The gamma rhythms

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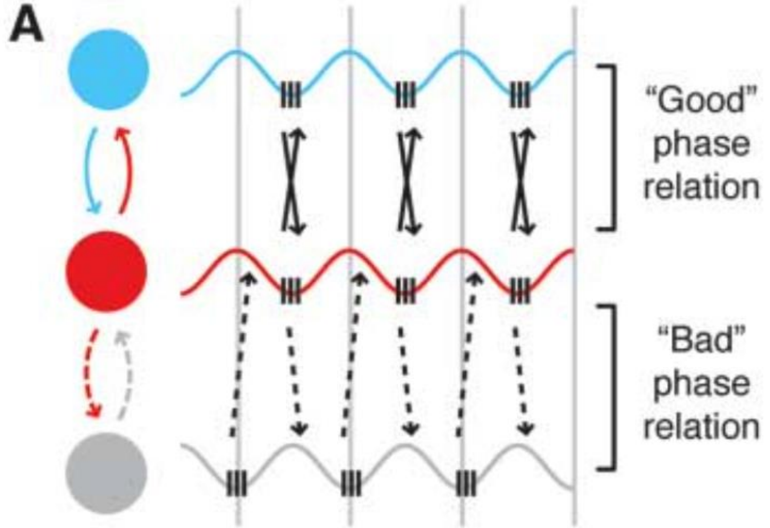
Today

Models of the gamma rhythms

Gamma rhythms



30-80 Hz

<u>functions</u>

- cell-assembly formation / synchronization

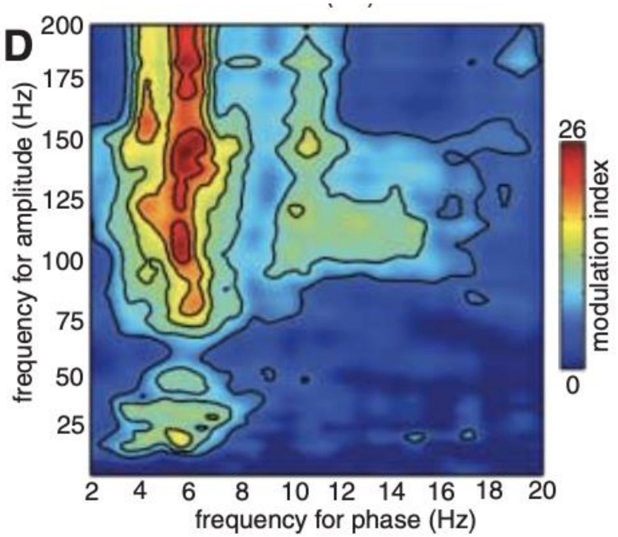
[Womelsdorf et al. "Modulation of Neuronal Interactions Through Neuronal Synchronization." Science , 2007]
[Fernández-Ruiz et al., "Gamma Rhythm Communication between Entorhinal Cortex and Dentate Gyrus Neuronal Assemblies.", Science, 2021]
[Canolty et al., "High Gamma Power Is Phase-Locked to Theta Oscillations in Human Neocortex.", Science, 2006]

- memory

[Lisman & Idiart. "Storage of 7 ± 2 Short-Term Memories in Oscillatory Subcycles." Science, 1995]
[Lundqvist et al., "Gamma and Beta Bursts Underlie Working Memory." Neuron, 2016]

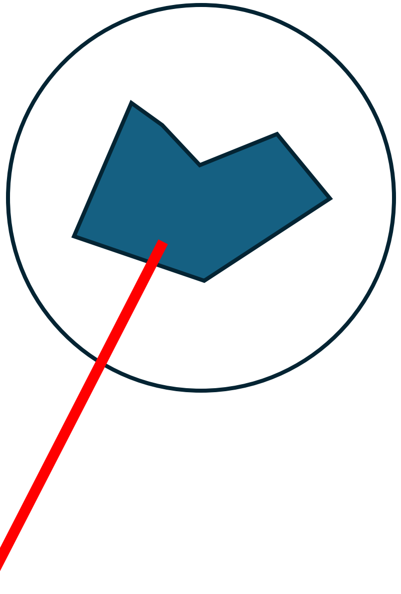
- plasticity

[Hadler et al, "Gamma Oscillation Plasticity Is Mediated via Parvalbumin Interneurons." Science Advances, 2024]



