Cross-frequency coupling

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Today

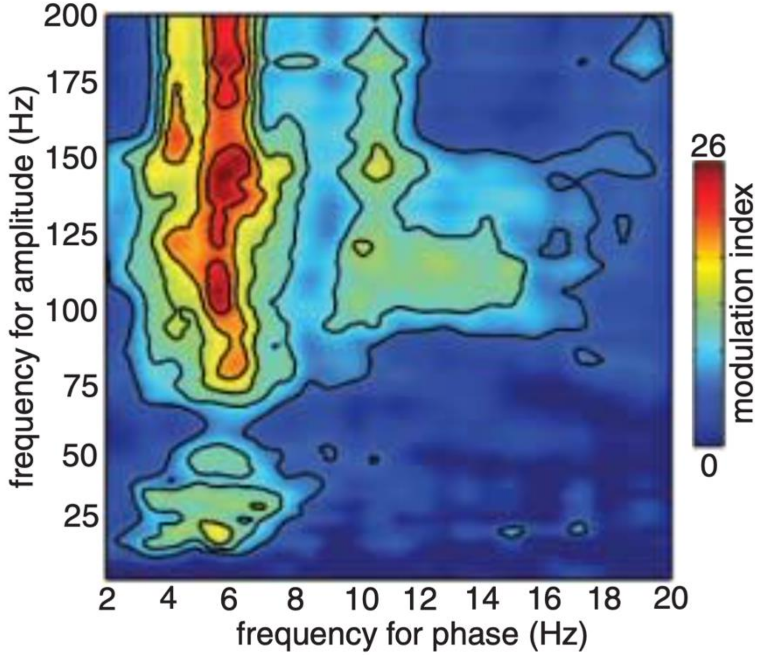
Cross-frequency coupling (one type of)

Remember, coherence

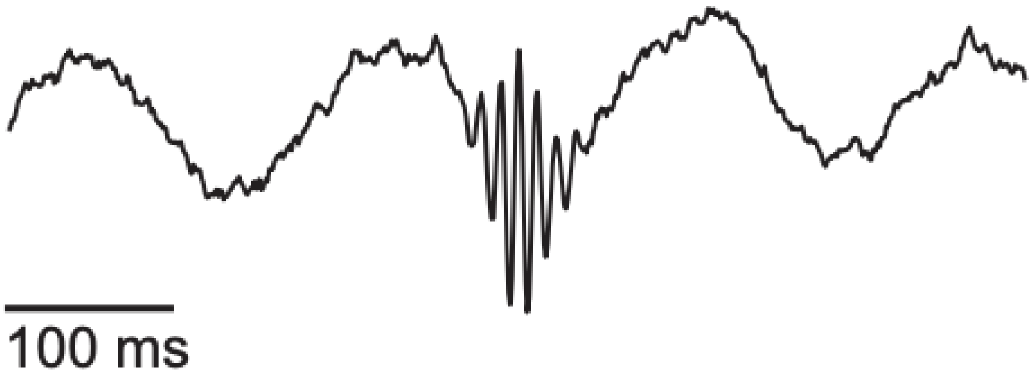
a constant phase relationship between two signals across trials,
at the same frequency

