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Predicting the Outcome of a Tennis Tournament: Based on Both Data and Judgments

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Abstract. This paper is about predicting the outcome of tennis matches of the Association of Tennis Professionals (ATP) and the Women's Tennis Association (WTA) using both data and judgments. There are many factors that influence that outcome. An important question is which factors have significant influence on the outcome. We have identified numerous factors and systematically prioritized them subjectively and objectively, so as to improve the accuracy of the prediction. We then used them to predict the win-lose outcome of the 2015 US OPEN tennis matches (63 men and 31 women's games) before they took place. The tennis match prediction in sports literature thus far reported an accuracy rate of 70%.The accuracy of our proposed model which combines data and judgment reaches 85.1%

Keywords: Prediction, tennis, data analysis, judgment

1. Introduction

Learning to predict outcomes in sports is a substantial step towards prediction in politics and in complex international conflicts. It enables one to show politicians how to deal with the world rationally with a greater degree of confidence. Uncertainty in the outcome of competitive games also increases the attractiveness and excitement in sports competition and makes them challenging for the science of prediction.

Predicting outcomes in sports competition has been given close and extensive attention for a long time now. Research on the subject has evolved into an important area of investigation whose major focus is the accuracy of a forecast. Forecasts have been divided into two types: predicting the champion team in a competition or a season, and predicting the winner of an individual game. So far, however, tennis has not received the attention it deserves that scholars have paid to football, basketball, and soccer. The few researchers have used statistical approaches based on data, or unqualified judgments which amounts to guessing to foretell the winner of tennis games.

It is commonly believed that group sports such as basketball, football, hockey and others are much harder to predict than individual sports like badminton, ping pong and tennis. Team sports include more factors and need to be considered more in research on sports competition, and the crucial factor in individual sports is the strength of the individual player. Among the primary factors that influence the outcome of tennis are: luck, psychology, surface type, refereeing, the strength of a player and her/his past performance, and so on. However, most factors are uncertain as to how much they influence the outcome of a game, and thus make the outcome hard to predict.

In this paper, we report on predicting the outcome of tennis matches of the Association of Tennis Professionals (ATP) and the Women's Tennis Association (WTA) using both data and judgments. This study aims at producing a highly accurate tennis match forecast as complete as possible.

The main innovations in this paper are as follows:

- We are the first to make prediction in tennis using a method combining data and judgments.
- The factors used in the prediction have been validated for their usefulness.
- The data analysis results along with subjective opinions, knowledge, and experience are used to form expert judgments
- Our prediction accuracy from the combined factors was 85.1%, which as we know is the highest accuracy in predicting matches in tennis tournaments. We believe it is due to the quantification and use of judgments on intangibles along with raw data metrics.

The rest of the paper is organized as follows: Section 2 provides some background and related research including accuracy of predictions in the field of sports. Section 3 includes an analysis of the key factors influencing the outcome of a tennis match and grouping all the key factors for our model. Section 4 provides the methodology for the prediction. Section 5 gives the details of our experimental approach to make predictions by incorporating both data and judgments. Section 6 describes the results of our predictions and how they compare with other methods used in other sports like basketball and soccer. Section 7 presents our conclusions about the effectiveness of this approach.

2. Literature Review

There is much literature on forecasting the outcome of sports matches. Forrest and Simmons (2000) did research on English professional soccer. They examined tipster predictions and the role of various data in forecasts used by forecasters in making judgments about outcomes. Lebovic and Sigelman (2001) predicted the outcome of games between ranked teams. Their model demonstrated that lower-ranked teams move up incrementally faster after a victory. All focal predictors in the model perform as expected in influencing a team's chances of moving up following a win. Boulier and Stekler (2003) also proved that rankings are valuable in predicting the outcomes of basketball games, NFL teams and tennis matches. Goddard (2005) compared two models of match outcome prediction. One of the two models includes the number of goals scored by each team and the other includes a win-draw-lose outcome by using the same data from the English league football games. They concluded that a hybrid of the two methods yields the highest performance outcome. Groll and Abedieh (2013) analyzed the role of bookmakers' odds together with many additional, potentially influential covariates with respect to a national team's success at European football championships and especially to detect covariates. They used the generalized linear mixed model (GLMM) approach to incorporate team-specific random effects. Later Groll et al. (2015) predicted soccer match results based on a regularized Poisson regression model that includes various covariates describing the national teams' success in previous FIFA World Cups. Leitner et al.(2010) proposed techniques for forecasting the results of the European football championship 2008, for which the consensus model based on bookmakers' odds outperforms the methods based on both the Elo rating and the FIFA/Coca Cola World rating. Gu et al. (2016) make a prediction of hockey games with AHP and data mining.

As for tennis match forecasts, previous literature on forecasting in tennis have used official rankings of players to predict whether the higher ranked player will win (Boulier and Stekler 1999, Clarke 2000, Klaassen and Magnus 2003, Del Corral and Prieto-Rodríguez 2010). Boulier and Stekler (1999) used statistical probit regressions with the difference in rankings as the predictor of the outcome of games for professional tennis and proved that the rankings are useful forecasting information. Clarke (2000) used the official ATP computer tennis rankings to predict a player's chance of winning with a logistic regression model. Del Corral and Prieto-Rodríguez (2010) tested whether the differences in rankings between individual players are good predictors. However, the method has been called into question by Scheibehenne and Bröder (2007). They pointed out that recognition heuristic works better in predicting the outcomes of the Wimbledon men's tennis competition than predictions based on official ATP rankings. They only rely on mere recognition of player names in the mass media coverage while ignoring many factors.

A handful of papers considered various factors influencing the outcome. Del Corral and Prieto-Rodríguez (2010) predicted the Grand Slam tennis outcomes with three groups of variables: a player's past performance, a player's physical characteristics, and match characteristics. They have estimated three alternative profit models for men and women separately. The results demonstrated that the ranking effect is statistically the same for men and women. Individual-tournament (individual competition) is only significant for men and the variable related to being a previous top-ten player is a more relevant predictor of victory among women than men.

McHale and Morton (2011) proposed a Bradley-Terry type model for forecasting the top tier of the Women's Tennis Association ATP competition. They considered surfaces (hardcourt, carpet, clay or grass) influence on match outcomes. The found that a model incorporating information on match score, play data and surface can give a higher accuracy of forecasting than ranking-based models. However, they did not consider the dependence between factors.

Klaassen and Magnus (2003) used a computer program and large data from Wimbledon to forecast the winner of tennis matches while the competition was taking place. Their work, especially in that they update the predictions during the events, is fit for a betting market. There is some literature research on the within-match sports betting markets (Borghesi 2007, Gil and Levitt. 2012, Easton and Uylangco 2007). Easton and Uylangco (2010) used Klaassen and Magnus's model and compared it with betting odds on a point-by-point basis. They verified that betting markets are a good predictor of outcomes of both men's and women's tennis matches.

Song et al. (2007) compared forecasts of the outcomes of NFL games by statistical models using expert's prediction. The results showed that the accuracy is almost the same statistically and the variation is higher in the success rates among experts.

Machine learning is used in other sports forecasting. Joseph et al.(2006) provide a Bayesian network model with expert knowledge. Min et al. (2008) proposed a framework with Bayesian inference for football game prediction. Their method is used to predict outcomes of matches between teams with few encounters. However, machine learning methods require adequate data. Lessmann et al.(2012) employed statistical models to predict the outcomes of competitive events. For tennis games as one of the most popular sports, Knottenbelt et al. (2012) developed a hierarchical Markov model to estimate the probability of each player's winning a professional tennis match. Chitnis and Vaidya (2014) considered performance assessment of professional tennis players using Data Envelopment Analysis in historical matches played in ATP. They provided a method for assessing the performance of tennis players. We made a comparison of our paper with previous literature in Table 1.

Green and Armstrong (2015) maintain that simplicity in forecasting requires that method, representation of cumulative knowledge, rela-

Table 1 Comparison of Our Paper with Previous Literature

Literature	Sport	Method	Contribution
Forrest & Simmons (2000)	English professional soccer	Logit	Investigated the role of newspaper tipsters' advice in forecasting sports
Boulier & Stekler (2003)	NFL games	Comparison of forecasts published in The New York Times and the betting market	Proved power scores is useful in predicting sports
Boulier & Stekler (1999)	Tennis	Statistical probit regressions	Proved ranking is useful information in predicting sports
Clarke (2000)	Tennis	Logistic regression model	Investigated the role of ranking in predicting player's chance of winning
Del Corral & Prieto-Rodríguez (2010)	Tennis	Probit models	Investigated the role of players' past performance, physical characteristics, and match characteristics in predicting
Reid, Crespo, Lay, & Berry (2007)	Tennis	Review of research and practice	Investigated the role of players' physical fitness in predicting tennis
Chitnis &Vaidya (2014)	Tennis	Data Envelopment Analysis	Evaluated the performance of tennis players
Our paper	Tennis	Data analysis and evidence-based judgments	Developed a comprehensive and highly accurate prediction framework based on data analysis and human judgments to forecast the outcome of a tennis match before it takes place

tionships in models, and relationships among models, forecasts, and decisions are all sufficiently uncomplicated and easily understood by decision-makers. In the book of Moneyball: the Art of Winning an Unfair Game, Lewis (2003) uses the team's analytical, evidence-based, sabermetric approach to assemble a competitive baseball team. Considering the importance of experts and past data evidence, we propose a new approach in this paper by combining experts' subjective judgments and historical data evidence. To incorporate the judgment and evidence, we take both tangible and intangible variables and conduct evidence based judgement by applying statistics and the (Analytic Network Process) ANP method, which is validated to be effective in forecasting and making decisions (Saaty 2013, Shang and Ergu 2016, Gu et al. 2016, Gu et al. 2017, Gu et al. 2018).

