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A production/remanufacturing inventory model with price and quality dependant return rate

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A B S T R A C T

Inventory management of produced, remanufactured/repaired and returned items has been receiving increasing attention in recent years. The available studies in the literature consider a production environment that consists of two shops. The first shop is for production and remanufacturing/repair, while the second shop is for collecting used (returned) items to be remanufactured in the first shop, where demand is satisfied from producing new and from remanufacturing/repairing returned items. Numerical and analytical results from these developed models suggested that a pure (bang–bang) policy of either no waste disposal (total remanufacturing) or no remanufacturing (pure production and total disposal) is the best strategy, while the mixed strategy (a mixture of production and remanufacturing) is the optimum case under certain limited assumptions. In practice, the quality of the returned items and the purchasing price that reflects this quality is what usually governs a collection (or return) policy of used items. Unlike those available models in the literature, this paper suggests that the flow of returned items is variable, and is controlled by two decision variables, which are the purchasing price for returned items corresponding to an acceptance quality level. Deterministic mathematical models are presented for multiple remanufacturing and production cycles.

1. Introduction

The flow of products in supply chains is from upstream to downstream, i.e., from the supplier's supplier to the customer's customer. Shorter product life cycles and changes in customers' consumption behaviors resulted in faster product flows and subsequently faster generation of waste and depletion of natural resources (e.g., Beamon, 1999). This gave rise to the drive towards collecting and remanufacturing used/returned products to extend their useable lives and thus reduce waste and conserve natural resources. Furthermore, economical incentives enticed and later governmental legislations compelled companies to initiate product recovery (e.g., remanufacturing, repairing, recycling, etc.) programs. Like supply chain, reverse logistics manages the flow of products, however in the opposite direction for remanufacturing or other purposes (i.e., from downstream to upstream), and therefore managing inventory in reverse logistics has been stressed in several studies (e.g., Fleischmann et al., 1997; Minner, 2001).

Although reverse logistics is relatively a new term, initial attempts to address the inventory of remanufactured items or products dates back to the 1960s, with Schrady (1967) being the first to investigate a repair–inventory system. He developed an EOQ model for repairable items which assumes that the manufacturing and recovery (repair) rates are instantaneous with no disposal cost. Schrady (1967) assumed a single manufacturing batch and multiple repair batches. Nahmias and Rivera (1979) extended Schrady's model to allow for a finite repair rate with the assumption of limited storage in the repair and production shops.

The production/remanufacturing inventory problem started taking a new direction in the 1990s. Richter (1996a, 1996b) investigated the EOQ model for stationary demand that is satisfied from producing items of a certain product using new materials and components, and from repairing used/returned items that are collected from the market at some rate. The production environment described in Richter (1996a, 1996b) consists of two shops; with the first shop is for production and recovery, while the second shop is for collecting used/returned items. Some of these collected used/returned items are disposed outside the second shop at a rate, which may display the ecological behavior of the producer. In a follow-up work, Richter (1997) extended the cost analysis of his earlier works (Richter, 1996a, 1996b) to show that a pure (bang–bang) policy of either no waste disposal (total repair) or no repair (total waste disposal) dominates a mixed strategy of waste disposal and repair. Richter and Dobos (1999) extended Richter's earlier work by considering an integer nonlinear programming problem with similar findings as before. Dobos and Richter (2003) investigated a production/recycling system with constant demand that is satisfied by non-instantaneous production and recycling with a single repair and a single production batch per interval. In a follow-up paper, Dobos and Richter (2004) generalized their earlier model (Dobos & Richter, 2003) by assuming multiple repair and

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production batches in a time interval. However, Dobos and Richter (2004) implied that their model has limitations since pure strategies of either no waste disposal (total repair) or no repair (total waste disposal) are technologically infeasible, and suggested that a more general and meaningful model would be to consider the quality of returned items.

Recently, and along the same line of research, Dobos and Richter (2006) extended their previous work by considering the quality of returned items. They considered two strategies to manage the collection of used items: (1) repurchase all used items and reuse only a maximal proportion of them (strategy 1) or (2) buyback only a proportion of the used items and decide how much of them to reuse (strategy 2). In their models, Dobos and Richter (2006) assumed that the proportion of the demand returned to be reused is dependent on two inter-dependent decision variables, which are: (1) buyback proportion and (2) use proportion. The product of these variables ($0 \leqslant$ (buyback proportion) $\times$ (use proportion) $\leqslant 1$) is the proportion of reusable items, which represents the return rate = (demand rate) $\times$ (buyback proportion) $\times$ (use proportion), and it was assumed by Dobos and Richter (2006) to be fixed at a value. That is, the buyback proportion and the use proportion vary, but their product is a fixed value. This assumption limits their model as it compared two strategies for a fixed rather than a variant return rate. In addition, and for the sake of argument, let us assume a case where no ecological constraints are considered and a decision on which strategy to adopt is solely based on economical feasibility. For such a case, if recycling (i.e., recovery) is expensive, then the strategy of pure production should be favored, which Dobos and Richter (2006) did not consider. In addition, in their model, Dobos and Richter (2006) assumed that a pure recycling/reuse strategy is more cost effective than a pure production strategy. In our opinion, this assumption limits the application of their model further, especially for the case when the cost of a pure recycling/reuse strategy is either equal to or more than a pure production strategy. This point will be discussed later in this paper. Furthermore, Dobos and Richter (2006) ignored the purchasing price of raw materials in the forward flow (production/remanufacture), and the purchasing price of collected used items in the backward flow (returns). Therefore, a major difference between the work of Dobos and Richter (2006) and the one presented herein is that this paper assumes the return rate of used items (a decision variable) is dependent on two decision variables, the purchasing price for returned items and its corresponding acceptance quality level.

The inventory management research in the reverse logistics context is not limited to the studies surveyed above. Other researchers have developed models along the same lines as Schrady (1967), Richter (1996a, 1996b, 1997), but with different assumptions. Examples of recent works, including, but not limited to, are those of Teunter (2001, 2004), Inderfurth, Lindner, and Rachaniotis (2005), Konstantaras and Papachristos (2006), Jaber and Rosen (2008), and El Saadany and Jaber (2008). These works and those surveyed in earlier paragraphs all assumed a constant return rate and ignored the factors that govern this rate. In practice, the purchasing price of a collected used (returned) item with a certain quality governs the usefulness of the remanufacture/repair process. For example, if the returned items are expensive or they have a poor quality, the whole remanufacture/repair process might not be economically feasible and therefore pure production with no returns might be the optimum solution. Although several researchers called for the need to differentiate the returned units according to their quality (Behret & Korugan, 2009; Blackburn, Guide, Souza, & Van Wassenhove, 2004; Bloemhof-Ruwaard, van Beek, Hordijk, & Van Wassenhove, 1995; Grubbström & Tang, 2006; Reimer, Sodhi, & Knight, 2000; Smith, Small, Dodds, Amagai, & Strong, 1996), there has been no work that modeled the collection rate of used items as price and quality dependent.

This paper extends the models developed in Dobos and Richter (2003, 2004) by assuming that the collection rate of used/returned items is dependent on the purchasing price (decision variable 1) and the acceptance quality level (decision variable 2) of these returns. This is done by incorporating a price–quality demand function, adopted from Vörös (2002), to model the collection rate of returned items. Vörös (2002) presented demand as a decreasing and increasing exponential functions of price and quality. Vörös (2002) integrated these functions into one function that describes the forward flow of a product, i.e., from the inventory system to the market, where demand increases as selling price (quality) decreases (increases). Vörös's demand function describes a general and known behavior that is well documented in the literature (e.g., Kalish, 1983; Teng & Thompson, 1996). Since this paper considers the price and quality in the reverse flow, therefore, the logic of the model presented in Vörös (2002) is switched. That is, in the reverse flow, the flow of used/returned items increases as the purchasing price increases, and decreases as the corresponding acceptance quality level increases.

In recent years, some researchers provided clearer definitions to the terms repair, reconditioning, remanufacturing, and recycling. For example, De Brito and Dekker (2004) differentiated between the terms repair and remanufacturing by industry. They suggested that if only a part of the product deteriorates, then recovery options like repair or part replacement or retrieval are considered. King, Burgess, Ijomah, and McMahon (2006) defined the term repair as the correction of specified faults in a product, where the quality of repaired products is inferior to those of remanufactured and reconditioned. This paper adopts the term "remanufacturing", which refers to repairing, reconditioning, refurbishing or remanufacturing.