Today, coupling **between different frequencies**

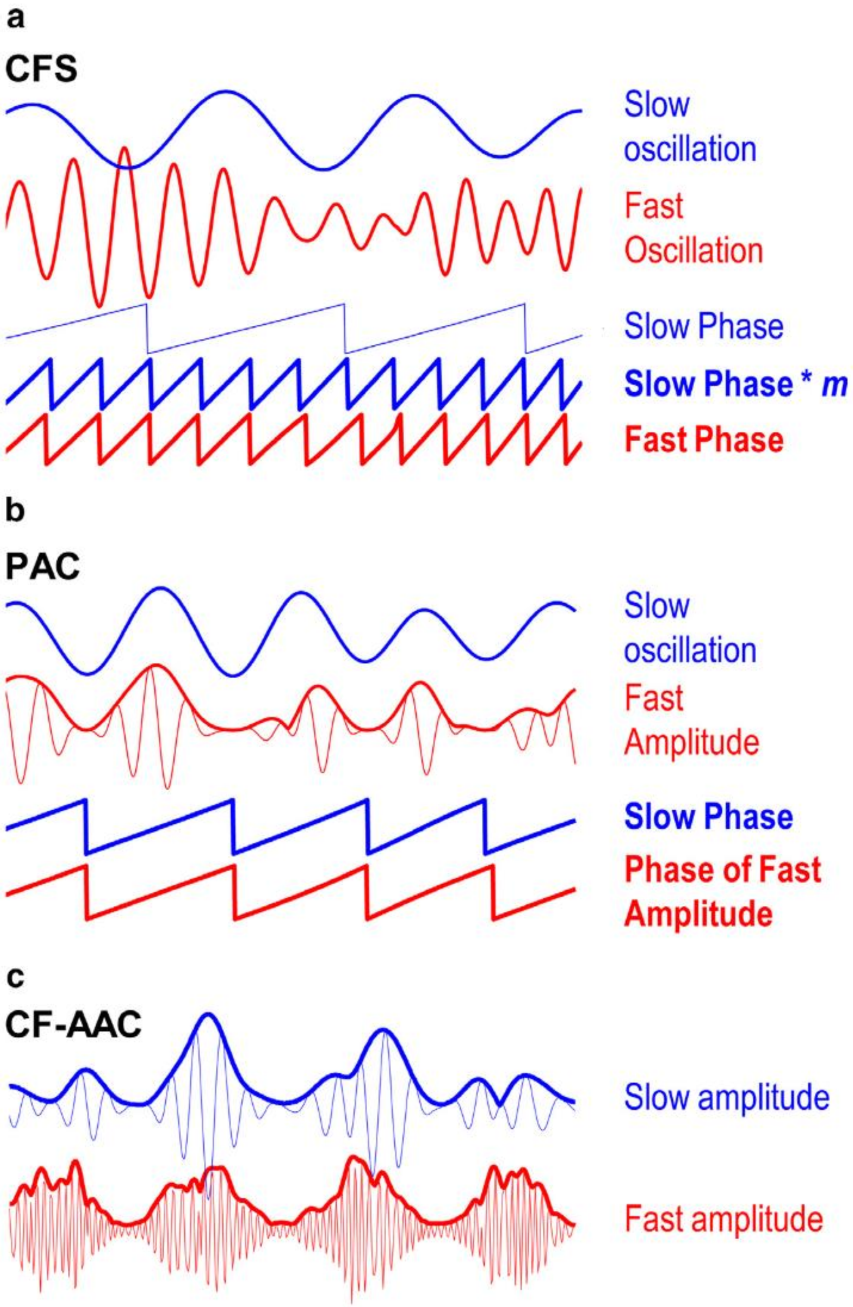
Examples



[Canolty et. al., Science, 2006]



[Tort et. al., PNAS, 2008]



[Palva & Palva, EJN, 2018]

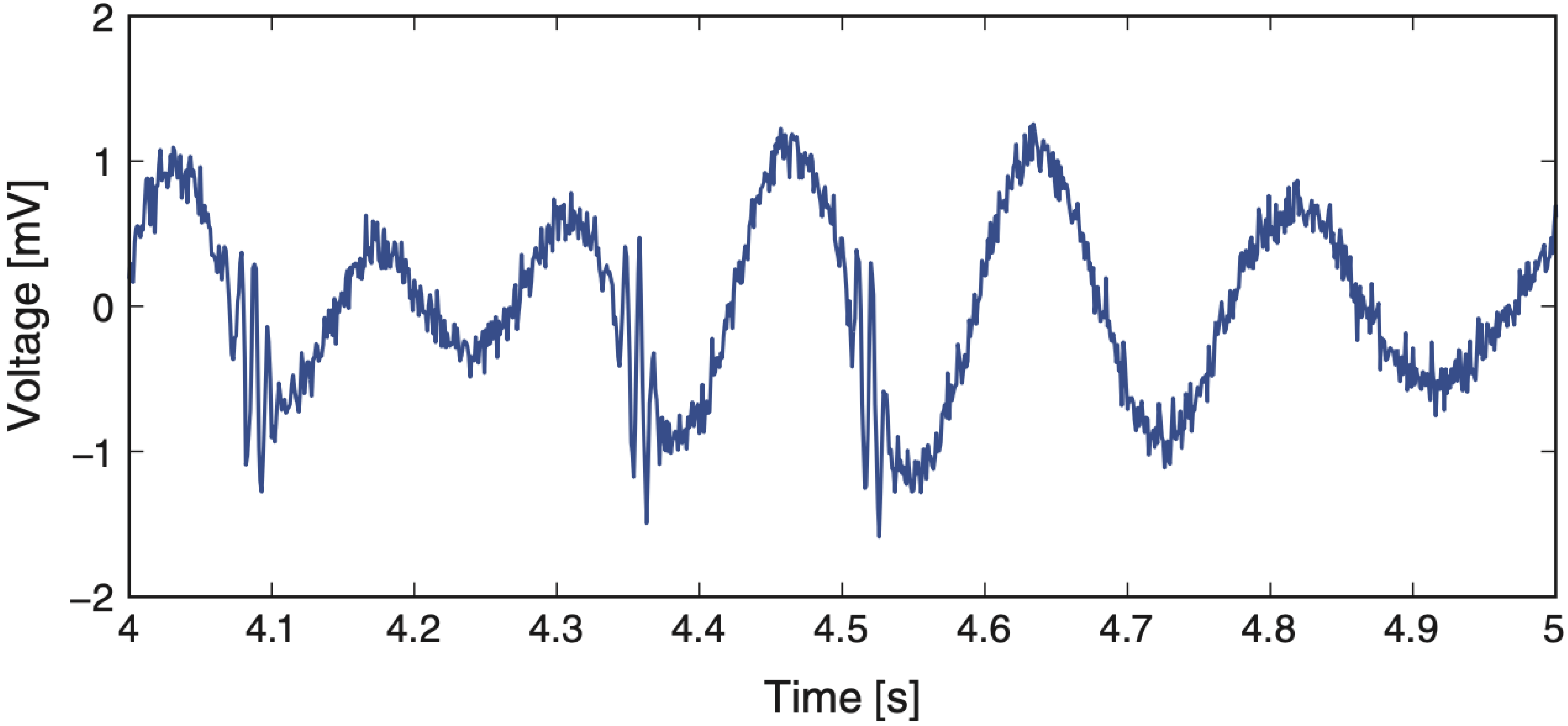
Outline

Data 100 s of local field potential data sampled at 1000 Hz.

Goal Characterize the coupling between rhythms of different frequency.

