

Rhythms

Analyzing Rhythms (Part 3)

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Today

Rhythms from spike trains

Review

spectrum \rightarrow which rhythms dominate a signal

$$S_{xx,j} \sim X_j X_j^*$$

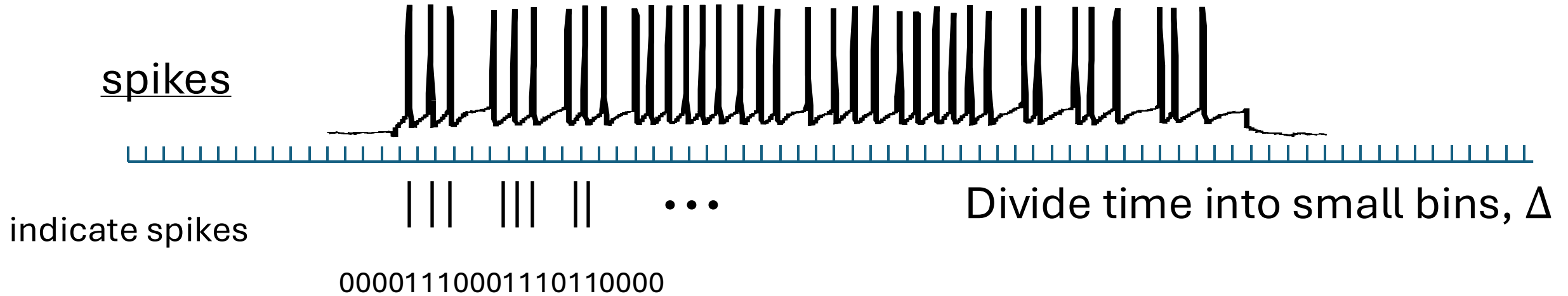
**Frequency
resolution**

$$df = \frac{1}{T} \quad \leftarrow \text{Duration of recording}$$

**Nyquist
frequency**

$$f_{\text{NQ}} = \frac{f_0}{2} \quad \leftarrow \text{Sampling frequency}$$

Today



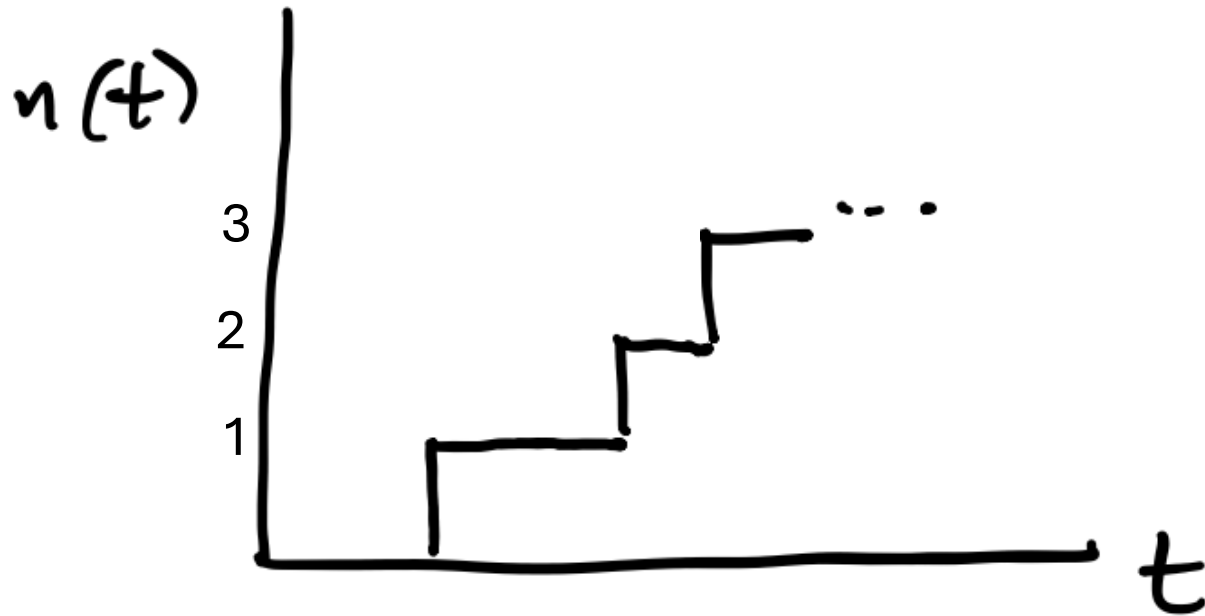
Define:

$$dn(t) = \begin{cases} 1 & \text{when a spike occurs in small time interval } \Delta \\ 0 & \text{otherwise} \end{cases}$$

Why?

