Cross-frequency coupling

Instructor: Mark Kramer

Today

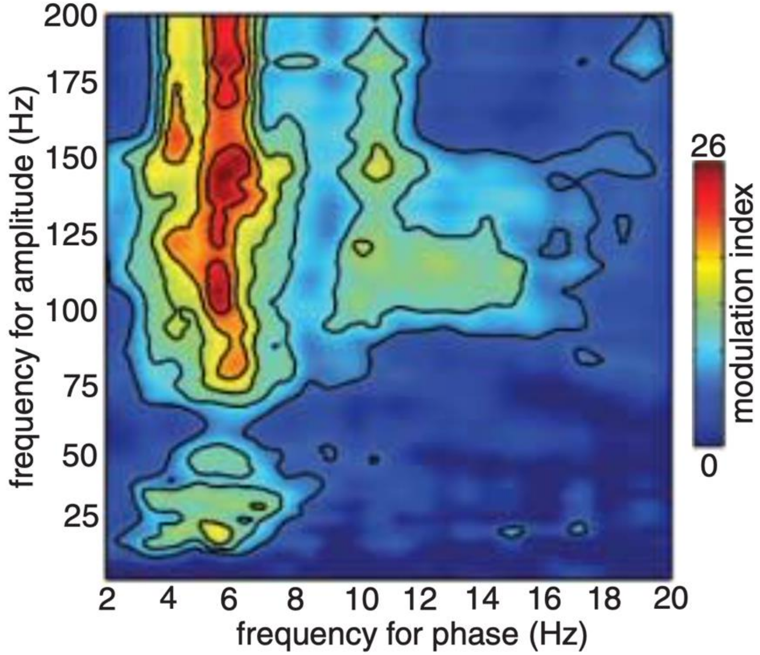
Cross-frequency coupling (one type of)

Remember, <u>coherence</u>

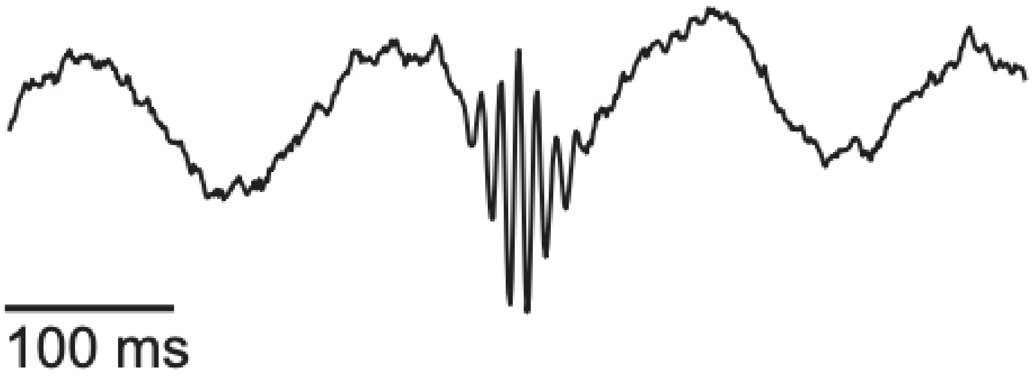
a constant phase relationship between two signals across trials,
at the same frequency