In this paper, production, remanufacture, and waste disposal EPQ (economic production quantity) type models are developed and analyzed, where a manufacturer serves a stationary demand by producing new items of a product as well as by remanufacturing collected used/returned items. In these developed models, the return rate of used items is modeled as a demand-like function of purchasing price and acceptance quality level of returns. The model developed herein is a decision tool that helps managers in determining the optimum acceptable acquisition quality level and its corresponding price for used items that are collected for recovery purposes and that minimizes the total system cost.

The remainder of this paper is organized as follows. The next section, Section 2, is for assumptions, notations and description of the production/remanufacturing inventory system that will be investigated in this paper. Section 3 is for mathematical modeling. Section 4 is for numerical examples and discussion of results. This paper summarizes and concludes in Section 5.

2. Assumptions and notations

2.1. Assumptions

This paper assumes: (1) finite production and remanufacturing rates, (2) remanufactured items are as good as new, (3) demand is known, constant and independent, (4) lead time is zero, (5) a single product case, (6) no shortages are allowed, (7) unlimited storage capacity is available and (8) infinite planning horizon.

2.2. Notations

2.2.1. Decision variables

- P purchasing price for a single returned item as a percentage of the cost of raw materials required to produce a new item of the product ($0 < P < 1$)
- q acceptance quality level of returned (collected used) items ($0 < q < 1$); representing the percentage of useful parts in a remanufacturable item

Subsequently, $R(P, q)$, or R for simplicity, is the portion of demand which is returned to the system for either remanufacturing or disposal, as a function of price and quality. The portion of returned demand R or $R(P, q)$ will be indicated as the return rate. Denote T as the length of a production and remanufacturing interval (units of time), which is itself dependent on P and q. The multi-attribute q quality measure for a returned item may be determined using some judgmental approaches. Some of these approaches are ranking, rating (scaling), and paired comparison (e.g., Ahn & Park, 2008; Barron & Barrett, 1996; Eckenrode, 1965; Jaccard, Brinberg, & Ackerman, 1986).

It is worth noting that the returned items are usually of varying quality. In this paper, it is assumed that a returned item with a quality less than the acceptance (optimum) quality level q^*, will be rejected. Only returned items of quality better than or equal to q^*, are accepted to flow in the reverse direction to be repaired. Returned item are purchased at an optimum price P^*.

2.2.2. Input parameters

D	demand rate (units per unit of time)
D/γ	remanufacturing rate ($0 < \gamma < 1$)
D/β	production rate ($0 < \beta < 1$)
S_r	remanufacturing setup cost
S_p	production setup cost
h_s	holding cost per unit per unit of time for serviceable (new and remanufactured) stock
h_r	holding cost per unit per unit of time for returned stock
C_n	cost of raw materials required to produce a newly produced unit, note that $P \times C_n$ is the purchasing price for a single returned item
C_r	remanufacturing cost per unit for qRT units
C_p	production cost per unit for $(D - qR)T$ units
C_w	waste disposal cost per unit for $(1 - q)RT$ units

Fig. 1 describes the production/remanufacturing inventory model of interest. This figure illustrates the flow of remanufactured and newly produced items from the system to the market, and the flow of returned items from the market to the system where they are screened to verify their quality and those not conforming to remanufacturing requirements are disposed outside the system.

3. Mathematical modelling

The models developed in this section extend the models of Dobos and Richter (2003, 2004) by assuming the return rate of used items follows a demand-like function dependent on two decision variables which are the purchasing price, P, and the acceptance quality level, q, for these returned items. In addition, this paper accounts for the cost of raw materials required to produce a single new unit of the product, C_n, where the monetary value of the purchasing price for a returned item is $P_M = P \times C_n$.

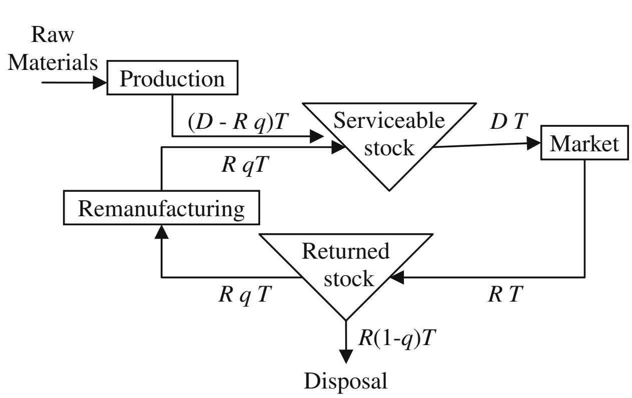

Fig. 1. Material flow in an interval of length T.

Two models are developed in this section. The first model assumes a single production cycle and a single remanufacturing cycle per interval T. The second model, a generalization of the first, assumes m remanufacturing cycles and n production cycles per interval T.

3.1. Model I: a single remanufacturing cycle and a single production cycle

In Fig. 1, market demand D is satisfied from the serviceable stock, which is a collection of newly produced and remanufactured items. Over an interval of length T, $R(P, q) \times T$ (or RT for simplicity) used/returned units are collected in the returned stock facility, where $0 < R/D < 1$, and $D > 0$. In this facility, activities such as disassembly and sorting are carried out. The waste disposal amount of the returned items is decided once the acceptance quality level is determined, i.e., disposal increases (decreases) as the acceptance quality level decreases (increases), with the number of used/returned items disposed per interval is $(1 - q)RT$. The remaining collected used/returned units, qRT, are transferred to the remanufacturing facility in the first shop. The term C_r here represents the cost to repair one unit (which includes cost components such as labor, energy, machinery, etc.) excluding the cost to purchase a used item $P_M = P \times C_n$. Like earlier works, this paper assumes that remanufactured used/returned items are considered as-good-as-new and are part of the serviceable stock. The remaining serviceable stock, $(D - qR)T$, is replenished by newly produced items, where DT represents the total demand in an interval of length T.

Note that the case when ($R > 0$ and $q = 0$) is technologically infeasible since it considers that all the returned/used items are non-remanufacturable and would be disposed. Although this case is valid mathematically, it is costly and therefore never optimal. On the other extreme, $q = 1$ means that a returned/used item must be of an identical quality to that of a newly produced one, for example, returns during trial periods or returns due to obsolete technology.

In this paper, the return rate of used/returned items, $R = R(P, q)$, is a portion of the demand rate D, i.e., ($0 < R(P, q)/D < 1$), where this portion is dependent on the purchasing price (P) and its corresponding level of acceptable quality (q) of returns. Adopting the demand function from Vörös (2002) and modifying it to capture the backward flow as discussed earlier, the price factor of the demand function is $f_P = (1 - ae^{-\theta P})$, where $0 < a < 1$ and $\theta > 1$ are parameters. This price factor models the behavior of returns for a fixed quality level. Fig. 2 illustrates the behavior of the price factor for the case when $a = 0.3$ and $\theta = 4$, where f_P is a monotonically increasing function over P since $df_P/dP > 0$ and $d^2f_P/dP^2 < 0$ $\forall P > 0$. The quality factor of the demand function is $f_q = be^{-\varphi q}$, where $0 < b < 1$ and $\varphi > 1$ are parameters. This quality factor models the behavior of returns for a fixed price level. Fig. 3 illustrates the behavior of the quality factor for the case when $b = 0.9$ and $\varphi = 5$, where f_P is a monotonically decreasing function over q since $df_q/dq < 0$ and $d^2f_q/dq^2 > 0$ $\forall q > 0$. Therefore, the return rate of used/returned items (demand of the reverse flow) is modeled as a function of price and quality factors f_P and f_q, and is expressed as $R = R(P, q) = D(1 - ae^{-\theta P})be^{-\varphi q}$.

There is one repair cycle of length T_R and one production cycle of length T_P in the time interval T, where $T = T_R + T_P$. The inventory levels of serviceable and returned stocks are shown in Fig. 4. The inventory of serviceable stock builds up at a rate of $(1/\gamma - 1)D$ units per unit of time with remanufacturing ceases when an inventory level of $I_{R,1} = (1 - \gamma)DT_R$ is attained. The production cycle commences once $I_{R,1}$ units are depleted. Similarly, the inventory of newly produced items builds at a rate of $(1/\beta - 1)D$ units per unit

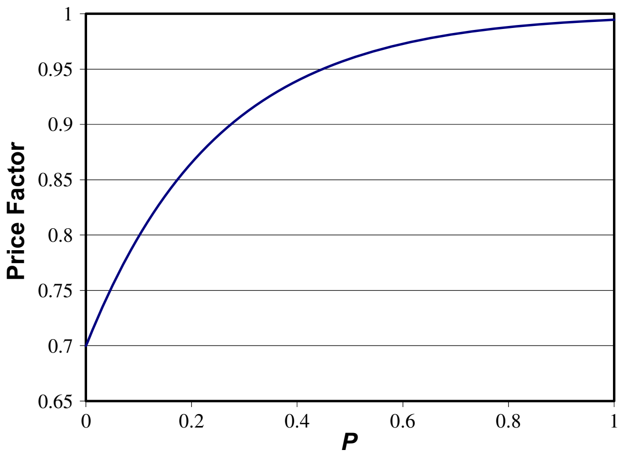

Fig. 2. Price factor plotted against price, *P*.

