Aperiodic Exponent

Instructor: Mark Kramer

Today

nature neuroscience

TECHNICAL REPORT

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Parameterizing neural power spectra into periodic and aperiodic components

Thomas Donoghue[1,7] ✉, Matar Haller[2,7], Erik J. Peterson[1,7], Paroma Varma[2], Priyadarshini Sebastian[1], Richard Gao[1], Torben Noto[1], Antonio H. Lara[2], Joni D. Wallis[2,3], Robert T. Knight[2,3], Avgusta Shestyuk[2,8] and Bradley Voytek[1,4,5,6,8] ✉

Nature Neuroscience **23**, 1655–1665 (2020) | Cite this article

Examples

Analysis of aperiodic activity in obsessive-compulsive disorder and major depression

Zhang, HC; Jahanian-Najafabadi, A; (...); Hommel, B
Nov 18 2025 | SCIENTIFIC REPORTS 15(1)

Task-induced 1/f slope modulation as a paradigm-independent marker of cognitive control in multiple sclerosis

Akbarian, F; Gyurkovics, M; (...); Van Schependom, J
Oct 30 2025 | IMAGING NEUROSCIENCE 3

Modulation of aperiodic EEG activity provides sensitive index of cognitive state changes during working memory task

Frelih, T; Matkovic, A; (...); Repovs, G
Nov 12 2025 | ELIFE 13

Age-related changes in 'cortical' 1/f dynamics are linked to cardiac activity

Schmidt, F; Danböck, SK; (...); Weisz, N
Oct 2 2025 | ELIFE 13

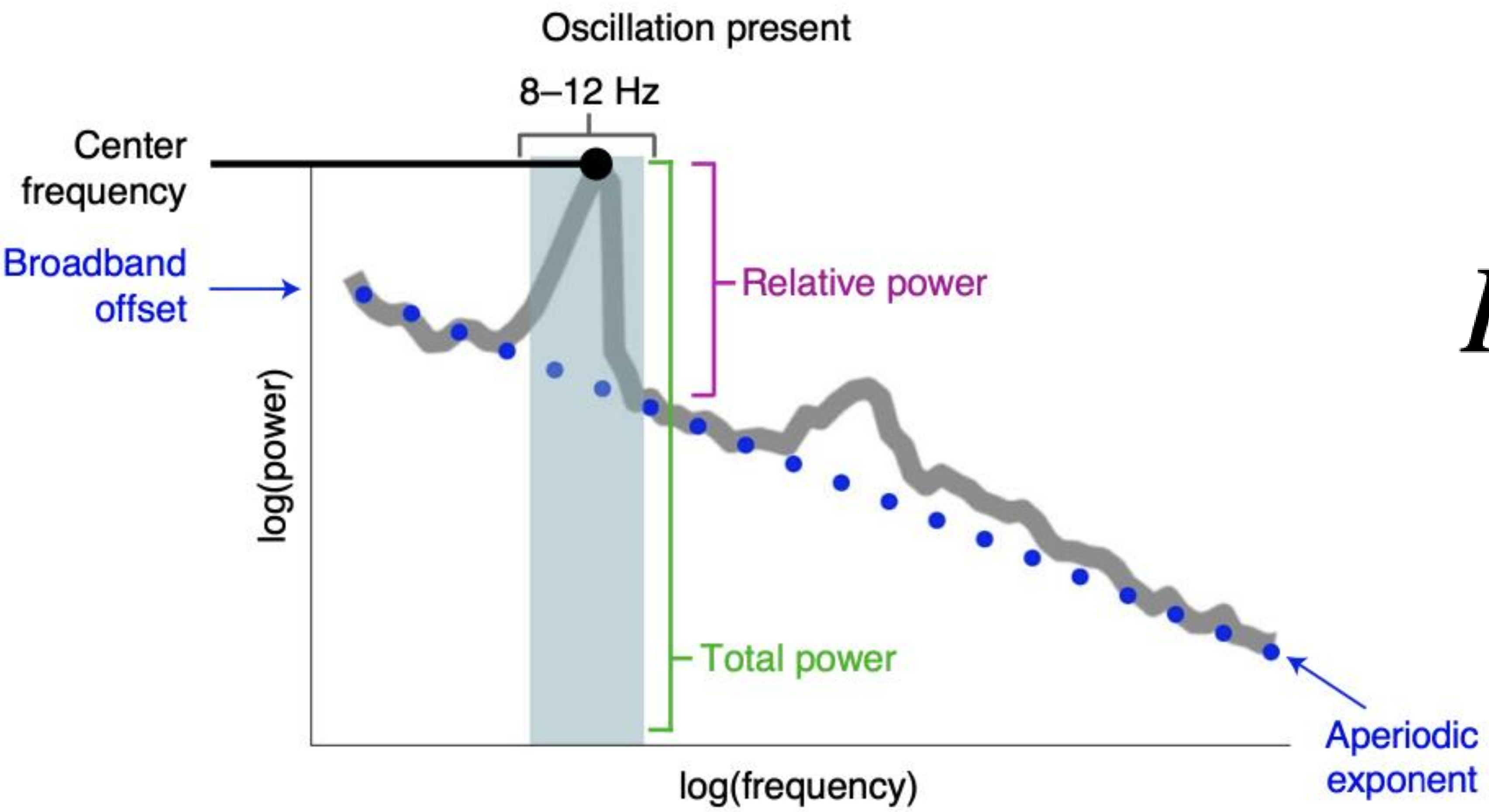
Aperiodic neural activity during speech comprehension in aging: Insights into cognitive effort

Woods, SJ; Silcox, JW and Payne, BR
Nov 2025 (Early Access) | COGNITIVE AFFECTIVE & BEHAVIORAL NEUROSCIENCE

Prestimulus Periodic and Aperiodic Neural Activity Shapes McGurk Perception

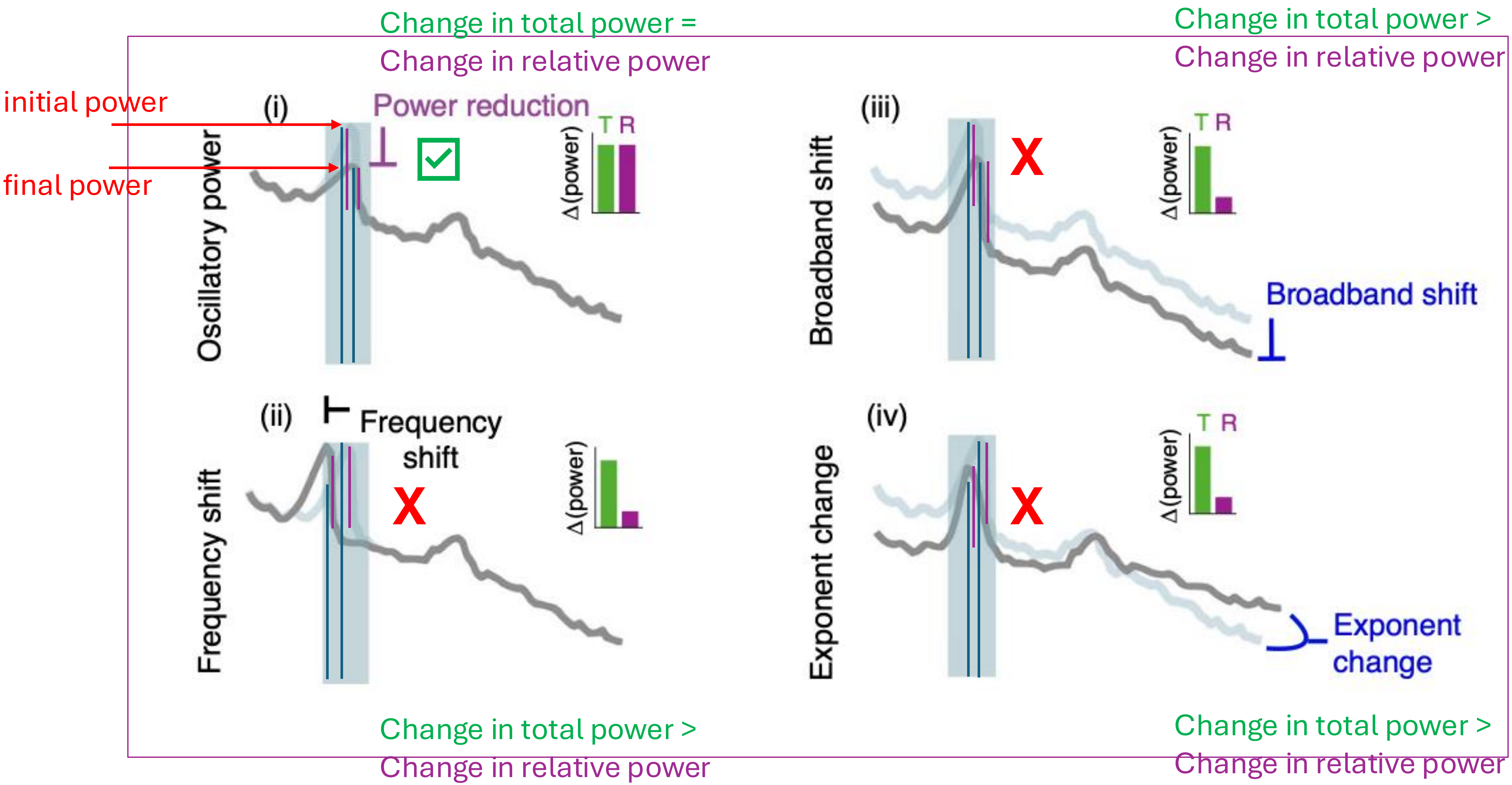
Singh, VAV; Kumar, VG; (...); Roy, D
Oct 2025 | ENEURO 12(10)

What is the aperiodic exponent?



