

MA666: Neural Networks and Learning

**Part 1
Backpropagation**

Today

This week we'll study learning in a “simple” neural network:

- Backpropagation

Remember, the Perceptron

Remember, the Perceptron

Cartoon & Cast of Characters

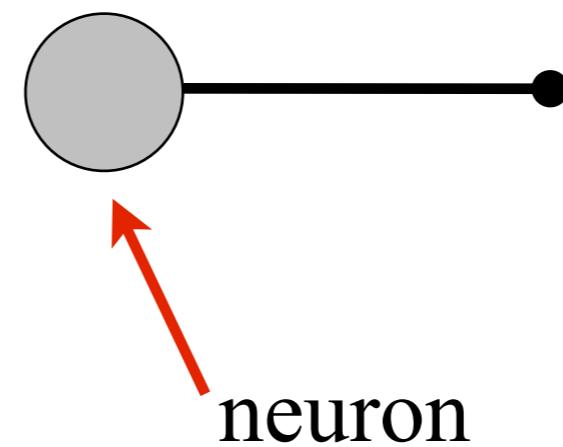
Remember, the Perceptron

Cartoon & Cast of Characters



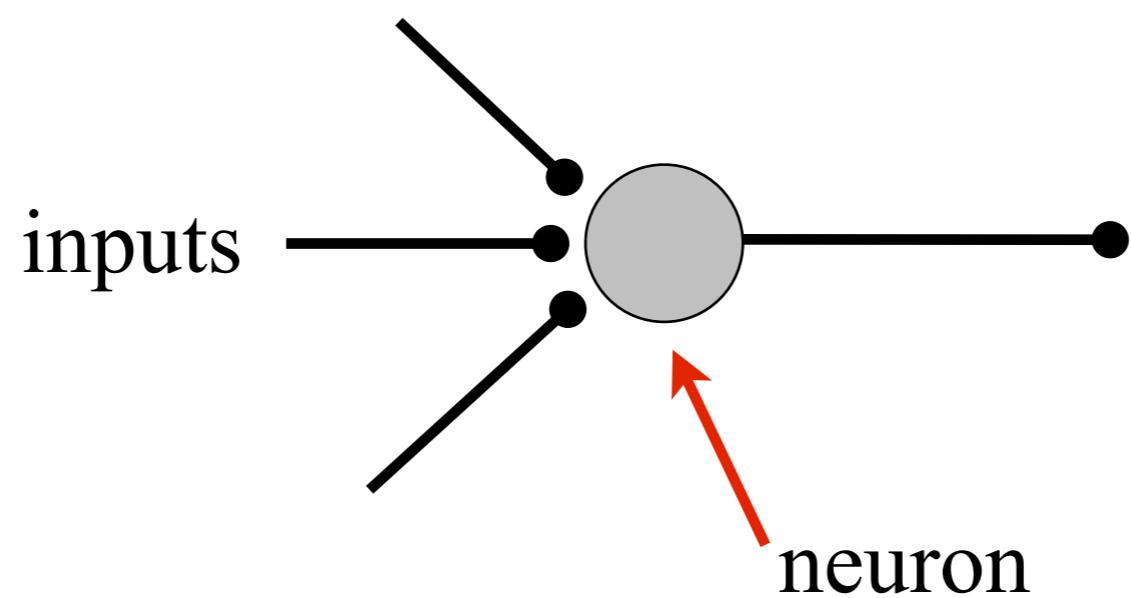
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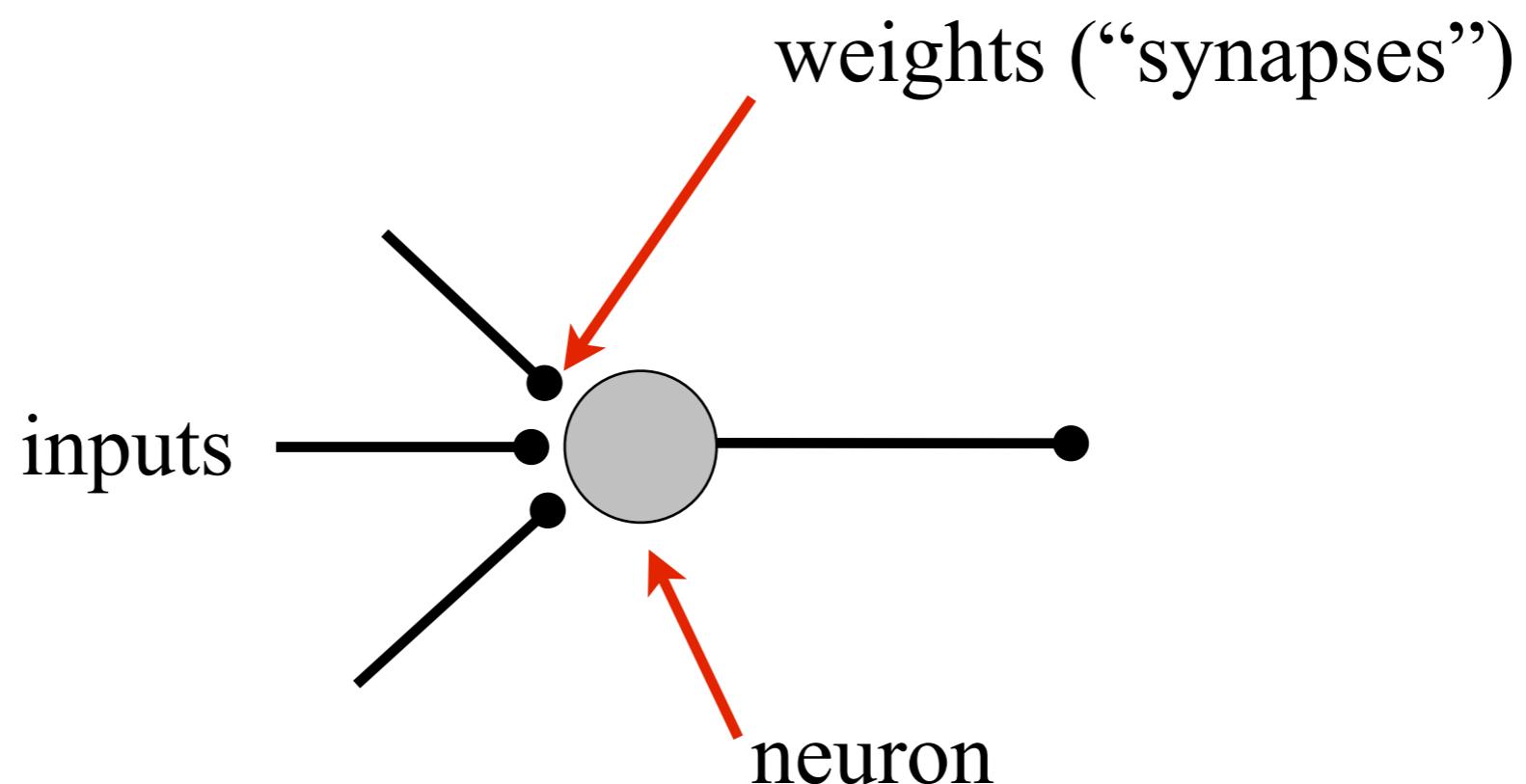
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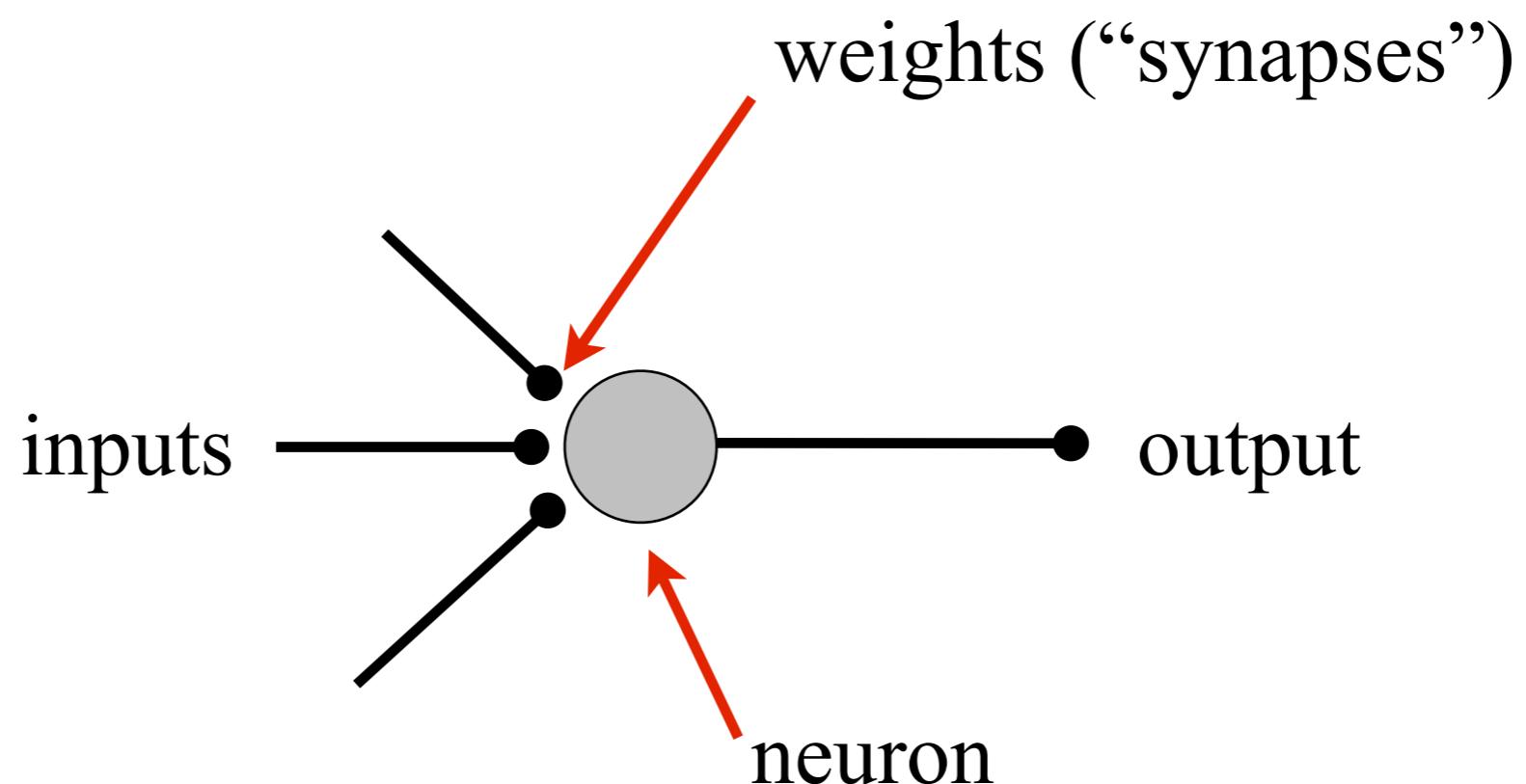
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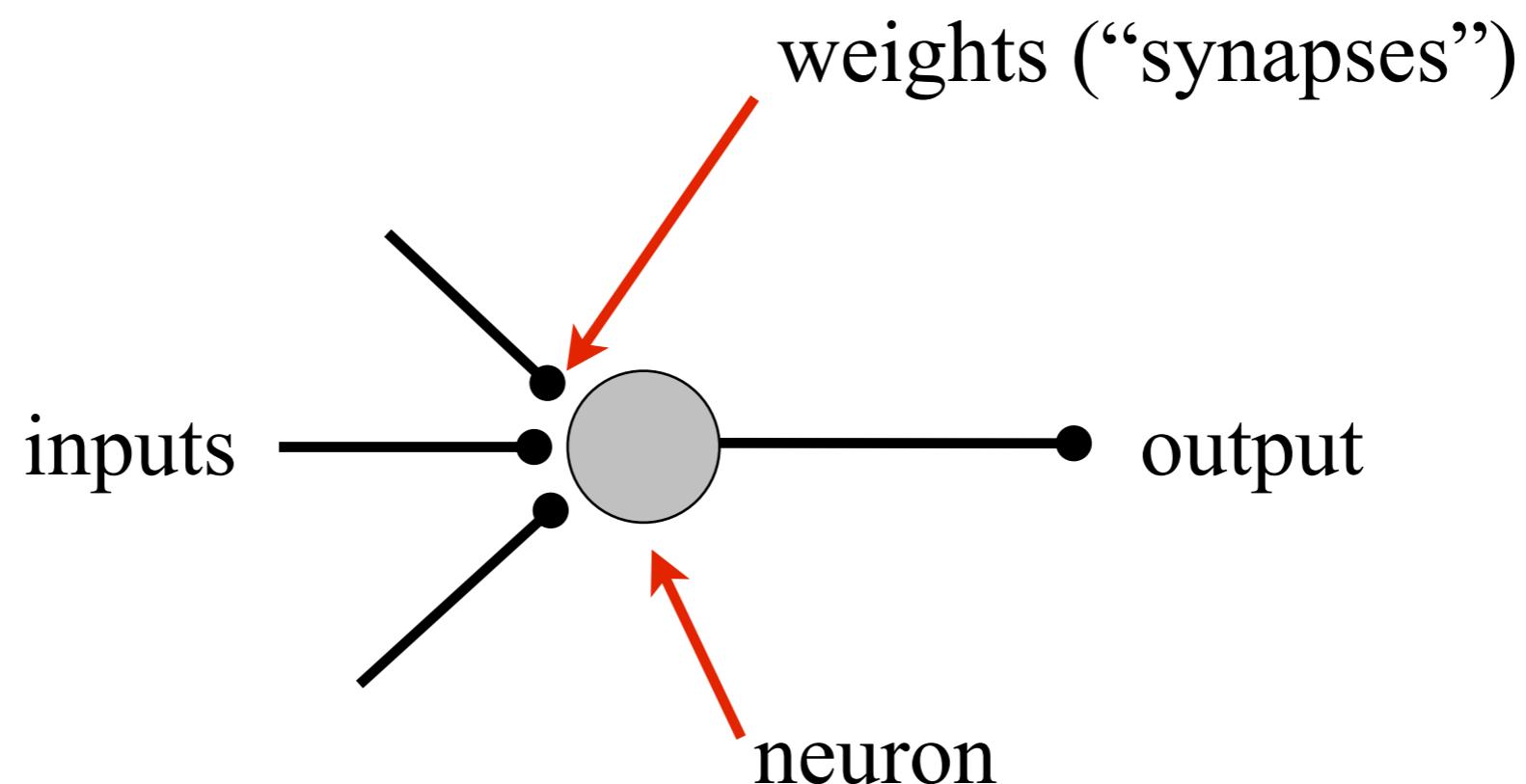
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Note: there's only one neuron.

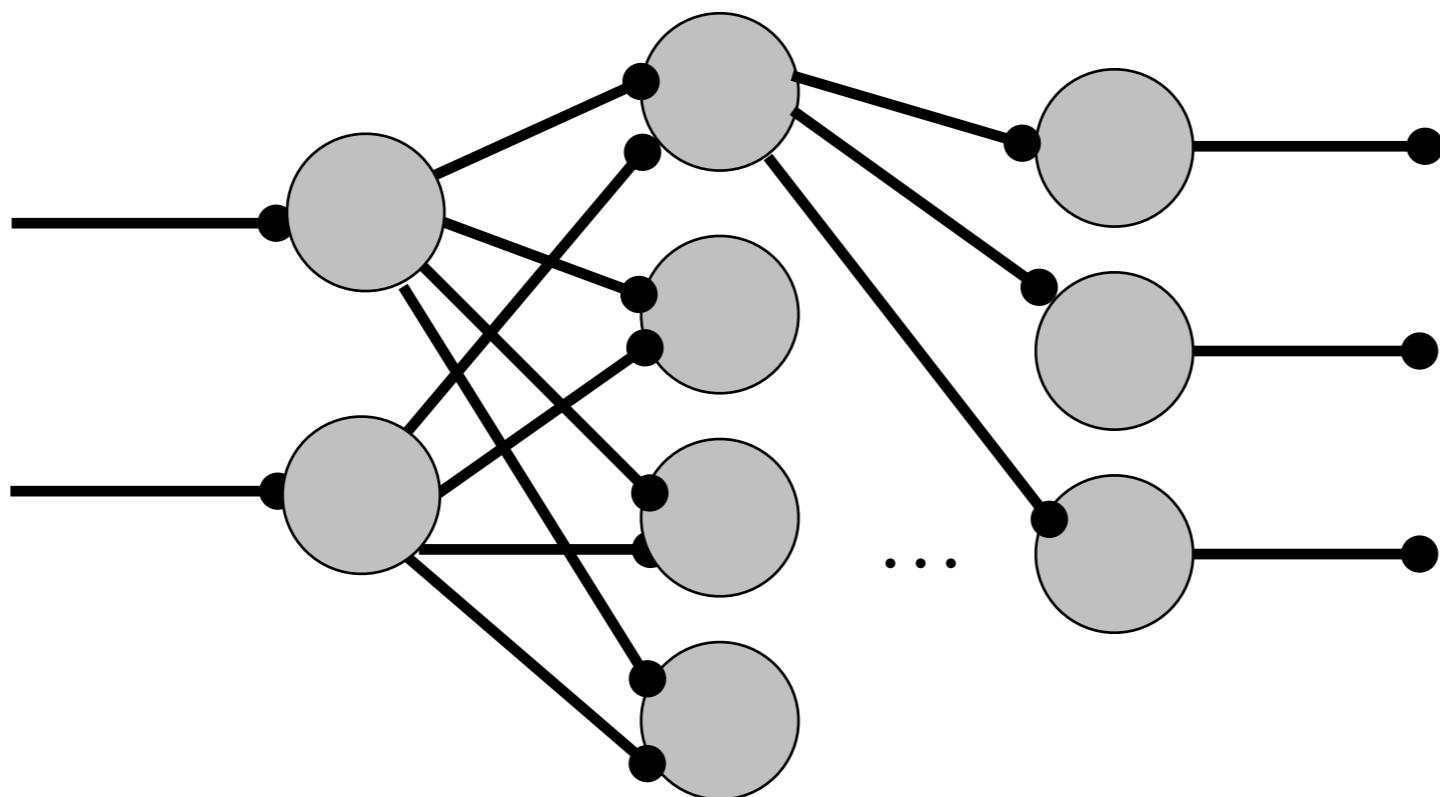
Today, a neural network

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Today, **neural networks**: collections of abstracted neurons connected to each other through weighted connections (simplified “synapses”).

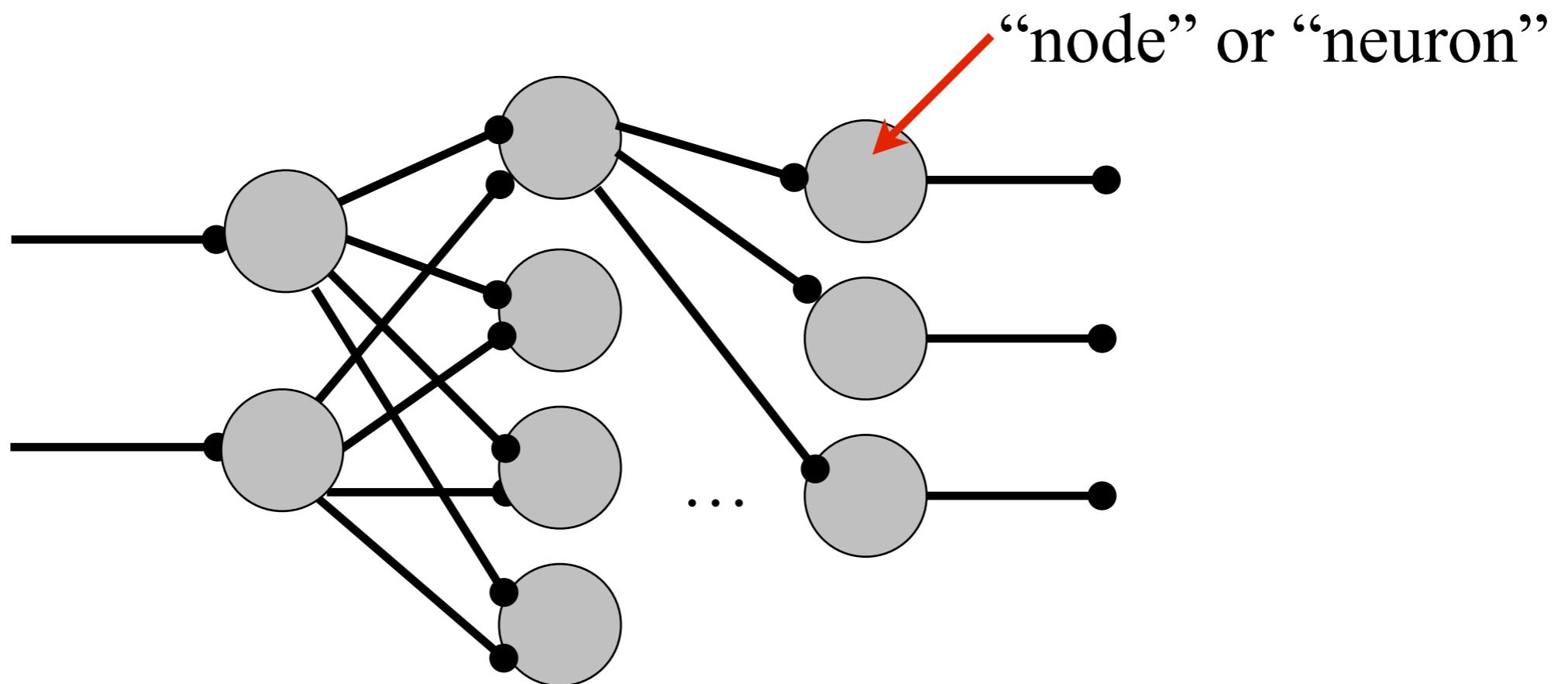
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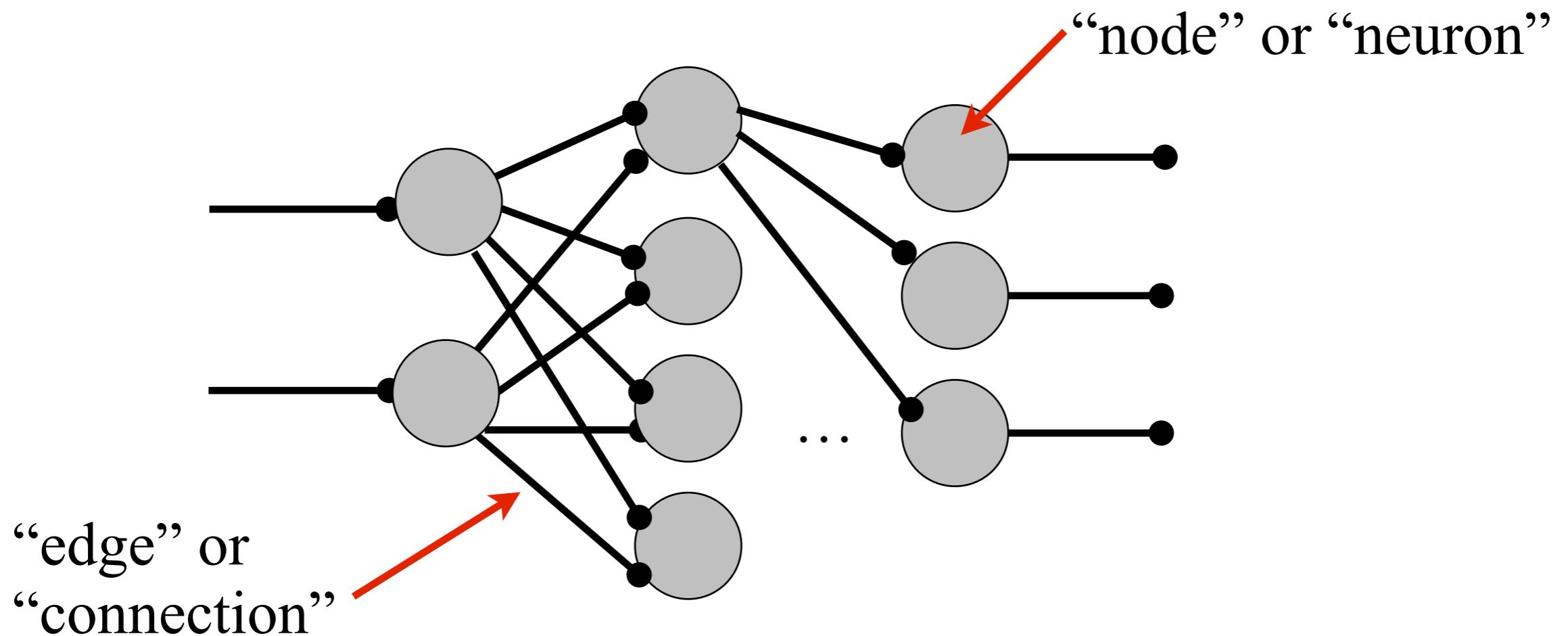
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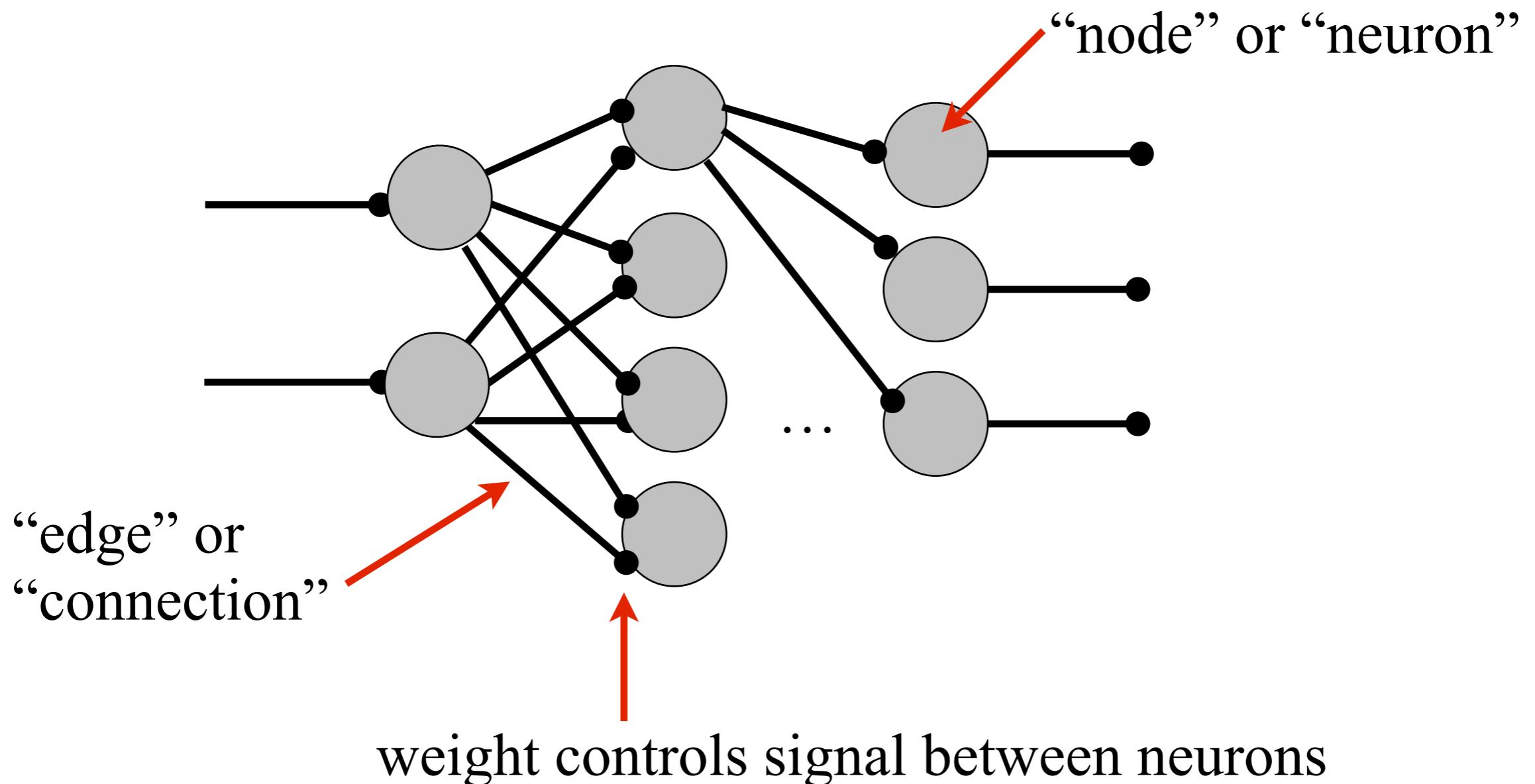
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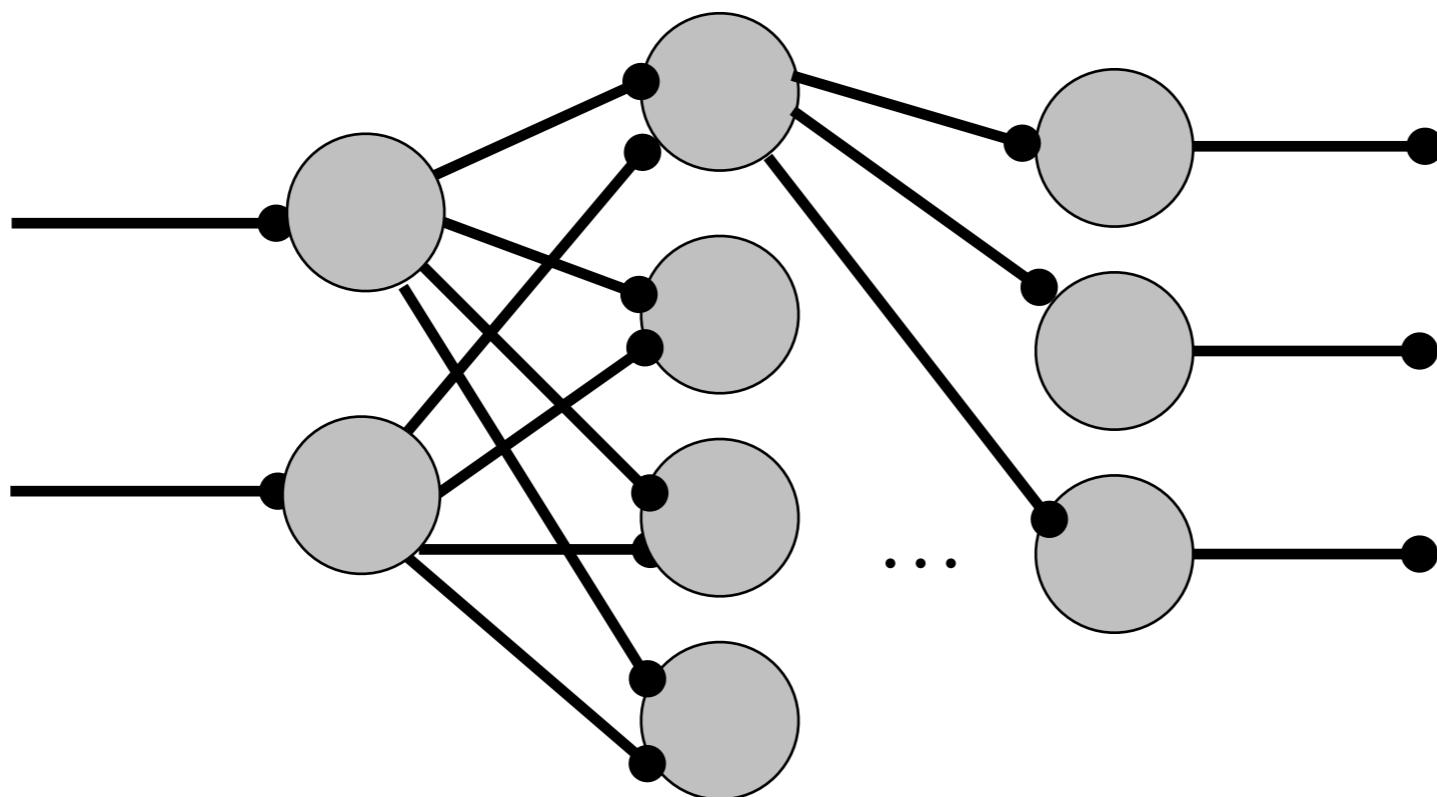
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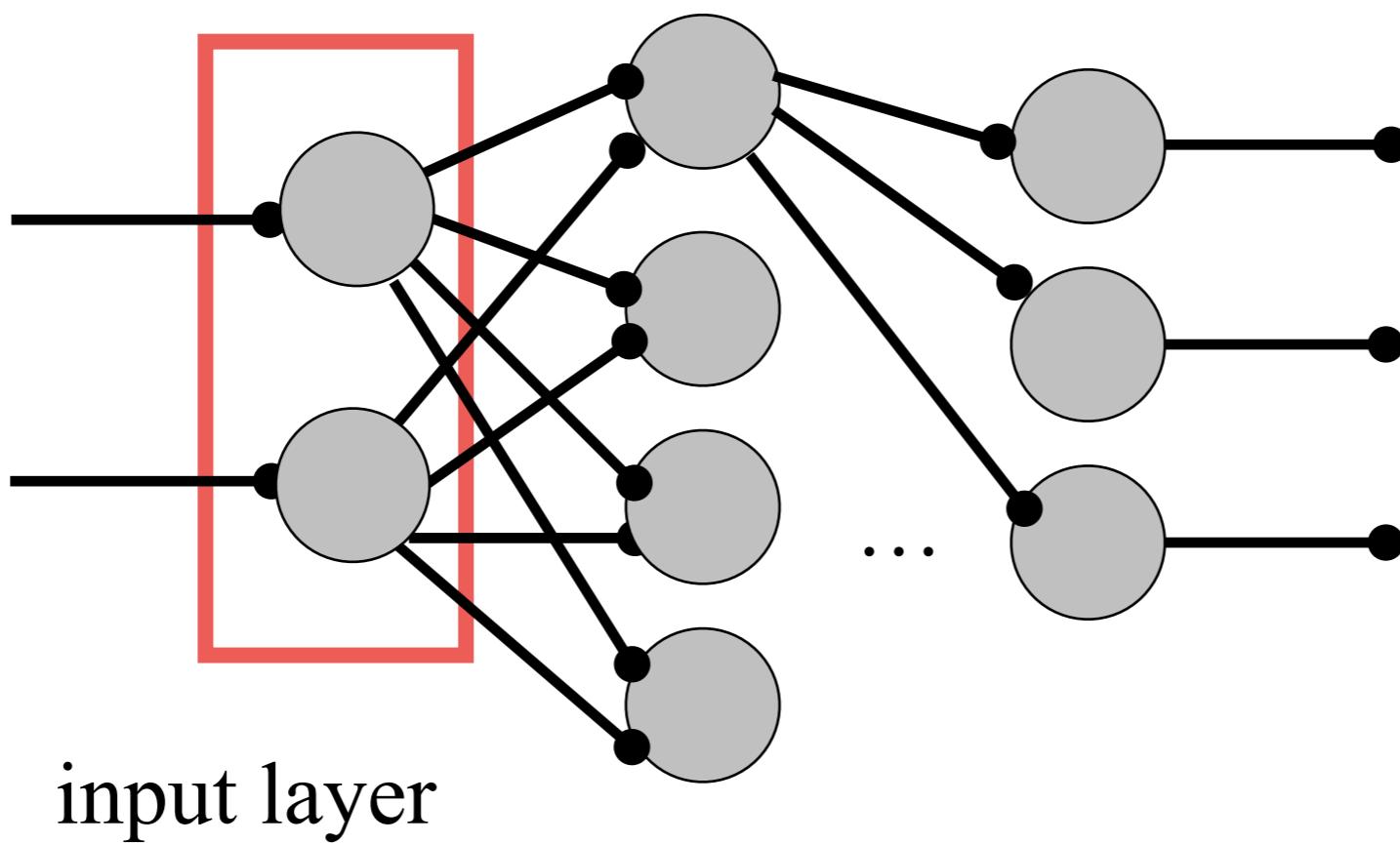
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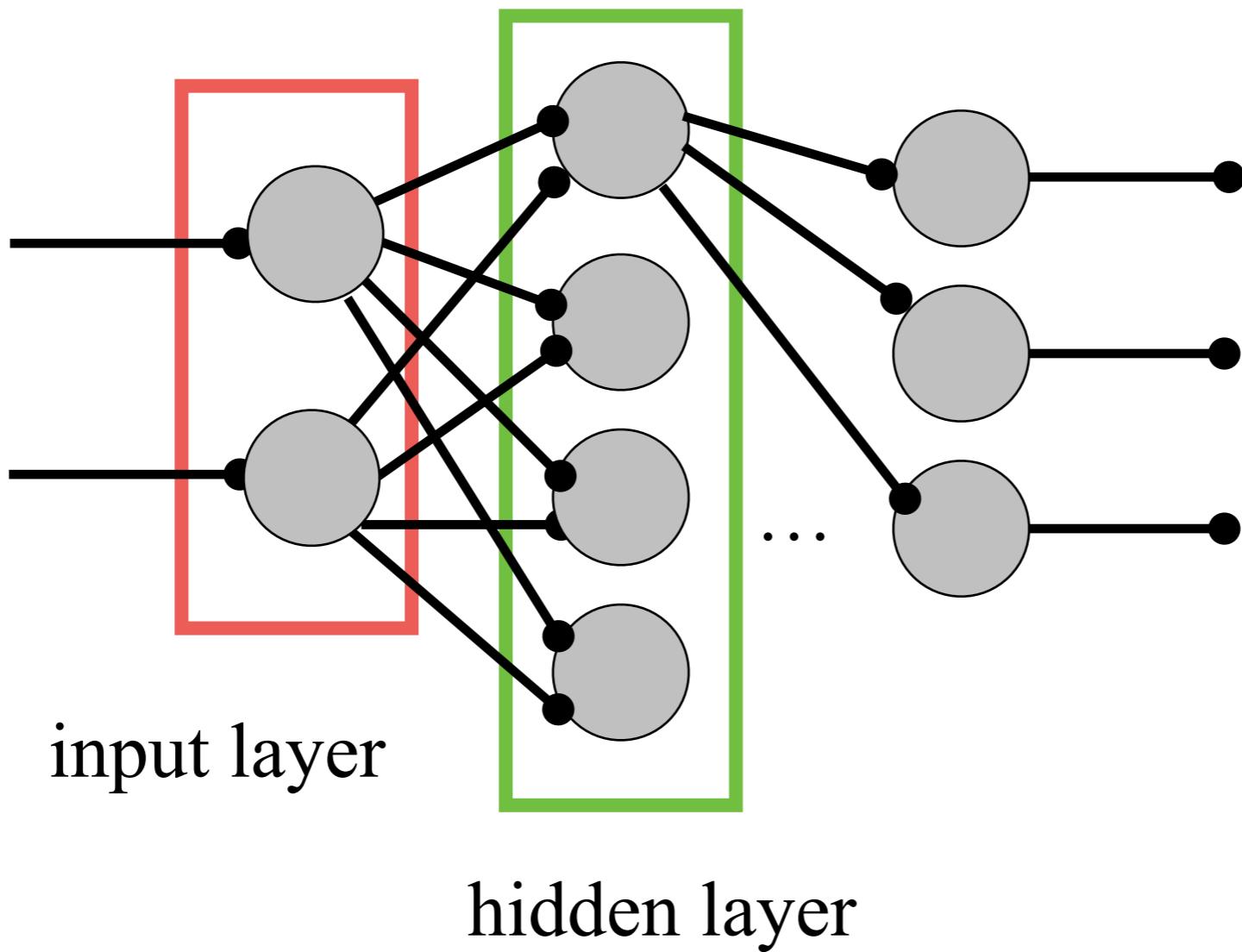
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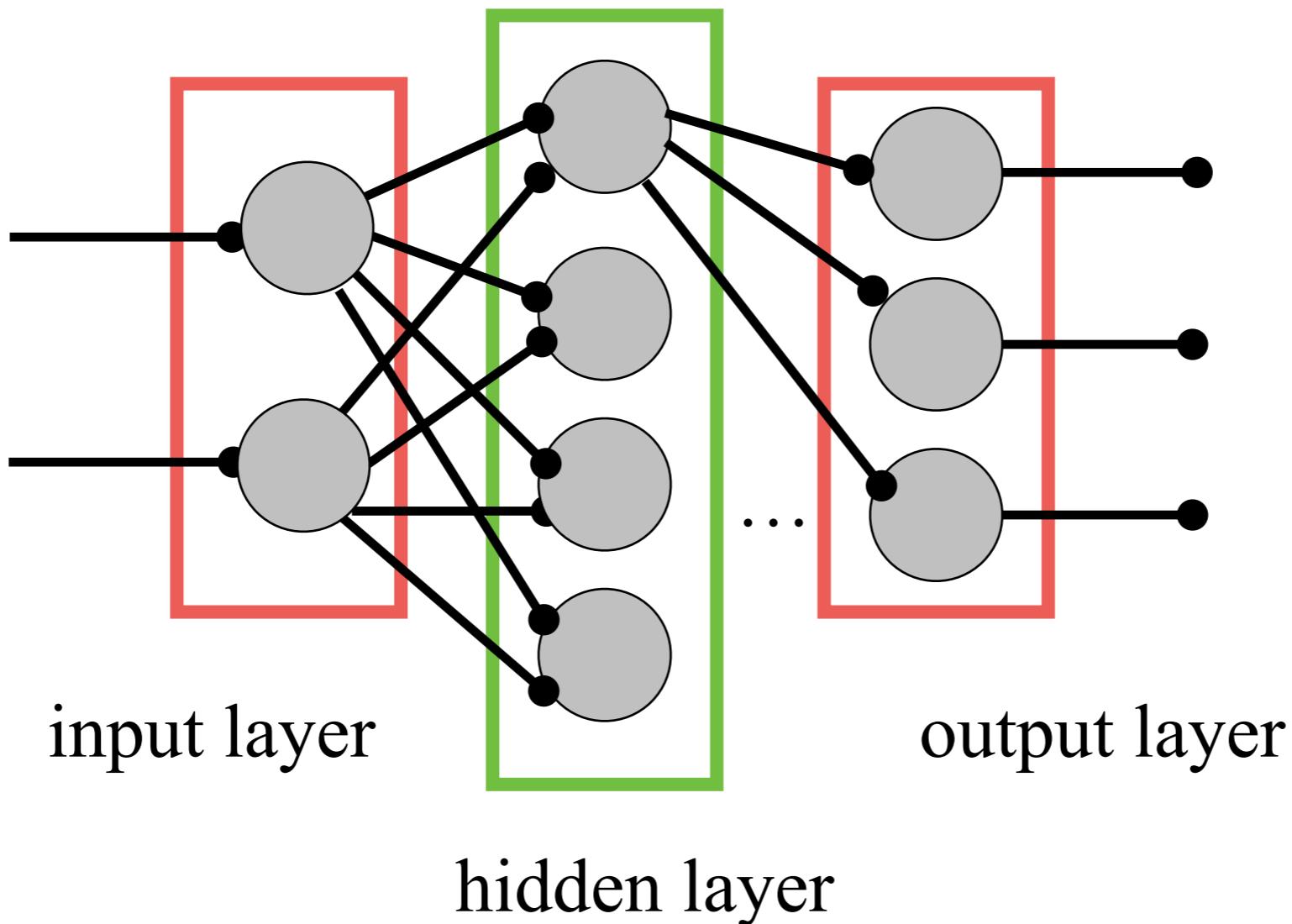
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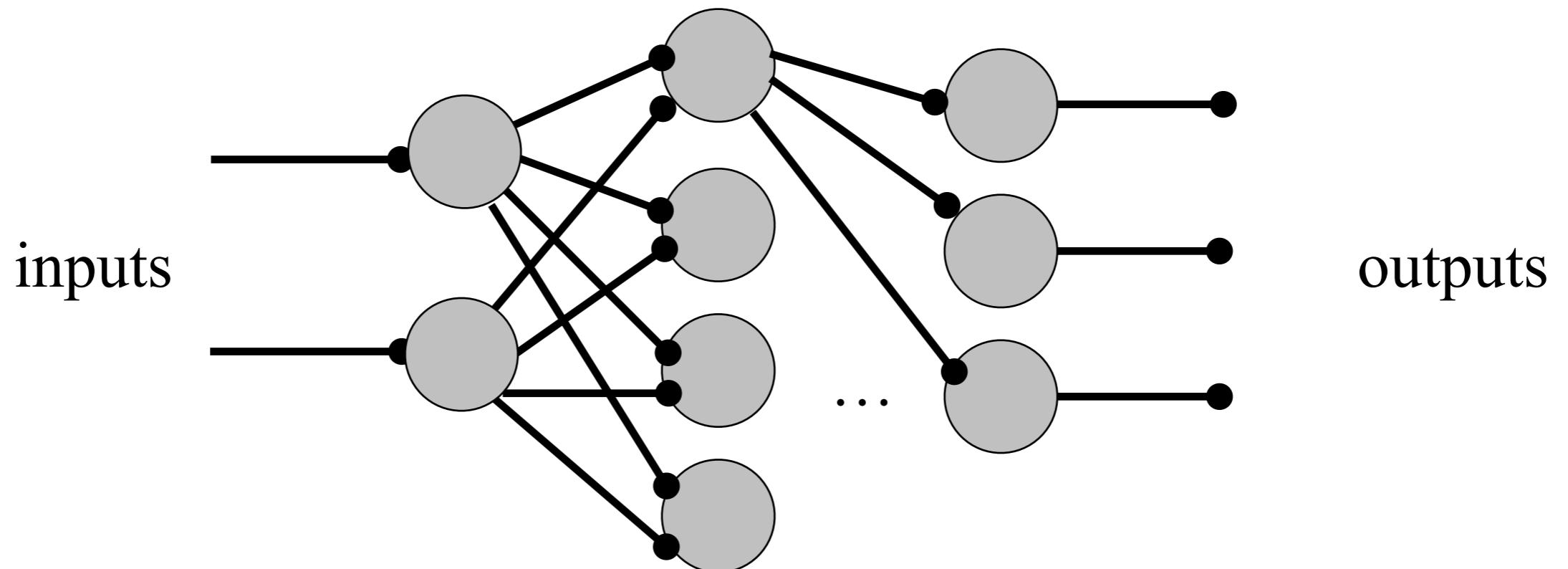
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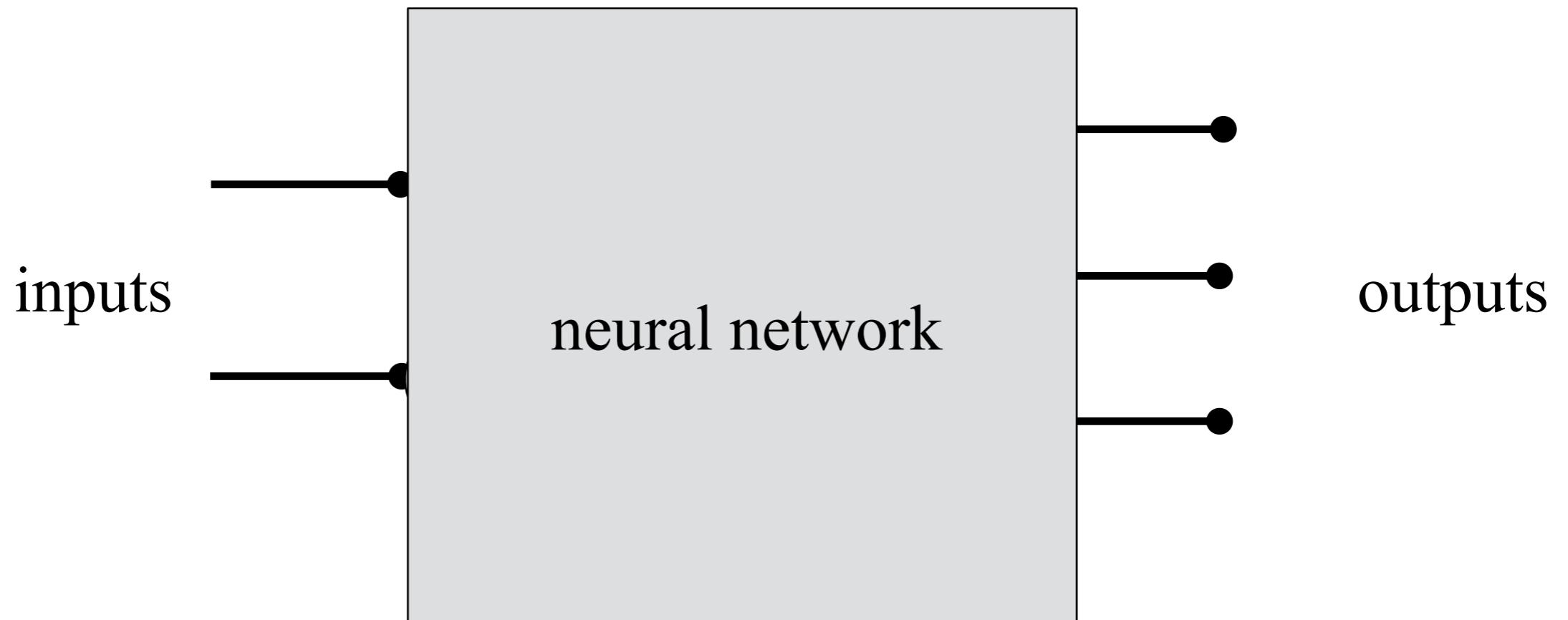
Information processed through the network

Abstractly:



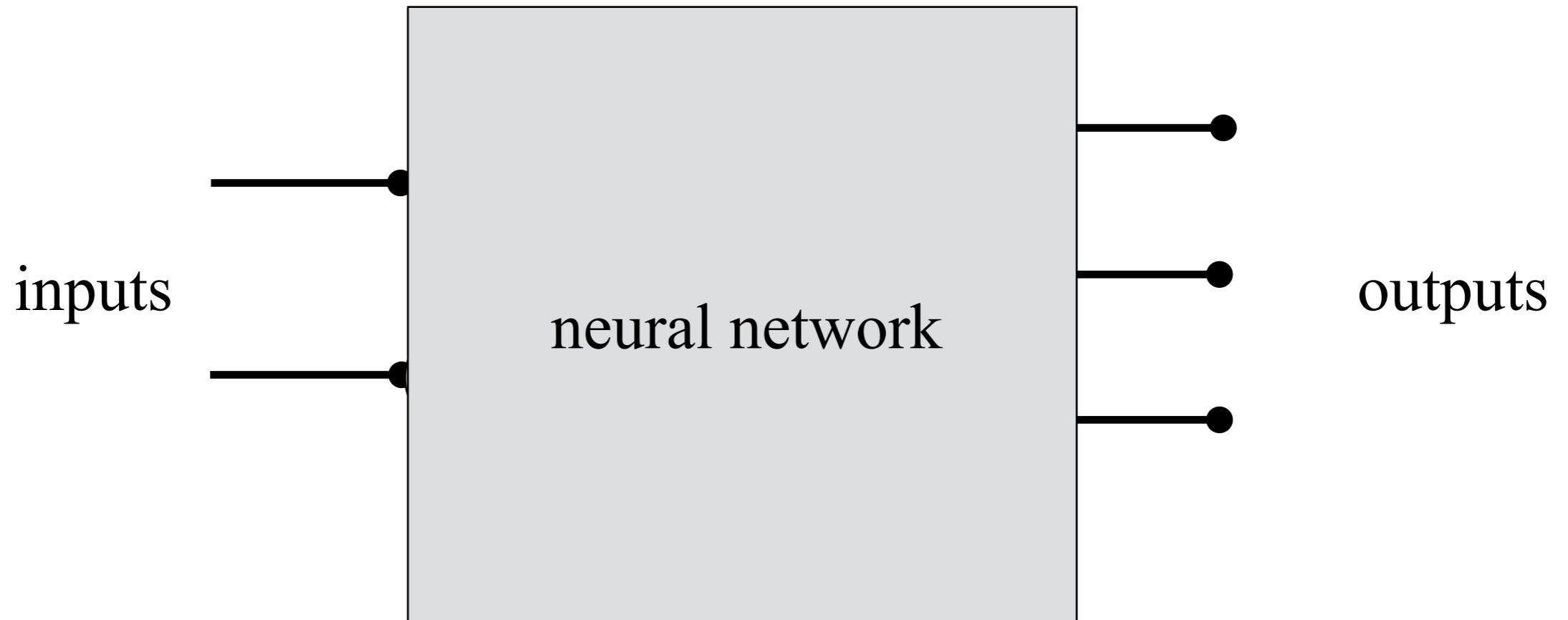
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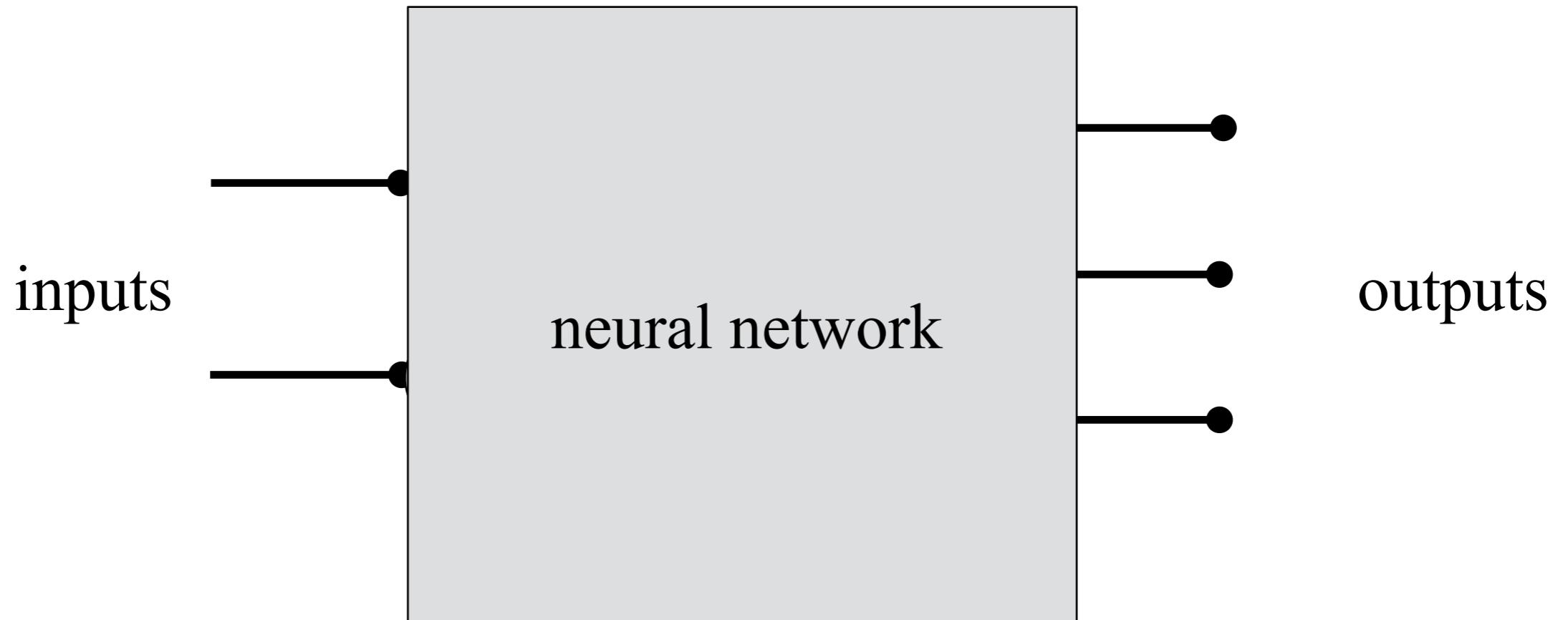
Abstractly:



Neural networks can exhibit rich behavior.