Gamma rhythms

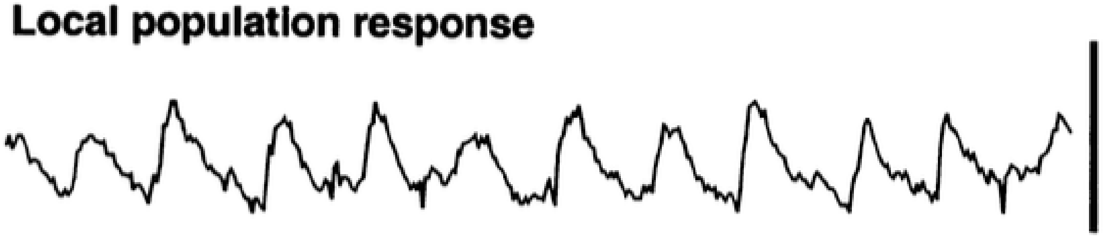
Mechanisms (via experimental models)



in vitro preparation

apply drugs (e.g., carbachol to increase excitability)

record LFP



<u>Facts</u>

- Block $GABA_A$ ➔ eliminate gamma
- Block AMPA ➔ eliminate gamma

involves ex & inh cells + synaptic interactions

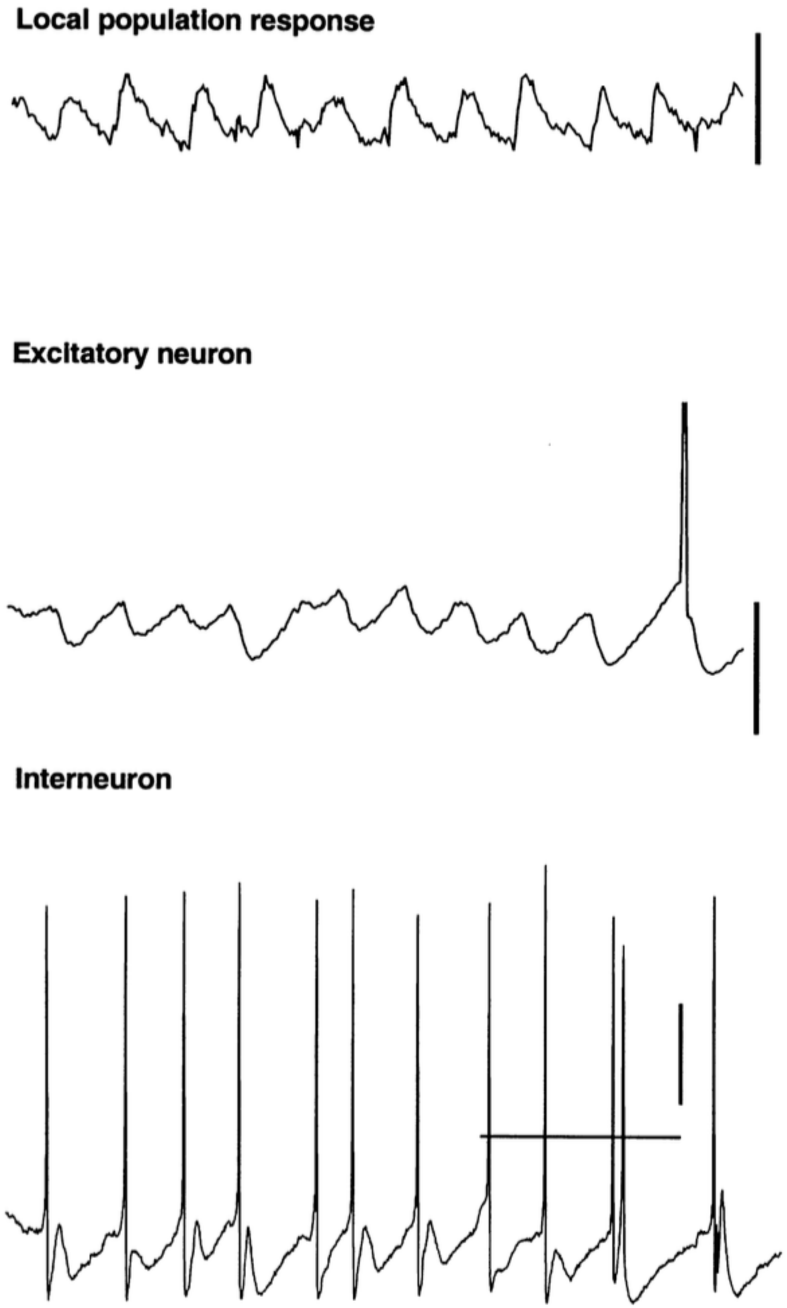
[Whittington et al., *Inhibition-Based Rhythms*, Int J Psychophysiol, 2000]

Gamma rhythms

Mechanisms (via experimental models)

<u>More Facts</u>

- rhythm frequency depends on $GABA_A$ kinetics (e.g., modulate with sedatives to alter period)
- pyramidal (ex) cells fire sparsely
- basket (inh) cells fire on most cycles



[Whittington et al., *Inhibition-Based Rhythms*, Int J Psychophysiol, 2000]

Models

Three types

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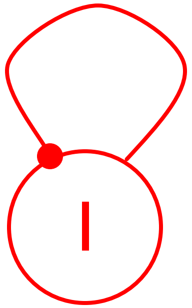
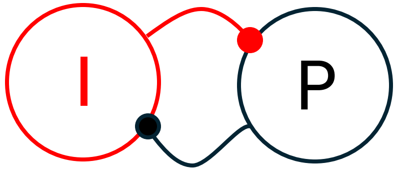
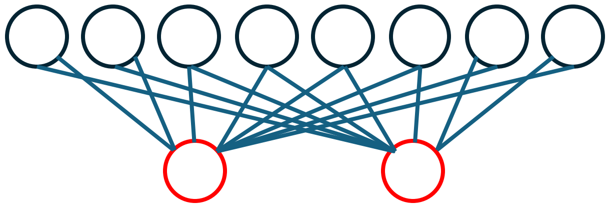