3. Addressing the Diversity of Information Available

We collected much data to predict the outcomes of tennis matches. A performance metric is usually defined as a number which aids in evaluating a player's performance. These numbers come in many different ways: aces scored, the number of successful first serves, percentages of break points saved and etc. It is even more complicated by the fact that sometimes larger numbers are better (aces scored),

Table 2 The ATP World Tour Comprises

Event category	Number	Total prize money (USD)	Winner's ranking points
Grand Slam	4	-	2000
ATP World Tour Finals	1	4450000	1100–1500
ATP World Tour Masters 1000	9	2,450,000 to 3,645,000	1000
ATP World Tour 500 series	13	755,000 to 2,100,000	500
ATP World Tour 250 series	39	416,000 to 1,024,000	250

while sometimes smaller numbers are better (double service fouls). Some performance metrics are basic and some are numbers obtained by counting or combining counts or statistics (Number of successful second serves = number of service points - number of successful first serve). Some metrics are not performance metrics, they are simply situational data about the matches played in a tournament, tournament date, tournament level, scores in the match, surface type, number of minutes and so on. They also include some basic information for the two players, such as a player's name, seed (a competitor who is given a preliminary ranking), handedness, height, age, ranking in the ATP.

3.1 Assembling the Data

The ATP World Tour comprises ATP World Tour Masters 1000, ATP World Tour 500 series, ATP World Tour 250 series and ATP Challenger Tour. Table 2 shows the amount of money won and the ranking points of winners.

We had different online sites from which we collected data for tennis matches. We collected all the tennis matches of the ATP and WTA World Tour from 1968 to Aug.25 2015. We found that there was no record of performances metrics of ATP until 1991 and of WTA until 2003 and thus we used the 82987 matches from Dec.31, 1990 for ATP and 35886 matches from Jan.6, 2003 for WTA. The metrics for each player in one match constitute a record and thus we have 165,974 records for ATP and 71772 records for WTA. We were able to find 44 metrics that were tracked for every ATP tennis match for the two players in each match. In addition, 11 metrics for each player are not listed in Table 3 as they are compound metrics obtained by combining or calculating basic metrics that is 22 metrics in each match. Besides, there are 5 metrics are meaningless metrics in each match. Then, there are total 71 metrics in each match. And the total is 5,892,077 data for ATP tennis matches and 2,547,906 data for WTA.

As the data are collected from different sites, it sometimes overlaps or has the same data presented in different forms. We had to filter and massage it to make it useable. The five meaningless metrics for result preduction are *draw_size* (capacity of the match), *match_num* (number of the match inside tournament), *winner_entry* (the entry method of the winner), *loser_entry* (the entry method if the loser), and *best_of* (either best of five or best of three).

We then noted that some of the important metrics were not included in the collected metrics, so we add some metrics calculated from the collected data (11 for each player). We ended up with 44 factors in 3 parts shown in Table 3 in which the first part consists of eight basic information factors of the matches (the tourney name, type, level, data and so on), the second part of nine descriptive parameters for each player in the match (the player's name, handedness, height, rank and so on), the third part of nine basic performances metrics for each player (number of aces, number of double service fouls, number of service points and so on). The fourth part of 11 performance metrics for each player that are compound metrics ob-

Table 3 The 44 Situational and Performance Data Factors

	Indicator	Explanation	Indicator	Explanation
Basic Information of the match (8 Indicators)	**T-ID**	Tourney ID	**Date**	Tourney date
	T-Name	Tourney name	**Score**	Score of the match
	Surface	Surface type	**Round**	Round
	Level	Tourney level	**Minutes**	Minutes
	WINNER		**LOSER**	
Descriptive parameters of the players (9 parameters for each player)	**W-ID**	Winner ID	**L-ID**	Loser ID
	W-Seed	Winner seed	**L-Seed**	Loser Seed
	W-Name	Winner name	**L-Name**	Loser name
	W-Hand	Winner handedness	**L-Hand**	Loser handedness
	W-HT	Winner height	**L-HT**	Loser height
	W-ioc	Winner country	**L-ioc**	Loser country
	W-Age	Winner age	**L-Age**	Loser age
	W-Rank	Winner rank	**L-Rank**	Loser rank
	W-Points	Winner rank points	**L-Points**	Loser rank points
Performances metrics of the players (9 basic variables for each player)	**W-Ace**	Number of aces won for winner	**L-Ace**	Number of aces won for loser
	W-DF	Number of double service fouls for winner	**L-DF**	Number of double service fouls for loser
	W-svpt	Number of service points for winner	**L-svpt**	Number of service points for loser
	W-1stIn	Number of successful first serve for winner	**L-1stIn**	Number of successful first serve for loser
	W-1stWon	Points won by first serve for winner	**L-1stWon**	points won by first serve for loser
	W-2ndWon	Points won by second serve for winner	**L-2ndWon**	points won by second serve for loser
	W-SvGms	Number of services games for winner	**L-SvGms**	Number of service games for loser
	W-bpSaved	Number of break points saved for winner	**L-bpSaved**	Number of break points saved for loser
	W-bpFaced	Number of break points faced for winner	**L-bpFaced**	Number of break points faced for loser

tained by combining some of 44 basic metrics (percentage of successful first serves, percentage of the second serve points won, percentage of first serve return points won and so on). Below Table 3 we give definitions of these 22 factors. So we have 66 metrics totally (20 performance metrics for each player, 9 descriptive parameters of each player, 8 indicators information of the match).

Some performances metrics (11 for each player) are not listed in Table 3 as they are compound metrics obtained by combining or calculating basic metrics (for the loser the same as for the winner)

- *winner_1stServe%* means the percentage of successful first serves for the winner, and is calculated from the winner_1stServe% =winner_1stIn/winner svpt;
- *winner_2ndIn* means the number of successful second serves for the winner, and it is calculated from the winner_2ndIn=winner_svpt-winner_1stIn;
- *winner_1st serve points won%* means the percentage of first serve points won by the winner, and is calculated from the winner_1st serve points won% =winner_1stwon/winner_1stIn;
- *winner_2nd serve points won%* means

Table 4 Results from the Wilcoxon Rank-sum Test

Factors	Direction	H	Z val	Rank sum	p
ace	+	TRUE	73.6	6.03E+09	0
df	−	TRUE	−63.53	4.92E+09	0
svpt	+	TRUE	−20.9	5.26E+09	5.21E-97
1stIn	+	TRUE	−4.18	5.39E+09	2.89E-05
1stServe%	+	TRUE	−40.01	5.10E+09	0
2ndIn	+	TRUE	31.63	7.23E+09	1.28E-219
1stWon	+	TRUE	62.74	5.94E+09	0
2ndWon	+	TRUE	53.54	5.87E+09	0
1st serve points won%	+	TRUE	−53.54	4.99E+09	0
2nd serve points won%	+	TRUE	−199.84	3.79E+09	0
1st serve return points won	+	TRUE	−188.83	3.88E+09	0
2nd serve return points won	+	TRUE	−106.04	5.88E+09	0
1st serve return points won%	+	TRUE	−95.86	5.98E+09	0
2nd serve return points won%	+	TRUE	−199.84	3.80E+09	0
SvGms	+	TRUE	11.71	5.52E+09	1.18E-31
bpSaved	+	TRUE	−82.88	4.76E+09	0
bpSaved%	+	TRUE	128.15	5.74E+09	0
bpFaced	−	TRUE	−166.96	4.07E+09	0
Break points won	+	TRUE	204.19	8.88E+09	0
Break points converted%	+	TRUE	166.95	6.79E+09	0

the percentage of second serve points won by the winner, and is calculated from the winner_2nd serve points won%=winner_2ndwon/winner_2ndIn;

- *winner_1st serve return points won* means the number of first serve return points won by the winner, and is calculated from the winner_1st serve return points won =loser_1stIn-loser_1stwon;
- *winner_2nd serve return points won* means the number of second serve return points won by the winner, and is calculated from the winner_2nd serve return points won =loser_2ndIn-loser_2ndwon;
- *winner_1st serve return points won%* means the percentage of first serve return points won by the winner, and is calculated from the winner_1st serve return points won%= w_1st serve return points won/ loser_1stIn;
- *winner_2nd serve return points won%* means the percentage of the second serve return points won by the winner, and is calculated from the winner_2nd serve return points won%= winner_2nd serve return points won/ loser_2ndIn;
- *winner_break points won%* means the percentage of break points won by the winner, and is calculated from the winner_break points won%=winner_ bpSaved/ winner_bpFaced;
- *winner_break points won* means the number of break points won by the winner, and is calculated from the winner_break points won=loser_bpfaced-loser_bpsaved;
- *winner_break points converted%* means the percentage of break points converted for the winner, and is calculated from the winner_break points converted% =winner_break Points won/loser_bpfaced.

3.2 Analyses of the Impact of Performance Metrics

We need to analyze the impact of the 20 performance metrics for each player on the outcome of the matches, which factor will influence the outcome of the match and which does not. So we can reduce the number of these factors. We conduct a statistical approach, the Wilcoxon rank-sum test (see Appendix), which is a nonparametric test of the null hypothesis that two samples come from the same population against an alternative hypothesis, to analyze the 20 performance metrics that were found by the test to be important to the outcome of the matches. For each performance metric data, we divide all the records into two groups: won and lost, then we use the rank-sum test to compare each performance metrics from all games won to those from all games lost. This is to check if a factor (i.e. number of aces) helps win the game. We reject the null hypothesis ($Z = -73.60$, $p < 0.001$), indicating that the outcomes of the games are not from the same population. Namely, the ace significantly impacts the outcome.

A rank-sum test was done to analyze the impact of these performance metrics on the outcome of the matches. All the 20 factors that were found to be key in affecting a match are shown in Table 4 as TRUE. Direction "+" means the indicator is positive (more is better, e.g., aces), direction "–" means the indicator is negative (less is better, e.g., df). It turned out that all 20 factors are important and so we used all of them in prediction.