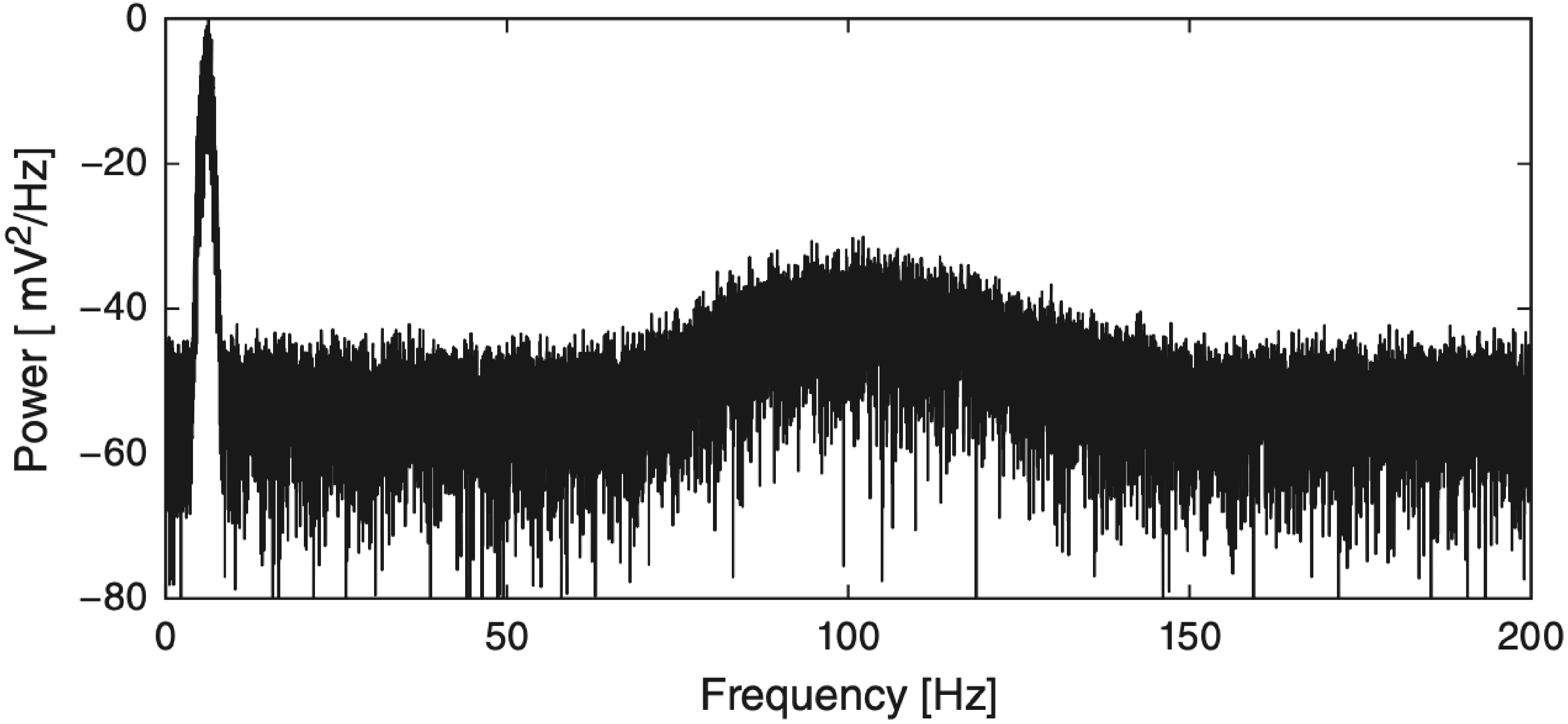
Tools Hilbert transform, analytic signal, instantaneous phase, cross-frequency coupling.