Information processed through the network

Abstractly:



Neural networks can exhibit rich behavior.

Cool example: playground.tensorflow.org

Neural networks can learn

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Neural networks are:

- **adaptive**

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 - internal structure changes based on information flowing through the network.

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We trained a perceptron ...

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We trained a perceptron ...

Now, we'll train a neural network to do what we want ...

Neural networks can learn

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Some terminology:

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- “*training a neural network*”
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For a set of weights & input, calculate output.

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For a set of weights & input, calculate output.
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Determine error in output, and adjust weights to decrease error.

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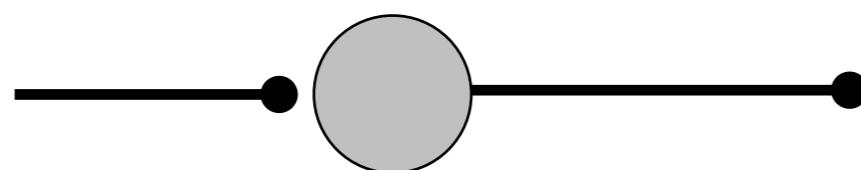
Let’s train a “simple” neural network to do our bidding ...

A “simple” neural network

Start with perceptron ...

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A “simple” neural network

Start with perceptron ...

add a node ...



A “simple” neural network

Start with perceptron ...

add a node ...

and label everything.

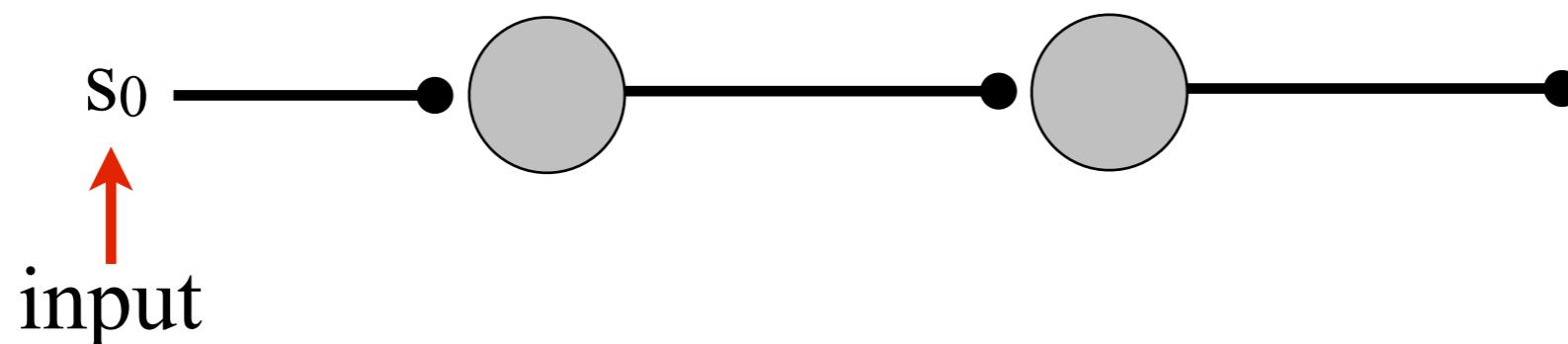


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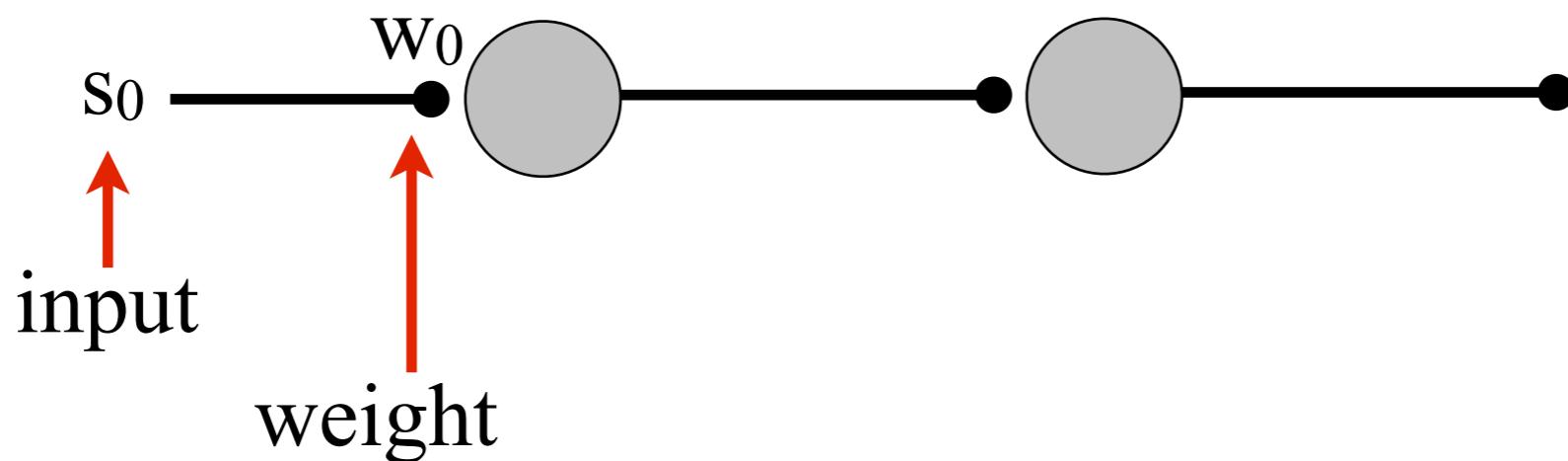


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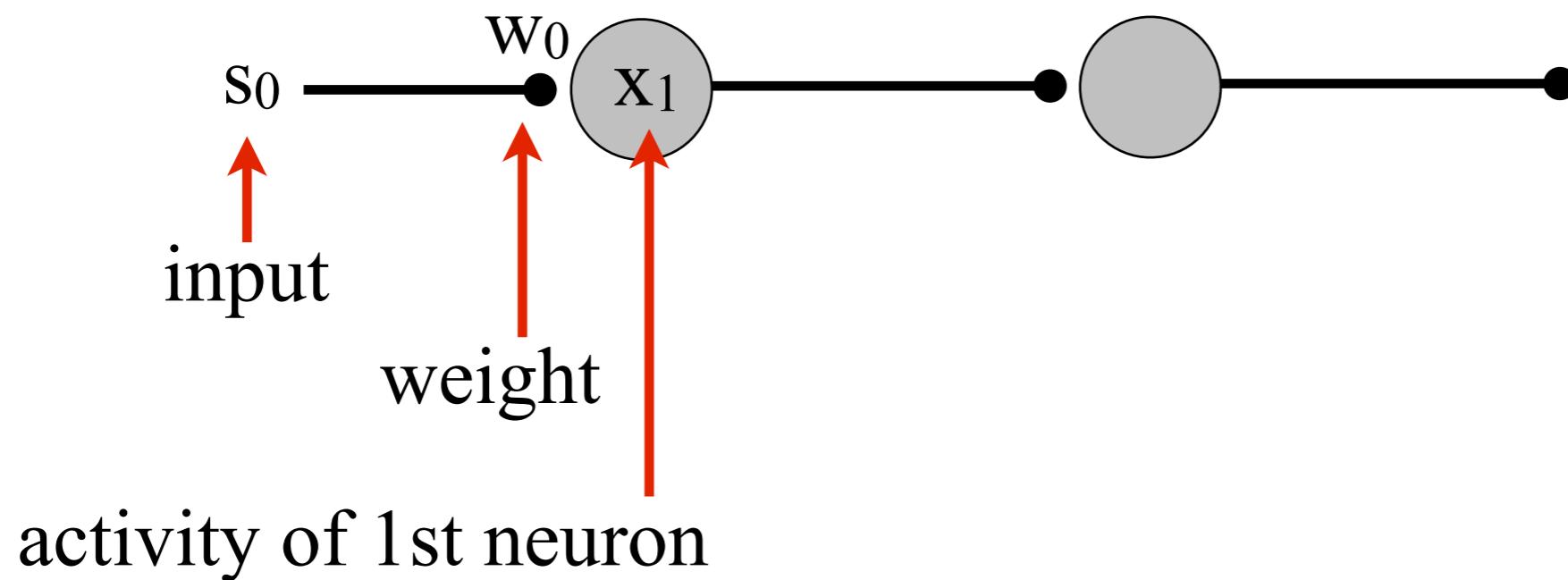


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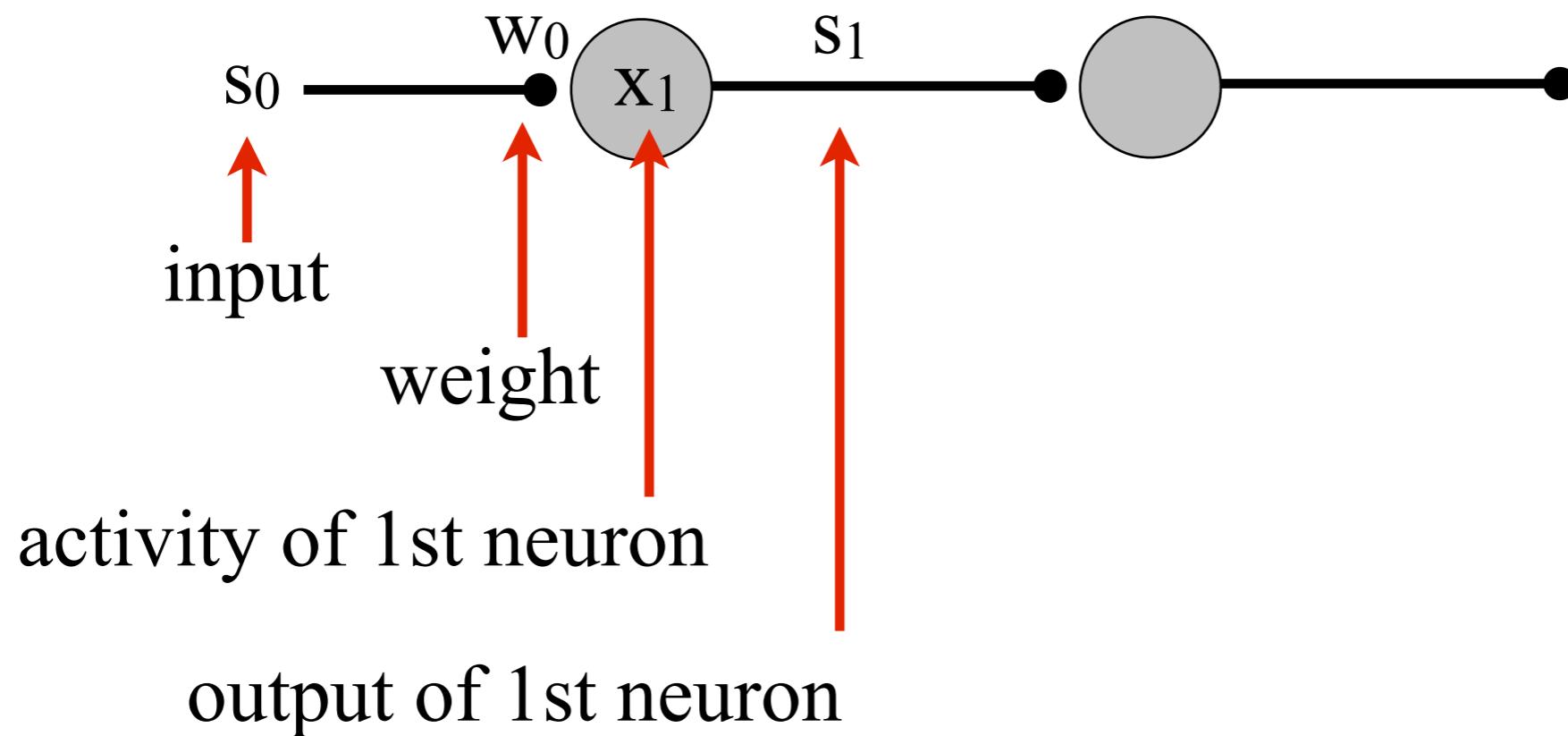


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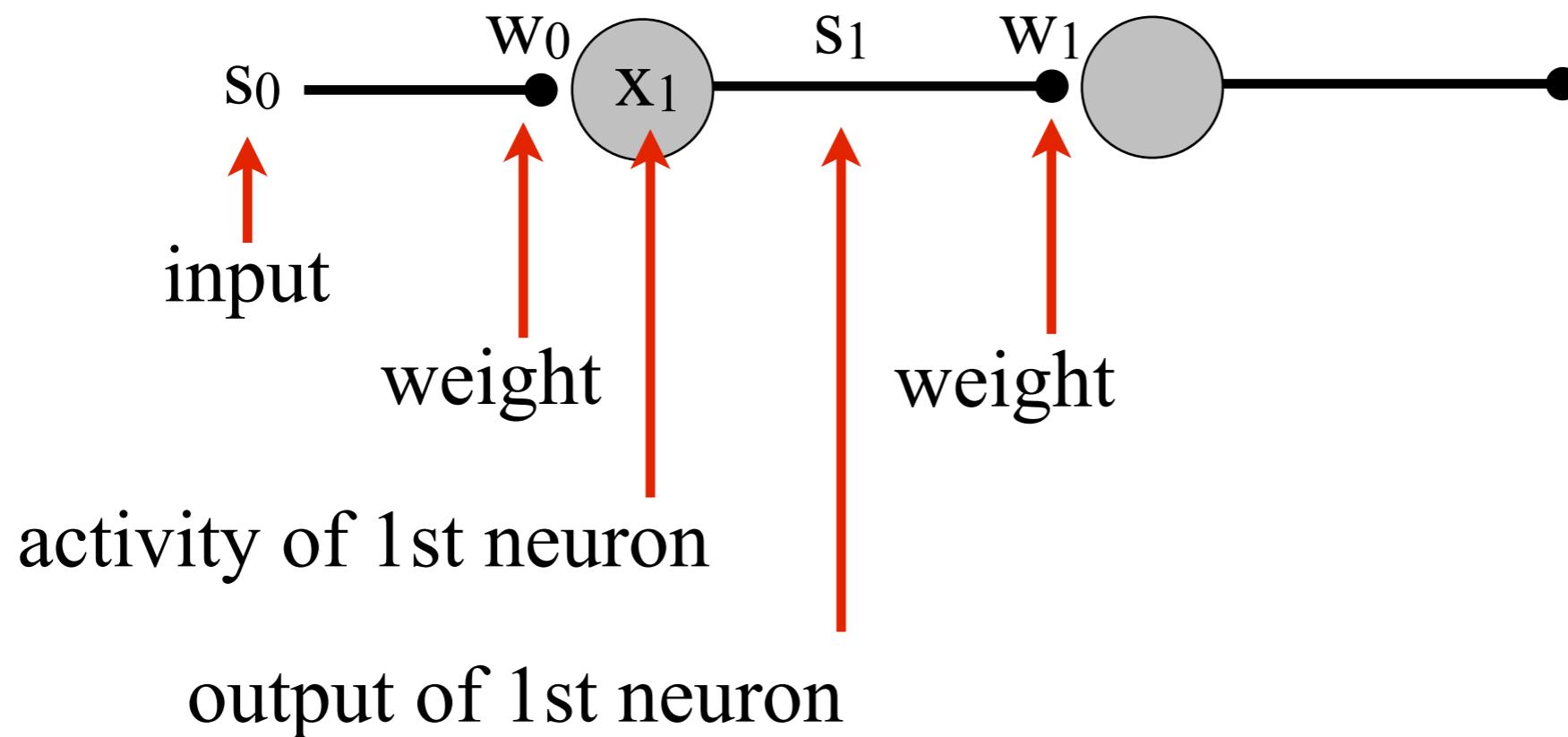


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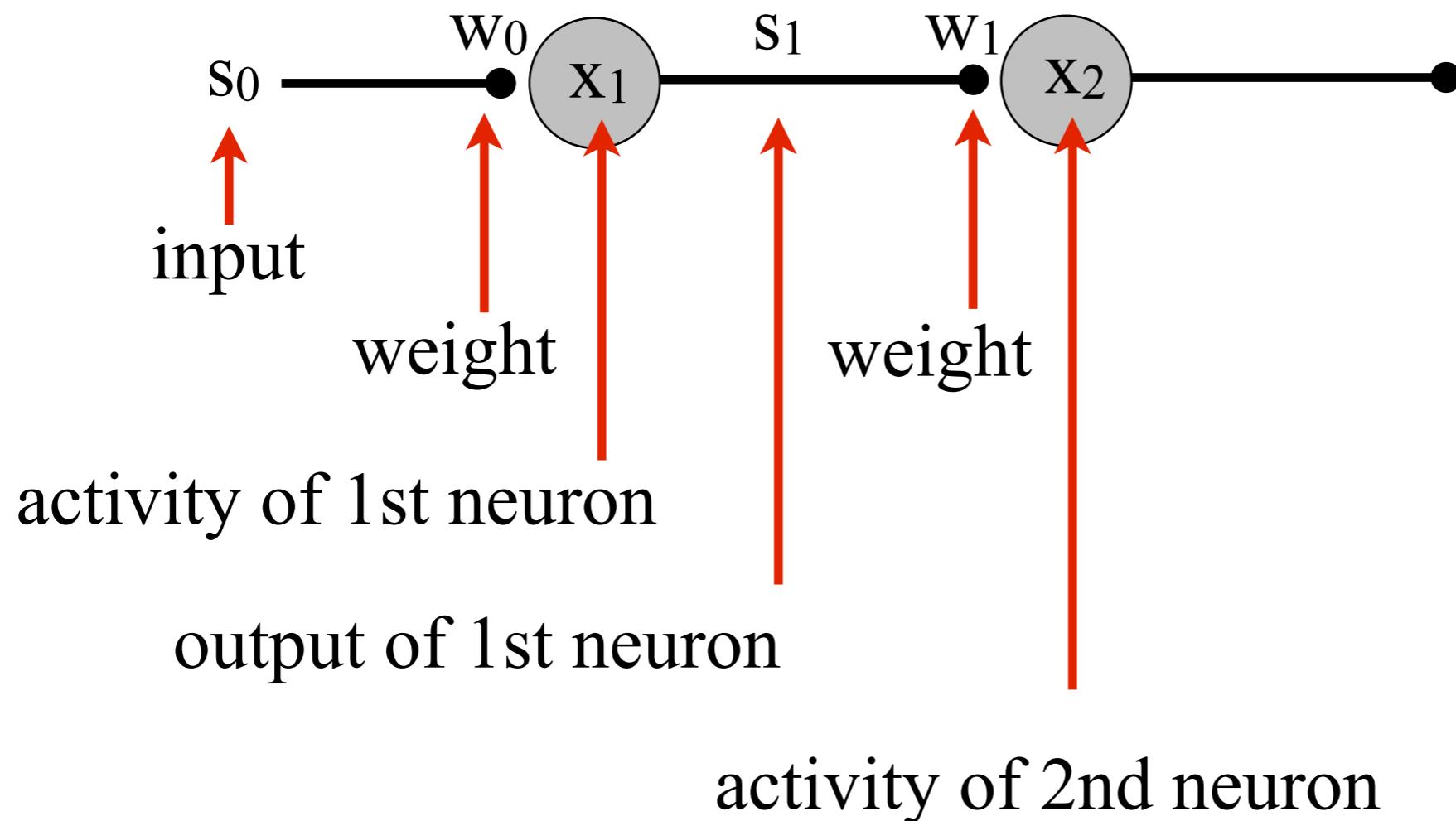


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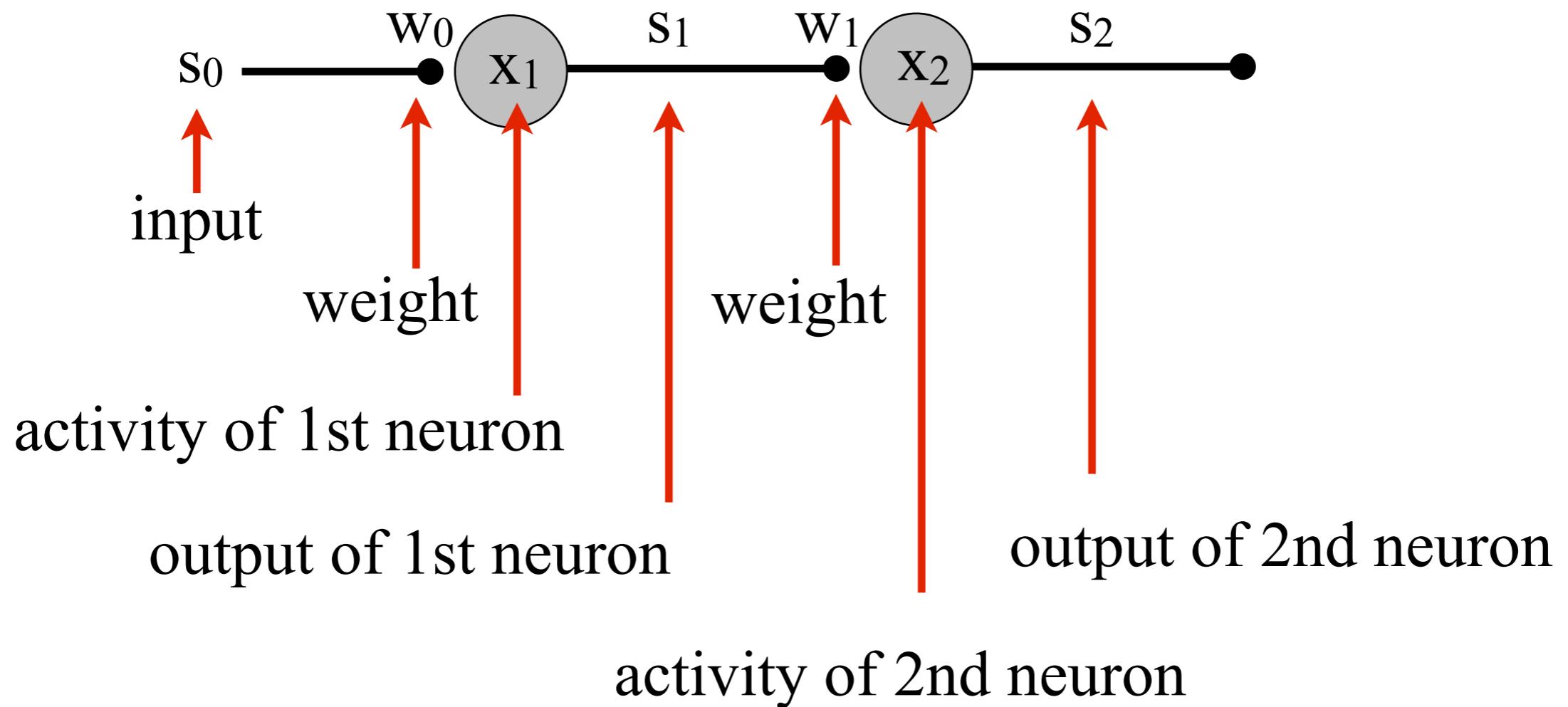


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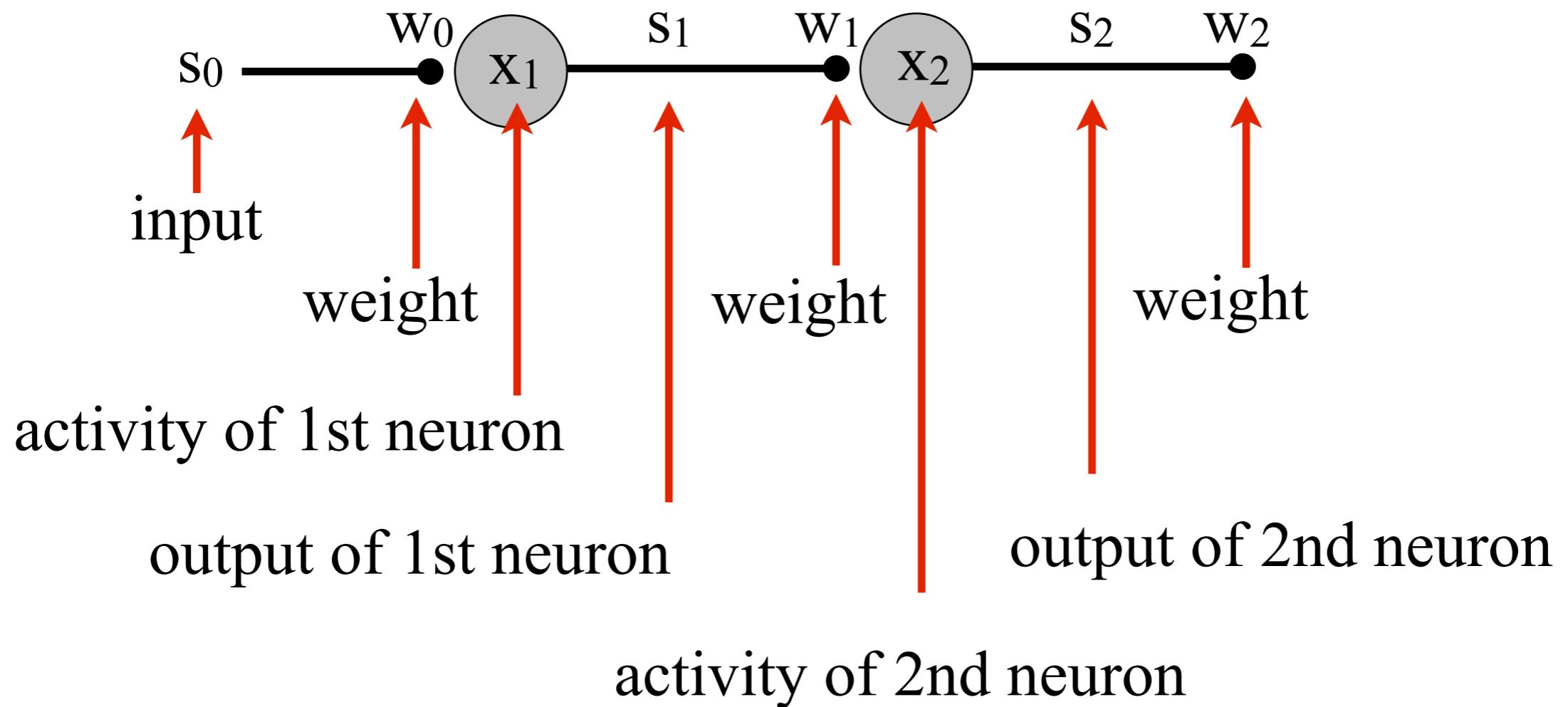


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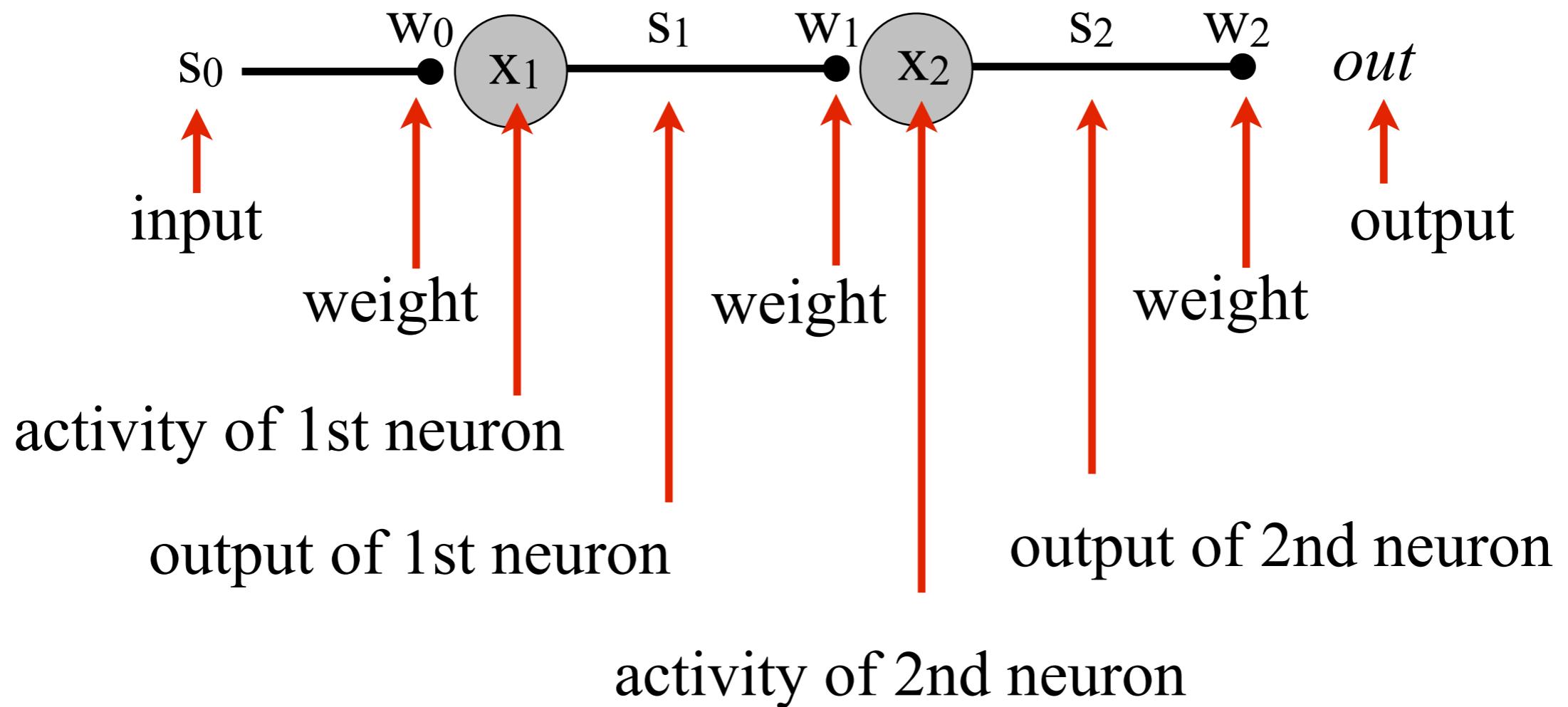


A “simple” neural network

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Activation function

Remember, the activation function:

Activation function

Remember, the activation function:

x (activity)

Activation function

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Here we'll use a **sigmoid** activation function:

Activation function

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$$S(x) = 1 / (1 + e^{-x})$$

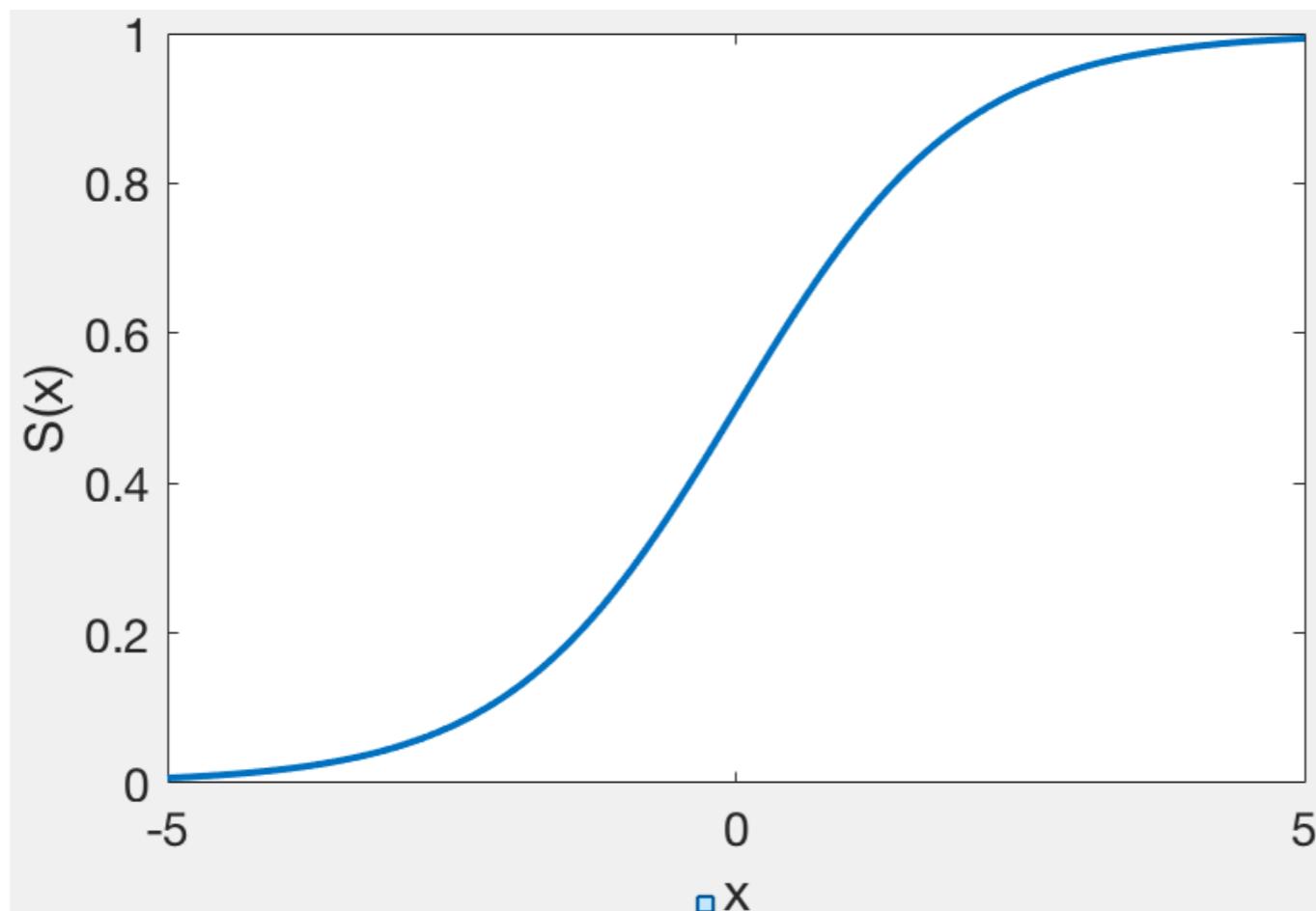
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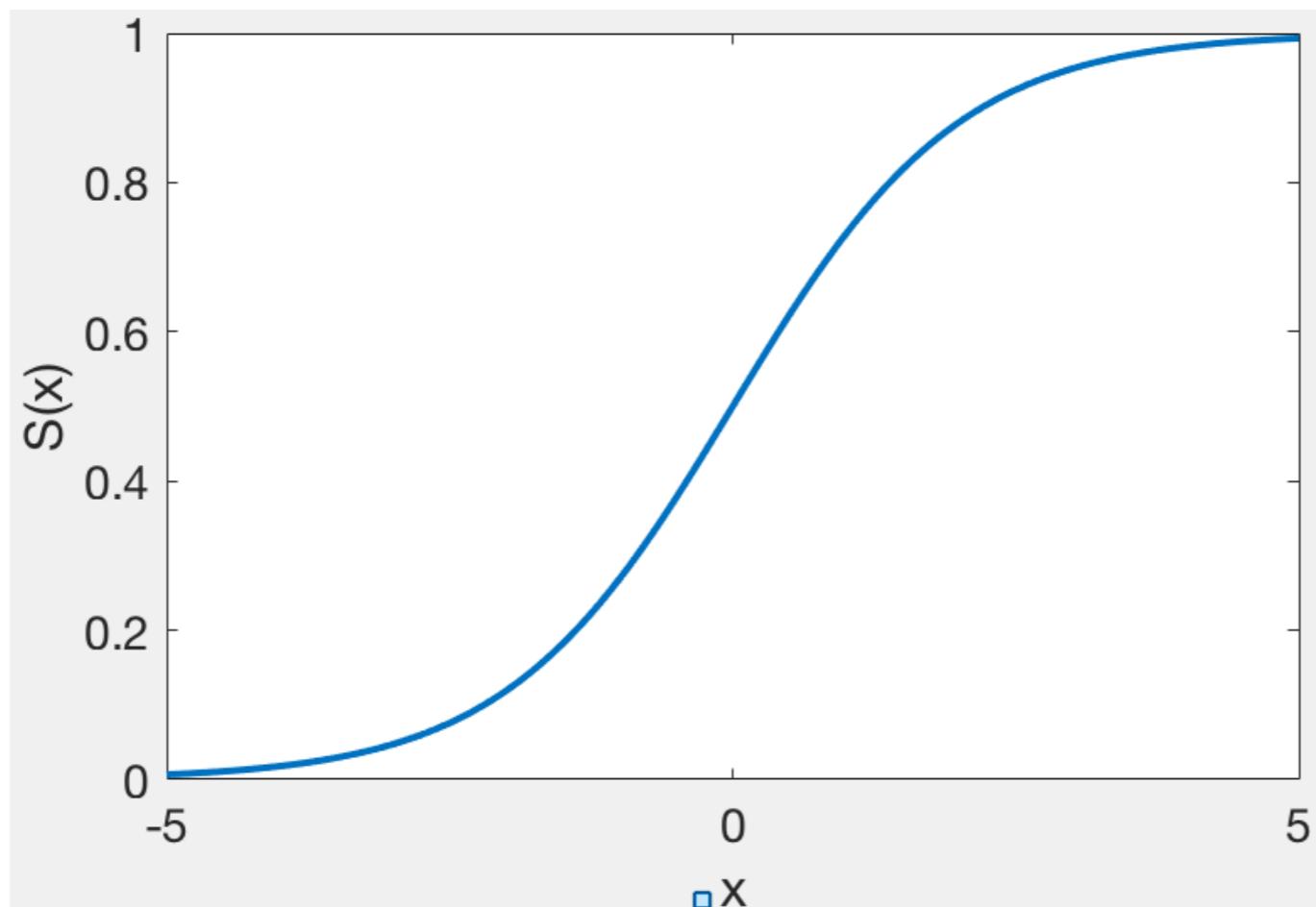
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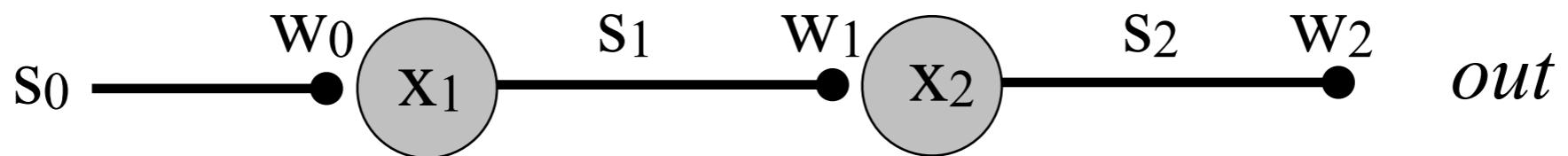
$$S(x) = 1 / (1 + e^{-x})$$



NOTE: It's like a “smoothed” binary threshold.

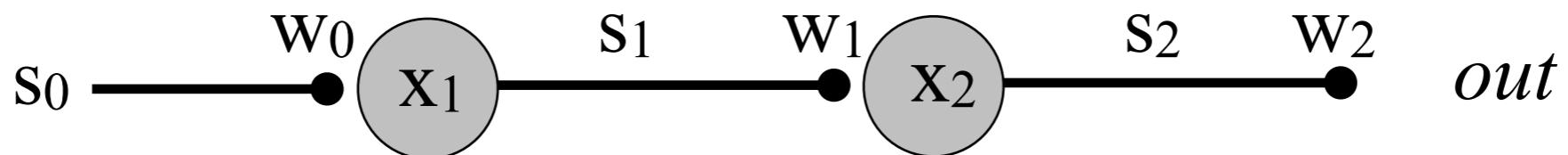
A “simple” neural network

We want our network to **learn** ...



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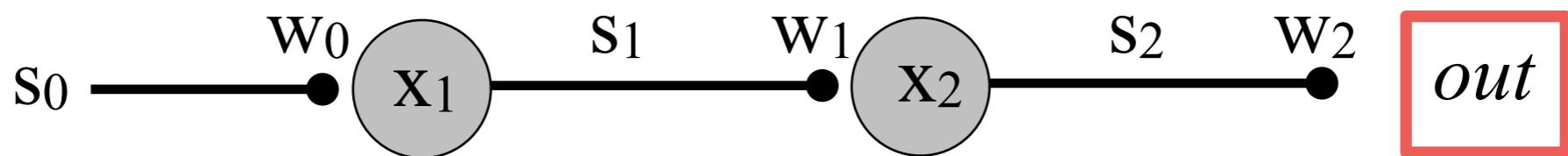
We want our network to **learn** ...



so that when then input $s_0=2$,

A “simple” neural network

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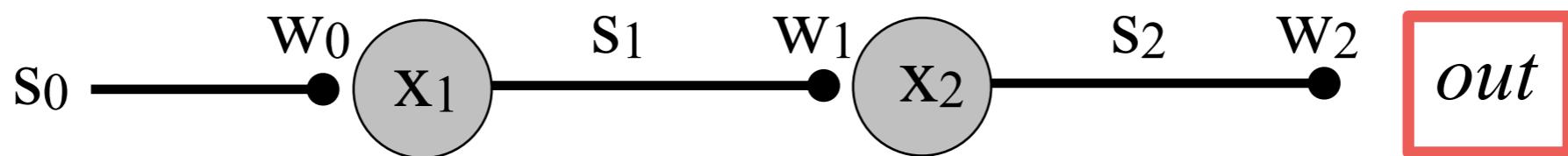


so that when then input $s_0=2$,

the input $out=0.7$

A “simple” neural network

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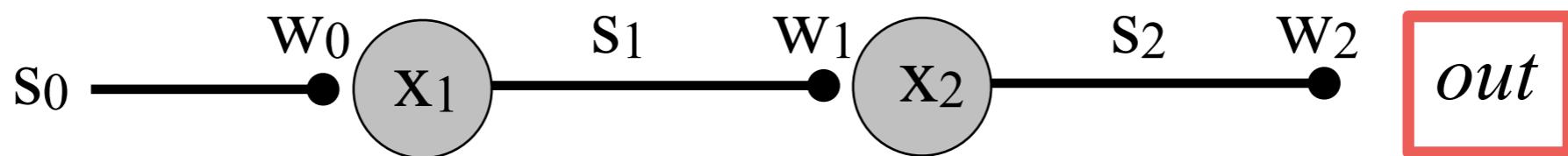
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Q: How do we do it?

A “simple” neural network

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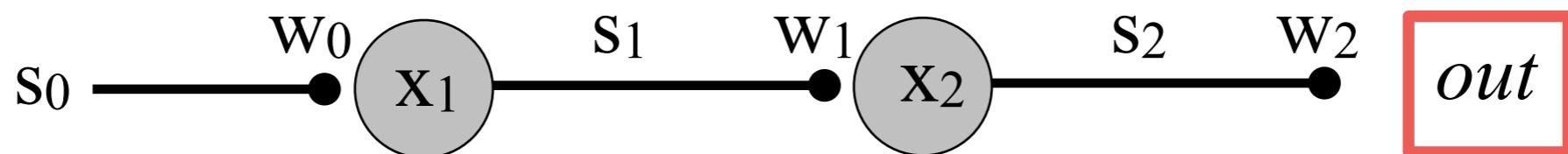
the input $out=0.7$

Q: How do we do it?

A: We need to choose the right weights: $w_0 \ w_1 \ w_2$

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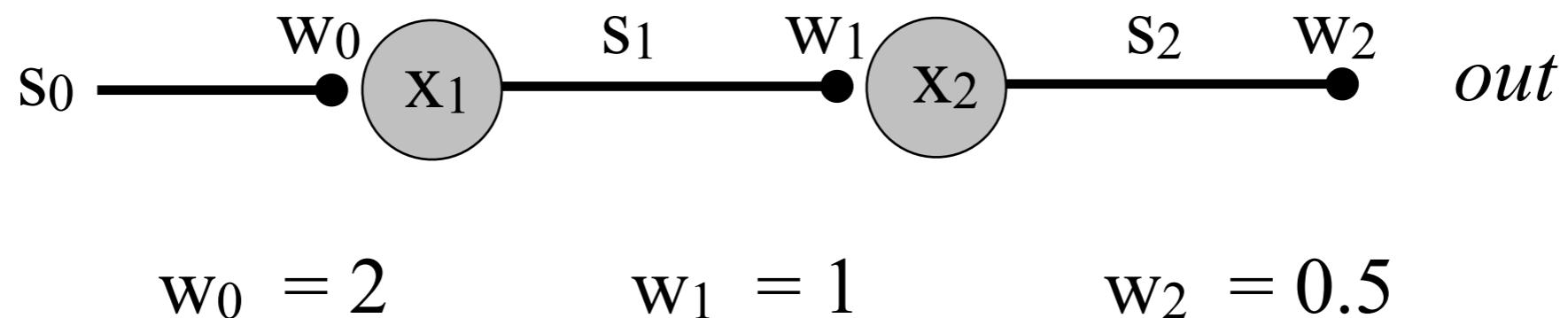
So, how do we find the right weights?

What are the right model weights?

Let's guess:

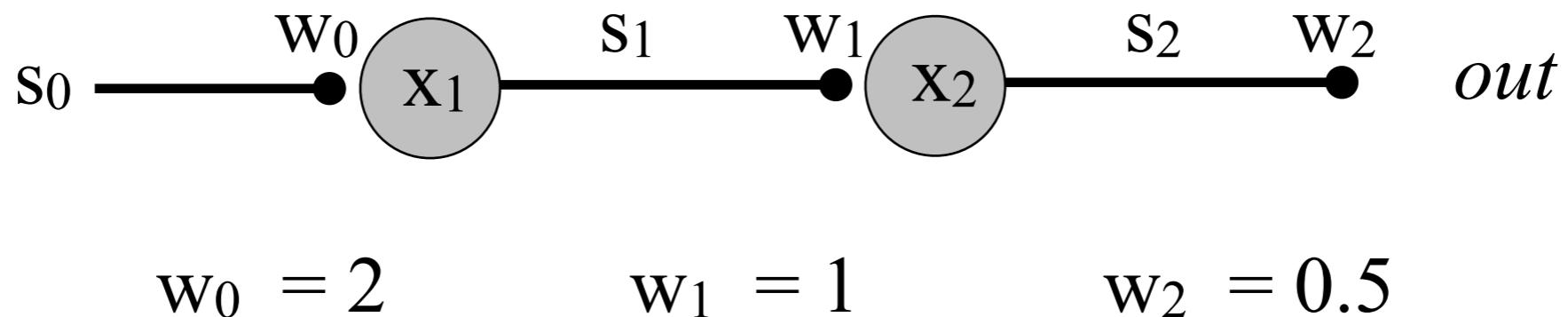
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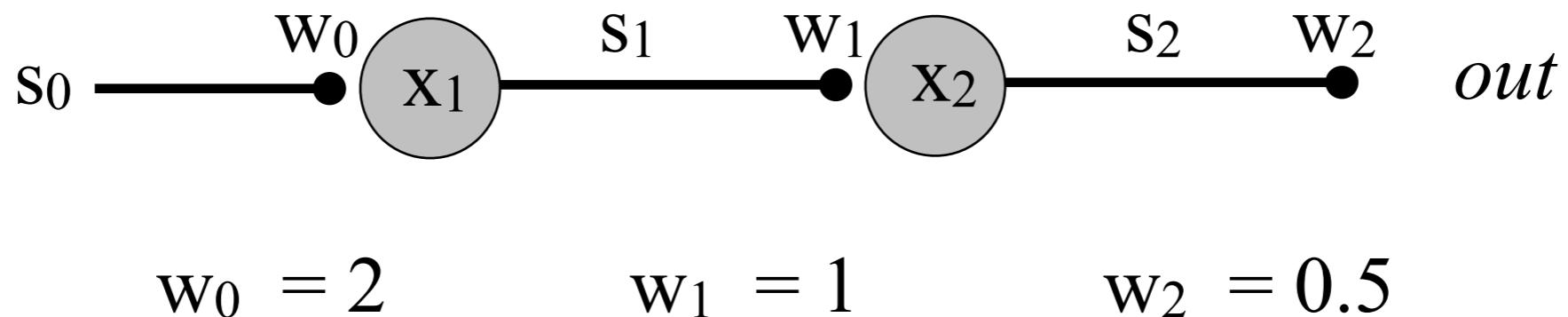
Let's guess:



Q: How did I choose these?