Q: Why $dn(t)$?

A: Consider $n(t)$ = the number of spikes that occur from 0 to t .



$dn(t)$ is the change in $n(t)$

Either 0 or 1 when a spike occurs

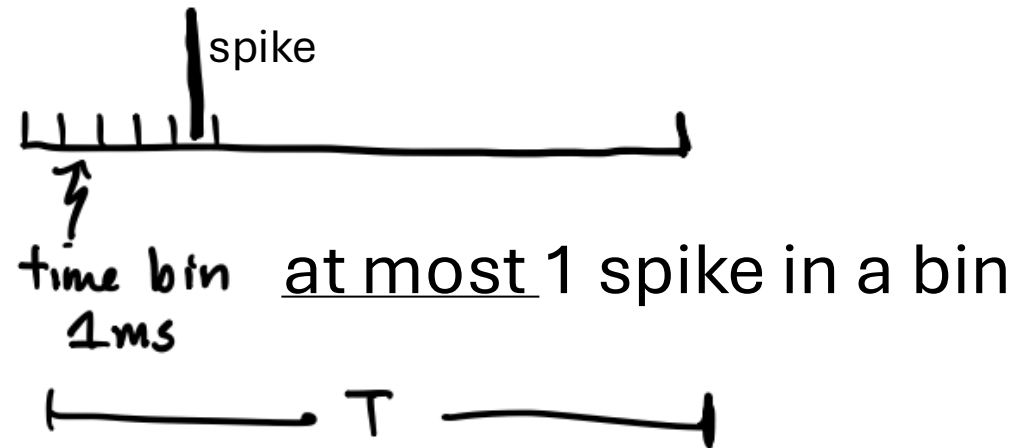
Subtract the mean

Define: $d\bar{n}(t) = dn(t) - \lambda_o \Delta$

where λ_o = mean spike rate

estimate λ_o from data $= \frac{n(T)}{T}$
 \leftarrow number of spikes after total time T
 \leftarrow total time

Δ = time bin (e.g., 1 ms)



$\lambda_o \Delta$ = expected number of spikes in a single time bin

Autocovariance (spike train)

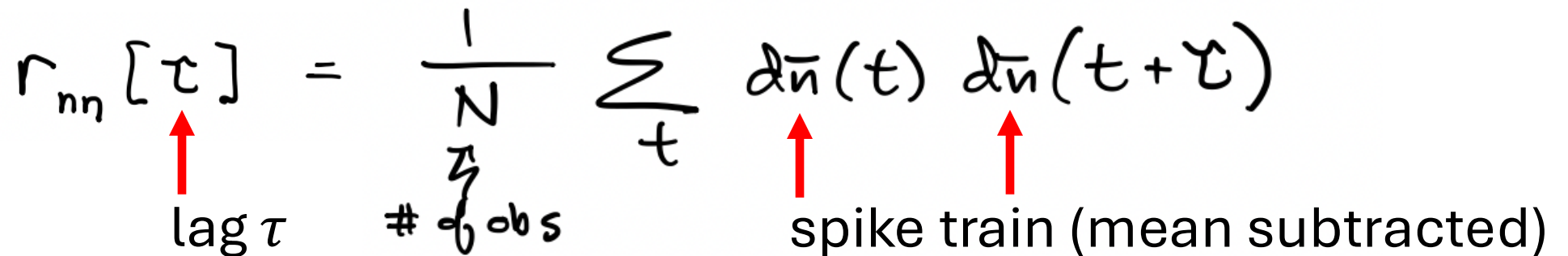
Given a spike occurred at time t , how likely is another spike at $t+\tau$?