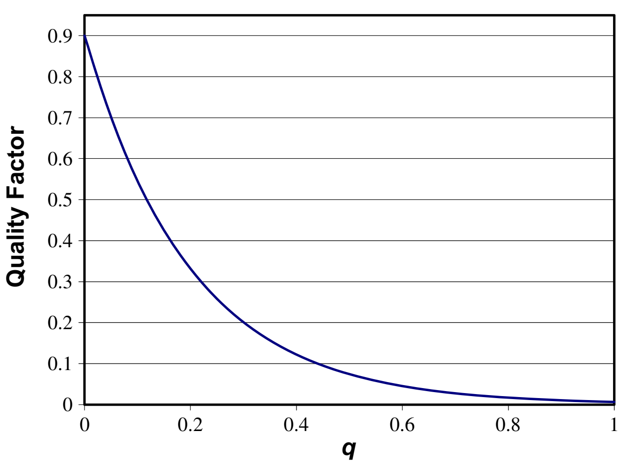

Fig. 3. Quality factor plotted against accepted quality, *q*.

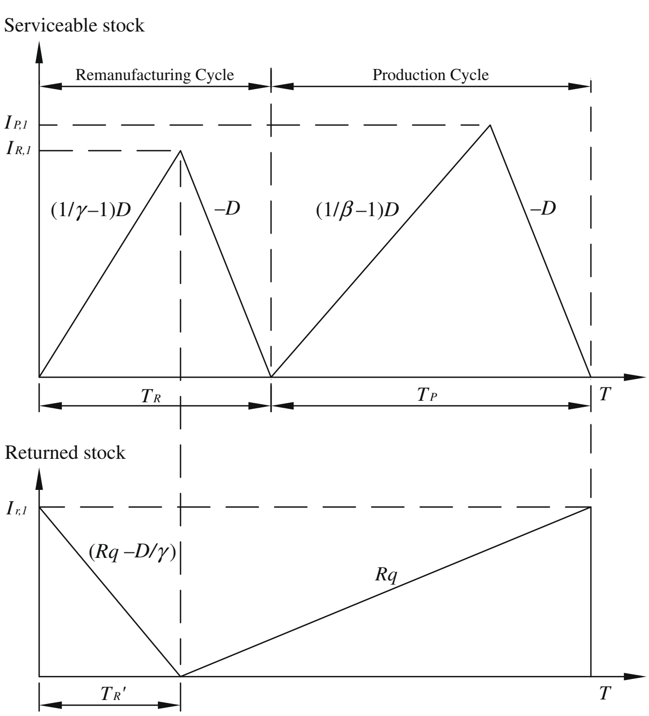

Fig. 4. Inventory status of serviceable and returned stocks.

of time with production ceasing when an inventory level of $I_{P,1} = (1-\beta)DT_P$ is attained. Once $I_{P,1}$ units are depleted, a new interval of length T is initiated.

A remanufacturing cycle commences once the inventory level of the returned stock reaches $I_{r,1} = qRT(1-qR\gamma/D)$, which depletes at a rate of $(qR - D/\gamma)$. By the end of a remanufacturing cycle, $I_{r,1}$ units would have been depleted, and a new collection cycle of used/returned items commences building up inventory at a rate of qR. It is assumed that the screening and sorting of collected used/returned items occur prior to storage, and items not conforming to quality standards are disposed, totaling $(1-q)RT$ units.

3.1.1. Holding cost expressions

Let h_s and h_r denote the inventory holding costs per unit per unit of time for serviceable and returned items, respectively. Also, denote H_P, H_R and H_r as the inventory costs for newly produced, remanufactured and returned items, respectively.

Now let $\lambda = \lambda(R, q) = qR/D$, where $0 < \lambda < 1$, denotes the ratio of repairable items to total demand, where $T_R = qRT/D = \lambda T$ and $T_P = (1 - qR/D)T = (1-\lambda)T$. The inventory holding cost expressions for newly produced, remanufactured and returned items are given, respectively, as

$$H_{P,1} = h_s \frac{I_{p,1}}{2} T_P = \frac{h_s}{2} T^2 (1-\lambda)^2 D(1-\beta) \tag{1}$$

$$H_{R,1} = h_s \frac{I_{R,1}}{2} T_R = \frac{h_s}{2} T^2 \lambda^2 D(1-\gamma) \tag{2}$$

$$H_{r,1} = h_r \frac{I_{r,1}}{2} T_r' = \frac{h_r}{2} T^2 D\lambda(1-\lambda\gamma) \tag{3}$$

Derivations of Eqs. (1)–(3) are provided in Appendix A.1. From (1)–(3), the total inventory holding cost per unit of time is given as

$$H_{T,1} = \frac{H_P + H_R + H_r}{T} = TD\frac{\psi(\lambda)}{2} \tag{4}$$

where the term $\psi(\lambda)$ is given as

$$\begin{aligned}\psi(\lambda) &= h_s(\lambda^2(1-\gamma) + (1-\lambda)^2(1-\beta)) + h_r\lambda(1-\lambda\gamma) \\ &= \lambda^2(h_s(2-\gamma-\beta) - h_r\gamma) + \lambda(h_r - 2(1-\beta)h_s) + h_s(1-\beta)\end{aligned} \tag{5}$$

3.1.2. Lot size dependant cost expressions

Let $S = S_r + S_p$, where S is the total setup cost and S_r and S_p are the remanufacturing and production setup costs, respectively. The cost per unit of time function is given from S and (5) as

$$C(\lambda, T) = \frac{S}{T} + TD\frac{\psi(\lambda)}{2} \tag{6}$$

where (6) is convex in T, i.e., $\partial^2 C(\lambda,T)/\partial T^2 > 0\ \forall T > 0$. Setting the first partial derivative to zero and solving for T we get

$$T^* = \sqrt{\frac{2S}{D\psi(\lambda)}} \tag{7}$$

Substituting (7) in (6) to get

$$C(P,q) = C(\lambda) = \sqrt{2SD\psi(\lambda)} \tag{8}$$

where $\lambda(R, q) = qR(P, q)/D$ and to simplify the presentation of the mathematics it will be referred to from this point onwards by λ. The optimal remanufacturing and production cycle times are given, respectively, from (7) as

$$T_R^* = \lambda T^* = \lambda\sqrt{\frac{2S}{D\psi(\lambda)}} \tag{9}$$

$$T_P^* = (1-\lambda)T^* = (1-\lambda)\sqrt{\frac{2S}{D\psi(\lambda)}} \tag{10}$$

The corresponding remanufacturing and production lot sizes are determined from (9) and (10), respectively, as $X_R^* = DT_R^*$ and $X_P^* = DT_P^*$.

3.1.3. Total cost expression

In this section, the overall costs are determined. The total cost per unit of time is the sum of the following unit time costs

Setup cost per unit time: $(S_r + S_p)/T = S/T$
Holding costs per unit of time: $TD\frac{\psi(\lambda)}{2}$, where $\psi(\lambda)$ is given from (5)
Disposal costs per unit of time: $(1-q)RC_w$
Remanufacturing costs per unit time: qRC_r
Production costs per unit time: $(D-qR)C_p$
Purchasing costs per unit time: $RPC_n + (D - Rq)C_n$

Therefore, total cost per unit of time is expressed as

$$\begin{aligned} C(P,q,T) &= \frac{S}{T} + TD\frac{\psi(\lambda)}{2} + (1-q)RC_w + qRC_r + (D-qR)C_p \\ &\quad + RPC_n + (D-Rq)C_n \\ &= \frac{S}{T} + TD\frac{\psi(\lambda)}{2} + R[q(C_r - C_w - C_p - C_n) \\ &\quad + C_w + PC_n] + D(C_p + C_n) \end{aligned} \tag{11}$$

where (11) is convex in T, i.e., $\partial^2 C(P, q, T)/\partial T^2 > 0$ $\forall T > 0$. Setting the first partial derivative to zero and solving for T we get

$$T^* = \sqrt{\frac{2S}{D\psi(\lambda)}} \tag{12}$$

Substituting (12) in (11) to get

$$\begin{aligned} C(P,q) &= \sqrt{2SD\psi(\lambda)} + R[q(C_r - C_w - C_p - C_n) + C_w + PC_n] \\ &\quad + D(C_p + C_n) \end{aligned} \tag{13}$$

where $\psi(\lambda)$ is given in (5), and

$$\lambda = qR(P,q)/D \tag{14}$$

$$R = R(P,q) = D(1 - ae^{-\theta P})be^{-\varphi q} \tag{15}$$

The convexity of the cost function (13) was validated numerically. That is, all the model input parameters were randomized and the Hessian matrix was computed for each specific data set generated. Sample examples are provided in Appendix A.2. In all of these numerical examples (totaling more than 10,000), the Hessian matrix held a positive value. It is therefore reasonable enough to conjecture that convexity of (13) holds. This approach is not surprising since this paper is not the first to do so (e.g., Agrawal & Nahmias, 1997; Silver & Costa, 1998).