What are the right model weights?

Let's guess:

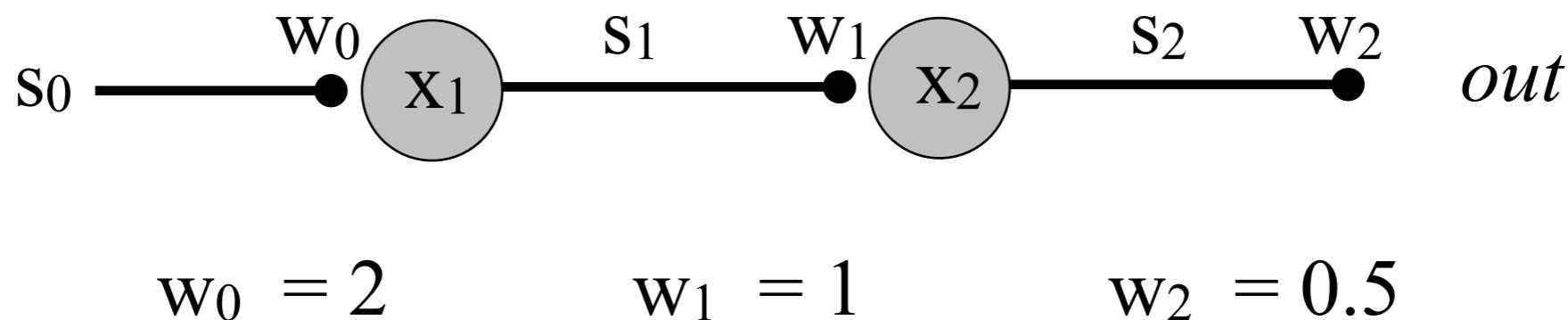


Q: How did I choose these?

Q: Do they work?

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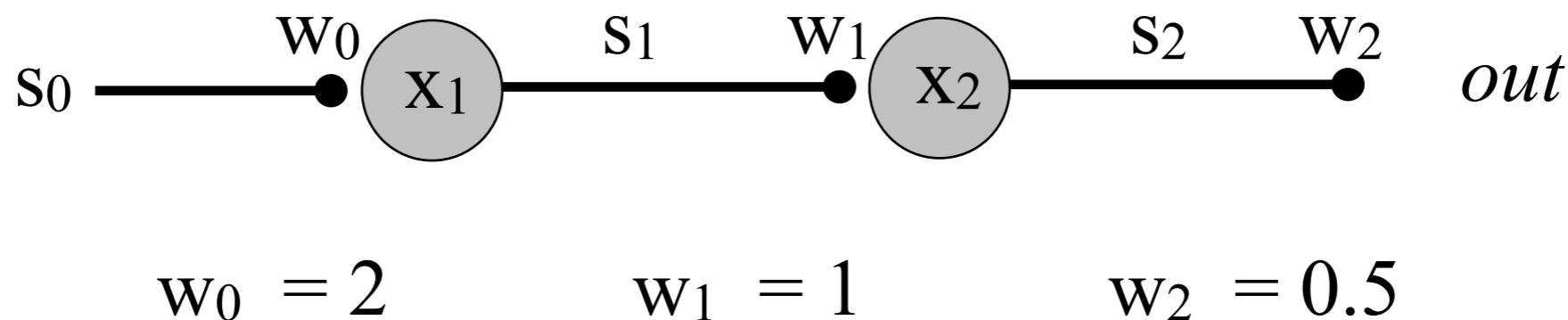
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A: Let's check ...

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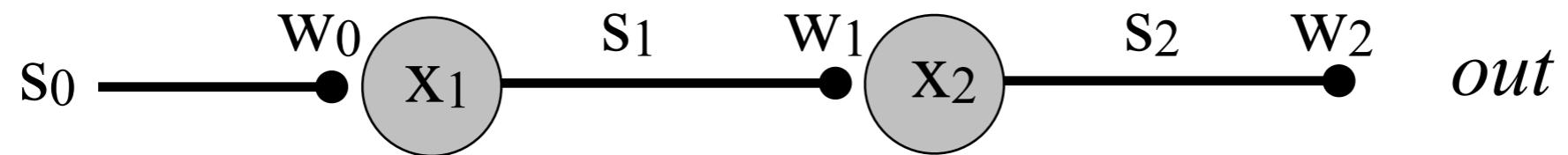
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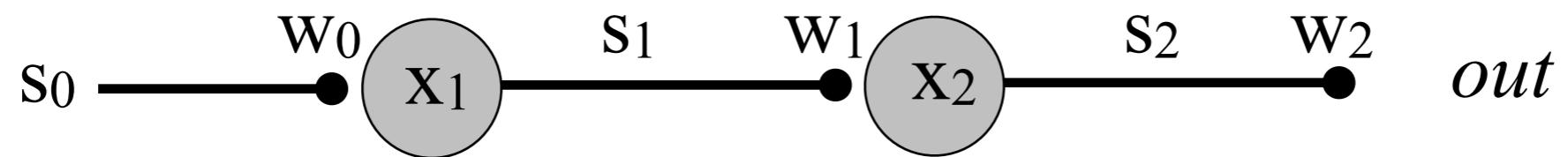
A: Let's check ...

forward propagation

Forward propagation

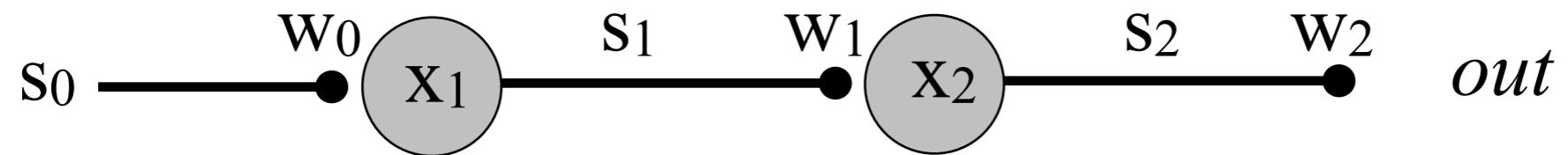


Forward propagation



$$s_0=2$$

Forward propagation



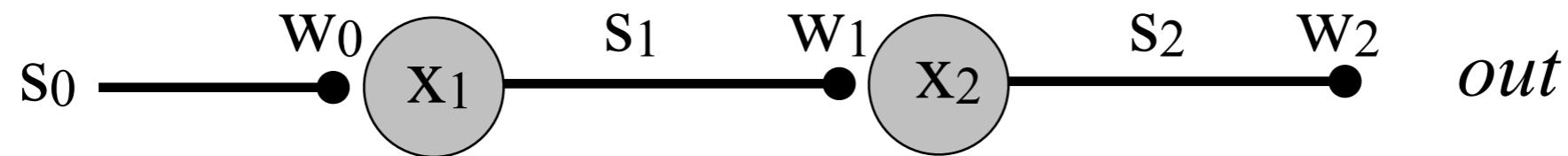
$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

Forward propagation



$$s_0 = 2$$

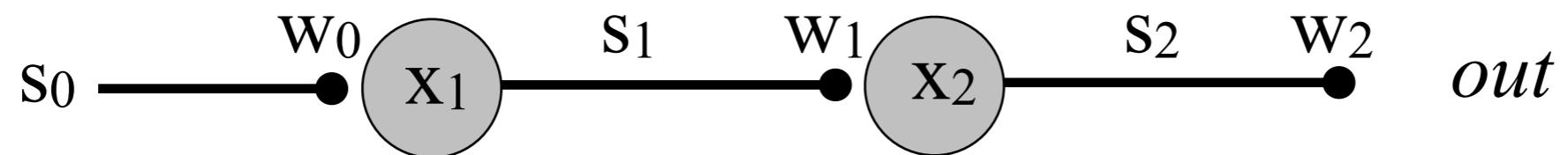
$$w_0 = 2$$

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target:
 $out = 0.7$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

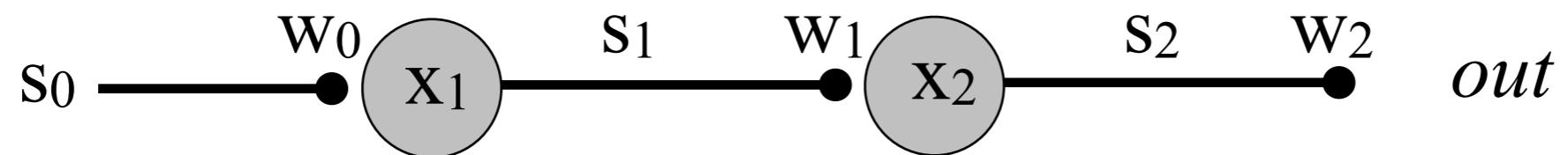
$$w_1 = 1$$

$$w_2 = 0.5$$

target:
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Let's do it.

Forward propagation



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$$w_1 = 1$$

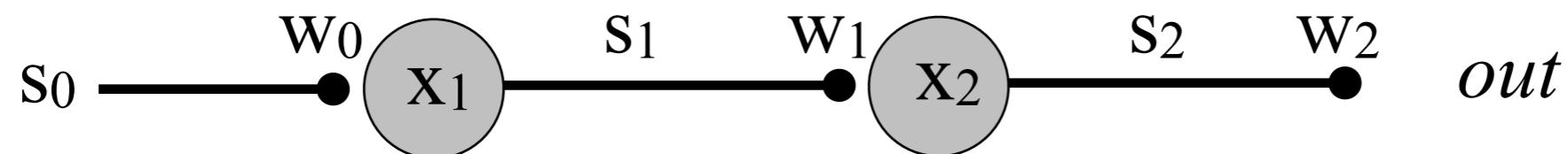
$$w_2 = 0.5$$

target:
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Let's do it.

$$x_1 =$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

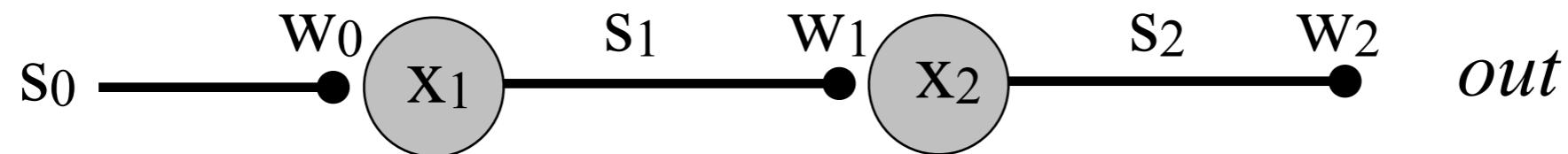
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Let's do it.

$$x_1 = w_0 s_0$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

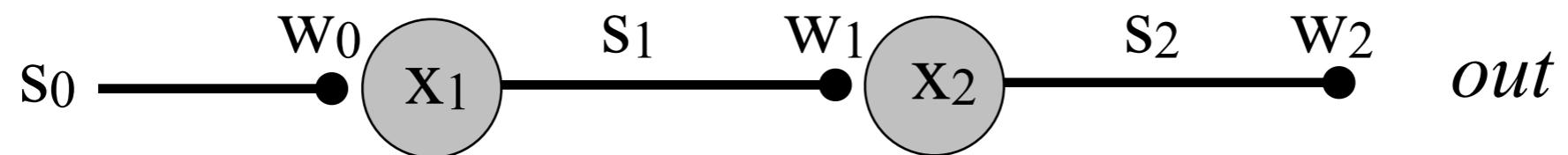
$$w_2 = 0.5$$

target:
 $out = 0.7$

Let's do it.

$$x_1 = w_0 s_0 = 2 * 2$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

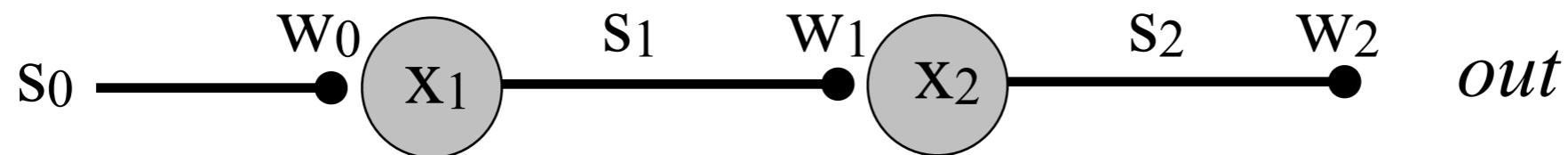
$$w_2 = 0.5$$

target:
 $out = 0.7$

Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

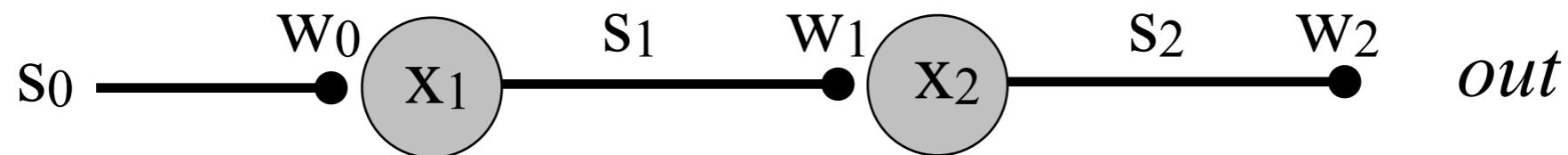
target:
 $out = 0.7$

Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

$$s_1 =$$

Forward propagation



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$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

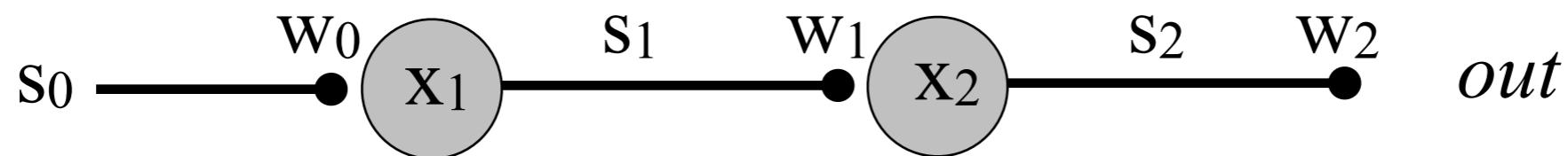
target:
 $out = 0.7$

Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

$$s_1 = S(x_1)$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

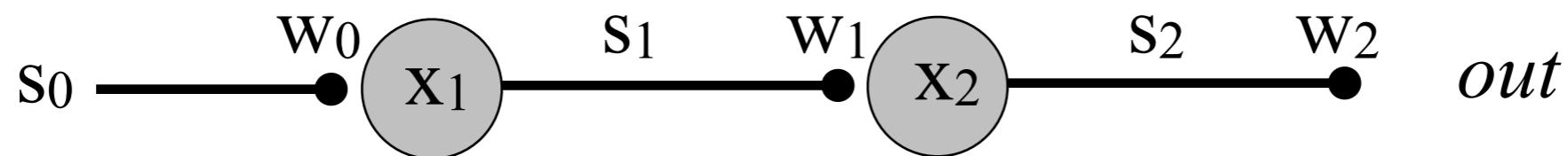
target:
 $out = 0.7$

Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

$$s_1 = S(x_1) = S(4)$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

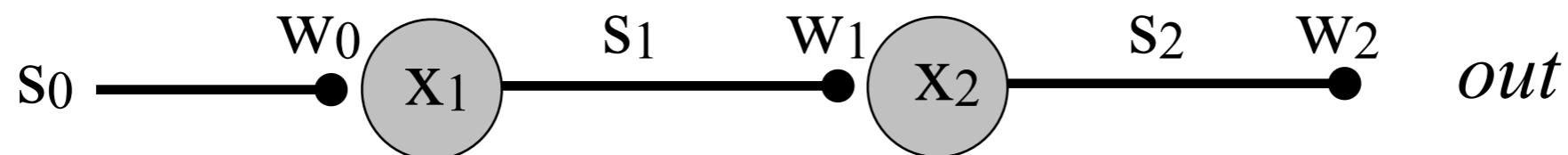
target:
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Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

$$s_1 = S(x_1) = S(4) = 0.982$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

target:
 $out = 0.7$

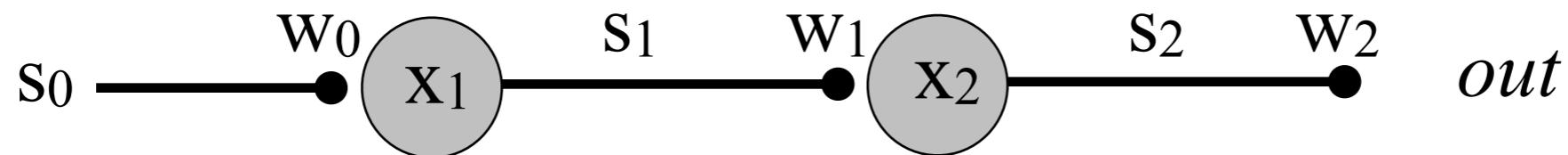
Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

$$s_1 = S(x_1) = S(4) = 0.982$$

$$x_2 =$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

target:
 $out = 0.7$

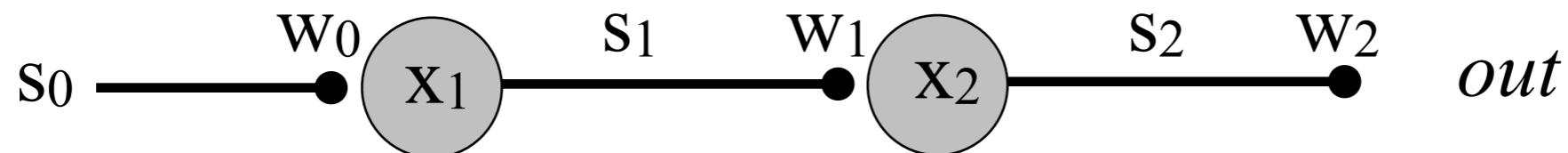
Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

$$s_1 = S(x_1) = S(4) = 0.982$$

$$x_2 = w_1 s_1$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

target:
 $out = 0.7$

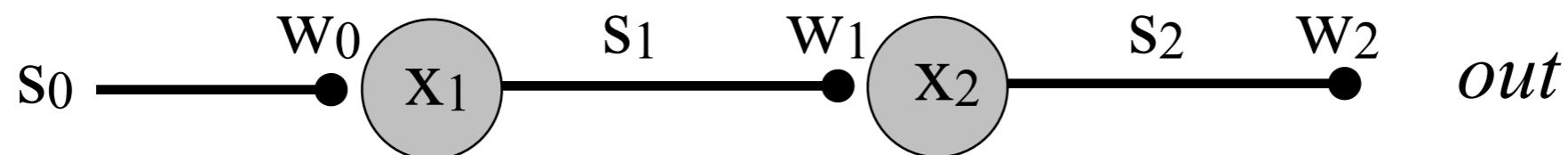
Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

$$s_1 = S(x_1) = S(4) = 0.982$$

$$x_2 = w_1 s_1 = 1 * 0.982$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

target:
 $out = 0.7$

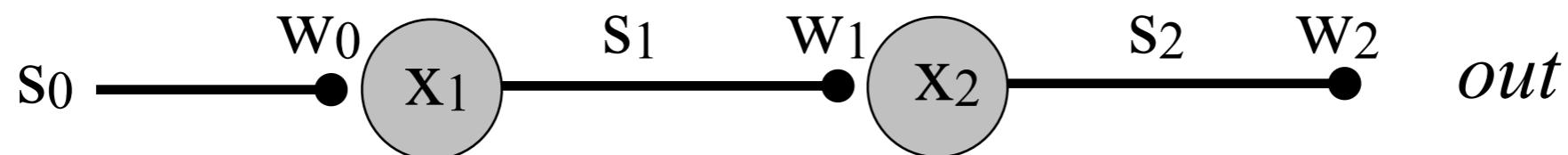
Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

$$s_1 = S(x_1) = S(4) = 0.982$$

$$x_2 = w_1 s_1 = 1 * 0.982 = 0.982$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

target:
 $out = 0.7$

Let's do it.

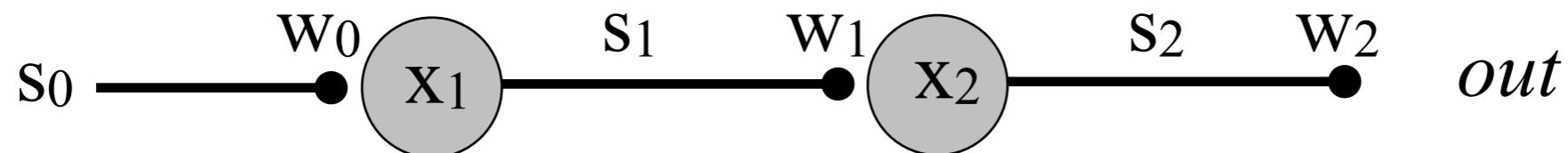
$$x_1 = w_0 s_0 = 2 * 2 = 4$$

$$s_1 = S(x_1) = S(4) = 0.982$$

$$x_2 = w_1 s_1 = 1 * 0.982 = 0.982$$

$$s_2 =$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

target:
 $out = 0.7$

Let's do it.

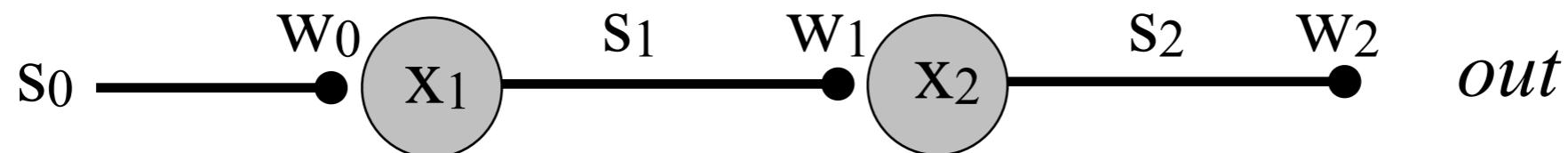
$$x_1 = w_0 s_0 = 2 * 2 = 4$$

$$s_1 = S(x_1) = S(4) = 0.982$$

$$x_2 = w_1 s_1 = 1 * 0.982 = 0.982$$

$$s_2 = S(x_2)$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

target:
 $out = 0.7$

Let's do it.

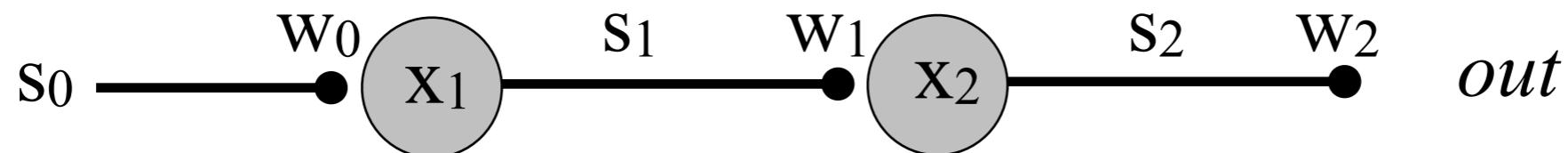
$$x_1 = w_0 s_0 = 2 * 2 = 4$$

$$s_1 = S(x_1) = S(4) = 0.982$$

$$x_2 = w_1 s_1 = 1 * 0.982 = 0.982$$

$$s_2 = S(x_2) = S(0.982)$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

target:
 $out = 0.7$

Let's do it.

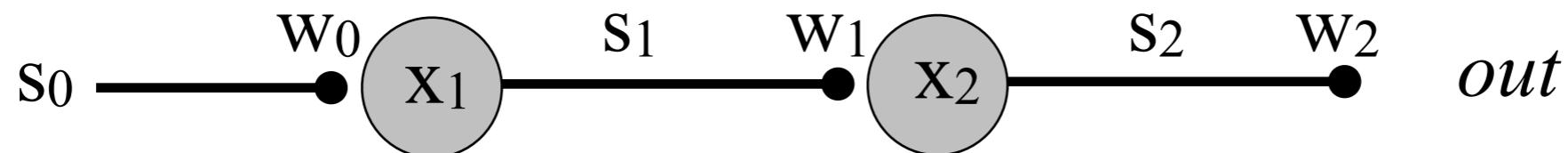
$$x_1 = w_0 s_0 = 2 * 2 = 4$$

$$s_1 = S(x_1) = S(4) = 0.982$$

$$x_2 = w_1 s_1 = 1 * 0.982 = 0.982$$

$$s_2 = S(x_2) = S(0.982) = 0.7275$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

target:
 $out = 0.7$

Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

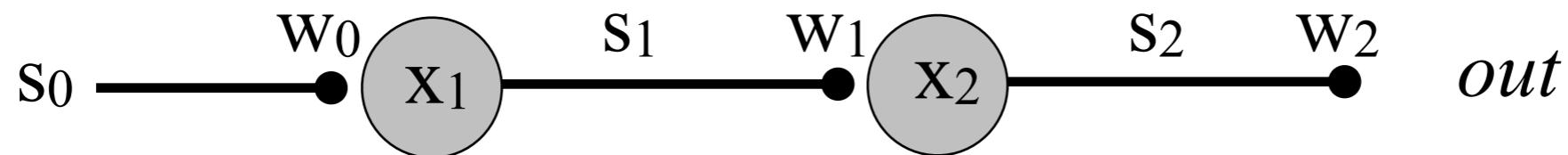
$$s_1 = S(x_1) = S(4) = 0.982$$

$$x_2 = w_1 s_1 = 1 * 0.982 = 0.982$$

$$s_2 = S(x_2) = S(0.982) = 0.7275$$

$$out =$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

target:
 $out = 0.7$

Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

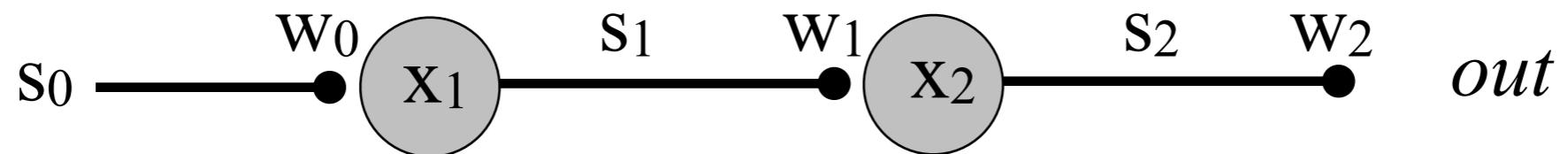
$$s_1 = S(x_1) = S(4) = 0.982$$

$$x_2 = w_1 s_1 = 1 * 0.982 = 0.982$$

$$s_2 = S(x_2) = S(0.982) = 0.7275$$

$$out = w_2 s_2$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

target:
 $out = 0.7$

Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

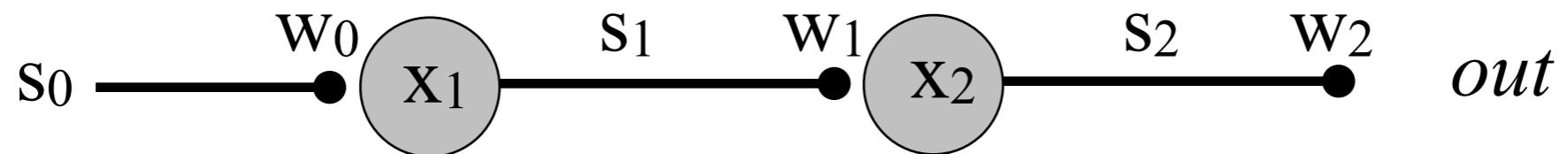
$$s_1 = S(x_1) = S(4) = 0.982$$

$$x_2 = w_1 s_1 = 1 * 0.982 = 0.982$$

$$s_2 = S(x_2) = S(0.982) = 0.7275$$

$$out = w_2 s_2 = 0.5 * 0.7275$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

target:
 $out = 0.7$

Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

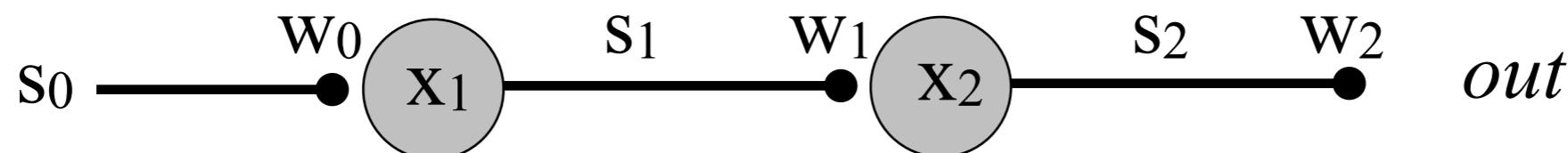
$$s_1 = S(x_1) = S(4) = 0.982$$

$$x_2 = w_1 s_1 = 1 * 0.982 = 0.982$$

$$s_2 = S(x_2) = S(0.982) = 0.7275$$

$$out = w_2 s_2 = 0.5 * 0.7275 = 0.3638$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

target:
 $out = 0.7$

Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

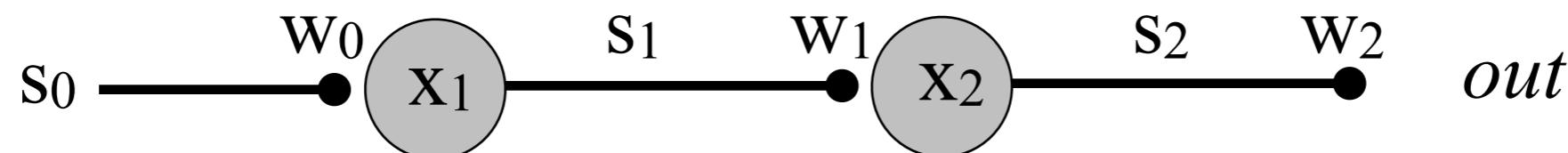
$$s_1 = S(x_1) = S(4) = 0.982$$

$$x_2 = w_1 s_1 = 1 * 0.982 = 0.982$$

$$s_2 = S(x_2) = S(0.982) = 0.7275$$

$$out = w_2 s_2 = 0.5 * 0.7275 = 0.3638 \rightarrow \text{Match?}$$

Forward propagation



$$s_0 = 2$$

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 0.5$$

target:
 $out = 0.7$

Let's do it.

$$x_1 = w_0 s_0 = 2 * 2 = 4$$

$$s_1 = S(x_1) = S(4) = 0.982$$

$$x_2 = w_1 s_1 = 1 * 0.982 = 0.982$$

$$s_2 = S(x_2) = S(0.982) = 0.7275$$

$$out = w_2 s_2 = 0.5 * 0.7275 = 0.3638 \rightarrow \text{Match?}$$

NO

How does a change in weight w_2 impact output?

Q: So now what?

How does a change in weight w_2 impact output?

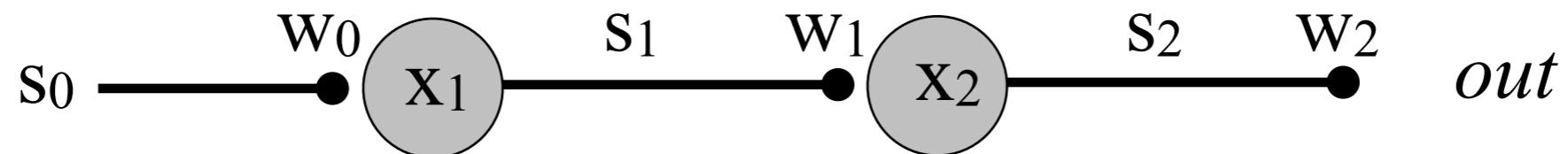
Q: So now what?

(intermediate) Goal: get output (*out*) closer to target (0.7)

How does a change in weight w_2 impact output?

Q: So now what?

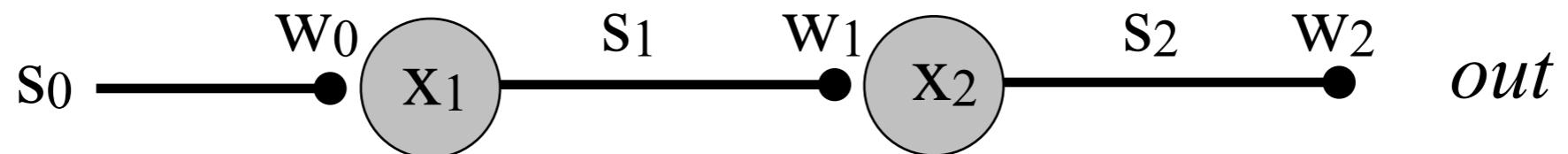
(intermediate) Goal: get output (out) closer to target (0.7)



How does a change in weight w_2 impact output?

Q: So now what?

(intermediate) Goal: get output (out) closer to target (0.7)

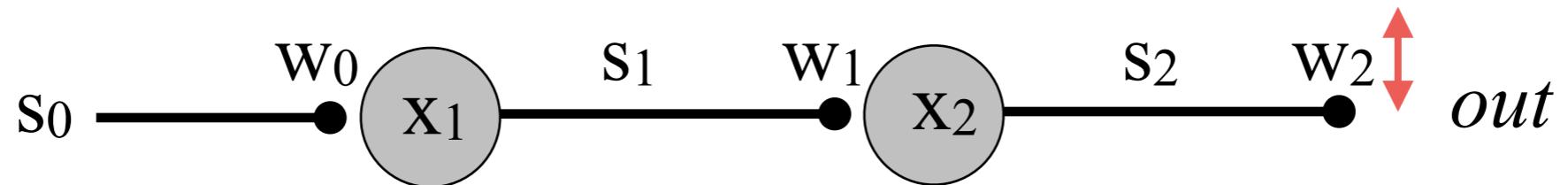


Q: How does a change in weight w_2 impact the output?

How does a change in weight w_2 impact output?

Q: So now what?

(intermediate) Goal: get output (out) closer to target (0.7)



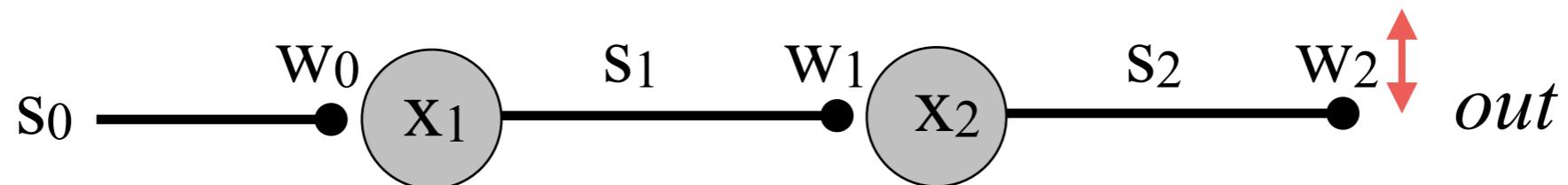
Q: How does a change in weight w_2 impact the output?

Idea: wiggle w_2

How does a change in weight w_2 impact output?

Q: So now what?

(intermediate) Goal: get output (out) closer to target (0.7)



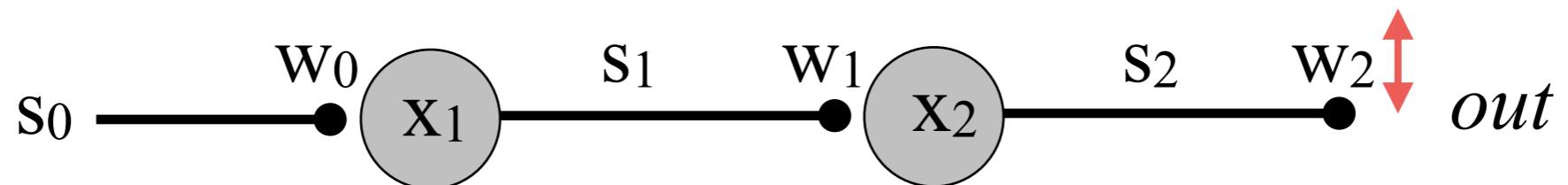
Q: How does a change in weight w_2 impact the output?

Idea: wiggle w_2 how does out change?

How does a change in weight w_2 impact output?

Q: So now what?

(intermediate) Goal: get output (out) closer to target (0.7)



Q: How does a change in weight w_2 impact the output?

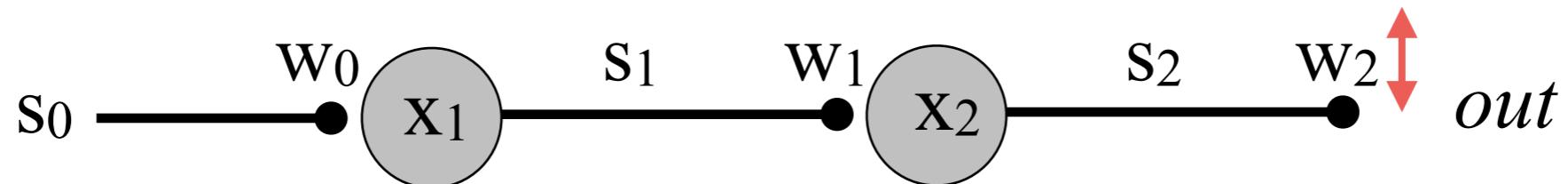
Idea: wiggle w_2 how does out change?

Mathematically,

How does a change in weight w_2 impact output?

Q: So now what?

(intermediate) Goal: get output (*out*) closer to target (0.7)



Q: How does a change in weight w_2 impact the output?

Idea: wiggle w_2 how does *out* change?

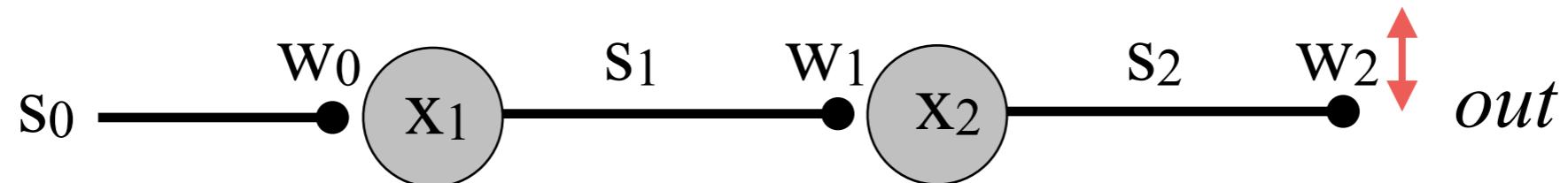
Mathematically,

$$\frac{d \text{ } out}{d \text{ } w_2}$$

How does a change in weight w_2 impact output?

Q: So now what?

(intermediate) Goal: get output (*out*) closer to target (0.7)



Q: How does a change in weight w_2 impact the output?

Idea: wiggle w_2 how does *out* change?

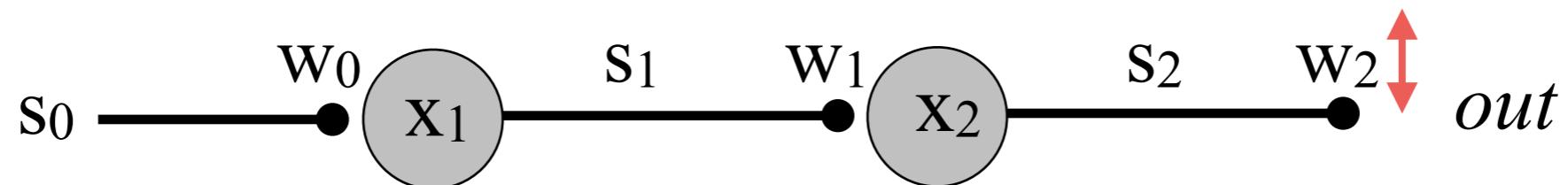
Mathematically,

$$\frac{d \text{ } out}{d \text{ } w_2} =$$

How does a change in weight w_2 impact output?

Q: So now what?

(intermediate) Goal: get output (*out*) closer to target (0.7)



Q: How does a change in weight w_2 impact the output?

Idea: wiggle w_2 how does *out* change? $out = s_2 w_2$

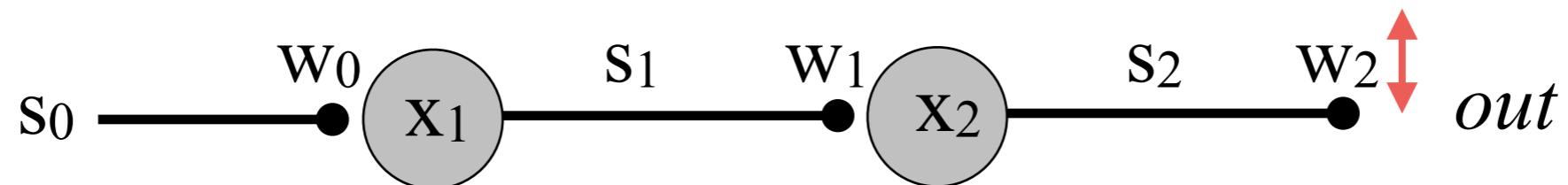
Mathematically,

$$\frac{d \text{ } out}{d \text{ } w_2} =$$

How does a change in weight w_2 impact output?

Q: So now what?

(intermediate) Goal: get output (*out*) closer to target (0.7)



Q: How does a change in weight w_2 impact the output?

Idea: wiggle w_2 how does *out* change? $out = s_2 w_2$

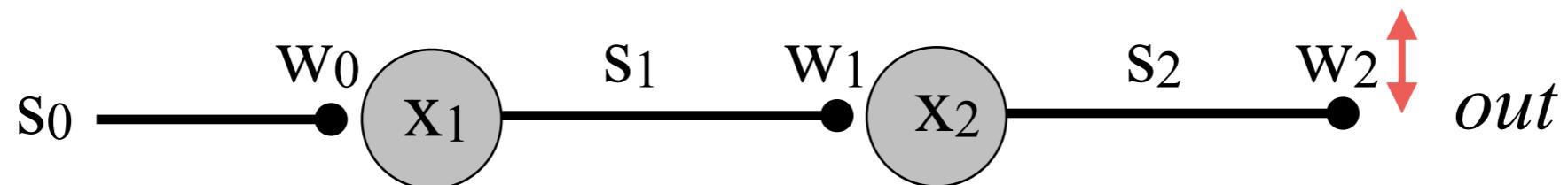
Mathematically,

$$\frac{d \text{ } out}{d \text{ } w_2} = \frac{d \text{ } (s_2 w_2)}{d \text{ } w_2}$$

How does a change in weight w_2 impact output?

Q: So now what?

(intermediate) Goal: get output (*out*) closer to target (0.7)



Q: How does a change in weight w_2 impact the output?

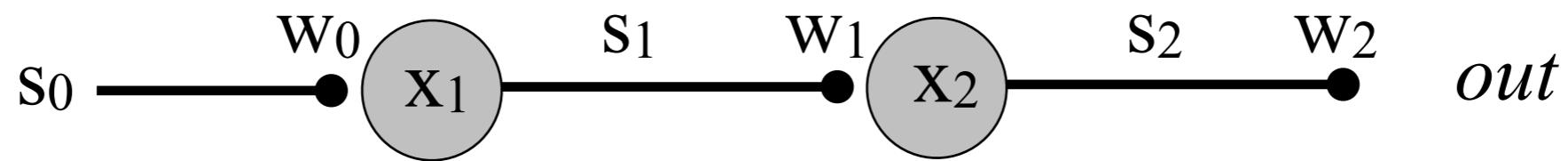
Idea: wiggle w_2 how does *out* change? $out = s_2 w_2$

Mathematically,

$$\frac{d \text{ } out}{d \text{ } w_2} = \frac{d \text{ } (s_2 w_2)}{d \text{ } w_2} = s_2$$

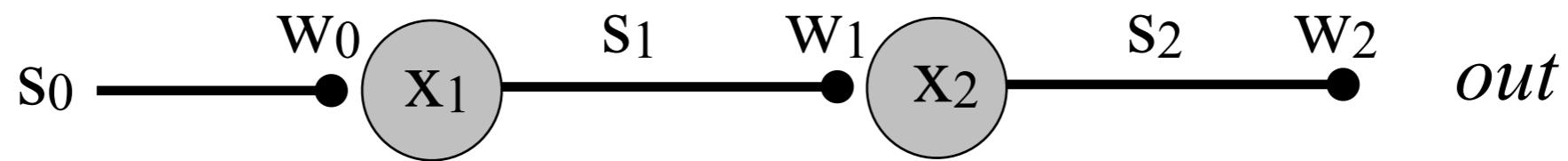
How does a change in weight w_1 impact output?

Let's keep going, working backwards.



How does a change in weight w_1 impact output?

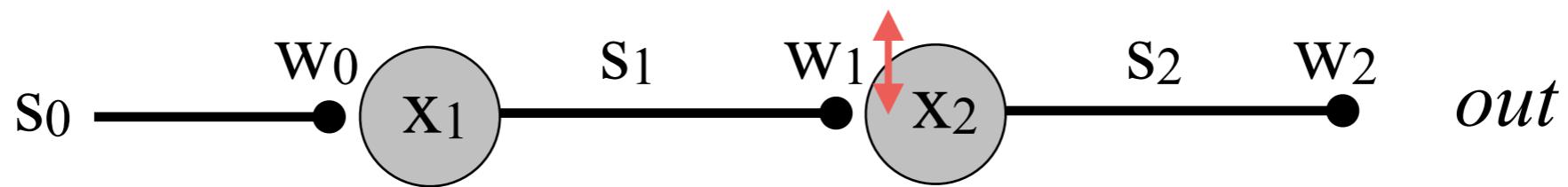
Let's keep going, working backwards.



Q: How does a change in weight w_1 impact the output?

How does a change in weight w_1 impact output?

Let's keep going, working backwards.

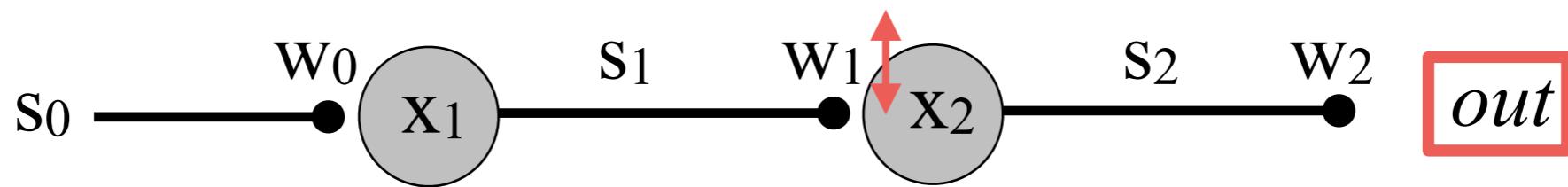


Q: How does a change in weight w_1 impact the output?

Idea: wiggle w_1

How does a change in weight w_1 impact output?

Let's keep going, working backwards.

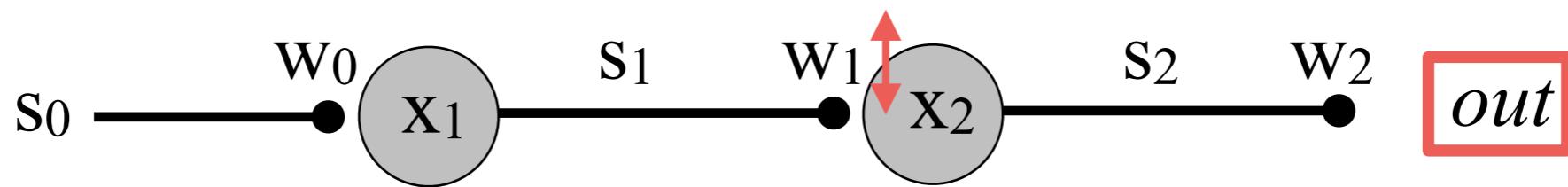


Q: How does a change in weight w_1 impact the output?

Idea: wiggle w_1 how does *out* change?

How does a change in weight w_1 impact output?

Let's keep going, working backwards.



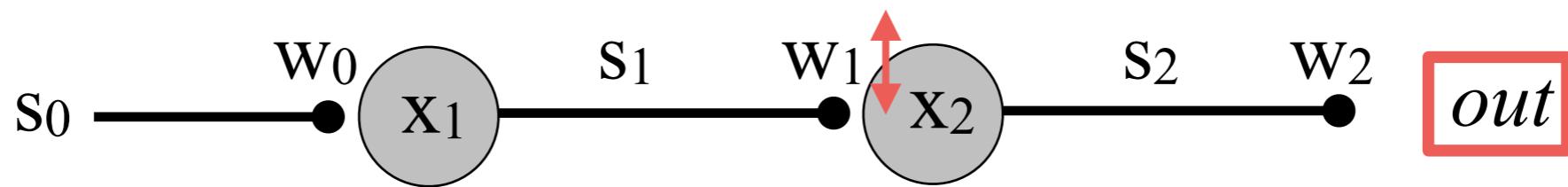
Q: How does a change in weight w_1 impact the output?

Idea: wiggle w_1 how does *out* change?

Mathematically,

How does a change in weight w_1 impact output?

Let's keep going, working backwards.



Q: How does a change in weight w_1 impact the output?

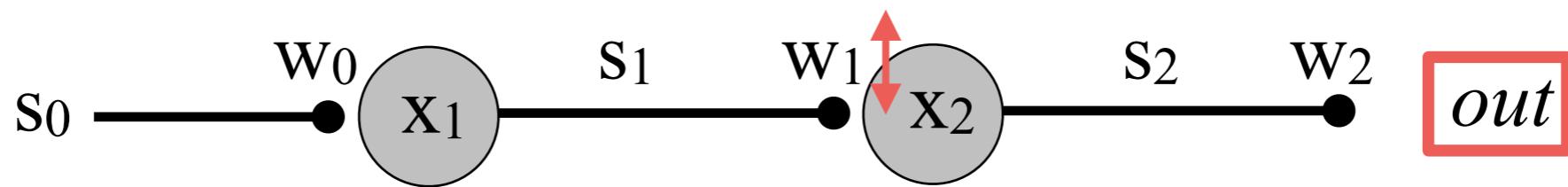
Idea: wiggle w_1 how does *out* change?

Mathematically,

$$\frac{d \text{ } out}{d \text{ } w_1} =$$

How does a change in weight w_1 impact output?

Let's keep going, working backwards.



Q: How does a change in weight w_1 impact the output?

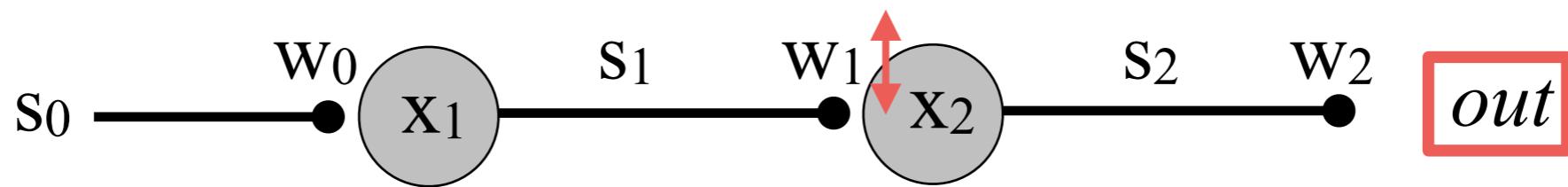
Idea: wiggle w_1 how does *out* change?

Mathematically,

$$\frac{d \text{ } out}{d \text{ } w_1} = \text{ Hmm ...}$$

How does a change in weight w_1 impact output?

Let's keep going, working backwards.



Q: How does a change in weight w_1 impact the output?