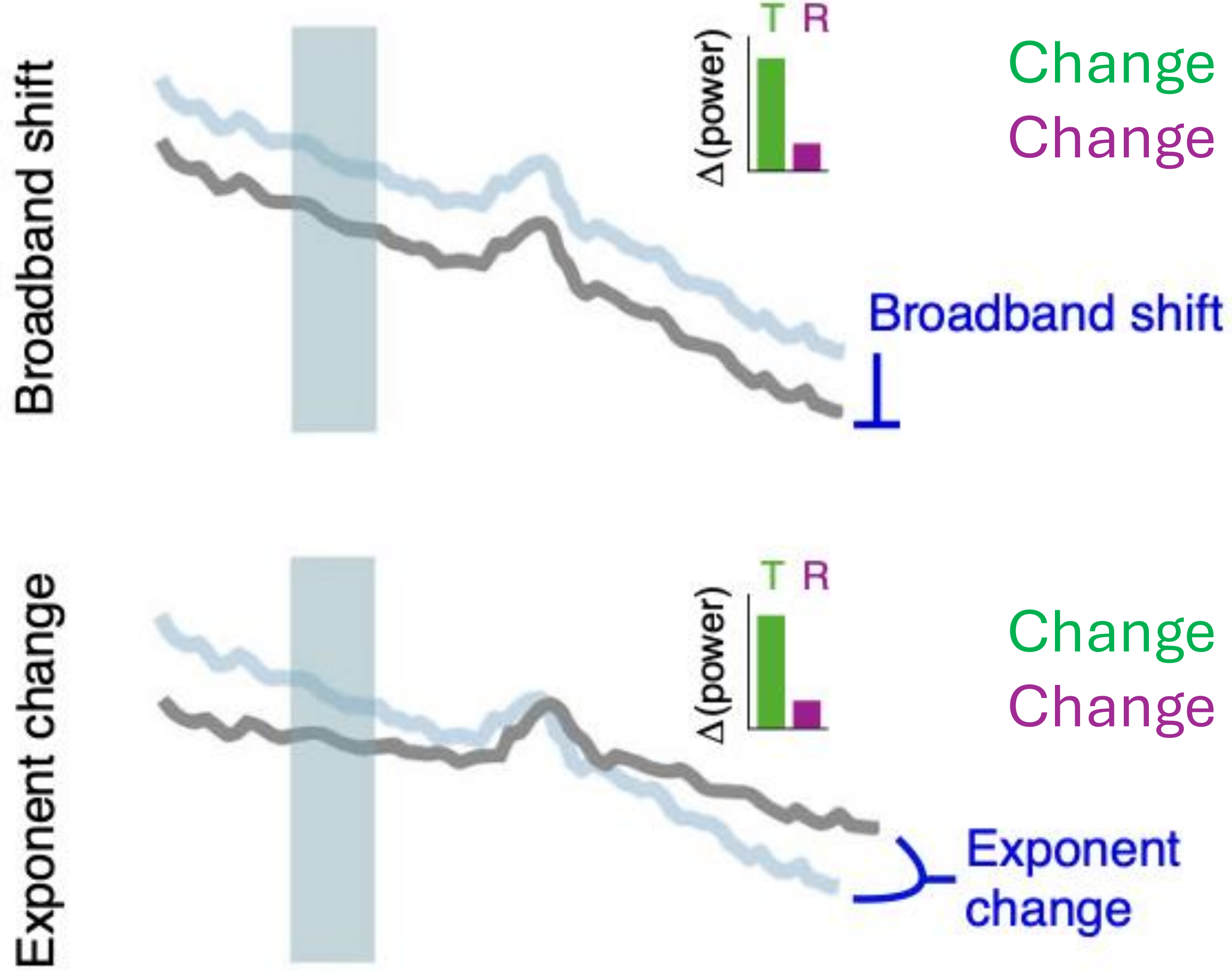
$$P \propto f^{\beta}$$

Confounds in power changes



Confounds in power changes

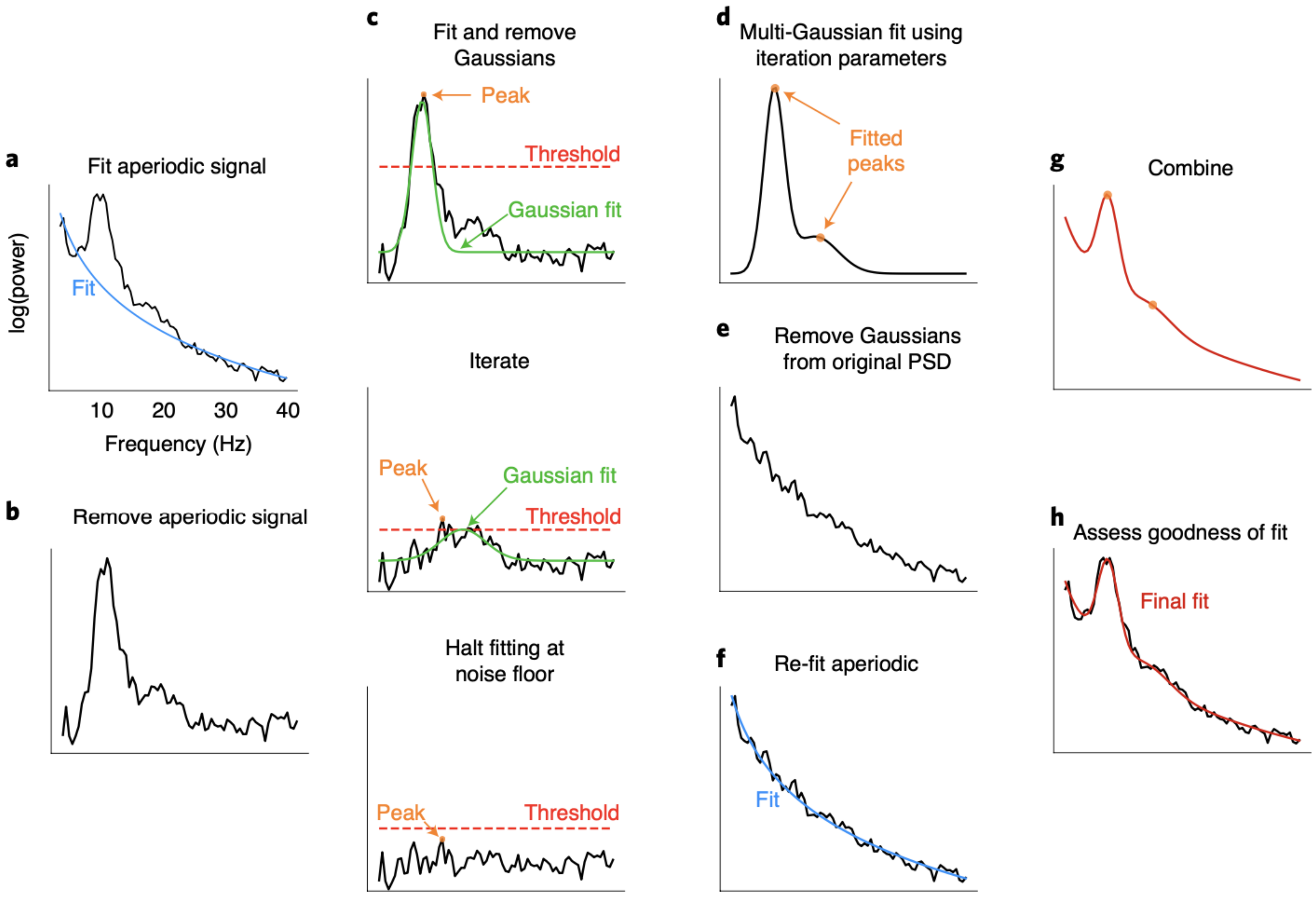
No peak



Change in total power >
Change in relative power

Change in total power >
Change in relative power

Aperiodic exponent method



Aperiodic exponent method

$$PSD = L + \sum_{n=0}^{N} G_n$$

aperiodic component *L* + *N* total Gaussians, *G*.
G_n is a Gaussian fit to a peak, for *N* total peaks extracted from spectrum.

$$G_n = a \times \exp\left(\frac{-(F-c)^2}{2w^2}\right)$$

a is the power of the peak, in log10(power) values
c is the center frequency, in Hz
w is the standard deviation of the Gaussian, also in Hz
and *F* is the vector of input frequencies.

$$L = b - \log(k + F^{\chi})$$

b is the broadband offset
χ is the exponent
k is the ‘knee’ parameter, controlling for bend in the aperiodic component
F as the vector of input frequencies.