3.2. Model II: multiple remanufacturing and production cycles

In this section, a generalized form of Model I is presented having m equal remanufacturing cycles and n equal production cycles in an interval of length T, where $T = mT_R + nT_P$ with T_R and T_P are defined earlier and $m \geqslant 1$ and $n \geqslant 1$ are positive integers. In this model, the decision variables are m, n, P, q, where n is the number of production cycles in an interval of length T, and m is the number of remanufacturing cycles in an interval of length T.

3.2.1. Holding cost expressions

For this case, the maximum inventory levels attained in a remanufacturing cycle and in a production cycle are given, respectively, as $I_{R,m} = (1-\gamma)DT_R = (1-\gamma)DT\lambda/m$ and $I_{P,n} = (1-\beta)DT_P = (1-\beta)DT(1-\lambda)/n$. Similarly, the maximum inventory level of repairable inventory is given as $I_{r,m} = T\lambda D(1 + \lambda(1-\gamma-m)/m)$. The inventory levels for the serviceable and returned stocks are shown in Fig. 5. The inventory holding cost expressions for newly produced, remanufactured and returned items are given, respectively, as

$$H_{P,n} = h_s\frac{I_{P,n}}{2}T_P = \frac{h_s}{2}T^2(1-\lambda)^2 D(1-\beta)/n \tag{16}$$

$$H_{R,m} = h_s\frac{I_{R,n}}{2}T_R = \frac{h_s}{2}T^2\lambda^2 D(1-\gamma)/m \tag{17}$$

$$H_{r,m} = h_r\frac{I_{r,n}}{2}T = \frac{h_r}{2}T^2 D\lambda(1+\lambda(1-\gamma-m)/m) \tag{18}$$

Derivations of equations (16)–(18) are provided in Appendix A.3. The total holding cost per unit of time unit is

$$H_{T,m,n} = \frac{H_{P,n} + H_{R,m} + H_{r,m}}{T} = \frac{TD\psi(m,n,\lambda)}{2} \tag{19}$$

$$\begin{aligned} \psi(m,n,\lambda) &= h_s(\lambda^2(1-\gamma)/m + (1-\lambda)^2(1-\beta)/n) \\ &\quad + h_r\lambda(1+\lambda(1-\gamma-m)/m) \end{aligned} \tag{20}$$

3.2.2. Lot size dependant costs

Let $S_{m,n} = mS_r + nS_p$, where $S_{m,n}$ is the total setup cost for m remanufacturing and n production cycles in an interval of length T units of time, where S_r is the cost per a remanufacturing setup and S_p is the cost per a production setup. The cost per unit of time is given from (20) as

$$C(m,n,\lambda,T) = \frac{S_{m,n}}{T} + \frac{TD\psi}{2}(m,n,\lambda) \tag{21}$$

where (21) is convex in T, i.e., $\partial^2 C(m, n, \lambda, T)/\partial T^2 > 0$ $\forall T > 0$. Setting the first partial derivative to zero and solving for T we get

$$T^*_{m,n} = \sqrt{\frac{2S_{m,n}}{D\psi(m,n,\lambda)}} \tag{22}$$

Substituting (22) in (21) to get

$$C(m,n,P,q) = C(m,n,\lambda) = \sqrt{2S_{m,n}D\psi(m,n,\lambda)} \tag{23}$$

The optimal remanufacturing and production cycle times are given, respectively, from (22) as

$$T^*_{R,m,n} = \frac{\lambda T}{m} = \frac{\lambda}{m}\sqrt{\frac{2S_{m,n}}{D\psi(m,n,\lambda)}} \tag{24}$$

$$T^*_{P,m,n} = \frac{(1-\lambda)T}{n} = \frac{(1-\lambda)}{n}\sqrt{\frac{2S_{m,n}}{D\psi(m,n,\lambda)}} \tag{25}$$

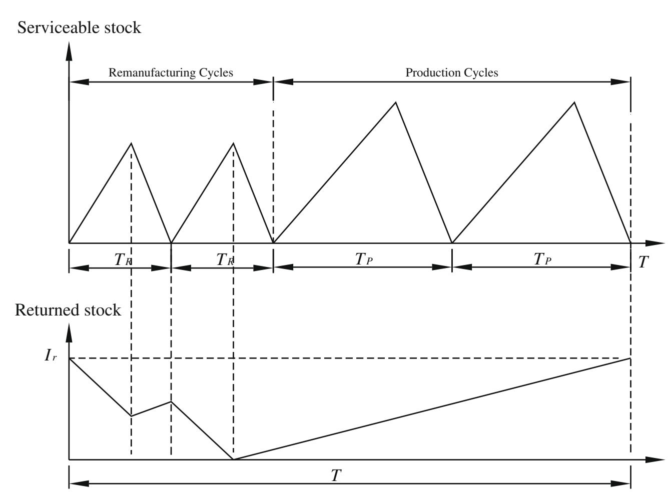

Fig. 5. Inventory status for the case of $m = 2$, $n = 2$.

The corresponding remanufacturing and production lot sizes are determined from (24) and (25), respectively, as $X^*_{R,m,n,} = DT^*_{R,m,n}$ and $X^*_{P,m,n} = DT^*_{P,m,n}$.

3.2.3. Total cost expression

As discussed in Section 3.1.3, the total cost per unit of time is the sum of all unit time costs, with all the cost components remaining the same except for the component of the holding costs per unit of time and is expressed as

$$H_{T,m,n} = \frac{TD}{2}\left[h_s(\lambda^2(1-\gamma)/m + (1-\lambda)^2(1-\beta)/n)\right.$$
$$\left. + h_r\lambda(1+\lambda(1-\gamma-m)/m)\right]$$

Therefore, and similar to the way in which (11) was developed, the total cost per unit of time is expressed from (21) as

$$C(m,n,P,q,T) = \frac{S_{m,n}}{T} + \frac{TD}{2}\psi(m,n,\lambda) + R(q(C_r - C_w - C_p - C_n) + C_w + PC_n) + D(C_p + C_n) \quad (26)$$

where (26) is convex in T, i.e., $\partial^2 C(m, n, P, qT)/\partial T^2 > 0\ \forall T > 0$. Setting the first partial derivative to zero and solving for T we get

$$T^* = \sqrt{\frac{2S_{m,n}}{D\psi(m,n,\lambda)}} \quad (27)$$

Substituting (27) in (26) to get

$$C(m,n,P,q) = \sqrt{2S_{m,n}D\psi(m,n,\lambda)} + R(q(C_r - C_w - C_p - C_n) + C_w + PC_n) + D(C_p + C_n) \quad (28)$$

where

$$\psi(m,n,\lambda) = h_s(\lambda^2(1-\gamma)/m + (1-\lambda)^2(1-\beta)/n) + h_r\lambda(1+\lambda(1-\gamma-m)/m) \quad (29)$$

while λ and R are given in (14) and (15), respectively.

It was shown in Appendix A.4 that a policy with both m and n being even integers is never be optimal.

3.2.4. Solution procedure

The cost function given in Eq. (28) can be optimized using the following solution procedure:

Step 1: For a set of input parameters, D, C_n, a, θ, b, φ, h_s, h_r, C_r, C_p, C_w, γ, β, S_r, and S_p. Set $n = 1$ and minimize $C(m, n, P, q)$ for $m = 1$ and $m = 2$ using an optimization tool (e.g., Solver from Microsoft Excel). With $C(m, n, P, q)$ being the objective function subject to $0 < P < 1$ and $0 < q < 1$, where P and q are decision variables. Record the minimum costs for the cases when $m = 1$ and $m = 2$ and their corresponding P and q values, where P and q are optimal for specific values of n and m.