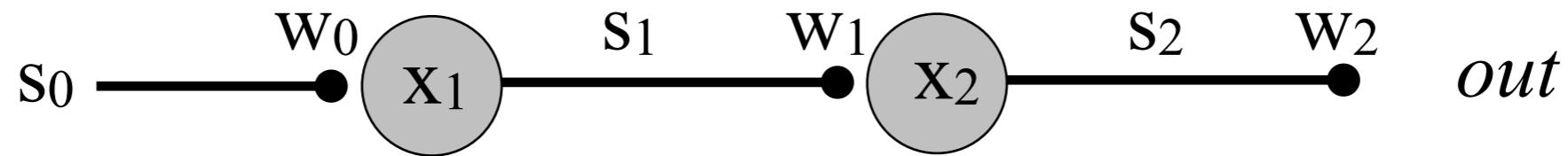
Idea: wiggle w_1 how does *out* change?

Mathematically,

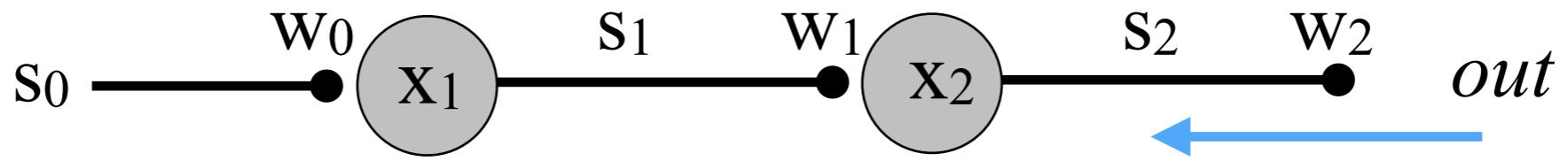
$$\frac{d \text{ } out}{d \text{ } w_1} = \text{ Hmm ...}$$

out does not depend directly on w_1

How does a change in weight w_1 impact output?

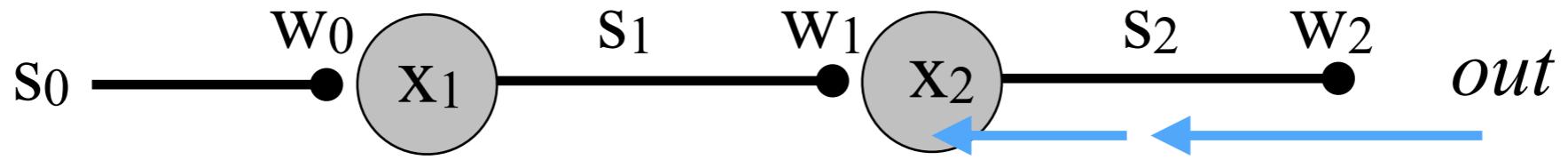


How does a change in weight w_1 impact output?



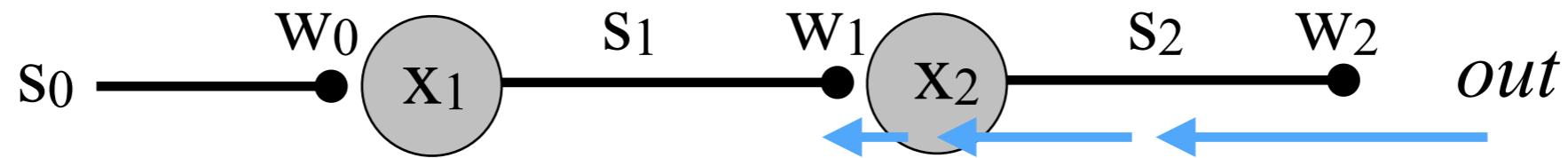
out does depend on s_2

How does a change in weight w_1 impact output?



out does depend on s_2 and s_2 depend on x_2

How does a change in weight w_1 impact output?

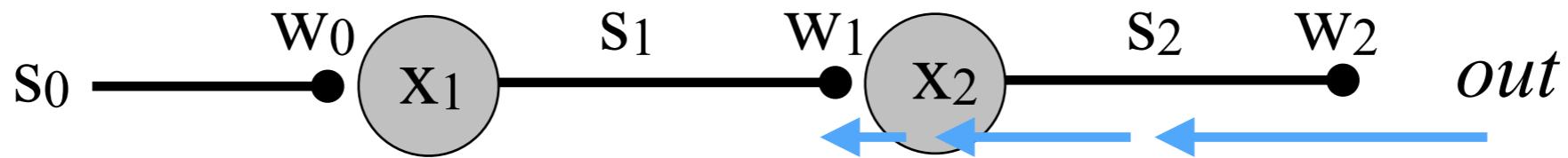


out does depend on s_2

and s_2 depend on x_2

and x_2 depend on w_1

How does a change in weight w_1 impact output?



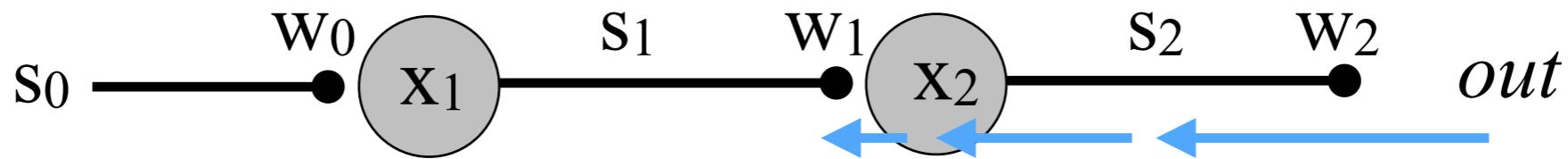
out does depend on s_2

and s_2 depend on x_2

and x_2 depend on w_1

Mathematically ...

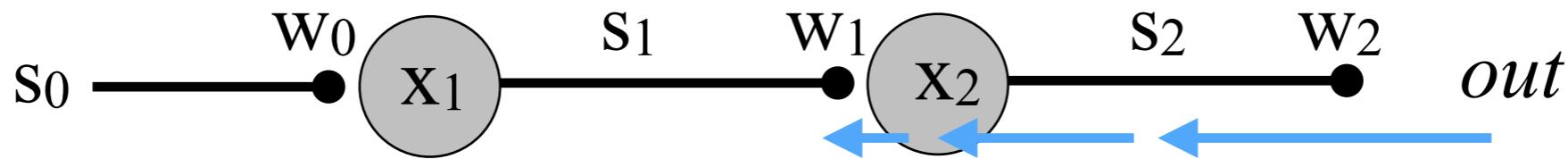
How does a change in weight w_1 impact output?



out does depend on s_2 and s_2 depend on x_2 and x_2 depend on w_1

Mathematically ... the **chain rule**

How does a change in weight w_1 impact output?

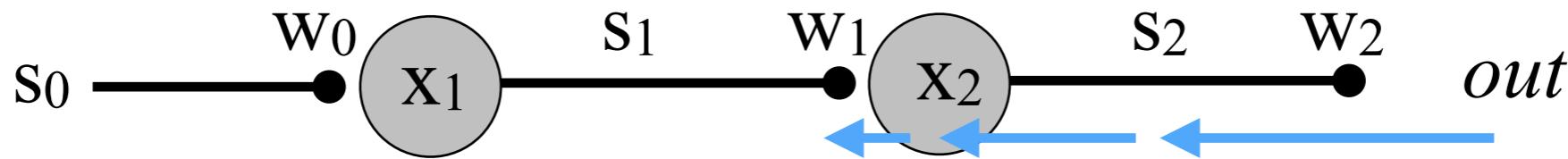


out does depend on s_2 and s_2 depend on x_2 and x_2 depend on w_1

Mathematically ... the **chain rule**

$$\frac{d \text{ } out}{d \text{ } w_1} =$$

How does a change in weight w_1 impact output?

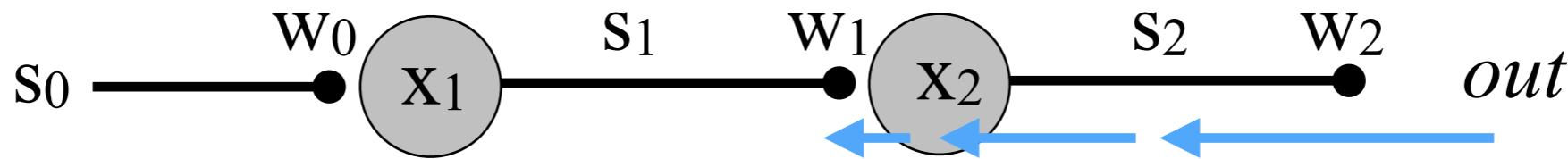


out does depend on s_2 and s_2 depend on x_2 and x_2 depend on w_1

Mathematically ... the **chain rule**

$$\frac{d \text{ } out}{d \text{ } w_1} = \frac{d \text{ } out}{d \text{ } s_2}$$

How does a change in weight w_1 impact output?

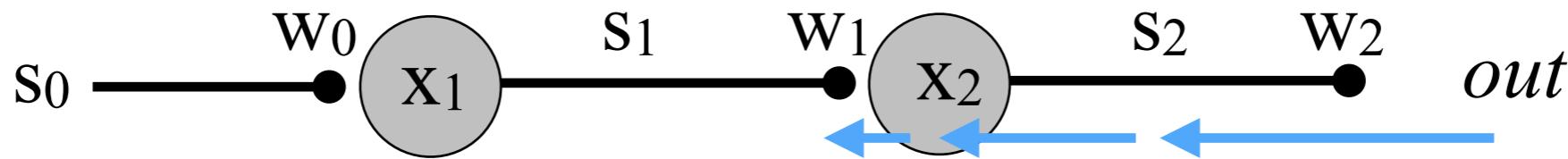


out does depend on s_2 and s_2 depend on x_2 and x_2 depend on w_1

Mathematically ... the **chain rule**

$$\frac{d \text{ } out}{d \text{ } w_1} = \frac{d \text{ } out}{d \text{ } s_2} \frac{d \text{ } s_2}{d \text{ } x_2}$$

How does a change in weight w_1 impact output?

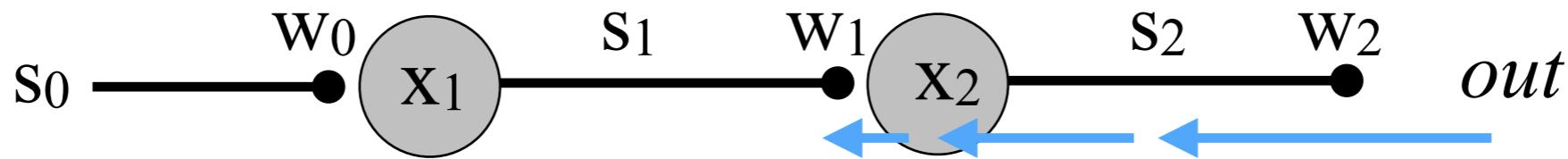


out does depend on s_2 and s_2 depend on x_2 and x_2 depend on w_1

Mathematically ... the **chain rule**

$$\frac{d \text{ } out}{d \text{ } w_1} = \frac{d \text{ } out}{d \text{ } s_2} \frac{d \text{ } s_2}{d \text{ } x_2} \frac{d \text{ } x_2}{d \text{ } w_1}$$

How does a change in weight w_1 impact output?

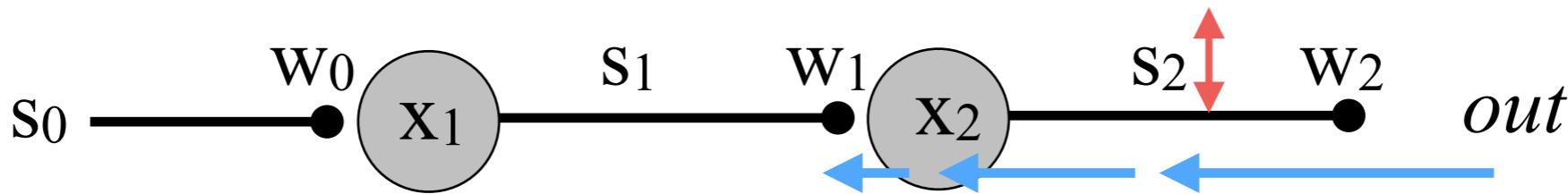


out does depend on s_2 and s_2 depend on x_2 and x_2 depend on w_1

Mathematically ... the **chain rule**

$$\frac{d \text{ } out}{d \text{ } w_1} = \boxed{\frac{d \text{ } out}{d \text{ } s_2}} \frac{d \text{ } s_2}{d \text{ } x_2} \frac{d \text{ } x_2}{d \text{ } w_1}$$

How does a change in weight w_1 impact output?



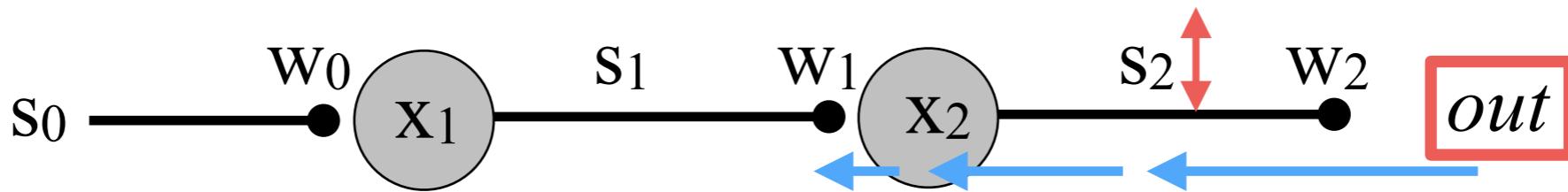
out does depend on s_2 and s_2 depend on x_2 and x_2 depend on w_1

Mathematically ... the **chain rule**

$$\frac{d \text{ } out}{d \text{ } w_1} = \boxed{\frac{d \text{ } out}{d \text{ } s_2}} \frac{d \text{ } s_2}{d \text{ } x_2} \frac{d \text{ } x_2}{d \text{ } w_1}$$

wiggle s_2

How does a change in weight w_1 impact output?



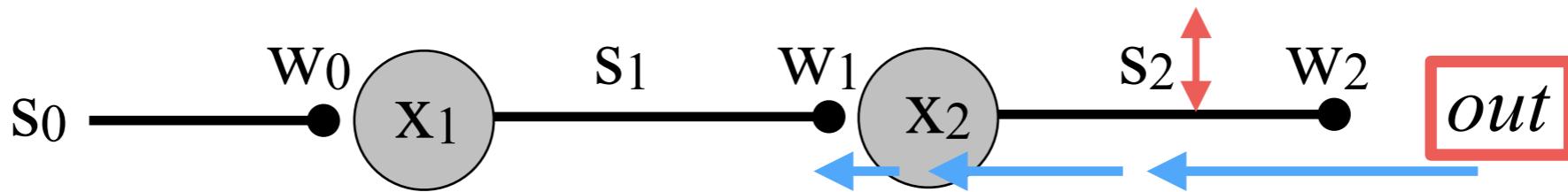
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wiggle s_2 and *out* changes

How does a change in weight w_1 impact output?



out does depend on s_2 and s_2 depend on x_2 and x_2 depend on w_1

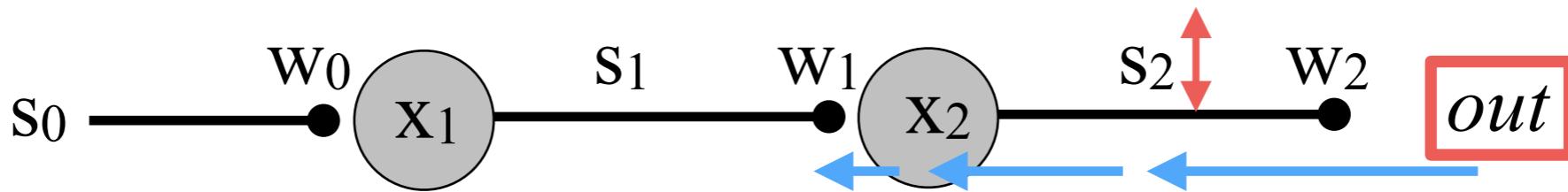
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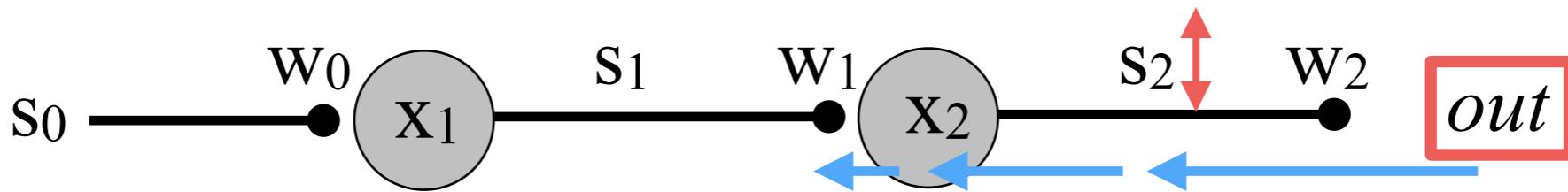
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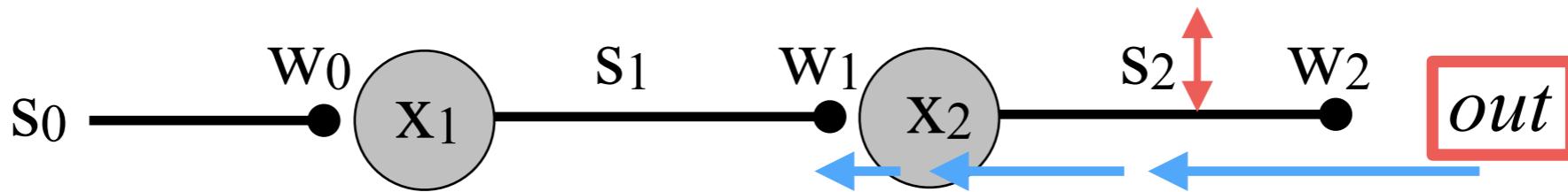
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wiggle s_2 and *out* changes

Remember: $out = s_2 w_2$

$$\frac{d \text{ } out}{d \text{ } s_2} = \frac{d \text{ } (s_2 w_2)}{d \text{ } s_2}$$

How does a change in weight w_1 impact output?



out does depend on s_2 and s_2 depend on x_2 and x_2 depend on w_1

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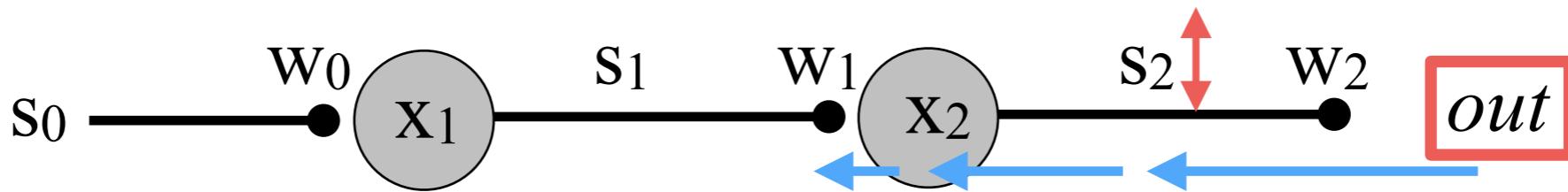
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wiggle s_2 and *out* changes

Remember: $out = s_2 w_2$

$$\frac{d \text{ } out}{d \text{ } s_2} = \frac{d (s_2 w_2)}{d \text{ } s_2} = w_2$$

How does a change in weight w_1 impact output?



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Mathematically ... the **chain rule**

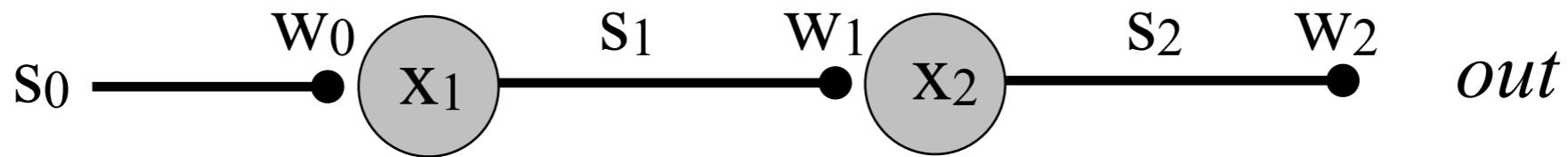
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wiggle s_2 and *out* changes

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$$\frac{d \text{ } out}{d \text{ } s_2} = \frac{d \text{ } (s_2 w_2)}{d \text{ } s_2} = \boxed{w_2}$$

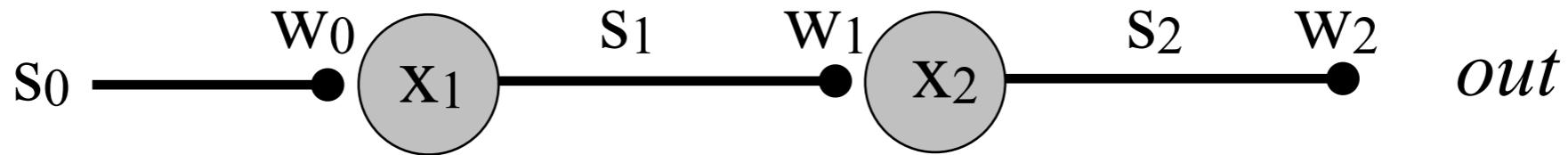
How does a change in weight w_1 impact output?



Continue the chain rule

$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

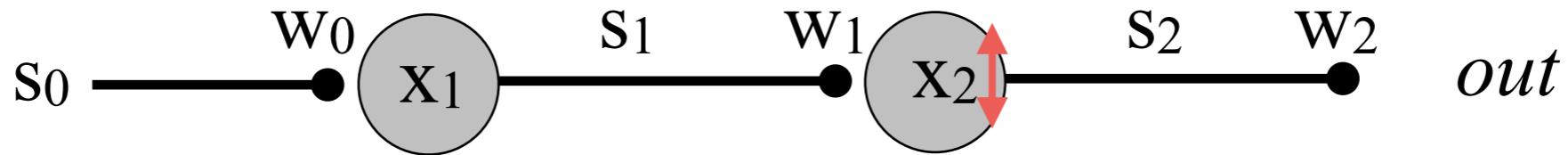
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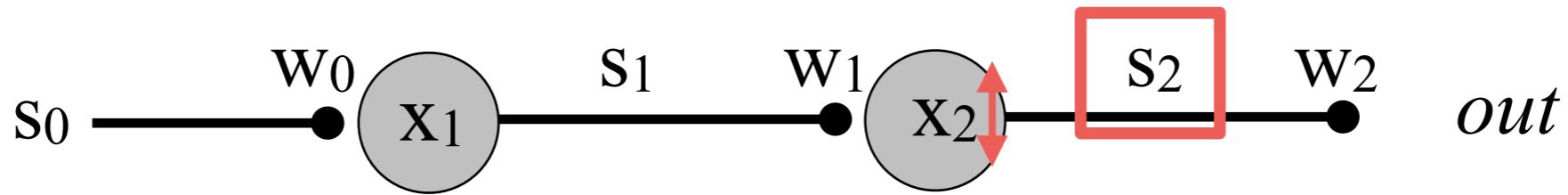


Continue the chain rule

$$\frac{d \text{out}}{d w_1} = \frac{d \text{out}}{d s_2} \boxed{\frac{d s_2}{d x_2}} \frac{d x_2}{d w_1}$$

wiggle x_2

How does a change in weight w_1 impact output?

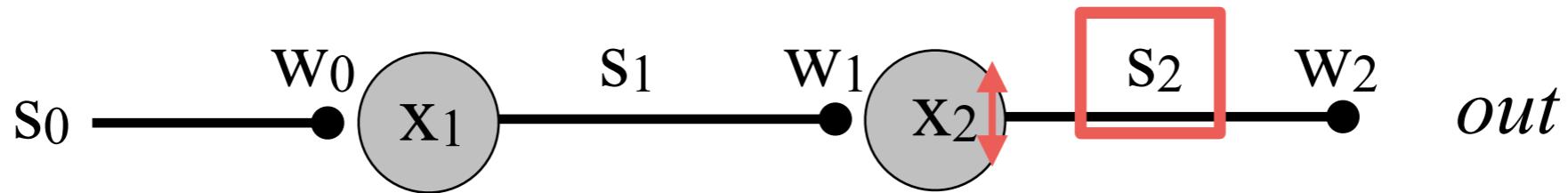


Continue the chain rule

$$\frac{d \text{out}}{d w_1} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

wiggle x_2 and s_2 changes

How does a change in weight w_1 impact output?



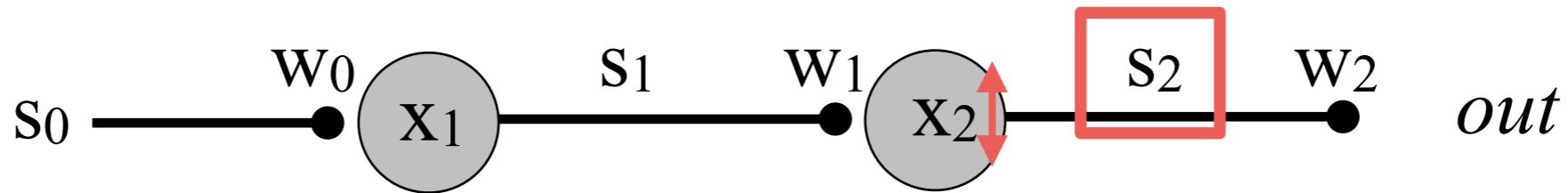
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$$s_2 =$$

How does a change in weight w_1 impact output?



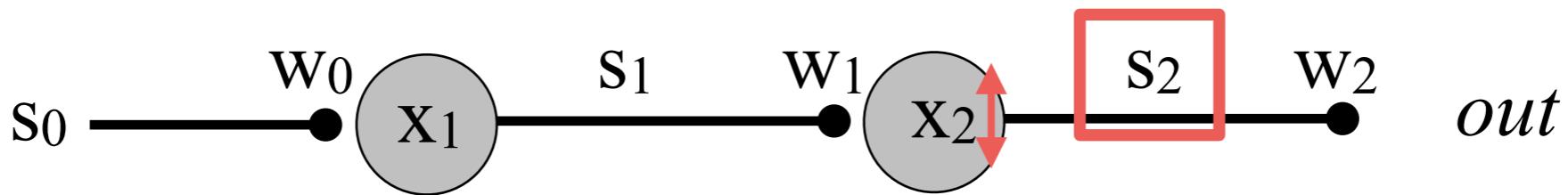
Continue the chain rule

$$\frac{d \text{out}}{d w_1} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

wiggle x_2 and s_2 changes

$$s_2 = S(x_2)$$

How does a change in weight w_1 impact output?



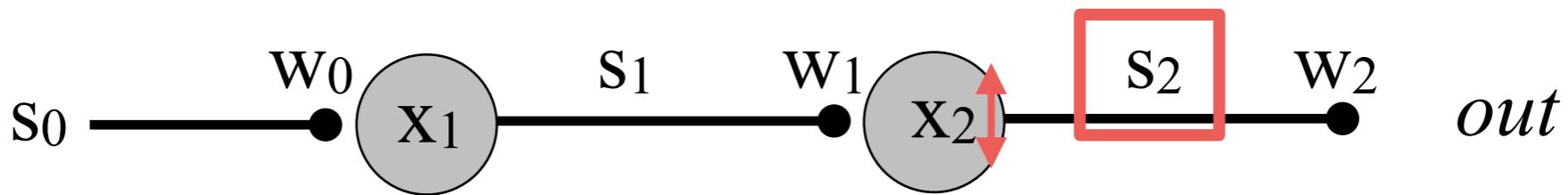
Continue the chain rule

$$\frac{d \text{out}}{d w_1} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

wiggle x_2 and s_2 changes

$$s_2 = S(x_2) = \left(\frac{1}{1 + e^{-x_2}} \right)$$

How does a change in weight w_1 impact output?



Continue the chain rule

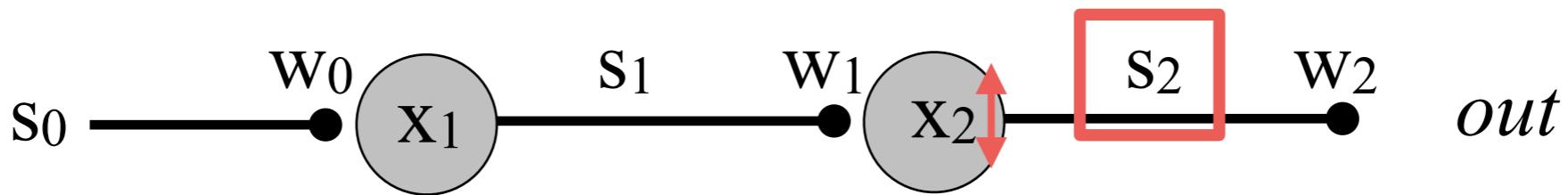
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so

How does a change in weight w_1 impact output?



Continue the chain rule

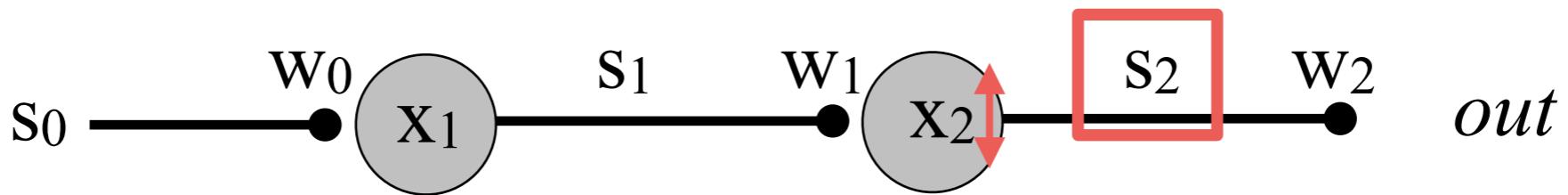
$$\frac{d \text{out}}{d w_1} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

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How does a change in weight w_1 impact output?



Continue the chain rule

$$\frac{d \text{out}}{d w_1} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

wiggle x_2 and s_2 changes

$$s_2 = S(x_2) = \left(\frac{1}{1 + e^{-x_2}} \right)$$

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A complicated derivative

We need to compute:

$$\frac{d}{dx_2} \left(\frac{1}{1 + e^{-x_2}} \right)$$

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$$\frac{d}{dx_2} \left(\frac{1}{1 + e^{-x_2}} \right) = \text{ Hmm ...}$$

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$$\begin{aligned}\frac{d}{dx_2} \left(\frac{1}{1 + e^{-x_2}} \right) &= \text{ Hmm ... Quotient Rule} \\ &= \frac{(1 + e^{-x_2}) \frac{d(1)}{dx_2} - (1) \frac{d(1 + e^{-x_2})}{dx_2}}{(1 + e^{-x_2})^2}\end{aligned}$$

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Q: What is $\frac{d(1 + e^{-x_2})}{dx_2}$?

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$$= \cancel{\frac{d(1)}{dx_2}^0} + \frac{d(e^{-x_2})}{dx_2} = (e^{-x_2}) \frac{d(-x_2)}{dx_2}$$

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Q: What is $\frac{d(1 + e^{-x_2})}{dx_2}$? = -1

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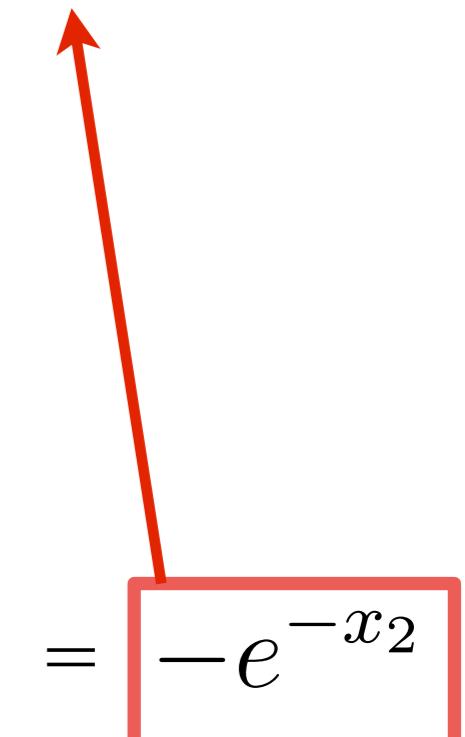
$$\frac{d}{dx_2} \left(\frac{1}{1 + e^{-x_2}} \right) = \text{ Hmm ... Quotient Rule}$$

$$= \frac{(1 + e^{-x_2}) \cancel{\frac{d(1)}{dx_2}^0} - (1) \boxed{\frac{d(1 + e^{-x_2})}{dx_2}}}{(1 + e^{-x_2})^2}$$

Q: What is $\frac{d(1 + e^{-x_2})}{dx_2}$?

$$= \frac{\cancel{d(1)}^0}{dx_2} + \frac{d(e^{-x_2})}{dx_2} = (e^{-x_2}) \boxed{\frac{d(-x_2)}{dx_2}} = \boxed{-e^{-x_2}}$$

$$= -1$$



A complicated derivative

So,

$$\frac{d}{dx_2} \left(\frac{1}{1 + e^{-x_2}} \right) =$$

A complicated derivative

So,

$$\frac{d}{dx_2} \left(\frac{1}{1 + e^{-x_2}} \right) = \frac{0 - (1)(-e^{-x_2})}{(1 + e^{-x_2})^2}$$

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So,

$$\begin{aligned}\frac{d}{dx_2} \left(\frac{1}{1 + e^{-x_2}} \right) &= \frac{0 - (1)(-e^{-x_2})}{(1 + e^{-x_2})^2} \\ &= \frac{e^{-x_2}}{(1 + e^{-x_2})^2}\end{aligned}$$

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Q: Can we simplify this expression?

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So,

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A: Yes, but requires faith ...

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$$\begin{aligned}\frac{d}{dx_2} \left(\frac{1}{1 + e^{-x_2}} \right) &= \frac{0 - (1)(-e^{-x_2})}{(1 + e^{-x_2})^2} \\ &= \frac{e^{-x_2}}{(1 + e^{-x_2})^2}\end{aligned}$$

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A: Yes, but requires faith ...

- Split up denominator: $\frac{e^{-x_2}}{(1 + e^{-x_2})^2} =$

A complicated derivative

So,

$$\begin{aligned}\frac{d}{dx_2} \left(\frac{1}{1 + e^{-x_2}} \right) &= \frac{0 - (1)(-e^{-x_2})}{(1 + e^{-x_2})^2} \\ &= \frac{e^{-x_2}}{(1 + e^{-x_2})^2}\end{aligned}$$

Q: Can we simplify this expression?

A: Yes, but requires faith ...

- Split up denominator: $\frac{e^{-x_2}}{(1 + e^{-x_2})^2} = \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{e^{-x_2}}{1 + e^{-x_2}} \right)$

A complicated derivative

- Add 0 to the second term:

A complicated derivative

- Add 0 to the second term:

$$\left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{e^{-x_2}}{1 + e^{-x_2}} \right) =$$

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$$\left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{e^{-x_2}}{1 + e^{-x_2}} \right) = \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2} - 1}{1 + e^{-x_2}} \right)$$

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- Add 0 to the second term:

Sum is 0

$$\left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{e^{-x_2}}{1 + e^{-x_2}} \right) = \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2} - 1}{1 + e^{-x_2}} \right)$$

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Q: WHY??????

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A: Let's organize terms ...

A complicated derivative

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$$\left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{e^{-x_2}}{1 + e^{-x_2}} \right) = \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2} - 1}{1 + e^{-x_2}} \right)$$

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$$= \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2}}{1 + e^{-x_2}} - \frac{1}{1 + e^{-x_2}} \right)$$

A complicated derivative

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Sum is 0

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Remember:

$$s_2 = \left(\frac{1}{1 + e^{-x_2}} \right)$$

A complicated derivative

- Add 0 to the second term:

Sum is 0

$$\left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{e^{-x_2}}{1 + e^{-x_2}} \right) = \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2} - 1}{1 + e^{-x_2}} \right)$$

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$$= \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2}}{1 + e^{-x_2}} - \frac{1}{1 + e^{-x_2}} \right)$$

$\quad \quad \quad = s_2$

Remember:

$$s_2 = \left(\frac{1}{1 + e^{-x_2}} \right)$$

A complicated derivative

- Add 0 to the second term:

Sum is 0

$$\left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{e^{-x_2}}{1 + e^{-x_2}} \right) = \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2} - 1}{1 + e^{-x_2}} \right)$$

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A: Let's organize terms ...

$$= \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2}}{1 + e^{-x_2}} - \frac{1}{1 + e^{-x_2}} \right)$$

$\quad \quad \quad = s_2$

Remember:

$$s_2 = \left(\frac{1}{1 + e^{-x_2}} \right)$$

A complicated derivative

- Add 0 to the second term:

Sum is 0

$$\left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{e^{-x_2}}{1 + e^{-x_2}} \right) = \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2} - 1}{1 + e^{-x_2}} \right)$$

Q: WHY??????

A: Let's organize terms ...

$$\begin{aligned} &= \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2}}{1 + e^{-x_2}} - \frac{1}{1 + e^{-x_2}} \right) \\ &\quad = s_2 \qquad \qquad \qquad = 1 \end{aligned}$$

Remember:

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A complicated derivative

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$$\left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{e^{-x_2}}{1 + e^{-x_2}} \right) = \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2} - 1}{1 + e^{-x_2}} \right)$$

Q: WHY??????

A: Let's organize terms ...

$$= \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2}}{1 + e^{-x_2}} - \frac{1}{1 + e^{-x_2}} \right)$$
$$= s_2 \quad \quad \quad = 1$$

Remember:

$$s_2 = \left(\frac{1}{1 + e^{-x_2}} \right)$$

A complicated derivative

- Add 0 to the second term:

Sum is 0

$$\left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{e^{-x_2}}{1 + e^{-x_2}} \right) = \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2} - 1}{1 + e^{-x_2}} \right)$$

Q: WHY??????

A: Let's organize terms ...

$$= \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2}}{1 + e^{-x_2}} - \frac{1}{1 + e^{-x_2}} \right)$$
$$= s_2 \quad \quad \quad = 1 \quad \quad \quad = s_2$$

Remember:

$$s_2 = \left(\frac{1}{1 + e^{-x_2}} \right)$$

A complicated derivative

- Add 0 to the second term:

Sum is 0

$$\left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{e^{-x_2}}{1 + e^{-x_2}} \right) = \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2} - 1}{1 + e^{-x_2}} \right)$$

Q: WHY??????

A: Let's organize terms ...

$$\begin{aligned} &= \left(\frac{1}{1 + e^{-x_2}} \right) \left(\frac{1 + e^{-x_2}}{1 + e^{-x_2}} - \frac{1}{1 + e^{-x_2}} \right) \\ &\quad = s_2 \qquad \qquad \qquad = 1 \qquad \qquad \qquad = s_2 \end{aligned}$$

$$= (s_2)(1 - s_2)$$

Remember:

$$s_2 = \left(\frac{1}{1 + e^{-x_2}} \right)$$

A complicated derivative

So, for our chain rule calculation:

$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

A complicated derivative

So, for our chain rule calculation:

$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

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And we found:

A complicated derivative

So, for our chain rule calculation:

$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \boxed{\frac{d s_2}{d x_2}} \frac{d x_2}{d w_1}$$

And we found:

$$\frac{d s_2}{d x_2} = \frac{d}{dx_2} \left(\frac{1}{1 + e^{-x_2}} \right)$$

A complicated derivative

So, for our chain rule calculation:

$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

And we found:

$$\begin{aligned}\frac{d s_2}{d x_2} &= \frac{d}{dx_2} \left(\frac{1}{1 + e^{-x_2}} \right) \\ &= s_2\end{aligned}$$

A complicated derivative

So, for our chain rule calculation:

$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

And we found:

$$\frac{d s_2}{d x_2} = \frac{d}{dx_2} \left(\frac{1}{1 + e^{-x_2}} \right) = \dots \text{many steps} \dots$$
$$= s_2$$

A complicated derivative

So, for our chain rule calculation:

$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

And we found:

$$\frac{d s_2}{d x_2} = \frac{d}{dx_2} \left(\frac{1}{1 + e^{-x_2}} \right) = \dots \text{many steps...} = s_2(1 - s_2)$$

$= s_2$

A complicated derivative

So, for our chain rule calculation:

$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

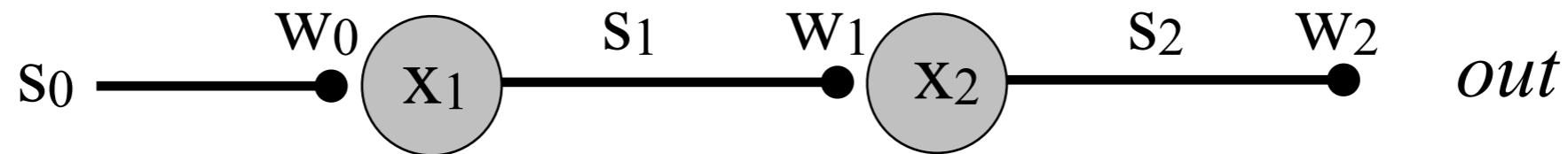
And we found:

$$\frac{d s_2}{d x_2} = \frac{d}{dx_2} \left(\frac{1}{1 + e^{-x_2}} \right) = \dots \text{many steps} \dots = s_2 (1 - s_2)$$

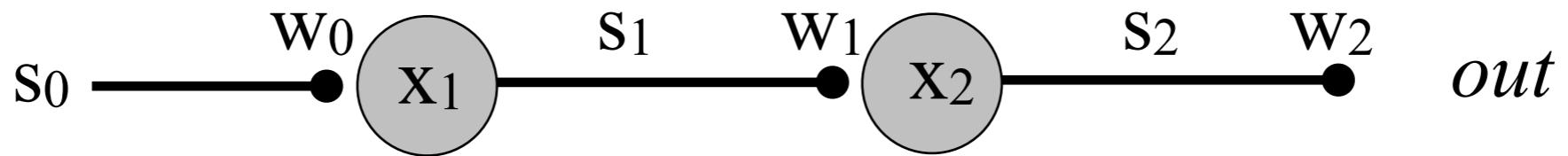
$= s_2$

To complete the chain rule, one more derivative ...

How does a change in weight w_1 impact output?

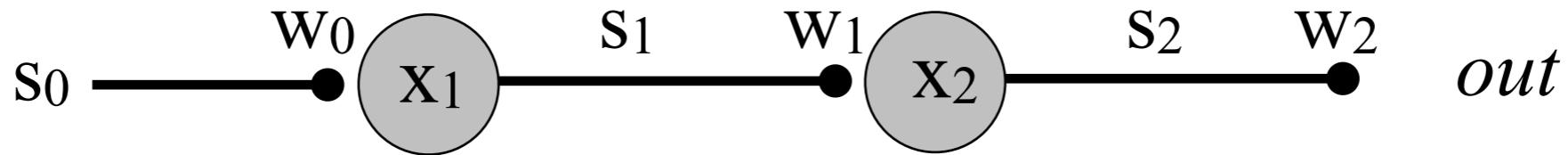


How does a change in weight w_1 impact output?



Continue the chain rule:

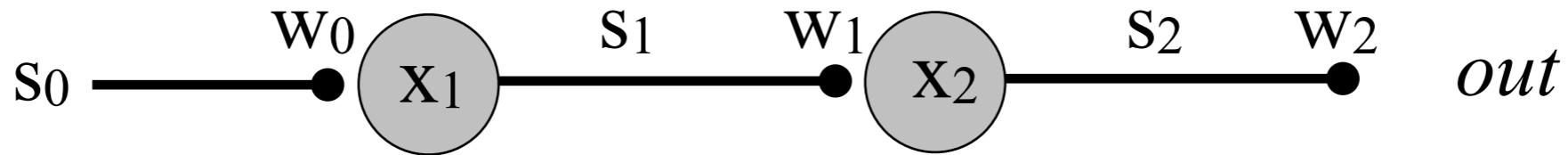
How does a change in weight w_1 impact output?



Continue the chain rule:

$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

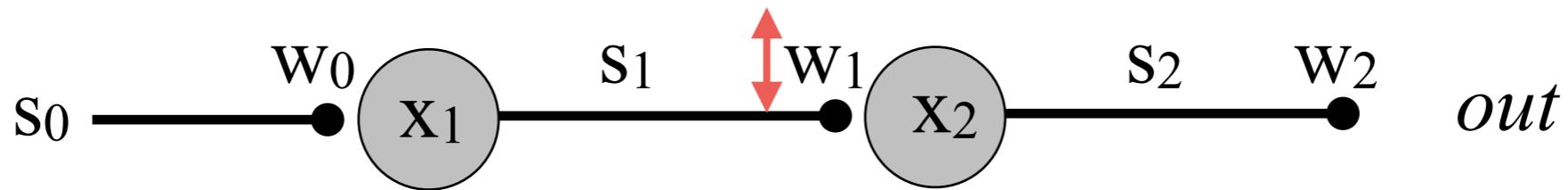
How does a change in weight w_1 impact output?



Continue the chain rule:

$$\frac{d \text{out}}{d w_1} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \boxed{\frac{d x_2}{d w_1}}$$

How does a change in weight w_1 impact output?

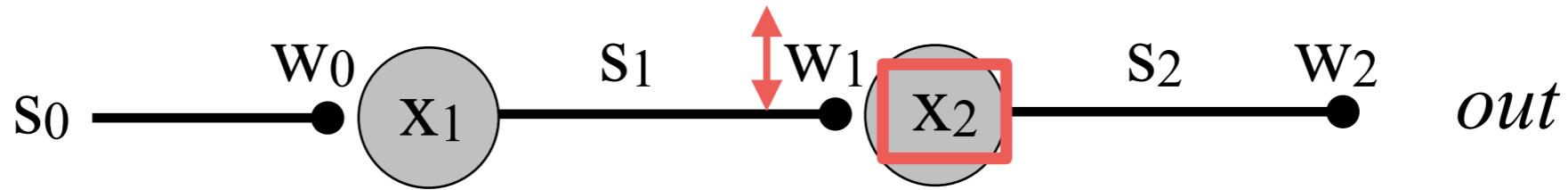


Continue the chain rule:

$$\frac{d \text{out}}{d w_1} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \boxed{\frac{d x_2}{d w_1}}$$

wiggle w_1

How does a change in weight w_1 impact output?

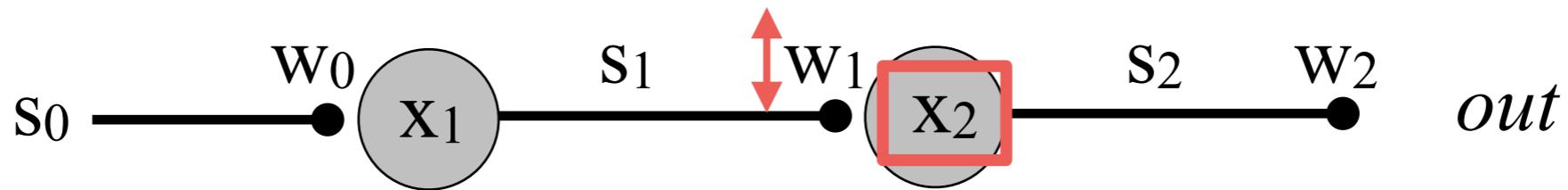


Continue the chain rule:

$$\frac{d \text{out}}{d w_1} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \boxed{\frac{d x_2}{d w_1}}$$

wiggle w_1 and x_2 changes

How does a change in weight w_1 impact output?



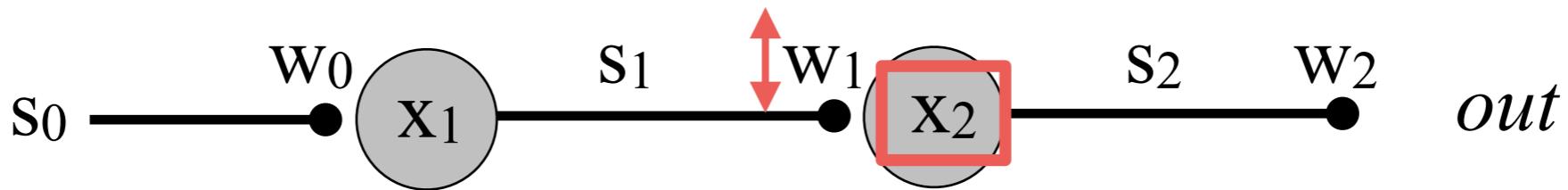
Continue the chain rule:

$$\frac{d \text{out}}{d w_1} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \boxed{\frac{d x_2}{d w_1}}$$

wiggle w_1 and x_2 changes

$$\frac{d x_2}{d w_1} =$$

How does a change in weight w_1 impact output?



Continue the chain rule:

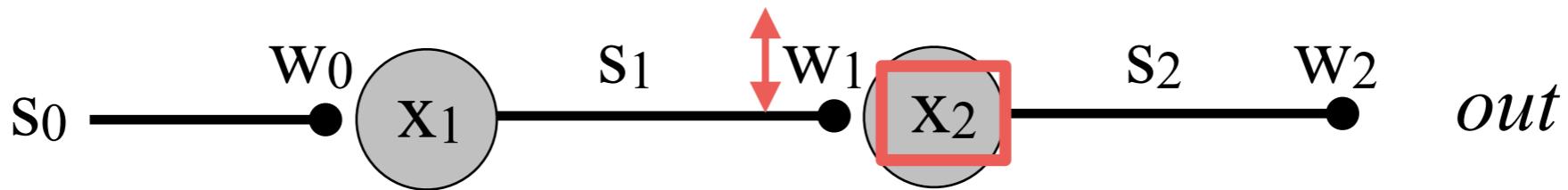
$$\frac{d \text{out}}{d w_1} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \boxed{\frac{d x_2}{d w_1}}$$

wiggle w_1 and x_2 changes

Remember: $x_2 = s_1 w_1$

$$\frac{d x_2}{d w_1} =$$

How does a change in weight w_1 impact output?



Continue the chain rule:

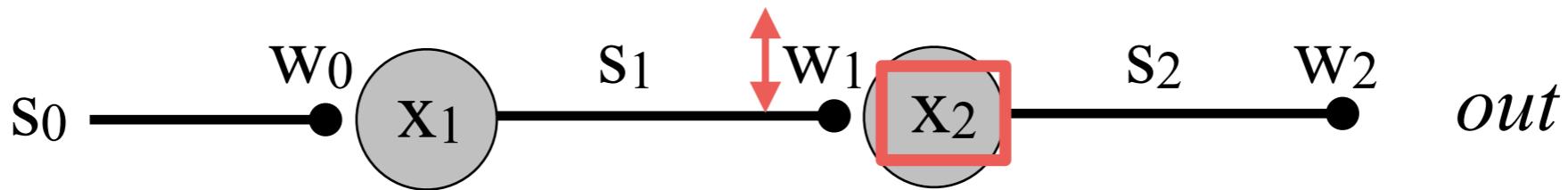
$$\frac{d \text{out}}{d w_1} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \boxed{\frac{d x_2}{d w_1}}$$

wiggle w_1 and x_2 changes

Remember: $x_2 = s_1 w_1$

$$\frac{d x_2}{d w_1} = \frac{d (s_1 w_1)}{d w_1}$$

How does a change in weight w_1 impact output?