3.3 Correlation Coefficients for Each of the Key Factors

We then used a statistical analysis method for the 165,974 record matches to find the regression coefficient (see Appendix) as the correlation coefficients for the key factors with respect to winning. The rank of these factors is obtained from the regression coefficient; ranking means which factor is more important in winning a match (Table 5).

Table 5 Sequence of Factors by Regression Coefficient

Rank	Factor	Coefficient (Absolute values)
1	1st serve return points won%	0.5668
2	2nd serve return points won%	0.2964
3	1stServe%	0.2594
4	1stIn	0.2594
5	Break points won	0.1236
6	2nd serve points won%	0.1109
7	bpSaved%	0.0746
8	bpFaced	0.0694
9	bpSaved	0.0666
10	2ndWon	0.0302
11	SvGms	0.0267
12	Break points converted%	0.0267
13	1st serve points won%	0.0267
14	1stWon	0.0254
15	svpt	0.0142
16	1st serve return points won	0.0028
17	df	0.001
18	ace	0.0008
19	2ndIn	0.0008
20	2nd serve return points won	0.0002

We employ the logistic model in order to understand the significance of each factor's contribution in winning the game. Since the game results are either lose or win (represented as "0"or "1"), the coefficient between game results and factors cannot be achieved with correlation analysis nor with simple linear regression. As a result, the logistic regression model was used, which has the form,

$$P(Y = 1|X = x) = \frac{e^{ax+b}}{1 + e^{ax+b}}$$

Where a denotes the parameter vector, b is constant, X is the factor vector,x is the factor values,$Y = 1$ represents winning the game and $P(Y = 1|X = x)$ denotes probability of winning the game when the factor value is x. All covariates are scaled to have the same variances.

We used MATLAB(https://www.mathworks.com/products/matlab.html) to run the logistic regression and obtained the coefficients of all the 20 factors as shown in

Table 5. For negative coefficients (ace, svpt, 2nd serve points won%, 2nd serve return points won and bpFaced), the coefficients' absolute values are displayed.

We acknowledge that multicollinearity may occur in the logistic regression. However, in our ANP model, the regression model serves to detect the association between the win/loss and each individual factor. The coefficients of the regression provide references for experts to conduct the pairwise comparison. The ANP users can thus employ a more educated judgment (anchored by these statistical results) to make use of the historical data, and determine the relative importance of each pair of factors.

3.4 Grouping the Factors for an ANP Structure

We structured an Analytic Network Process (ANP) model (see later) by first grouping the key performance factors into similar categories. Then we added some factors that are intangible and so variable that they were not candidates for data collection that we would evaluate by making judgments. The data factors from Section 3.1 are grouped into clusters in the ANP, as shown in Table 6. They form the first four clusters in Table 6 as data type. Note that, in the first four clusters of the ANP model, we reduce it from 20 factors to 12 factors, since we can attain compound metrics by combining (through calculation) some metrics. The compound metrics include: 1stServe%=1stIn/svpt, 1st serve points won%=1stwon/1stIn, 2nd serve points won%=2ndwon/2nd In, Break points won=bpfaced-bpsaved and Break points converted%=Break Points won/bpfaced. In this way, 8 factors have been reduced and only 12 factors are included in the ANP model.

Then we collected other factors that are not data type and grouped them into the other three clusters in Table 6. There are three different kinds of clusters: (1) Serve, Serve Return and Positive and Negative metrics are obtained by simply counting clearly indisputable data; (2) Condition and Physical is comprised of fact situational indicators; (3) Strength is subjective and are the factors that require human judgment, such as tactics, state and performance, psychology, brainpower and experience. There are 7 clusters in all containing 24 indicators as shown in Table 6. These factors yielded high accuracy in predicting outcomes of encounters in tennis matches.

Serving involves several factors that are effective in a match. Serve return is the set of factors that are effective for returning the ball. Positive is the set of factors that are effective for

Table 6 Clusters for Prediction (24 Factors)

Clusters	Factors	Type
Serve	Ace (+)	Data
	1st Serve %(+)	Data
	1st Serve Points Won %(+)	Data
	2nd Serve Points Won %(+)	Data
Serve Return	1st Serve Return Points Won %(+)	Data
	2nd Serve Return Points Won %(+)	Data
Positive	Break Points Saved(+)	Data
	Break Points Saved %(+)	Data
	Break Points Won(+)	Data
	Break Points Converted %(+)	Data
Negative	Break Points Faced(−)	Data
	Double Service Fouls(−)	Data
Condition	Surface type	*Fact*
	Weather	*Fact*
	Home/Road	*Fact*
	Referee	*Fact*
Physical	Height	*Fact*
	Age	*Fact*
	Stamina	*Judgment*
Strength	Tactics	*Judgment*
	State and Performance	*Judgment*
	Psychology	*Judgment*
	Brainpower	*Judgment*
	Experience	*Judgment*

winning the match. Negative, like mistakes in the match, is the set of factors that are effective in losing the match. These factors are smaller in number than the positive factors. These four clusters of factors are all the data factors. We use them as technical indicators to describe and estimate a player's performance.

Condition is the set of factors that describe the basic information about the match and physical is the set of factors that describe the basic information of the two players. These two clusters of factors are known as fact factors that we use as indicators to describe a match's situation or, and some of them will influence a player's performance. Home/Road indicates whether the game for the player is home game or road game. Home game is a sports game where the specified team or player is the host in the place and venue or have more support than opponent. Relatively, a road game is a sports game where the specified team or player is not the host and must travel to another venue.

Strength is the set of factors that reflect the strength of the players. It includes the tactics, state and performance, psychology, brainpower and experience. These factors are input judgments of an expert. We use them as indicators to describe indirectly and estimate a player's performance.

4. Methodology

According to factor analysis, the seven groups of factors (see in Table 6) are influencing the outcome of a tennis match. They include tangibles and intangibles and some of them are interdependent. In order to investigate the effect of all factors on the outcome of a tennis game and predict the outcome of games, the ANP model has been applied to form a methodology for predicting the match results. Each group of factors is considered as a cluster and factors inside groups are components of the network structure. In this way, the prediction of a tennis game is structured, then using a system of pairwise comparisons we can measure the priorities of the factors influencing the game results and finally to rank the alternatives or predict the winners. In this section, we introduce the formation of the methodology in detail.

Saaty (1980) developed the Analytic Hierarchy Process (AHP), a new scientific decision method based on hierarchical structures and making judgments. Saaty (1996) extended it to the Analytic Network Process (ANP), which involves network structures with dependence and feedback. In the ANP, networks of clusters of elements are used instead of the hierarchic levels of elements of the AHP. An ANP model can offer a solution to complex multi-criteria problems that have little objective supporting data (Saaty, 2005), thus it is very appropriate for analyzing a complex system like predicting sports game outcomes.

To predict the winner of a match before the match, an expert answers questions about the relative importance of the factors and about the relative performance of the players based on historical data. Using judgments, we can calculate the priorities of elements influencing alternative outcomes with respect to different control criteria and predict who will win in each match. Thus, the ANP can be used as a prediction tool for tennis matches.

4.1 Formulation of the ANP Decision-making Network

The decision-making network of criteria and alternative outcomes was constructed as shown in Figure 1. The object is to predict which player will win in the next tennis match by incorporating expert judgment with historical data about the two players. There are seven clusters of key factors shown in Table 6 that were chosen using statistical methods explained before. The factors are organized into clusters including serve, serve return, positive, negative, condition, physical and strength such as tactics, state and performance, psychology, brainpower, experience. In the net-

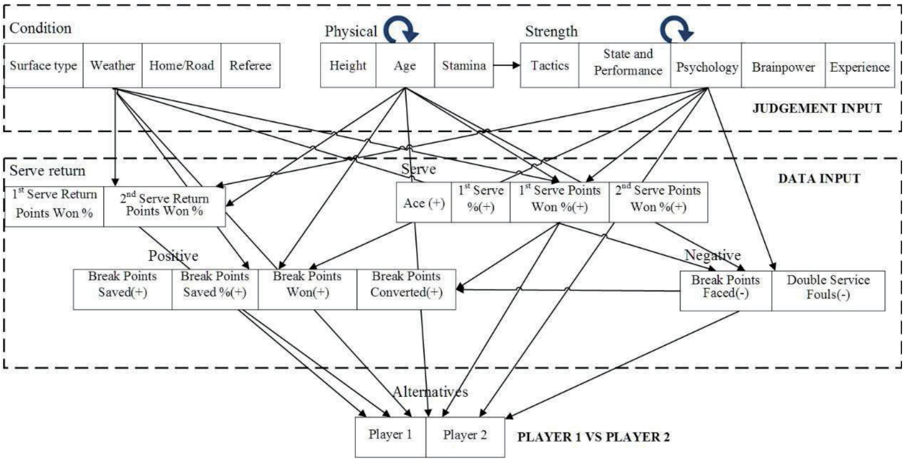

Figure 1 The ANP Network for Predicting Tennis Matches

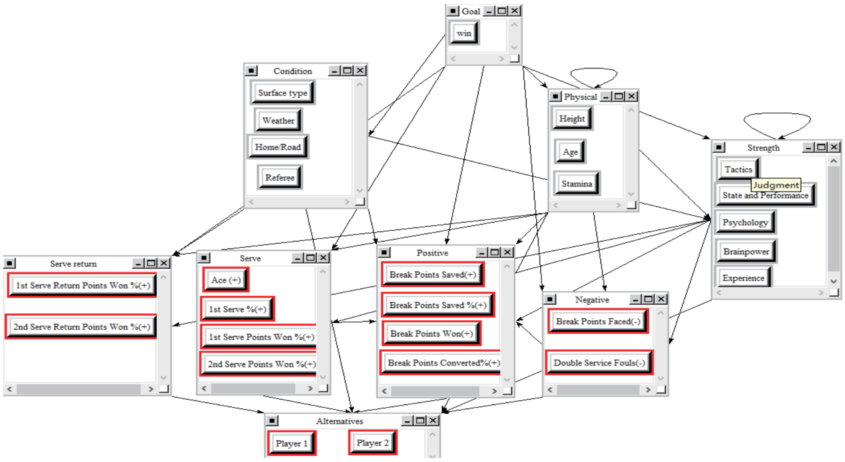

Figure 2 Factors Linked from and to Tactics

work, factors in clusters are connected. For example, both ace and double service fouls influence the break points saved. We also include the influence of surface type as it influences many kinds of technical performance including serve, serve return, positive and negative. As we stated in the previous section, stamina relies on age and tactics relies on age and stamina. Physical and strength indicators affect technical indicators whereby physical is tangible and strength indicators are intangibles. What is more, state and performance, brainpower and experience affect the tactics of players. State and performance influence the experience of players. These are feedback relations in the strength cluster.