Step 2: Compare $C(m, n, P, q)$ value for ($m = 1$) to that of ($m = 2$). If $C(1, 1, P, q) < C(2, 1, P, q)$, terminate the search for ($n = 1$) and record the value. If $C(1, 1, P, q) > C(2, 1, P, q)$, repeat for ($m = 3$), ($m = 4$), etc. Terminate once $C(m_1^* - 1, 1, P, q) > C(m_1^*, 1, P, q) < C(m_1^* + 1, 1, P, q)$, where m_1^* is the optimal value for the number of remanufacturing cycles when there is 1 production cycle. Record the values of $C(m_1^*, 1, P, q)$, m_1^*, P and q for $n = 1$.

Step 3: For the same set of parameters in Step 1, set $n = 2$, however, optimize $C(m, n, P, q)$ for $m = 1$ and $m = 3$. (The case when m and n being even integers is never optimal. Refer to Theorem 1 in Appendix A.4).

Step 4: Compare $C(m, n, P, q)$ values for ($m = 1$) to that of ($m = 3$). If $C(1, 2, P, q) < C(3, 2, P, q)$, terminate the search for ($n = 2$) and record the value. If $C(1, 2, P, q) > C(3, 2, P, q)$, repeat for ($m = 5$), ($m = 7$), etc. Terminate once $C(m_2^* - 2, 2, P, q) > C(m_2^*, 2, P, q) < C(m_2^* + 2, 2, P, q)$, where m_2^* is the optimal value for the number of remanufacturing cycles when there are two production cycles. Record the values of $C(m_2^*, 2, P, q)$, m_2^*, P and q, for $n = 2$.

Step 5: Compare $C(m, n, P, q)$ values for ($n = 1$) to that of ($n = 2$). If $C(m_1^*, 1, P, q) < C(m_2^*, 2, P, q)$, terminate the search and $C(m_1^*, 1, P, q)$ is the minimum cost. If $C(m_1^*, 1, P, q) > C(m_2^*, 2, P, q)$, then drop the value of $C(m_1^*, 1, P, q)$ and repeat steps 1 and 2 for $n = 3$. Compare $C(m,n,P,q)$ values for ($n = 2$) to that of ($n = 3$). If $C(m_2^*, 2, P, q) < C(m_3^*, 3, P, q)$, terminate the search and $C(m_2^*, 2, P, q)$ is the minimum cost. If $C(m_2^*, 2, P, q) > C(m_3^*, 3, P, q)$, then drop the value of $C(m_2^*, 2, P, q)$ and repeat steps 3 and 4 for $n = 4$.

Step 6: Repeat steps 1 to 5. Terminate the search once $C(m_{i-1}^*, i-1, P, q) > C(m_i^*, i, P, q) < C(m_{i+1}^*, i+1, P, q)$, where $i = 1, 2, 3, 4$, etc.

For the case of Model I ($m = 1$, $n = 1$), the solution procedure will be reduced to step 1 only.

4. Numerical examples

This section provides four numerical examples to illustrate the behaviors of Models I and II and to draw some conclusions.

Richter (1997), Teunter (2001), and Dobos and Richter (2003, 2004) concluded that the optimal inventory holding strategy in the production–recycling model they discussed is a pure strategy (bang–bang strategy). That is, it is either buyback all used/returned items for remanufacture/recycle with no production option, or produce new items with no buyback or remanufacturing/recycling option. These pure strategies are valid under limited conditions; when there are only two purchasing prices and one quality, i.e., the purchasing price of a used/returned item is different from the (collective) purchasing price of raw materials used to produce a new unit, while used/returned and new units have the same quality. Logically, when returns are less expensive to acquire than raw materials required producing a new item, repair with no production is a better policy, and the opposite is true. This corroborates the finding of Dobos and Richter (2004) who wrote (p.322): "*Probably these pure strategies are technologically not feasible and some used products will not return or even more as the sold ones will come back, some of them will be not recycled*". Models I and II illustrate this limitation in numerical example 1.

Practically, collected used/returned items are of varying prices and qualities. The models developed in this paper, when the purchasing price of returned items is fixed, an increase in the acceptance quality level of returns decreases flow in the reverse direction, allowing customers to return higher quality products only. Similarly, when the acceptance quality level of returns is fixed, an increase in the purchasing price for returns increases the flow in the reverse direction encouraging customers to return more products. Furthermore, in reality, it is nearly unattainable to reach a 100% buyback and use rate (remanufacture/recycle every produced item), therefore, what if a decision maker cannot attain a 100% buyback and use rates, but he/she can only attain say 90% or 95%? Is such an acceptance percentage close to the optimal solution? Or, is it better to switch to a pure production policy? These questions are addressed in numerical example 2.

In Dobos and Richter (2006), it was assumed that a pure recycling/reuse strategy (pure remanufacturing) to be more cost effective than a pure production strategy. They did not consider the cases when the cost of a pure remanufacturing strategy is either equal to or more than a pure production strategy. In addition, Dobos and Richter (2003, 2004, 2006) assumed a fixed return rate. These limitations to the work of Dobos and Richter (2003, 2004,

2006) are addressed in example 3. The convexity of the presented models is discussed in example 4.

4.1. Example 1 (Model I)

A comparison between Dobos and Richter (2003) and Model I developed in this paper is presented. Let $D = 1000$, $h_s = 1.6$, $h_r = 1.2$, $\gamma = 0.3$, $\beta = 0.6$, $S_p = 2400$, $S_r = 1600$, $C_r = 1.2$, $C_w = 0.1$, $C_p = 2$, $C_n = 5$. First, these values are substituted in the model of Dobos and Richter (2003), where the buyback proportion is 0.231 and the use proportion is 0.829, which is equivalent to a reusable proportion = (α = buyback proportion) × (δ = use proportion) × D = 0.231 × 0.829 × 1000 = 191.5 units or 19.15% of demand. In Dobos and Richter (2003), the cost for a mixed production and remanufacturing strategy is 9305, for a pure production strategy ($\alpha = \delta = 0$) is 8752, and for a pure remanufacturing ($\alpha = \delta = 1$) is 8704. This suggests that a pure remanufacturing strategy is the cheapest of the three strategies (8704 < 8752 < 9305).

Second, the values determined above are substituted in Model I, with the parameters of $R = R(P,q) = D(1 - ae^{-\theta P})be^{-\varphi q}$ adjusted to $a = 0.5$, $b = 0.95$, $\theta = 8$, $\varphi = 1.5$, so that $\alpha = 0.231$ and $\delta = 0.829$, where $R(P,q)/D = (1 - ae^{-\theta P})be^{-\varphi q} = 0.231$. Solving Model I, a mixed production and remanufacturing/recycling policy has the lowest cost of 8386 of the other two strategies, where the cost of a pure production is 8752 and the cost of pure remanufacturing is 8704. The cost 8386 of a mixed production repair strategy is attained when $P^* = 0.146$ and $q^* = 0.829$. To illustrate, an optimum mixed strategy was found to be optimum because some of the returned items to be remanufactured/repaired are good quality items ($q^* = 0.829$ or more) that are purchased at a low price (Purchasing Price = $P^* \times C_n = 0.146 \times 5 = 0.73$). Such a case makes a mixed strategy advantageous to the pure strategies of either pure production or pure remanufacturing.

4.2. Example 2 (Model I)

Let $D = 1000$, $h_s = 850$, $h_r = 80$, $\gamma = 2/3$, $\beta = 2/3$, $S_p = 1960$, $S_r = 440$ (Dobos & Richter, 2004). According to the calculations provided by Dobos and Richter, the pure recycling costs 16,516 and pure production costs 33,326. Therefore, it is economical to recycle with the buyback and use all returned items. What if the buyback or use rate is not 100%, is it then profitable to buyback and use all the available items?

It is worth noting that the pure policy means either to include the setup cost for remanufacturing or the setup cost for production, but not both. The mixture of remanufacturing and production means the inclusion of both costs leading to higher costs than in a pure strategy (either pure production or pure remanufacturing). Fig. 6 illustrates the behavior of the total cost for Dobos and Richter's model when the marginal buyback and use rates are varied simultaneously (from 0% to 100%). As discussed earlier, since a 100% buyback rate and 100% use rate cannot be attained, values for these rates close to 100% are considered, where these values are far from being optimal, as shown in Fig. 6. However, a mixed remanufacturing and production strategy suggesting an optimal buyback and use rates of 70% is optimal as shown. Dobos and Richter (2003, 2004, 2006) did not address this issue. To illustrate further, and for example, if the buyback rate is 99% and the decision maker decides to accept all returns, then such a policy would be 38% more expensive than the solution attained at 70% buyback and reuse rate.