PING



sparse PING



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Interneuron **N**etwork **G**amma

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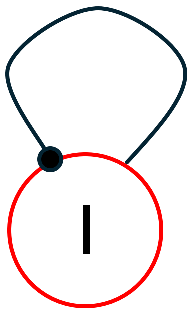

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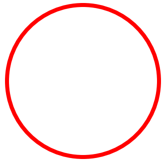
Interneuron Network Gamma

Experimental observations

(1) Excitation (driven cells)
(2) $GABA_A$ critical
(3) Altering $GABA_A$ kinetics changes frequency

Model

1 cell



Load with standard HH currents

$$\frac{dV}{dt} = I_{Na} + I_K + I_L + I_{inj}$$

To mimic (1) make I_{inj} large → depolarize neuron → fast spiking

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Interneuron Network Gamma

Experimental observations

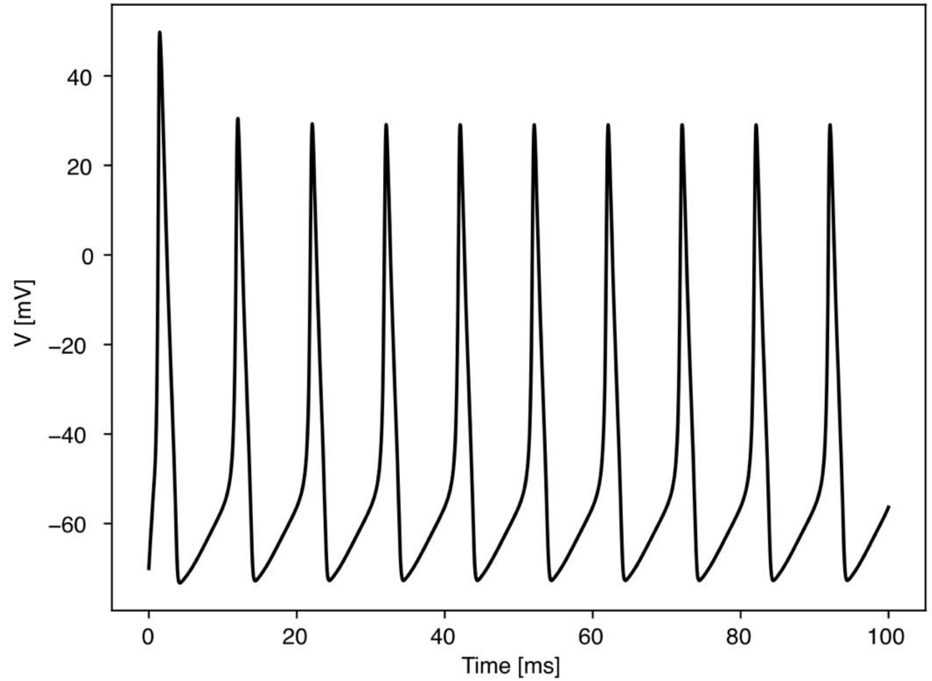
(1) Excitation (driven cells)

(2) $GABA_A$ critical

(3) Altering $GABA_A$ kinetics changes frequency

To mimic (1) make I_{inj} large ➔ depolarize neuron ➔ fast spiking

Ex. I_{inj} = 30



Q. What sets the timescale of spiking?

A. Dynamics of intrinsic currents (Na, K)

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Interneuron Network Gamma

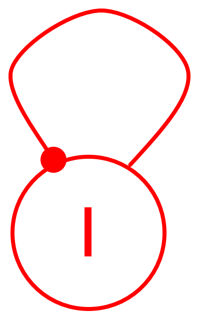
Experimental observations

(1) Excitation (driven cells)

(2) GABA_A critical

(3) Altering GABA_A kinetics changes frequency

To mimic (2) add an inhibitory synapse



autapse (presynaptic neuron = postsynaptic neuron)

Q. Realistic?

Then
$$\frac{dV}{dt} = I_{Na} + I_K + I_L + I_{inj} + \boxed{I_{synapse}}$$

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Interneuron Network Gamma

$$\frac{dV}{dt} = I_{Na} + I_K + I_L + I_{inj} + \boxed{I_{synapse}}$$

Synaptic current

$$I_{synapse} = g_I\, s_I\, (E_I - V)$$

maximal conductance

inh. synapse gate

equilibrium voltage for inh. synapse (-80 mV)

neuron voltage

Experimental observations

(1) Excitation (driven cells)