Data



Q. How to make sense of these data?

Spectrum



Q. What do you see?

CFC in three steps

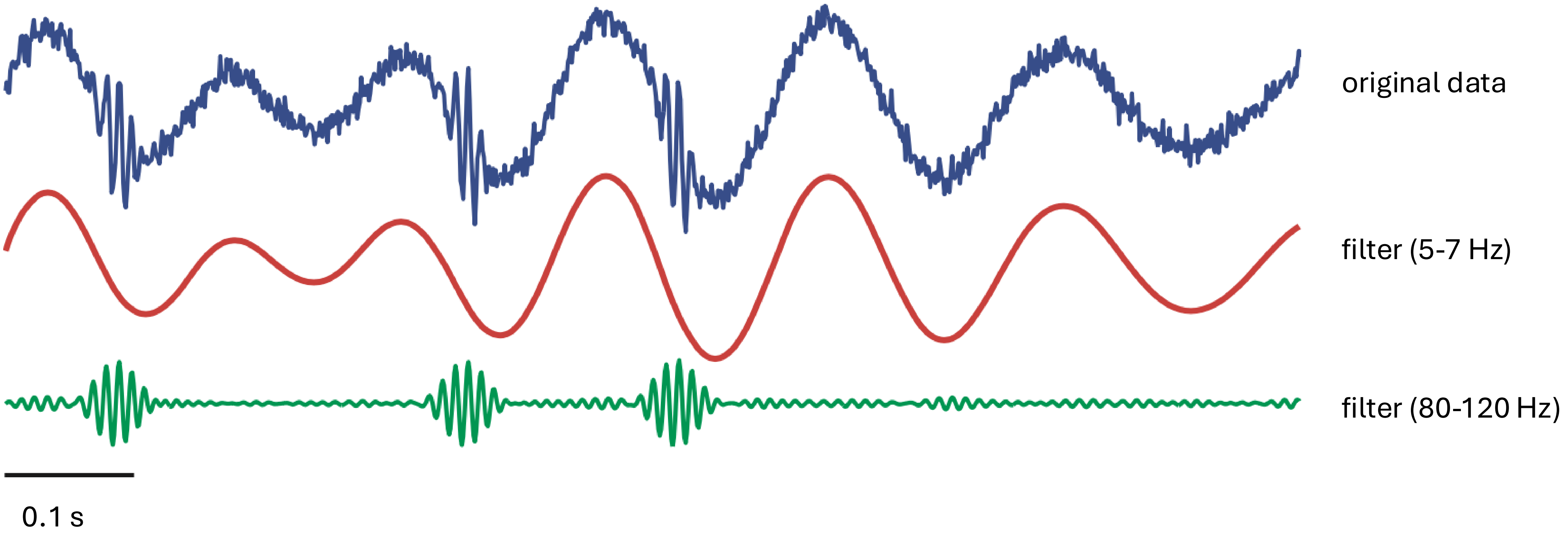
CFC analysis steps

1. Filter the data into high- and low-frequency bands.
2. Extract the amplitude and phase from the filtered signals.
3. Determine if the phase and amplitude are related.

Let’s perform each step …

CFC – Step 1

Filter the Data into High- and Low-Frequency Bands

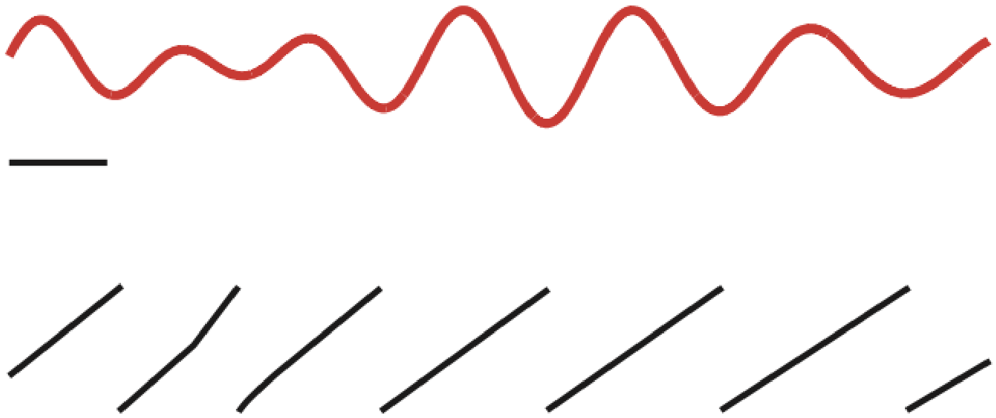


Q. Why did we filter in these bands?

CFC – Step 2

Extract the Amplitude and Phase from Filtered Signals

filter (5-7 Hz)



phase (of low frequency band)

filter (80-120 Hz)

amplitude envelope (of high frequency band)

Q. How?

Hilbert transform

$$y = H(x)$$

$$H(x) = \begin{cases} -\pi/2 \text{ phase shift} & \text{if } f > 0, \\ 0 \text{ phase shift} & \text{if } f = 0, \\ \pi/2 \text{ phase shift} & \text{if } f < 0. \end{cases}$$

The Hilbert transform $H(x)$ of the signal x produces a phase shift of ± 90 degrees for $\mp$ frequencies of x.

Hilbert transform

Define: Analytic signal z

$$z = x + iy = x + iH(x)$$

Impact: remove negative frequencies from z

Q. How?

Hilbert transform

Q. What does it do?

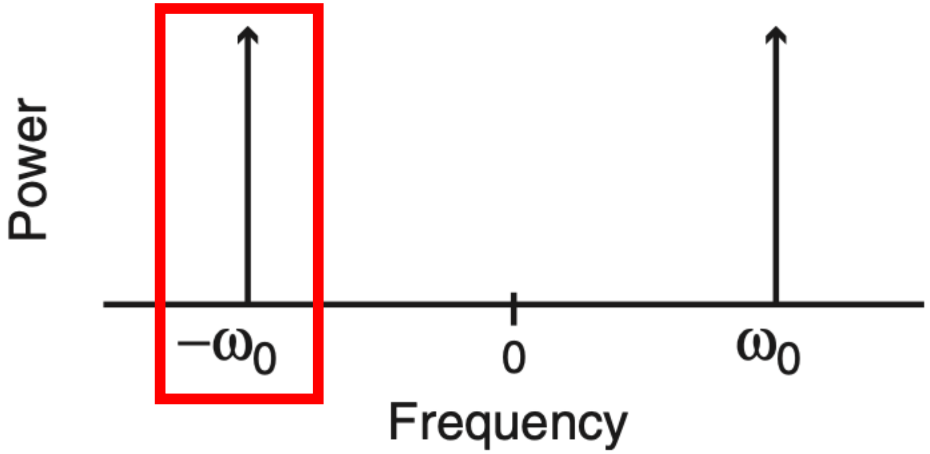
<u>Ex.</u>

$$x_0 = 2\cos(2\pi f_o t) = 2\cos(\omega_0 t) \quad \text{where} \quad \omega_0 = 2\pi f_o$$