Today, coupling **between different frequencies**

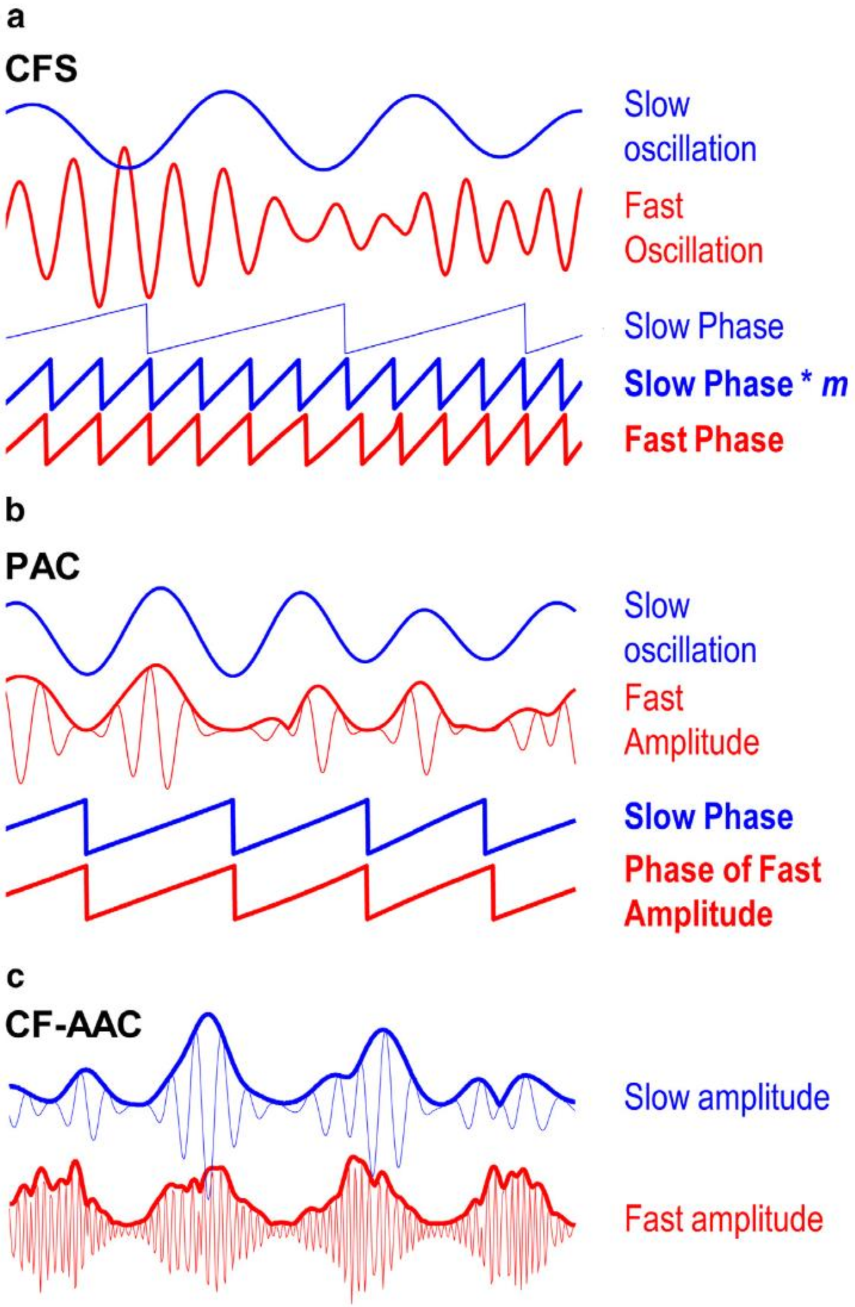
Examples



[Canolty et. al., Science, 2006]



[Tort et. al., PNAS, 2008]



[Palva & Palva, EJN, 2018]

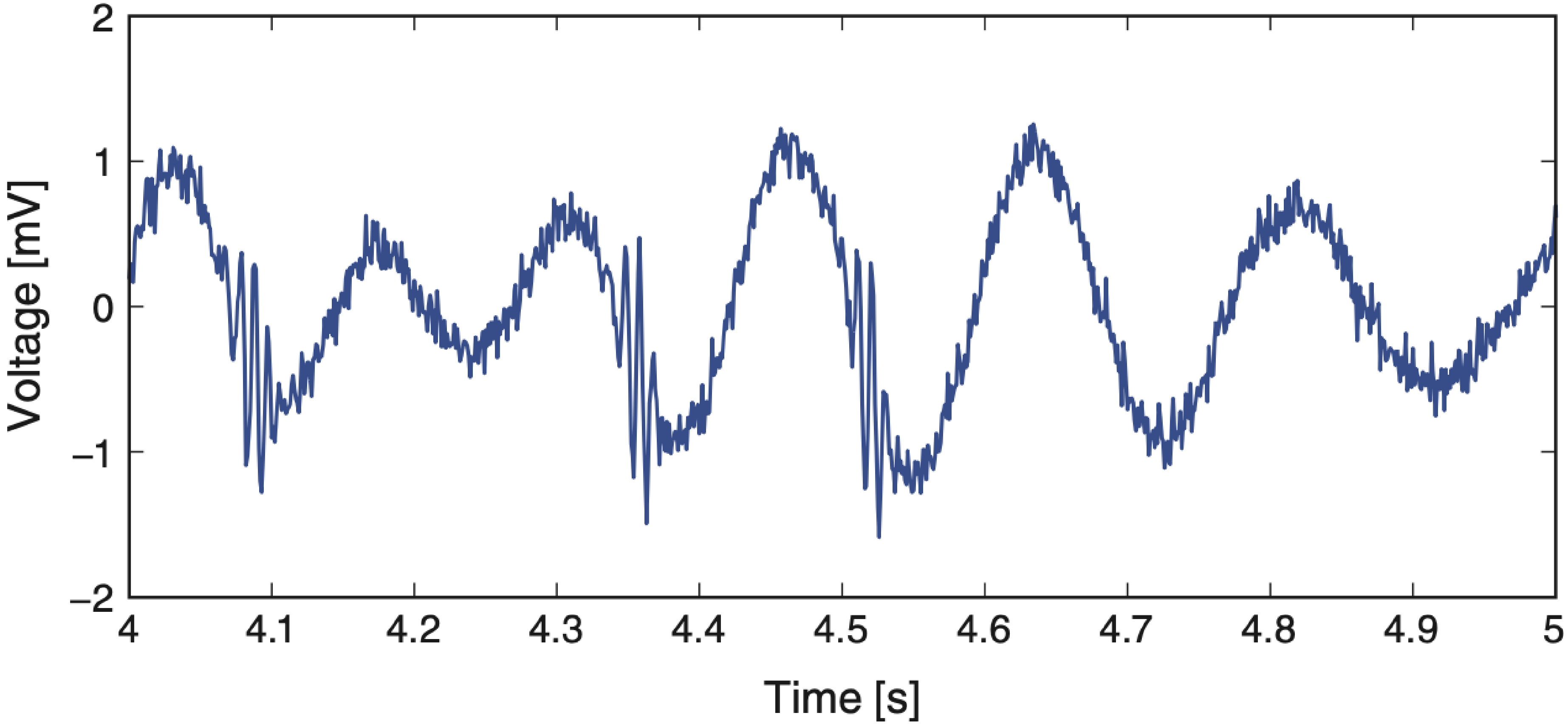
Outline

Data 100 s of local field potential data sampled at 1000 Hz.

Goal Characterize the coupling between rhythms of different frequency.