Idea: Compare spike train to a shifted version of itself ... match?

previously
$$r_{xx}[L] = \frac{1}{N} \sum_{n=1}^{N-L} (x_{n+L} - \bar{x})(x_n - \bar{x})$$

Notation change

$$r_{nn}[\tau] = \frac{1}{N} \sum_t \tilde{x}_n(t) \tilde{x}_n(t+\tau)$$



lag τ # of obs spike train (mean subtracted)

Autocovariance (spike train)

Ex.



Q. $r_{nn}[0]$?

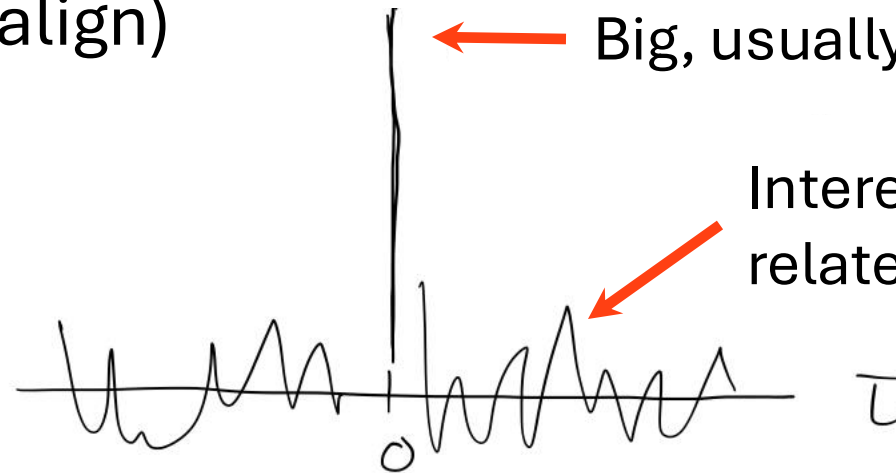


copy ($\tau = 0$)

$r_{nn}[0]$ = big (spikes align)

$r_{nn}[\neq 0] = ?$

Plot $r_{nn}[\tau]$



← Big, usually ignore it

← Interesting (how spike now relates to future spikes)

Autocovariance (spike train)

Remember: spectrum is the Fourier transform of the autocovariance

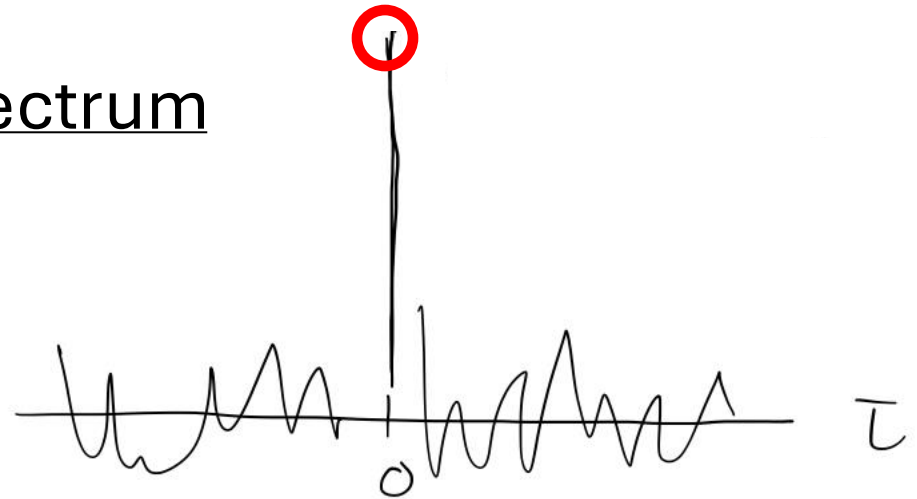
$$S_{nn,j} = 2 \Delta FT\{r_{nn}\}$$

↑
function of frequency

↑
function of time

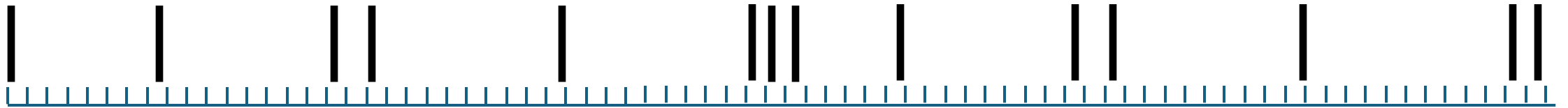
So, the peak in r_{nn} at lag 0 impacts the spectrum