Substituting the same values of the parameters in Dobos and Richter (2004) example in Model I with $a = 0.9$, $b = 0.9$, $\theta = 6$, $\varphi = 2$, while varying price and quality simultaneously, the cost and its corresponding return rate for are plotted in Fig. 7. A mixed strategy of repair and production reaches its least cost (not optimal) at a price and quality of 0.6 corresponding to a return rate of 26%. Optimizing Model I by varying the values of purchasing price and acceptance quality level independently, the optimal solution was attained at a price of 0.69 × C_n = 0.69 × 5 and a quality index of 0.53 corresponding to 30.7% returns.

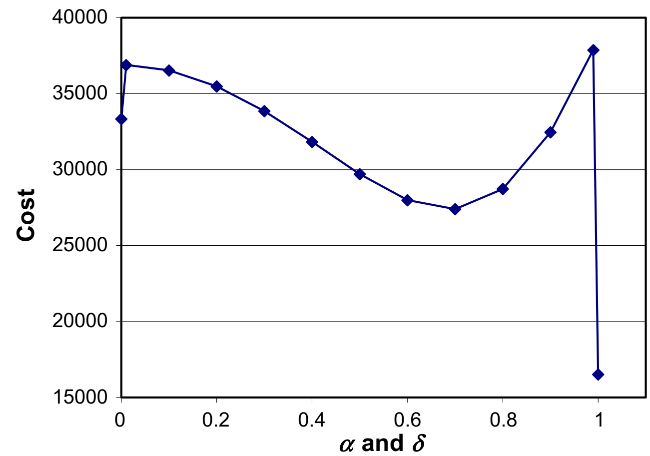

Fig. 6. The behavior Dobos and Richter's (2004) cost function for varying marginal buyback rate, α, and marginal use, δ, rate.

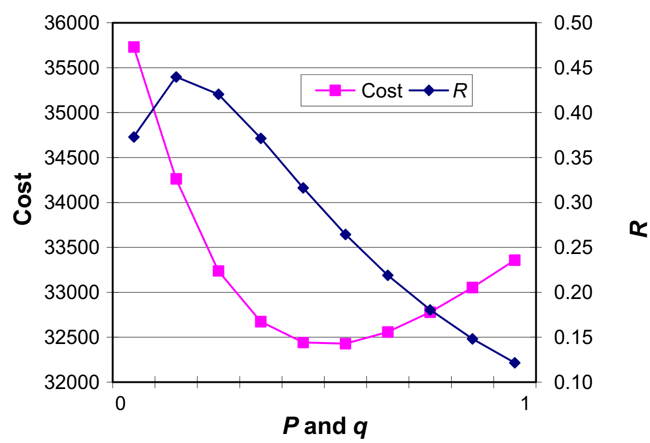

Fig. 7. Behaviors of the total cost and the return rate of Model I for varying price and quality values.

Note that the average values produced from the model of Dobos and Richter (2004) are less than the average solutions produced by the presented model, because the calculations in Dobos and Richter (2004) example did not consider the purchasing price of returned items, while price of raw materials C_n is considered equal in both examples. The second example is presented to demonstrate the infeasibility of the bang–bang policy.

4.3. Example 3 (Model II)

Let $D = 1000$, $a = 0.9$, $b = 0.9$, $\theta = 6$, $\varphi = 2$, $h_s = 4$, $h_r = 4$, $\gamma = 0.8$, $\beta = 0.5$, $S_p = 6$, $S_r = 6$, $C_r = 2$, $C_w = 0.15$, $C_p = 2$, $C_n = 0.95$. For this data set, pure production costs, which is satisfying demand with 0% returns, has a total cost of 3105, while a pure remanufacturing, which satisfies demand from 100% of returns, has a total cost of 3155. Here is a case where pure strategy recycling (pure remanufacturing) is not as cost effective as a pure strategy production. Substituting the above parameters and optimizing equation (28), the total cost is $C = 3085.5$, where $P^* = 0.21$ and $q^* = 0.87$. Although the case of pure remanufacturing is desirable, it is technologically unattainable. Therefore, assuming that all produced items are returned, and that all of these returns are remanufactured is not considered further. However, in some cases, it may be possible to accept an amount of returns of high quality and low purchasing price, which will deliver a better solution than either pure production or pure remanufacturing. Even in the case, when pure remanufacturing is more expensive than pure production, a pure production strategy is not optimal.

4.4. Example 4 (Model II)

As example 1, let $D = 1000$, $a = 0.9$, $b = 0.9$, $\theta = 6$, $\varphi = 2$, $h_s = 4$, $h_r = 3$, $\gamma = 0.8$, $\beta = 0.5$, $S_p = 6$, $S_r = 4$, $C_r = 0.1$, $C_w = 0.15$, $C_p = 2$, $C_n = 10$. Substituting these parameters and optimizing equation (28), the total cost is $C = 11160.7$. The total cost function is convex with respect to purchasing price, $P \times C_n$, and acceptance quality level, q, of returns as shown in Fig. A-2.1. The Hessian matrix is positive and an optimum solution is obtained when $m = 1$, $n = 2$, $P = 0.71$ (corresponding to an optimal purchasing price $= P^* \times C_n = 0.71 \times 10 = 7.1$) and $q^* = 0.2365$. The optimal solution is obtained using the solution procedure described in Section 3.2.4. Table 1 illustrates this solution procedure numerically.

The convexity of expression (28) was demonstrated for varying values of m, n, P, and q to investigate the behavior of the cost function in (28). Several plots of the cost function in (28) were generated and showed to have almost identical behaviors to that shown in Fig. A-2.1 (Model I) in the appendix. Avoiding duplication and preferring to be succinct, these figures are therefore not shown here.

The above numerical example was replicated for varying values of a, b, θ, φ, h_s, h_r, γ, C_r, S_r, S_p, C_w, β, and C_p, where expression (28) was optimized for 10,000 data sets. All of these data sets confirmed Theorem 1, i.e., a solution is never optimal when m and n are even. Of the 10,000 replications two numerical examples generated optimal solutions when $m > 1$ and $n > 1$ and m and n are not even (e.g., $m = 3$, $n = 2$), with the remaining examples having either m or n equal to one. To illustrate, since T is a decision variable dependent on m, n, P and q, the optimal solution tends to reside with smaller values of m and n that meets a specific return rate while minimizing the holding costs of serviceable and repairable inventories.

5. Summary and conclusions

This paper extended upon the production, remanufacturing/repair and waste disposal model of Dobos and Richter (2003, 2004, 2006) by assuming a variable return rate of used items that follows a demand-like function of purchasing price and acceptance quality level of returns. Two mathematical models were developed. The first assumes a single remanufacturing cycle and a single production cycle, with the second being a generalized version of the first assuming multiple remanufacturing and production cycles. A solution procedure was introduced with an enhanced search technique that eliminates solution branches that do guarantee an optimal solution. This enhanced solution procedure was supported by a theorem, which shows that having even numbers of remanufacturing (m) and production (n) cycles in an interval never produces an optimal solution.

Numerical results showed that when considering the return rate of used items to be dependent on the purchasing price and acceptance quality level of these returns, a pure (bang–bang) policy of either no waste disposal (total repair) or no repair (total waste disposal) as advocated in Dobos and Richter (2003, 2004) is not optimal. The limitation considered in Dobos and Richter (2006) that a pure strategy recycling should be more cost effective than pure strategy production was addressed in this paper. Results showed that a mixed (production + remanufacturing) strategy is optimal, when compared to either a pure strategy recycling (pure remanufacturing) or a pure strategy production.

An immediate extension of the work presented herein is to integrate the production–remanufacturing system into a multistage supply chain (say supplier–manufacturer–retailer), where used items are collected from the market by the manufacturer to be disassembled for reuse. In this case, the production–remanufacturing process will be supplied by components from the disassembly process and from the manufacturer's supplier as needed. A second extension is to assume that the production and remanufacturing processes are imperfect where defective items are either reworked or scrapped. A third extension is to assume demand to be stochastic.

Table 1
A numerical example illustrating the solution procedure.

