Rhythms

Analyzing Rhythms (Part 3)

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Today

Rhythms from spike trains

Review

spectrum → which rhythms dominate a signal

$$S_{xx,j} \sim X_j X_j^*$$

Frequency resolution

$$df = \frac{1}{T}$$

Duration of recording

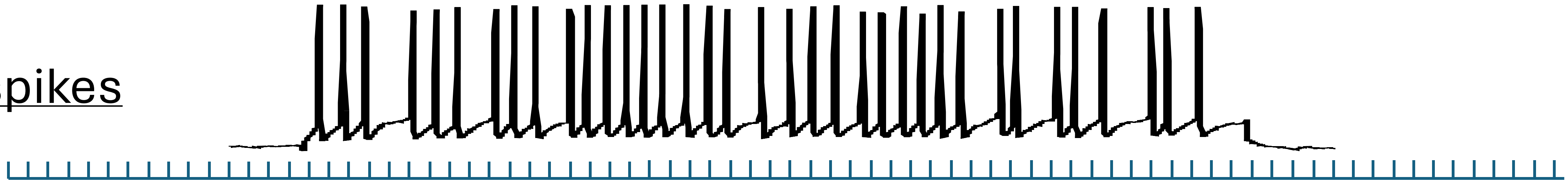
Nyquist frequency

$$f_{\mathrm{NQ}} = \frac{f_0}{2}$$

Sampling frequency

Today

spikes



indicate spikes ||| ||| || •••

Divide time into small bins, Δ

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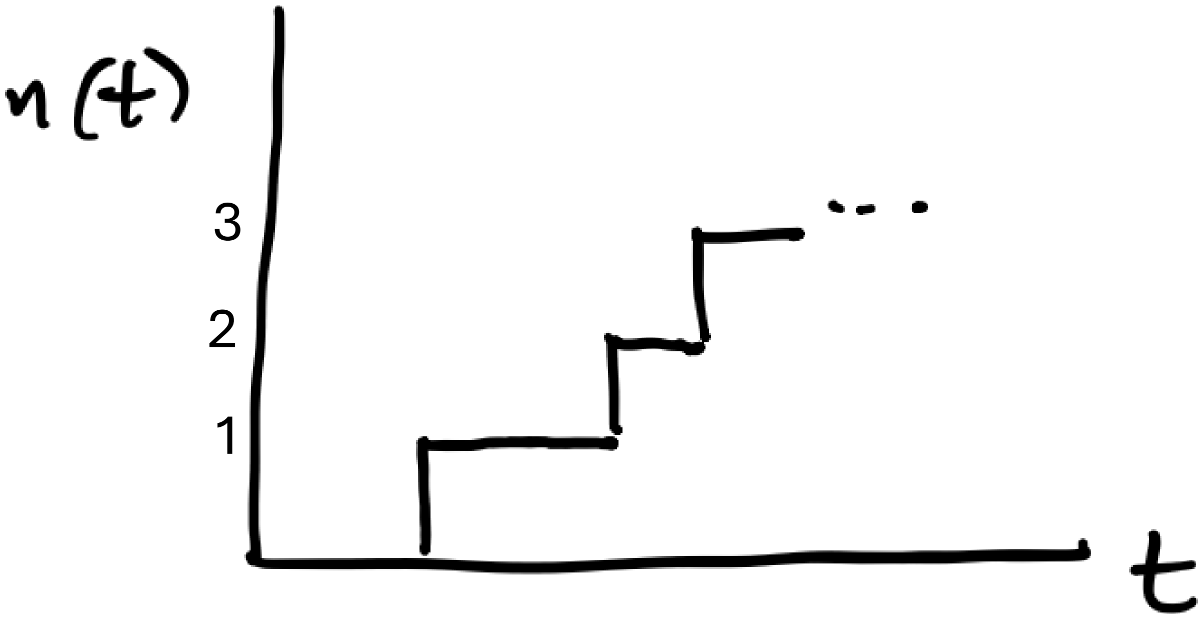
Define:

$$dn(t) = \begin{cases} 1 \text{ when a spike occurs in small time interval } \Delta \\ 0 \text{ otherwise} \end{cases}$$

Why?