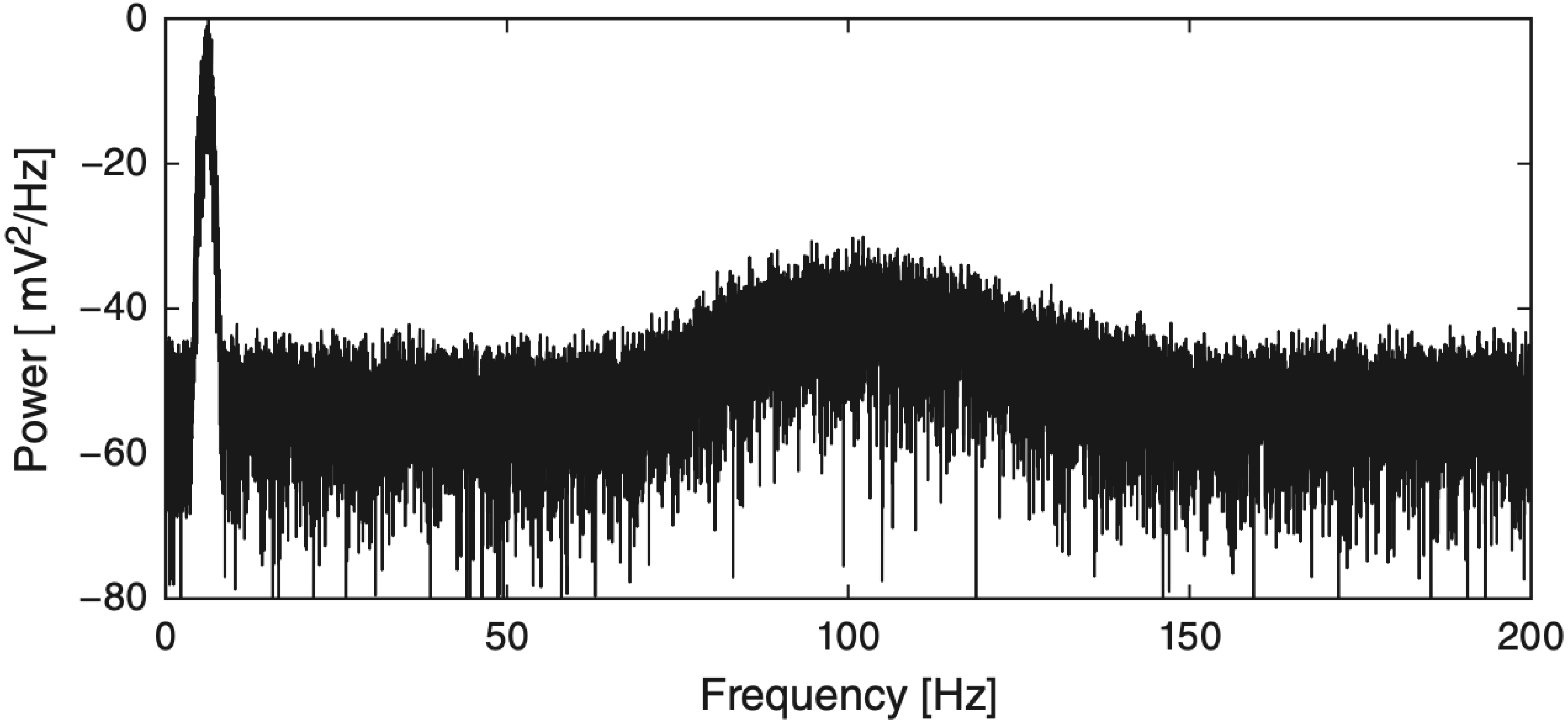
Tools Hilbert transform, analytic signal, instantaneous phase, cross-frequency coupling.