Continue the chain rule:

$$\frac{d \text{out}}{d w_1} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \boxed{\frac{d x_2}{d w_1}}$$

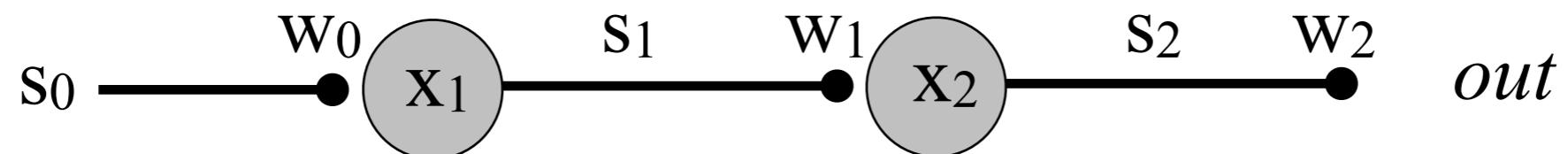
wiggle w_1 and x_2 changes

Remember: $x_2 = s_1 w_1$

$$\frac{d x_2}{d w_1} = \frac{d (s_1 w_1)}{d w_1} = \boxed{s_1}$$

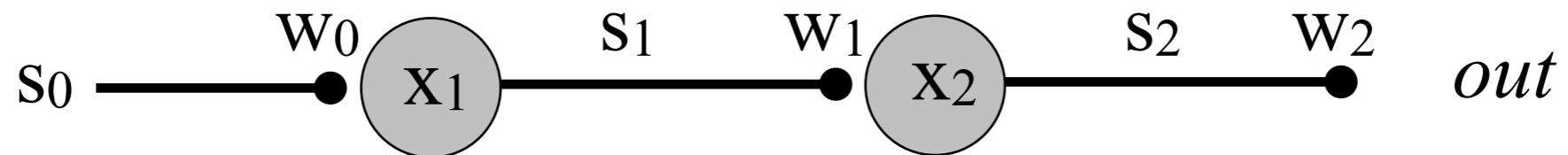
How does a change in weight w_1 impact output?

Back to our original question:



How does a change in weight w_1 impact output?

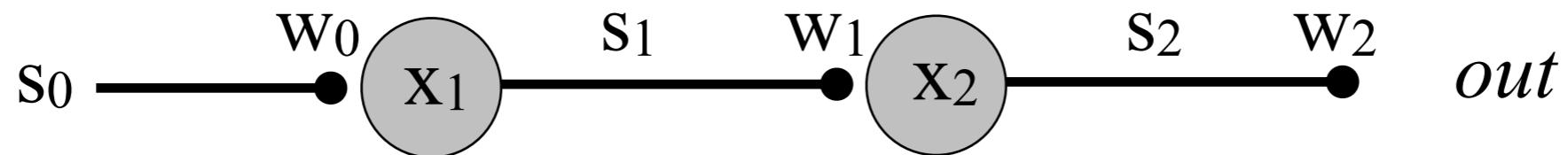
Back to our original question:



Q: How does a change in weight w_1 impact the output?

How does a change in weight w_1 impact output?

Back to our original question:

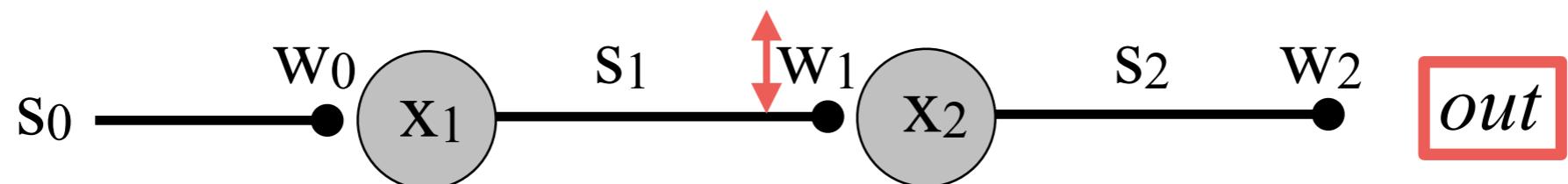


Q: How does a change in weight w_1 impact the output?

Mathematically ... **the chain rule**

How does a change in weight w_1 impact output?

Back to our original question:



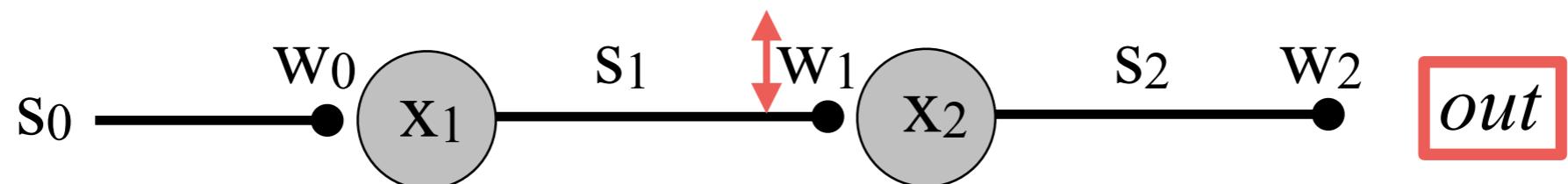
Q: How does a change in weight w_1 impact the output?

Mathematically ... the **chain rule**

$$\frac{d \text{ out}}{d w_1}$$

How does a change in weight w_1 impact output?

Back to our original question:



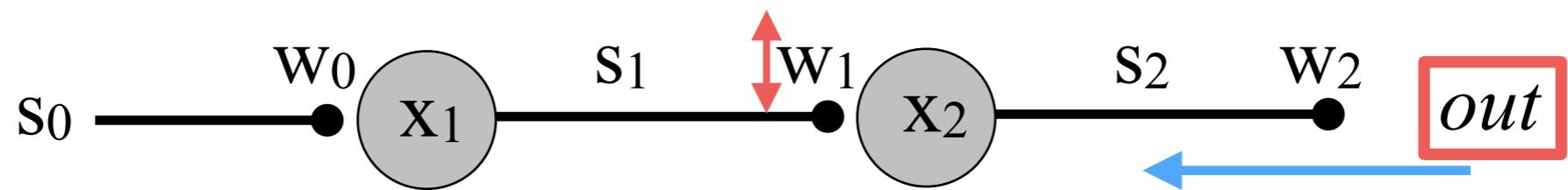
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Mathematically ... the **chain rule**

$$\frac{d \text{ out}}{d w_1} =$$

How does a change in weight w_1 impact output?

Back to our original question:



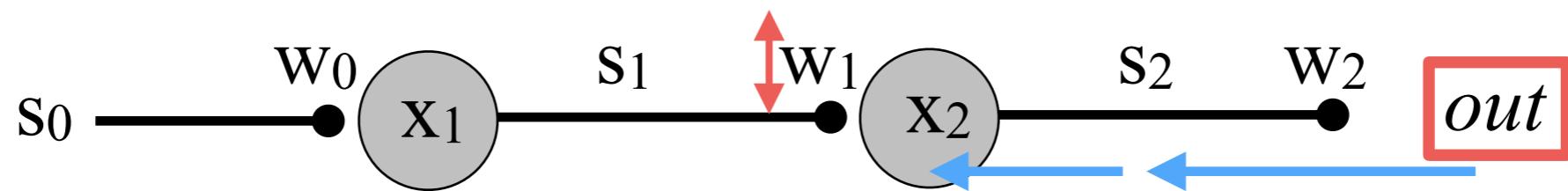
Q: How does a change in weight w_1 impact the output?

Mathematically ... the **chain rule**

$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2}$$

How does a change in weight w_1 impact output?

Back to our original question:



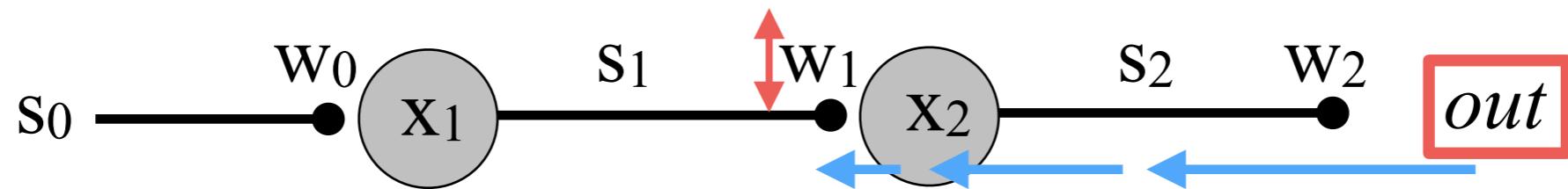
Q: How does a change in weight w_1 impact the output?

Mathematically ... the **chain rule**

$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \frac{d s_2}{d x_2}$$

How does a change in weight w_1 impact output?

Back to our original question:



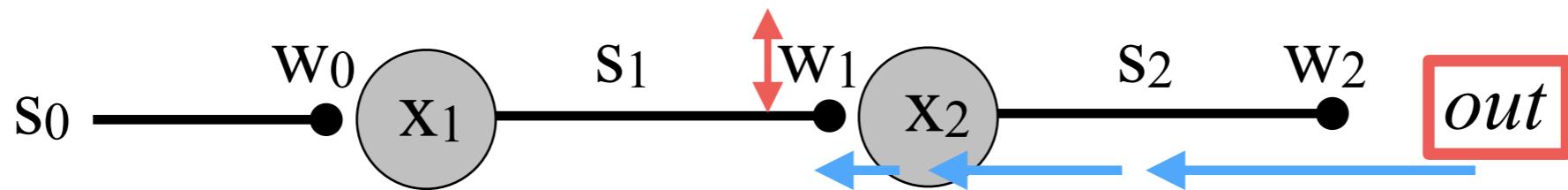
Q: How does a change in weight w_1 impact the output?

Mathematically ... the **chain rule**

$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \cdot \frac{d s_2}{d x_2} \cdot \frac{d x_2}{d w_1}$$

How does a change in weight w_1 impact output?

Back to our original question:



Q: How does a change in weight w_1 impact the output?

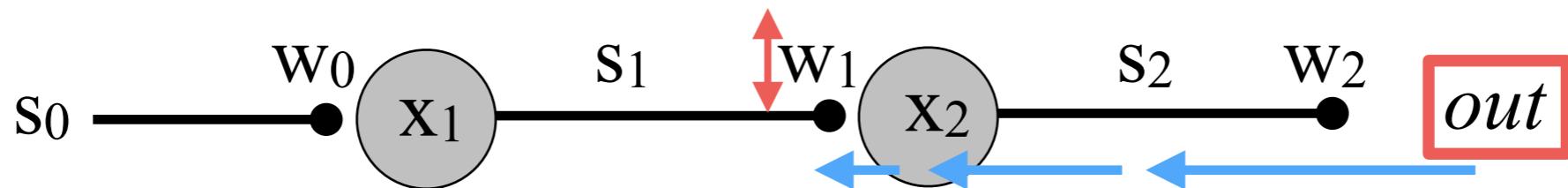
Mathematically ... the **chain rule**

$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \cdot \frac{d s_2}{d x_2} \cdot \frac{d x_2}{d w_1}$$

$$\frac{d \text{ out}}{d w_1} =$$

How does a change in weight w_1 impact output?

Back to our original question:



Q: How does a change in weight w_1 impact the output?

Mathematically ... the **chain rule**

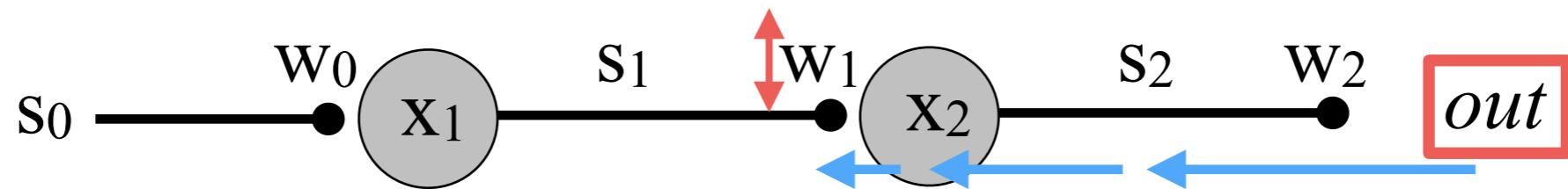
$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \cdot \frac{d s_2}{d x_2} \cdot \frac{d x_2}{d w_1}$$

$$\frac{d \text{ out}}{d w_1} = w_2$$

Slide 16

How does a change in weight w_1 impact output?

Back to our original question:



Q: How does a change in weight w_1 impact the output?

Mathematically ... the **chain rule**

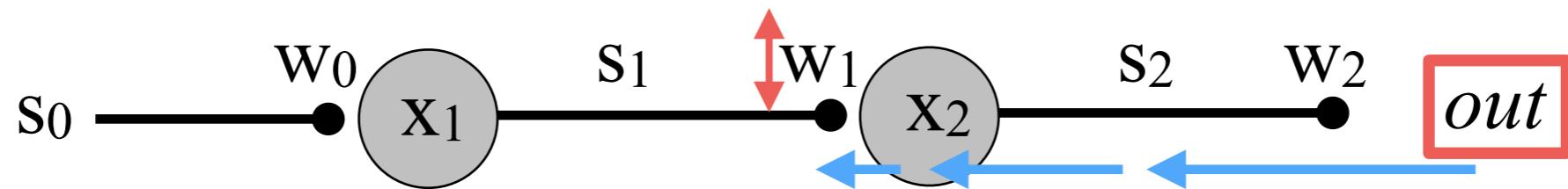
$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

$$\frac{d \text{ out}}{d w_1} = w_2 s_2 (1 - s_2)$$

Slide 16 Slide 21

How does a change in weight w_1 impact output?

Back to our original question:



Q: How does a change in weight w_1 impact the output?

Mathematically ... the **chain rule**

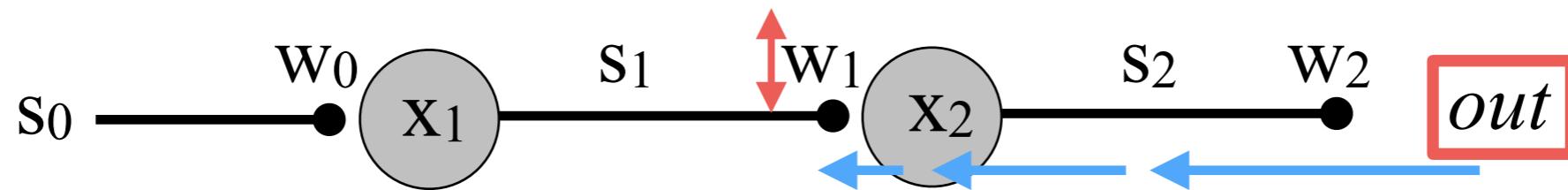
$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

$$\frac{d \text{ out}}{d w_1} = w_2 s_2 (1 - s_2) s_1$$

Slide 16 *Slide 21* *Slide 22*

How does a change in weight w_1 impact output?

Back to our original question:



Q: How does a change in weight w_1 impact the output?

Mathematically ... the **chain rule**

$$\frac{d \text{ out}}{d w_1} = \frac{d \text{ out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d w_1}$$

$$\frac{d \text{ out}}{d w_1} = w_2 \frac{s_2(1 - s_2)}{s_1}$$

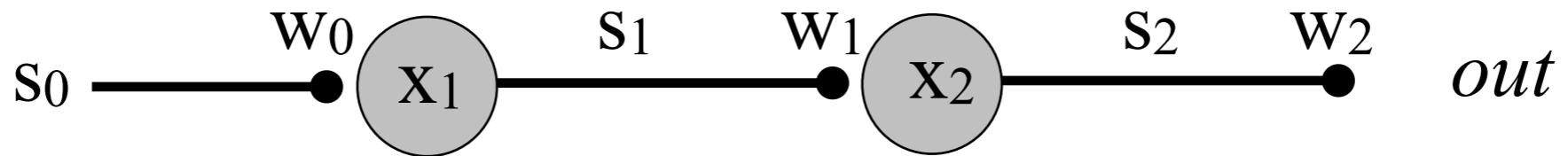
Slide 16

Slide 21

Slide 22

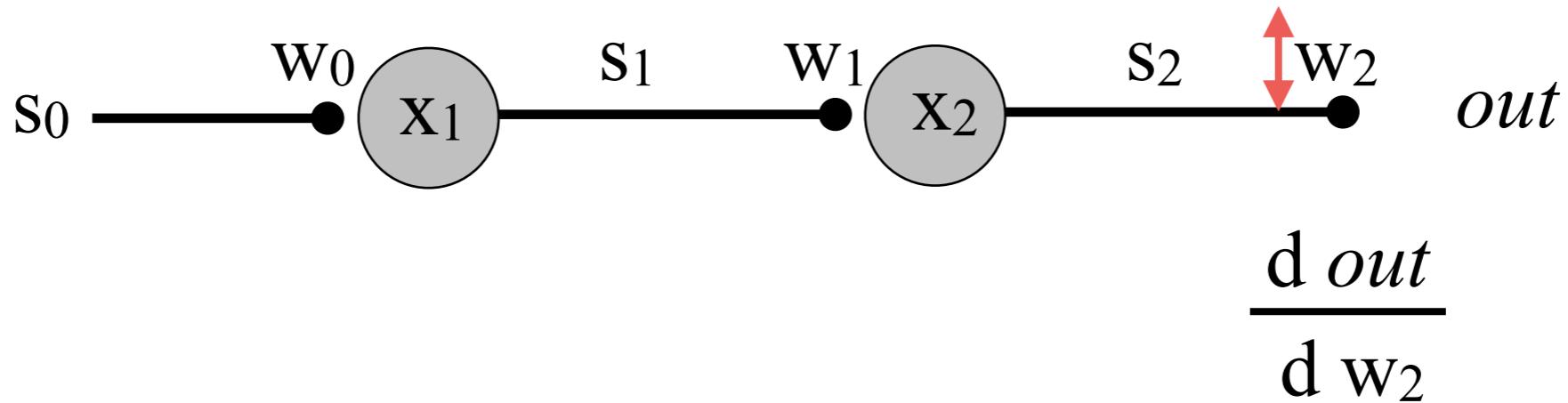
How does a change in weight w_0 impact output?

We're almost there ...



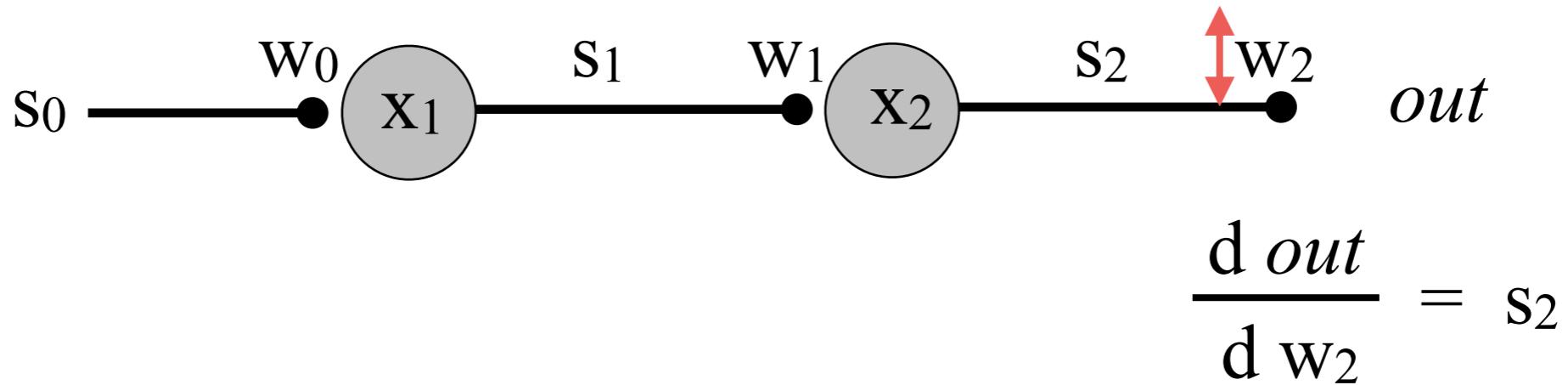
How does a change in weight w_0 impact output?

We're almost there ...



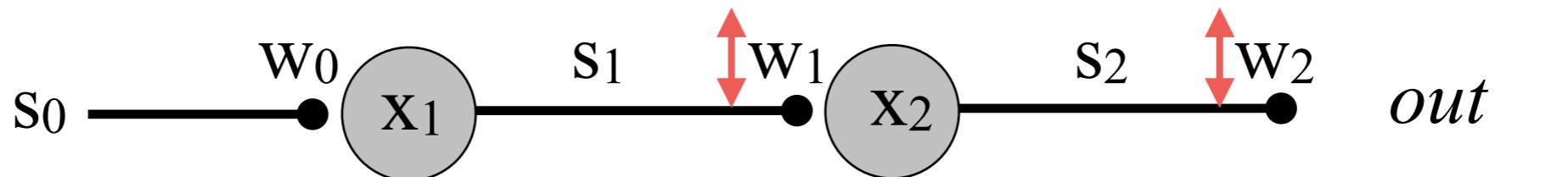
How does a change in weight w_0 impact output?

We're almost there ...



How does a change in weight w_0 impact output?

We're almost there ...

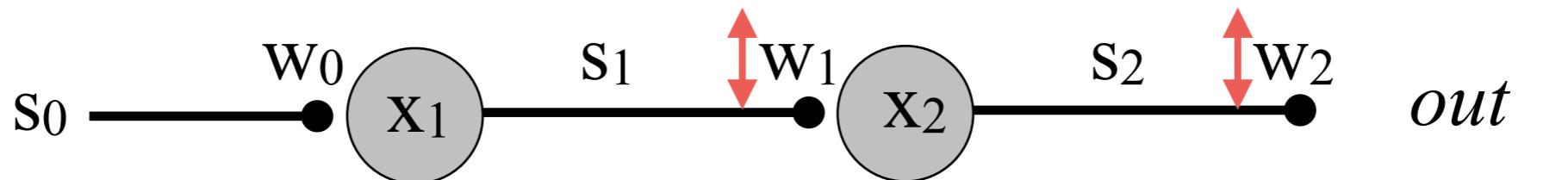


$$\frac{d \text{ } out}{d \text{ } w_2} = s_2$$

$$\frac{d \text{ } out}{d \text{ } w_1}$$

How does a change in weight w_0 impact output?

We're almost there ...

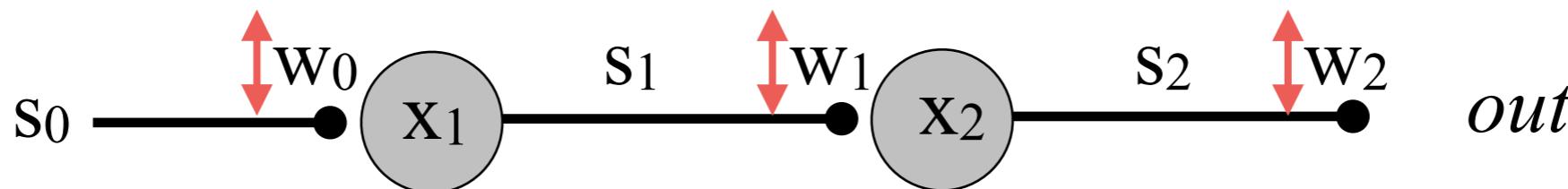


$$\frac{d \text{out}}{d w_2} = s_2$$

$$\frac{d \text{out}}{d w_1} = w_2 s_2 (1 - s_2) s_1$$

How does a change in weight w_0 impact output?

We're almost there ...



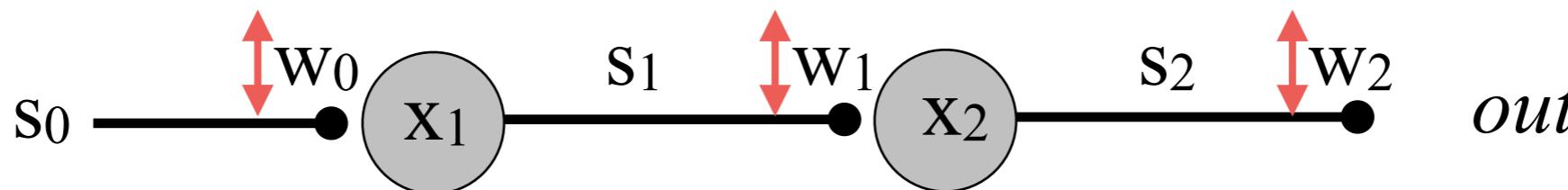
$$\frac{d \text{out}}{d w_2} = s_2$$

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Q: How does a change in weight w_0 impact the output?

How does a change in weight w_0 impact output?

We're almost there ...



$$\frac{d \text{out}}{d w_2} = s_2$$

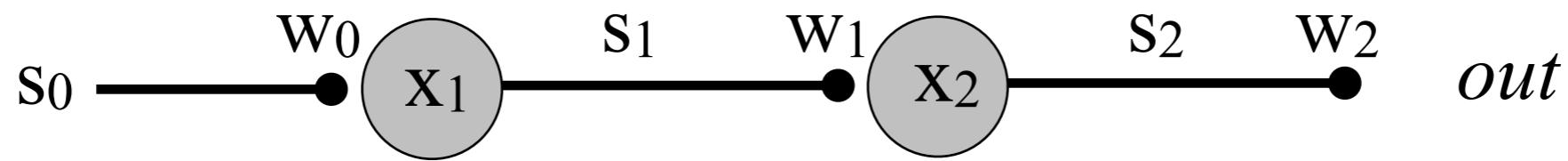
$$\frac{d \text{out}}{d w_1} = w_2 s_2 (1 - s_2) s_1$$

Q: How does a change in weight w_0 impact the output?

A: Chain rule ...

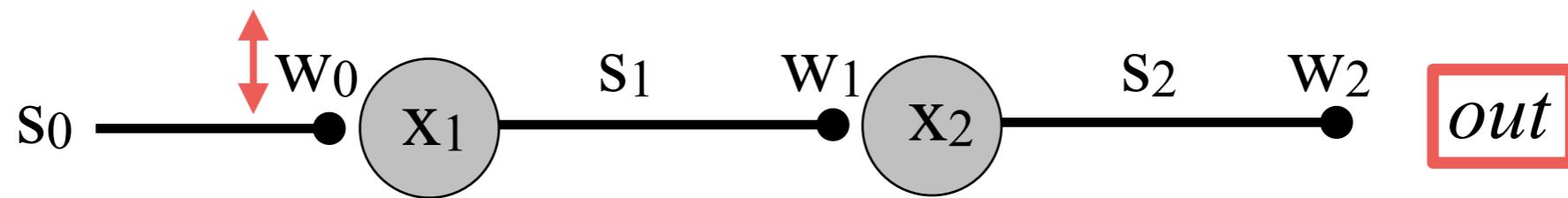
How does a change in weight w_0 impact output?

Q: How does a change in weight w_0 impact the output?



How does a change in weight w_0 impact output?

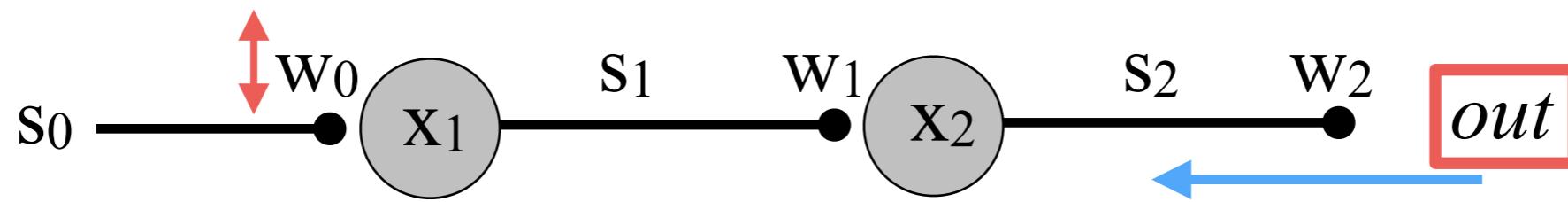
Q: How does a change in weight w_0 impact the output?



$$\frac{d \text{ out}}{d w_0} =$$

How does a change in weight w_0 impact output?

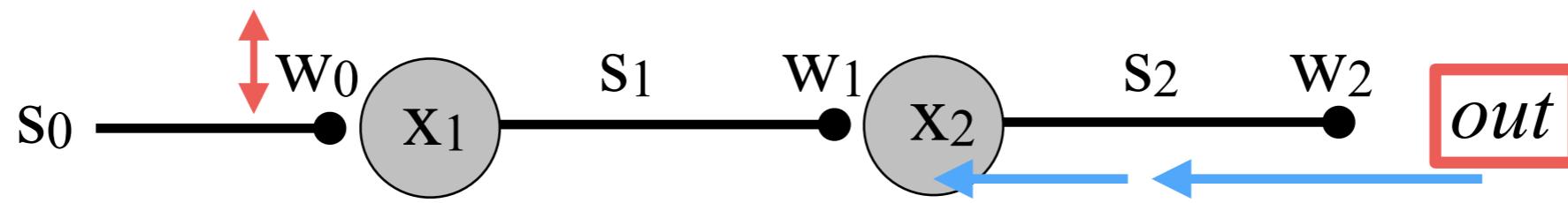
Q: How does a change in weight w_0 impact the output?



$$\frac{d \text{ } out}{d \text{ } w_0} = \frac{d \text{ } out}{d \text{ } s_2}$$

How does a change in weight w_0 impact output?

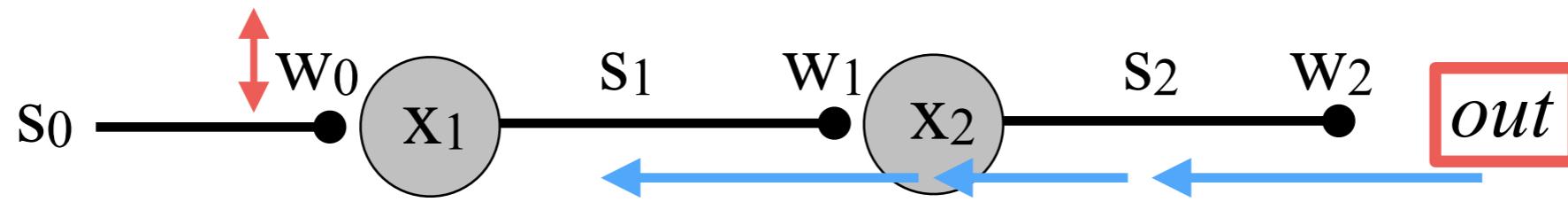
Q: How does a change in weight w_0 impact the output?



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2}$$

How does a change in weight w_0 impact output?

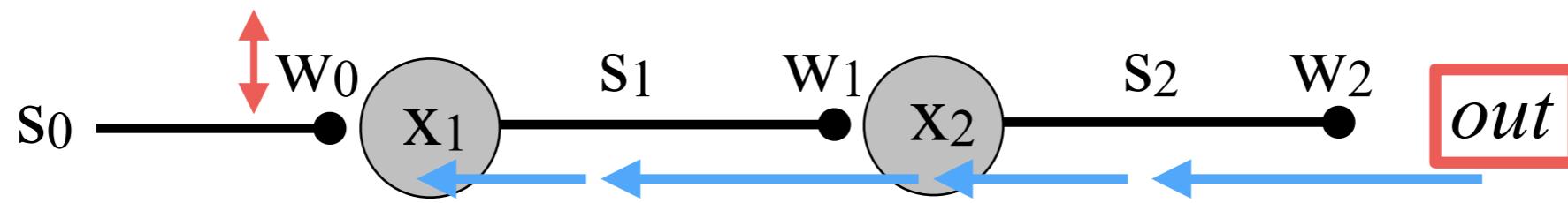
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$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1}$$

How does a change in weight w_0 impact output?

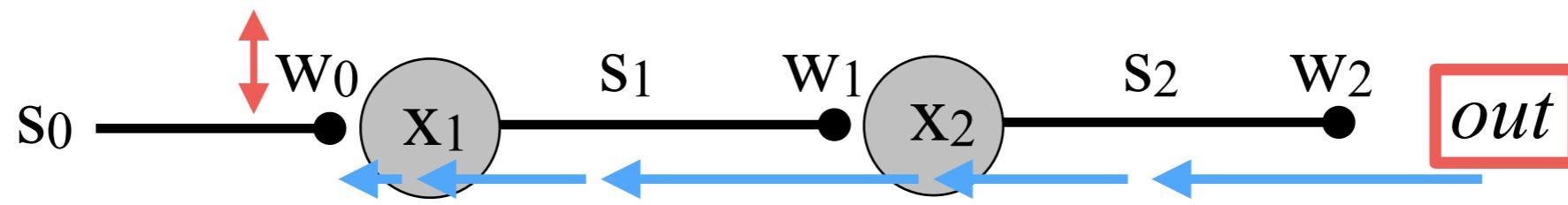
Q: How does a change in weight w_0 impact the output?



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1}$$

How does a change in weight w_0 impact output?

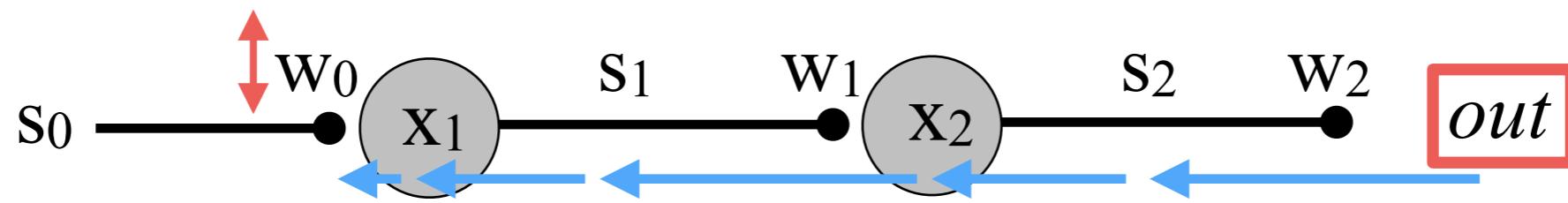
Q: How does a change in weight w_0 impact the output?



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \frac{d x_1}{d w_0}$$

How does a change in weight w_0 impact output?

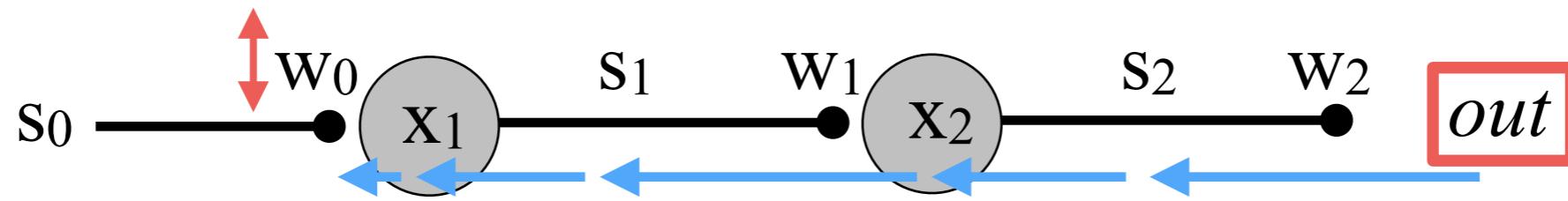
Q: How does a change in weight w_0 impact the output?



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \frac{d x_1}{d w_0} \quad \text{Ugh ...}$$

How does a change in weight w_0 impact output?

Q: How does a change in weight w_0 impact the output?

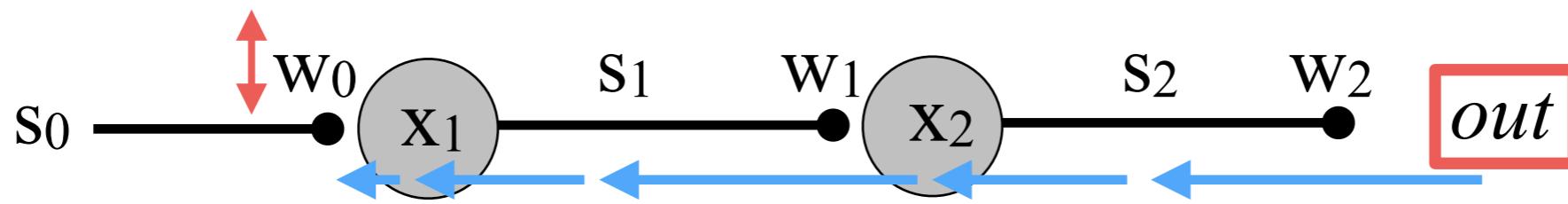


$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \frac{d x_1}{d w_0} \quad \text{Ugh ...}$$

Luckily, we've already calculated two of these.

How does a change in weight w_0 impact output?

Q: How does a change in weight w_0 impact the output?



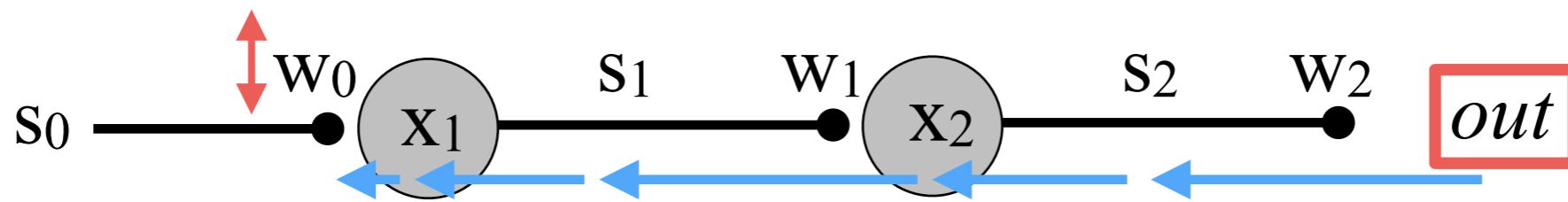
$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \frac{d x_1}{d w_0} \quad \text{Ugh ...}$$

Luckily, we've already calculated two of these.

w_2

How does a change in weight w_0 impact output?

Q: How does a change in weight w_0 impact the output?



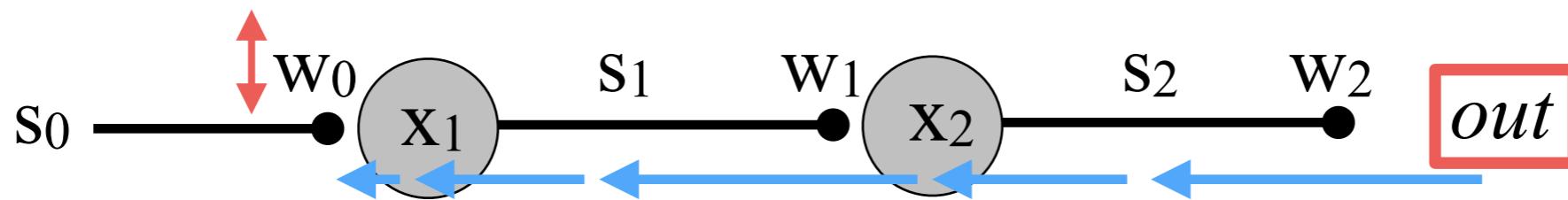
$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \frac{d x_1}{d w_0} \quad \text{Ugh ...}$$

Luckily, we've already calculated two of these.

$$w_2 \quad s_2(1 - s_2)$$

How does a change in weight w_0 impact output?

Q: How does a change in weight w_0 impact the output?



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \frac{d x_1}{d w_0}$$

Ugh ...

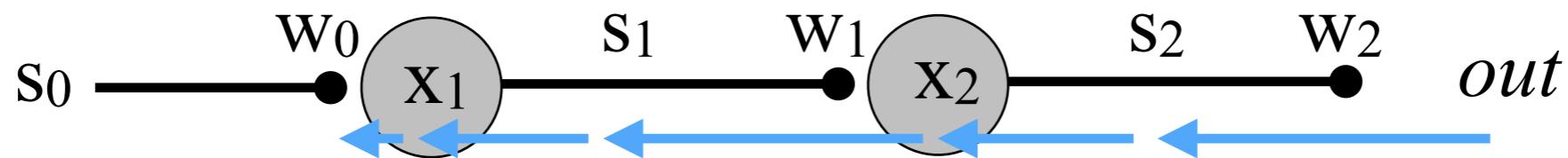
Luckily, we've already calculated two of these.

$$w_2 \quad s_2(1 - s_2)$$

Let's compute the last 3 terms ...

How does a change in weight w_0 impact output?

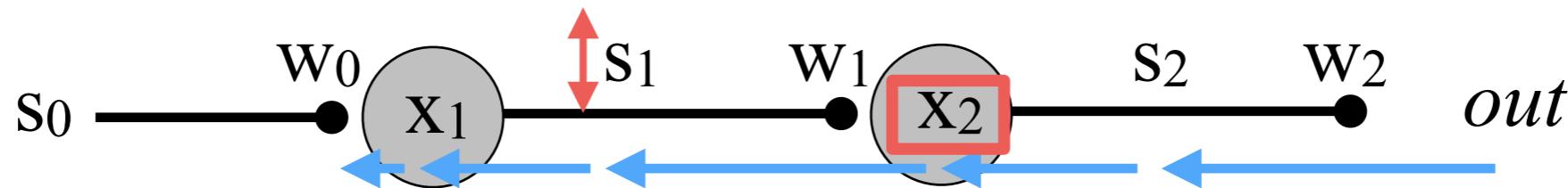
3rd term:



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \cdot \frac{d s_2}{d x_2} \cdot \frac{d x_2}{d s_1} \cdot \frac{d s_1}{d x_1} \cdot \frac{d x_1}{d w_0}$$

How does a change in weight w_0 impact output?

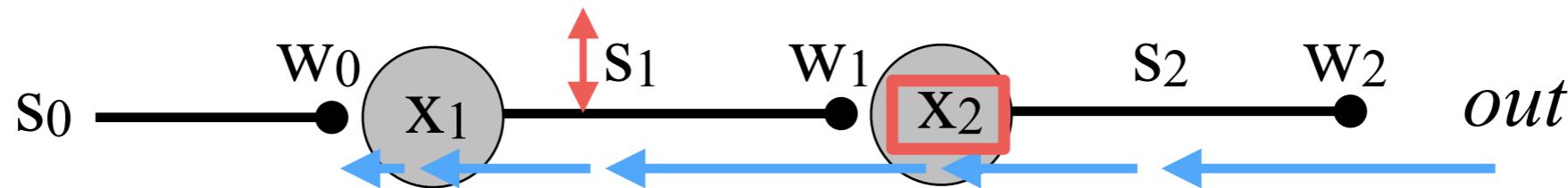
3rd term:



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \cdot \frac{d s_2}{d x_2} \cdot \frac{d x_2}{d s_1} \cdot \frac{d s_1}{d x_1} \cdot \frac{d x_1}{d w_0}$$

How does a change in weight w_0 impact output?

3rd term:

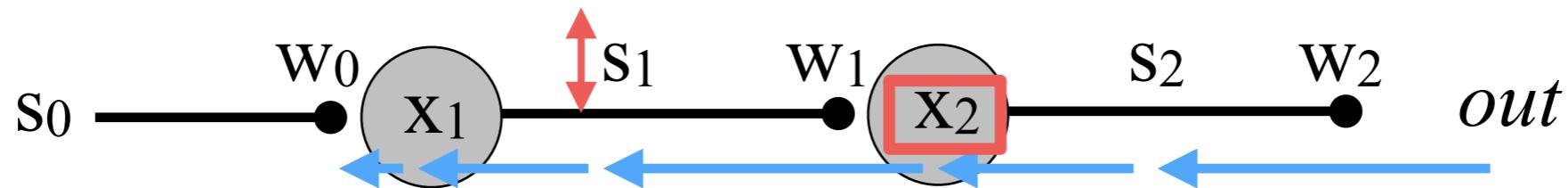


$$\frac{d out}{d w_0} = \frac{d out}{d s_2} \cdot \frac{d s_2}{d x_2} \cdot \frac{d x_2}{d s_1} \cdot \frac{d s_1}{d x_1} \cdot \frac{d x_1}{d w_0}$$

Remember: $x_2 = s_1 w_1$

How does a change in weight w_0 impact output?

3rd term:



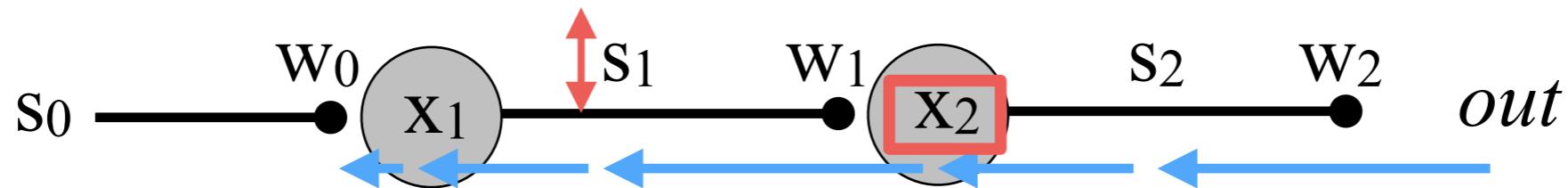
$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \frac{d x_1}{d w_0}$$

Remember: $x_2 = s_1 w_1$

$$\frac{d x_2}{d s_1} =$$

How does a change in weight w_0 impact output?

3rd term:



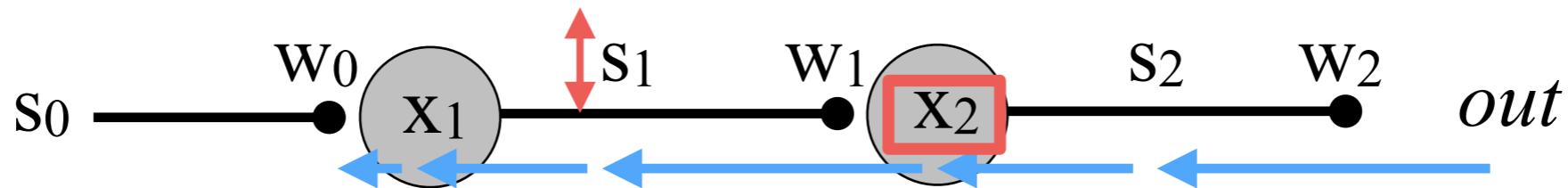
$$\frac{d out}{d w_0} = \frac{d out}{d s_2} \cdot \frac{d s_2}{d x_2} \cdot \boxed{\frac{d x_2}{d s_1}} \cdot \frac{d s_1}{d x_1} \cdot \frac{d x_1}{d w_0}$$

Remember: $x_2 = s_1 w_1$

$$\frac{d x_2}{d s_1} = \frac{d (s_1 w_1)}{d s_1}$$

How does a change in weight w_0 impact output?

3rd term:



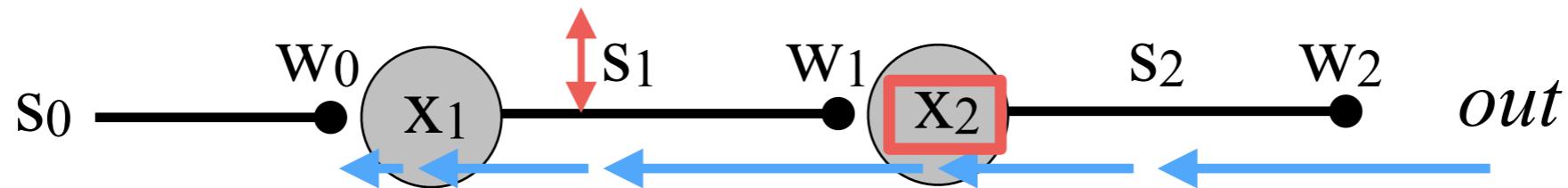
$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \frac{d x_1}{d w_0}$$

Remember: $x_2 = s_1 w_1$

$$\frac{d x_2}{d s_1} = \frac{d (s_1 w_1)}{d s_1} = w_1$$

How does a change in weight w_0 impact output?

3rd term:



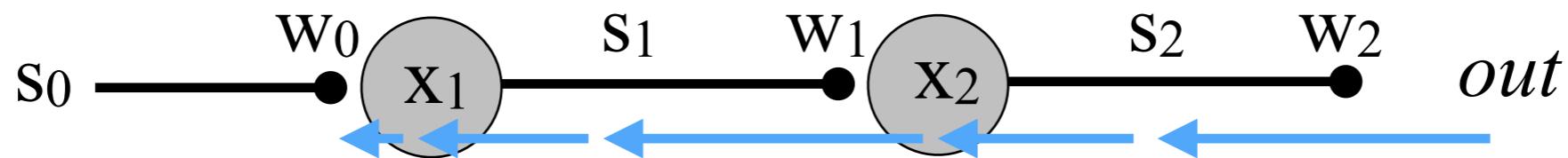
$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \frac{d x_1}{d w_0}$$

Remember: $x_2 = s_1 w_1$

$$\frac{d x_2}{d s_1} = \frac{d (s_1 w_1)}{d s_1} = w_1$$

How does a change in weight w_0 impact output?

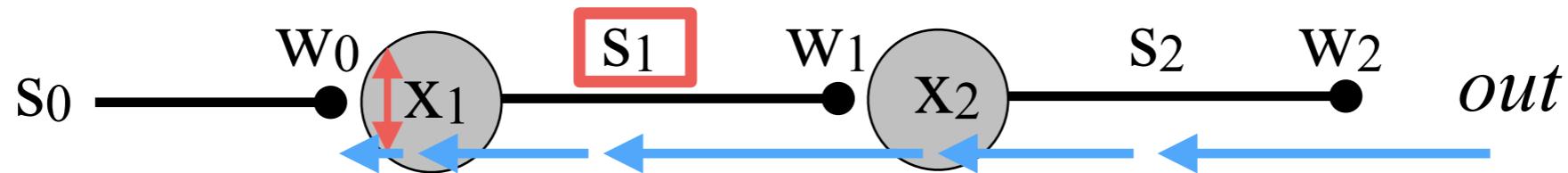
4th term:



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \cdot \frac{d s_2}{d x_2} \cdot \frac{d x_2}{d s_1} \cdot \frac{d s_1}{d x_1} \cdot \frac{d x_1}{d w_0}$$

How does a change in weight w_0 impact output?

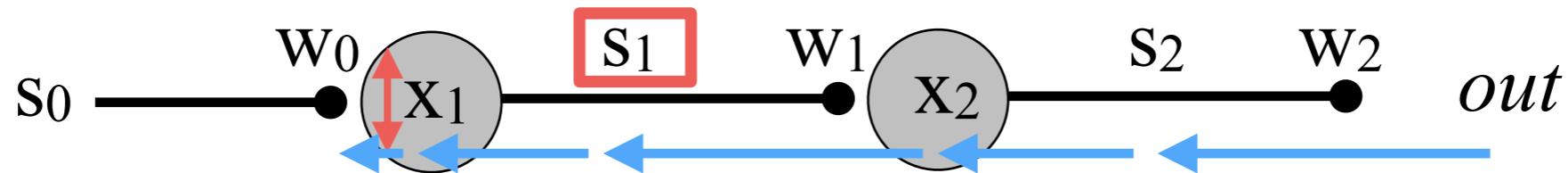
4th term:



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \boxed{\frac{d s_1}{d x_1}} \frac{d x_1}{d w_0}$$

How does a change in weight w_0 impact output?

4th term:

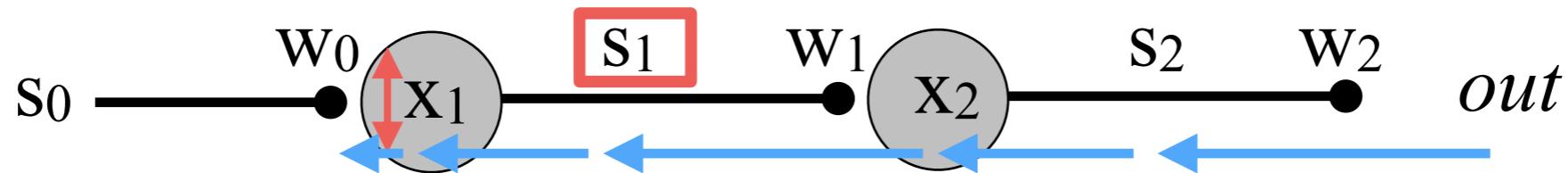


$$\frac{d out}{d w_0} = \frac{d out}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \boxed{\frac{d s_1}{d x_1}} \frac{d x_1}{d w_0}$$

We found earlier that:

How does a change in weight w_0 impact output?