Euler's formula

$$x_0 = \underbrace{e^{i\omega_0 t}}_{\text{positive frequency}} + \underbrace{e^{-i\omega_0 t}}_{\text{negative frequency}}$$

we usually ignore this one



<u>Note</u>: The spectrum has <u>two peaks</u>

Hilbert transform

Apply the Hilbert transform to x_0.

$$x_0 = e^{i\omega_0 t} + e^{-i\omega_0 t}$$

$$H(x) = \begin{cases} -\pi/2 \text{ phase shift} \quad \text{if } f > 0, \\ \end{cases}$$

$\boxed{\rightarrow \text{ multiply positive frequency part of } x \text{ by } -i}$

Q. Really?

Consider $e^{i\omega_0 t}$

positive frequency part of x

Shift $e^{i\omega_0 t}$ by $-\frac{\pi}{2}$

$\rightarrow e^{i(\omega_0 t - \frac{\pi}{2})} \quad \rightarrow e^{i\omega_0 t} e^{-i\pi/2} \quad \rightarrow e^{i\omega_0 t}(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}) \rightarrow \boxed{e^{i\omega_0 t}(-i)}$

Hilbert transform

Apply the Hilbert transform to x_0.

$$x_0 = e^{i\omega_0 t} + e^{-i\omega_0 t}$$

$$H(x) = \begin{cases} -\pi/2 \text{ phase shift} & \text{if } f > 0, \\ 0 \text{ phase shift} & \text{if } f = 0, \\ \pi/2 \text{ phase shift} & \text{if } f < 0. \end{cases} \rightarrow H(x) = \begin{cases} -ix & \text{if } f > 0, \\ x & \text{if } f = 0, \\ ix & \text{if } f < 0. \end{cases}$$

So, $$x_0 = \underset{\text{multiply by } -i}{e^{i\omega_0 t}} + \underset{\text{multiply by } i}{e^{-i\omega_0 t}}$$

$$y_0 = H(x_0) = -ie^{i\omega_0 t} + ie^{-i\omega_0 t}$$

Euler's formula

$$\boxed{= 2\sin(\omega_0 t)}$$

Hilbert Transform of x_0 (a cosine function) is a sine function.