Data



Q. How to make sense of these data?

Spectrum



Q. What do you see?

CFC in three steps

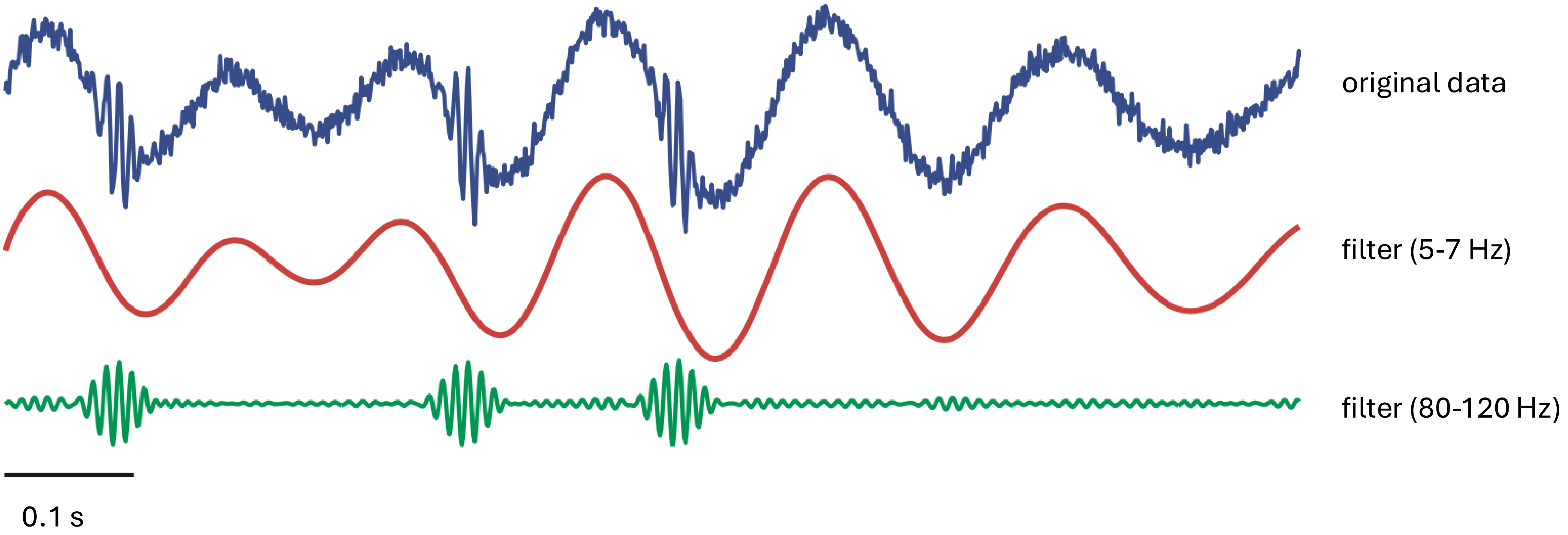
CFC analysis steps

1. Filter the data into high- and low-frequency bands.
2. Extract the amplitude and phase from the filtered signals.
3. Determine if the phase and amplitude are related.

Let's perform each step …

CFC – Step 1

Filter the Data into High- and Low-Frequency Bands

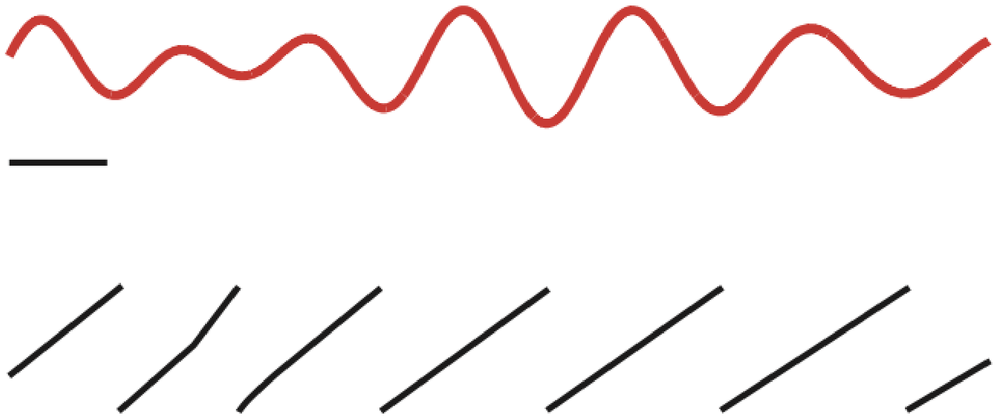


Q. Why did we filter in these bands?

CFC – Step 2

Extract the Amplitude and Phase from Filtered Signals

filter (5-7 Hz)



filter (80-120 Hz)

phase (of low frequency band)

amplitude envelope (of high frequency band)

Q. How?

Hilbert transform

$$y = H(x)$$

$$H(x) = \begin{cases} -\pi/2 \text{ phase shift} & \text{if } f > 0, \\ 0 \text{ phase shift} & \text{if } f = 0, \\ \pi/2 \text{ phase shift} & \text{if } f < 0. \end{cases}$$

The Hilbert transform $H(x)$ of the signal x produces a phase shift of ± 90 degrees for $\mp$ frequencies of x.

Hilbert transform

Define: Analytic signal z

$$z = x + iy = x + iH(x)$$

Impact: remove negative frequencies from z

Q. How?

Hilbert transform

Q. What does it do?

<u>Ex.</u>

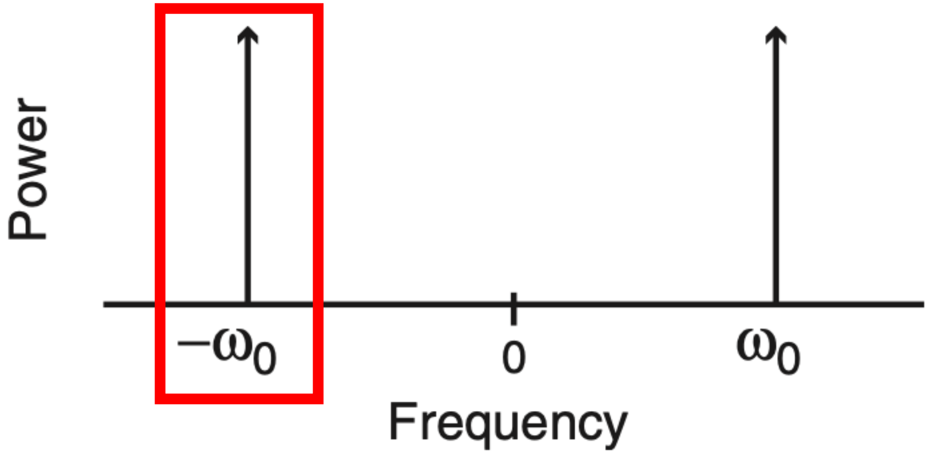
$$x_0 = 2\cos(2\pi f_o t) = 2\cos(\omega_0 t) \quad \text{where} \quad \omega_0 = 2\pi f_o$$

Euler's formula

$$x_0 = e^{i\omega_0 t} + e^{-i\omega_0 t}$$

positive frequency   negative frequency

we usually ignore this one



<u>Note</u>: The spectrum has <u>two peaks</u>

Hilbert transform

Apply the Hilbert transform to x_0.