4th term:



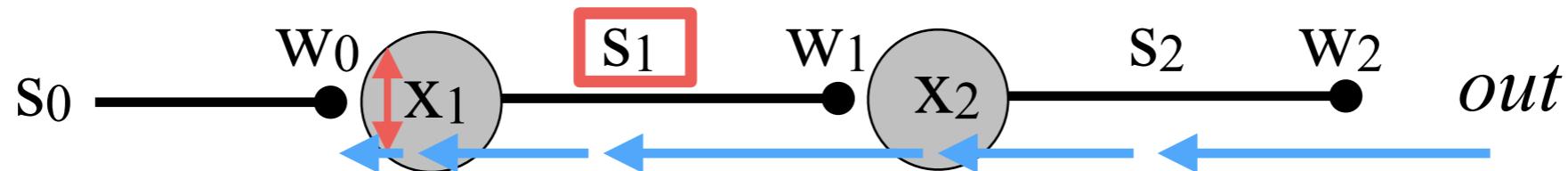
$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \boxed{\frac{d s_1}{d x_1}} \frac{d x_1}{d w_0}$$

We found earlier that:

$$\frac{d s_2}{d x_2} =$$

How does a change in weight w_0 impact output?

4th term:



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \boxed{\frac{d s_1}{d x_1}} \frac{d x_1}{d w_0}$$

We found earlier that:

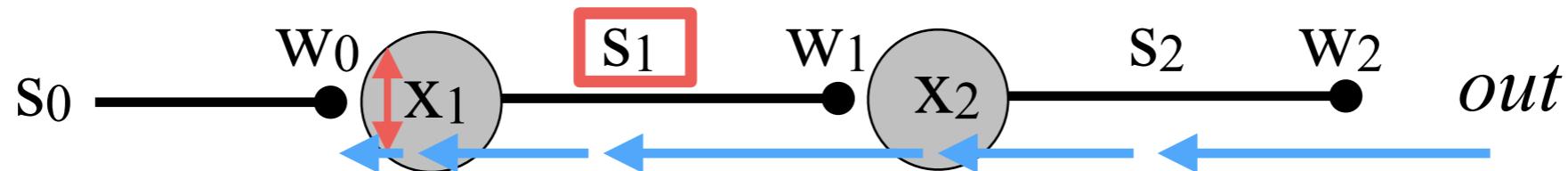
$$\frac{d s_2}{d x_2} = s_2(1 - s_2)$$

↑

This involved many steps!

How does a change in weight w_0 impact output?

4th term:



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \boxed{\frac{d s_1}{d x_1}} \frac{d x_1}{d w_0}$$

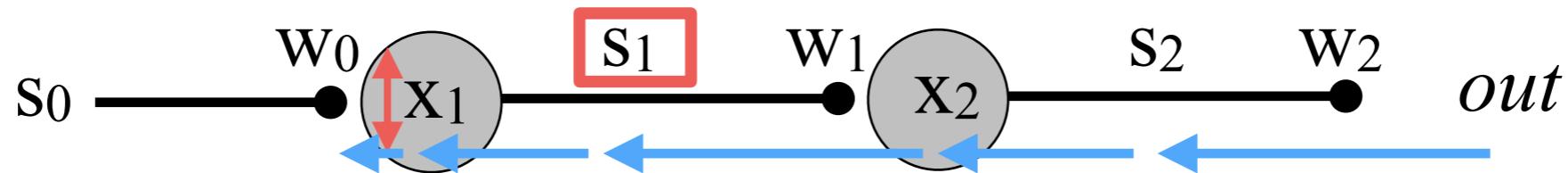
We found earlier that:

$$\frac{d s_2}{d x_2} = s_2(1 - s_2) \quad \dots \text{so} \dots$$

This involved many steps!

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4th term:



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \boxed{\frac{d s_1}{d x_1}} \frac{d x_1}{d w_0}$$

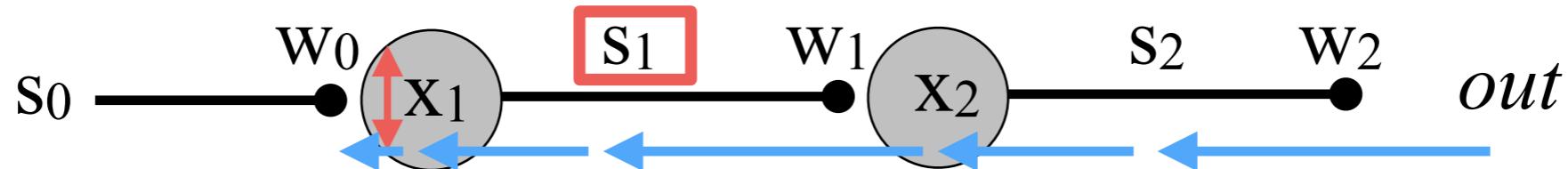
We found earlier that:

$$\frac{d s_2}{d x_2} = s_2(1 - s_2) \quad \dots \text{so} \dots \quad \frac{d s_1}{d x_1} =$$

This involved many steps!

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4th term:



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \boxed{\frac{d s_1}{d x_1}} \frac{d x_1}{d w_0}$$

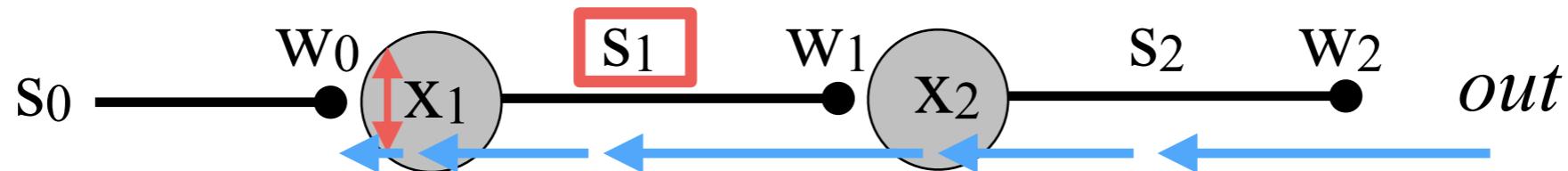
We found earlier that:

$$\frac{d s_2}{d x_2} = s_2(1 - s_2) \quad \dots \text{so} \dots \quad \frac{d s_1}{d x_1} = s_1(1 - s_1)$$

This involved many steps!

How does a change in weight w_0 impact output?

4th term:



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \frac{d x_1}{d w_0}$$

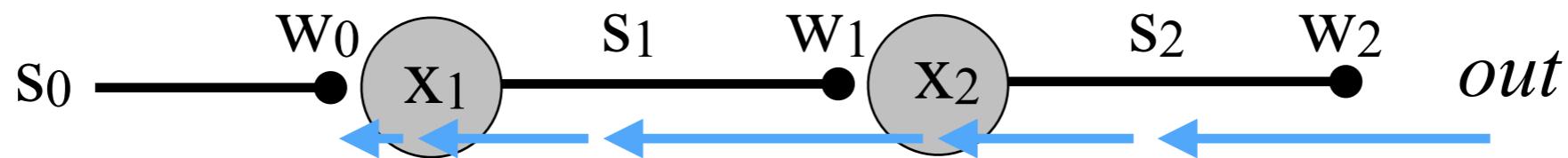
We found earlier that:

$$\frac{d s_2}{d x_2} = s_2(1 - s_2) \quad \dots \text{so} \dots \quad \frac{d s_1}{d x_1} = \boxed{s_1(1 - s_1)}$$

This involved many steps!

How does a change in weight w_0 impact output?

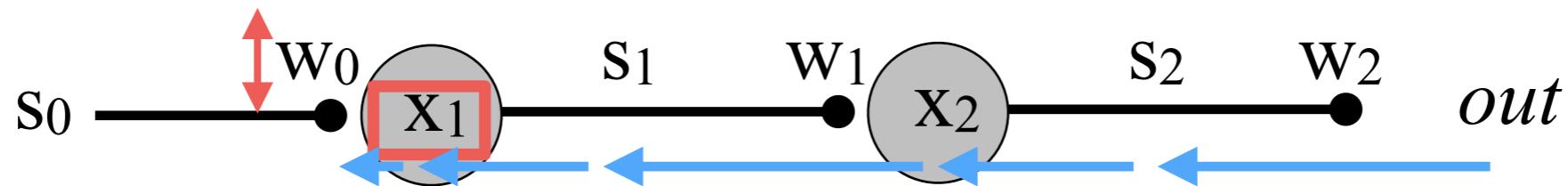
5th term:



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \cdot \frac{d s_2}{d x_2} \cdot \frac{d x_2}{d s_1} \cdot \frac{d s_1}{d x_1} \cdot \frac{d x_1}{d w_0}$$

How does a change in weight w_0 impact output?

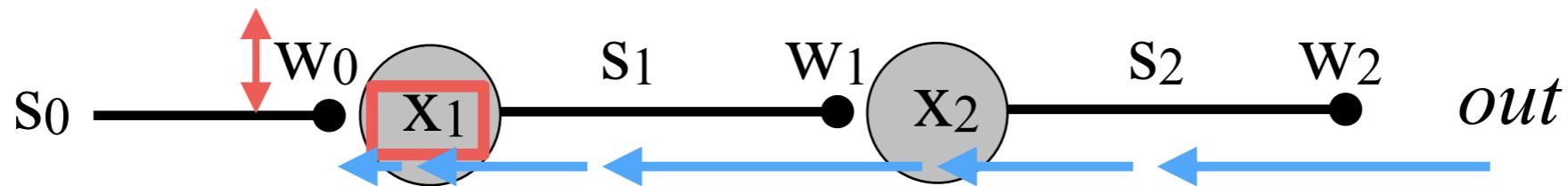
5th term:



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \boxed{\frac{d x_1}{d w_0}}$$

How does a change in weight w_0 impact output?

5th term:

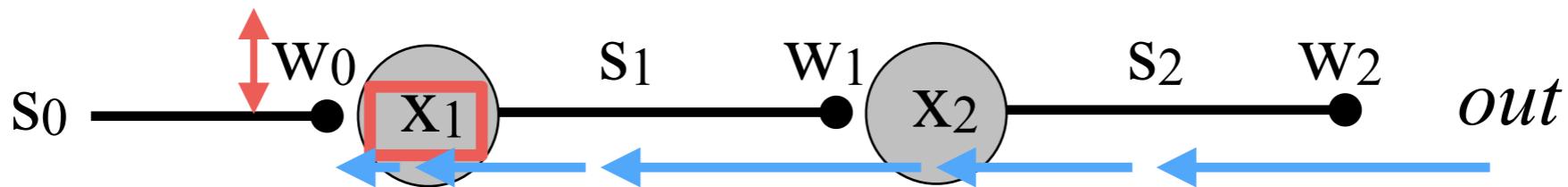


$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \boxed{\frac{d x_1}{d w_0}}$$

Remember: $x_1 = s_0 w_0$

How does a change in weight w_0 impact output?

5th term:



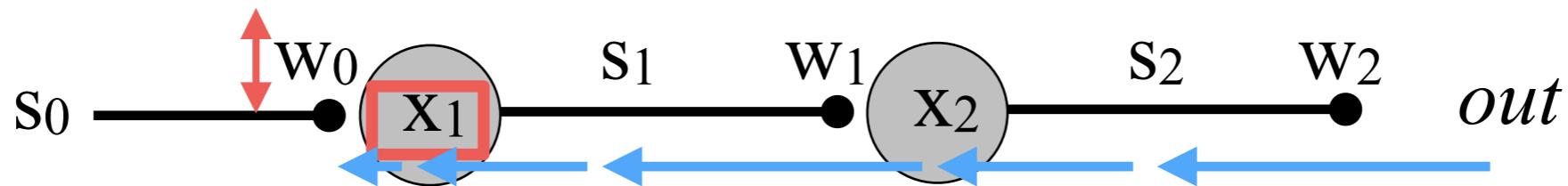
$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \frac{d x_1}{d w_0}$$

Remember: $x_1 = s_0 w_0$

$$\frac{d x_1}{d w_0} =$$

How does a change in weight w_0 impact output?

5th term:



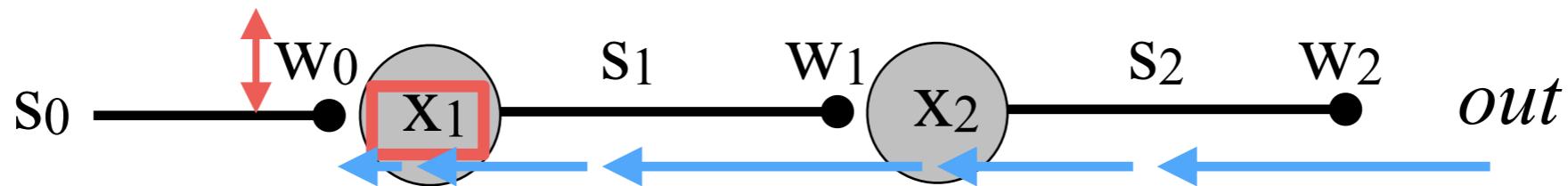
$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \boxed{\frac{d x_1}{d w_0}}$$

Remember: $x_1 = s_0 w_0$

$$\frac{d x_1}{d w_0} = \frac{d (s_0 w_0)}{d w_0}$$

How does a change in weight w_0 impact output?

5th term:



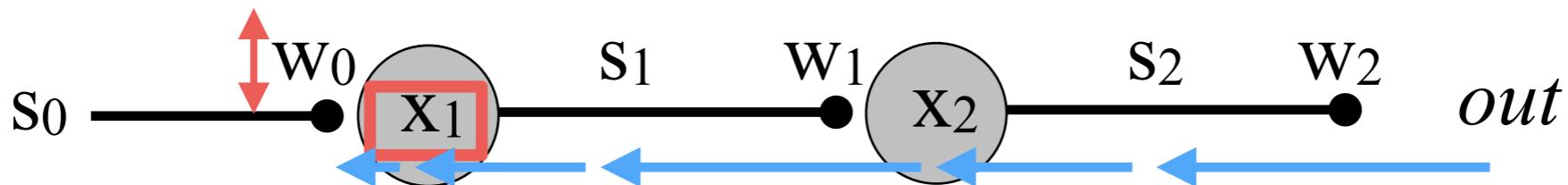
$$\frac{d \text{ out}}{d w_0} = \frac{d \text{ out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \boxed{\frac{d x_1}{d w_0}}$$

Remember: $x_1 = s_0 w_0$

$$\frac{d x_1}{d w_0} = \frac{d (s_0 w_0)}{d w_0} = s_0$$

How does a change in weight w_0 impact output?

5th term:



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \boxed{\frac{d x_1}{d w_0}}$$

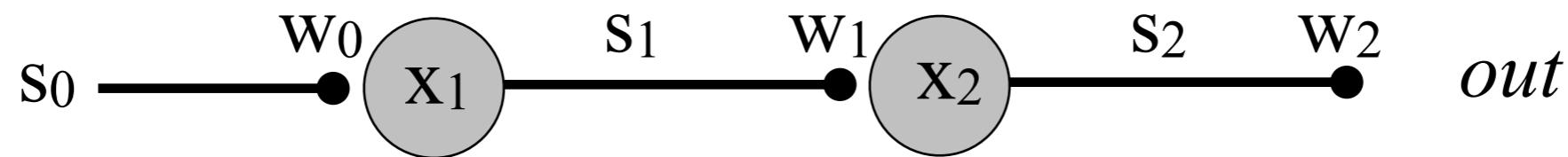
Remember: $x_1 = s_0 w_0$

$$\frac{d x_1}{d w_0} = \frac{d (s_0 w_0)}{d w_0} = \boxed{s_0}$$

How does a change in weight w_0 impact output?

We now have the pieces to answer:

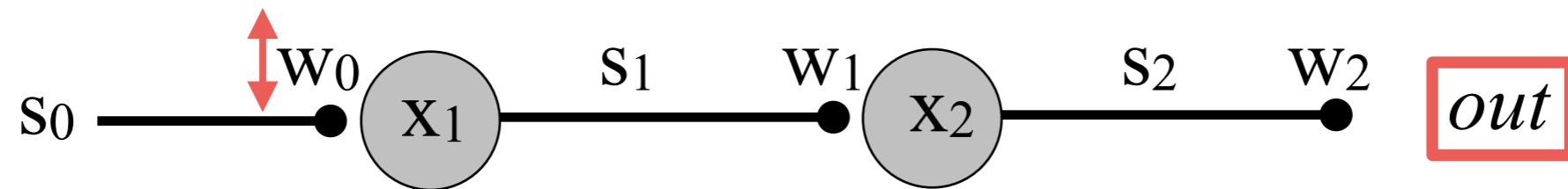
Q: How does a change in weight w_0 impact the output?



How does a change in weight w_0 impact output?

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Q: How does a change in weight w_0 impact the output?

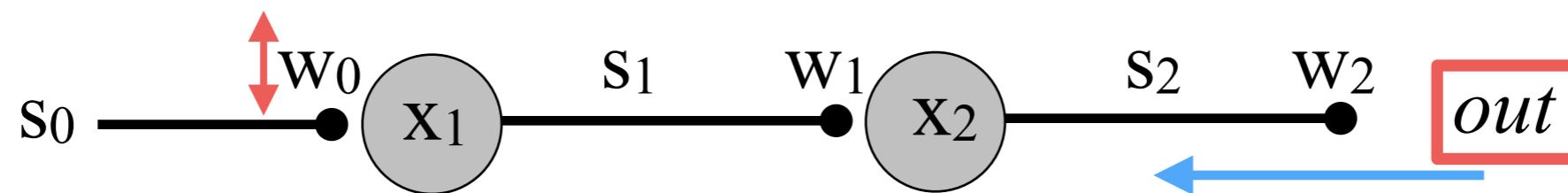


$$\frac{d \text{ } out}{d \text{ } w_0} =$$

How does a change in weight w_0 impact output?

We now have the pieces to answer:

Q: How does a change in weight w_0 impact the output?

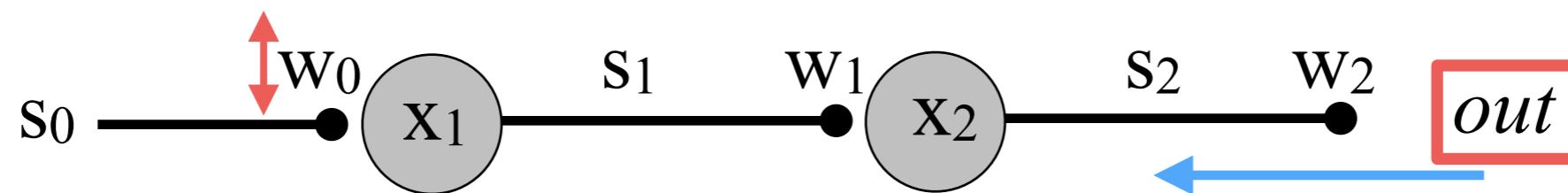


$$\frac{d \text{ out}}{d w_0} = \frac{d \text{ out}}{d s_2}$$

How does a change in weight w_0 impact output?

We now have the pieces to answer:

Q: How does a change in weight w_0 impact the output?



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2}$$



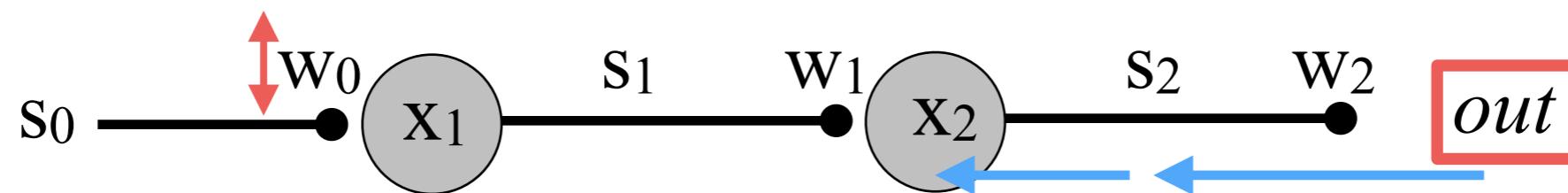
$$= w_2$$

Slide 16

How does a change in weight w_0 impact output?

We now have the pieces to answer:

Q: How does a change in weight w_0 impact the output?



$$\frac{d out}{d w_0} = \frac{d out}{d s_2} \frac{d s_2}{d x_2}$$



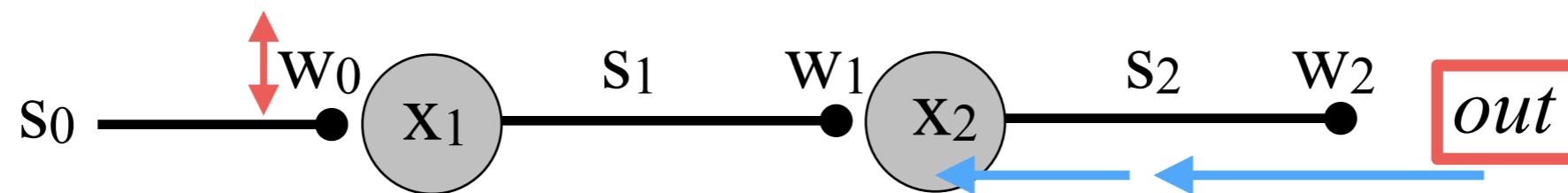
$$= w_2$$

Slide 16

How does a change in weight w_0 impact output?

We now have the pieces to answer:

Q: How does a change in weight w_0 impact the output?



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2}$$



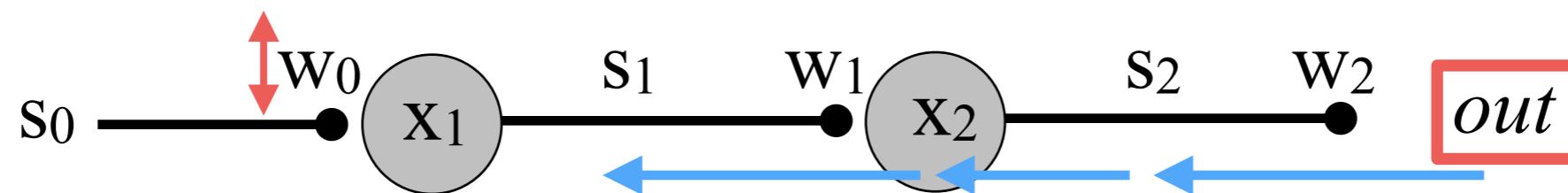
$$= w_2 s_2 (1 - s_2)$$

Slide 16 Slide 21

How does a change in weight w_0 impact output?

We now have the pieces to answer:

Q: How does a change in weight w_0 impact the output?



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1}$$



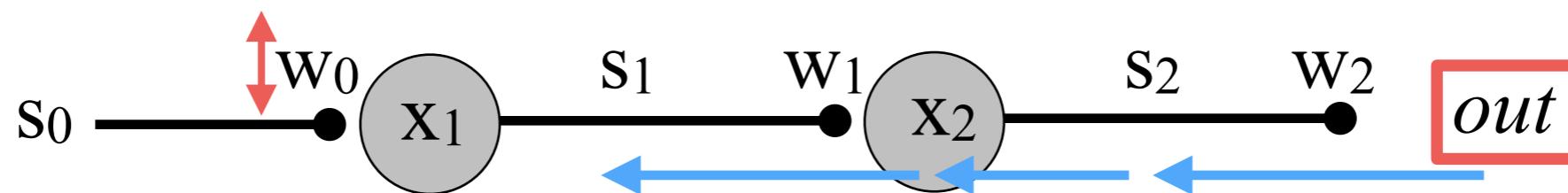
$$= w_2 s_2 (1 - s_2)$$

Slide 16 Slide 21

How does a change in weight w_0 impact output?

We now have the pieces to answer:

Q: How does a change in weight w_0 impact the output?



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1}$$

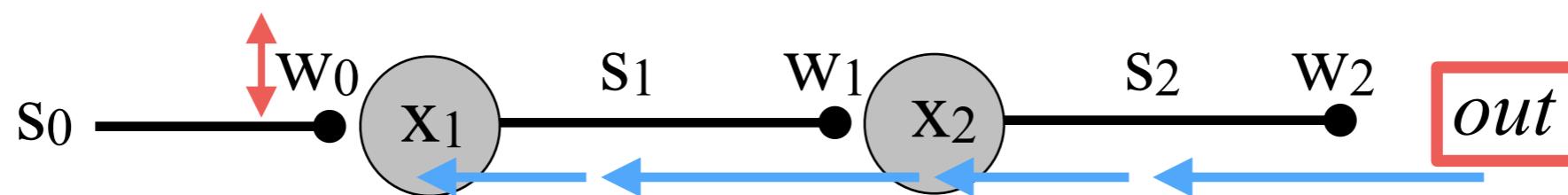
$$= w_2 \quad s_2(1 - s_2) \quad w_1$$

Slide 16 Slide 21 Slide 26

How does a change in weight w_0 impact output?

We now have the pieces to answer:

Q: How does a change in weight w_0 impact the output?



$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1}$$

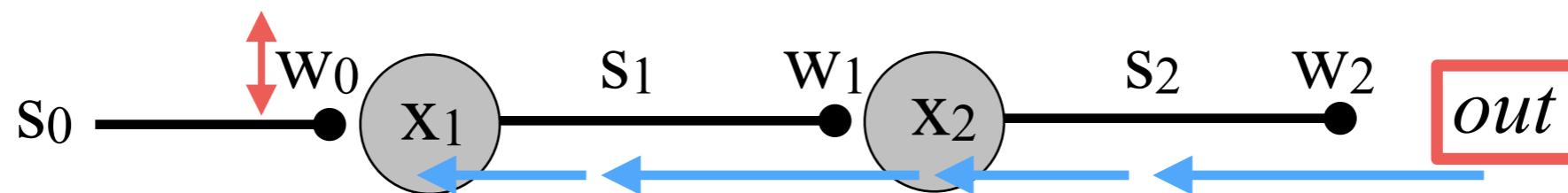
$$= w_2 s_2 (1 - s_2) w_1$$

Slide 16 Slide 21 Slide 26

How does a change in weight w_0 impact output?

We now have the pieces to answer:

Q: How does a change in weight w_0 impact the output?



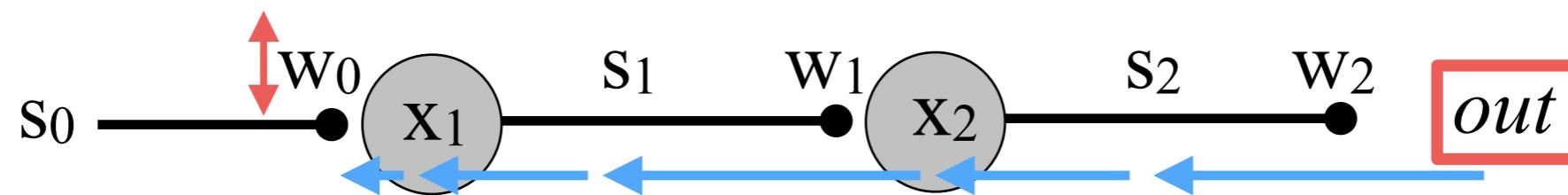
$$\begin{aligned}\frac{d \text{out}}{d w_0} &= \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \\&= w_2 s_2 (1 - s_2) w_1 s_1 (1 - s_1)\end{aligned}$$

Slide 16 Slide 21 Slide 26 Slide 27

How does a change in weight w_0 impact output?

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$$\frac{d \text{out}}{d w_0} = \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \frac{d x_1}{d w_0}$$

$$= w_2 \quad s_2(1 - s_2) \quad w_1 \quad s_1(1 - s_1)$$

Slide 16

Slide 21

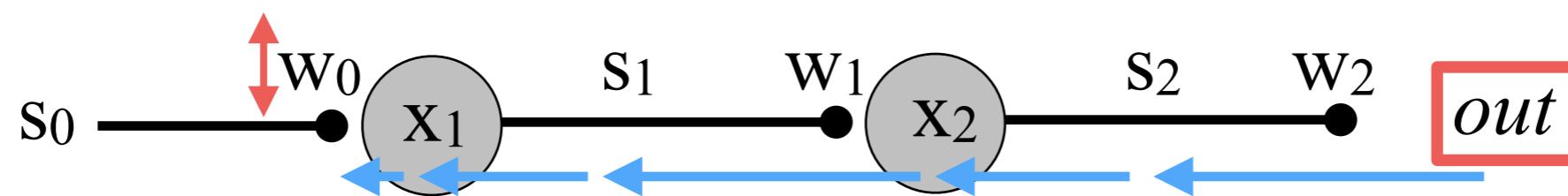
Slide 26

Slide 27

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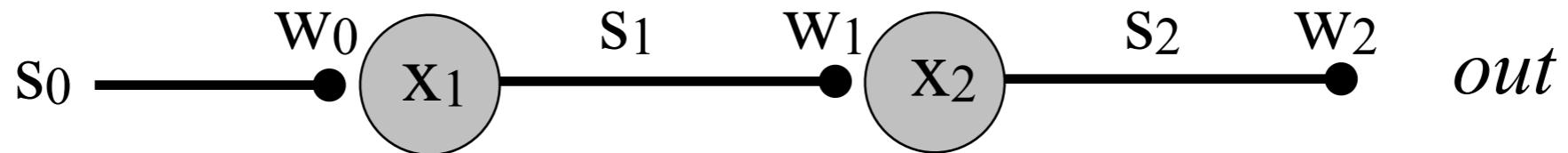
$$\begin{aligned}\frac{d \text{out}}{d w_0} &= \frac{d \text{out}}{d s_2} \frac{d s_2}{d x_2} \frac{d x_2}{d s_1} \frac{d s_1}{d x_1} \frac{d x_1}{d w_0} \\ &= w_2 \quad s_2(1 - s_2) \quad w_1 \quad s_1(1 - s_1) \quad s_0\end{aligned}$$

Slide 16 Slide 21 Slide 26 Slide 27 Slide 28

How does a change in weight impact output?

To summarize:

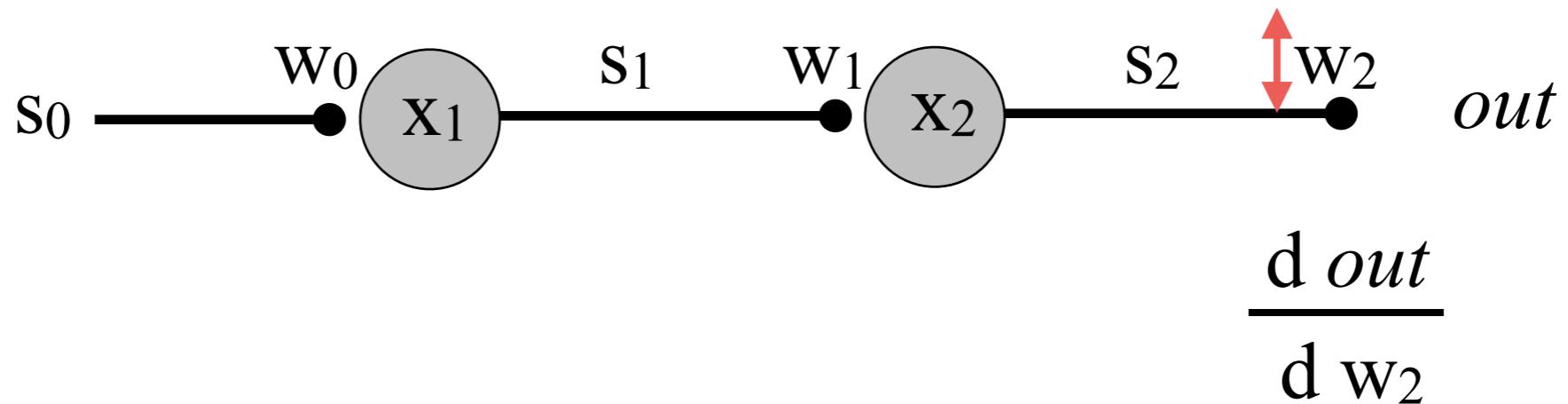
- We've found how changes in model weights impact output.



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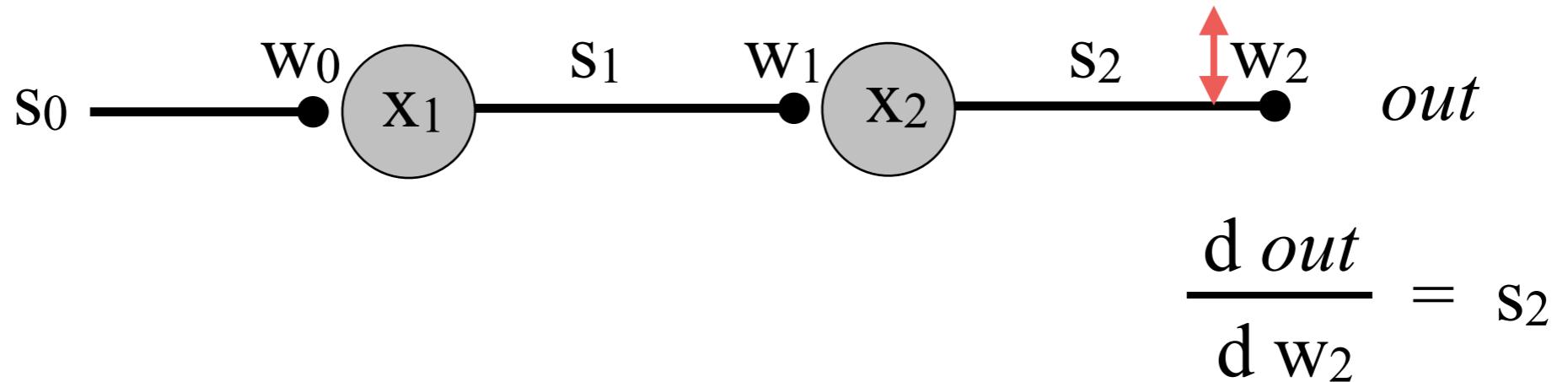
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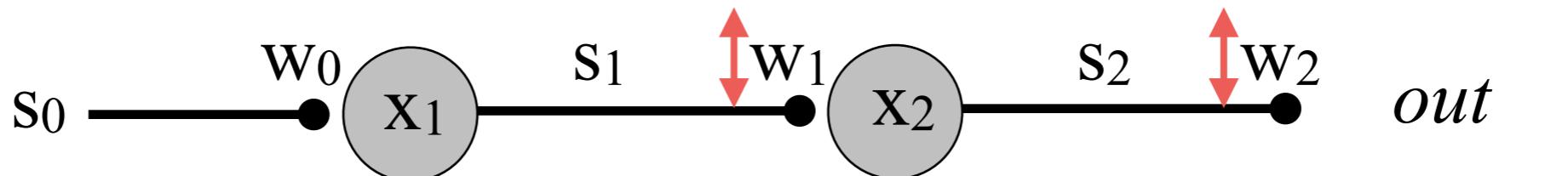
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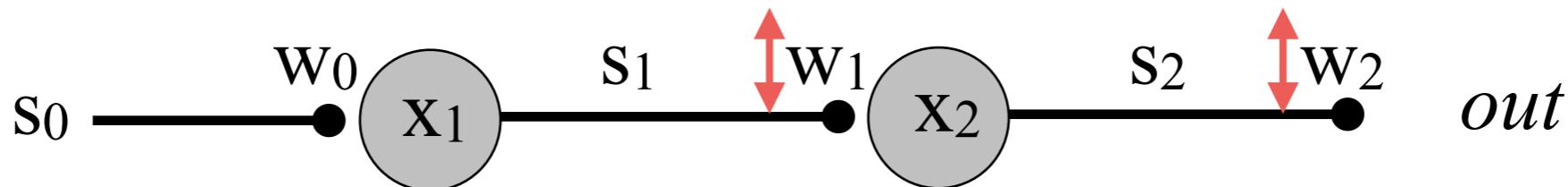
$$\frac{d \text{ out}}{d w_2} = s_2$$

$$\frac{d \text{ out}}{d w_1}$$

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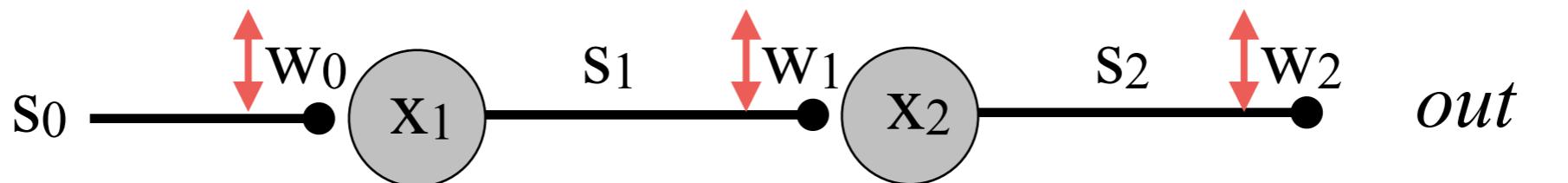
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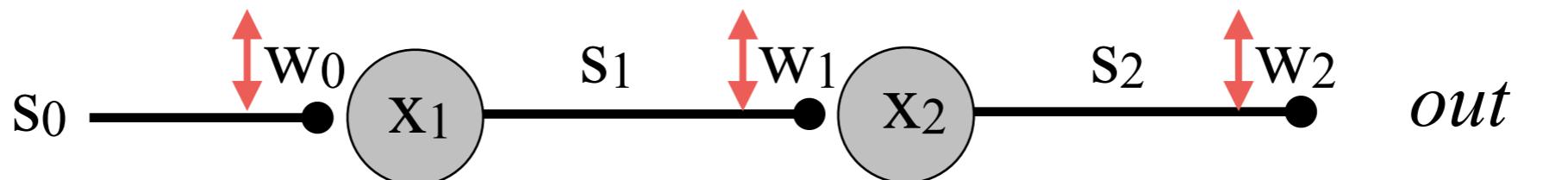
$$\frac{d \text{ out}}{d w_1} = w_2 s_2 (1 - s_2) s_1$$

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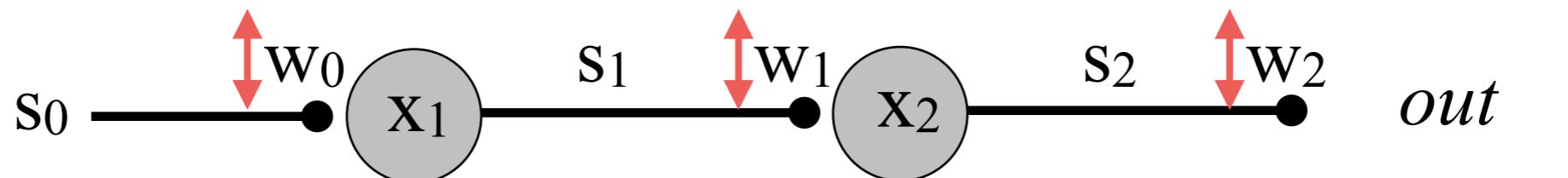
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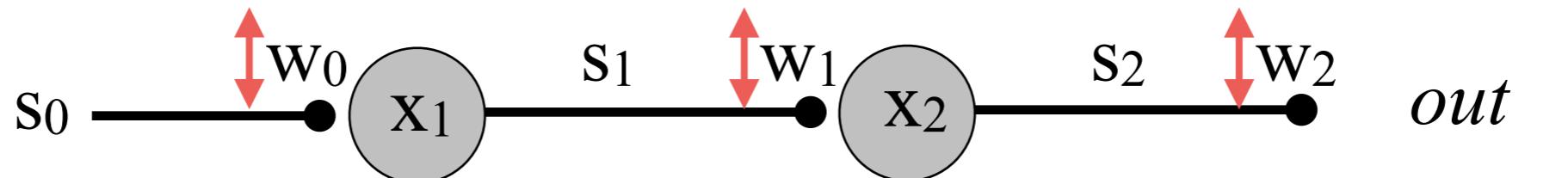
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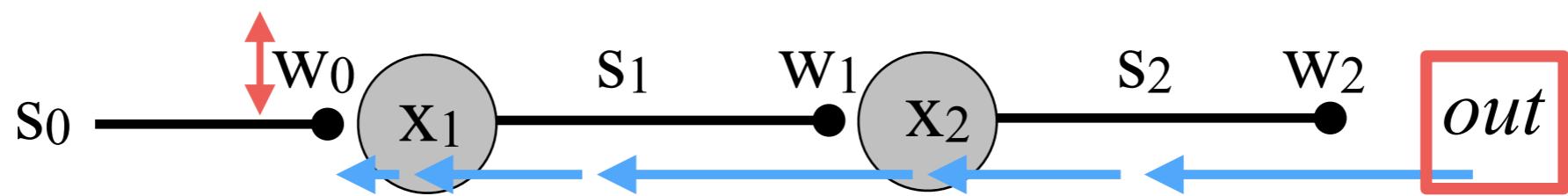
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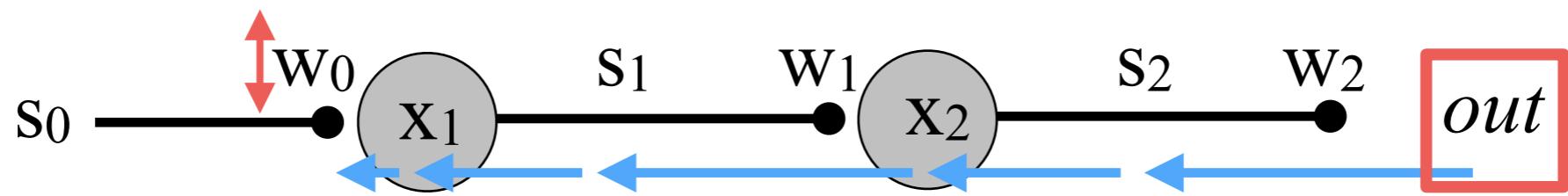
So, how does a change in weight impact output? **backpropagation!**

How does a change in weight impact output?



Backpropagation:

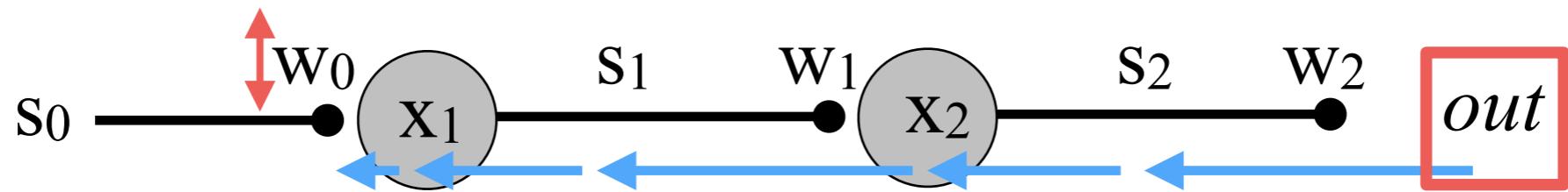
How does a change in weight impact output?



Backpropagation:

Work “**backwards**” from output to weight, computing derivatives along the way

How does a change in weight impact output?



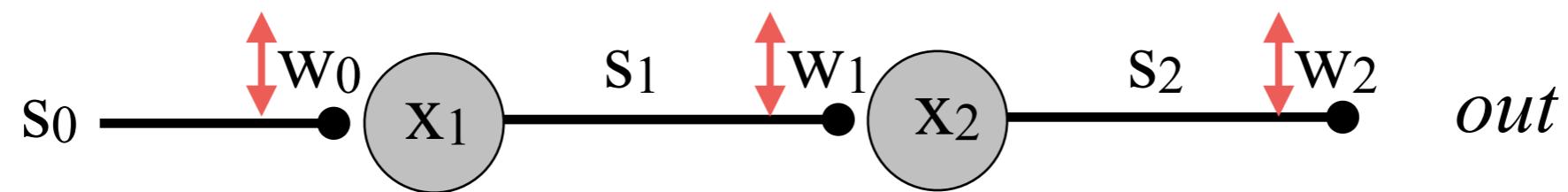
Backpropagation:

Work “**backwards**” from output to weight, computing derivatives along the way

Q: How do these derivatives help us update weights and obtain desired output?

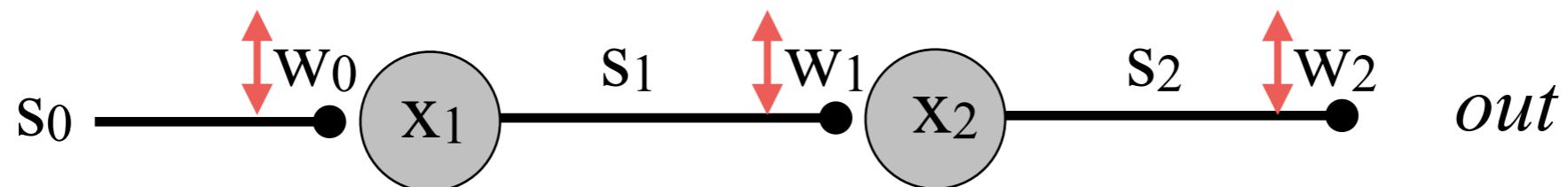
Define our goal

We want:



Define our goal

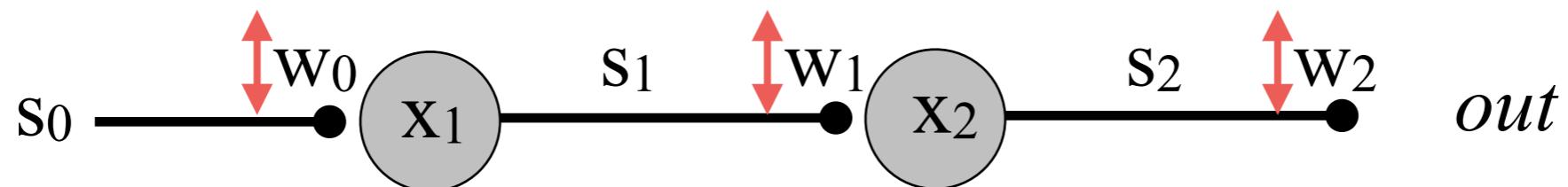
We want:



$out = \text{target}$

Define our goal

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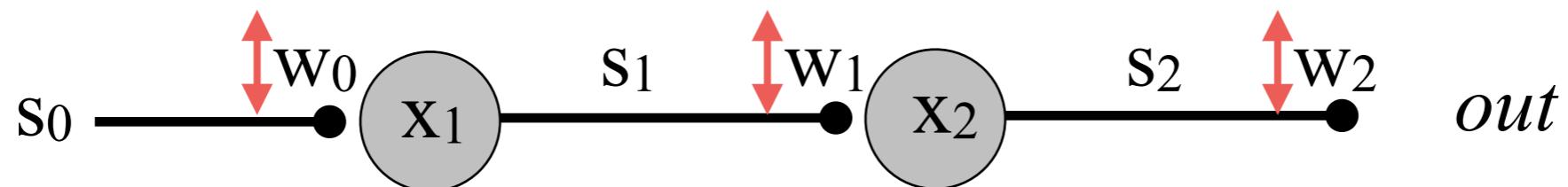


out = target

rearrange

Define our goal

We want:

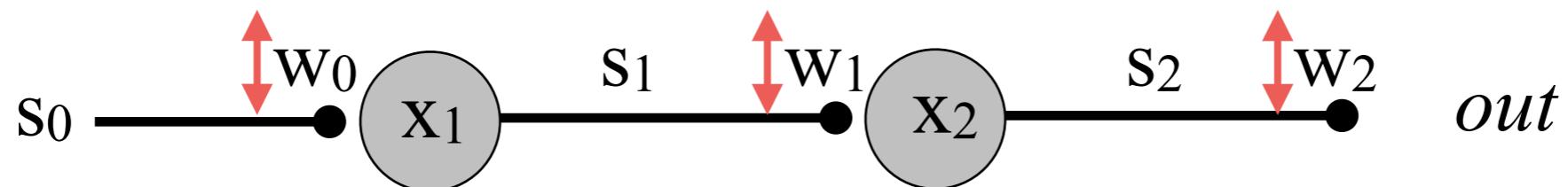


$out = \text{target}$

rearrange $out - \text{target} = 0$ **GOAL**

Define our goal

We want:



out = target

rearrange *out* - target = 0 **GOAL**

Let's use this to define a **cost function** ...