(2) GABA_A critical

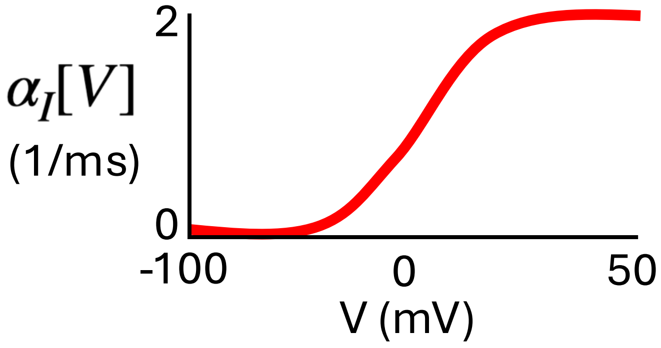
(3) Altering GABA_A kinetics changes frequency

Synaptic gate dynamics

$$\frac{ds_I}{dt} = \alpha_I[V](1 - s_I) - \beta_I[V]s_I$$

forward rate fxn

backward rate fxn



$$\beta_I[V] = \beta_I = \frac{1}{\tau_d}$$

constant

decay time $\approx$ 10 ms

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Interneuron Network Gamma

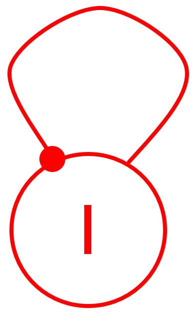
Experimental observations

(1) Excitation (driven cells)

(2) $GABA_A$ critical

(3) Altering $GABA_A$ kinetics changes frequency

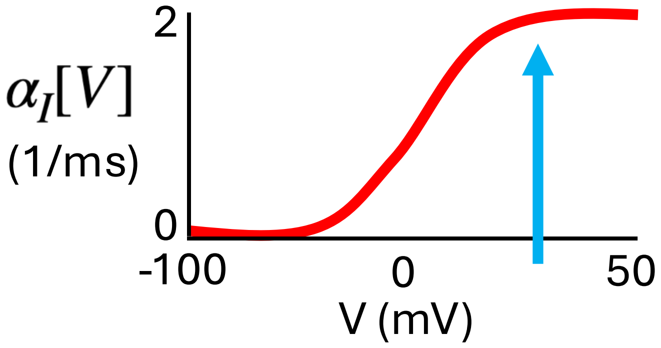
$$\frac{ds_I}{dt} = \alpha_I[V](1 - s_I) - \beta_I[V]s_I$$



Q. What happens?

- neuron spikes (V > 0)
- $\alpha_I[V]$ → 2
- s_I → 1 (open)
- Cl^- flows in → hyperpolarize cell (push to -80 mV)

Note: $[Cl^-]_{out} >> [Cl^-]_{in}$



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Interneuron Network Gamma

Experimental observations

(1) Excitation (driven cells)

(2) $GABA_A$ critical

(3) Altering $GABA_A$ kinetics changes frequency

Model

$$C\frac{dV}{dt} = I_{\text{input}}(t) - \bar{g}_{\text{K}} n^4 (V - V_{\text{K}}) - \bar{g}_{\text{Na}} m^3 h (V - V_{\text{Na}}) - \bar{g}_{\text{L}} (V - V_{\text{L}}) \boxed{- g_I s_I (V - E_I)}$$

$$\frac{dn}{dt} = -\frac{n - n_\infty(V)}{\tau_n(V)}$$

$$\frac{dm}{dt} = -\frac{m - m_\infty(V)}{\tau_m(V)}$$

$$\frac{dh}{dt} = -\frac{h - h_\infty(V)}{\tau_h(V)}$$

$$\boxed{\frac{ds_I}{dt} = \alpha_I[V](1 - s_I) - \beta_I[V] s_I}$$

5 variables

5 differential equations

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Interneuron Network Gamma

Q. How does it work?

Experimental observations

(1) Excitation (driven cells)

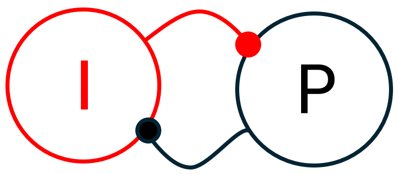
(2) $GABA_A$ critical

(3) Altering $GABA_A$ kinetics changes frequency

Python

PING

Pyramidal **I**nterneuron **N**etwork **G**amma

PING 

PING

Using ING

<u>Experimental observations</u>

- ☑ (1) Excitation (driven cells)
- ☑ (2) $GABA_A$ critical
- ☑ (3) Altering $GABA_A$ kinetics changes frequency
- ✗ (4) AMPA critical