Step #	n	m	P	q	C(m, n, P, q)	Notes
1	1	1	0.237	0.709	11,166	
2	1	2	0.238	0.708	11,201	$m_1^* = 1$ and $C(m_1^*, 1, P, q) = 11,166$
3	2	1	0.236	0.71	11,161	
4	2	3	0.236	0.709	11,202	$m_2^* = 1$ and $C(m_2^*, 2, P, q) = 11,161$
5						Repeat steps 1 and 2
1	3	1	0.235	0.711	11,165	
2	3	2	0.235	0.711	11,182	$m_3^* = 1$and $C(m_3^*, 3, P, q) = 11,165$
6	2	1	0.236	0.71	11,161	Terminate. Optimum solution is attained when $C(m, n, P, q)$ is minimum

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Appendix A

A.1. Appendix

The inventory holding cost for a production/remanufacturing cycle is computed by multiplying the holding cost per unit per unit of time by the average inventory level over a production/remanufacturing cycle length. The average inventory level in a cycle (production, remanufacturing or returned) is half the peak of inventory multiplied by the length of cycle time and given as

$$\begin{aligned}
H_{P,1} &= h_s \cdot 1/2.I_{P,1} \cdot T_P = h_s \cdot 1/2.(1/\beta - 1)D(\beta T_P) \cdot T_P \\
&= (1/2)h_s T_P^2 D(1-\beta) = 1/2 h_s T^2 (1-\lambda)^2 D(1-\beta) \\
H_{R,1} &= h_s \cdot 1/2.I_{R,1} \cdot T_R = h_s \cdot 1/2 \cdot (1/\gamma - 1)D(\gamma T_R) \cdot T_R \\
&= (1/2)h_s T_R^2 D(1-\gamma) = h_s T^2 \lambda^2 D(1-\gamma) \\
H_{r,1} &= h_r \cdot (1/2.I_{r,1} \cdot \gamma T_R + 1/2.I_{r,1} \cdot (T - \gamma T_R)) \\
&= h_r \cdot 1/2\left(\left(\frac{D}{\gamma} - Rq\right)\gamma T_R \cdot \gamma T_R + Rq(T - \gamma T_R) \cdot (T - \gamma T_R)\right) \\
H_{r,1} &= h_r \cdot 1/2 \cdot TRq(1 - Rq\gamma/D) \cdot T = 1/2.h_r T^2 \lambda D(1 - \lambda\gamma)
\end{aligned}$$

A.2. Appendix

To prove that the total costs function (13), which is a function in two variables, is convex, the Hessian matrix has to be computed and to be proven positive. To compute the Hessian matrix for a general case (without substituting with values other than P and q), the outcome turned to be so messy.

Table 2
Sample calculations of randomly generated examples of Model I.

a	b	θ	ϕ	h_s	h_r	γ	C_r	S_r	S_p	C_w	β	C_p	C_n	R	P	q	Cost	Hessian
0.506	0.967	9.282	5.535	0.159	0.427	0.683	0.537	1.435	6.703	0.138	0.755	10.9	0.531	173.6	0.033	0.226	11,109	2939,89,637
0.6	0.83	9.098	8.873	0.126	0.374	0.637	1.235	7.256	3.667	0.035	0.606	9.333	1.103	123.1	0.02	0.137	10,342	2085,09,397
0.449	0.932	8.004	3.702	0.66	0.251	0.813	1.956	5.659	1.126	0.128	0.871	9.257	0.53	183.7	0.031	0.322	9427	1131,45,997
0.965	0.347	8.92	3.338	0.321	0.344	0.88	0.065	3.395	3.86	0.14	0.78	9.793	0.492	56.62	0.146	0.452	10,142	130,72,114
0.659	0.923	7.047	7.916	0.578	0.34	0.7	1.473	7.468	4.525	0.191	0.774	2.938	1.991	87.38	0.046	0.216	4945	86,71,911
0.453	0.24	5.985	1.528	0.734	0.096	0.482	0.785	6.356	6.347	0.052	0.7	5.31	1.196	57.72	0.146	0.797	6361	11,61,911
0.731	0.398	4.997	2.431	0.543	0.338	0.532	0.069	3.377	2.351	0.196	0.516	1.724	1.528	73.92	0.261	0.602	3204	484,774
0.81	0.639	3.462	9.953	0.159	0.553	0.618	1.559	3.213	4.855	0.132	0.578	1.141	1.464	13.71	0.021	0.245	2636	32177.06
0.695	0.023	5.894	5.229	0.333	0.503	0.636	1.614	3.257	6.164	0.16	0.854	1.995	1	1.576	0.038	0.358	3024	485.7093
0.707	0.001	8.422	3.114	0.439	0.247	0.448	0.317	7.523	5.497	0.184	0.671	1.307	0.553	0.157	0.14	0.532	1921	1.880676

The convexity of (13) was demonstrated numerically. More than 10,000 input parameters datasets were randomly generated and the Hessian matrix was computed for each dataset (numerical example). For all of these numerical examples the Hessian matrix held positive values suggesting that it is reasonable to conjuncture that Eq. (13) is convex. A sample calculation of 10 randomly selected numerical are represented in Table 2, where a, b, θ, ϕ, h_s, h_r, γ, C_r, S_r, S_p, C_w, β, C_p and P_n are randomly generated.

Example A.1. Fig. A-2.1 illustrates the behavior of the cost function for the following input parameters: $D = 1000$, $a = 0.9$, $b = 0.9$, $\theta = 6$, $\varphi = 2$, $h_s = 4$, $h_r = 3$, $\gamma = 0.8$, $\beta = 0.5$, $S_r = 4$, $S_p = 6$, $C_r = 0.1$, $C_w = 0.15$, $C_p = 2$, $P_n = 1$. The total cost function is convex with respect to price, P, as well as quality, q, where optimal values are $P = 0.370929$ and $q = 0.668266$.

A.3. Appendix

The inventory holding cost for a production or manufacturing cycle is computed by multiplying the holding cost per unit per unit of time by the average inventory level over a production/remanufacturing cycle length. The average inventory level in a cycle is half the peak of inventory multiplied by the length of cycle time.

$$H_{P,n} = (1/2)h_s(1/\beta - 1)D(\beta T_P)T_P n = \frac{h_s}{2}T_P^2 D(1-\beta)n$$
$$= 1/2h_s T^2(1-\lambda)^2 D(1-\beta)n/n^2$$
$$H_{P,n} = 1/2h_s T^2(1-\lambda)^2 D(1-\beta)/n$$
$$H_{R,m} = (1/2)h_s(1/\gamma - 1)D(\gamma T_R)T_R m = h_s T_R^2 D(1-\gamma)m$$
$$= 1/2h_s T^2\lambda^2 D(1-\gamma)m/m^2$$
$$H_{R,m} = 1/2h_s T^2\lambda^2 D(1-\gamma)/m$$

For $H_{r,m}$, the area is divided into m A triangles, B $(m-1)$ triangles, a single C triangle and $(m-1)$ D squares, as shown in Fig. A-3.1.

Area of triangle A is $T_A = \gamma\frac{T_R}{2}(D/\gamma - Rq)\cdot\gamma T_R = \frac{T_R^2}{2}\gamma(D - Rq\gamma)$.
Area of triangle B is $T_B = 1/2(1-\gamma)T_R(Rq)\cdot(1-\gamma)T_R = 1/2RqT_R^2(1-\gamma)^2$.
Area of triangle C is $T_C = (1/2)(T - mT_R + (1-\gamma)T_R)\cdot Rq(T - mT_R + (1-\gamma)T_R)$, reducing to

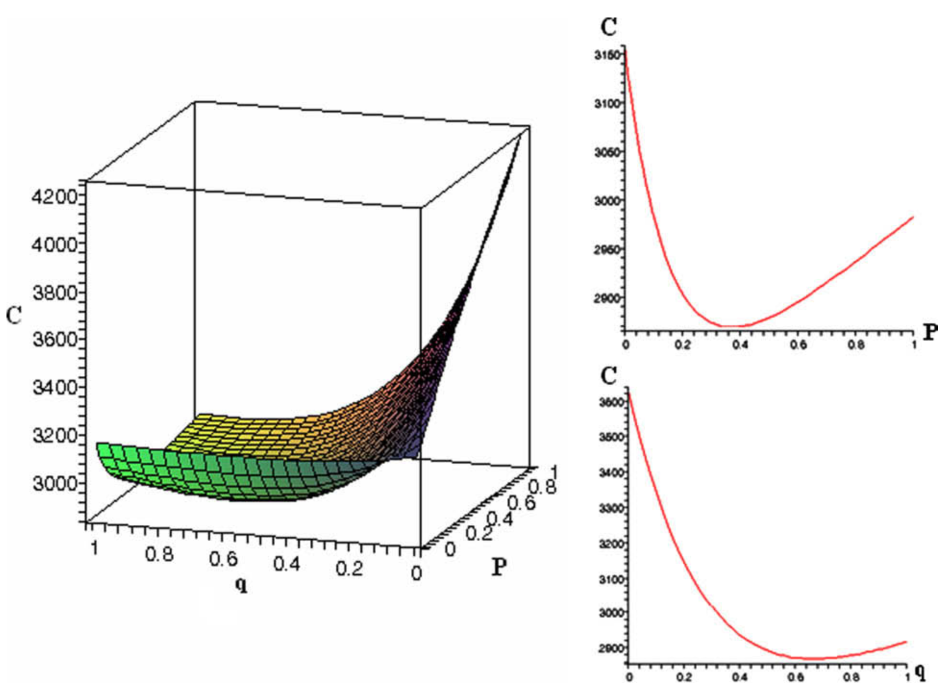

Fig. A-2.1. Cost plotted versus price and quality.