Hilbert transform

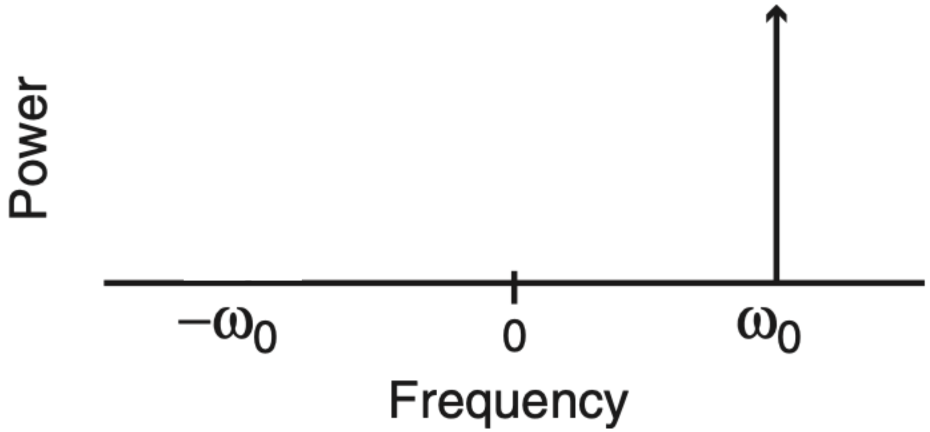
Analytic signal z

$$z = x + iy = x + iH(x)$$

$$2\cos(\omega_0 t) \qquad 2\sin(\omega_0 t)$$

$$= 2\cos(\omega_0 t) + i\,2\sin(\omega_0 t)$$

$$\boxed{= 2e^{i\omega_0 t}}$$



The analytic signal contains
<u>no negative frequencies</u>

Hilbert transform

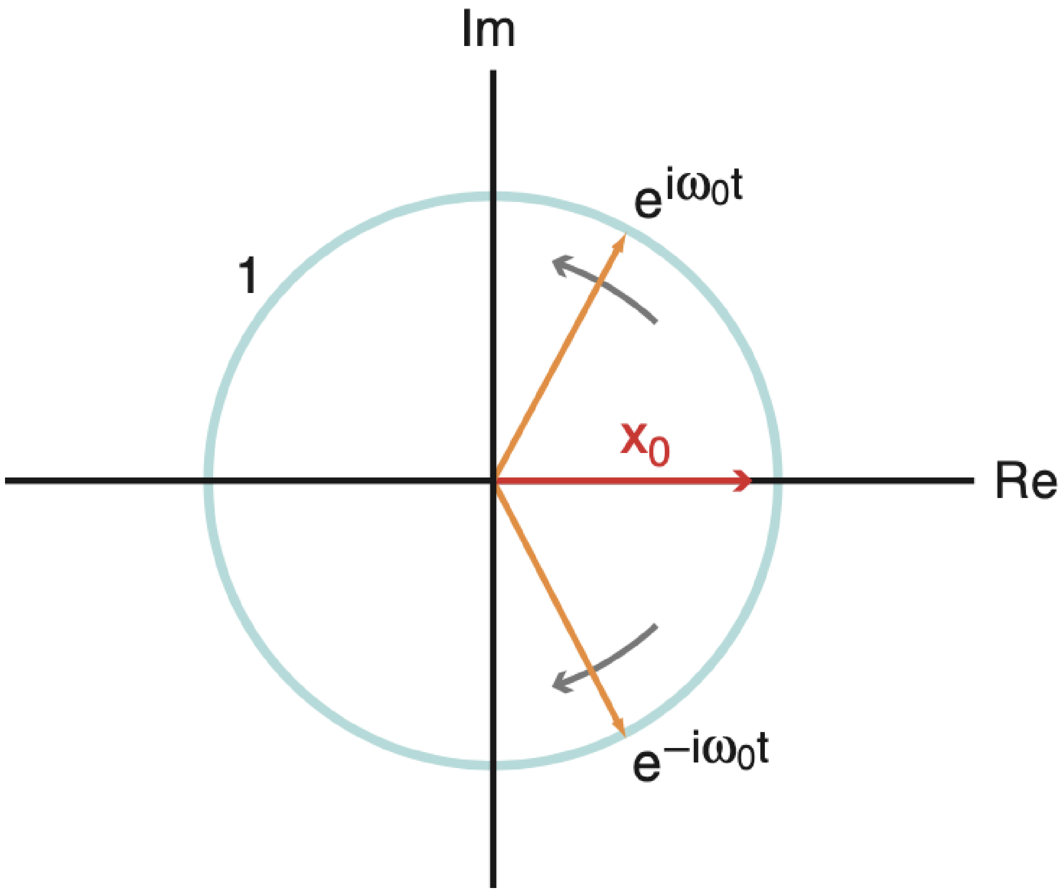
Original signal $x_0 = 2\cos(2\pi f_o t) = e^{i\omega_0 t} + e^{-i\omega_0 t}$

Complicated (2 complex exponentials)

Analytic signal $z_0 = 2e^{i\omega_0 t}$

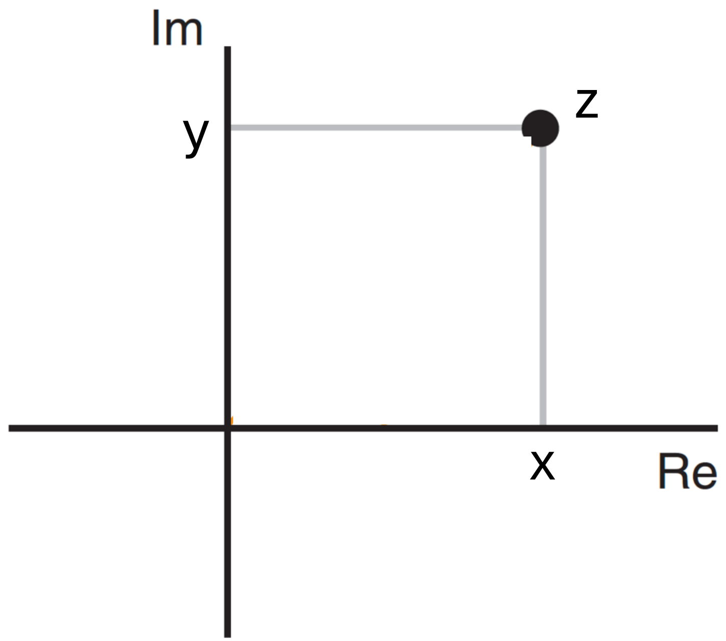
Simple (1 complex exp)

A point in the complex plane



Hilbert transform

Analytic signal $z = x + i\,y$ A point in the complex plane



$$z(t) = A(t)\, e^{i\,\phi(t)}$$

amplitude phase

Get the **amplitude** and **phase** from the analytic signal

Ex.

$z_0(t) = 2e^{i\omega_0 t}$ $A(t) = 2$ $\phi(t) = \omega_0\, t$

CFC in three steps

CFC analysis steps

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CFC – Step 3

Determine if the Phase and Amplitude are Related

Define the two-column vector

$$\begin{pmatrix} \phi(1) & A(1) \\ \phi(2) & A(2) \\ \phi(3) & A(3) \\ \vdots & \vdots \end{pmatrix}$$