New model

+ include excitatory (pyramidal) cell

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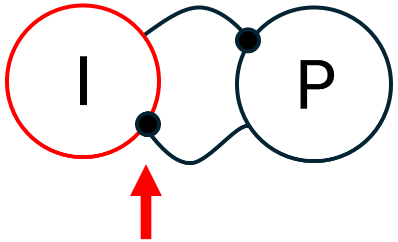

include AMPA synapse

<u>Idea</u>: cells collaborate to produce gamma

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Connect cells with synapses

when I-cell spikes → hyperpolarize P-cell
inhibitory (GABA) synapse

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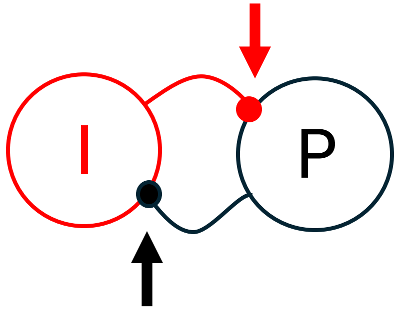
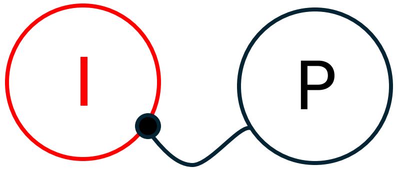


excitatory (AMPA) synapse
when P-cell spikes → depolarize I-cell

Build the model: HH + synapses

PING

Include synapses



$$\frac{dV_I}{dt} = I_{Na} + I_K + I_L + I_{inj} + \boxed{I_{synapse\ P \to I}}$$

I-cell voltage

Synaptic current

$$I_{synapse\ P \to I} = g_P\, s_P\, (E_P - V_I)$$

maximal conductance

ex. synapse gate

equilibrium voltage for ex. synapse (0 mV)

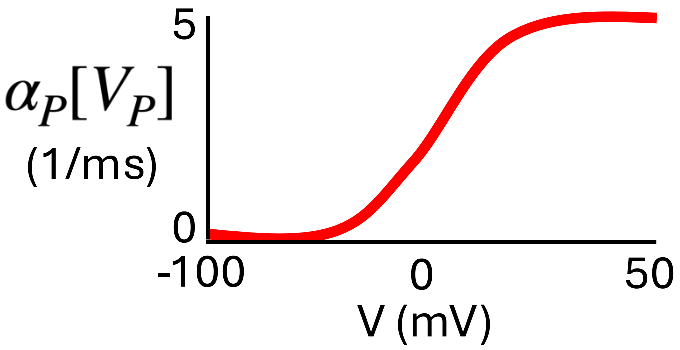
I-cell voltage **post-synaptic** cell

Synaptic gate dynamics

$$\frac{ds_P}{dt} = \alpha_P[V_P](1 - s_P) - \beta_P[V_P]s_P$$

forward rate fxn, **pre-synaptic** V

backward rate fxn



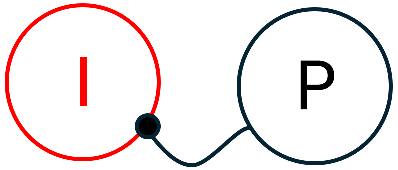
$$\beta_P[V_P] = \beta_P = \frac{1}{\tau_d}$$

constant

decay time $\approx$ 2 ms

Note: faster than inh. synapse

PING

Include synapses 

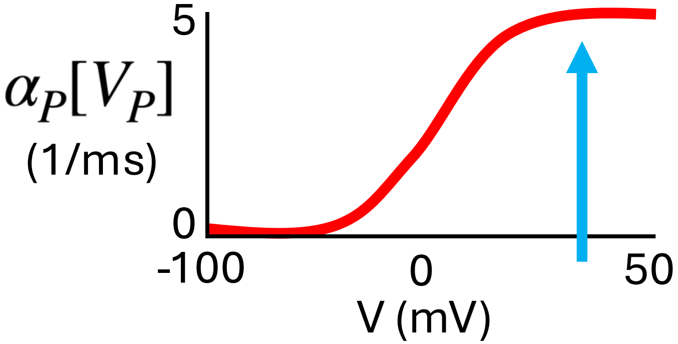
$$\frac{dV_I}{dt} = I_{Na} + I_K + I_L + I_{inj} + I_{synapse\, P\to I}$$

$$\frac{ds_P}{dt} = \alpha_P[V_P](1 - s_P) - \beta_P[V_P]s_P$$

Q. What happens?

- P-cell spikes ($V_P > 0$)
- $\alpha_P[V_P] \rightarrow 5$
- $s_P \rightarrow 1$ (open)
- charge (Na^+) flows in → depolarize I-cell (push to 0 mV)

<u>Note</u>: $[Na^+]_{out} >> [Na^+]_{in}$



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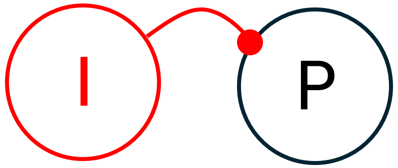
Include synapses



$$\frac{dV_P}{dt} = I_{Na} + I_K + I_L + I_{inj} + \boxed{I_{synapse\, I\rightarrow P}}$$

P-cell voltage

Synaptic current

$$I_{synapse\, I\rightarrow P} = g_I\, s_I\, (E_I - V_P)$$

maximal conductance

inh. synapse gate

equilibrium voltage for inh. synapse (-80 mV)