Aperiodic exponent method

Software https://github.com/fooof-tools/fooof

README Code of conduct Apache-2.0 license

Spectral Parameterization

repo status Active | pypi v1.1.0 | build passing | codecov 97% | license Apache License, 2.0
python 3.6 | 3.7 | 3.8 | 3.9 | 3.10 | 3.11 | paper nn10.1038

Spectral parameterization (specparam, formerly fooof) is a fast, efficient, and physiologically-informed tool to parameterize neural power spectra.

WARNING: this Github repository has been updated to a major update / breaking change from previous releases, which were under the fooof name, and now contains major breaking update for the new specparam version of the code. The new version is not fully released, though a test version is available (see installation instructions below).

Overview

Aperiodic exponent method

Let’s try it

Stable Version

To install the latest stable release, use pip:

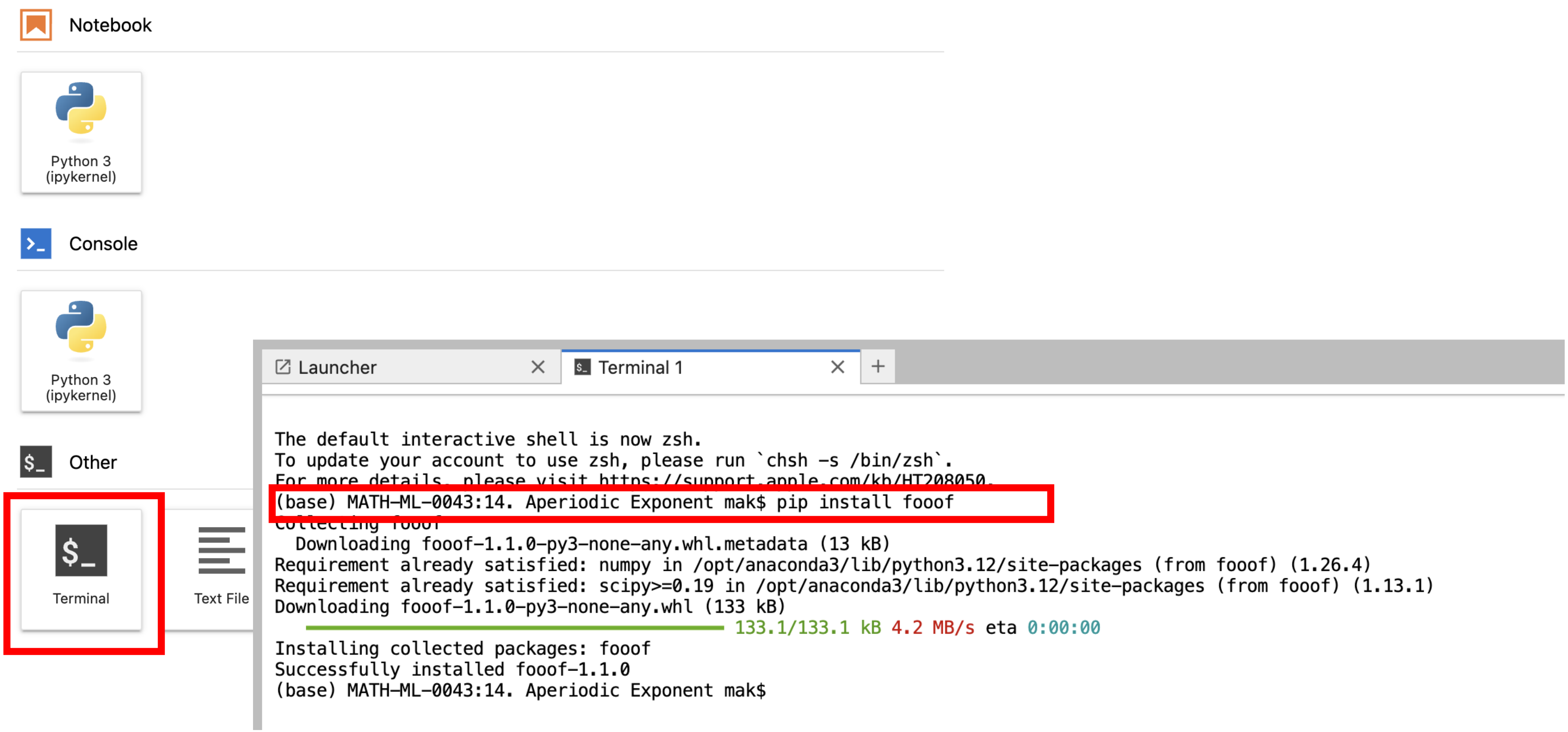
```
$ pip install fooof
```

The module can also be installed with conda, from the conda-forge channel:

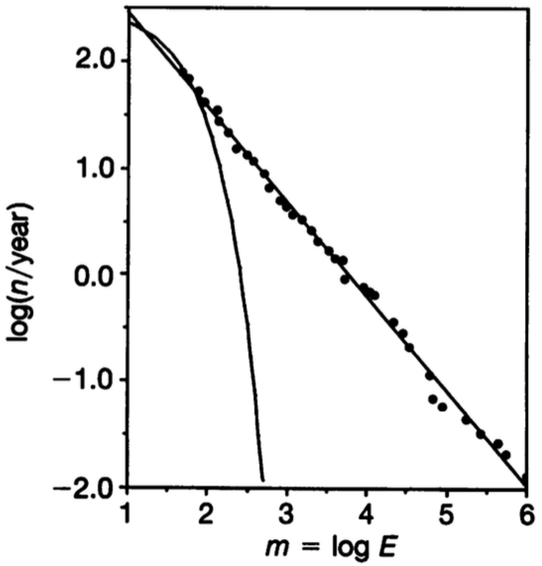
```
$ conda install -c conda-forge fooof
```

Python

Aperiodic exponent method



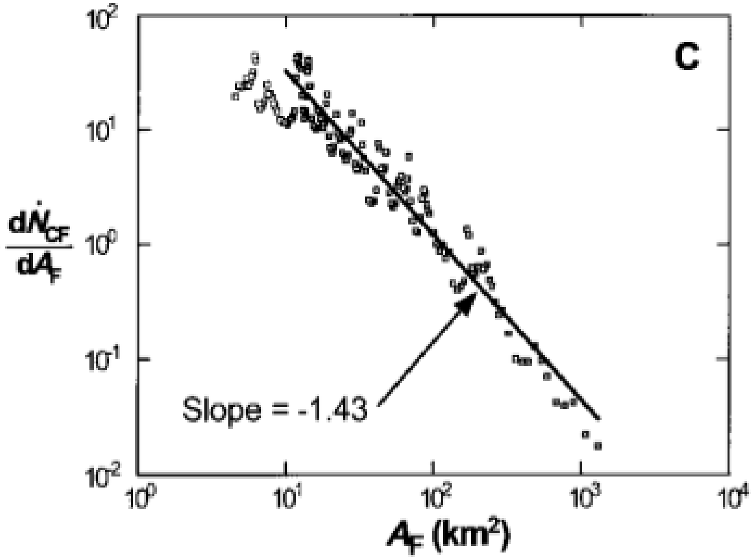
Power laws are common in nature



Energy of earthquake

Bak & Paczuski, PNAS, 1995

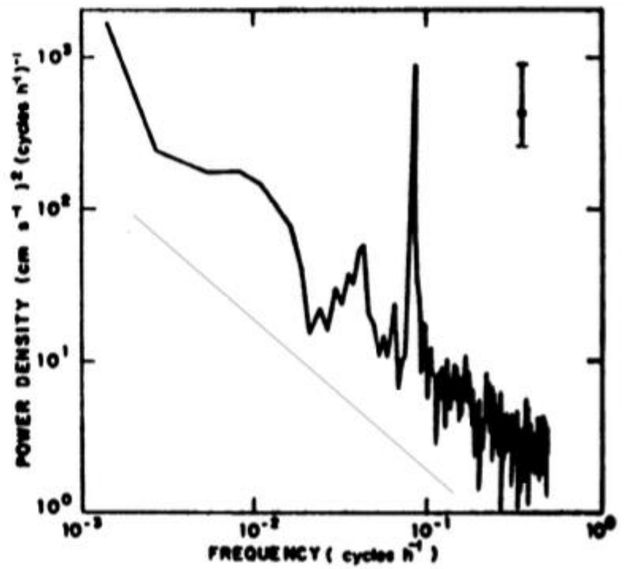
Number of forest fires



Number of trees

Malamud et al, Science, 1998

power spectrum of the east-west component of ocean current velocity



a.