$$x_0 = e^{i\omega_0 t} + e^{-i\omega_0 t}$$

$$H(x) = \begin{cases} -\pi/2 \text{ phase shift} \quad \text{if } f > 0, \end{cases}$$

$\boxed{\rightarrow \text{ multiply positive frequency part of } x \text{ by } -i}$

Q. Really?

Consider $e^{i\omega_0 t}$

positive frequency part of x

Shift $e^{i\omega_0 t}$ by $-\frac{\pi}{2}$

$\rightarrow e^{i(\omega_0 t - \frac{\pi}{2})} \quad \rightarrow e^{i\omega_0 t} e^{-i\pi/2} \quad \rightarrow e^{i\omega_0 t}(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}) \rightarrow \boxed{e^{i\omega_0 t}(-i)}$

Hilbert transform

Apply the Hilbert transform to x_0.

$$x_0 = e^{i\omega_0 t} + e^{-i\omega_0 t}$$

$$H(x) = \begin{cases} -\pi/2 \text{ phase shift} & \text{if } f > 0, \\ 0 \text{ phase shift} & \text{if } f = 0, \\ \pi/2 \text{ phase shift} & \text{if } f < 0. \end{cases} \Rightarrow H(x) = \begin{cases} -ix & \text{if } f > 0, \\ 0 & \text{if } f = 0, \\ ix & \text{if } f < 0. \end{cases}$$

So, $$x_0 = \underset{\text{multiply by } -i}{e^{i\omega_0 t}} + \underset{\text{multiply by } i}{e^{-i\omega_0 t}}$$

$$y_0 = H(x_0) = -ie^{i\omega_0 t} + ie^{-i\omega_0 t}$$

Euler's formula

$$\boxed{= 2\sin(\omega_0 t)}$$

Hilbert Transform of x_0 (a cosine function) is a sine function.

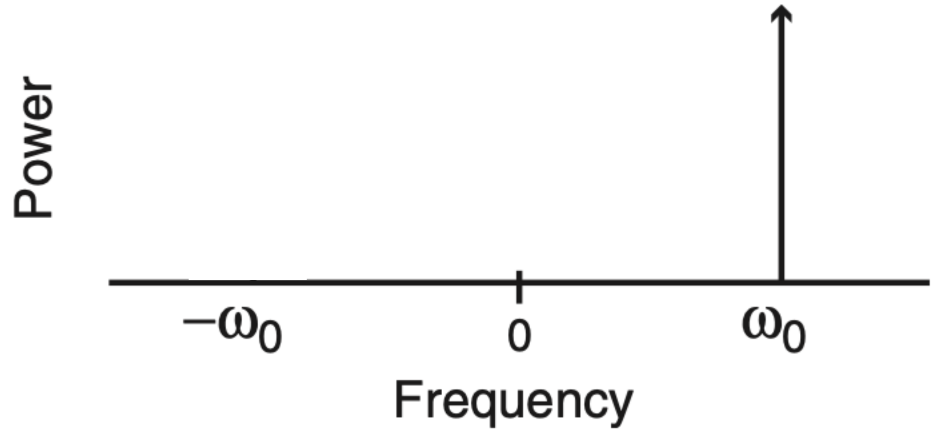
Hilbert transform

Analytic signal z

$$z = x + iy = \underset{2\cos(\omega_0 t)}{x} + i\,\underset{2\sin(\omega_0 t)}{H(x)}$$

