

Cross-frequency coupling

Instructor: Mark Kramer

Today

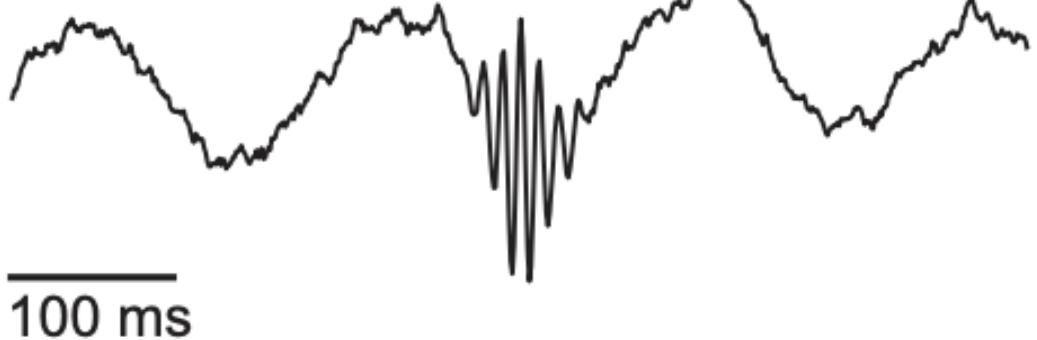
Cross-frequency coupling (one type of)

Remember, coherence

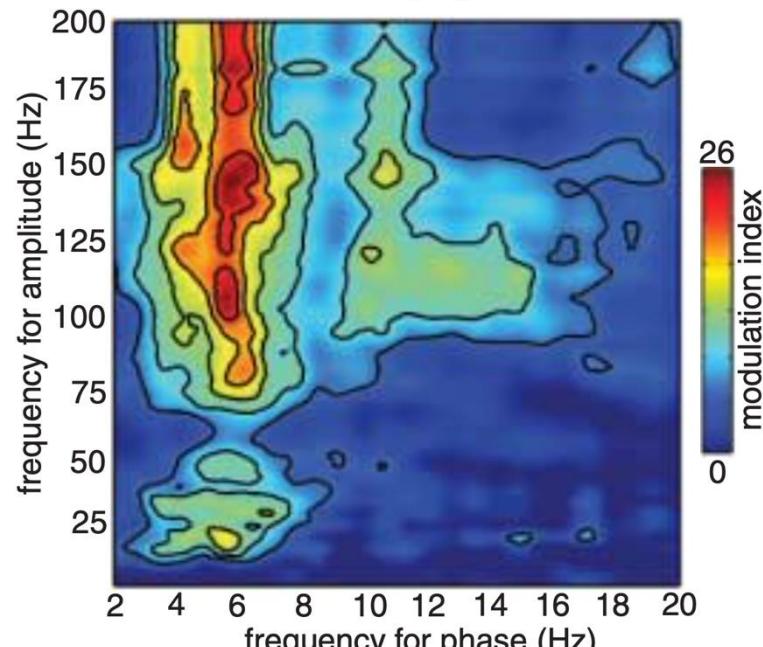
a constant phase relationship between two signals across trials,
at the same frequency