phase of low frequency band activity

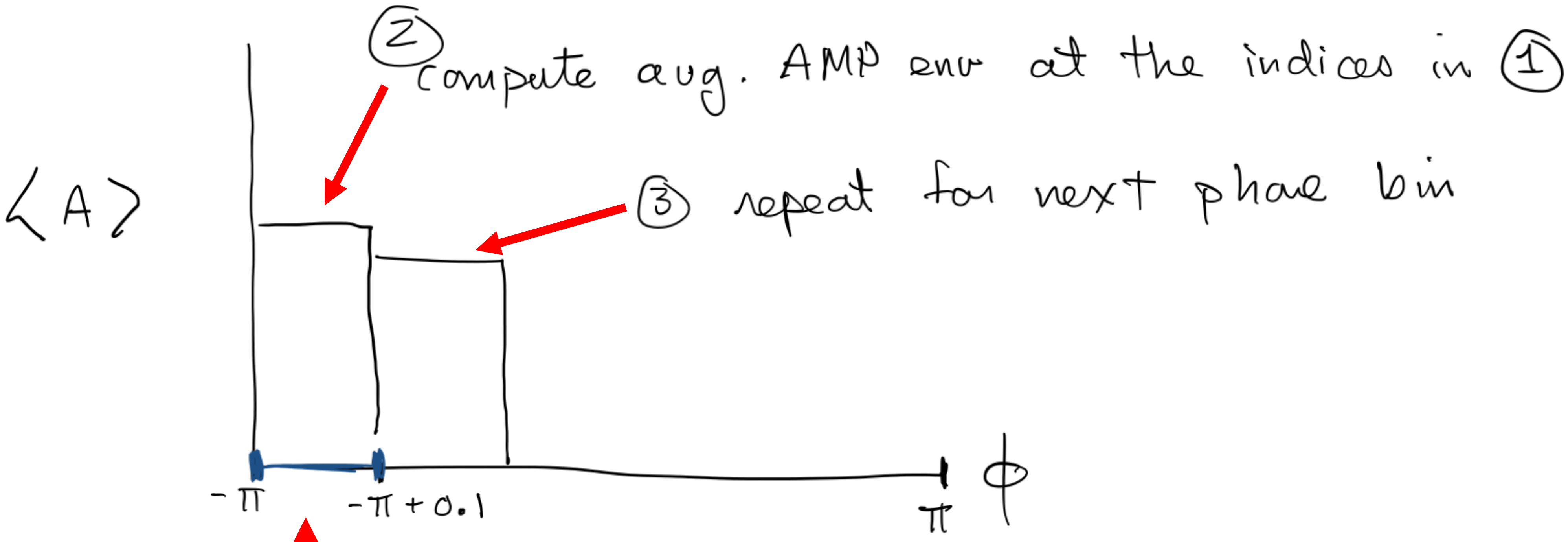
amplitude of high frequency band activity

Make a histogram

CFC – Step 3

Determine if the Phase and Amplitude are Related

Divide the phase into bins of size 0.1

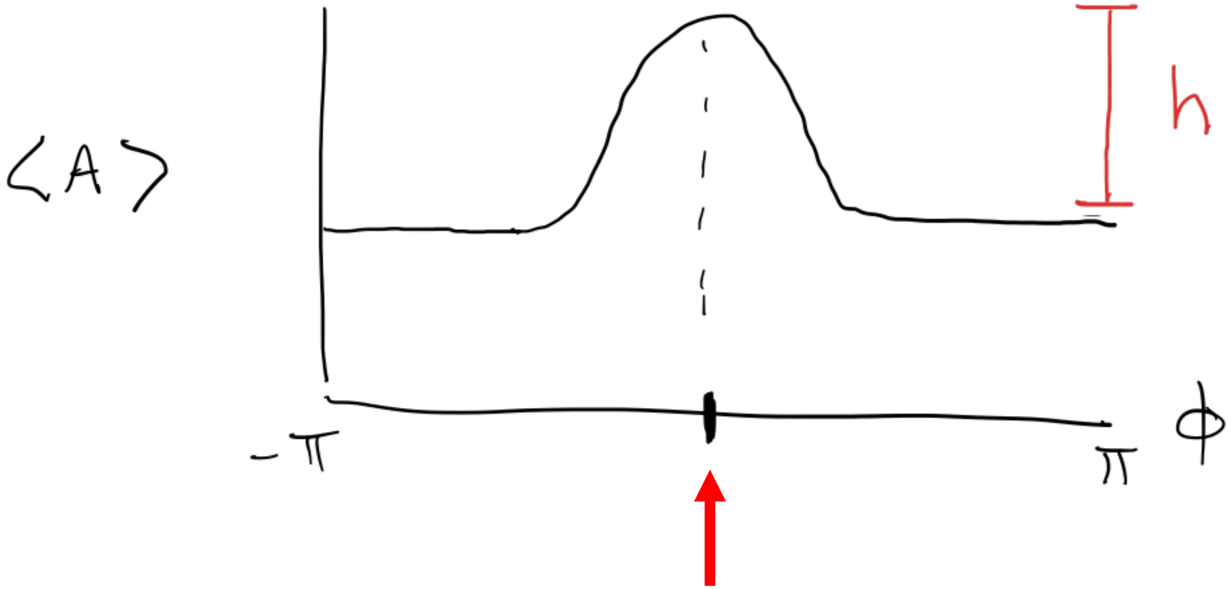


① Find all ϕ in this range $[-\pi, -\pi + 0.1]$
& get indices $\{1, 2, 3, 10, 11, 12, 41, 42, 43, \ldots\}$