Q. How?



Autocovariance (spike train)

Ex. Random spiking at a fixed rate



N samples, time step Δ

each moment in time: a (biased) coin flip

No dependence on past or future

Q. What is the autocovariance $r_{nn}[\tau]$?

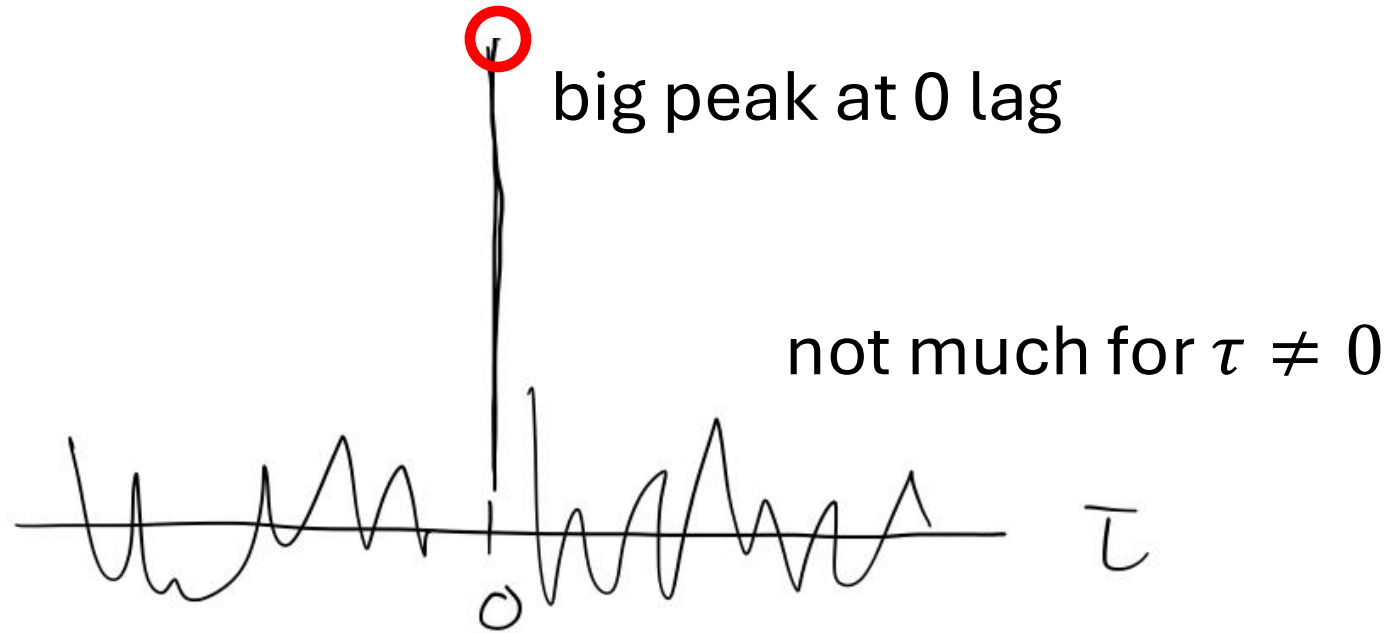
Q. What is the spectrum $S_{nn,j}$?