The factor stamina is linked to the factors in five clusters: Serve return, Serve, Positive, Negative, and strength clusters in Figure 2. All these factors must be pairwise compared with respect to the factor stamina. In this particular set of comparisons, the judgments were made by experts about the relative importance of the factors. The experts refer to the data and analysis results (e.g. Table 5), and interpret them quantitatively using the AHP/ANP fundamental scale as shown in Table 7.

They made judgment by choosing a number from the fundamental scale to represent the intensity of the relationship between the factors with respect to the criteria. Pairwise comparisons result in a judgment matrix, from which the vector of priorities is derived as the principal eigenvector of the judgment matrix.

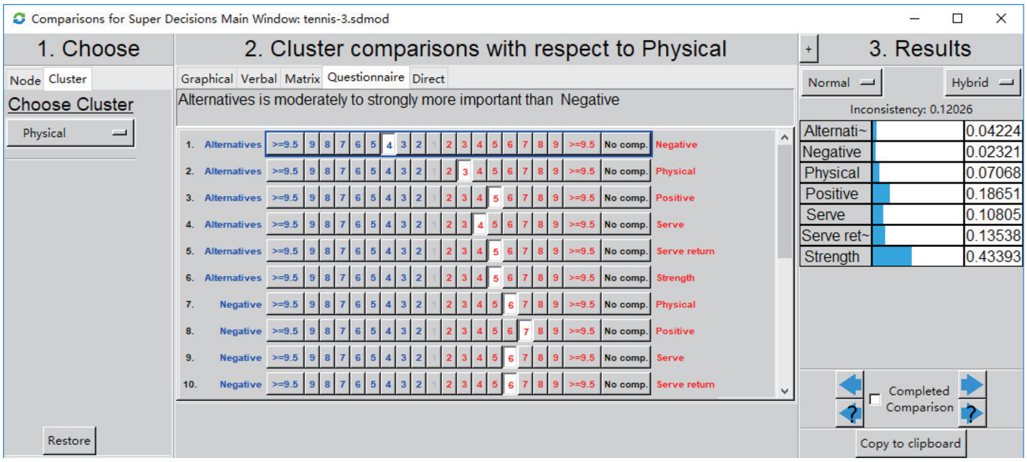

Figure 3 Entering the Judgments Based on Expert Judgment and Data

Table 7 Fundamental Scale (Saaty, 1982)

1	Equal importance
3	Moderate importance of one over another
5	Strong or essential importance
7	Very strong or demonstrated importance
9	Extreme importance
2,4,6,8	Intermediate values
	Use reciprocals for inverse comparisons

The data about the factors was transformed into correlation coefficients derived from the aggregate performance of all the players in the tennis match from 1991 to Aug, 2015. This is explained in the following section.

4.2 The Quantitative Judgment Process for Both Data and for Intangibles

To collect judgments from tennis experts about the matches, the players and so on, based on their knowledge and experience, we used a questionnaire format that the experts filled out. The experts interpreted these numbers and combined them with their general knowledge of the match to make judgments on the importance of the factors with respect to winning (May, 2013, Saaty & Shang ,2011). For these judgments, we used expert judgment based on knowledge combined with information from the correlation coefficient for the factors from the factor analysis (see Section 3.2). For example, 2nd serve return points won% with value 0.296 was considered moderately more important than 1st Serve In with value 0.259. It is more important for a player to have a high percentage of second serve return points won than to get first serves in. Figure 3 shows the merged expert judgments with data.

4.3 Formation of the Supermatrix

A network involves a grouping of elements (scenarios, environmental factors, actors, objectives, actions) into clusters that are not organized in any particular way. The network of the ANP model can deal with control structures. What is a control hierarchy It is a hierarchy with criteria, called control criteria that serve as a basis for making pairwise comparisons about influence. Analysis of priorities in a system can be thought of in terms of a control hierarchy with dependence among its bottom-level subsystem arranged as a network. Assume there are $p_1, \ldots_c, \ldots_n (c = 1, 2 \ldots)$ criteria in the control level of an ANP model. We have a system of clusters (or components) as a network layer, whereby the elements in each component interact, or have an impact on other elements, or on themselves with respect

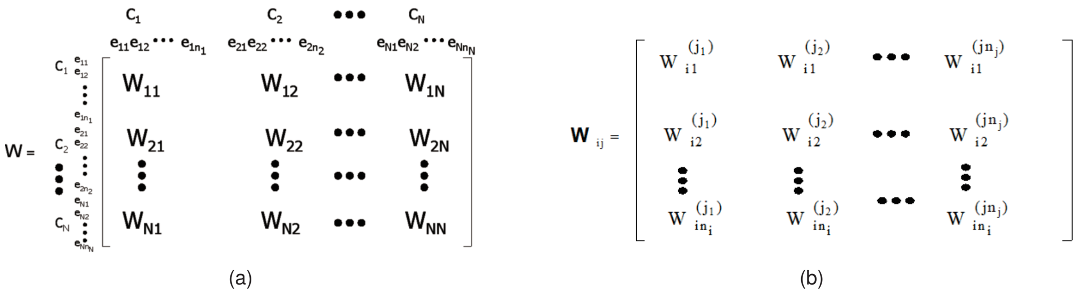

Figure 4 The Supermatrix of A Network

to a property governing the interactions of the components.Assume that component, denoted by C_h,$h = 1, ..., N$, has n_h elements that we denote by $e_{k1}, e_{k2}, ..., e_{kn}$. A priority vector is derived from paired comparisons by calculating eigenvalues and eigenvectors of judgment matrix. It represents the impact of a given set of elements in a component on another element in the system. When an element has no influence on another element, its influence priority is zero because there is no link to it (Tjader et al., 2014).

The priority vectors derived from pairwise comparison matrices are each entered as a part of some column of a supermatrix. The supermatrix represents the influence priority of an element on the left of the matrix on an element at the top of the matrix. A supermatrix along with an example of one of its general entryi, j block are shown in Figure 4. The component C_i alongside the supermatrix includes all the priority vectors derived for nodes that are "parent" nodes in the C_i cluster.

As an example of the pairwise comparison here, tactics is one of the key factors that can influence the serve, serve return, the negative and positive aspects of players and matches. Choose the serve return cluster, with respect to tactics, to make pairwise comparisons of the indicators. The priority columns constructed unweighted supermatrix. In this case, in the supermatrix, the left are elements in the serve return cluster and the top are elements is tactics cluster. The priorities in the supermatrix indicate the relative significance of the left elements with respect to the top elements. In the same way, input judgment from all the experts were taken and combined by the geometric mean and the priority vector was derived. Consistency test results turned out to be all less than 0.1 in which case the comparison matrix is acceptable and the unweighted supermatrix in formed.

Under the criterionp_c, we made a comparison of the relative importance of the clusters $C_1, . . . , C_n$, with respect to p_c and used them to weight the corresponding blocks of the supermatrix. By using p_c in the comparison matrices of the relative importance of the clusters $C_1, . . . , C_n$ on each of the clusters, yields the weighting matrix $A = (a_{ij})$ of clusters. Then the weighted supermatrix becomes $\overline{W} = (\overline{W}_{ij})$ where $W_{ij} = a_i j W_i j, i = 1, . . . , N, j = 1, . . . , N$. The weighted supermatrix is column stochastic and thus its powers converge to limiting priorities as any column of the limit supermatrix, because in the limit all the columns are the sameThe limit supermatrix is derived from by raising the weighted supermatrix to powers until it stabilizes in that all columns are identical. The final priorities of the elements can be obtained from the limit supermatrix.

5. Example of Predicting the Outcome of a Tennis Match

We used judgments and data in an ANP model to predict the outcome of each of the US OPEN men and women tennis matches in 2015(127 matches in all). We used the same ANP structure for every match but customized it with

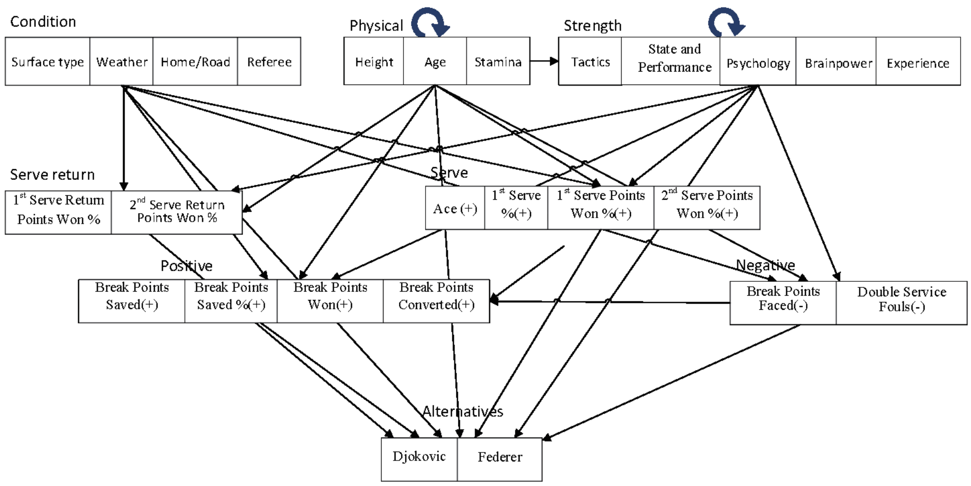

Figure 5 The ANP Network for Predicting Tennis Matches between Djokovic and Federer

data for the two players playing. The process is explained in the following section.