Create a cost function

Define the cost function:

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$$C = \frac{1}{2} (\text{target} - \text{out})^2$$

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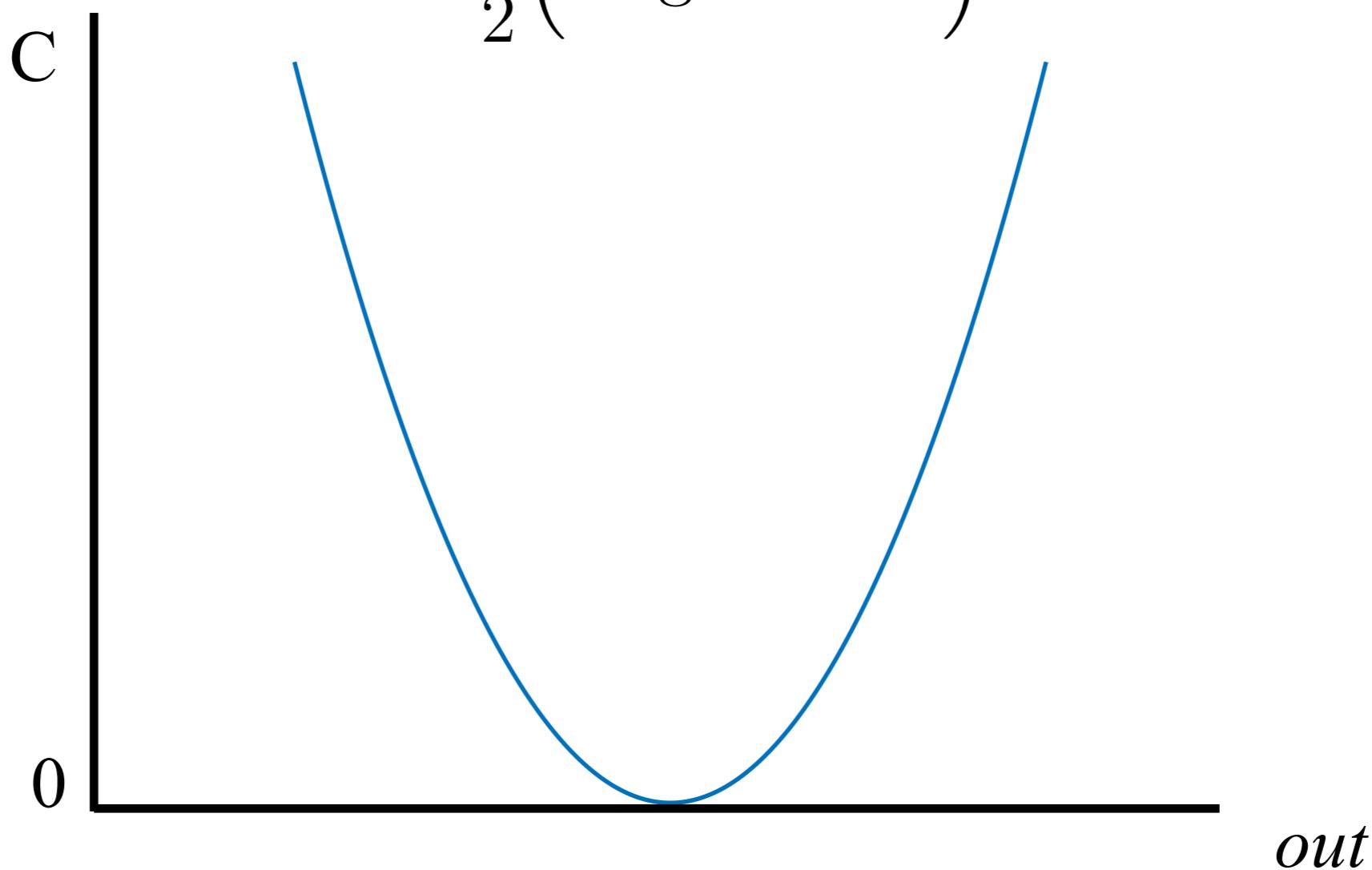


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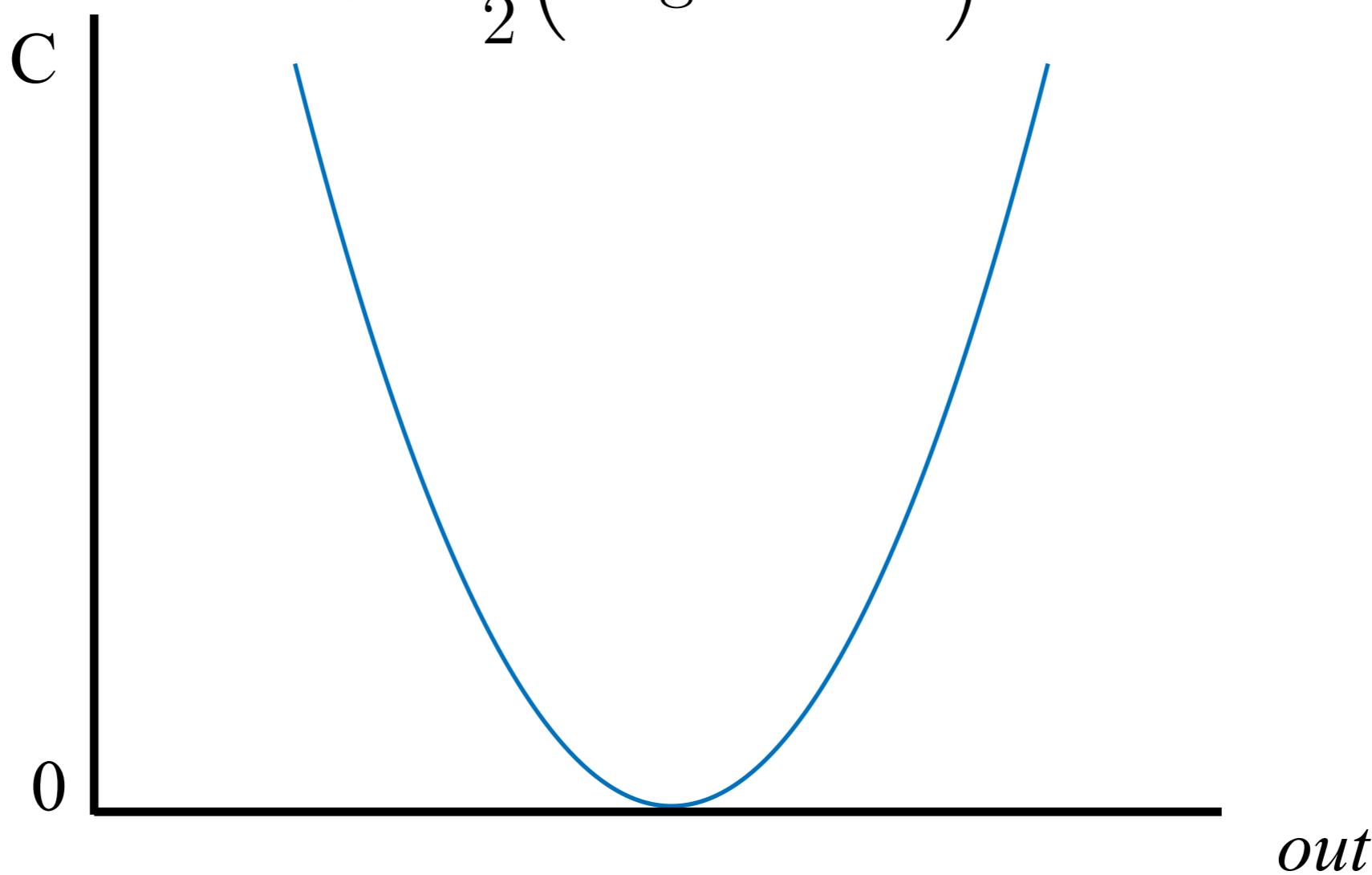


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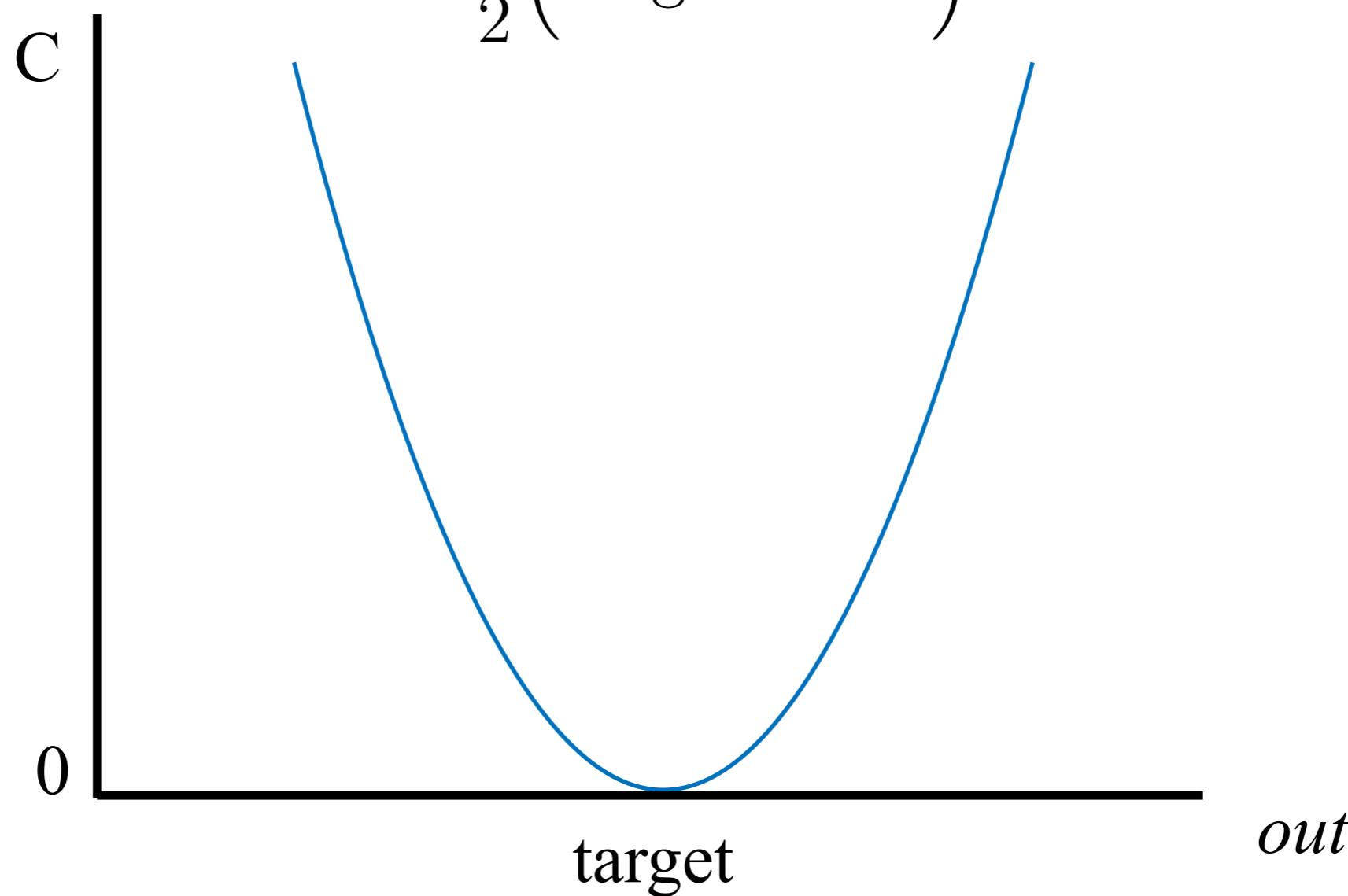
Q: Where is the cost zero?

Create a cost function

Define the cost function:

$$C = \frac{1}{2}(\text{target} - \text{out})^2$$

Plot it:



Q: Where is the cost zero?

A: When $\text{out} = \text{target}$.

Create a cost function

Q: Why this cost function?

$$C = \frac{1}{2} (\text{target} - \text{out})^2$$

Create a cost function

Q: Why this cost function?

$$C = \frac{1}{2} (\text{target} - \text{out})^2$$

- Minimum (the “lowest cost”) when $\text{out} = \text{target}$.

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- It’s convenient (a quadratic).

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- It steadily increases as out deviates from target.

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- Minimum (the “lowest cost”) when $\text{out} = \text{target}$.
- It’s convenient (a quadratic).
- It steadily increases as out deviates from target.
- It’s “easy” to compute derivatives.

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Q: How does the cost function change due to changes in *out*?

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$$\frac{dC}{dout} = \frac{d}{out} \left[\frac{1}{2} \left(\text{target} - out \right)^2 \right]$$

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Chain rule ...

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Chain rule ...

$$\frac{dC}{dout} = 2 \frac{1}{2} \left(\text{target} - out \right)^1 \left(\frac{d(-out)}{dout} \right)$$

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1

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$$\frac{dC}{dout} = - \left(\text{target} - out \right)^1$$

$$\boxed{\frac{dC}{dout} = out - \text{target}}$$

Create a cost function

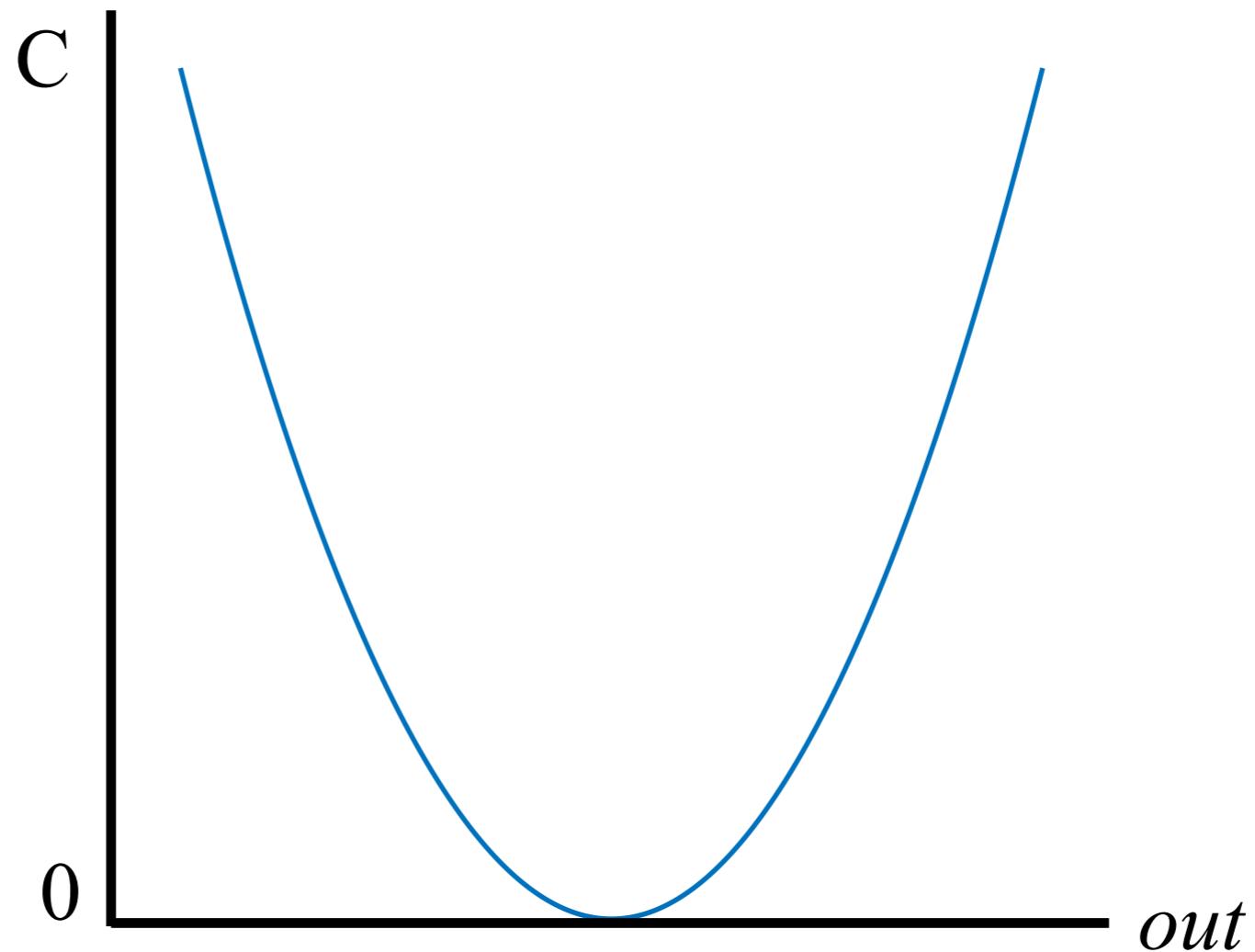
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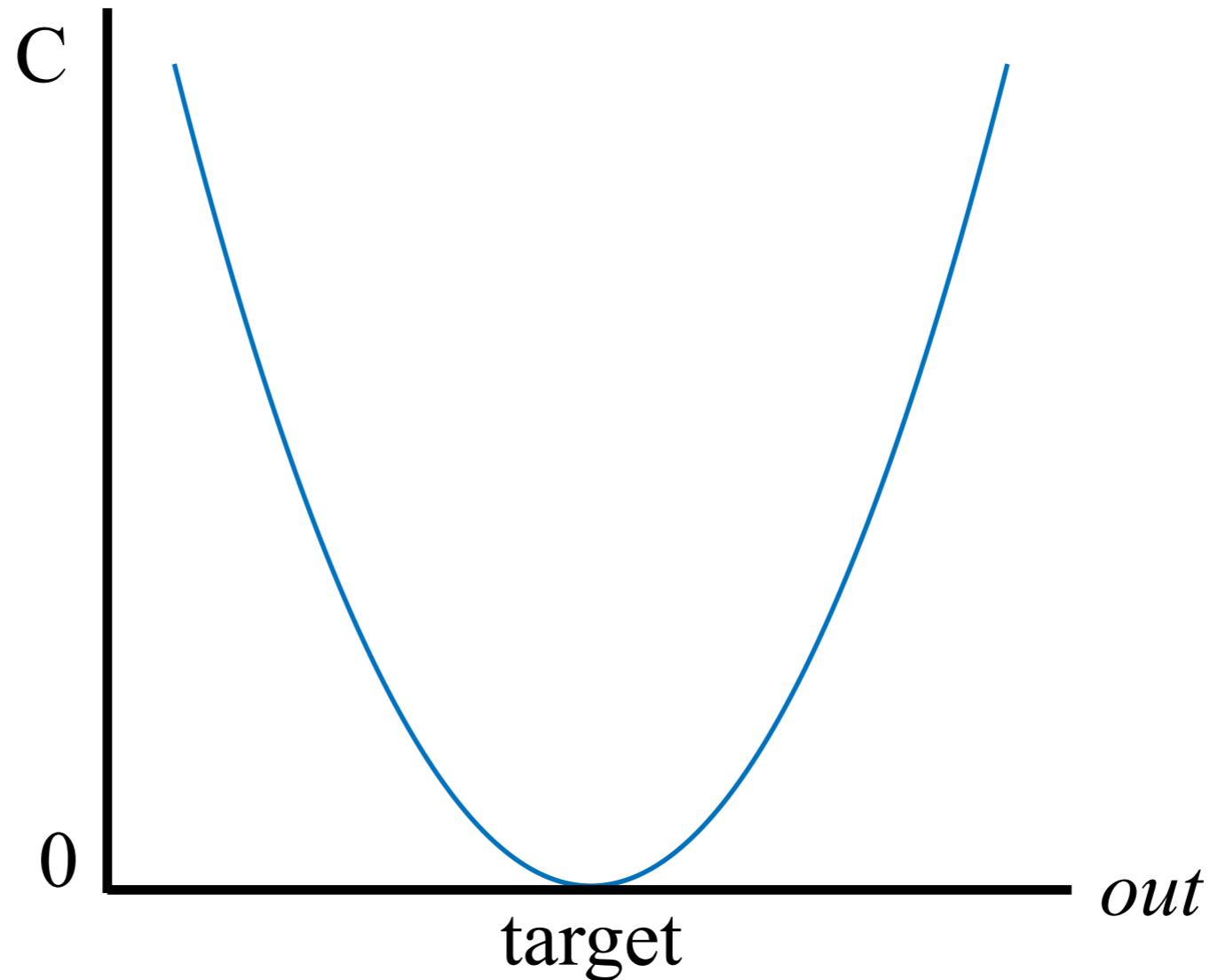
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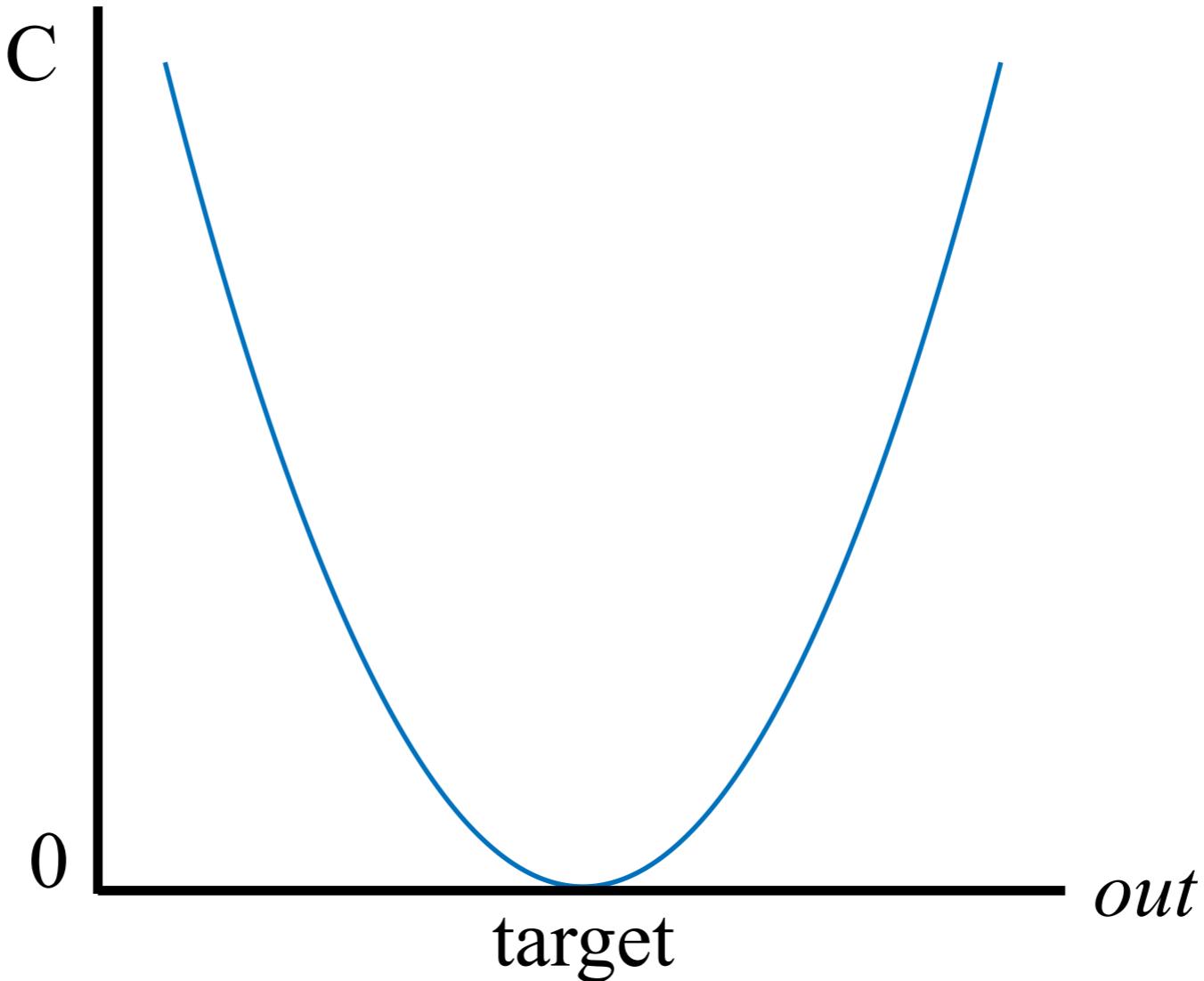


Create a cost function

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When $out < \text{target}$



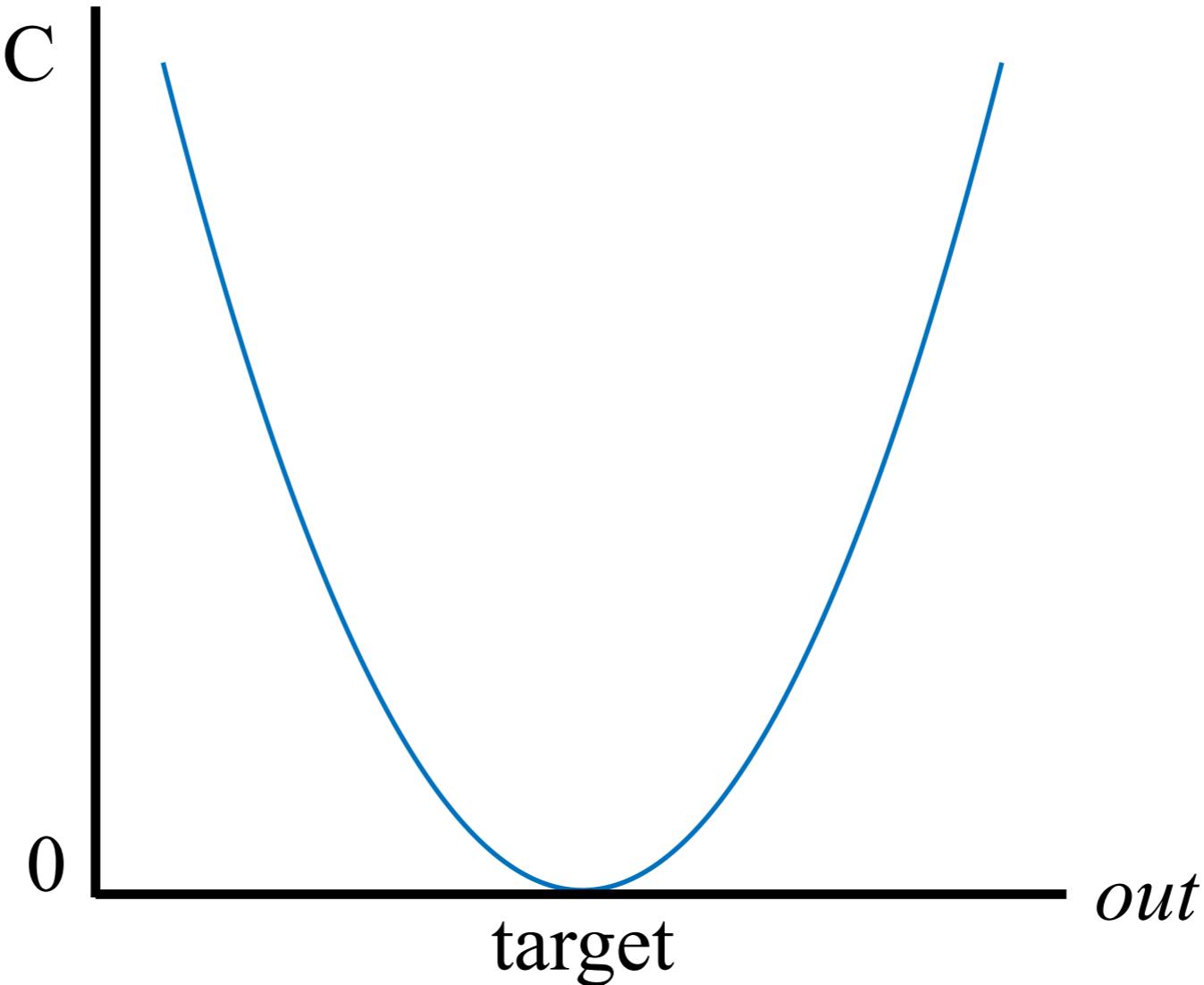
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$$\frac{dC}{dout}$$



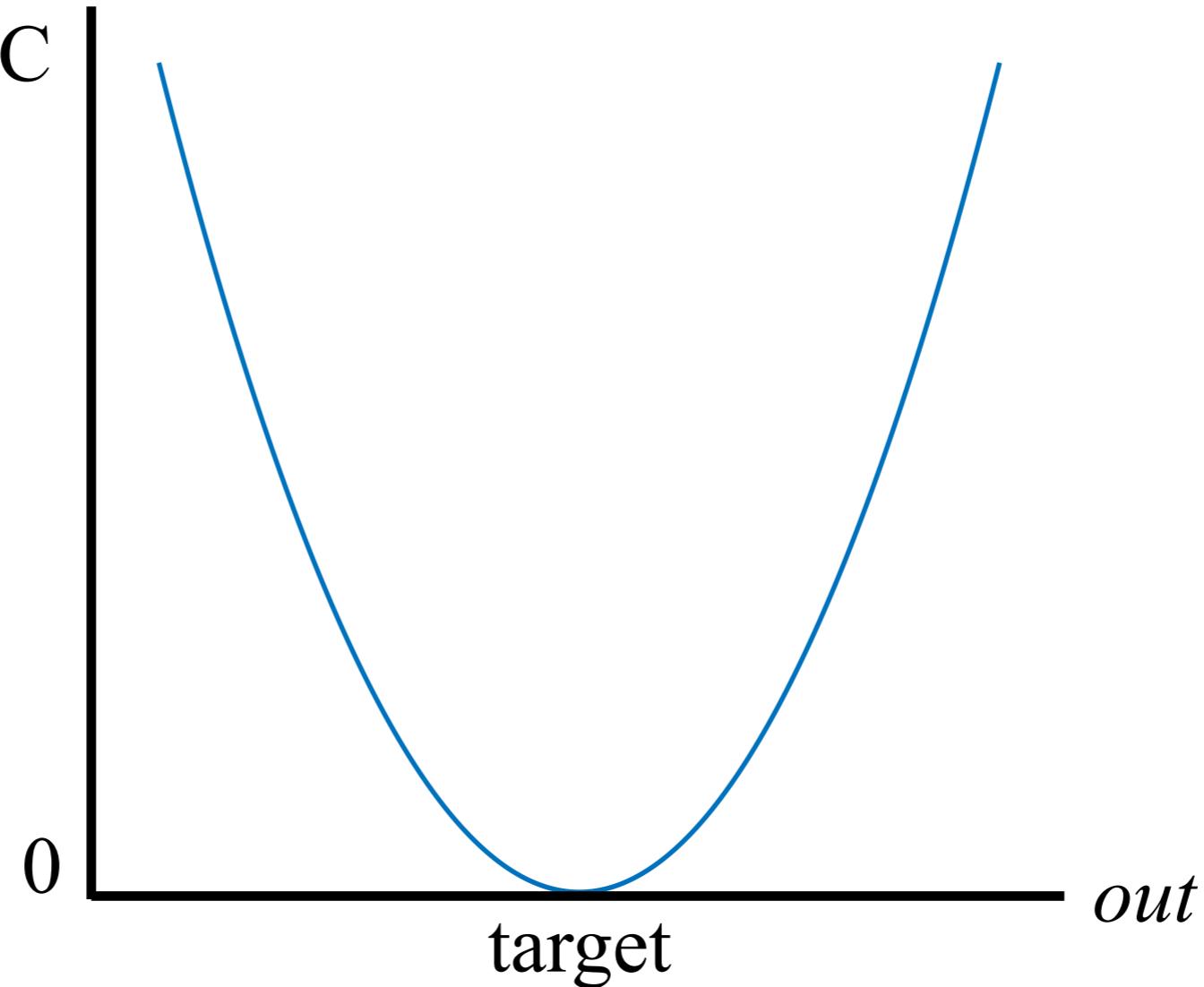
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$$\frac{dC}{dout} < 0$$



Create a cost function

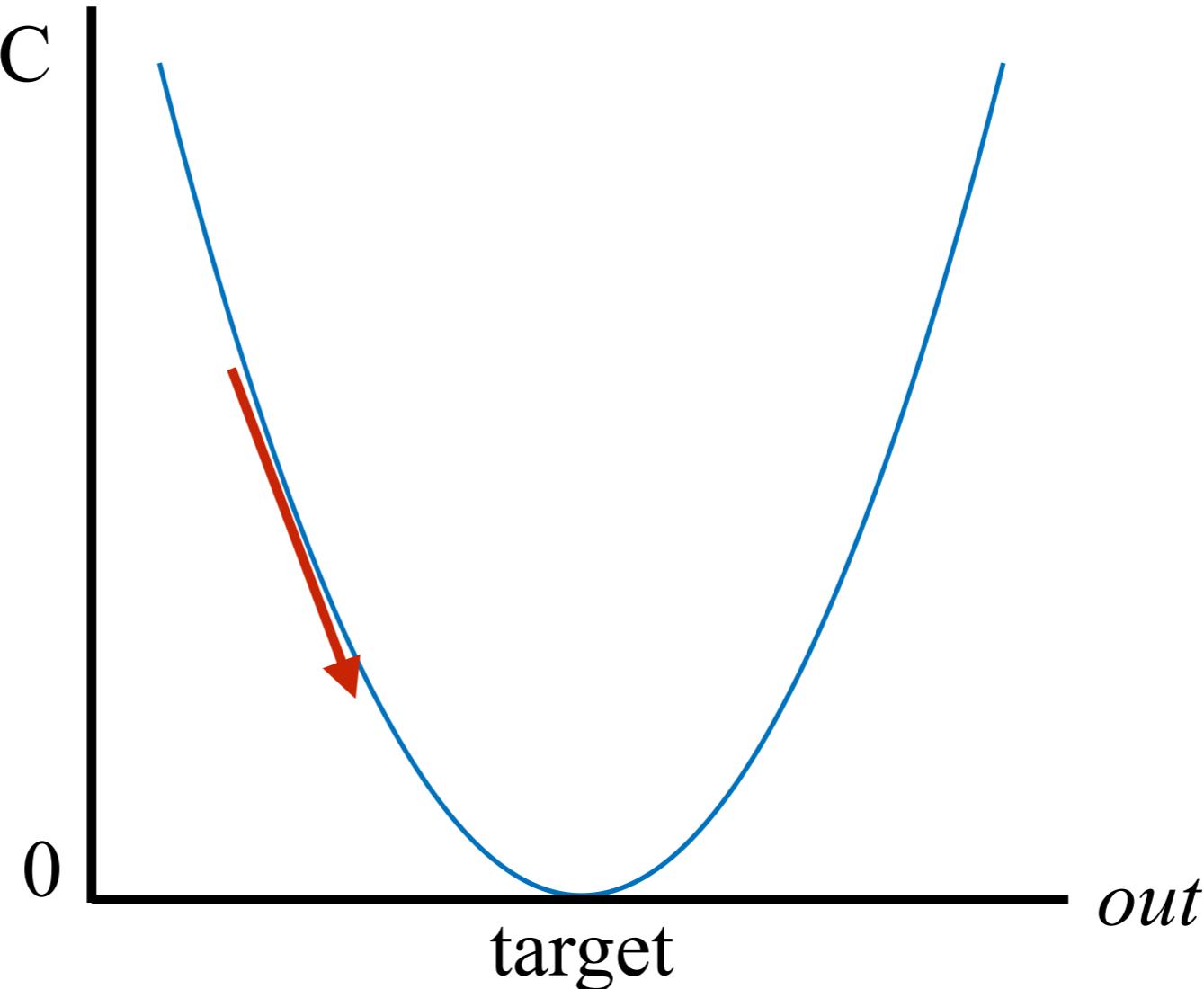
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slope < 0



Create a cost function

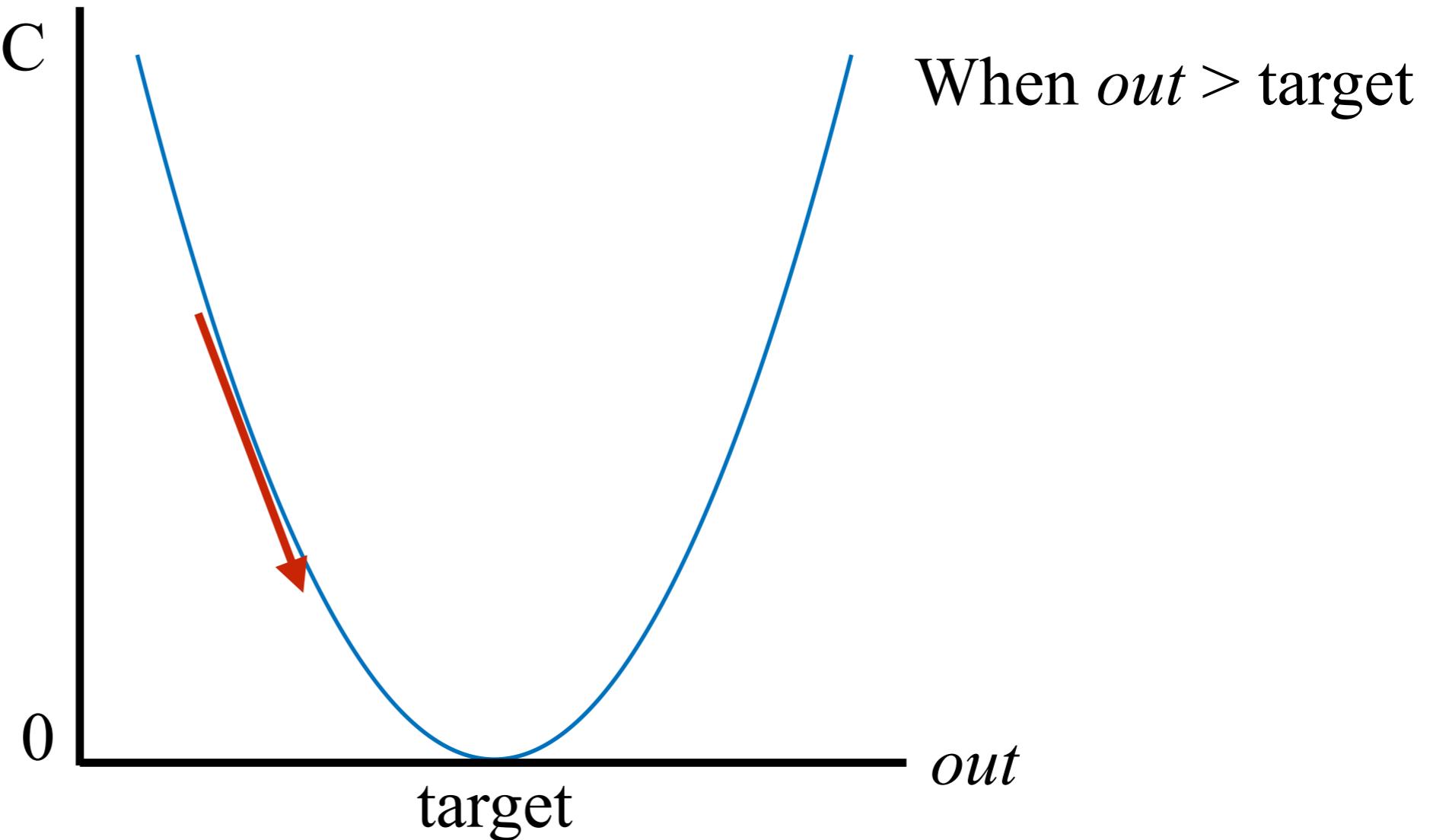
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When $out > \text{target}$

Create a cost function

Q: Does this derivative make sense?

$$\frac{dC}{dout} = out - \text{target}$$

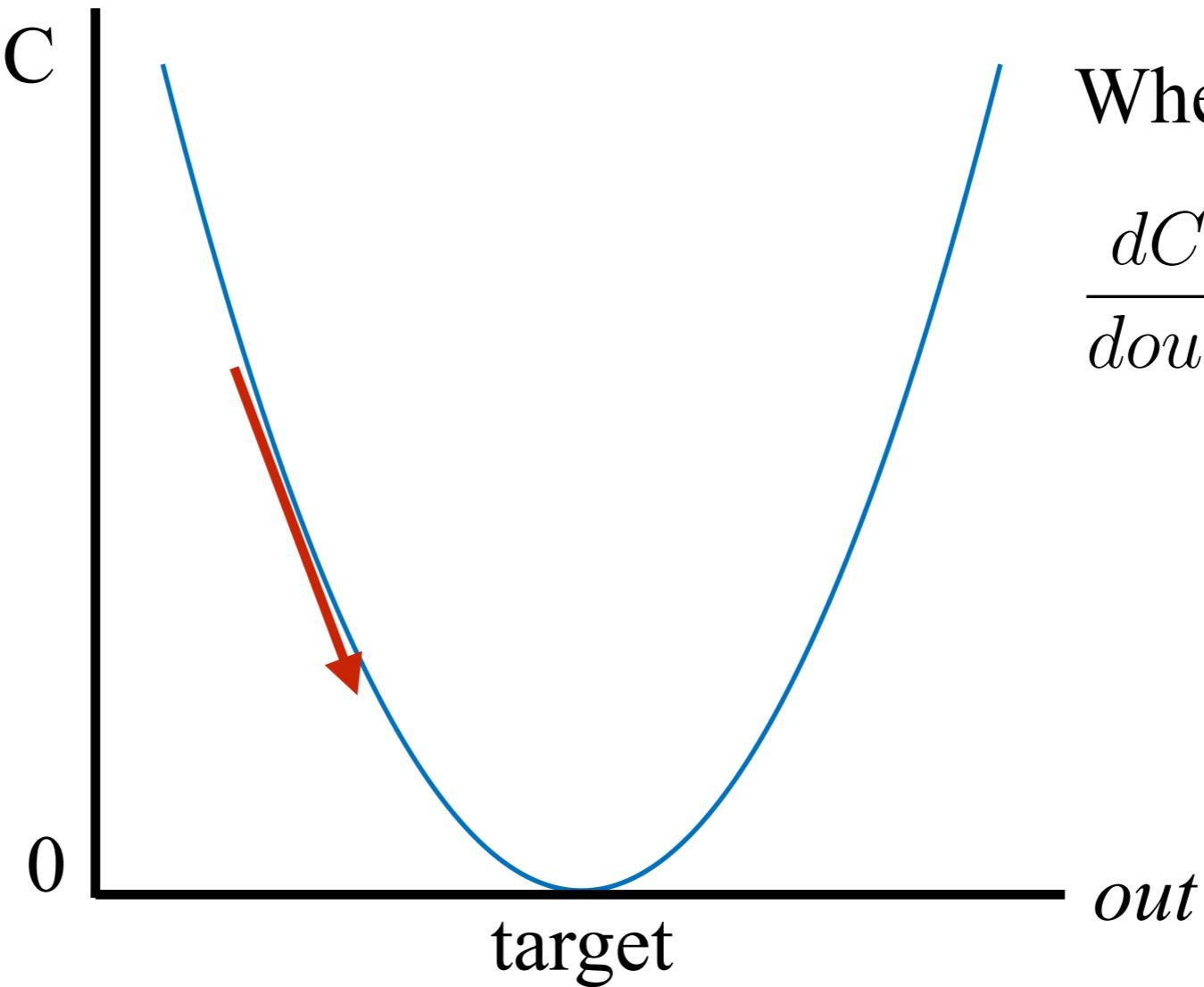
When $out < \text{target}$

$$\frac{dC}{dout} < 0$$

slope < 0

When $out > \text{target}$

$$\frac{dC}{dout}$$



Create a cost function

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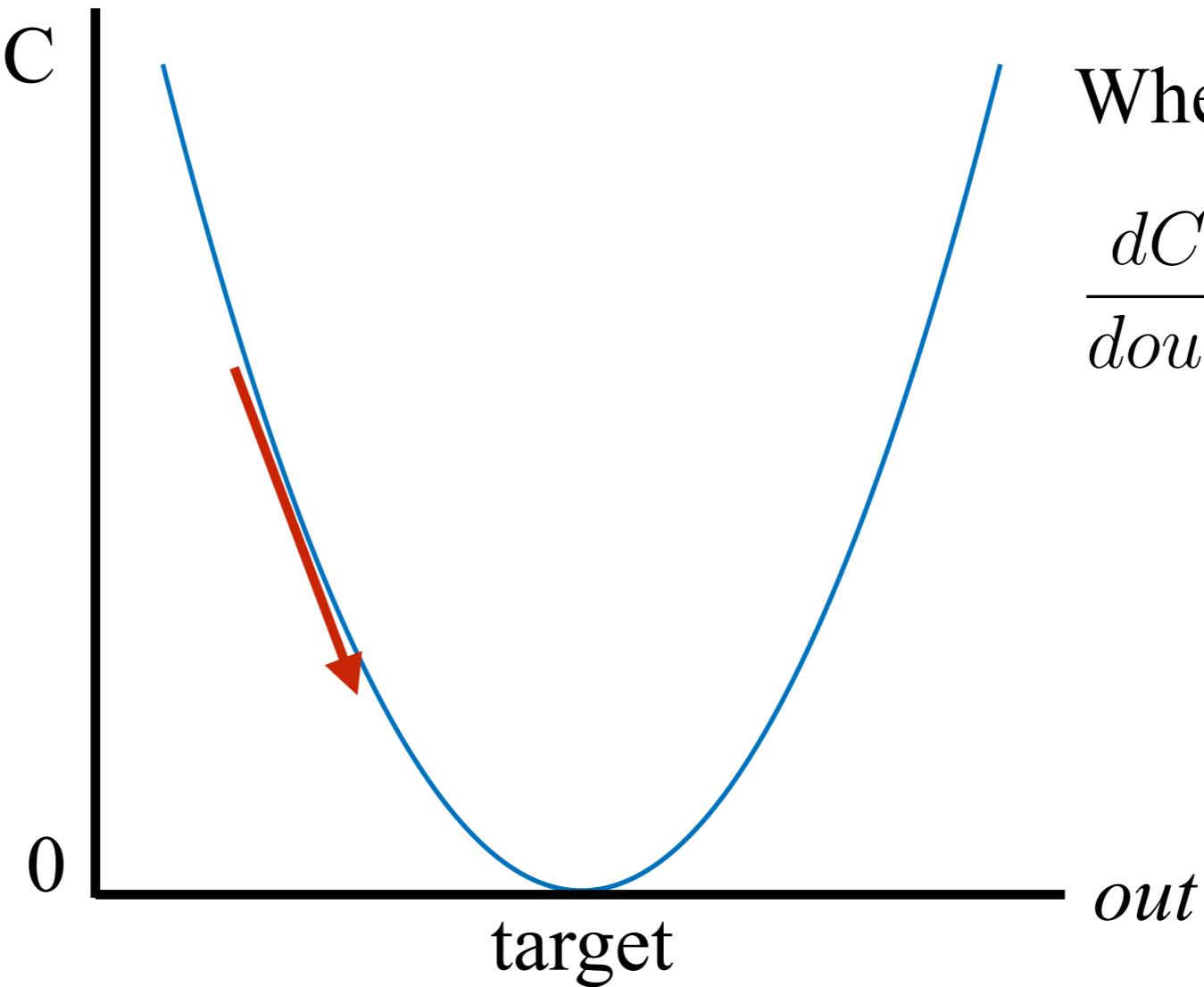
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slope < 0

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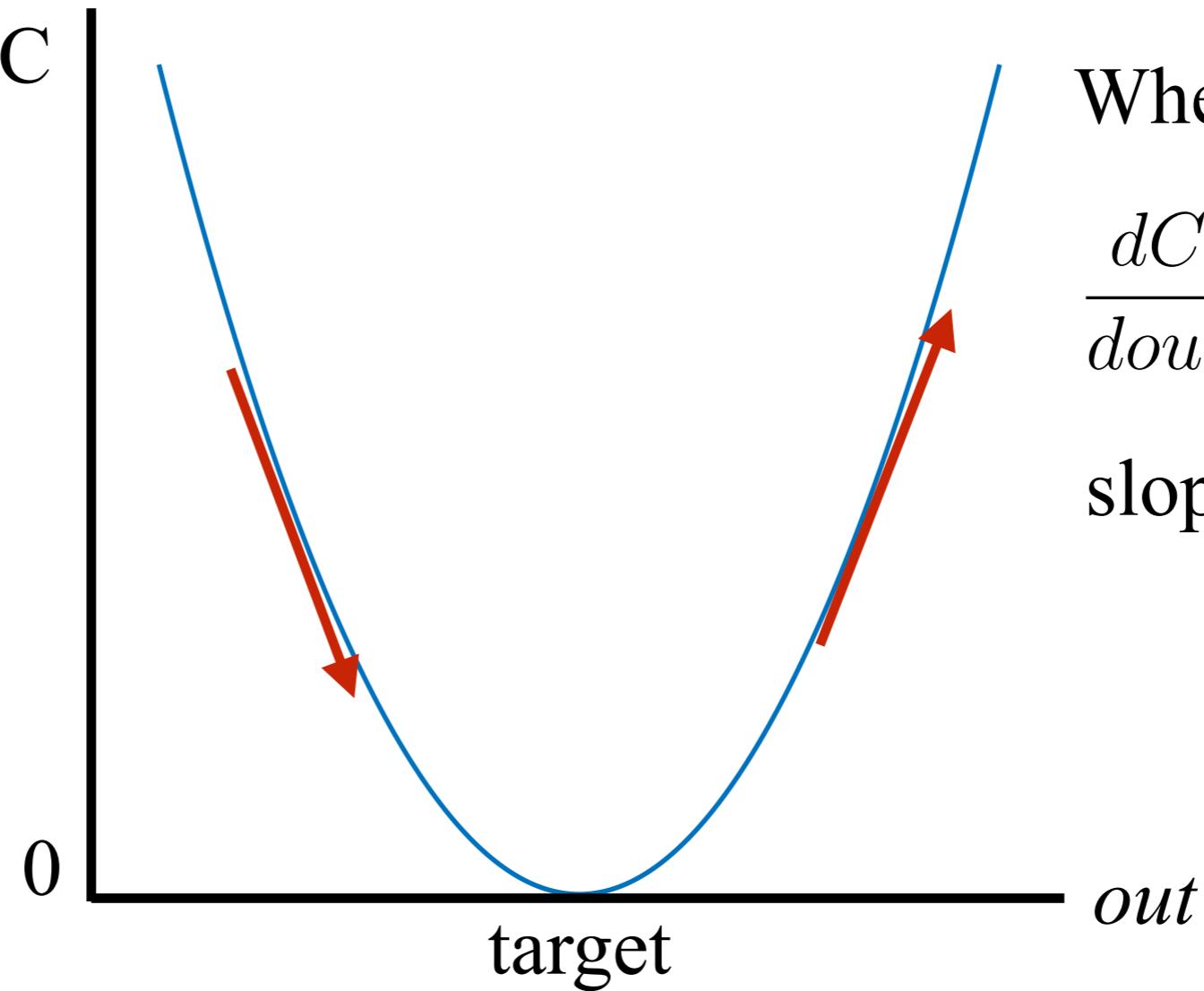
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When $out > \text{target}$

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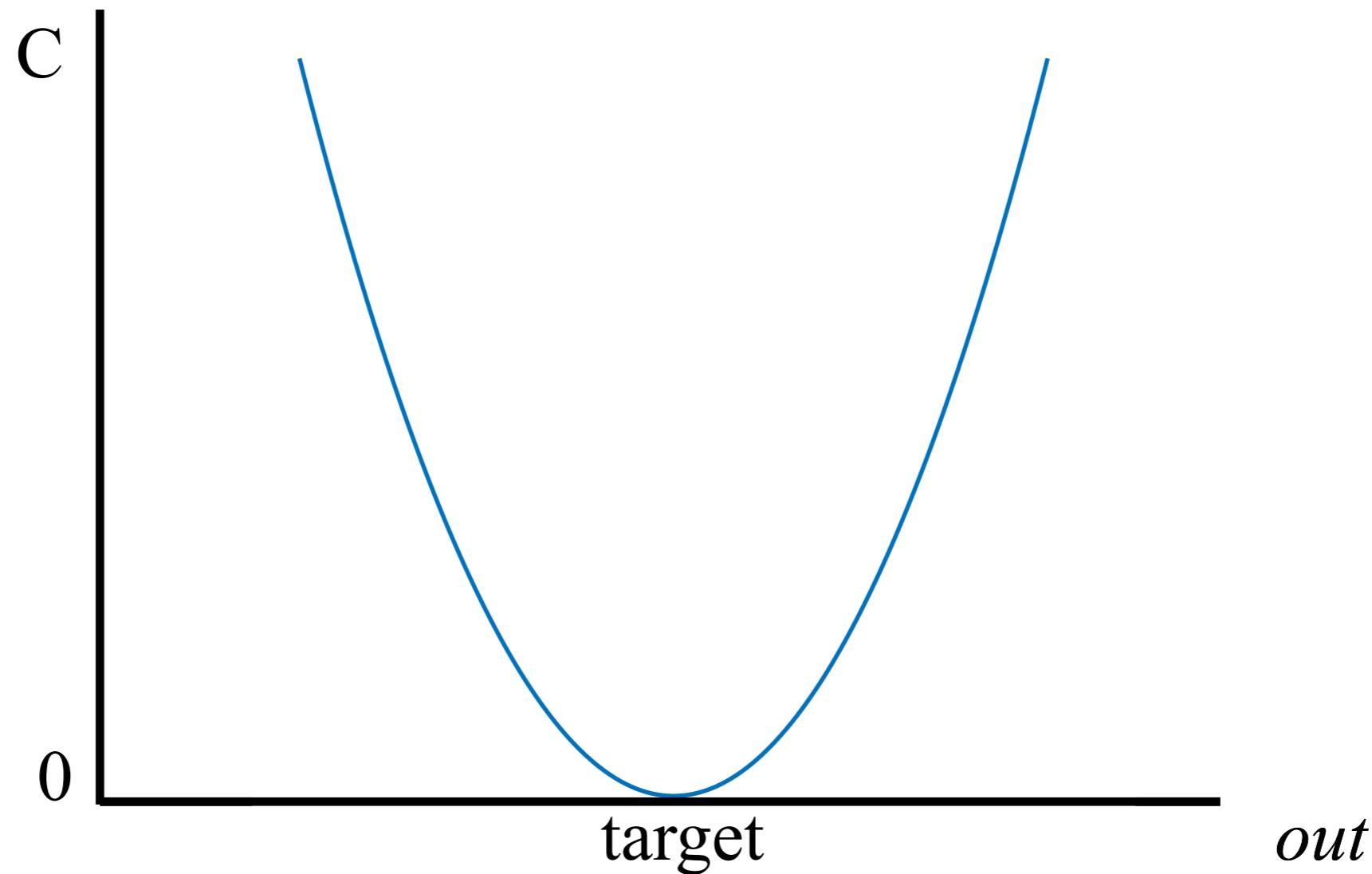
Create a cost function