Autocovariance (spike train)

Ex. Random spiking at a fixed rate



Plot $r_{nn}[\tau]$



Q. What is $r_{nn}[0]$?

Autocovariance (spike train)

Ex. Random spiking at a fixed rate

Q. What is $r_{nn}[0]$?

$$r_{nn}[0] = \frac{1}{N} \sum_t \underbrace{d\bar{n}(t)^2}_{\substack{\approx 0 \text{ or } 1}} \approx \frac{\text{total \# spikes}}{\text{total \# time bins}} = \frac{n(T)}{N}$$

Remember the mean spike rate: $\lambda_0 = \frac{n(T)}{T}$ so $n(T) = \lambda_0 T$

$$r_{nn}[0] = \frac{\lambda_0 T}{N} \leftarrow T = N \Delta$$

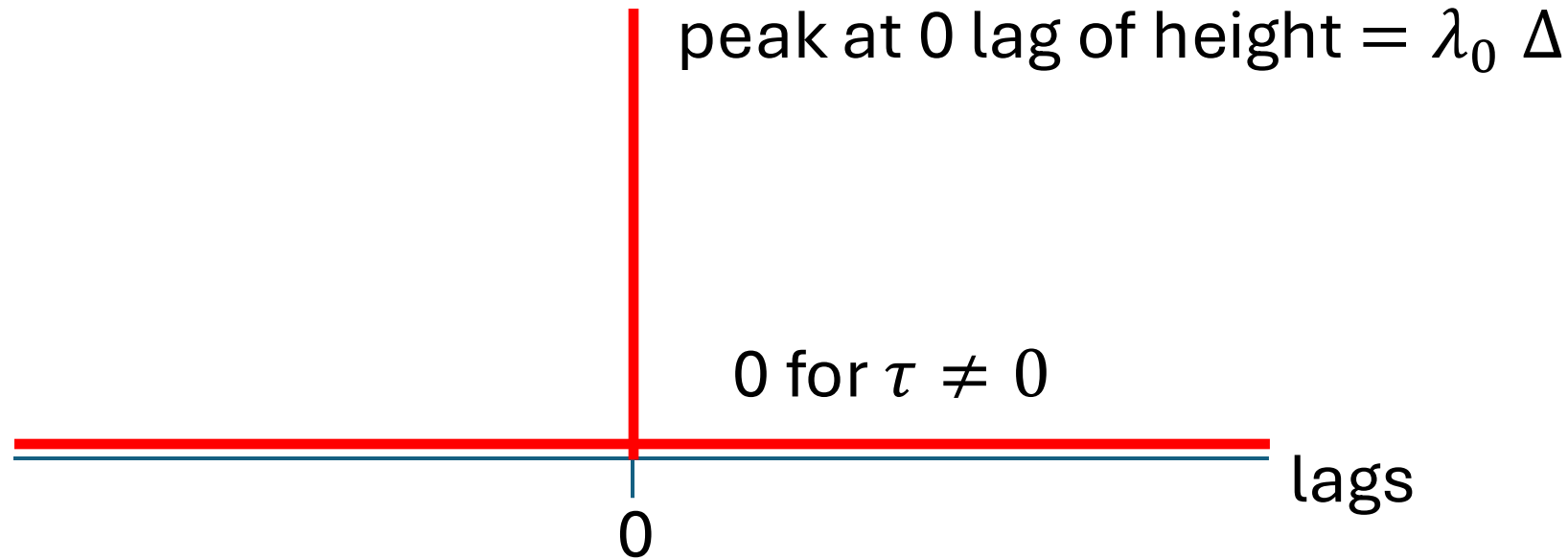
$$r_{nn}[0] = \lambda_0 \Delta$$

expected number of spikes in a time bin

Autocovariance (spike train)

Ex. Random spiking at a fixed rate

Plot idealized $r_{nn}[\tau]$



$r_{nn}[\tau] = \lambda_0 \Delta \delta[\tau]$ where $\delta[\tau] = \begin{cases} 1 & \text{when } \tau = 0 \\ 0 & \text{otherwise} \end{cases}$ **Q.** What is the spectrum?