Milotti, 2002

What does it mean (in the brain)?

Aperiodic exponent: Example 1

The Journal of Neuroscience, September 23, 2015 • 35(38):13257–13265 • **13257**

Behavioral/Cognitive

Age-Related Changes in 1/*f* Neural Electrophysiological Noise

Bradley Voytek,[1] Mark A. Kramer,[3] John Case,[4] Kyle Q. Lepage,[3] Zechari R. Tempesta,[1] Robert T. Knight,[4,5] and Adam Gazzaley[1,2]

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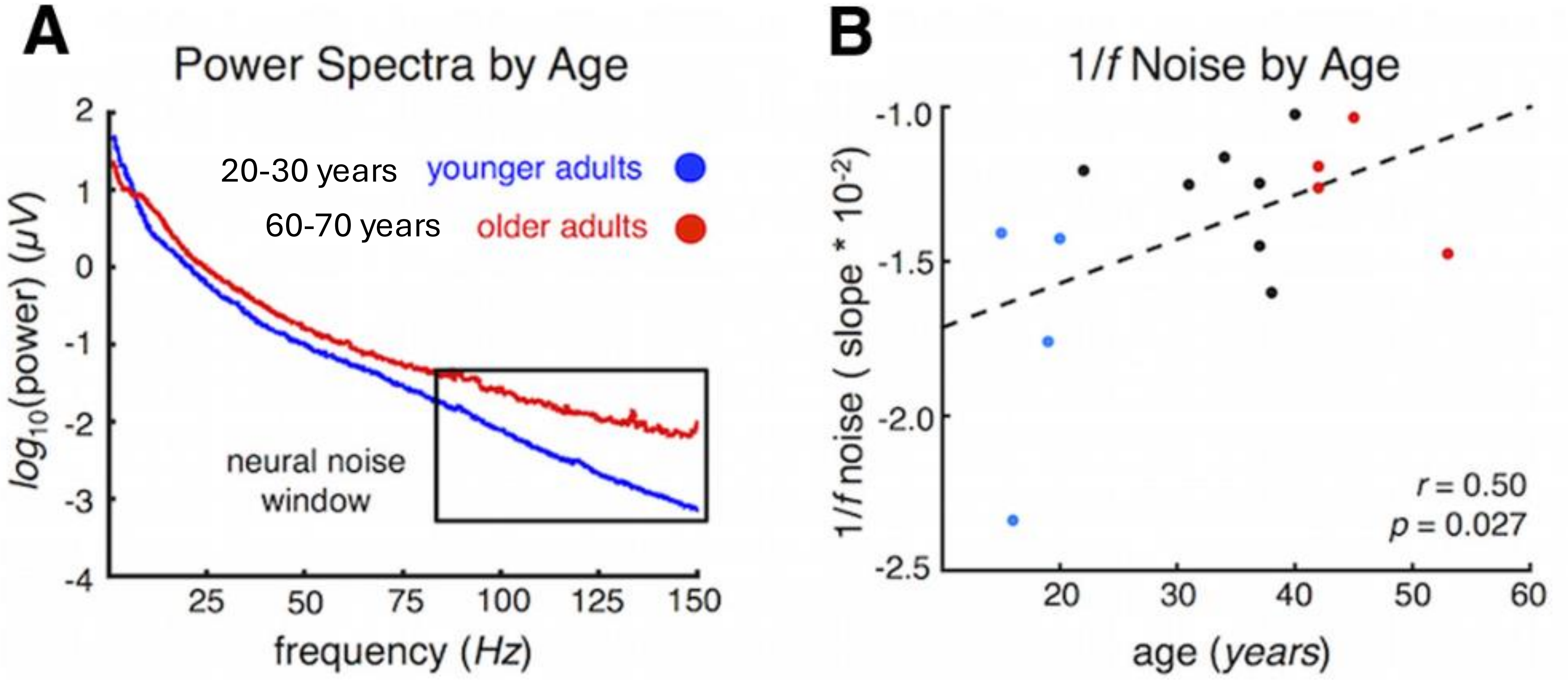
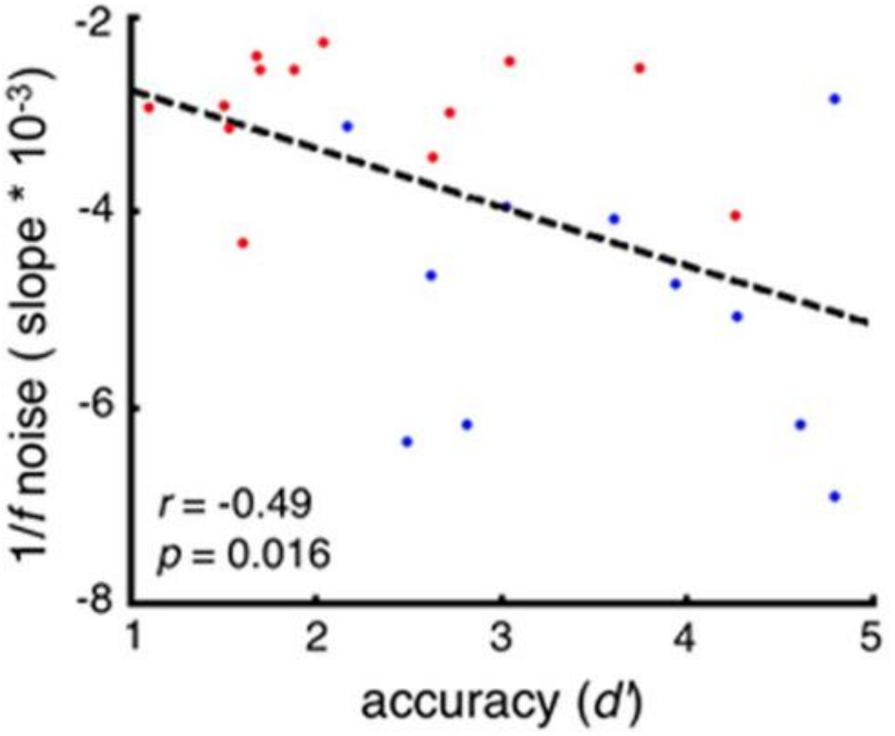
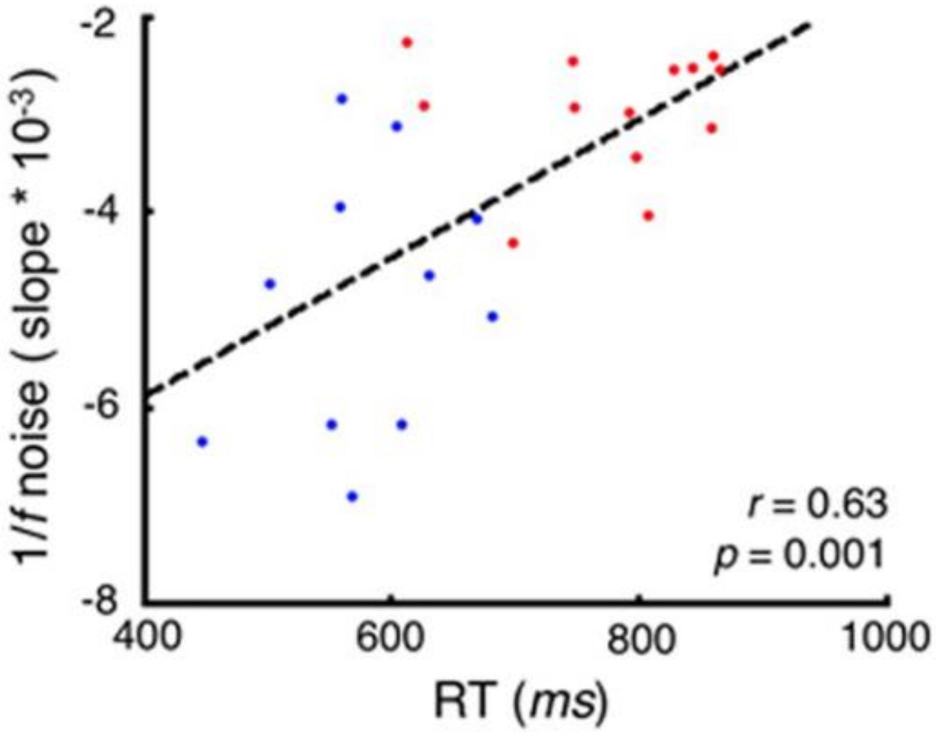
Aperiodic exponent: Example 1