Today, coupling **between different frequencies**

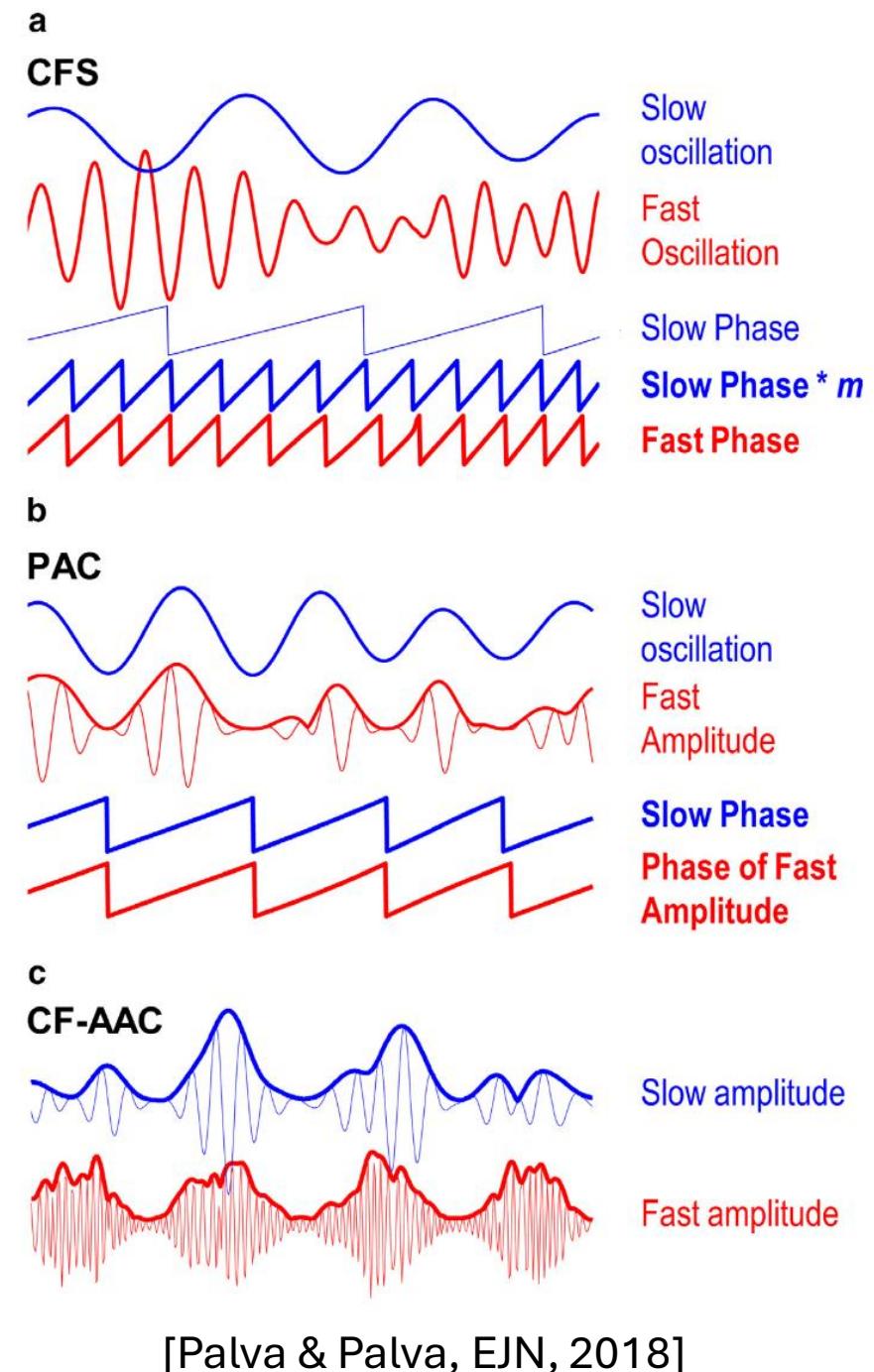
Examples



[Tort et. al., PNAS, 2008]



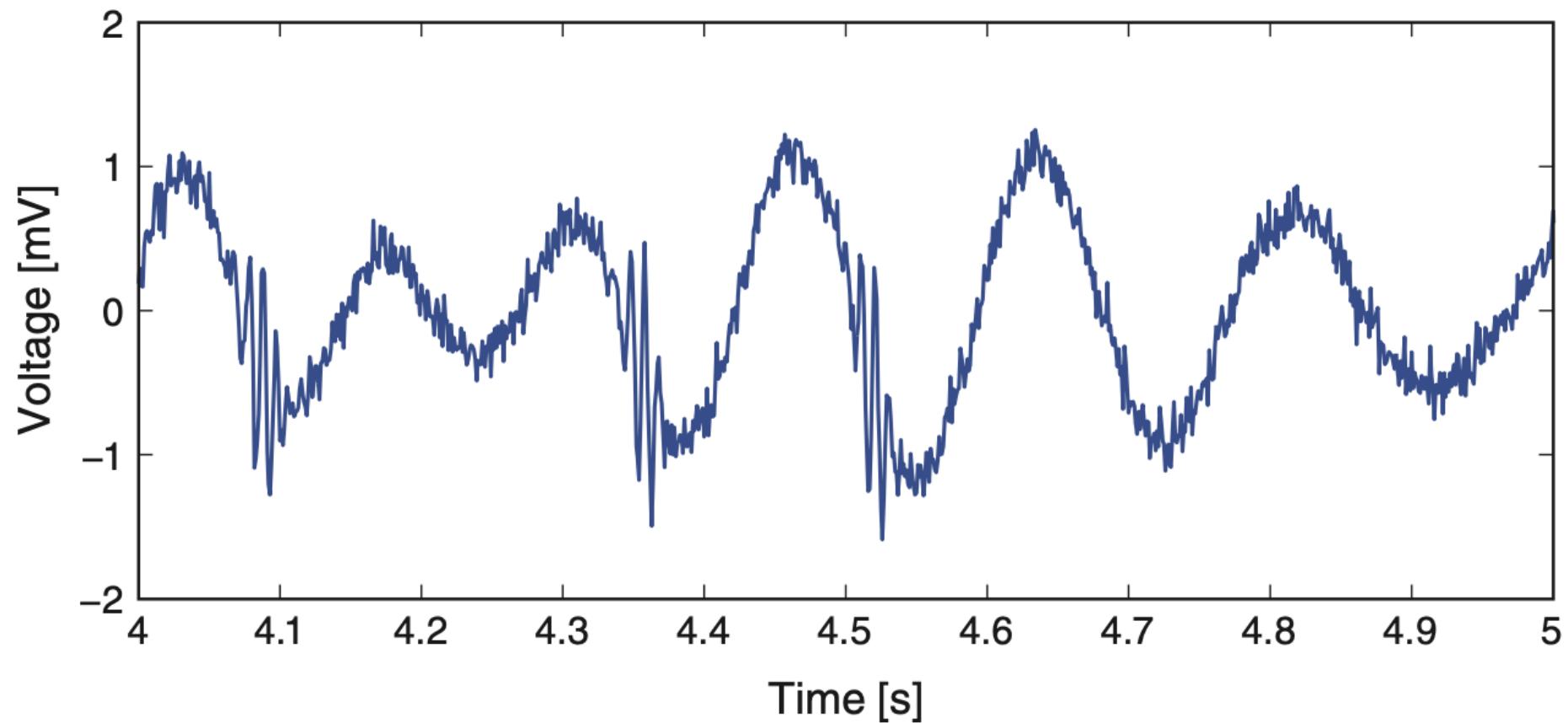
[Canolty et. al., Science, 2006]