In the experiment, the alternatives are the world's top two tennis players who are about to face each other in the US open final match. Before this match, these two players played 41 matches from Apr17, 2006 at the Monte Carlo Masters, Federer won 21 (51.2%) matches and Djokovic won 20 (48.8%) matches, and their ranks are very close (Djokovic:1, Federer:3), so it was hard to predict who will be the winner.

5.1 Judgment Process for the Example

The decision-making network of criteria and alternative outcomes was constructed as shown in Figure 5. The object is to predict which player will win the final match in the US OPEN 2015 by incorporating expert judgment with historical data about the two players.

Djokovic and Federer, who is better in physical and strength aspects? "Height", "Age", "Stamina", "Tactics", "State and Performance","Psychology", "Brainpower" and "Experience" (tangible and intangible) factors are linked to the two players. Thus, the data factors have one set of priorities resulting from comparisons for their importance with respect to the goal, and two more sets of priorities when they are compared with respect to Player A and Player B. In this feedback type of comparison, the question to ask is, for example, "Who is better at tactics and how strongly better?" This is a subjective judgment that takes into account the current state of the players. The indicators will lead to priorities of the influence of the players' performance for Djokovic and Federer, respectively. Such judgments are made throughout the network as required by the connections. The limiting supermatrix of the ANP combines all the priorities in the network and synthesizes them to give the outcome in the form of priorities for Djokovic and Federer. For example, the synthesis might be 0.52 for Djokovic and 0.48 for Federer. Thus, it is more likely that Djokovic should win, which then also would be the prediction match outcome.

All of these factors must be pairwise compared with respect to the factor stamina for importance. In this particular set of comparisons, the judgments were made by experts who arrived at their consensus judgment by examining data about the importance of these factors and interpreting it in the form of an AHP/ANP fundamental scale judgment (numbers from 1 to 9). The data about the factors was transformed into correlation coefficients derived from the aggregate performance of all the players in the tennis matches from 1991 to Aug,

Table 8 Example of Pairwise Comparing Positive Cluster Elements for Importance with Respect to Tactics

	Break points converted%	Break points saved%	Break points saved	Break points won	Priorities
Break points converted%	1	1/3	1/2	1/5	0.1346
Break points saved%	3	1	1	1/2	0.2946
Break points saved	2	1	1	1/2	0.1571
Break points won	5	2	2	1	0.4136

Inconsistency index =0.08062

Table 9 Cluster Matrix

	Alternatives	Condition	Negative	Physical	Positive	Serve	Serve return	Strength
Alternatives	0	0.058463	0.166667	0.042239	1	0.125	1	0.037093
Condition	0	0	0	0	0	0	0	0
Negative	0	0	0	0.023211	0	0	0	0.022558
Physical	0	0	0	0.070678	0	0	0	0
Positive	0	0.55309	0.833333	0.186507	0	0.875	0	0.111802
Serve	0	0	0	0.108047	0	0	0	0.127285
Serve return	0	0.136636	0	0.135383	0	0	0	0.172351
Strength	0	0.251811	0	0.433934	0	0	0	0.528912

2015. This is explained in the following section.

5.2 The Super Matrix of the Example

The comparisons are made by experts' judgments. Expert make paired comparisons based on knowledge/experience as well as information from correlation coefficients of historical data. An example of the pairwise comparisons regarding positive cluster elements is shown in Table 8. Following the ANP comparison process, experts assess the importance regarding the criteria by answering the question like "Between Break points converted% and Break points saved%, which is more important with respect to tactics, and how more important according to 1-9 scale in Table 7?" The number 1/3 means Break points saved% is moderately (3 times) more important than Break points converted%. The priorities of the criteria are calculated following the standard ANP procedure for the matrix. Thus, through pairwise comparisons we can derive the weighting matrix A for each of the clusters, which are shown in Table 9.

Judgments were entered this way for all the data factors with respect to the goal. This resulted in one set of priorities for the data factors. The next step in our ANP model was to link all the data factors to the alternative players (Djokovic and Federer) and enter judgments based on comparing their statistical data (see Table 10).

We provide a head to head data in Table 11 to help the expert make the decision. Table 11 has two parts of data. The first part is the basic information data describing the facts about the two players, and the second part is the average performances metrics data calculated for the 41 matches played before.

Figure 6 and Figure 7 show the merged expert judgments with data. Tennis commen-

Table 10 The Winning Rate of Players on Different Type of Surface

Surface type	Winning rate	
	Djokovic	Federer
Hard	0.842	0.795
Grass	0.886	0.824
Clay	0.824	0.802
Carpet	0.767	0.711

Table 11 Head to Head Data between Novak Djokovic and Roger Federer in the 2015 US OPEN

Novak Djokovic		Roger Federer	Novak Djokovic		Roger Federer
1	**Seed**	2	28 (1987.05.22)	**Age**	34 (1981.08.08)
Right-Handed/Two-Handed	**Hand**	Right-Handed/One-Handed	1	**Rank**	3
6′2" (188 cm)	**Height**	6′1" (185 cm)	14865	**Points**	9065
5.375	**ace**	8.625	15.075	**1st srp won**	19.35
3.25	**df**	2.025	17.65	**2nd srp won**	16.275
96.925	**svpt**	92.65	0.263303	**1st srp won%**	0.313499
61.95	**1stIn**	56.625	0.485596	**2nd srp won%**	0.470458
0.639154	**1stServe**	0.611171	14.5	**SvGms**	14.575
34.975	**2ndIn**	36.025	4.95	**bpSaved**	4.175
42.6	**1stWon**	41.55	7.825	**bpFaced**	6.975
18.7	**2ndWon**	18.375	0.566959	**bpSaved%**	0.568676
0.686501	**1st sp won%**	0.736697	2.8	**bpwon**	2.875
0.529542	**2nd sp won%**	0.514404	0.381324	**Bp converted%**	0.433041

Table 12 Limit Matrix

	Player1	Player2	Home/Road	Reference	Surfacetype	Weather	Bpf(-)	DF(-)	Age	Height	Stamina	Bpc%(+)	BpSaved%	BpSaved(+)	bpWon(+)	1stServe%(+)	1stspWon%	2ndspW%	Ace(+)	1stspWon%	2ndspW%	BrainPower	ExperIence	Psychology	Performance	Tactics
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	0	0	0.17	0.18	0.2	0.21	0.33	0.27	0.18	0.19	0.2	0.25	0.5	0.67	0.33	0.83	0.25	0.83	0.26	0.25	0.83	0.21	0.21	0.22	0.27	0.24
2	0	0	0.16	0.15	0.26	0.16	0.67	0.27	0.14	0.18	0.21	0.75	0.5	0.33	0.67	0.17	0.75	0.17	0.28	0.75	0.17	0.19	0.19	0.17	0.23	0.27
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0.01	0.01	0	0	0	0	0.01	0.01	0.01	0	0	0	0	0	0	0	0	0	0	0.01	0.01	0.01	0.01	0.02
8	0	0	0.01	0.01	0	0.01	0	0	0.01	0.01	0.01	0	0	0	0	0	0	0	0	0	0	0.01	0.01	0.01	0.02	0.01
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0.04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0.02	0.02	0.12	0.02	0	0.04	0.02	0.02	0.04	0	0	0	0	0	0	0	0.06	0	0	0.04	0.04	0.02	0.03	0.05
13	0	0	0.04	0.03	0.05	0.04	0	0.12	0.03	0.04	0.04	0	0	0	0	0	0	0	0.22	0	0	0.03	0.03	0.05	0.07	0.03
14	0	0	0.02	0.02	0.04	0.03	0	0.22	0.02	0.02	0.03	0	0	0	0	0	0	0	0.15	0	0	0.03	0.02	0.03	0.04	0.02
15	0	0	0.01	0.01	0.08	0.01	0	0.07	0.02	0.02	0.05	0	0	0	0	0	0	0	0.04	0	0	0.02	0.03	0.02	0.03	0.04
16	0	0	0.02	0.02	0.01	0.02	0	0	0.02	0.03	0.02	0	0	0	0	0	0	0	0	0	0	0.03	0.03	0.02	0.04	0.06
17	0	0	0.01	0.01	0.01	0.01	0	0	0.01	0.01	0.03	0	0	0	0	0	0	0	0	0	0	0.03	0.03	0.02	0.01	0.04
18	0	0	0.02	0.03	0.01	0.02	0	0	0.02	0.02	0.02	0	0	0	0	0	0	0	0	0	0	0.03	0.03	0.04	0.04	0.02
19	0	0	0.02	0.02	0.01	0.02	0	0	0.02	0.02	0.03	0	0	0	0	0	0	0	0	0	0	0.02	0.02	0.03	0.04	0.02
20	0	0	0.06	0.05	0.07	0.07	0	0	0.07	0.08	0.09	0	0	0	0	0	0	0	0	0	0	0.07	0.07	0.08	0.12	0.14
21	0	0	0.04	0.05	0.03	0.04	0	0	0.04	0.04	0.04	0	0	0	0	0	0	0	0	0	0	0.07	0.07	0.07	0.06	0.05
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0.21	0.27	0	0.07	0	0	0.08	0.04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0.15	0.11	0.03	0.25	0	0	0.17	0.21	0.14	0	0	0	0	0	0	0	0	0	0	0.04	0.05	0	0	0
26	0	0	0.03	0.04	0.09	0.01	0	0	0.05	0.09	0.05	0	0	0	0	0	0	0	0	0	0	0.18	0.16	0	0	0

tators and pundits were saying Djokovic was in good shape recently. Information like this from several sources was considered and used to form the judgment entered.