High frequency “flattens out” in older adults: more “noise” in older brains

Aperiodic exponent: Example 1





low accuracy
more positive slope
more neural noise

long reaction time
more positive slope
more neural noise

Aperiodic exponent: Example 2

NeuroImage 158 (2017) 70–78

Contents lists available at ScienceDirect

NeuroImage

journal homepage: www.elsevier.com/locate/neuroimage

Inferring synaptic excitation/inhibition balance from field potentials

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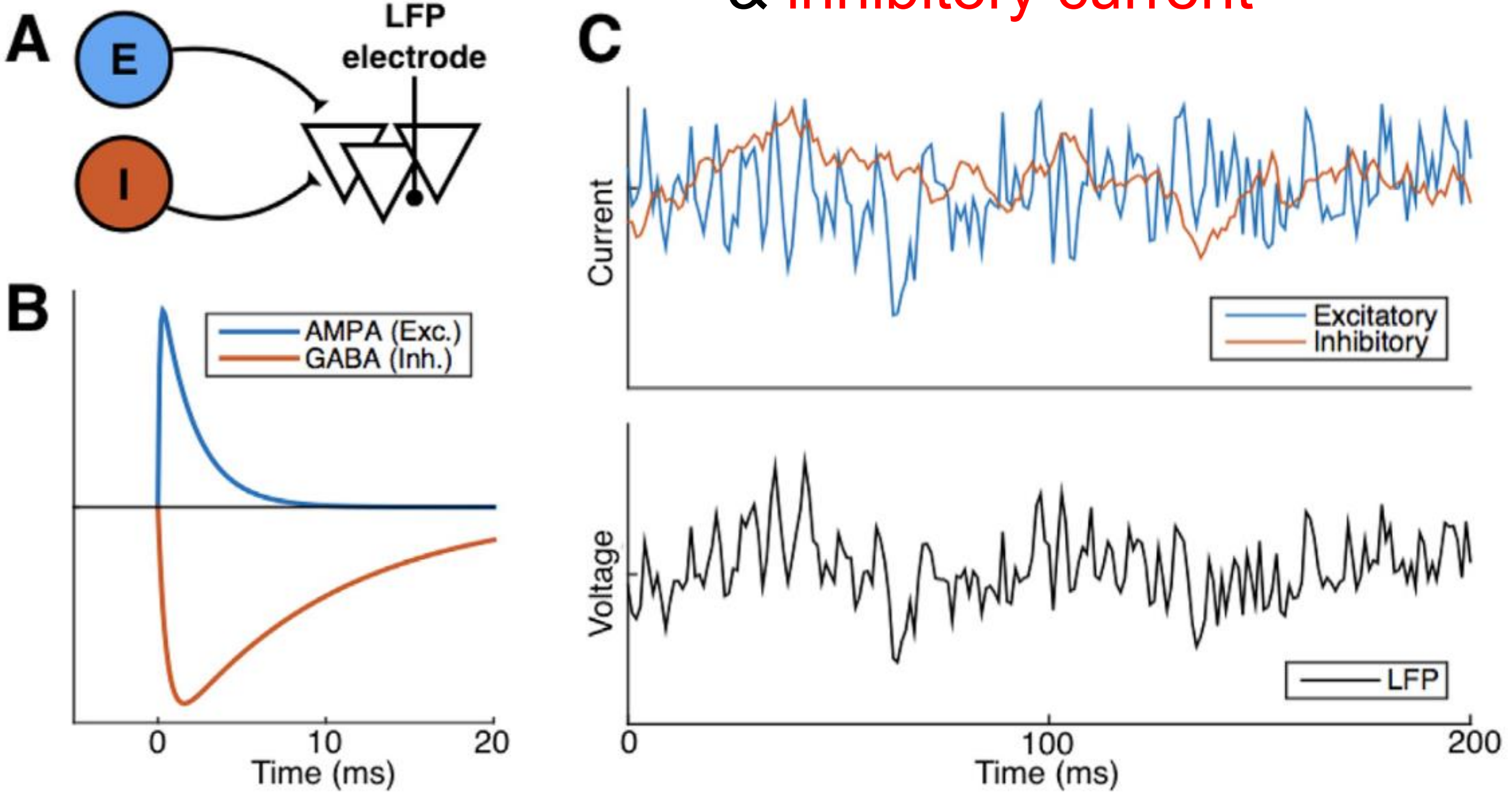
Aperiodic exponent: Example 2

Simulate the LFP

Coin flip

total excitatory current
& inhibitory current

Spectra from (C)

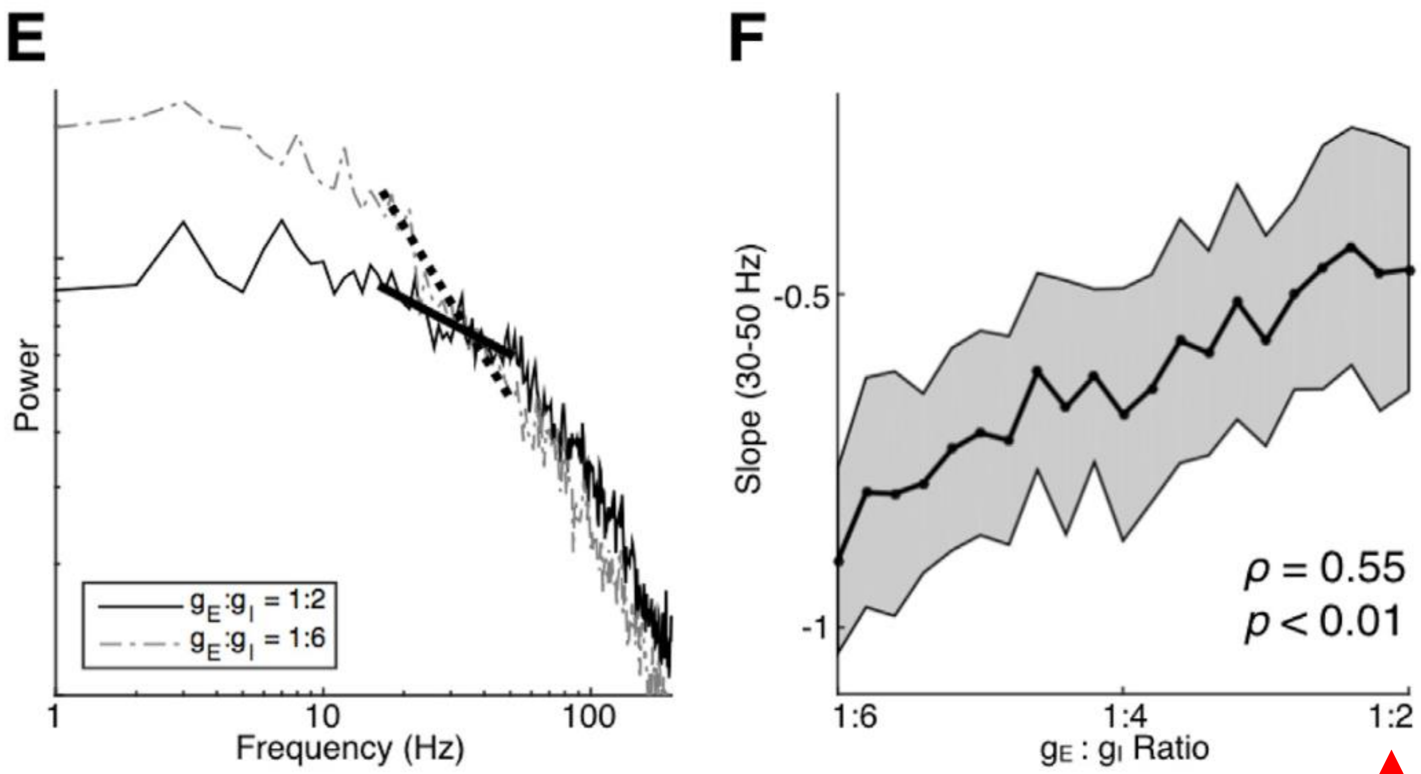


Convolve

LFP = sum total excitatory
& inhibitory current

Inhibitory = steeper slope

Aperiodic exponent: Example 2



More inhibition → steeper slope

More excitation
→ shallower slope

Conclusion: aperiodic exponent reflects E/I balance

Aperiodic exponent: Example 3

LETTER Communicated by Peter J. Thomas

A General, Noise-Driven Mechanism for the 1/f-Like Behavior of Neural Field Spectra