$$= 2\cos(\omega_0 t) + i\,2\sin(\omega_0 t)$$

$$\boxed{= 2e^{i\omega_0 t}}$$

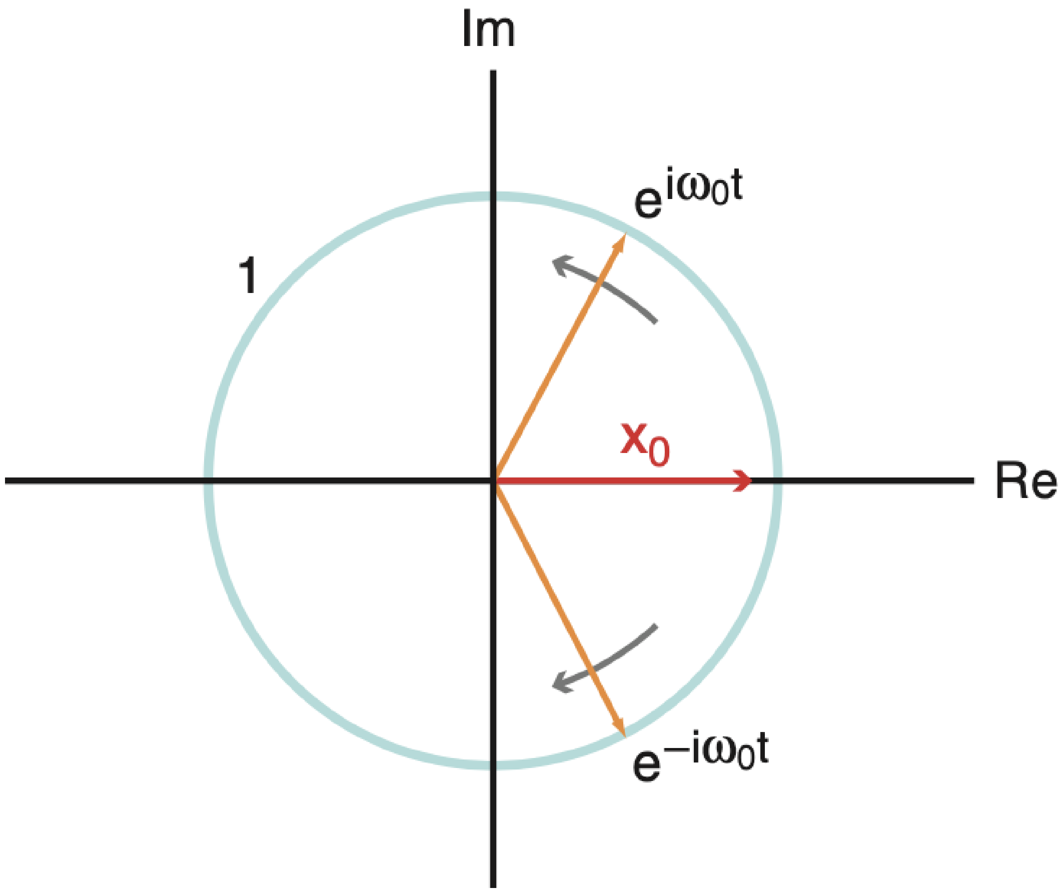


The analytic signal contains
<u>no negative frequencies</u>

Hilbert transform

<u>Original signal</u> $x_0 = 2\cos(2\pi f_o t) = e^{i\omega_0 t} + e^{-i\omega_0 t}$

Complicated (2 complex exponentials)



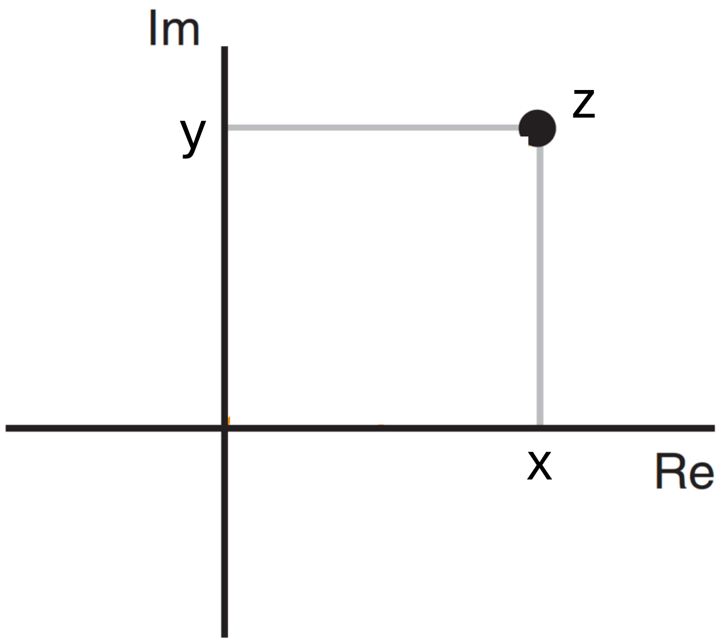
<u>Analytic signal</u> $z_0 = 2e^{i\omega_0 t}$

Simple (1 complex exp)

A point in the complex plane

Hilbert transform

Analytic signal $\quad z = x + i\,y \quad$ A point in the complex plane



$$z(t) = \underbrace{A(t)}_{\text{amplitude}}\, e^{i\,\overbrace{\phi(t)}^{}}$$

amplitude $\qquad$ phase

$\boxed{\text{Get the } \textbf{amplitude} \text{ and } \textbf{phase} \text{ from the analytic signal}}$

Ex.

$z_0(t) = 2e^{i\omega_0 t} \qquad A(t) = 2 \qquad \phi(t) = \omega_0\, t$

CFC in three steps

CFC analysis steps

1. Filter the data into high- and low-frequency bands.
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CFC – Step 3

Determine if the Phase and Amplitude are Related

Define the two-column vector

$$\begin{pmatrix} \phi(1) & A(1) \\ \phi(2) & A(2) \\ \phi(3) & A(3) \\ \vdots & \vdots \end{pmatrix}$$