Autocovariance (spike train)

Ex. Random spiking at a fixed rate

Q. What is the spectrum?

In our case, we need $FT\{\delta[\tau]\}$

in our example

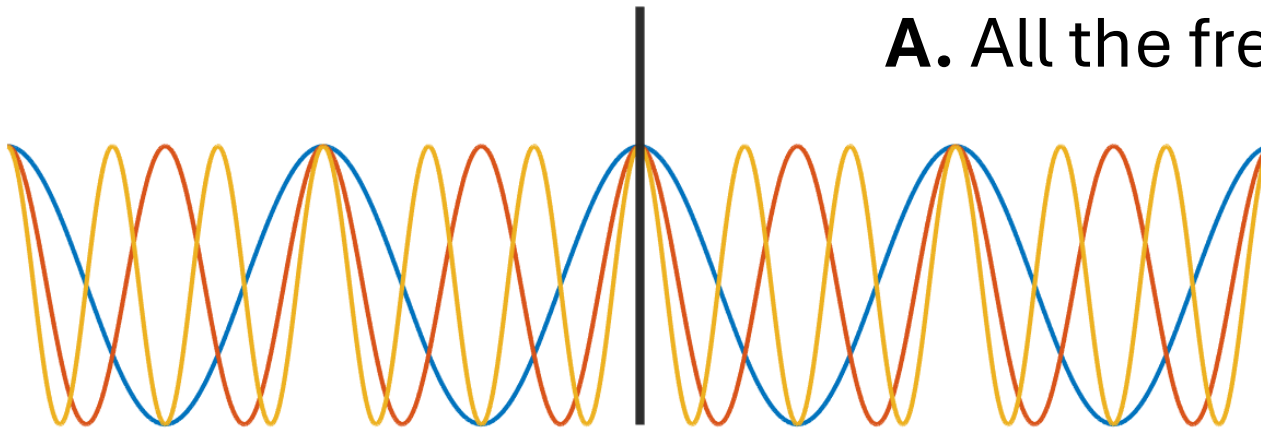
$$r_{nn}[\tau] = \lambda_0 \Delta \delta[\tau]$$



$$\begin{aligned} \text{Use } S_{nn,j} &= 2 \Delta FT\{r_{nn}\} = 2 \Delta FT\{\lambda_0 \Delta \delta[\tau]\} \\ &= 2 \Delta^2 \lambda_0 FT\{\delta[\tau]\} \end{aligned}$$

Q. What frequency sinusoids do we need?

A. All the frequencies



$$FT\{\delta[\tau]\} = 1$$

Need contribution from each frequency to capture this sharp event

Autocovariance (spike train)