Outline

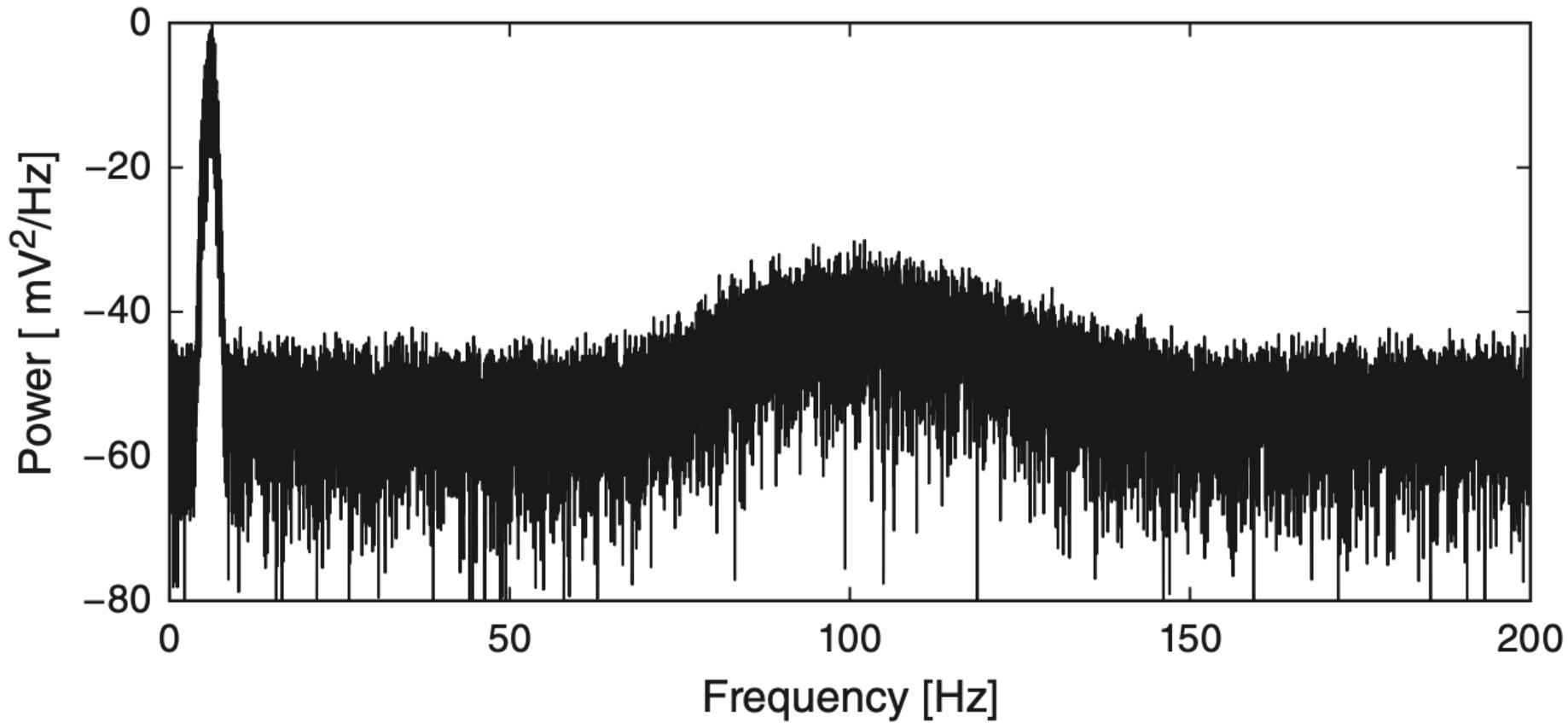
- Data** 100 s of local field potential data sampled at 1000 Hz.
- Goal** Characterize the coupling between rhythms of different frequency.
- Tools** Hilbert transform, analytic signal, instantaneous phase, cross-frequency coupling.

Data



Q. How to make sense of these data?

Spectrum



Q. What do you see?

CFC in three steps

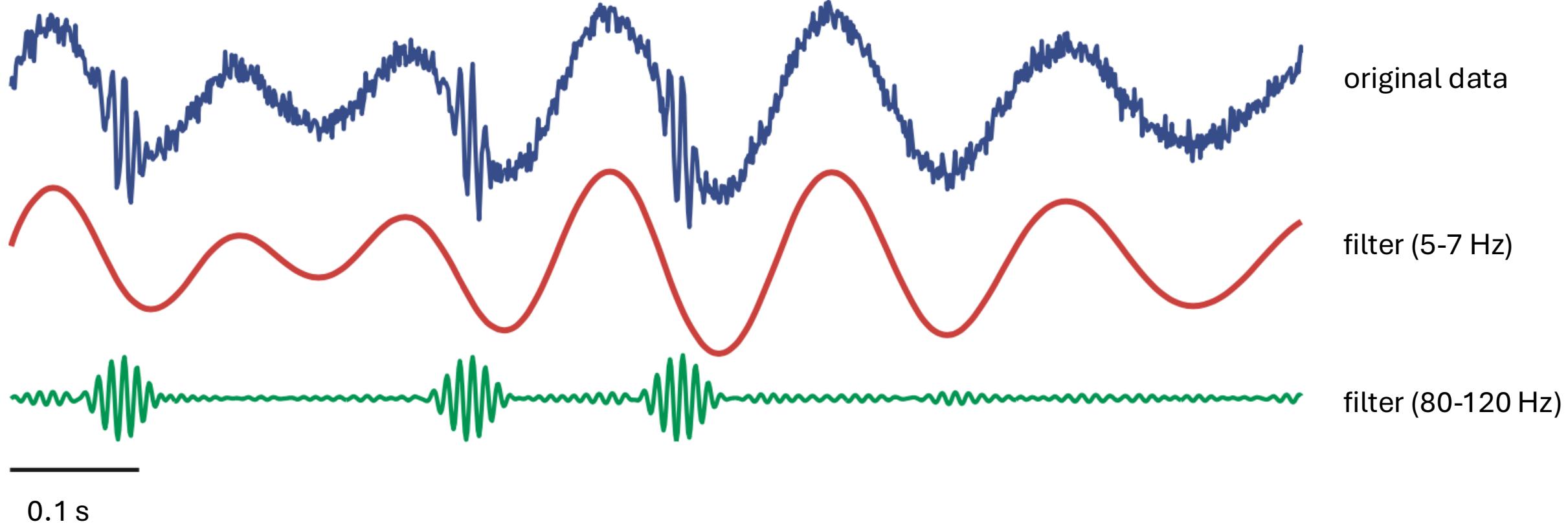
CFC analysis steps

1. Filter the data into high- and low-frequency bands.
2. Extract the amplitude and phase from the filtered signals.
3. Determine if the phase and amplitude are related.

Let's perform each step ...