Q: Why $dn(t)$?

A: Consider $n(t)$ = the number of spikes that occur from 0 to t.



$dn(t)$ is the change in $n(t)$

Either 0 or 1 when a spike occurs

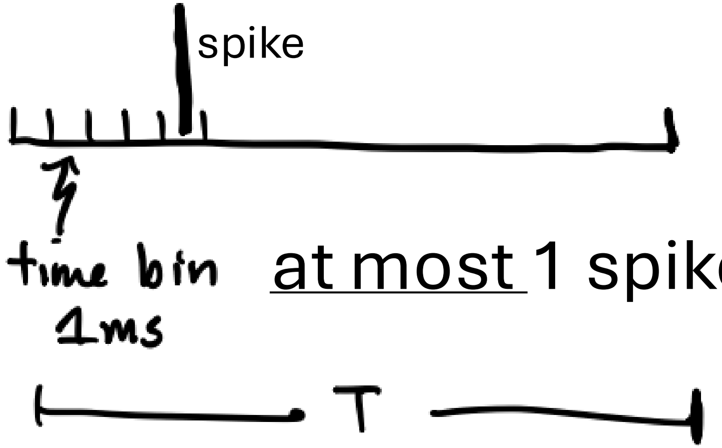
Subtract the mean

Define: $d\bar{n}(t) = dn(t) - \lambda_o \Delta$

where λ_o = mean spike rate

estimate λ_o from data $= \dfrac{n(T)}{T}$ ← number of spikes after total time T; ← total time

Δ = time bin (e.g., 1 ms)



<u>at most</u> 1 spike in a bin

$\lambda_o \Delta$ = expected number of spikes in a single time bin

Autocovariance (spike train)

Given a spike occurred at time t, how likely is another spike at t+τ ?

<u>Idea</u>: Compare spike train to a shifted version of itself … match?

previously
$$r_{xx}[L] = \frac{1}{N}\sum_{n=1}^{N-L}(x_{n+L}-\bar{x})(x_n-\bar{x})$$

Notation change

$$r_{nn}[\tau] = \frac{1}{N}\sum_{t} d\bar{n}(t)\, d\bar{n}(t+\tau)$$

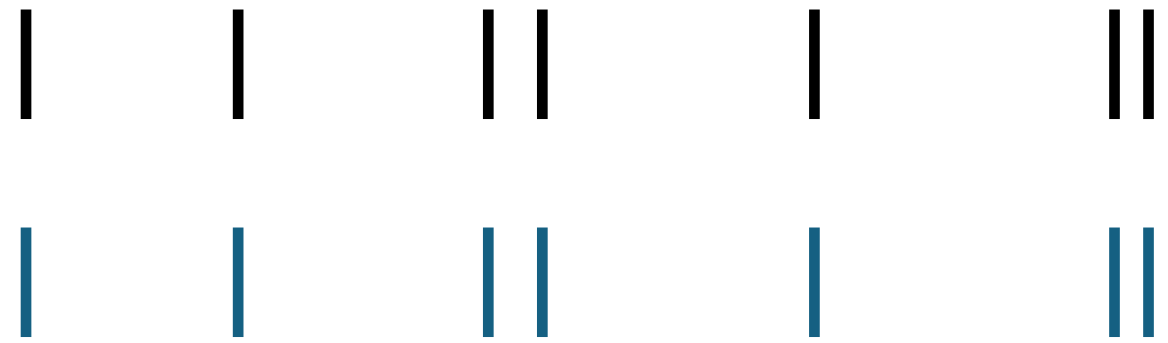
lag τ

\# of obs

spike train (mean subtracted)

Autocovariance (spike train)

Ex.