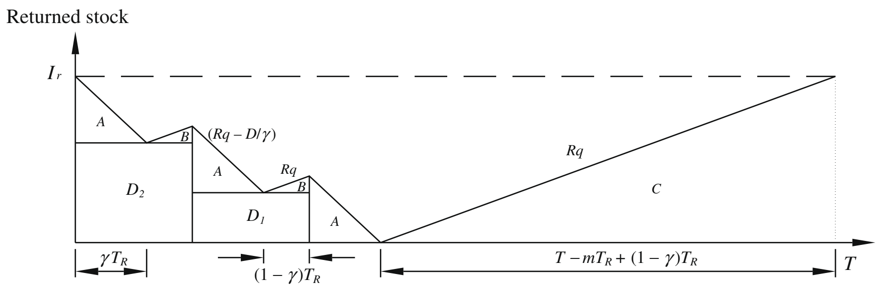

Fig. A-3.1. Inventory calculations for H_r.

$$T_C = \frac{Rq}{2D}D\left(T - m\frac{\lambda T}{m} + (1-\gamma)\frac{\lambda T}{m}\right)^2 = \frac{T^2}{2}\lambda D(1+\lambda(1-\gamma-m)/m)^2$$

Area of square $T_{D_i} = ((D/\gamma - Rq)\gamma T_R - Rq(1-\gamma)T_R)\cdot i\cdot T_R = T_R^2(D-Rq)i$

$$\begin{aligned} H_{r,m} &= mT_A + (m-1)T_B + T_C + \sum_{i=1}^{m-1} T_{D_i} \\ &= (1/2)T_R^2[m\gamma(D - Rq\gamma) + (m-1)Rq(1-\gamma)^2 \\ &\quad + \lambda D(1+\lambda(1-\gamma-m)/m)^2 + (D-Rq)(m-1)m] \\ H_{r,m} &= 1/2 h_r T^2 D\lambda(1+\lambda(1-\gamma-m)/m). \end{aligned}$$

As shown in Fig. A-3.1, increasing m or n increases average inventory in returned stock, while it has no effect on average inventory in serviceable stock. This signifies the tendency of the presented models to produce optimal solutions with smaller m and n. This point is emphasised in Appendix A.4.

A.4. Appendix

Theorem 1. A policy $C(m, n, \lambda, T)$[1] with both m and n being even integers can never be optimal, since the total cost rate associated with policy $C(m/2, n/2, \lambda, T/2)$ is smaller.

Proof. In Figs. A-4.1a and A-4.1b, the inventories associated with policies $C(m, n, \lambda, T)$, *Case a*, and $C(m/2, n/2, \lambda, T/2)$, *Case b*, are depicted by an example.In *Case a*, the parameters S_r, S_p, $S_{m,n}$, D, T, m, n, R, q, h_s, h_r, β, γ, correspond, respectively, to parameters S_r, S_p, $S_{m,n}/2$, D, $T/2$, $m/2$, $n/2$, R, q, h_s, h_r, β, γ, in *Case b*. For *Case a*, the cost per unit of time unit is $C_a(m, n, \lambda, T) = S_{m,n}/T + TD\psi_a(m, n, \lambda)/2$, where

$$\begin{aligned} \psi_a(m,n,\lambda) &= h_s\left(\lambda^2(1-\gamma)\frac{1}{m} + (1-\lambda)^2(1-\beta)\frac{1}{n}\right) \\ &\quad + h_r\lambda\left(1+\lambda(1-\gamma-m)\frac{1}{m}\right) \end{aligned}$$

For *Case b*, the cost per unit of time unit is $C_b(m,n,\lambda,T) = \frac{S_{m,n}/2}{T/2} + 1/2\frac{T}{2}D\psi_b(m,n,\lambda)$, where

$$\begin{aligned} \psi_b(m,n,\lambda) &= h_s\left(\lambda^2(1-\gamma)\frac{2}{m} + (1-\lambda)^2(1-\beta)\frac{2}{n}\right) \\ &\quad + h_r\lambda\left(1+\lambda\left(1-\gamma-\frac{m}{2}\right)\frac{2}{m}\right) \end{aligned}$$

The difference in costs for *Case a* and *Case b* is given as

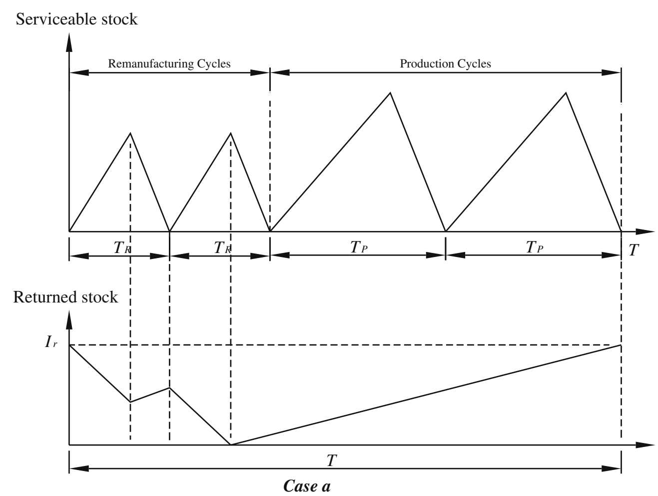

Fig. A-4.1a. The behavior of inventory for Case *a*: C (m, n, λ, T).

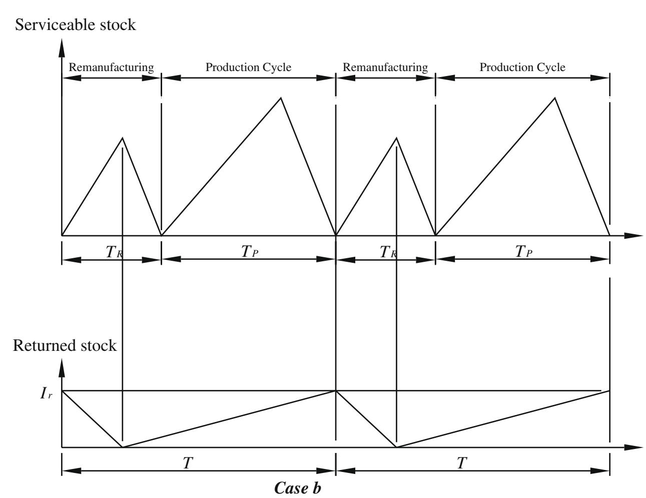

Fig. A-4.1b. The behavior of inventory for Case *b*: C ($m/2$, $n/2$, λ, $T/2$).

$$\begin{aligned} &C_a(m,n,\lambda,T) - C_b(m,n,\lambda,T) \\ &= \frac{S_{m,n}}{T} + \frac{T}{2}D\psi_a(m,n,\lambda) - \frac{S_{m,n}/2}{T/2} - \frac{T}{2}D\psi_b(m,n,\lambda) \\ &= \frac{T}{2}D(\psi_a(m,n,\lambda) - 1/2\psi_b(m,n,\lambda)) \\ &= \frac{TD}{2}\left(h_r\lambda\left(1+\lambda(1-\gamma-m)\frac{1}{m}\right) - 1/2h_r\lambda\left(1+\lambda\left(1-\gamma-\frac{m}{2}\right)\frac{2}{m}\right)\right) \\ &= \frac{TD}{4}h_r\lambda(1-\lambda). \end{aligned} \tag{30}$$

$\because 0 < \lambda < 1, \quad \therefore (1-\lambda) > 0$ and

[1] $C(m, n, \lambda, T)$ is equivalent to $C(m, n, P, q)$, as Eq. (27) is equivalent to Eq. (29).

$$C_a(m,n,\lambda,T) - C_b(m,n,\lambda,T) = \frac{TD}{4} h_r \lambda(1-\lambda) > 0,$$
$$\forall m,n,\lambda > 0. \quad \square$$

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