CFC – Step 1

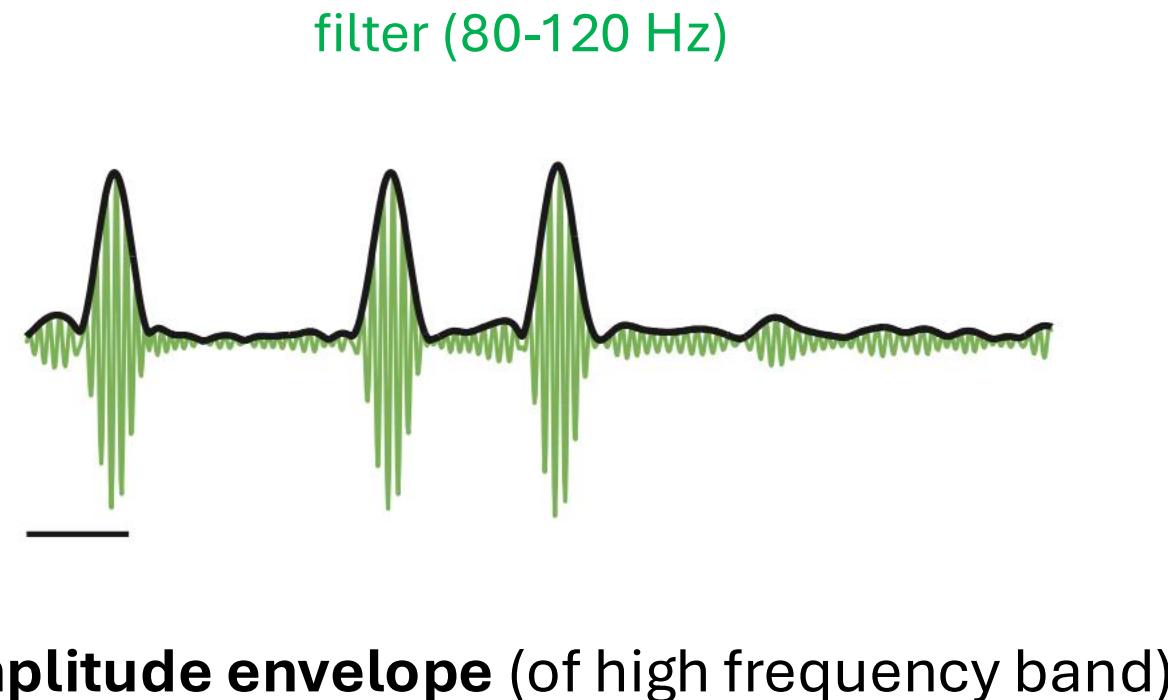
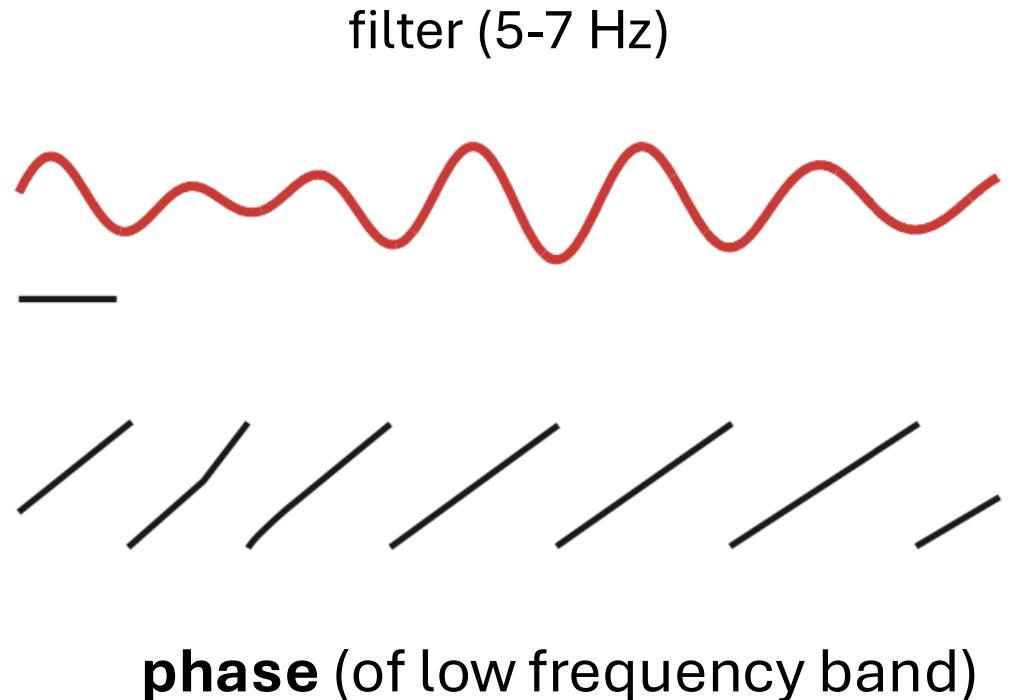
Filter the Data into High- and Low-Frequency Bands



Q. Why did we filter in these bands?

CFC – Step 2

Extract the Amplitude and Phase from Filtered Signals



Q. How?

Hilbert transform

$$y = H(x)$$

$$H(x) = \begin{cases} -\pi/2 \text{ phase shift} & \text{if } f > 0, \\ 0 \text{ phase shift} & \text{if } f = 0, \\ \pi/2 \text{ phase shift} & \text{if } f < 0. \end{cases}$$

The Hilbert transform $H(x)$ of the signal x produces a phase shift of ± 90 degrees for \mp frequencies of x .