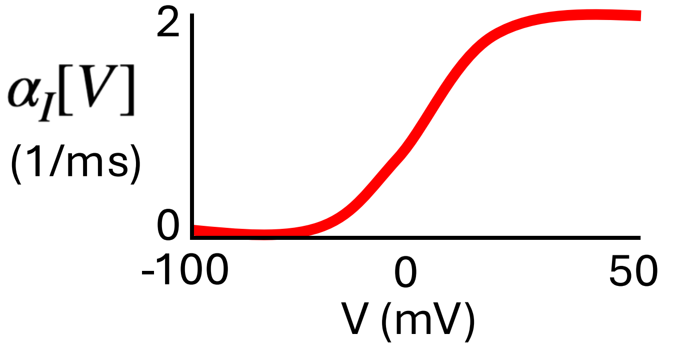
P-cell voltage **post-synaptic** cell

Synaptic gate dynamics

$$\frac{ds_I}{dt} = \alpha_I[V_I](1 - s_I) - \beta_I[V_I]s_I$$

forward rate fxn, **pre-synaptic** V

backward rate fxn



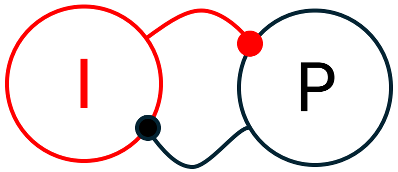
$$\beta_I[V] = \beta_I = \frac{1}{\tau_d}$$

constant

decay time $\approx$ 10 ms

PING

Put it all together



10 variables

$$\frac{dV_P}{dt} = I_{Na} + I_K + I_\ell + I_{inj,P} + \overbrace{g_I S_I (E_I - V_P)}^{\text{inh. synaptic input}}$$

$$\left.\begin{aligned} \frac{dm_P}{dt} &= \\ \frac{dh_P}{dt} &= \\ \frac{dn_P}{dt} &= \end{aligned}\right\} \text{HH}$$

$$\frac{ds_P}{dt} = \alpha_P[V_P](1 - s_P) - \beta_P[V_P]\, s_P \qquad (\text{ex. gate dynamics}).$$

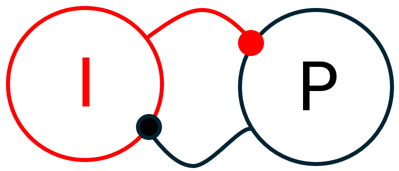
$$\frac{dV_I}{dt} = I_{Na} + I_K + I_\ell + I_{inj,I} + \underbrace{g_P S_P (E_P - V_I)}_{\text{ex. synaptic input}}$$

$$\left.\begin{aligned} \frac{dm_I}{dt} &= \\ \frac{dh_I}{dt} &= \\ \frac{dn_I}{dt} &= \end{aligned}\right\} \text{HH}$$

$$\frac{ds_I}{dt} = \alpha_I[V_I](1 - s_I) - \beta_I[V_I]\, s_I \qquad (\text{inh. gate dynamics})$$

PING

Q. How does this generate a gamma rhythm?



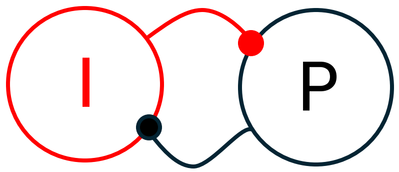
Assume P-cell has $I_{inj,P}$ big enough to spike repeatedly in isolation

t=0	P-cell spikes	→ excitation to I-cell	→ I-cell spikes
t≈0	I-cell spikes	→ inhibition to P-cell	
t=25	P-cell recovers	→ P-cell spikes	

Repeat …

PING

Q. Consistent with experimental observations?



Experimental observations

? (1) Excitation (driven cells)

? (2) $GABA_A$ critical

? (3) Altering $GABA_A$ kinetics changes frequency

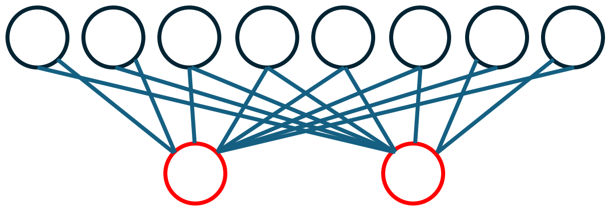
? (4) AMPA critical

Python *Homework*

Sparse PING

Sparse Pyramidal **I**nterneuron **N**etwork **G**amma

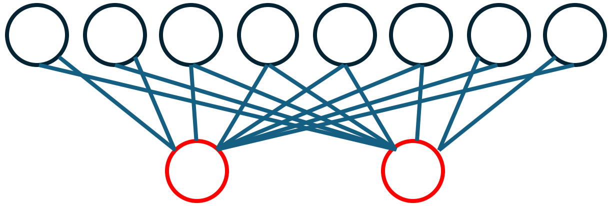
sparse
PING



Sparse PING

Idea: update the PING model to include a population of P&I cells.