Ex. Random spiking at a fixed rate

Q. What is the spectrum?

$$S_{nn,j} = 2 \Delta FT\{r_{nn}\}$$



$$S_{nn,j} = 2 \Delta FT\{\lambda_0 \Delta \delta[\tau]\}$$

$$r_{nn}[\tau] = \lambda_0 \Delta \delta[\tau]$$

$$S_{nn,j} = 2 \Delta (\lambda_0 \Delta) FT\{\delta[\tau]\}$$

$$S_{nn,j} = 2 \Delta^2 \lambda_0 \quad 1$$

$$S_{nn,j} = 2 \Delta^2 \lambda_0$$

This is the spectrum of random spiking at fixed rate

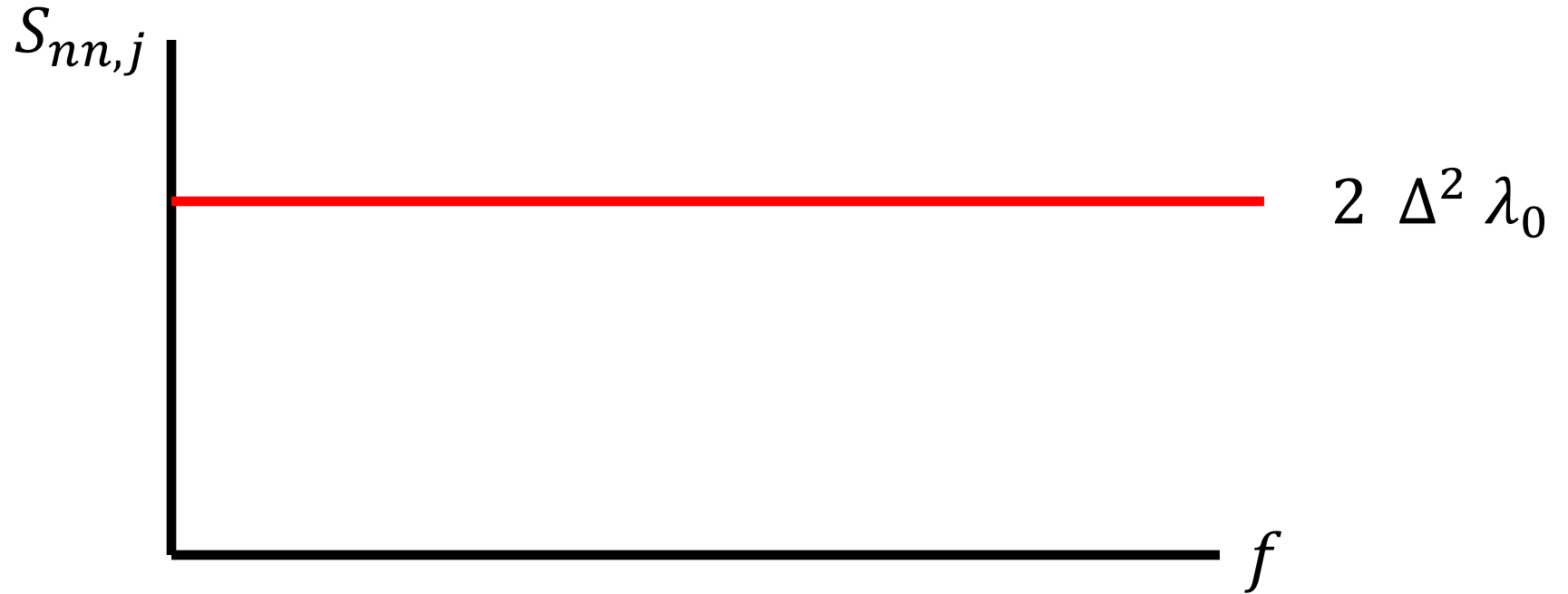
Autocovariance (spike train)

Ex. Random spiking at a fixed rate

Q. What is the spectrum?

$$S_{nn,j} = 2 \Delta^2 \lambda_0$$

Plot it



Spectrum (spike train)

Use autocovariance to gain intuition

Alternative: compute spectrum directly.

$$S_{nn,j} = \frac{2\Delta^2}{T} D_j D_j^*$$

Fourier transform of **spike train**

$$D_j = \sum_{n=1}^N d\bar{n}(t) \exp(-2\pi i f_j t_n)$$

complex exponentials at frequency f_j

Like our approach for “fields”
- new notation

spike train with mean subtracted

Spectrum (spike train)