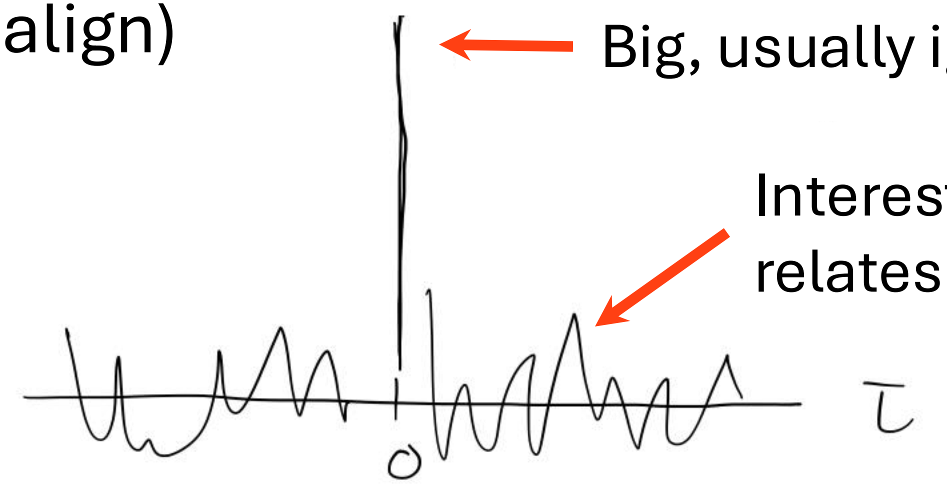
copy ($\tau = 0$)

Q. $r_{nn}[0]$?

$r_{nn}[0]$ = big (spikes align)

$r_{nn}[\neq 0]$ = ?

Plot $r_{nn}[\tau]$



Autocovariance (spike train)

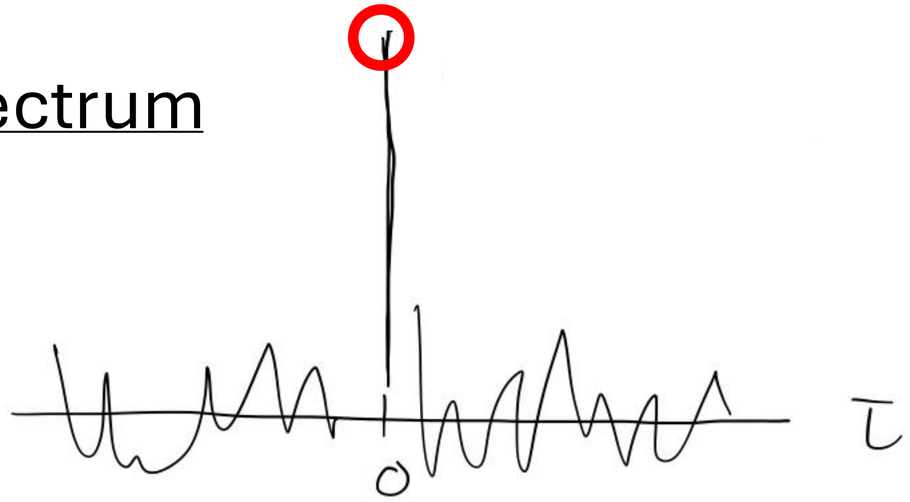
<u>Remember</u>: spectrum is the Fourier transform of the autocovariance

$$S_{nn,j} = 2\,\Delta\, FT\{\, r_{nn}\}$$

function of frequency

function of time

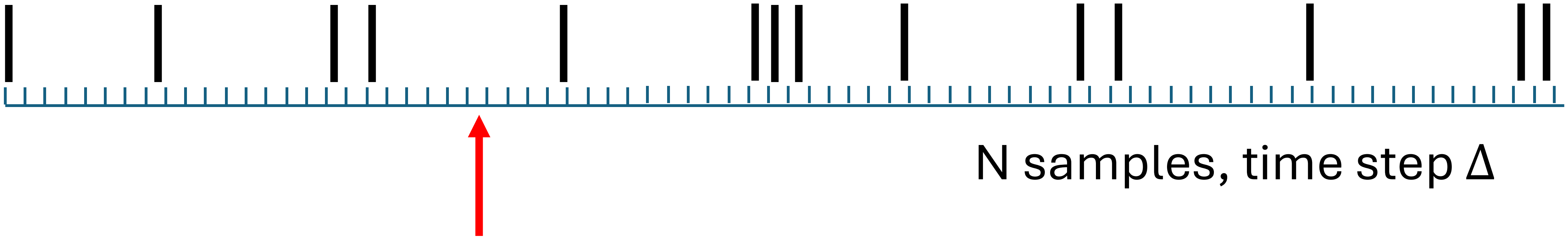
So, the peak in r_{nn} at lag 0 <u>impacts the spectrum</u>