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Aperiodic exponent: Example 3

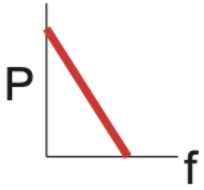
Many proposed mechanisms

excitatory/inhibitory balance [*Gao et al, NeuroImage, 2017*]
low-pass frequency filtering by dendrites [*Buzsáki et al, Nat Rev Neuro, 2012*]
low-pass frequency filtering by the extracellular medium [*Bédard et al, PRL, 2006*]
nonideal resistance in cell membranes [*Bédard & Destexhe, Biophys J, 2008*]
stochastic firing of neurons [*Miller et al, PLOS Comp Bio, 2009*]
stochastic synaptic conductances [*Rudolph et al, J Neurophys, 2005*]
stochastically driven damped oscillators [*Evertz et al, PLOS Comp Bio, 2022*]
local homogenous connectivity [*Jirsa, Philos Trans A Math, 2009*]
transient oscillations at different frequencies & amplitudes [*He et al, Neuron, 2010*]
network mechanisms & neuronal recruitment [*Buzsáki et al, Nat Rev Neuro, 2012*]
fractal properties [*Pritchard, Int J Neurosci, 1992*]
critical transitions [*O'Bryne & Jerbi, Trends in Neurosci, 2022*]
self-organized criticality [*Cocchi et al, Progress in Neurobiology, 2017*]

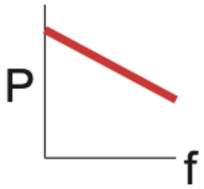
Aperiodic exponent:

β	Reference	Recording Modality	Frequency	Experimental Condition
−0.08	(Colombo et al., 2019)	Scalp EEG ($n = 5$)	20–40 Hz	Anesthesia (ketamine)
−1.12	(Lanzone et al., 2022)	Scalp EEG ($n = 16$)	1–40 Hz	Eyes closed
−1.3	(Adelhöfer et al., 2021)	Scalp EEG ($n = 74$)	2–40 Hz	Behavioral experiments
−1.44	(Adelhöfer et al., 2021)	Scalp EEG ($n = 74$)	2–40 Hz	Behavioral experiments
−1.48	(Lanzone et al., 2022)	Scalp EEG ($n = 18$)	1–40 Hz	Stroke patients
−1.51	(Robertson et al., 2019)	Scalp EEG ($n = 78$)	4–50 Hz	Resting state
−1.67	(Robertson et al., 2019)	Scalp EEG ($n = 76$)	4–50 Hz	Resting state
−1.84	(Lendner et al., 2020)	Scalp EEG ($n = 9$)	30–45 Hz	Wakefulness
−1.86	(Fransson et al., 2013)	Scalp EEG ($n = 7$)	0.2–30 Hz	Sleep
−1.87	(Lendner et al., 2020)	Scalp EEG ($n = 14$)	30–45 Hz	Resting state
	bo et al., 2019)	Scalp EEG ($n = 5$)	20–40 Hz	Wakefulness
	on et al., 2013)	Scalp EEG ($n = 15$)	0.2–30 Hz	Sleep
	an et al., 2000)	Intracranial EEG ($n = 5$)	0.5–150 Hz	Resting state
	et al., 2021)	Scalp EEG ($n = 175$)	2–48 Hz	NREM sleep
	l., 2010)	Intracranial EEG ($n = 5$)	1–100 Hz	Wakefulness
	bo et al., 2019)	Scalp EEG ($n = 5$)	20–40 Hz	Wakefulness
	h et al., 2022)	Scalp EEG ($n = 251$)	2–48 Hz	NREM sleep
	et al., 2021)	Scalp EEG ($n = 175$)	2–48 Hz	NREM sleep
	er et al., 2020)	Intracranial EEG ($n = 12$)	30–45 Hz	Wakefulness
	l., 2010)	Intracranial EEG ($n = 5$)	1–100 Hz	Slow wave sleep
	er et al., 2020)	Intracranial EEG ($n = 10$)	30–45 Hz	Wakefulness
−3.1	(Lendner et al., 2020)	Scalp EEG ($n = 9$)	30–45 Hz	Anesthesia
−3.13	(Colombo et al., 2019)	Scalp EEG ($n = 5$)	20–40 Hz	Wakefulness
−3.46	(Lendner et al., 2020)	Scalp EEG ($n = 14$)	30–45 Hz	N3 Sleep
−3.59	(Colombo et al., 2019)	Scalp EEG ($n = 5$)	20–40 Hz	Anesthesia (xenon)
−3.67	(Lendner et al., 2020)	Scalp EEG ($n = 14$)	30–45 Hz	N2 sleep
−3.69	(Lendner et al., 2020)	Intracranial EEG ($n = 10$)	30–45 Hz	N3 sleep
−4	(Miller et al., 2009)	Intracranial EEG ($n = 20$)	80–500 Hz	Behavioral experiments
−4.15	(Lendner et al., 2020)	Intracranial EEG ($n = 10$)	30–45 Hz	REM sleep
−4.34	(Lendner et al., 2020)	Intracranial EEG ($n = 12$)	30–45 Hz	Anesthesia
−4.36	(Colombo et al., 2019)	Scalp EEG ($n = 5$)	20–40 Hz	Anesthesia (propofol)
−4.73	(Lendner et al., 2020)	Scalp EEG ($n = 14$)	30–45 Hz	REM sleep

For humans, at f > 20 Hz:



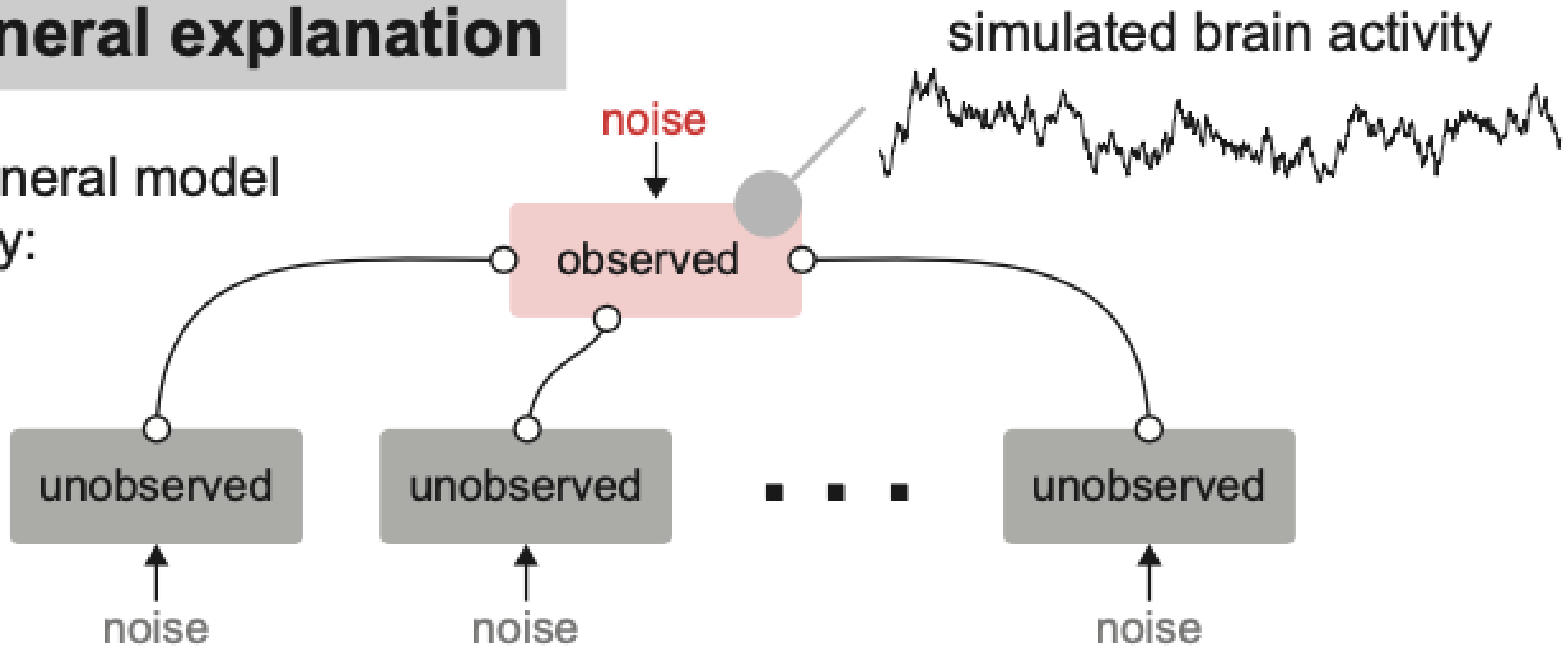
$$\boxed{-4 < \beta < -2}$$



Aperiodic exponent: Example 3

A more general explanation

Consider a general model of brain activity:



Aperiodic exponent: Example 3

For nearly any model

noise small $-4 < \beta < -2$ noise big

β reflects noise in the observed & unobserved dynamics.