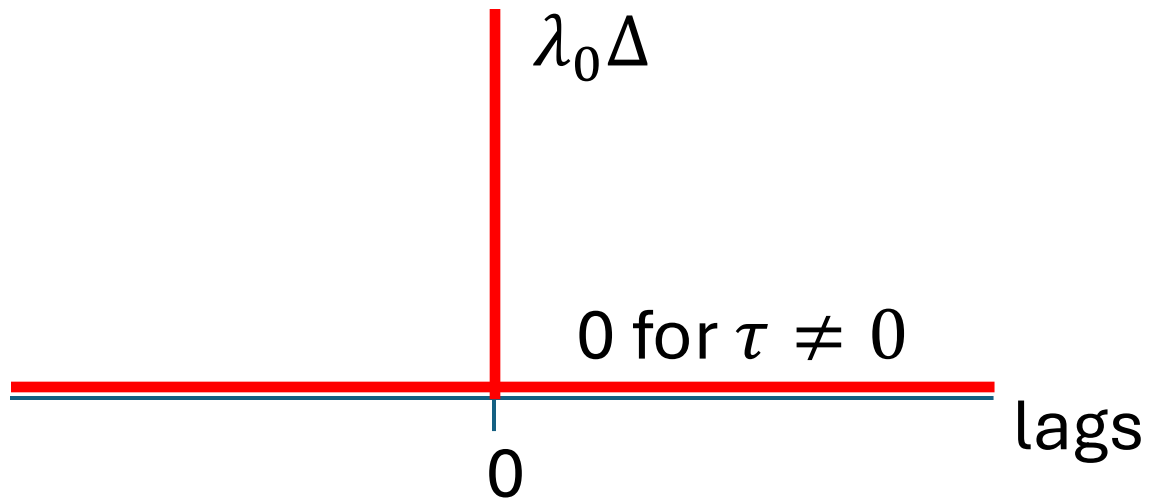
Use autocovariance to gain intuition

Random spiking

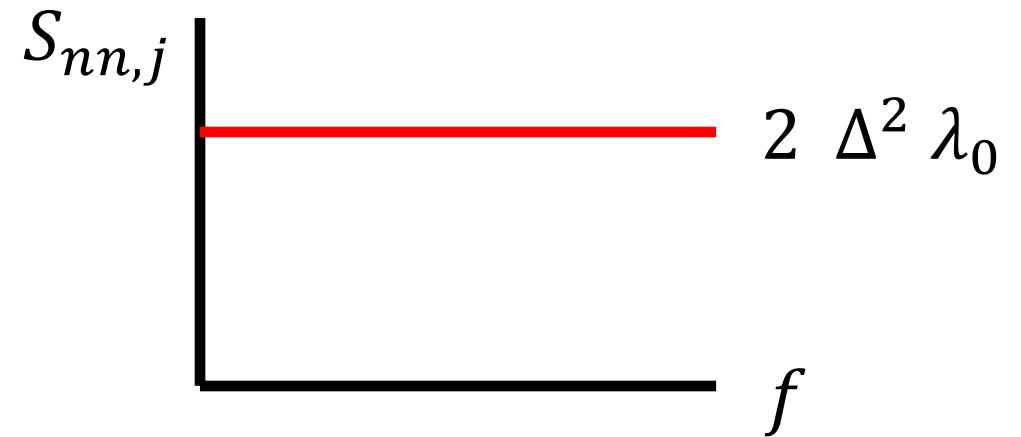


$$S_{nn,j} = 2 \Delta FT\{r_{nn}\}$$

Autocovariance



Spectrum



Autocovariance (spike train)

Ex. Random spiking at a fixed rate

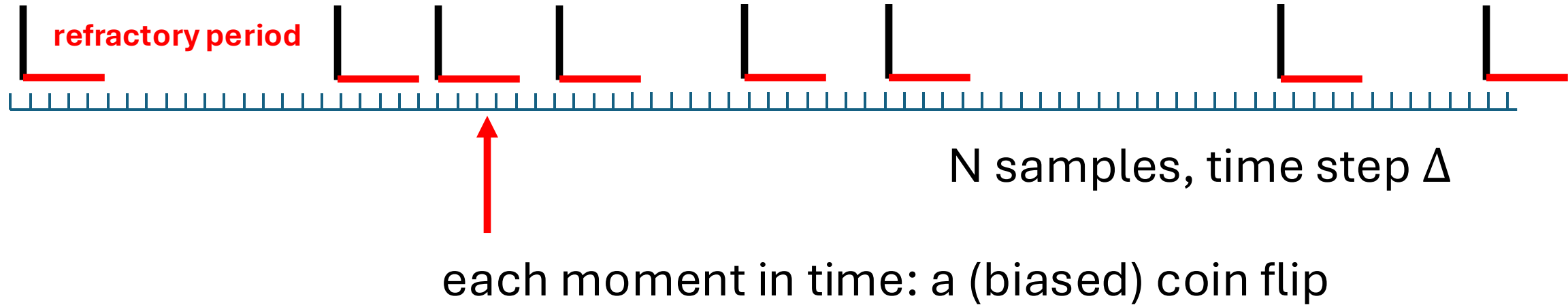
Q. What is the autocovariance?

Q. What is the spectrum?

Python

Autocovariance (spike train)

Ex. Random spiking with refractory period.



Q. What is the autocovariance $r_{nn}[\tau]$?

Q. What is spectrum $S_{nn,j}$?

Autocovariance (spike train)

Ex. Random spiking with refractory period.

Q. What is the autocovariance $r_{nn}[\tau]$?

[sketch]



Autocovariance (spike train)

Ex. Random spiking with refractory period.

Q. What is spectrum $S_{nn,j}$?

[sketch]



Autocovariance (spike train)

Ex. Random spiking with refractory period.

Q. What is the autocovariance $r_{nn}[\tau]$?

Q. What is spectrum $S_{nn,j}$?

Python