The judgment is made to evaluate the relative ability of pairs of players on the key factors. The experts' judgments relied on data of past performance. Let us take for our example the match between Novak Djokovic and Roger Federer to demonstrate the judgment process. First there are some factors that are certain such as surface type, age, height for which experts can form judgments deterministically. For other performance factors experts can do the judgment according to historical performance data (shown in the Table 11), which is helpful for the experts to determine through this factor which player is bet-

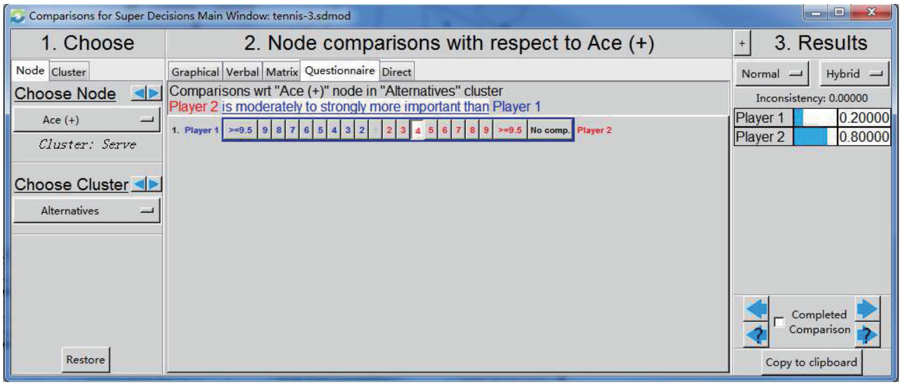

Figure 6 The ANP Network for Predicting Tennis Matches between Djokovic and Federer

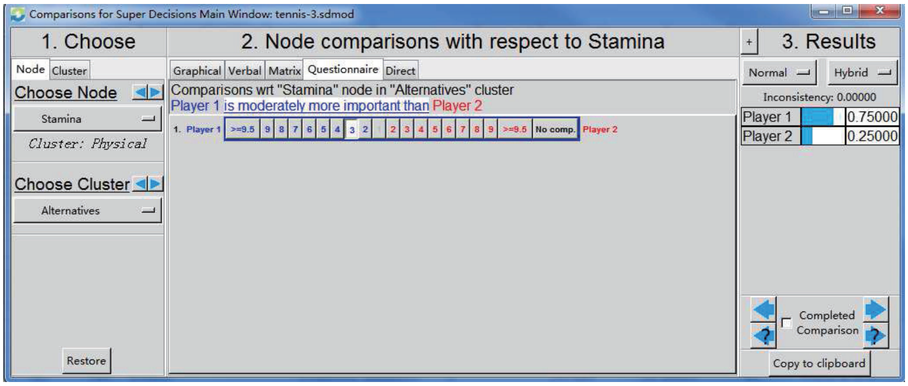

Figure 7 The ANP network for Predicting Tennis Matches between Djokovic and Federer

ter. In positive, Federer (43%) has moderate advantage over Djokovic (38%) with respect to Break points converted%. While Djokovic (4.95) has equal to moderate advantage over Federer (4.175) when considering break points saved. Therefore, we have the judgment that Djokovic is equally to moderately more dominant with a value of 2 than Federer with respect to break points saved; Federer is moderately more dominant with a value of 3 than Djokovic with respect to Break points converted%.

There are no data to help the experts to make judgment about the other factors, such as psychology, brainpower and experience, so they must use judgment according to their best understanding of the match, of the players, and of the current situation. As for the experience of the players, Federer is moderately more experienced than Djokovic though Federer is older than Djokovic and played more matches than Djokovic, and Djokovic has moderately more stamina than Federer.

The priorities in this paper were calculated by the Super Decisions (SD) Software (get from www.superdecisions.com). The supermatrix is obtained through pairwise comparisons of the importance of the impact of each cluster. The weighted supermatrix is derived from the unweighted supermatrix by multiplying the priority weights of the clusters. Every column of the weighted supermatrix adds to one. Finally, the limit supermatrix (see Table 12) will be obtained by multiplying the weighted supermatrix by itself until it does not change any more, and we obtain the priority of each element. All the calculation process can be done by Super Decision (https://www.superdecisions.com/) in seconds of time. See Saaty (2013) for detail definition and calculation.

Table 13 Priority Results of the Example of the Match between Djokovic and Federer

Name	Normalized By Cluster	Limiting
Player 1 (Djokovic)	0.52	0.179
Player 2 (Federer)	0.48	0.165
Home/Road	0.25	0.003
Referee	0.25	0.003
Surface type	0.25	0.003
Weather	0.25	0.003
Break Points Faced(-)	0.481	0.008
Double Service Fouls(-)	0.519	0.009
Age	0.32	0.016
Height	0.32	0.016
Stamina	0.361	0.018
Break Points Converted %(+)	0.243	0.031
Break Points Saved %(+)	0.297	0.038
Break Points Saved(+)	0.25	0.032
Break Points Won(+)	0.21	0.027
1st Serve %(+)	0.271	0.027
1st Serve Points Won %(+)	0.231	0.023
2nd Serve Points Won %(+)	0.264	0.026
Ace (+)	0.234	0.023
1st Serve Return Points Won %(+)	0.57	0.062
2nd Serve Return Points Won %(+)	0.43	0.047
Brainpower	0.122	0.03
Experience	0.129	0.032
Psychology	0.162	0.04
State and Performance	0.308	0.076
Tactics	0.279	0.069

5.3 Predicting the Result of the Example

In the experiment, the alternatives are the world's top two tennis players who are about to face each other in the US open final match. The object is to win the match. Table 13 shows priorities of each element derived in the ANP model. The limiting priority indicates the priority of each element in the complete network and they sum up to one. The normalized propriety by cluster shows the weight of each element in each cluster and each of them in one cluster sum up to one. The result shows that Djokovic will win over Federer with 52% advantage. After the actual match, we collected information about the match. The actual match information between Djokovic and Federer is shown in Table 14. It showed that Djokovic won over Federer with 3:1 and the actual performance statistics are shown in Table 14. The priority results are shown in Table 13. We predicted that finally Djokovic will win Federer, though we cannot predict the score and performance yet, but we can predict the winner correctly.

Table 14 Actual Match Information

winner	**Novak Djokovic**	
score	(3:1) 6-4; 5-7; 6-4; 6-4	
Novak Djokovic		**Roger Federer**
3	ace	11
5	df	5
155	svpt	137
96	1stIn	87
0.619355	1stServe	0.635036
59	2ndIn	50
63	1stWon	62
32	2ndWon	23
0.65625	1st sp won%	0.712644
0.542373	2nd sp won%	0.46
25	1st srp won	33
27	2nd srp won	27
0.287356	1st srp won%	0.34375
0.54	2nd srp won%	0.457627
21	SvGms	21
19	bpSaved	7
23	bpFaced	13
0.826087	bpSaved%	0.538462
6	bpwon	4
0.461538	Bp converted%	0.173913

6. Results

With data analysis and our judgment model we predicted the outcomes of 63 men tennis matches after the first round and 31 women tennis matches after the second round of the 2015 US OPEN. The results of our prediction are shown in Tables 16 and 17. There were 63 men matches of which we predicted in advance the correct outcomes of 55 matches and also predicted 25 correct outcomes of 31 women tennis matches, giving an average accuracy rate of 84%(men:87.3%, women:80.6%). Compared with the classical prediction research on tennis, the result appeared to achieve a considerably

Table 15 Comparison of Prediction Accuracy of Sports Game

Paper Work	Sports	Method	Accuracy
McHale & Morton, 2011	Tennis (ATP)	Bradley-Terry model	66.90%
Scheibehenne & Bröder, 2007	Wimbledon 2005	Predictions based on recognition rankings aggregated over all participants	70%
Knottenbelt, Spanias, & Madurska, 2012	Tennis	Hierarchical Markov model	Not verified
Klaassen & Magnus, 2003	Tennis	TENNISPROB	Not verified
Easton & Uylangco, 2010	Tennis	Betting odds	Not verified
Our paper	***Tennis (ATP&WTA)***	***Data and judgment***	***85.10%***

higher accuracy than predicted by others.

For more comparisons of how our results stand compared to others, we refer to previous literature about prediction in sports (Table 15). Table 16 shows Prediction results for the men's and women's tennis matches of the US Open 2015 (bold values in the table indicates the wrong prediction item and the others are predicted correctly).

7. Conclusions

Compared with previous methods used in prediction in sports, there are some advantages of our method. It can be seen from the results of this paper that:

1) We are the first to do prediction in tennis matches using a combined method based on data and judgments. Prior to the study, little information was available and there were few inputs that commentators could use before a tennis match. They guessed which player is better and is more likely to win. That means we can do prediction not only by the data or machine learning method (historical standing between the two players) but also using judgments by experts.

2) The factors that we used in the prediction have been validated for their usefulness in producing an accurate outcome. Based on data analysis we analyzed the correlation coefficients between the factors and winning a match. This means that we found how much these factors can explain why a player wins the match and have a way to bring the importance of the factors into making judgments in the ANP.