CFC – Step 3

Determine if the Phase and Amplitude are Related

If phase modulate amplitude



summarize extent of modulation

Q. What does no phase-amplitude coupling look like in the plot?

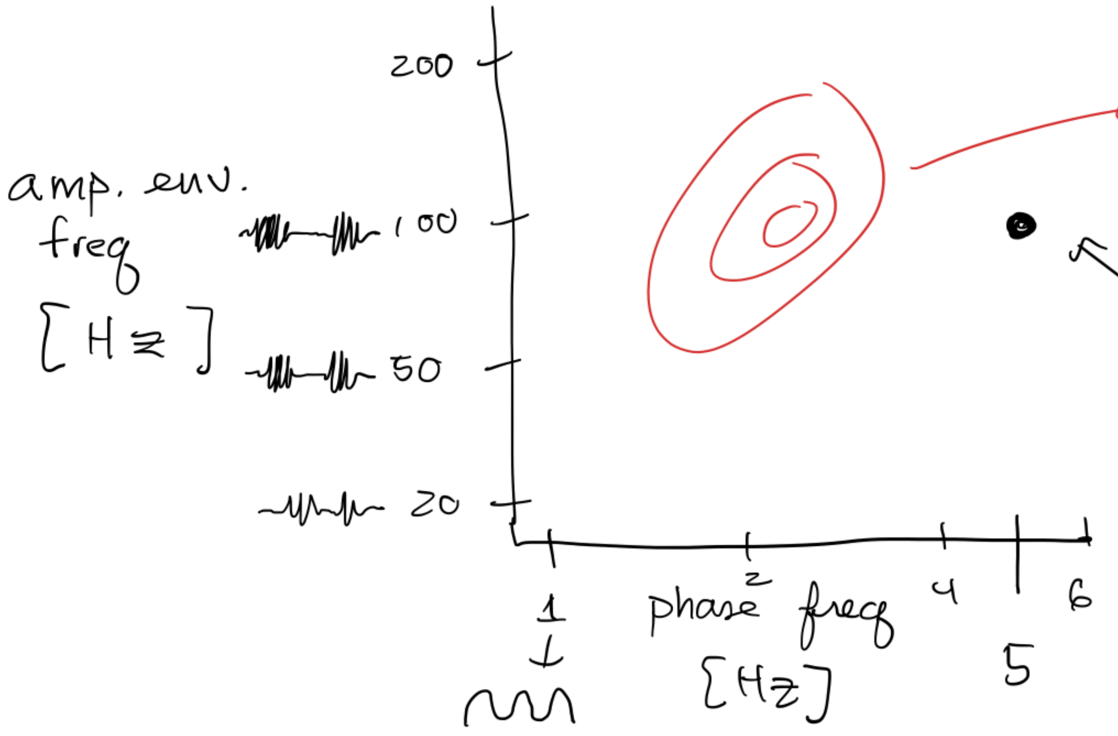
At this phase, amplitude envelope is big

➔ phase modulates amplitude

CFC – Step 4

(optional): Repeat for other frequencies.

Summarize in a <u>comodulogram</u>



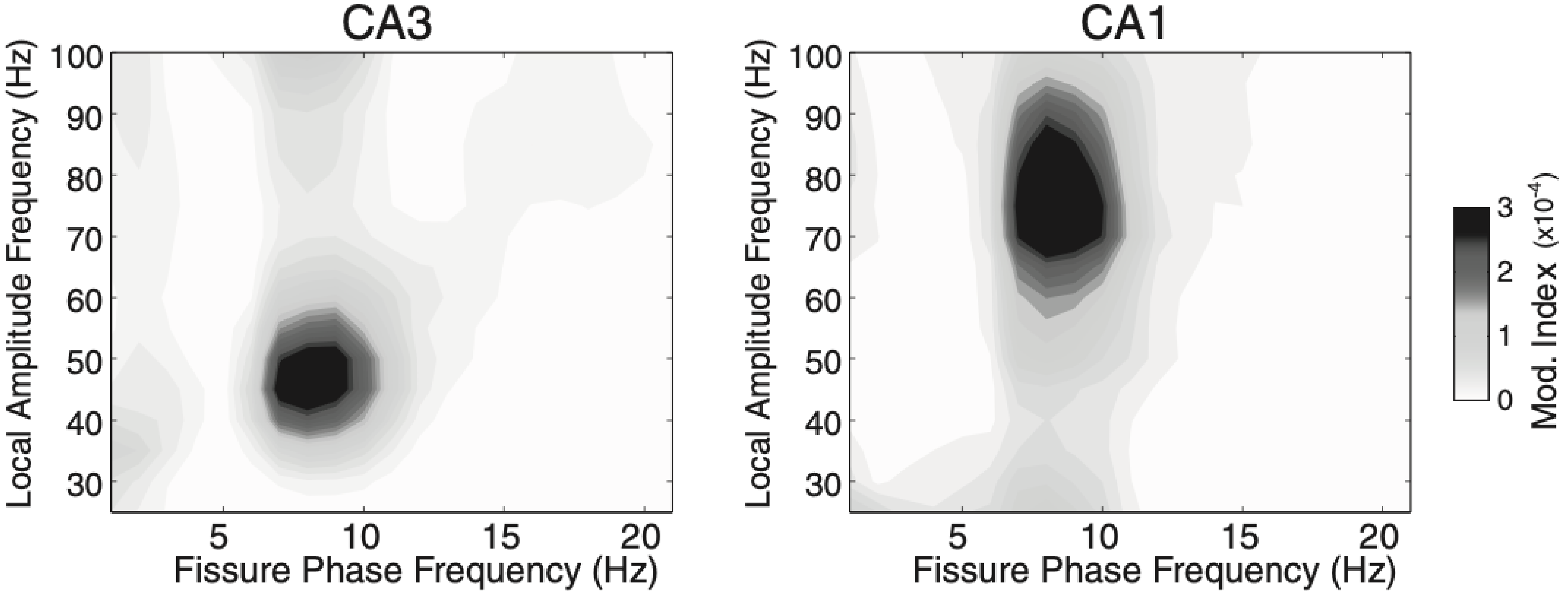
indicate h
at each
low freq, high freq
<u>pair</u>

h for
$\theta = 5$ Hz, $\gamma = 100$ Hz

CFC – Step 4

(optional): Repeat for other frequencies.

Summarize in a comodulogram



[Tort et al., J Neurophysiol, 2010]

CFC in three steps

CFC analysis steps

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Python