Ex. 80 P cells & 20 I cells



Each a HH model

P1

$$C\frac{dV}{dt} = I_{\text{input}}(t) - \bar{g}$$
$$\frac{dn}{dt} = -\frac{n - n_\infty(V}{\tau_n(V)}$$
$$\frac{dm}{dt} = -\frac{m - m_\infty(}{\tau_m(V)}$$
$$\frac{dh}{dt} = -\frac{h - h_\infty(V}{\tau_h(V)}$$

P2

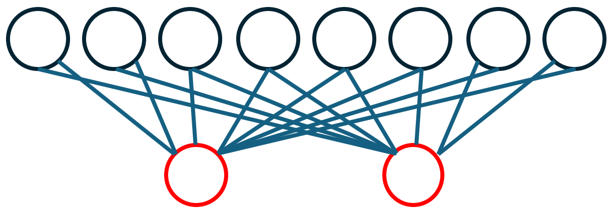
$$C\frac{dV}{dt} = I_{\text{input}}(t) - \bar{g}_{\text{K}}$$
$$\frac{dn}{dt} = -\frac{n - n_\infty(V)}{\tau_n(V)}$$
$$\frac{dm}{dt} = -\frac{m - m_\infty(V}{\tau_m(V)}$$
$$\frac{dh}{dt} = -\frac{h - h_\infty(V)}{\tau_h(V)}$$

P3

$$C\frac{dV}{dt} = I_{\text{input}}(t) - \bar{g}_{\text{K}}n^4(V - V_{\text{K}}) - \bar{g}_{\text{Na}}m^3h(V - V_{\text{Na}}) - \bar{g}_{\text{L}}(V - V_{\text{L}})$$
$$\frac{dn}{dt} = -\frac{n - n_\infty(V)}{\tau_n(V)}$$
$$\frac{dm}{dt} = -\frac{m - m_\infty(V)}{\tau_m(V)}$$
$$\frac{dh}{dt} = -\frac{h - h_\infty(V)}{\tau_h(V)}$$

400 differential equations

Sparse PING



Connect with synapses

Each P → all I (with ex. synapses)

Each I → all P (with inh. synapses)

Then (for P1)

$$C\frac{dV}{dt} = I_{\text{input}}(t) - \bar{g}_{\text{K}} n^4 (V - V_{\text{K}}) - \bar{g}_{\text{Na}} m^3 h (V - V_{\text{Na}}) - \bar{g}_{\text{L}} (V - V_{\text{L}}) \boxed{+ I_{syn\ I1 \to P1} + I_{syn\ I2 \to P1} + \ \ldots}$$

many terms

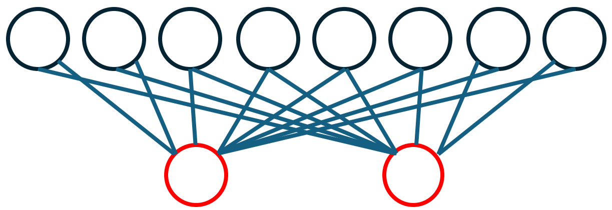
$$\frac{dn}{dt} = -\frac{n - n_\infty(V)}{\tau_n(V)}$$

$$\frac{dm}{dt} = -\frac{m - m_\infty(V)}{\tau_m(V)}$$

$$\frac{dh}{dt} = -\frac{h - h_\infty(V)}{\tau_h(V)}$$

$$\boxed{\frac{ds_{I1}}{dt} = \ldots, \frac{ds_{I2}}{dt} = \ldots, \frac{ds_{I3}}{dt} = \ldots}$$

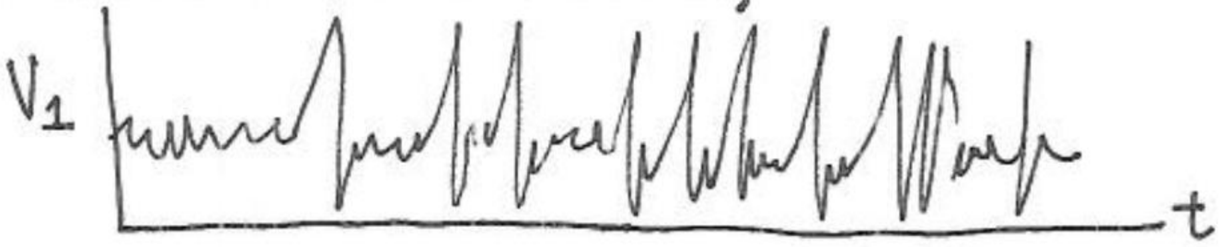
Sparse PING



Q. How will the model work?

<u>**Idea**</u>:
- Give P cells enough depolarizing input to spike at high rate in isolation.
- Include noise in dynamics.