3) When the experts make the judgments in the ANP model, they can use the data from the data analysis and base their judgments on them, as was shown in the way we used the correlation coefficients between the factors. The experts interpreted these numbers and combined them with their general knowledge of the match to make judgments as to the importance of the factors with respect to the Goal of winning a match. Further, when the experts make judgments to compare two players based on performance factors, on data analysis, matchup data of these factors can be provided between the two players. Thus it helps the experts to make judgments that are more numerically-based in order to make the prediction more accurate.

The method proposed in this research is a systematic approach which considers both tangible and intangible data. Armstrong et al. (2015) maintains that "The Golden Rule of forecasting is consistent with cumulative knowledge about the present and the past. To be conservative, forecasters must seek all knowledge relevant to the problem, and use methods that have been validated for the situation." Following his advice, we make use of expert knowledge and data analysis.

Table 16 Prediction Results for the Men's Tennis Matches of the US Open 2015

Date	P1seed	Player1 (winner)	P1Rank	P1R_ points	P2s eed	Player2 (loser)	P2rank	P2_points	Score	Prediction Result	round
9/2/2015	1	Novak Djokovic	1	14865		Andreas Haider Maurer	52	870	6-4 6-1 6-2	0.89:0.11	R64
9/2/2015	25	Andreas Seppi	25	1430		Teymuraz Gabashvili	53	867	3-6 6-3 7-6(3) 6-1	0.68:0.32	R64
9/2/2015	23	Roberto Bautista Agut	23	1510		Pablo Carreno Busta	54	855	4-6 6-4 6-0 2-6 6-4	0.66:0.34	R64
9/2/2015	14	David Goffin	15	2130		Ricardas Berankis	78	661	5-7 6-4 3-6 6-2 6-1	0.78:0.22	R64
9/2/2015	10	Milos Raonic	10	2880		Fernando Verdasco	42	1020	6-2 6-4 6-7(5) 7-6(1)	0.69:0.31	R64
9/2/2015	18	Feliciano Lopez	19	1665		Mardy Fish	581	55	2-6 6-3 1-6 7-5 6-3	0.88:0.12	R64
9/2/2015	32	Fabio Fognini	32	1165		Pablo Cuevas	40	1065	6-3 6-4 6-4	0.54:0.46	R64
9/2/2015	8	Rafael Nadal	8	3680		Diego Sebastian Schwartzman	74	685	7-6(5) 6-3 7-5	0.76:0.24	R64
9/2/2015		Benoit Paire	41	1052		Marsel Ilhan	84	618	6-3 3-6 6-4 6-3	0.72:0.28	R64
9/2/2015	26	Tommy Robredo	26	1405		Samuel Groth	55	843	6-4 7-6(3) 6-4	0.77:0.23	R64
9/2/2015	19	Jo Wilfried Tsonga	18	1675		Marcel Granollers	77	665	6-3 6-4 6-3	0.81:0.19	R64
9/2/2015		Sergiy Stakhovsky	60	804		Illya Marchenko	120	465	6-4 7-6(2) 4-6 6-4	0.75:0.25	R64
9/2/2015	9	Marin Cilic	9	3550		Evgeny Donskoy	139	402	6-2 6-3 7-5	0.88:0.12	R64
9/2/2015		***Mikhail Kukushkin***	***56***	***842***	***17***	***Grigor Dimitrov***	***17***	***1735***	***6-3 7-6(2) 2-6 4-6 6-4***	***0.42:0.58***	***R64***
9/2/2015	27	Jeremy Chardy	27	1300		Martin Klizan	36	1125	7-5 6-4 7-6(1)	0.56:0.44	R64

Date	P1seed	Player1 (winner)	P1Rank	P1R_ points	P2s eed	Player2 (loser)	P2rank	P2_points	Score	Prediction Result	round
9/3/2015	7	David Ferrer	7	3695		Filip Krajinovic	102	534	7-5 7-5 7-6(4)	0.85:0.15	R64
9/3/2015	5	Stanislas Wawrinka	5	5710		Hyeon Chung	69	721	7-6(2) 7-6(4) 7-6(6)	0.84:0.16	R64
9/3/2015		***Ruben Bemelmans***	***107***	***520***	***28***	***Jack Sock***	***28***	***1250***	***4-6 4-6 6-3 2-1 (RET)***	***0.35:0.65***	***R64***
9/3/2015	22	Viktor Troicki	22	1559		Rajeev Ram	88	570	7-6(10) 6-4 3-6 6-3	0.73:0.27	R64
9/3/2015		Donald Young	68	730		Aljaz Bedene	57	841	3-6 6-4 6-4 6-2	0.55:0.45	R64
9/3/2015	15	Kevin Anderson	14	2160		Austin Krajicek	117	474	6-3 6-4 6-2	0.86:0.14	R64
9/3/2015	20	Dominic Thiem	20	1645		Denis Istomin	70	716	6-4 6-4 1-0 (RET)	0.74:0.26	R64
9/3/2015	30	Thomaz Bellucci	30	1190		Yoshihito Nishioka	128	430	6-0 6-3 6-4	0.72:0.28	R64
9/3/2015	3	Andy Murray	3	8840		Adrian Mannarino	35	1140	5-7 4-6 6-1 6-3 6-1	0.78:0.22	R64
9/3/2015	6	Tomas Berdych	6	5230		Jurgen Melzer	132	426	7-6(2) 6-1 6-3	0.88:0.12	R64
9/3/2015	31	Guillermo Garcia Lopez	31	1190		Nicolas Mahut	64	767	6-4 6-2 6-7(4) 6-1	0.68:0.32	R64
9/3/2015	24	Bernard Tomic	24	1465		Lleyton Hewitt	355	130	6-3 6-2 3-6 5-7 7-5	0.87:0.13	R64
9/3/2015	12	Richard Gasquet	12	2240		Robin Haase	79	645	4-6 6-3 7-6(4) 6-4	0.72:0.28	R64
9/3/2015	13	John Isner	13	2235		Mikhail Youzhny	93	546	6-3 6-4 6-4	0.77:0.23	R64
9/3/2015		***Jiri Vesely***	***48***	***962***	***21***	***Ivo Karlovic***	***21***	***1620***	***7-6(3) 3-6 3-6 6-2 7-6(4)***	***0.46:0.54***	***R64***

Date	P1seed	Player1 (winner)	P1Rank	P1R_ points	P2seed	Player2 (loser)	P2rank	P2_points	Score	Prediction Result	round
9/3/2015	29	Philipp Kohlschreiber	29	1230		Lukas Rosol	87	582	7-6(4) 6-2 6-2	0.71:0.29	R64
9/4/2015	2	Roger Federer	2	9065		Steve Darcis	66	750	6-1 6-2 6-1	0.83:0.17	R64
9/4/2015	1	Novak Djokovic	1	14865	25	Andreas Seppi	25	1430	6-3 7-5 7-5	0.78:0.22	R32
9/4/2015	23	Roberto Bautista Agut	23	1510	14	David Goffin	15	2130	2-6 5-7 6-3 3-1 (RET)	0.53:0.47	R32
9/4/2015	18	Feliciano Lopez	19	1665	10	Milos Raonic	10	2880	6-2 7-6(4) 6-3	0.52:0.48	R32
9/4/2015	***32***	***Fabio Fognini***	***32***	***1165***	***8***	***Rafael Nadal***	***8***	***3680***	***3-6 4-6 6-4 6-3 6-4***	***0.39:0.61***	***R32***
9/4/2015		***Benoit Paire***	***41***	***1052***	***26***	***Tommy Robredo***	***26***	***1405***	***7-6(3) 6-1 6-1***	***0.45:0.55***	***R32***
9/4/2015	19	Jo Wilfried Tsonga	18	1675		Sergiy Stakhovsky	60	804	6-3 7-5 6-2	0.76:0.24	R32
9/5/2015	9	Marin Cilic	9	3550		Mikhail Kukushkin	56	842	6-7(5) 7-6(1) 6-3 6-7(3) 6-1	0.77:0.23	R32
9/5/2015	***27***	***Jeremy Chardy***	***27***	***1300***	***7***	***David Ferrer***	***7***	***3695***	***7-6(6) 4-6 6-3 6-1***	***0.43:0.57***	***R32***
9/5/2015	5	Stanislas Wawrinka	5	5710		Ruben Bemelmans	107	520	6-3 7-6(5) 6-4	0.86:0.14	R32
9/5/2015		***Donald Young***	***68***	***730***	***22***	***Viktor Troicki***	***22***	***1559***	***4-6 0-6 7-6(3) 6-2 6-4***	***0.39:0.61***	***R32***
9/5/2015	15	Kevin Anderson	14	2160	20	Dominic Thiem	20	1645	6-3 7-6(3) 7-6(3)	0.54:0.46	R32
9/5/2015	3	Andy Murray	3	8840	30	Thomaz Bellucci	30	1190	6-3 6-2 7-5	0.78:0.22	R32
9/5/2015	6	Tomas Berdych	6	5230	31	Guillermo Garcia Lopez	31	1190	6-7(2) 7-6(7) 6-3 6-3	0.78:0.22	R32
9/5/2015	12	Richard Gasquet	12	2240	24	Bernard Tomic	24	1465	6-4 6-3 6-1	0.58:0.42	R32