Q. How?

Autocovariance (spike train)

Ex. Random spiking at a fixed rate



N samples, time step Δ

each moment in time: a (biased) coin flip

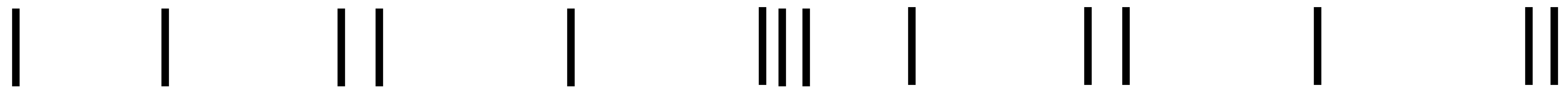
No dependence on past or future

Q. What is the autocovariance $r_{nn}[\tau]$?

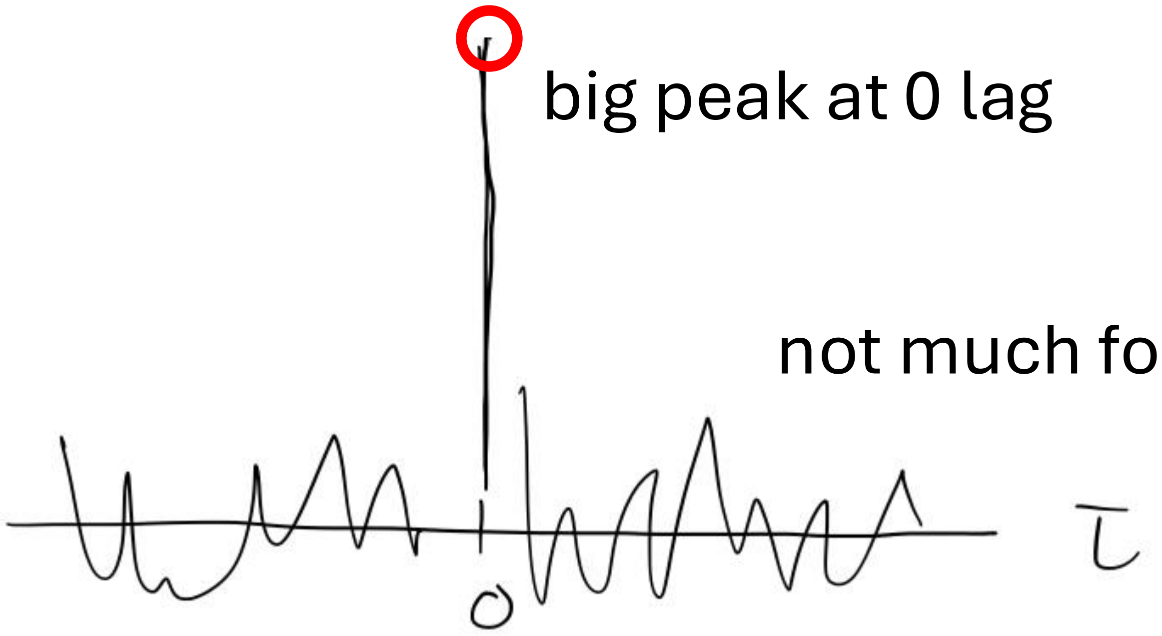
Q. What is the spectrum $S_{nn,j}$?