Then, for some P-cell. in isolation,

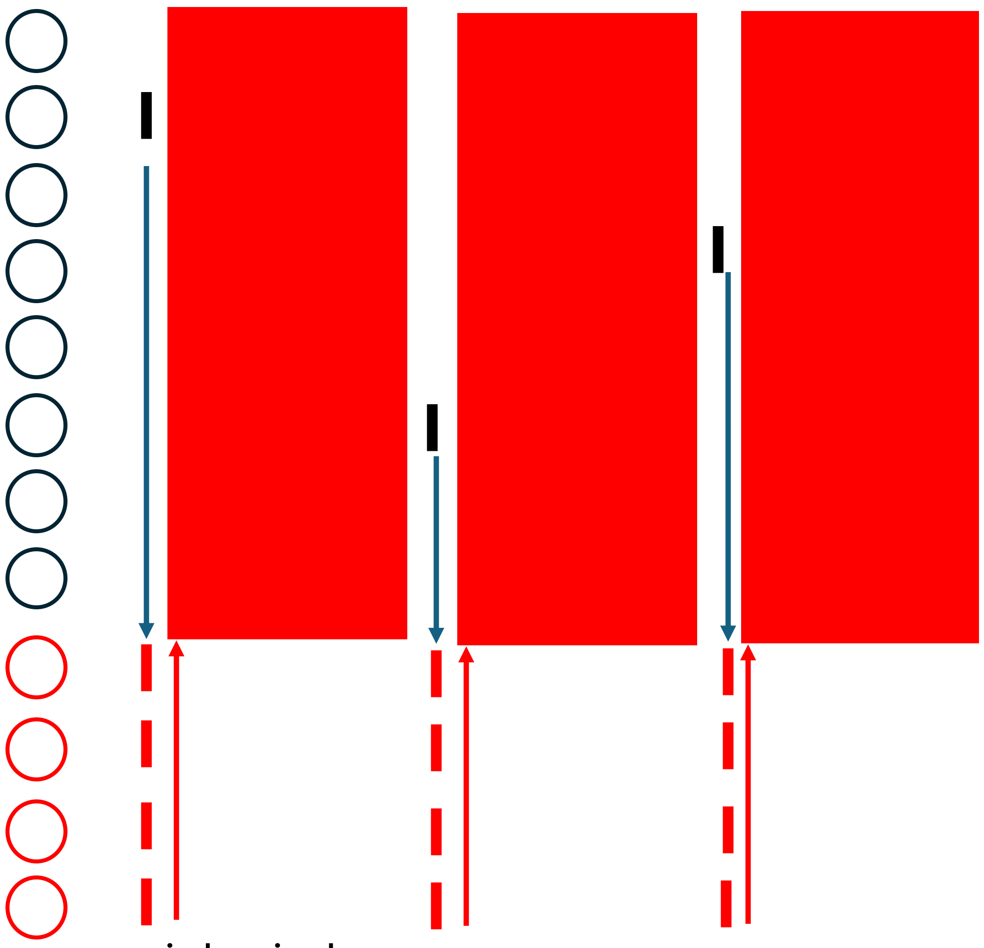


P-cell spikes.

Time between spikes varies due to noise.

- Make synapses strong

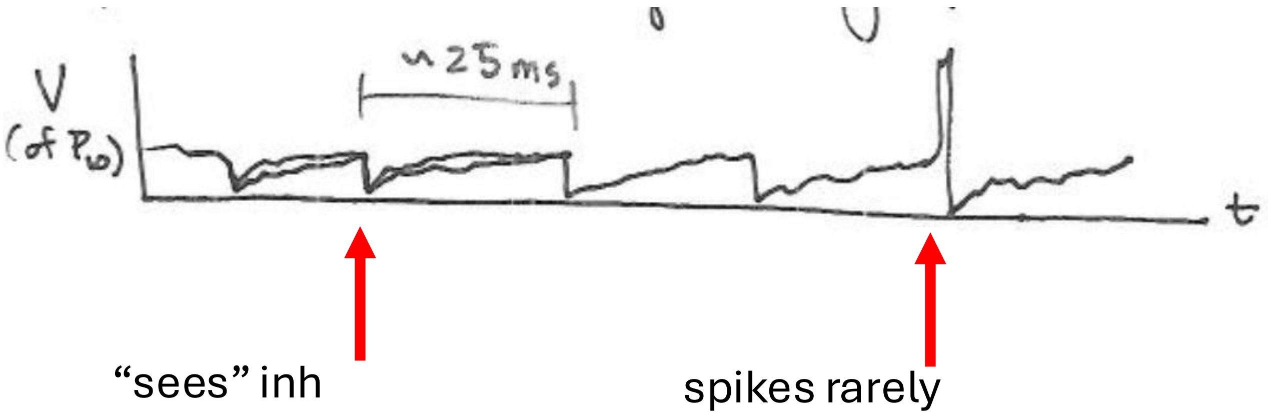
Sparse PING



inh window
depends on decay time
of inh. synapse

Note: a different P-cell can spike on each cycle

Plot V for a P-cell



Each P-cell fires sparsely ... "sparse PING"
Match experimental observation
Cost: more complexity.