Date	P1seed	Player1 (winner)	P1Rank	P1R_ points	P2seed	Player2 (loser)	P2rank	P2_points	Score	Prediction Result	round
9/5/2015	13	John Isner	13	2235		Jiri Vesely	48	962	6-3 6-4 (RET)	0.72:0.28	R32
9/6/2015	2	Roger Federer	2	9065	29	Philipp Kohlschreiber	29	1230	6-3 6-4 6-4	0.73:0.27	R32
9/6/2015	1	Novak Djokovic	1	14865	23	Roberto Bautista Agut	23	1510	6-3 4-6 6-4 6-3	0.79:0.21	R16
9/7/2015	***15***	***Kevin Anderson***	***14***	***2160***	***3***	***Andy Murray***	***3***	***8840***	***7-6(5) 6-3 6-7(2) 7-6(0)***	***0.38:0.62***	***R16***
9/6/2015	18	Feliciano Lopez	19	1665	32	Fabio Fognini	32	1165	6-3 7-6(5) 6-1	0.58:0.42	R16
9/6/2015	19	Jo Wilfried Tsonga	18	1675		Benoit Paire	41	1052	6-4 6-3 6-4	0.64:0.36	R16
9/7/2015	9	Marin Cilic	9	3550	27	Jeremy Chardy	27	1300	6-3 2-6 7-6(2) 6-1	0.59:0.41	R16
9/7/2015	5	Stanislas Wawrinka	5	5710		Donald Young	68	730	6-4 1-6 6-3 6-4	0.63:0.37	R16
9/8/2015	12	Richard Gasquet	12	2240	6	Tomas Berdych	6	5230	2-6 6-3 6-4 6-1	0.52:0.48	R16
9/8/2015	2	Roger Federer	2	9065	13	John Isner	13	2235	7-6(0) 7-6(6) 7-5	0.61:0.39	R16
9/8/2015	1	Novak Djokovic	1	14865	18	Feliciano Lopez	19	1665	6-1 3-6 6-3 7-6(2)	0.68:0.32	QF
9/9/2015	9	Marin Cilic	9	3550	19	Jo Wilfried Tsonga	18	1675	6-4 6-4 3-6 6-7(3) 6-4	0.58:0.42	QF
9/9/2015	5	Stanislas Wawrinka	5	5710	15	Kevin Anderson	14	2160	6-4 6-4 6-0	0.62:0.38	QF
9/10/2015	2	Roger Federer	2	9065	12	Richard Gasquet	12	2240	6-3 6-3 6-1	0.65:0.35	QF
9/11/2015	1	Novak Djokovic	1	14865	9	Marin Cilic	9	3550	6-0 6-1 6-2	0.68:0.32	SF
9/12/2015	2	Roger Federer	2	9065	5	Stanislas Wawrinka	5	5710	6-4 6-3 6-1	0.58:0.42	SF
9/13/2015	1	Novak Djokovic	1	14865	2	Roger Federer	2	9065	6-4 5-7 6-4 6-4	0.52:0.48	F

Table 17 Prediction Results for the Women's Tennis Matches of the US Open 2015

Date	P1seed	Player1 (winner)	P1Rank	P1R_ points	P2seed	Player2 (loser)	P2rank	P2_ points	score	Prediction Result	Round
9/4/2015	1	Serena Williams	1	12721		Bethanie Mattek Sands	101	609	3-6 7-5 6-0	0.88:0.12	3rd Round
9/4/2015	19	Madison Keys	19	2275	15	Agnieszka Radwanska	15	2760	6-3 6-2	0.54:0.46	3rd Round
9/4/2015	23	Venus Williams	23	2072	12	Belinda Bencic	12	3035	6-3 6-4	0.52:0.48	3rd Round
9/4/2015		***Anett Kontaveit***	***152***	***348***		***Madison Brengle***	***47***	***1224***	***6-2 3-6 6-0***	***0.35:0.65***	***3rd Round***
9/4/2015		Kristina Mladenovic	40	1335		Darya Kasatkina	133	426	6-2 6-3	0.68:0.32	3rd Round
9/4/2015	13	Ekaterina Makarova	13	2920	17	Elina Svitolina	17	2530	6-3 7-5	0.56:0.44	3rd Round
9/5/2015		Roberta Vinci	43	1260		Mariana Duque Marino	96	640	6-1 5-7 6-2	0.71:0.29	3rd Round
9/5/2015	25	Eugenie Bouchard	25	1887		Dominika Cibulkova	50	1066	7-6(9) 4-6 6-3	0.64:0.36	3rd Round
9/5/2015	5	Petra Kvitova	4	4995	32	Anna Karolina Schmiedlova	32	1451	6-2 6-1	0.73:0.27	3rd Round
9/5/2015		***Johanna Konta***	***97***	***638***	***18***	***Andrea Petkovic***	***18***	***2450***	***7-6(2) 6-3***	***0.31:0.69***	***3rd Round***
9/5/2015	22	Samantha Stosur	22	2135	16	Sara Errani	16	2610	7-5 2-6 6-1	0.52:0.48	3rd Round
9/5/2015	26	Flavia Pennetta	26	1747		Petra Cetkovska	149	354	1-6 6-1 6-4	0.79:0.21	3rd Round
9/5/2015		Varvara Lepchenko	46	1234		Mona Barthel	53	1035	1-6 6-3 6-4	0.54:0.46	3rd Round
9/5/2015	20	Victoria Azarenka	20	2271	11	Angelique Kerber	11	3150	7-5 2-6 6-4	0.52:0.48	3rd Round
9/6/2015	24	Sabine Lisicki	24	1945		Barbora Zahlavova Strycova	42	1290	6-4 4-6 7-5	0.62:0.38	3rd Round
9/6/2015	2	Simona Halep	2	6130		Shelby Rogers	154	347	6-2 6-3	0.87:0.13	3rd Round
9/6/2015	1	Serena Williams	1	12721	19	Madison Keys	19	2275	6-3 6-3	0.68:0.32	4th Round
9/6/2015	23	Venus Williams	23	2072		Anett Kontaveit	152	348	6-2 6-1	0.71:0.29	4th Round
9/7/2015		Kristina Mladenovic	40	1335	13	Ekaterina Makarova	13	2920	7-6(2) 4-6 6-1	0.53:0.47	4th Round
9/7/2015		***Roberta Vinci***	***43***	***1260***	***25***	***Eugenie Bouchard***	***25***	***1887***	***W/O***	***0.46:0.54***	***4th Round***
9/7/2015	5	Petra Kvitova	4	4995		Johanna Konta	97	638	7-5 6-3	0.78:0.22	4th Round

Date	P1seed	Player1 (winner)	P1Rank	P1R_ points	P2seed	Player2 (loser)	P2rank	P2_ points	score	Prediction Result	Round
9/7/2015	26	Flavia Pennetta	26	1747	22	Samantha Stosur	22	2135	6-4 6-4	0.58:0.42	4th Round
9/7/2015	20	Victoria Azarenka	20	2271		Varvara Lep-chenko	46	1234	6-3 6-4	0.62:0.38	4th Round
9/8/2015	2	Simona Halep	2	6130	24	Sabine Lisicki	24	1945	6-7(6) 7-5 6-2	0.65:0.35	4th Round
9/8/2015	1	Serena Williams	1	12721	23	Venus Williams	23	2072	6-2 1-6 6-3	0.72:0.38	Quarter finals
9/9/2015		Roberta Vinci	43	1260		Kristina Mladen-ovic	40	1335	6-3 5-7 6-4	0.53:0.47	Quarter finals
9/9/2015	***26***	***Flavia Pennetta***	***26***	***1747***	***5***	***Petra Kvitova***	***4***	***4995***	***4-6 6-4 6-2***	***0.42:0.58***	***Quarterfinals***
9/9/2015	2	Simona Halep	2	6130	20	Victoria Azarenka	20	2271	6-3 4-6 6-4	0.66:0.34	Quarter finals
9/11/2015		***Roberta Vinci***	***43***	***1260***	***1***	***Serena Williams***	***1***	***12721***	***2-6 6-4 6-4***	***0.35:0.65***	***Semi finals***
9/11/2015	***26***	***Flavia Pennetta***	***26***	***1747***	***2***	***Simona Halep***	***2***	***6130***	***6-1 6-3***	***0.37:0.73***	***Semi finals***
9/12/2015	26	Flavia Pennetta	26	1747		Roberta Vinci	43	1260	7-6(4) 6-2	0.58:0.42	The Final

For tangible variables, we collected all the tennis matches of the ATP and WTA World Tour from Jan.19 1968 to Aug.25 2015. After data integrity checking, 165,974 records are used as past records and therefore 5,892,077 data for ATP tennis matches and 2,547,906 data for WTA are analyzed in data analysis process. We apply rank-sum test, a simple nonparametric test in statistics through which 20 performance metrics were shown to be significant for the prediction of a match outcome. Accordingly, these 20 validated factors are parts of the key indicators in the proposed model. For tangible variables, we seek commonsensical (e.g. height, age) and variables that have been investigated in previous literature by their role in winning a game (e.g. past performance, home/road game).

In this research, we use ANP to incorporate subjective judgment and intangible measures through the help of historical information. In ANP the sub-criteria under a cluster are supposed to be related, so they can be sensibly compared. This is very different from the regression perspective, where factors of high associations should not be incorporated in the same model. Depending on the data involved, multicollinearity may take place from time to time. This opens an interesting research topic that ANP is very comprehensive and all-inclusive, i.e. taking all relevant factors into consideration including their interactions, feedback, being closely related or remotely interconnected. On the other hand, conventional statistics maintain that one should always choose the parsimonious approach to reduce the variables necessary to make predictions, due to time and cost concerns in data collection and accuracy. In short, the ANP expects factors under the same cluster to have some correlations, while regression expects factors to have low associations/correlation in order to avoid multicollinearity

There are also limitations in our research that need to be improved in future work. The judgments relied on the experts' background, knowledge and experience directly so in the future we can explore whether subjective judgments could be improved by using evidence-based historical data. In future research into predictions in sports, perhaps "big data" can provide richer references for experts to make the judgments; we can also combine the judgments from very knowledgeable experts in prediction then use group decision procedures like the geometric mean to combine them. By using group questionnaires for the experts with access to evidence-based data we may be able to achieve a higher accuracy. Finally, distinguishing the similarities, differences, and interaction between the ANP and regression model will be an important, interesting, and very relevant topic for researchers to take advantages of the strengths of both methods and help improve the evidence-based judgment in decision making and forecasting. We will make this a future research direction and address such dilemma and potentials.

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