Hilbert transform

Define: Analytic signal z

$$z = x + iy = x + iH(x)$$

Impact: remove negative frequencies from z

Q. How?

Hilbert transform

Q. What does it do?

Ex.

$$x_0 = 2 \cos(2\pi f_o t) = 2 \cos(\omega_0 t) \quad \text{where } \omega_0 = 2\pi f_o$$

Euler's formula

$$x_0 = e^{i\omega_0 t} + e^{-i\omega_0 t}$$

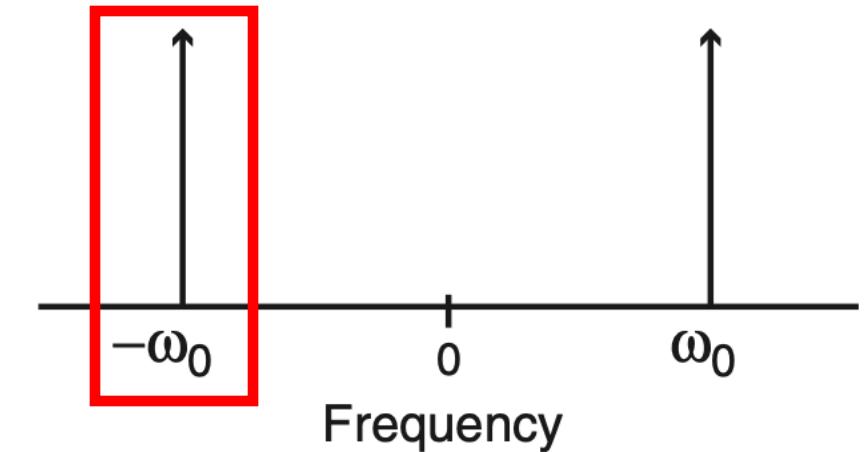


positive frequency



negative frequency

we usually ignore this one



Note: The spectrum has two peaks

Hilbert transform

Apply the Hilbert transform to x_0 .

$$x_0 = e^{i\omega_0 t} + e^{-i\omega_0 t}$$

$$H(x) = \begin{cases} -\pi/2 \text{ phase shift if } f > 0, \\ \end{cases}$$

→ multiply positive frequency part of x by $-i$

Q. Really?

Consider $e^{i\omega_0 t}$

positive frequency part of x



Shift $e^{i\omega_0 t}$ by $-\frac{\pi}{2}$

$$\rightarrow e^{i(\omega_0 t - \frac{\pi}{2})} \rightarrow e^{i\omega_0 t} e^{-i\pi/2} \rightarrow e^{i\omega_0 t} (\cos \frac{\pi}{2} - i \sin \frac{\pi}{2}) \rightarrow e^{i\omega_0 t} (-i)$$

Hilbert transform

Apply the Hilbert transform to x_0 .

$$x_0 = e^{i\omega_0 t} + e^{-i\omega_0 t}$$

$$H(x) = \begin{cases} -\pi/2 \text{ phase shift} & \text{if } f > 0, \\ 0 \text{ phase shift} & \text{if } f = 0, \\ \pi/2 \text{ phase shift} & \text{if } f < 0. \end{cases} \quad \rightarrow \quad H(x) = \begin{cases} -ix & \text{if } f > 0, \\ 0 & \text{if } f = 0, \\ ix & \text{if } f < 0. \end{cases}$$

So, $x_0 = e^{i\omega_0 t} + e^{-i\omega_0 t}$

\uparrow \uparrow

multiply by $-i$ multiply by i

Euler's formula

$= 2 \sin(\omega_0 t)$

Hilbert Transform of x_0 (a cosine function) is a sine function.