phase of low frequency band activity

amplitude of high frequency band activity

Make a histogram

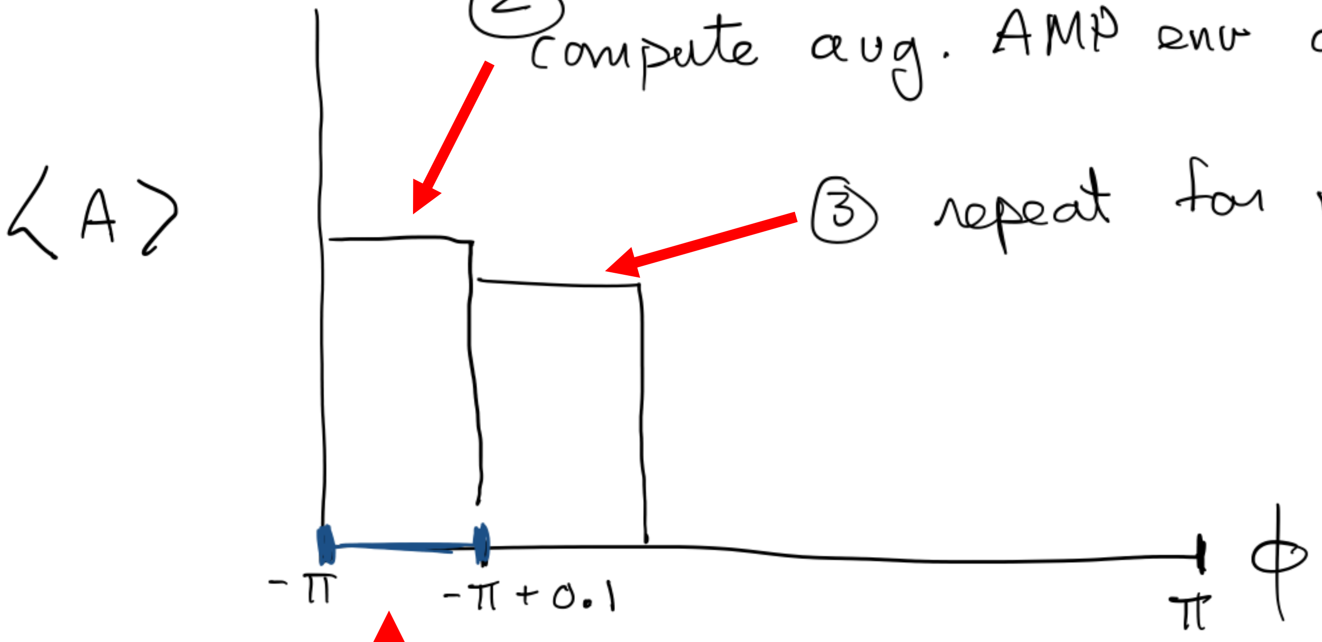
CFC – Step 3

Determine if the Phase and Amplitude are Related

Divide the phase into bins of size 0.1

(2) compute avg. AMP env at the indices in (1)

(3) repeat for next phase bin

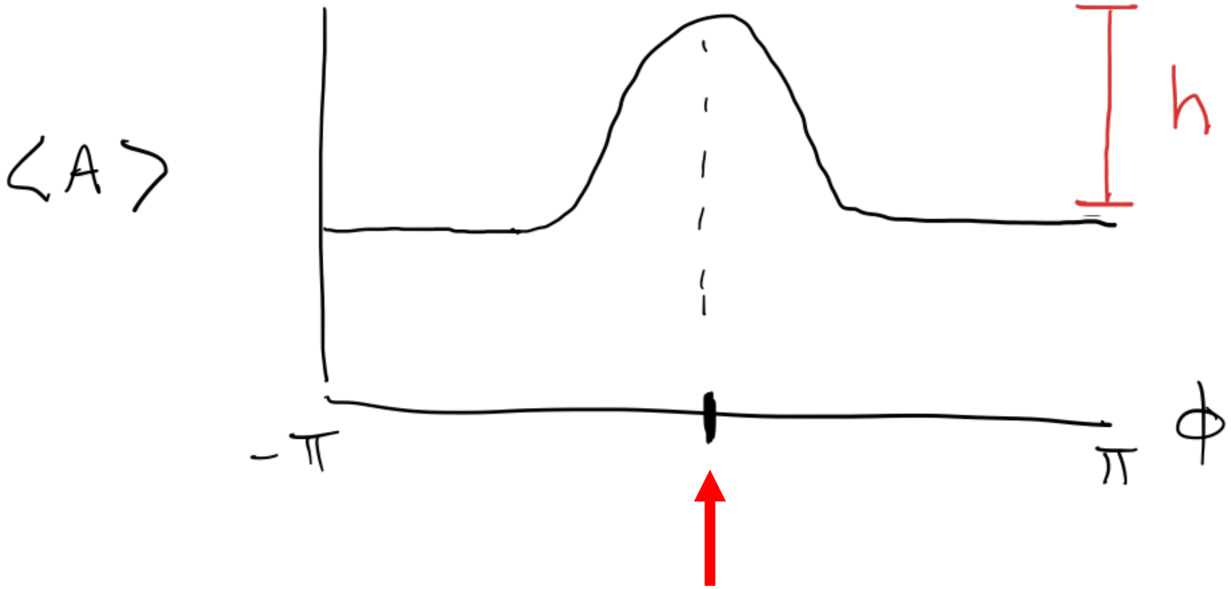


(1) Find all ϕ in this range $[-\pi, -\pi + 0.1]$
& get indices $\{1, 2, 3, 10, 11, 12, 41, 42, 43, \ldots\}$

