per_group = 6,
loadings_g = "medium", loadings_s = "medium",
crossloadings = 0, pure = FALSE,
generals_rho = 0, groups_rho = 0,
method = "minres", fit = "rmsr", misfit = 0,
error_method = "cudeck", efa = FALSE,
lambda = NULL, Phi = NULL, Psi = NULL)
```

Arguments

- n\_generals      Number of general factors.
- groups\_per\_general      Number of group factors per general factor.
- items\_per\_group      Number of items per group factor.
- loadings\_g      Loadings' magnitude on the general factors: "low", "medium" or "high". Defaults to "medium".
- loadings\_s      Loadings' magnitude on the group factors: "low", "medium" or "high". Defaults to "medium".
- crossloadings      Magnitude of the cross-loadings among the group factors. Defaults to 0.
- pure      Fix a pure item on each general factor. Defaults to FALSE.

generals_rho	Correlation among the general factors. Defaults to 0.
groups_rho	Correlation among the group factors. Defaults to 0.
method	Method used to generate population error: "minres" or "ml".
fit	Fit index to control the population error.
misfit	Misfit value to generate population error.
error_method	Method used to control population error: c("yuan", "cudeck"). Defaults to "cudeck".
efa	Reproduce the error with EFA or CFA. Defaults to FALSE (CFA).
lambda	Custom loading matrix. If Phi is NULL, then all the factors will be correlated at the value given in groups_rho.
Phi	Custom Phi matrix. If lambda is NULL, then Phi should be conformable to the loading matrix specified with the above arguments.
Psi	Custom Psi matrix.

### Details

sim\_factor generates bi-factor and generalized bifactor patterns with cross-loadings, pure items and correlations among the general and group factors. When crossloading is different than 0, one cross-loading is introduced for an item pertaining to each group factor. When pure is TRUE, one item loading of each group factor is removed so that the item loads entirely on the general factor. To maintain the item communalities constant upon these modifications, the item loading on the other factors may shrink (if adding cross-loadings) or increase (if setting pure items).

Loading magnitudes may range between 0.3-0.5 ("low"), 0.4-0.6 ("medium") and 0.5-0.7 ("high").

### Value

List with the following objects:

lambda	Population loading matrix.
Phi	Population factor correlation matrix.
R	Population correlation matrix.
R_error	Population correlation matrix with error.
uniquenesses	Vector of population uniquenesses.
delta	Minimum of the objective function that correspond to the misfit value.

### References

Jiménez, M., Abad, F.J., Garcia-Garzon, E., Garrido, L.E. (2021, June 24). Exploratory bi-factor analysis with multiple general factors. Under review. Retrieved from [https://osf.io/7aszj/?view\\_only=8f7bd98025104347a96](https://osf.io/7aszj/?view_only=8f7bd98025104347a96)

---

sl	<i>Schmid-Leiman Transformation.</i>
----	--------------------------------------

---

**Description**

Schmid-Leiman transformation into a bi-factor pattern with one or multiple general factors.

**Usage**

```
sl(X, n_generals, n_groups, cor = "pearson", nobs = NULL, first_efa = NULL,
   second_efa = NULL, cores = 1L)
```

**Arguments**

X	Raw data matrix or correlation matrix.
n_generals	Number of general factors.
n_groups	Number of group factors.
cor	Correlation method. Available correlations: c("pearson", "poly"). Defaults to "pearson".
nobs	Sample size. Defaults to NULL.
first_efa	Arguments to pass to efast in the first-order factor extraction. See efast for the default arguments.
second_efa	Arguments to pass to efast in the second-order factor extraction. See efast for the default arguments.
cores	Number of cores for the polychorics estimation.

**Details**

First, a hierarchical factor model is fitted using a second-order factor analysis on the factor correlation obtained from a first-order factor analysis. Then, the item loadings on the general factors are assumed to be the direct effects of the general factors according to such hierarchical model. On the other hand, the item loadings on the group factors become the originally first-order loadings post-multiplied by the diagonal matrix containing the root of the item uniquenesses.

Obviously, the first-order factor analysis should be oblique to perform a second exploratory factor analysis.

If the second-order solution does not use an orthogonal projection, then the correlation matrix among the general factors for the Schmid-Leiman solution is simply that obtained from such second-order solution.

**Value**

loadings	Loading matrix of the Schmid-Leiman solution.
first_order_solution	Object of class efast with the first-order solution.

`second_order_solution`      Object of class `efast` with the second-order solution.

`uniquenesses`      Vector of uniquenesses.

`Rhat`      Correlation matrix predicted by the (hierarchical) model.

## References

Jiménez, M., Abad, F. J., Garcia-Garzon, E., & Garrido, L. E. (2023). Exploratory Bi-factor Analysis with Multiple General Factors. *Multivariate behavioral research*, 1–18. Advance online publication. <https://doi.org/10.1080/00273171.2023.2189571>

## Examples

```
## Not run:
# Simulate data:
sim <- sim_factor(n_generals = 2, groups_per_general = 3, items_per_group = 5)
lambda <- sim$lambda
Target <- ifelse(lambda > 0, 1, 0)

# Target rotation for the first-order efa and oblimin for the second-order efa:
first <- list(rotation = "target", projection = "oblq", Target = Target[, -c(1:2)])
second <- list(rotation = "oblimin", projection = "oblq", gamma = 0)

SL <- sl(sim$R, n_generals = 2, n_groups = 6, nobs = 100,
         first_efa = first, second_efa = second)

## End(Not run)
```

---

summarys

*S3Methods for summarizing*

---

## Description

summarys for bifactor objects

## Usage

```
## S3 method for class 'efa'
summary(object, nobs=NULL, suppress=0, order=FALSE, digits=2, ...)

## S3 method for class 'bifactor'
summary(object, nobs=NULL, suppress=0, order=FALSE, digits=2, ...)
```



**Arguments**

object	Object from bifactor package.
nobs	Optional number of observations. If not provided, Chi-squared-based statistics will not be computed.
suppress	Hide the loadings which absolute magnitudes are smaller than this cutoff. Defaults to 0.
order	Order the columns of the pattern matrix according to the variance they account for. Defaults to FALSE.
digits	Number of digits to display in the loading and factor correlation matrices.
...	Arguments to be passed to or from other methods.

**Value**

summarys bifactor object

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