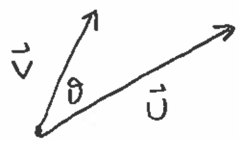
CSCI 378 HW

In this homework, the goal is to prove a "geometric" interpretation of dot product. Specifically, given two vectors $\vec{u}$ and $\vec{v}$ that are separated by angle θ:



(*)

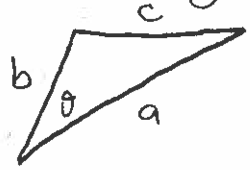
We want to prove:

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

where $\|\vec{x}\| = \sqrt{x_1^2 + \ldots + x_n^2}$ is the length (L2-norm) of vector $\vec{x}$ and $\vec{u} \cdot \vec{v} = u_1 v_1 + \ldots + u_n v_n$ is the (algebraic) dot product of $\vec{u}$ and $\vec{v}$.

You may use the following results:

- distributive law of dot product: $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$
- symmetry of dot product: $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
- the <u>Law of Cosines</u>, which is a more general version of the Pythagorean theorem for any triangle:



(**)

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

Observe that if $\theta = 90°$, then this reduces to the Pythagorean theorem.

Hint: why did I draw the diagrams (*) and (**) in such a similar manner?