CFC – Step 3

Determine if the Phase and Amplitude are Related

If phase modulate amplitude



summarize extent of modulation

Q. What does no phase-amplitude coupling look like in the plot?

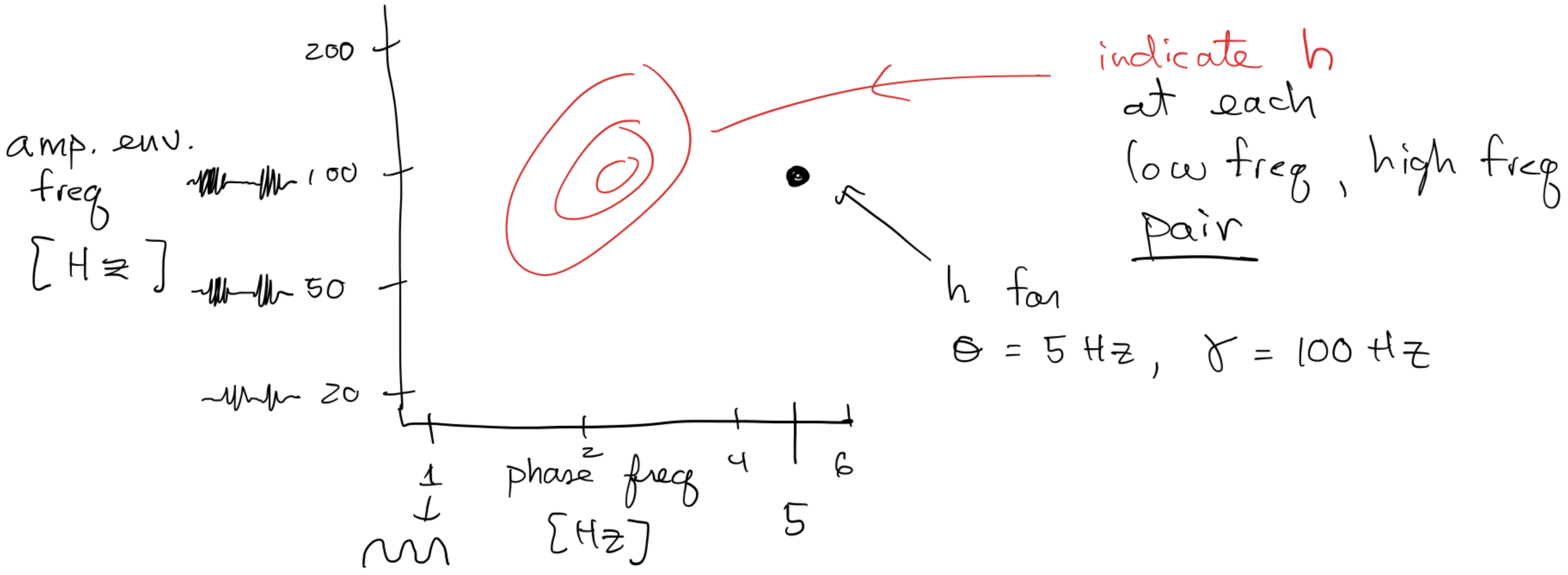
At this phase, amplitude envelope is big

➔ phase modulates amplitude

CFC – Step 4

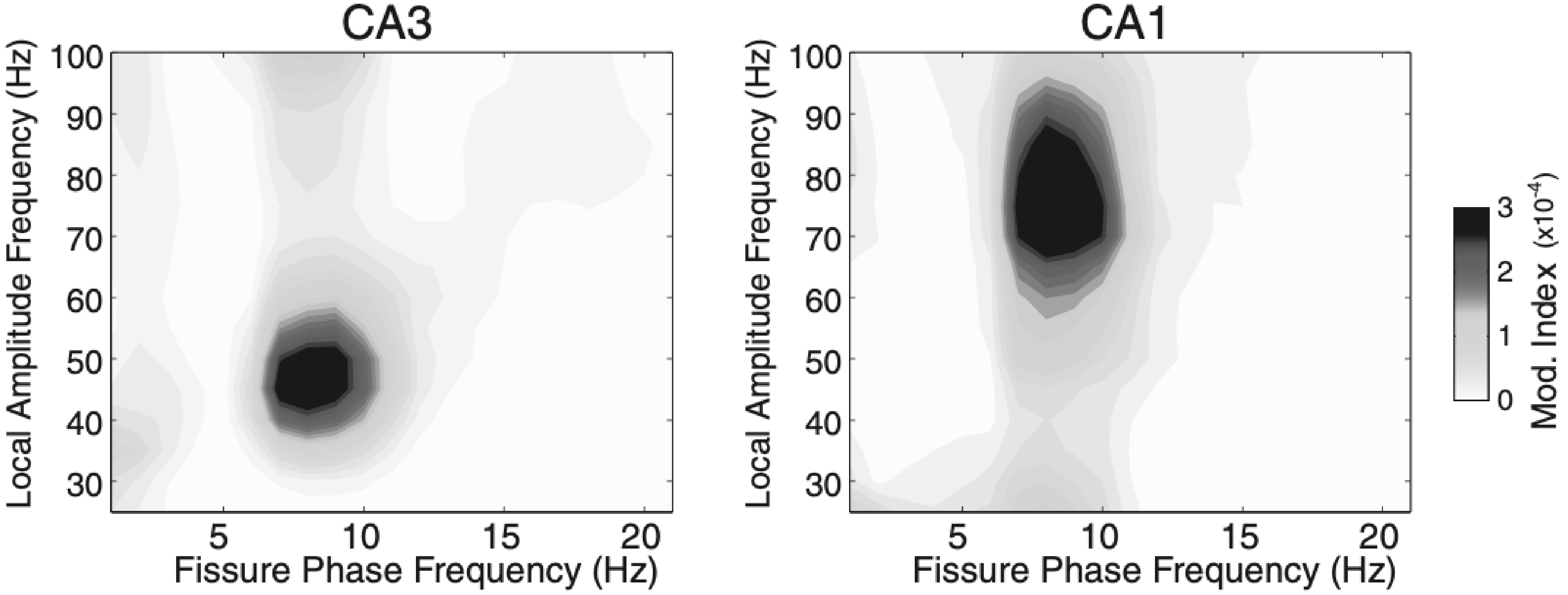
(optional): Repeat for other frequencies.

Summarize in a <u>comodulogram</u>



CFC – Step 4 (optional): Repeat for other frequencies.

Summarize in a <u>comodulogram</u>



[Tort et al., J Neurophysiol, 2010]

CFC in three steps

CFC analysis steps

1. Filter the data into high- and low-frequency bands.
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Python