Autocovariance (spike train)

Ex. Random spiking at a fixed rate



Plot $r_{nn}[\tau]$



big peak at 0 lag

not much for $\tau \neq 0$

Q. What is $r_{nn}[0]$?

Autocovariance (spike train)

Ex. Random spiking at a fixed rate

Q. What is $r_{nn}[0]$?

$$r_{nn}[0] = \frac{1}{N} \sum_t \underbrace{d\bar{n}(t)^2}_{\approx\, 0 \text{ or } 1} \approx \frac{\text{total \# spikes}}{\text{total \# time bins}} = \frac{n(T)}{N}$$

Remember the mean spike rate: $\lambda_o = \dfrac{n(T)}{T}$ so $n(T) = \lambda_o T$

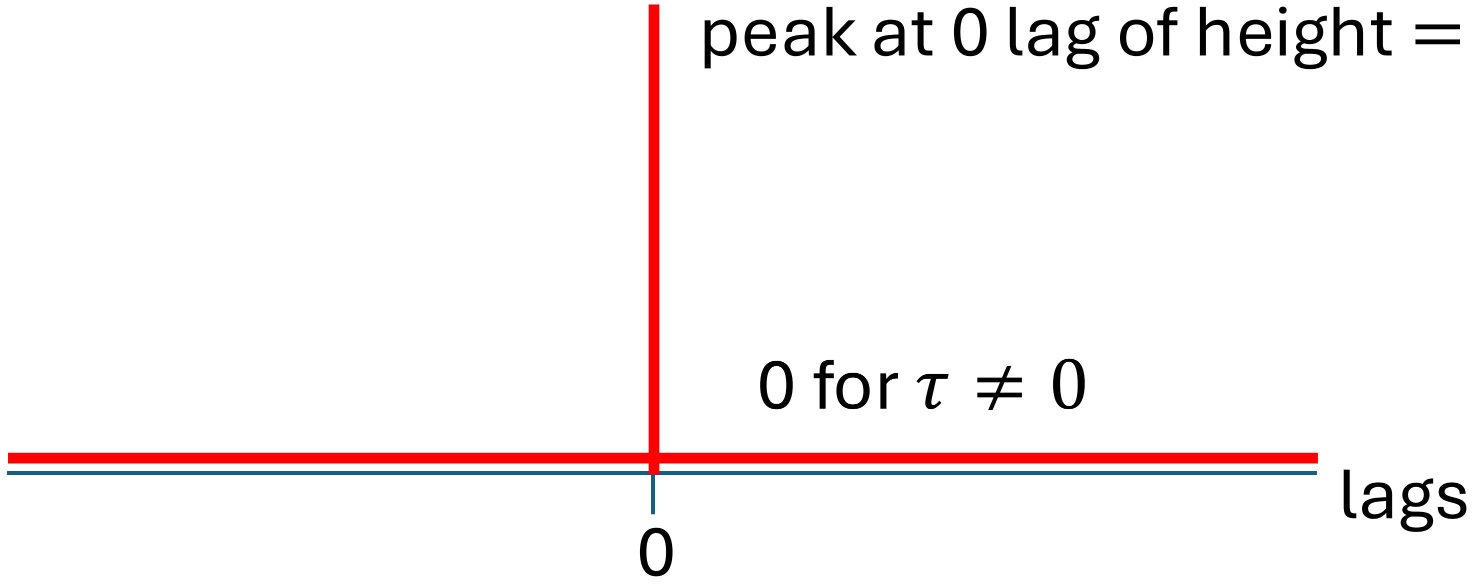
$$r_{nn}[0] = \frac{\lambda_0 T}{N} \leftarrow T = N\Delta$$

$$\boxed{r_{nn}[0] = \lambda_0 \Delta}$$ expected number of spikes in a time bin

Autocovariance (spike train)

Ex. Random spiking at a fixed rate

Plot <u>idealized</u> $r_{nn}[\tau]$



$\boxed{r_{nn}[\tau] = \lambda_0\ \Delta\ \delta[\tau]}$ where $\delta[\tau] = \begin{cases} 1 & \text{when } \tau = 0 \\ 0 & \text{otherwise} \end{cases}$