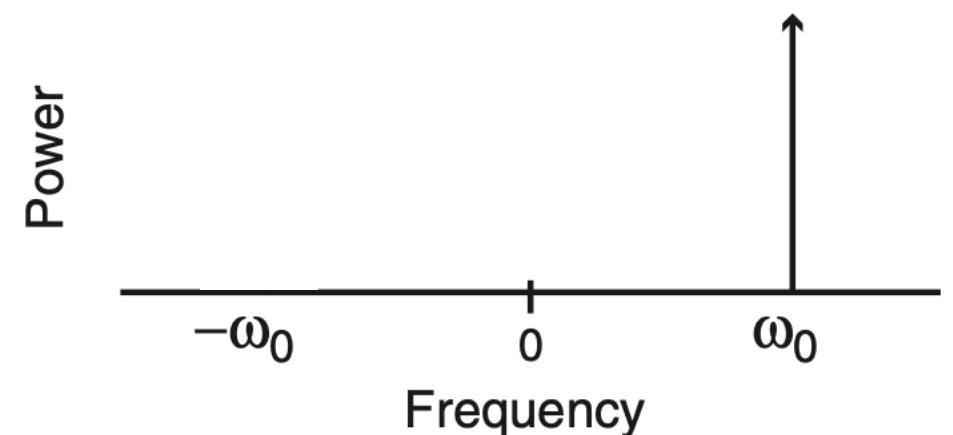
Hilbert transform

Analytic signal z

$$z = x + iy = x + iH(x)$$
$$= 2\cos(\omega_0 t) + i2\sin(\omega_0 t)$$

$$= 2e^{i\omega_0 t}$$

The analytic signal contains
no negative frequencies



Hilbert transform

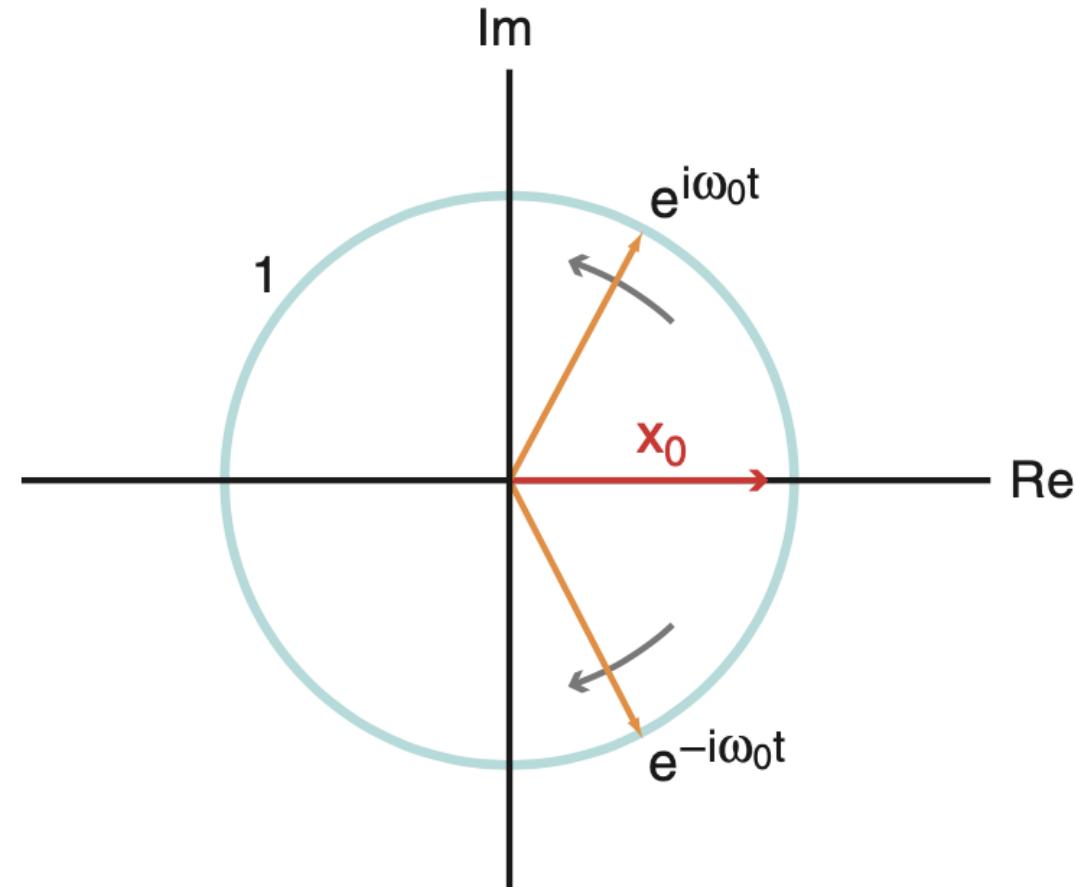
Original signal $x_0 = 2 \cos(2\pi f_o t) = e^{i\omega_0 t} + e^{-i\omega_0 t}$

Complicated (2 complex exponentials)

Analytic signal $z_0 = 2e^{i\omega_0 t}$

Simple (1 complex exp)

A point in the complex plane

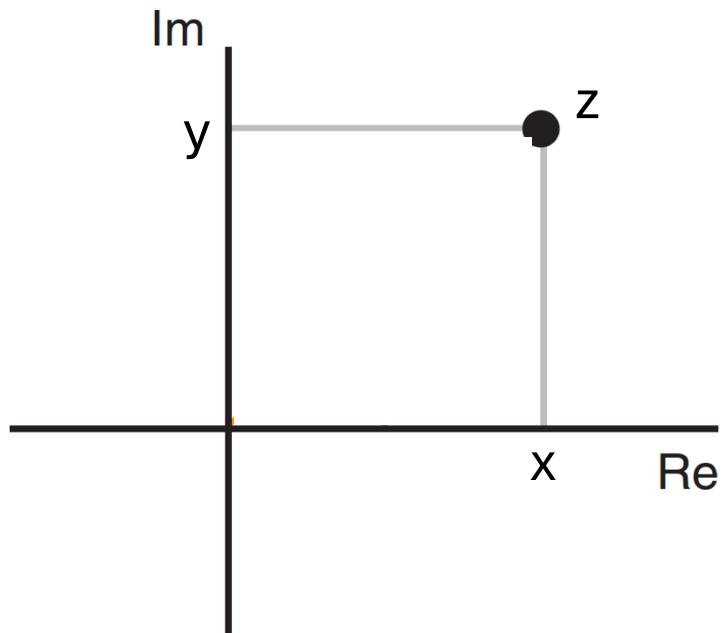


Hilbert transform

Analytic signal

$$z = x + iy$$

A point in the complex plane



$$z(t) = A(t) e^{i \phi(t)}$$

amplitude

phase

Get the amplitude and phase from
the analytic signal

Ex.

$$z_0(t) = 2e^{i\omega_0 t}$$

$$A(t) = 2$$

$$\phi(t) = \omega_0 t$$

CFC in three steps

CFC analysis steps

- 1. Filter the data into high- and low-frequency bands.
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CFC – Step 3

Determine if the Phase and Amplitude are Related

Define the two-column vector

$$\begin{pmatrix} \phi(1) & A(1) \\ \phi(2) & A(2) \\ \phi(3) & A(3) \\ \vdots & \vdots \end{pmatrix}$$