Values consistent with *in vivo* data do **not** require specific biological mechanisms or collective critical behavior.

Aperiodic exponent: Example 3

Consider a general, noise-driven dynamical system

variable
in your
system

$$\frac{dX_k}{dt} = f_k(X_1, X_2, \ldots, X_n) + \sum_{j=1}^{m} B_{kj}\epsilon_{X,j}$$

you choose

observed

unobserved

noise to variable X_k
(can be multiple noise sources)

Aperiodic exponent: Example 3

Spectrum of the observable variable X_1 near a steady-state

$$S_{11}[\omega] = (B_{11}^2 + B_{12}^2 + \cdots + B_{1m}^2)\mathcal{O}(\omega^{-2}) + \mathcal{O}(\omega^{-4})$$

1/f^2 1/f^4

Without noise drive to the observable variable X_1 (all B = 0)

$$S_{11}[\omega] = \mathcal{O}(\omega^{-4}) \qquad \beta \approx -4$$

With stochastic drive to the observable variable X_1

$$S_{11}[\omega] = \mathcal{O}(\omega^{-2}) \qquad \beta \approx -2$$

So

$-4 < \beta < -2$

no matter your model

Aperiodic exponent: Example 3

Consider an example model

noise to observable V

$$\frac{dV}{dt} = I_0 + g_M M(E_M - V) + \epsilon_V,$$

$$\frac{dM}{dt} = \alpha_M[V](1 - M) - \beta_M[V]M + \epsilon_M$$

noise to unobservable M

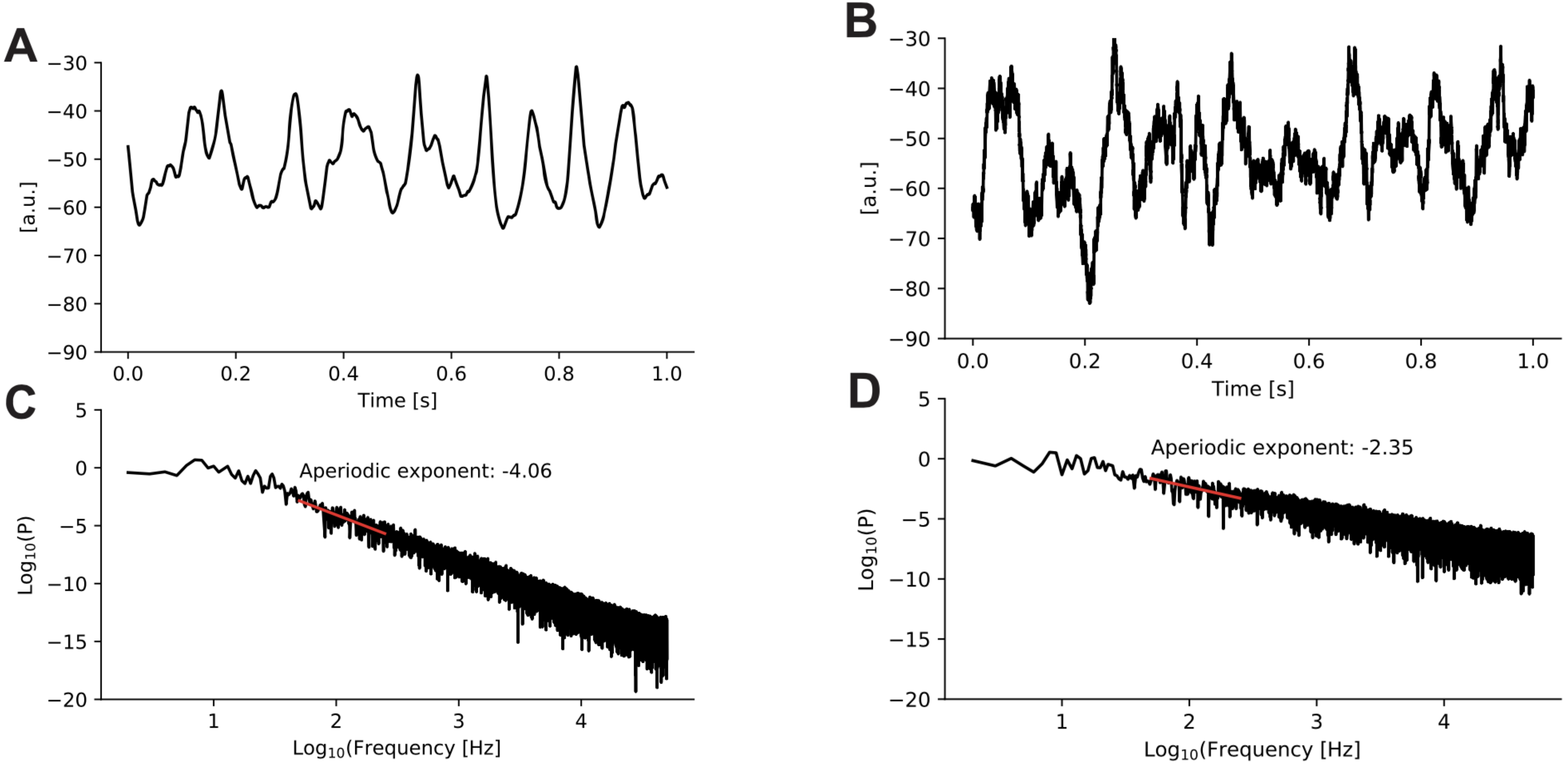
$$\alpha_M[V] = \frac{0.02}{1 + \exp\left(\frac{-V-20}{5}\right)},$$