Q. What is the <u>spectrum</u>?

Autocovariance (spike train)

in our example

$$r_{nn}[\tau] = \lambda_0 \, \Delta \, \delta[\tau]$$

Ex. Random spiking at a fixed rate

Q. What is the spectrum?

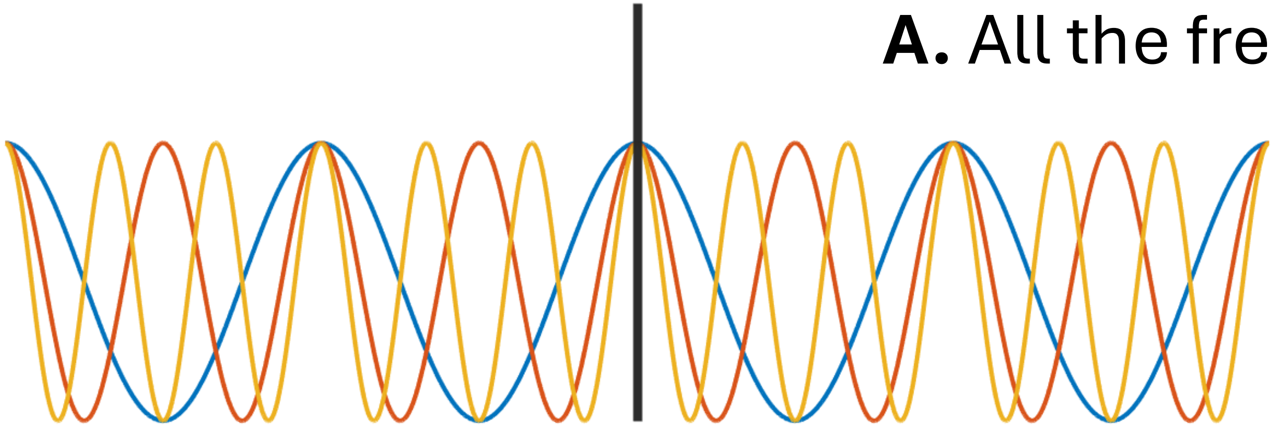
Use $S_{nn,j} = 2\,\Delta\, FT\{\, r_{nn}\} \;= 2\,\Delta\, FT\{\lambda_0 \Delta\, \delta[\tau]\}$

$= 2\,\Delta^2 \lambda_0 \; FT\{\delta[\tau]\}$

In our case, we need $FT\{\delta[\tau]\}$

Q. What frequency sinusoids do we need?

A. All the frequencies



$\boxed{FT\{\delta[\tau]\} = 1}$

Need contribution from each frequency to capture this sharp event

Autocovariance (spike train)