phase of low frequency band activity

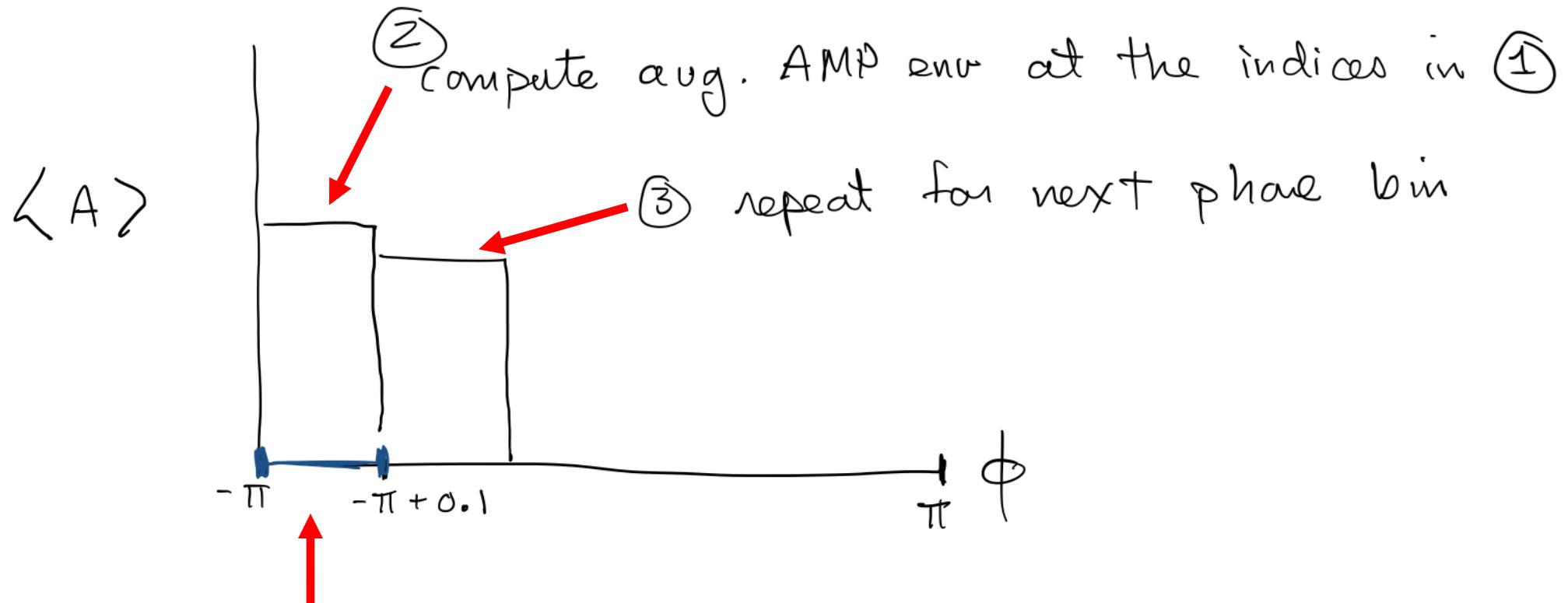
amplitude of high frequency band activity