$$\beta_M[V] = 0.01\exp\left(\frac{-V - 43}{18}\right)$$

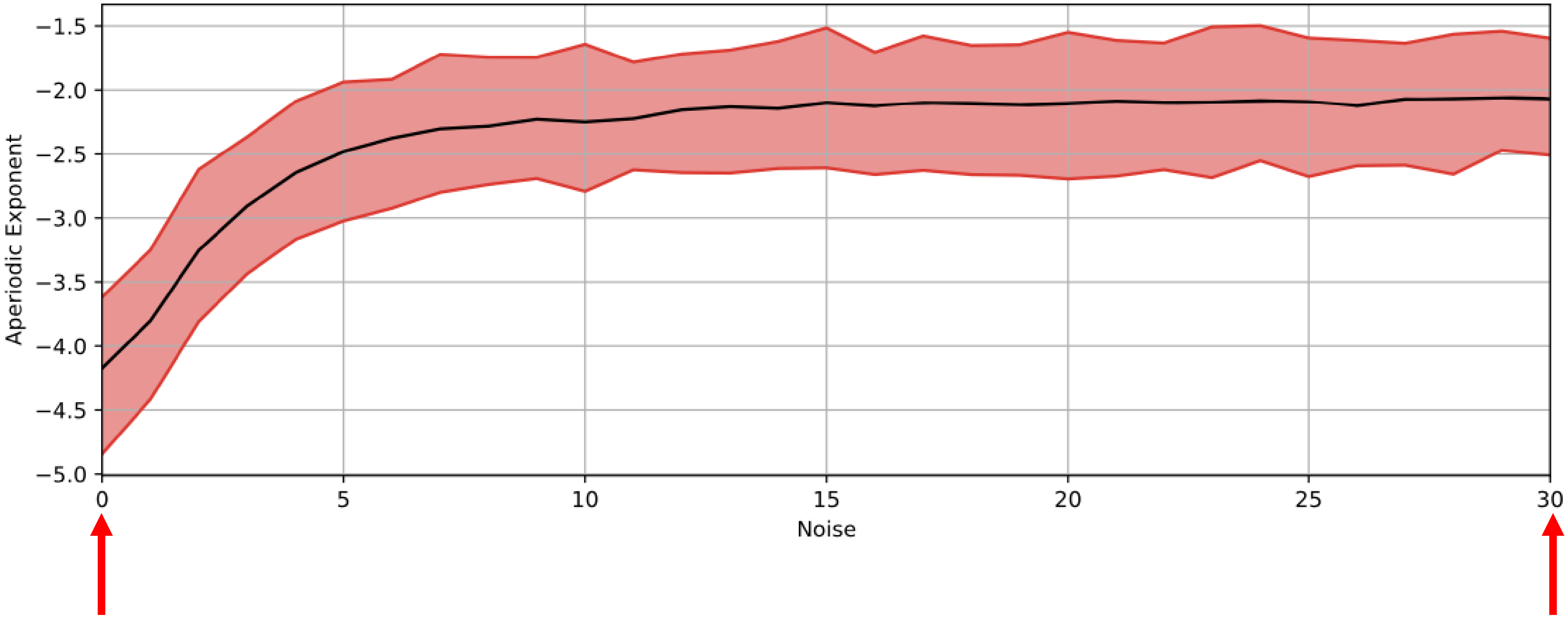
Aperiodic exponent: Example 3

No noise to V

Noise to V



Aperiodic exponent: Example 3



no noise to observable V

strong noise to observable V

Aperiodic exponent: Example 3

Let's try it

https://github.com/Mark-Kramer/Aperiodic-Exponent-Model/blob/main/Figure-1.ipynb

Python

Aperiodic Exponent: Example 4

Consider this example

noise to observable x

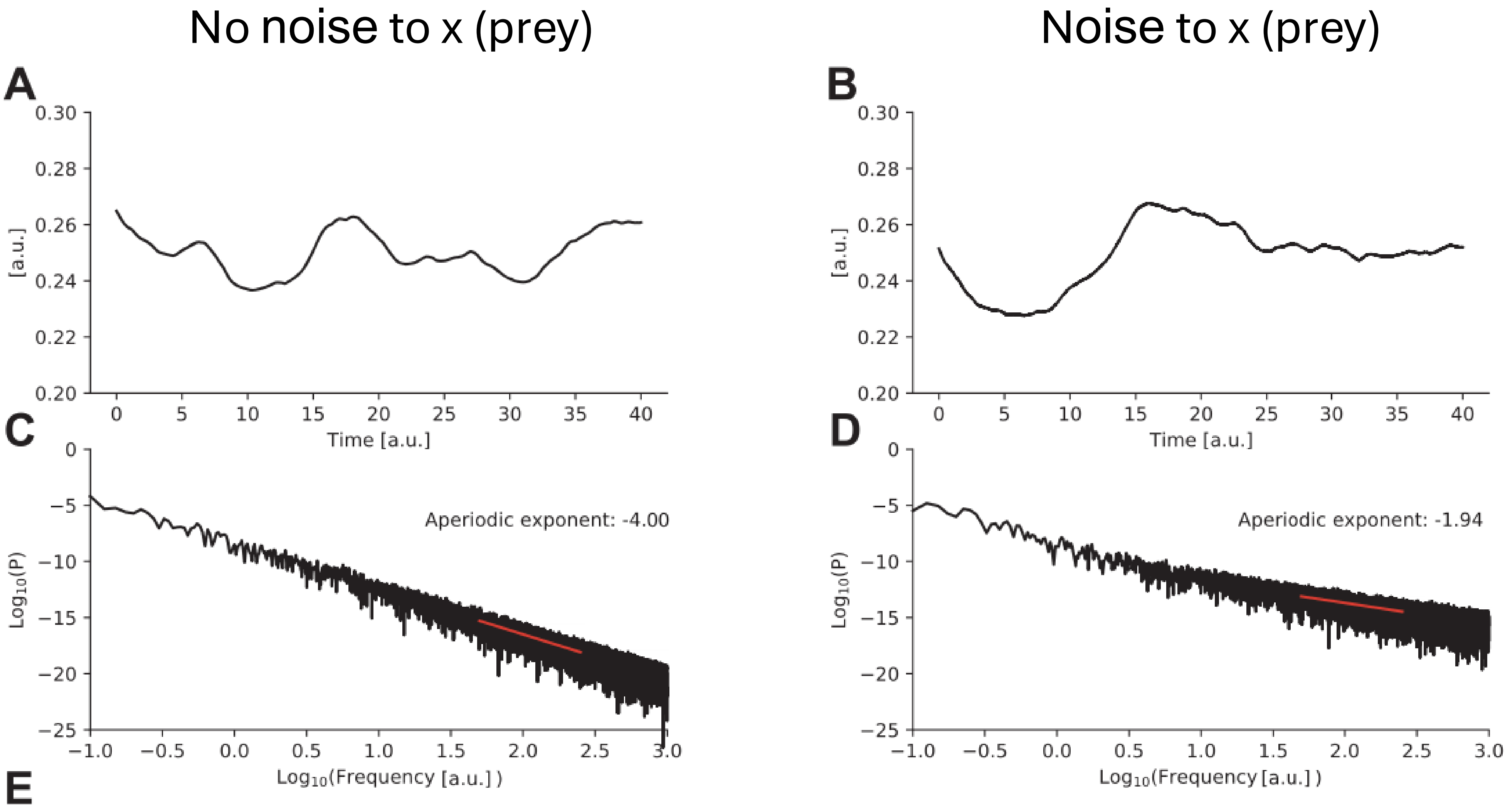
$$\frac{dx}{dt} = \frac{x(\gamma - x)}{\gamma} - x\,y + \epsilon_x,$$

$$\frac{dy}{dt} = -\alpha\,y + x\,y + \epsilon_y,$$

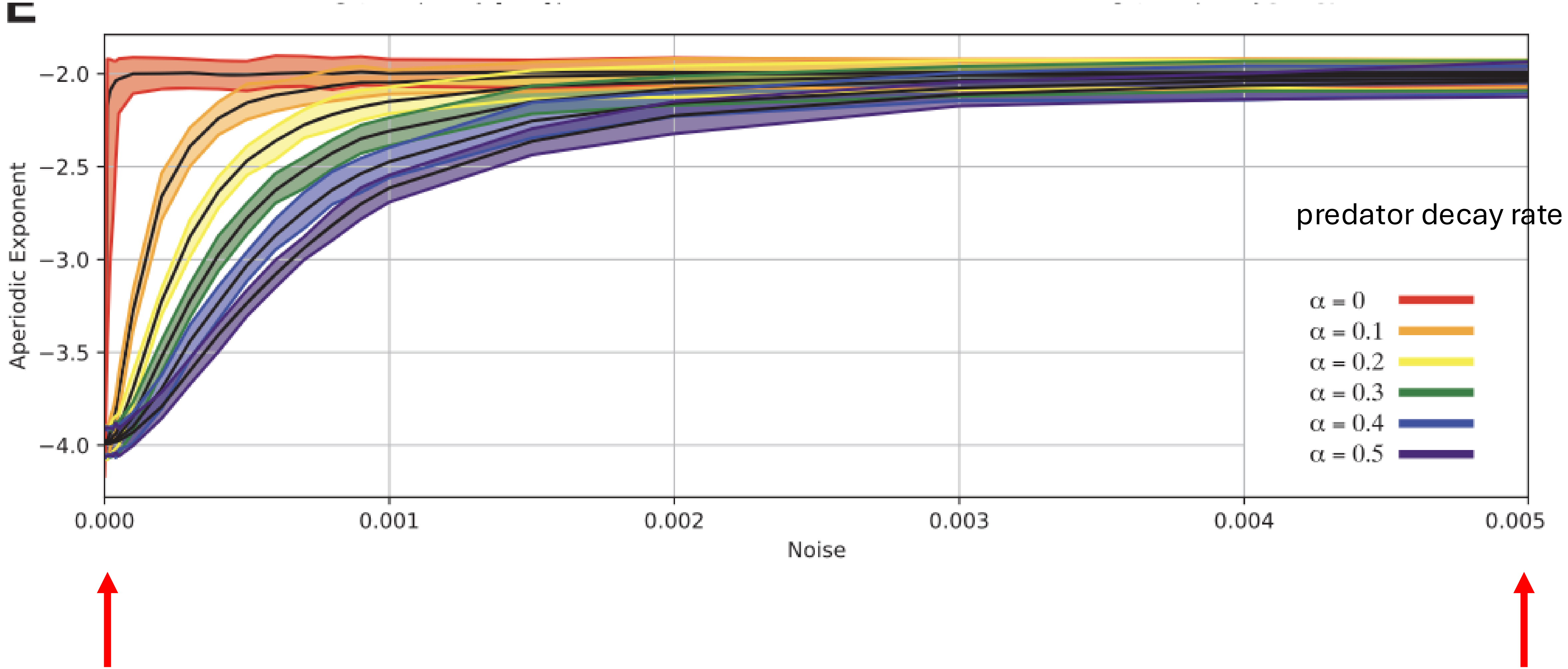
noise to unobservable y

x = prey population
y = predator population

Aperiodic Example 4



Aperiodic exponent: Example 4



no noise to observable x

strong noise to observable x

Aperiodic exponent: Example 3

Conclusions

Noise-driven models of brain activity produce 1/f-like spectra.

No specific biological mechanism is required to produce the 1/f-like spectra observed in brain recordings.

Be careful interperting the 1/f-like neural spectra and any proposed mechanisms.