Ex. Random spiking at a fixed rate

Q. What is the spectrum?

$$S_{nn,j} = 2\,\Delta\, FT\{\, r_{nn}\}$$

$$r_{nn}[\tau] = \lambda_0\,\Delta\,\delta[\tau]$$

$$S_{nn,j} = 2\,\Delta\, FT\{\lambda_0 \Delta\,\delta[\tau]\}$$

$$S_{nn,j} = 2\,\Delta\,(\lambda_0 \Delta)\, FT\{\delta[\tau]\}$$

$$S_{nn,j} = 2 \quad \Delta^2\,\lambda_0 \qquad 1$$

$$\boxed{S_{nn,j} = 2\;\Delta^2\,\lambda_0}$$

This is the spectrum of random spiking at fixed rate

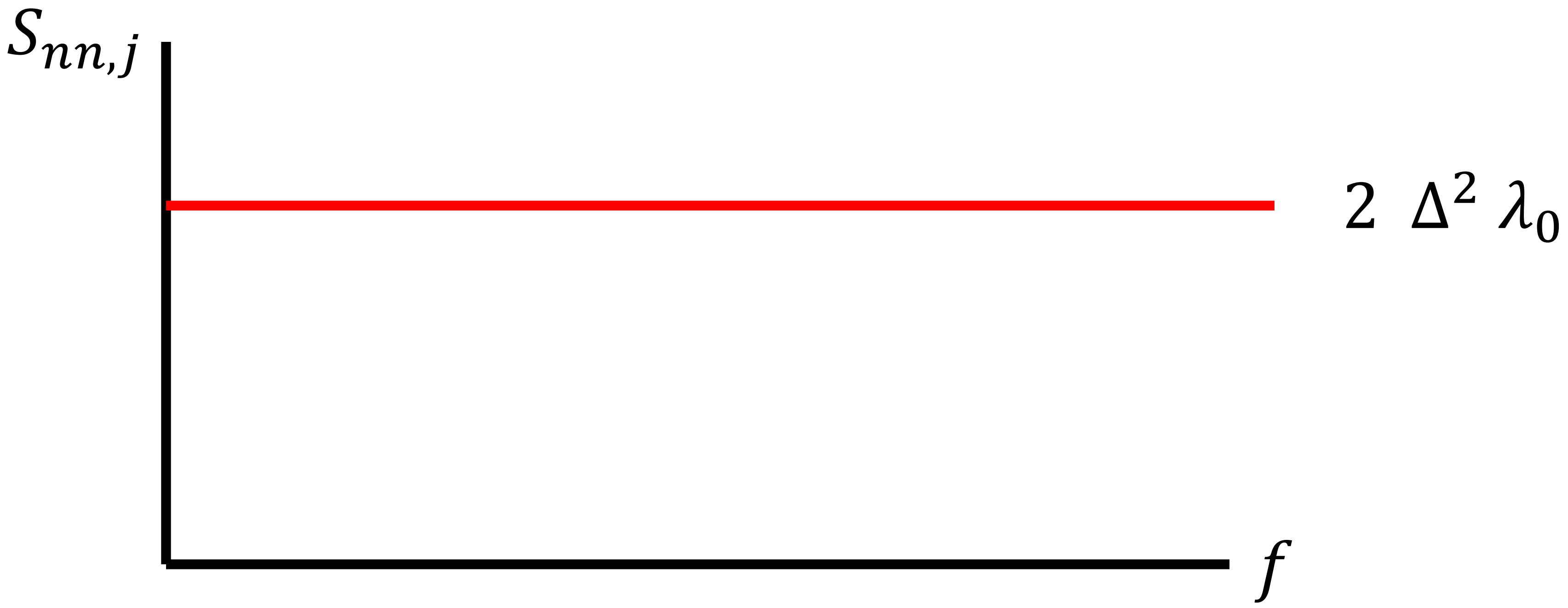
Autocovariance (spike train)

Ex. Random spiking at a fixed rate

Q. What is the spectrum?

$$\boxed{S_{nn,j} = 2\ \Delta^2\ \lambda_0}$$

Plot it



Spectrum (spike train)

Use autocovariance to gain intuition

<u>Alternative</u>: compute spectrum directly.

$$S_{nn,j} = \frac{2\Delta^2}{T} D_j D_j^*$$

Fourier transform of **spike train**

Like our approach for “fields”

- new notation

$$D_j = \sum_{n=1}^{N} d\bar{n}(t) \boxed{\exp(-2\pi i f_j t_n)}$$

complex exponentials at frequency f_j

spike train with mean subtracted

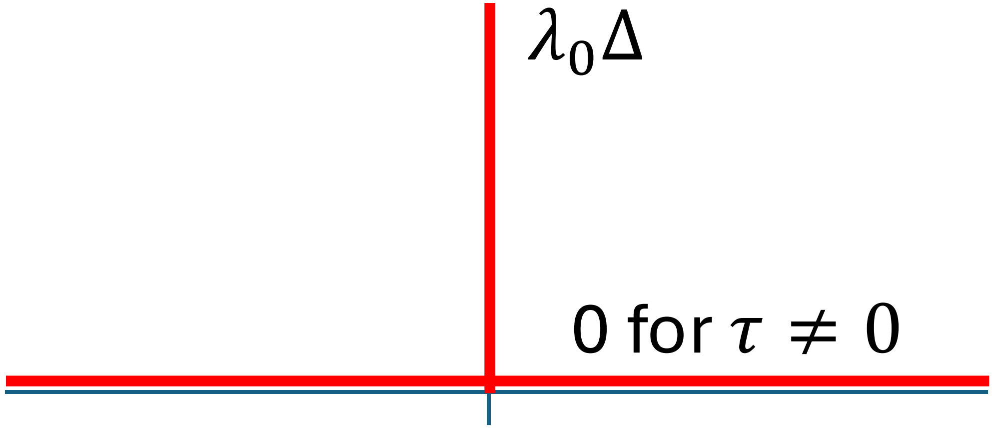
Spectrum (spike train)