Make a histogram

CFC – Step 3

Determine if the Phase and Amplitude are Related

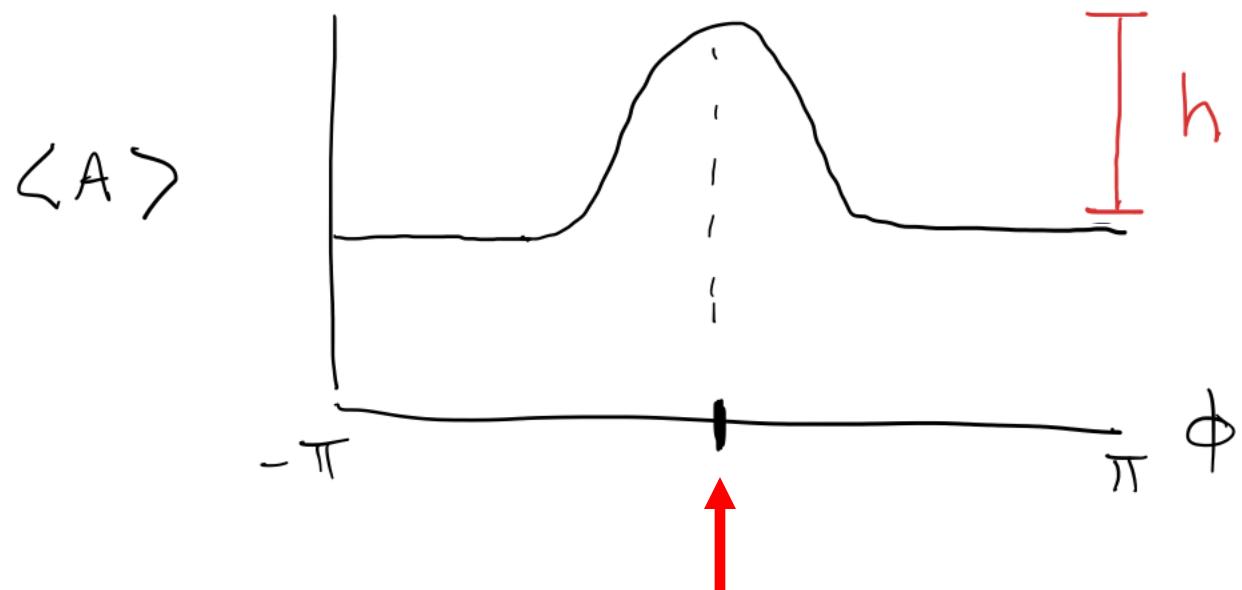
Divide the phase into bins of size 0.1



CFC – Step 3

Determine if the Phase and Amplitude are Related

If phase modulate amplitude



summarize extent of modulation

At this phase, amplitude envelope is big

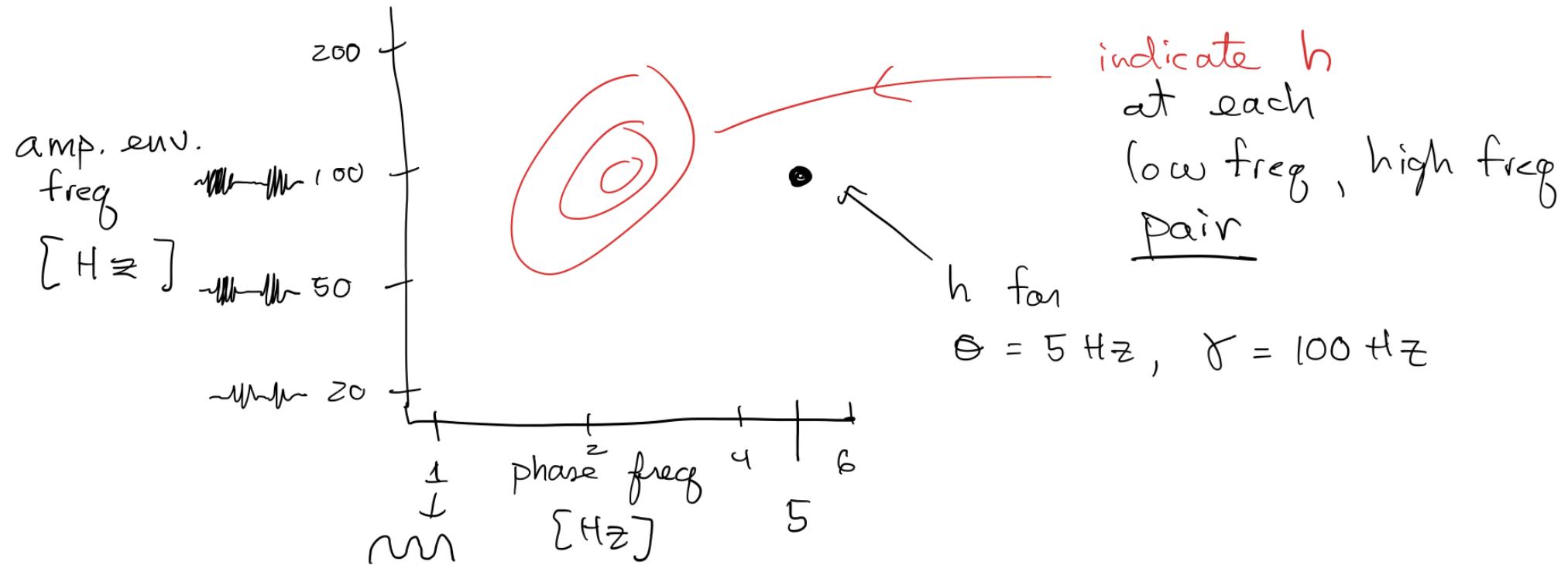
→ phase modulates amplitude

Q. What does no phase-amplitude coupling look like in the plot?

CFC – Step 4

(optional): Repeat for other frequencies.

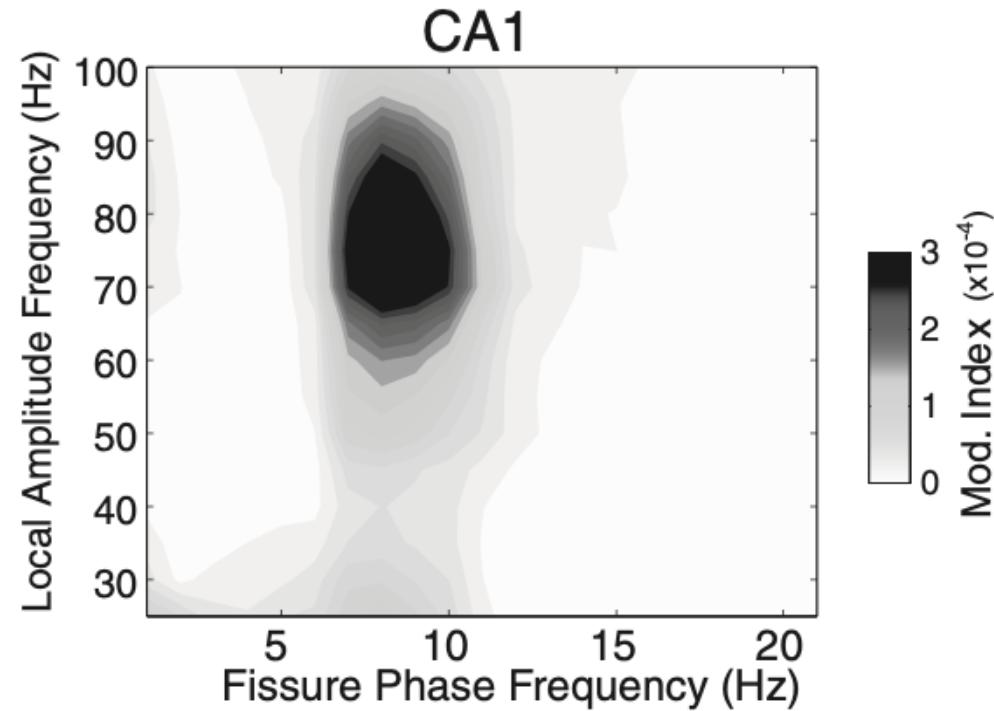
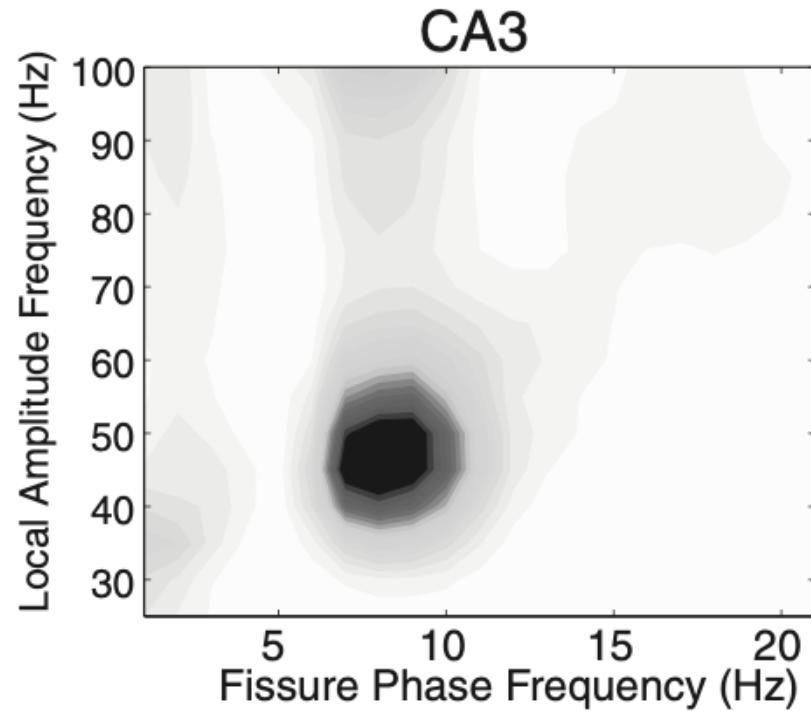
Summarize in a comodulogram



CFC – Step 4

(optional): Repeat for other frequencies.

Summarize in a comodulogram



[Tort et al., J Neurophysiol, 2010]

CFC in three steps

CFC analysis steps

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Python