Use autocovariance to gain intuition

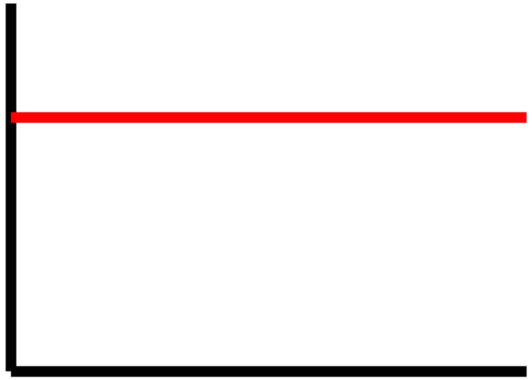
Random spiking

$$S_{nn,j} = 2\,\Delta\,FT\{\,r_{nn}\}$$

Autocovariance



Spectrum



Autocovariance (spike train)

Ex. Random spiking at a fixed rate

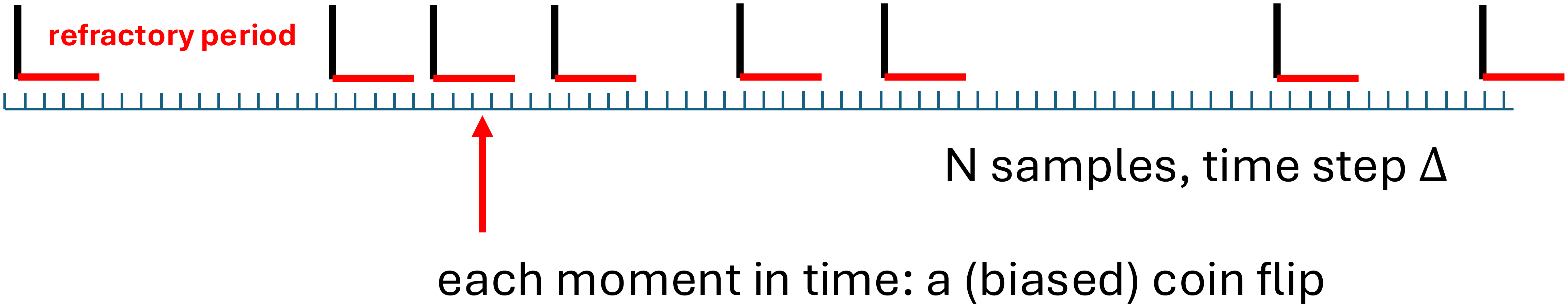
Q. What is the autocovariance?

Q. What is the spectrum?

Python

Autocovariance (spike train)

Ex. Random spiking **<u>with refractory period</u>**.



Q. What is the autocovariance $r_{nn}[\tau]$?

Q. What is spectrum $S_{nn,j}$?

Autocovariance (spike train)

Ex. Random spiking <u>**with refractory period**</u>.

Q. What is the <u>autocovariance</u> $r_{nn}[\tau]$? [sketch]

Autocovariance (spike train)

Ex. Random spiking <u>**with refractory period**</u>.

Q. What is <u>spectrum</u> $S_{nn,j}$? [sketch]

Autocovariance (spike train)

Ex. Random spiking **with refractory period**.

Q. What is the autocovariance $r_{nn}[\tau]$?

Q. What is spectrum $S_{nn,j}$?

Python