Now, our goal: Choose weights to minimize the cost function

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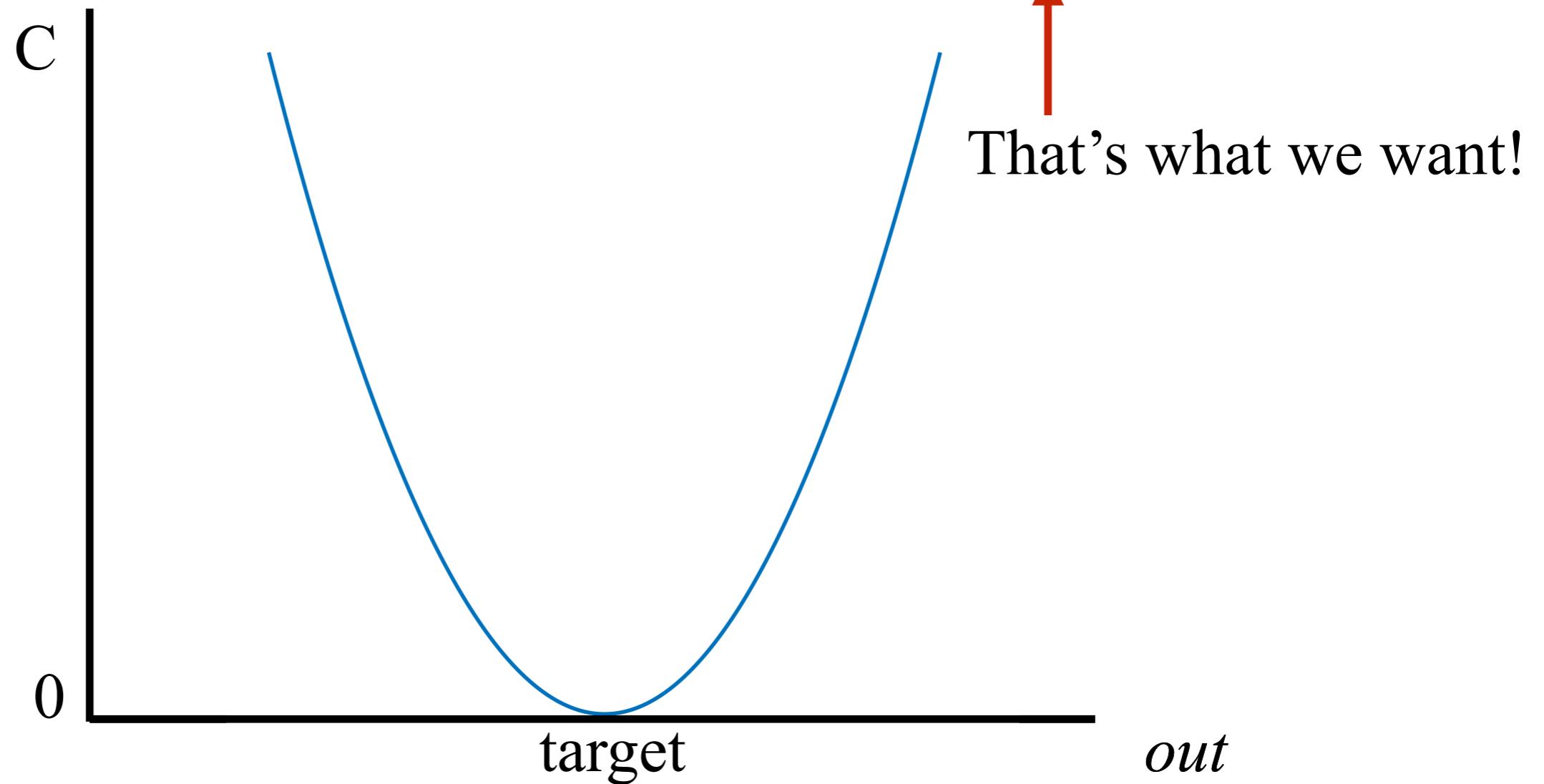
Remember, cost minimized when $out = \text{target}$



Create a cost function

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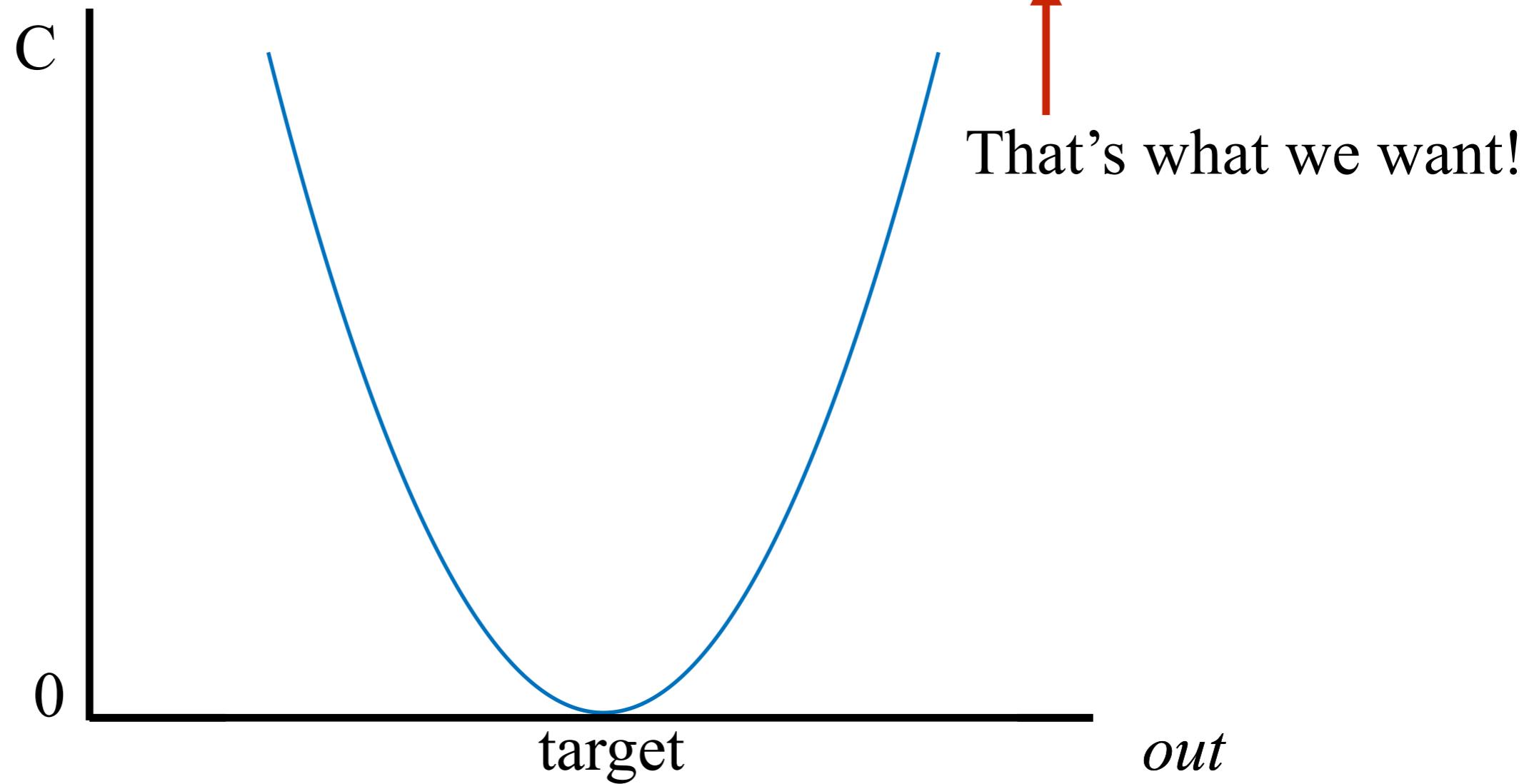
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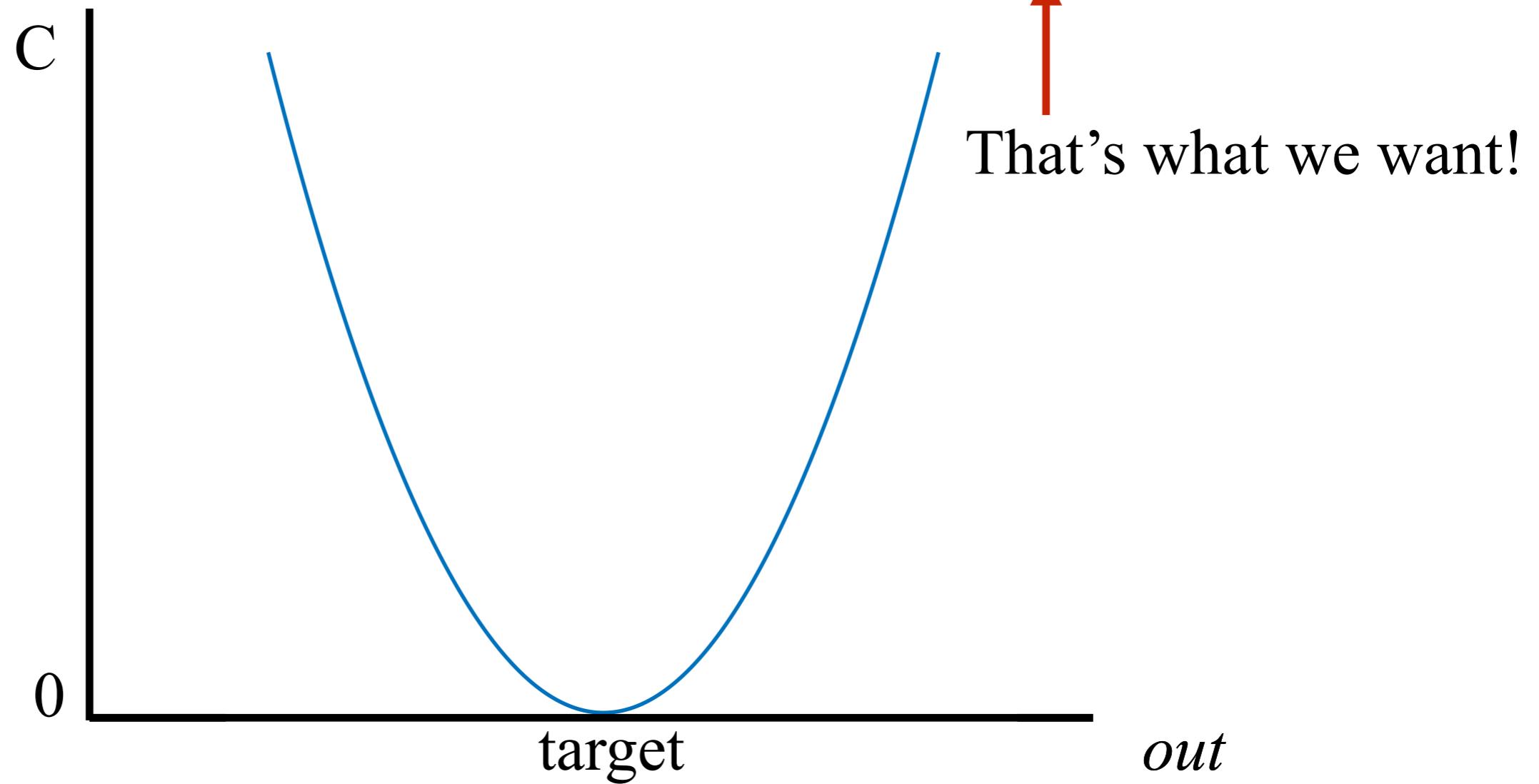


Here, we plot C versus out .

Create a cost function

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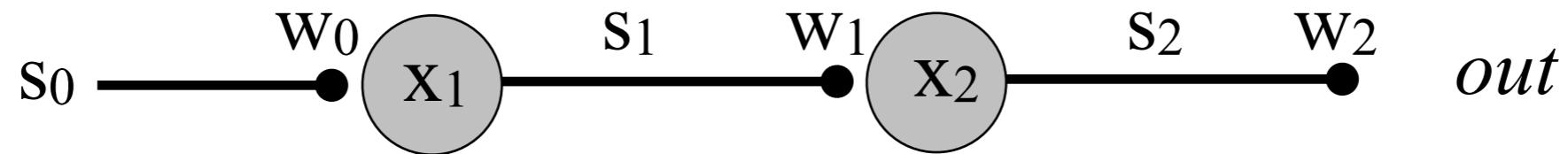
But out depends on weights ...

Create a cost function

Q: How does out depend on weights?

Create a cost function

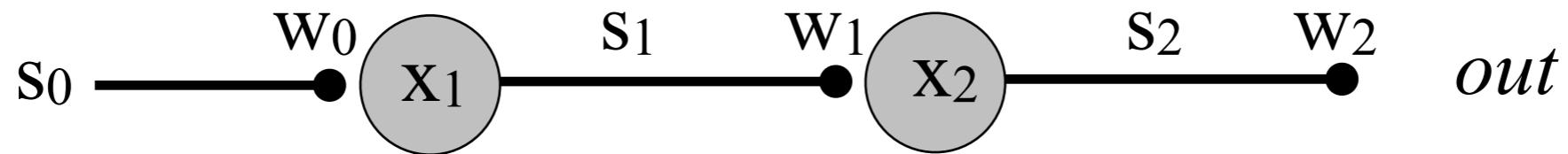
Q: How does *out* depend on weights?



A: It's complicated.

Create a cost function

Q: How does *out* depend on weights?

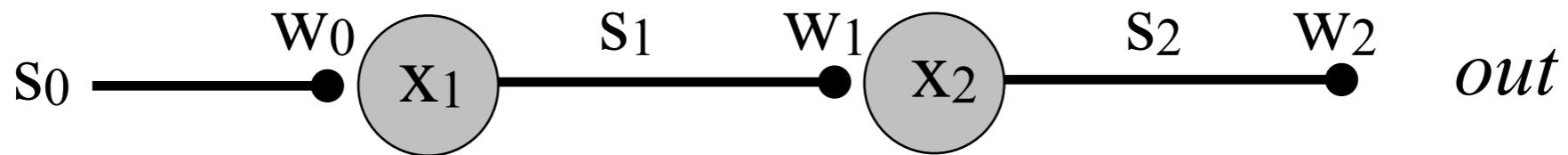


A: It's complicated.

So, if *out* depends on weights, so does the cost ...

Create a cost function

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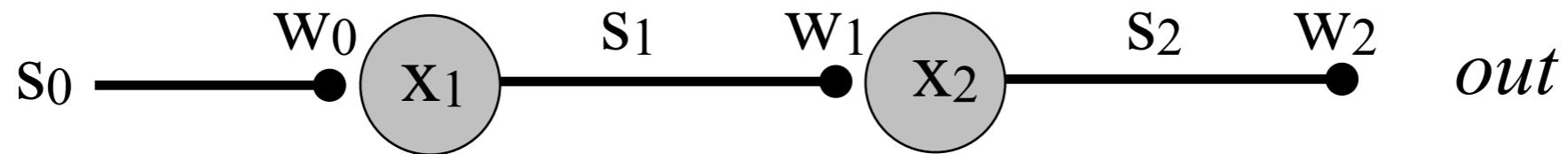


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Create a cost function

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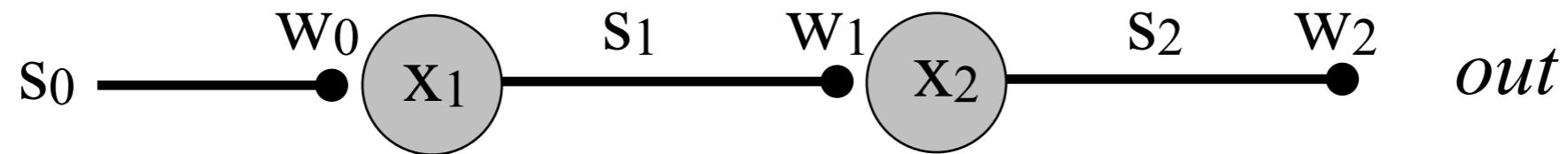
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Consider feedforward solution ...

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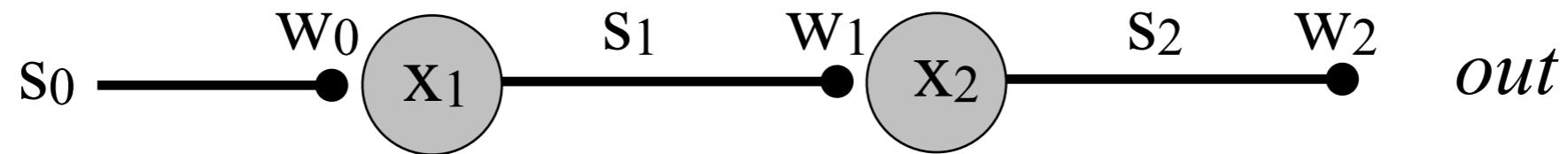
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$$X_1 = S_0 W_0$$

Create a cost function

Q: How does *out* depend on weights?



A: It's complicated.

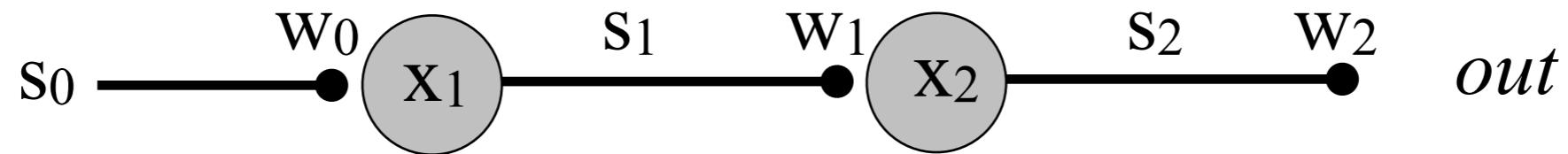
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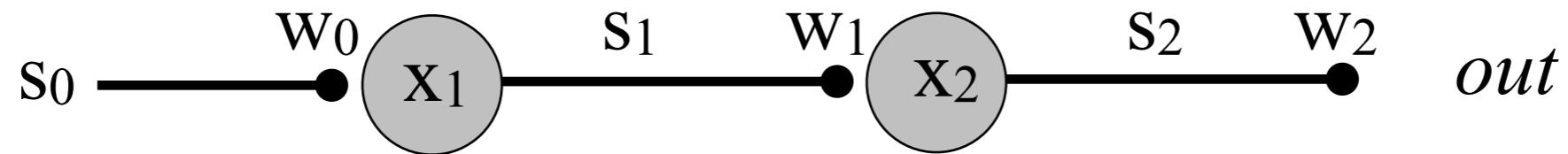
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$$x_1 = s_0 \quad w_0 \longrightarrow s_1 = S(x_1)$$

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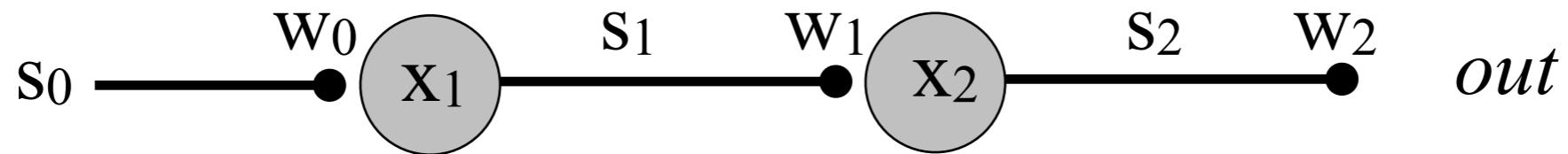
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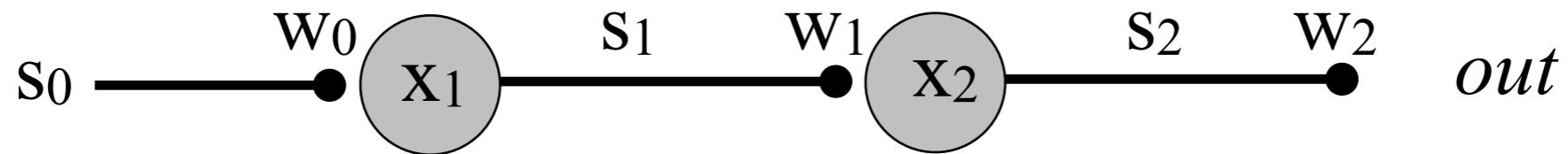
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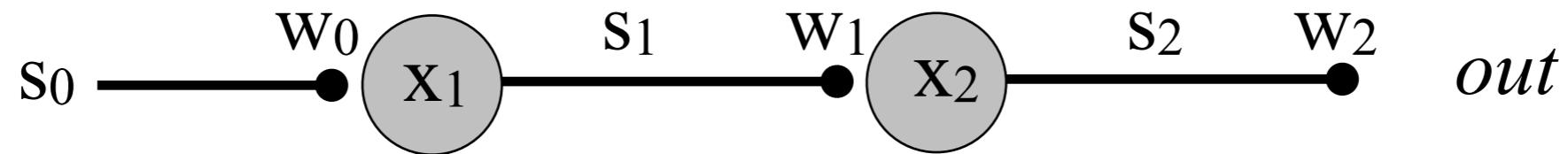
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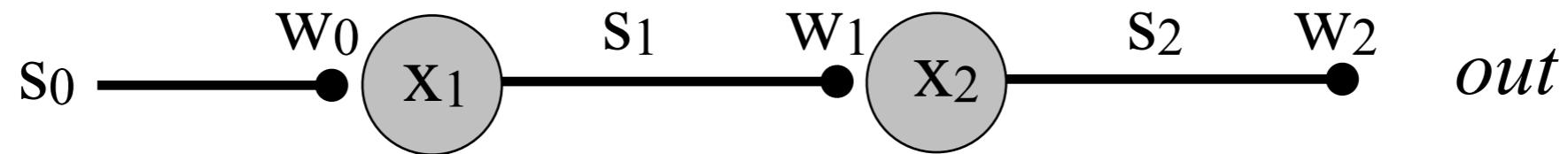
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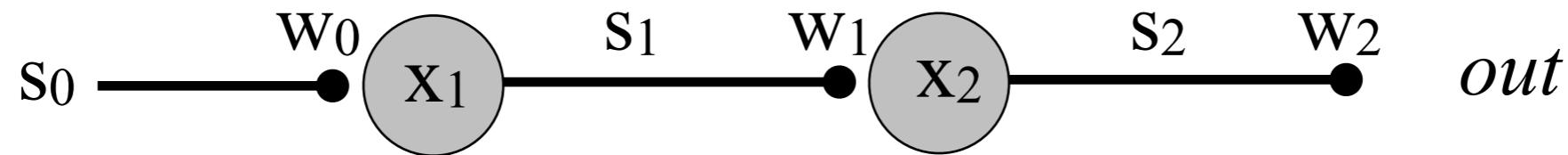
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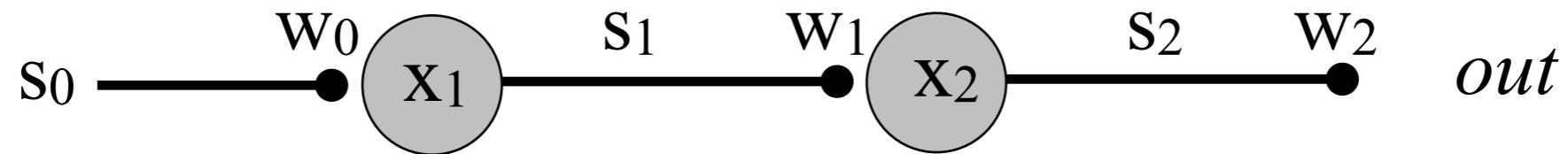
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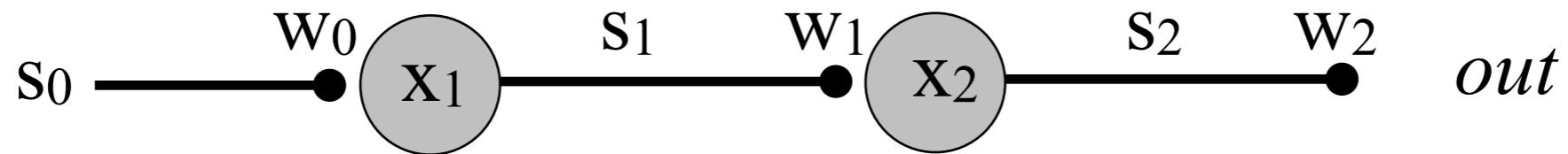
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Create a cost function

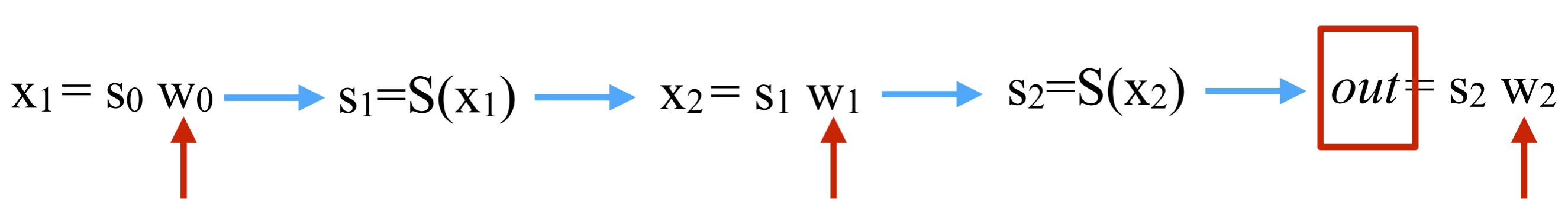
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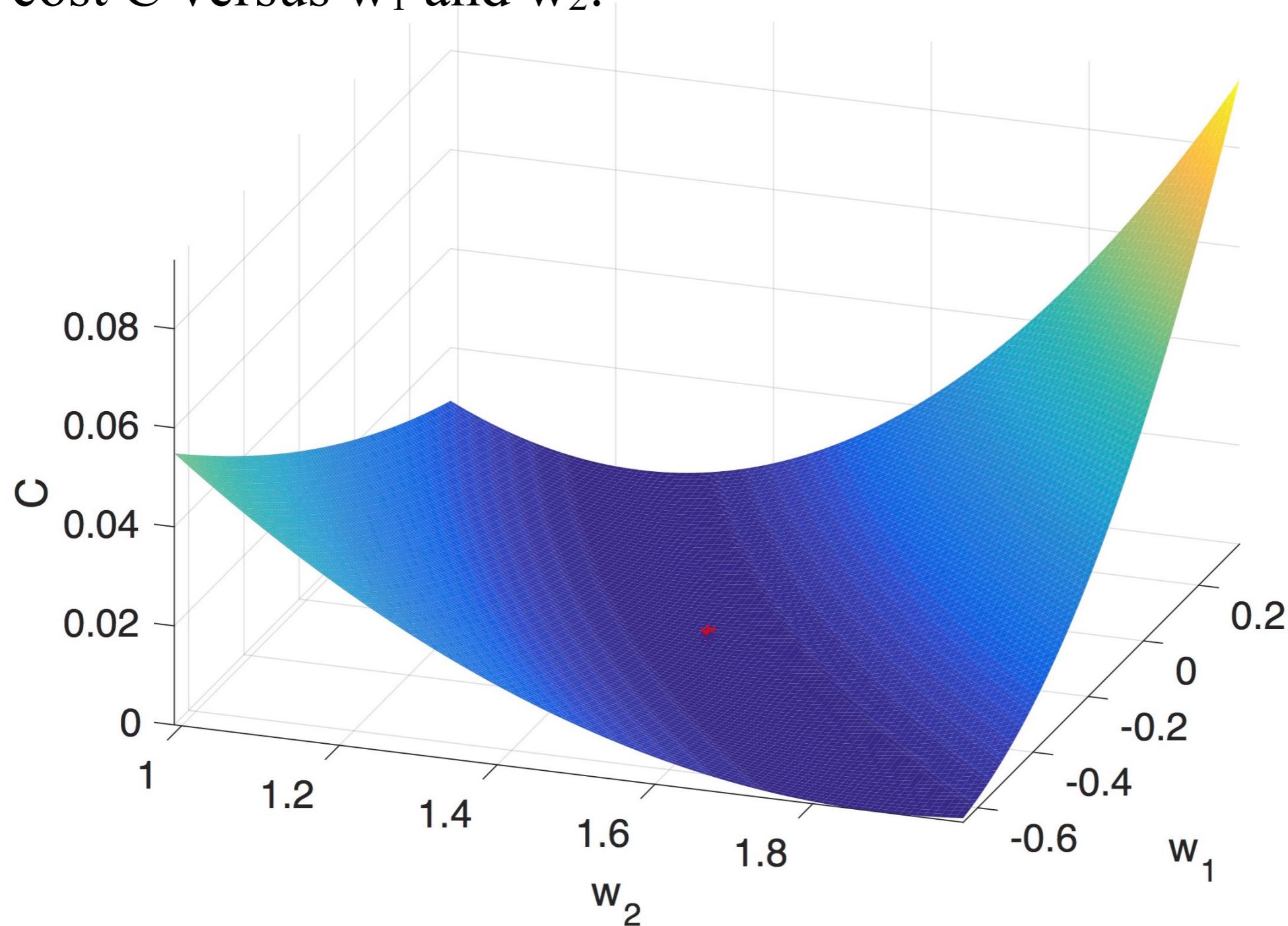


Create a cost function

Plot cost C versus w_1 and w_2 :

Create a cost function

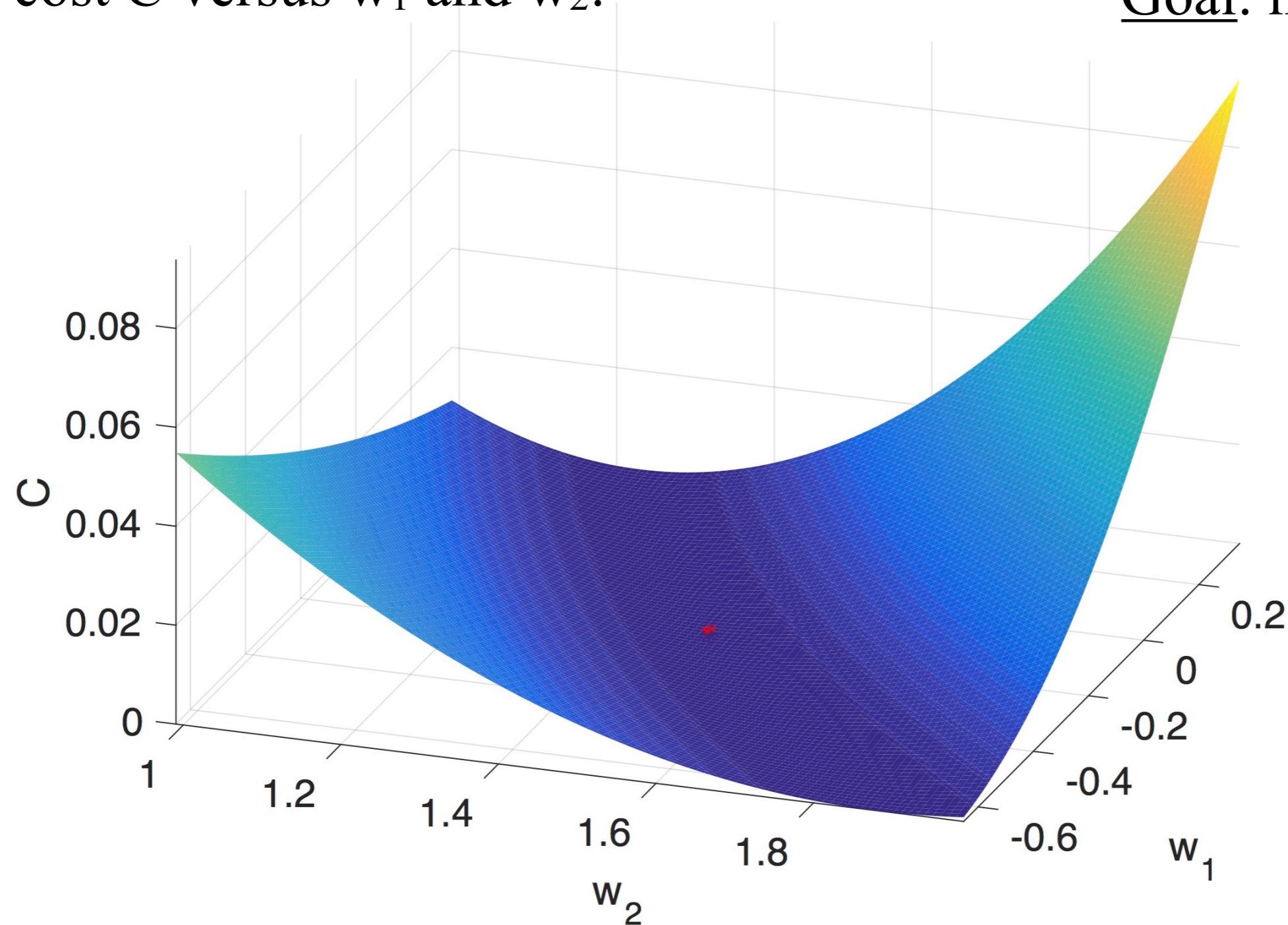
Plot cost C versus w_1 and w_2 :



Create a cost function

Plot cost C versus w_1 and w_2 :

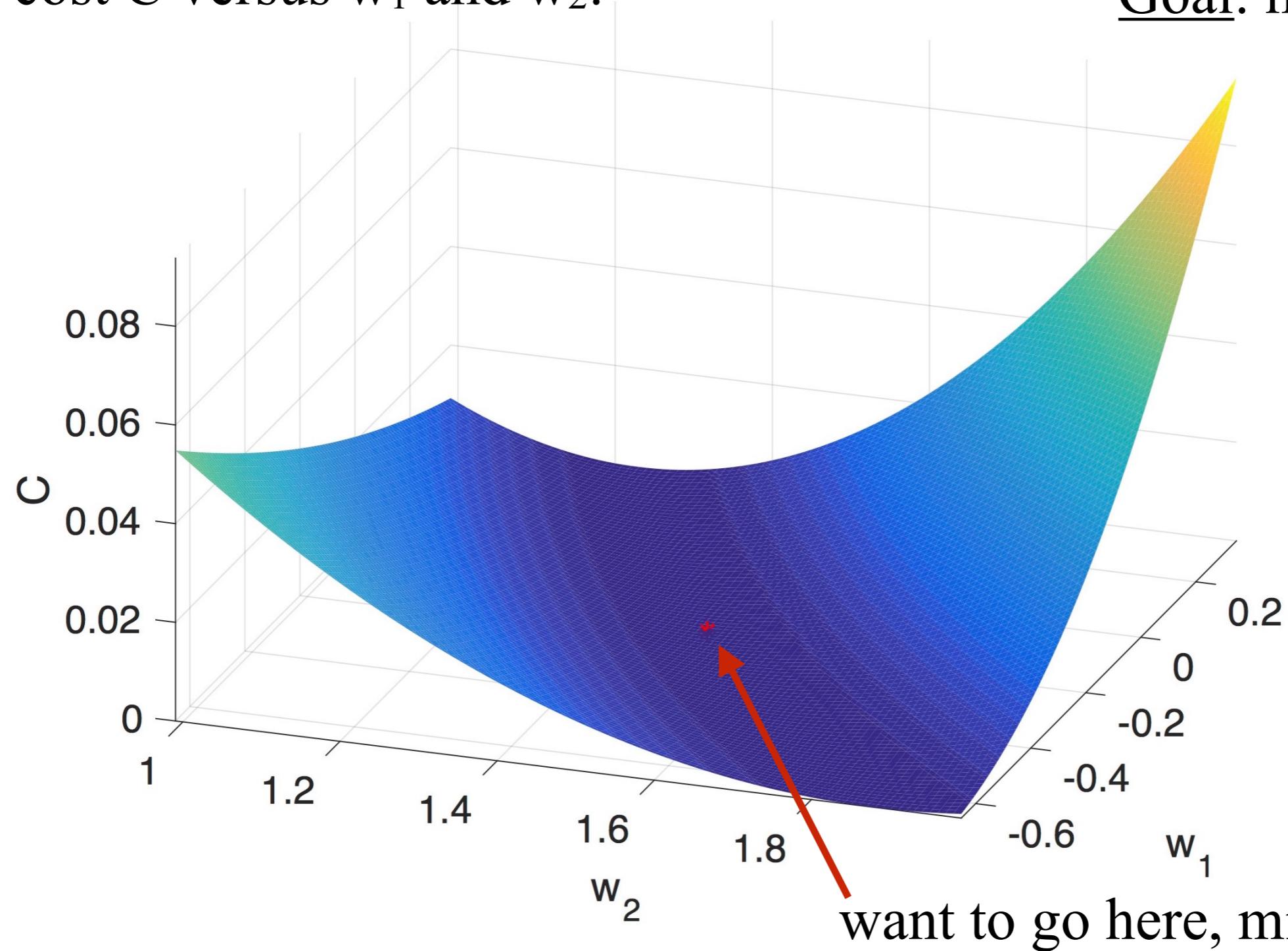
Goal: minimize cost.



Create a cost function

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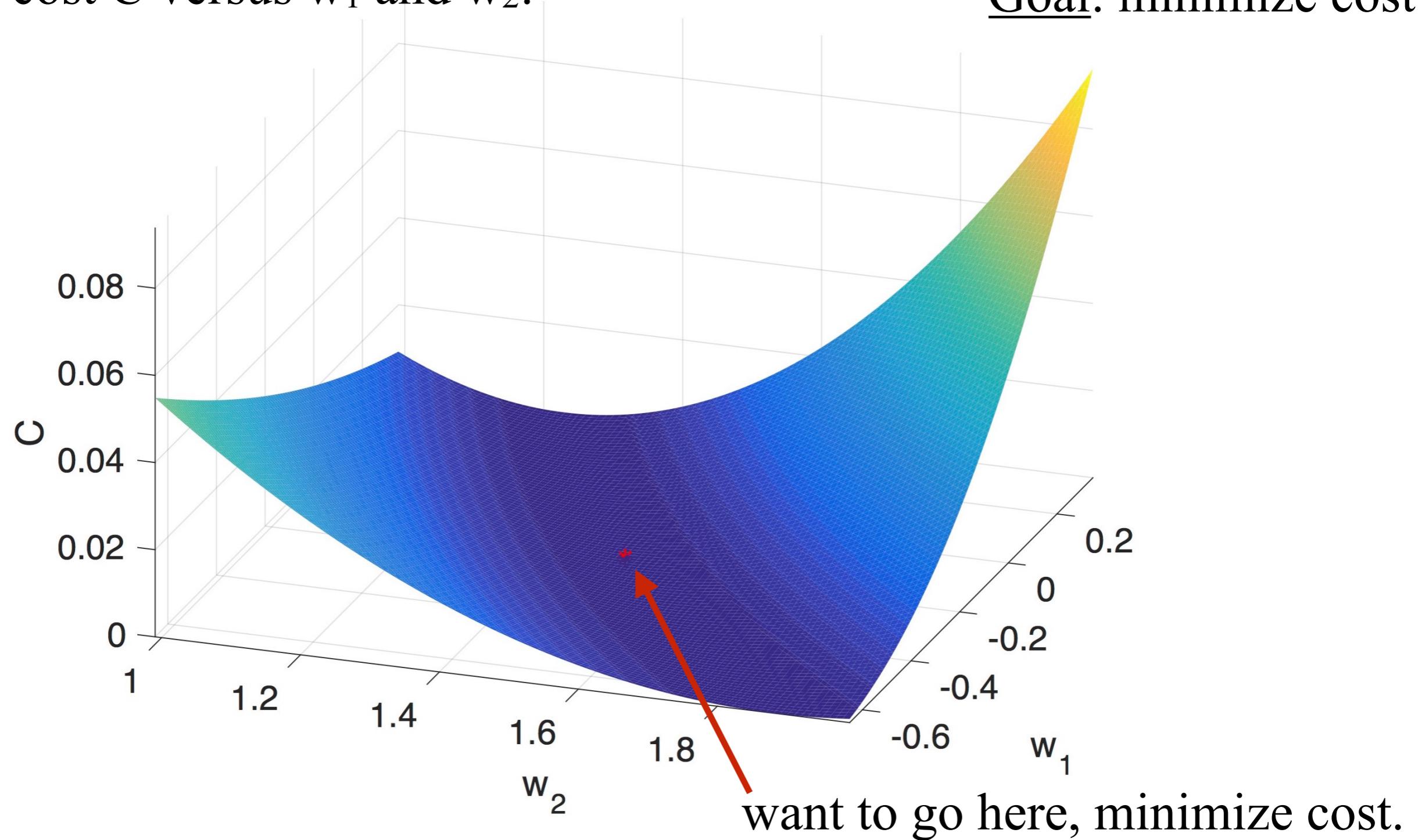
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Create a cost function

Plot cost C versus w_1 and w_2 :

Goal: minimize cost.



Imagine a “top-down” view ...

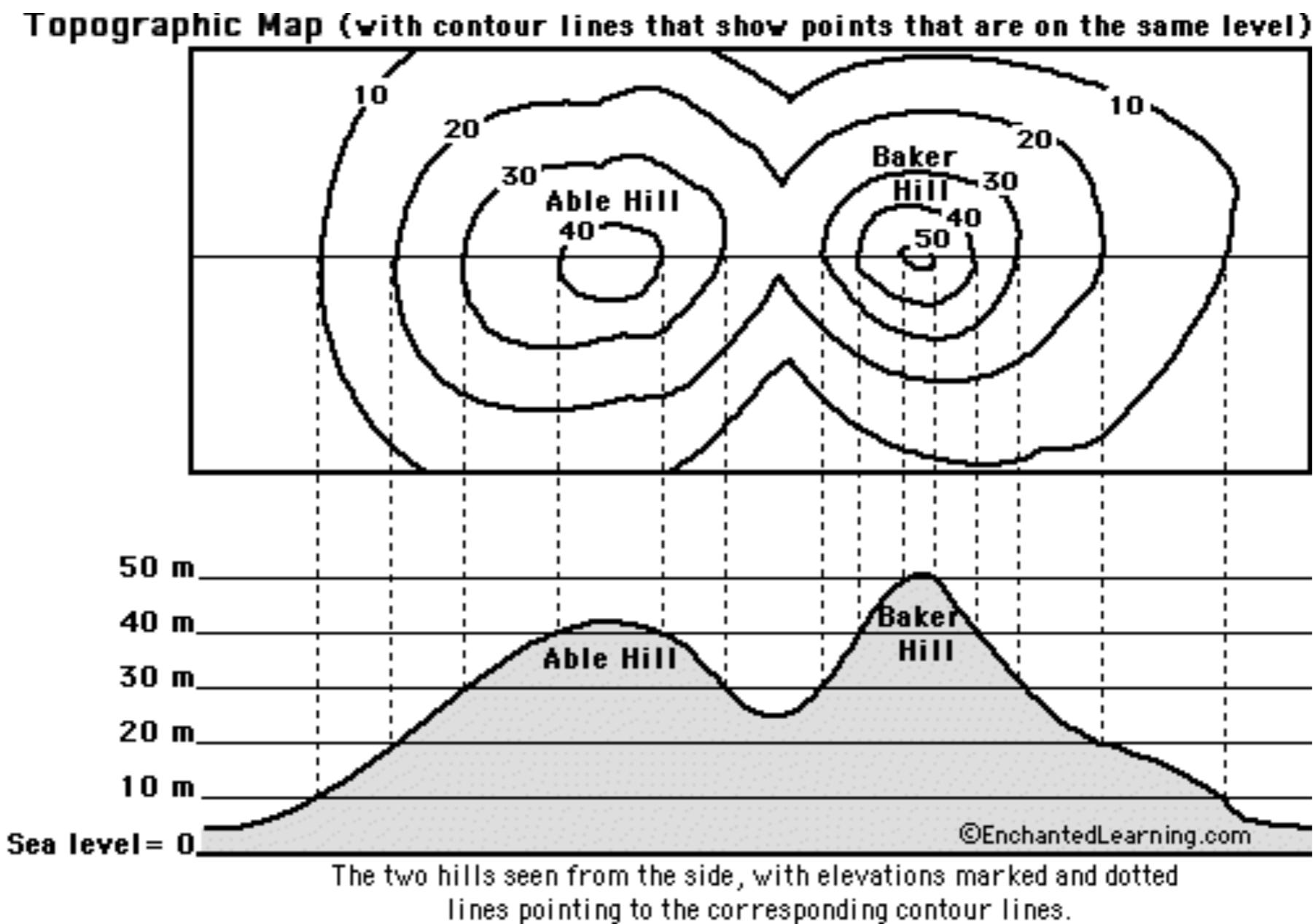
Create a cost function

Contour map or topographic map of cost function:

Create a cost function

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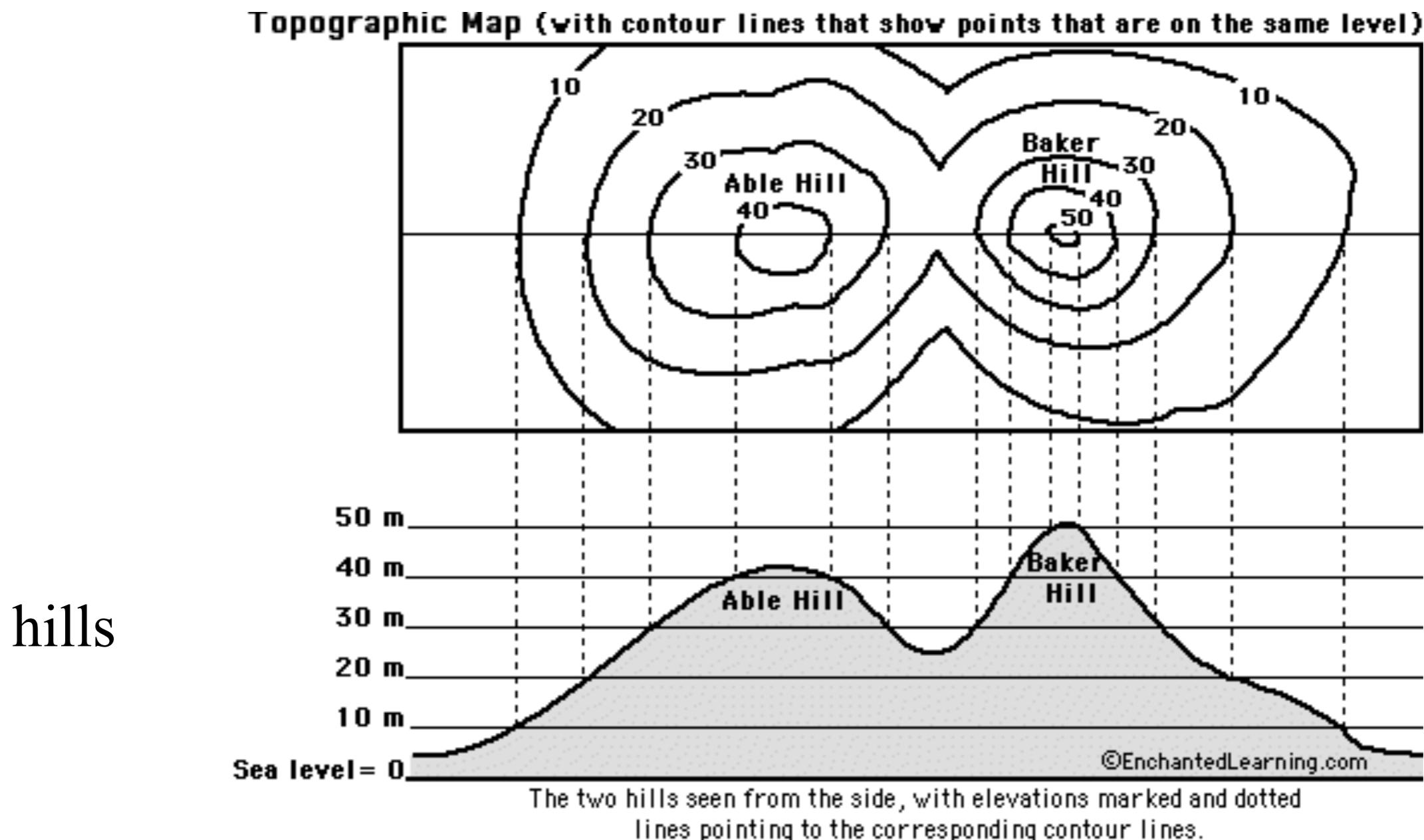
Related example:



Create a cost function

Contour map or topographic map of cost function:

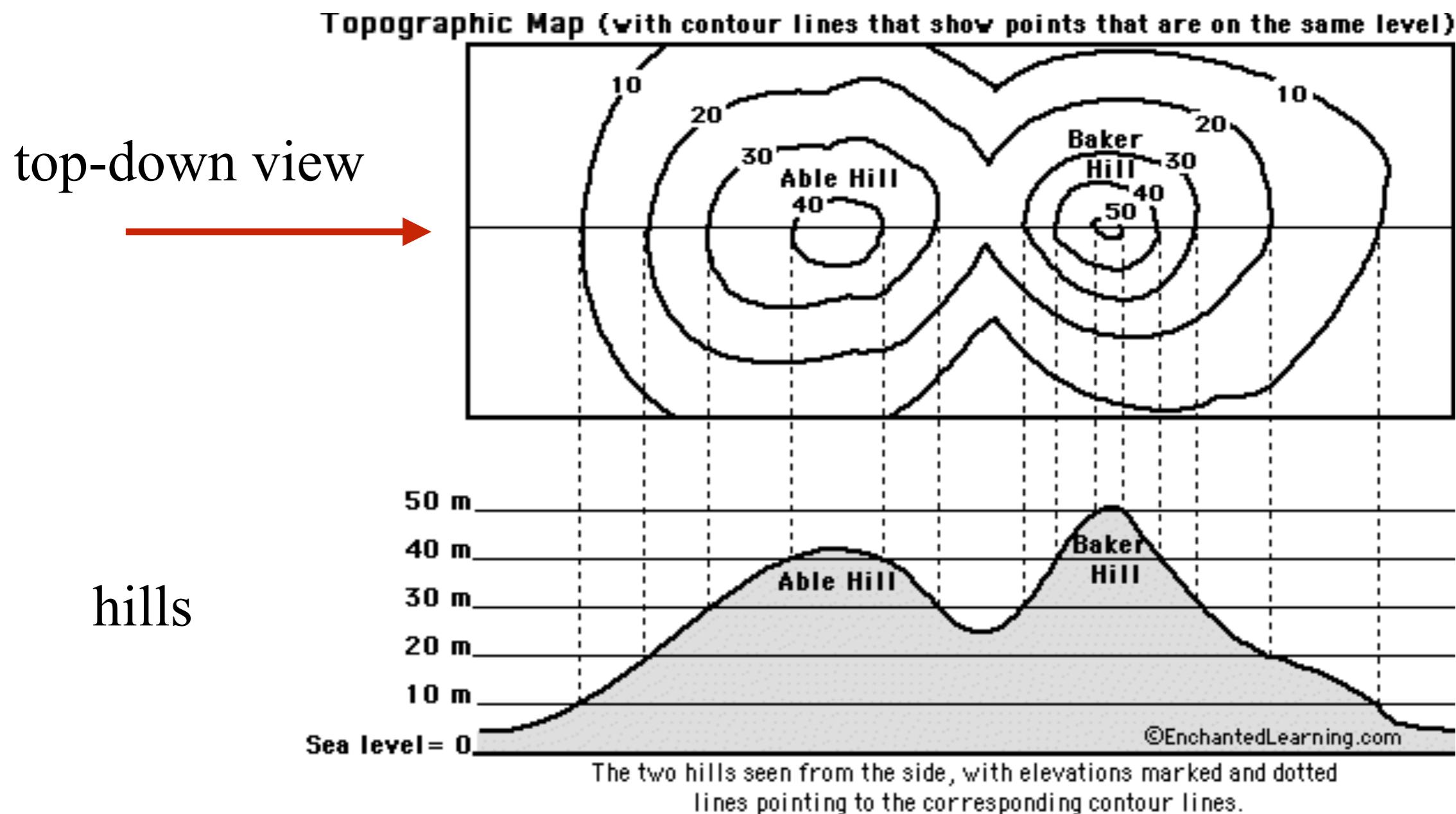
Related example:



Create a cost function

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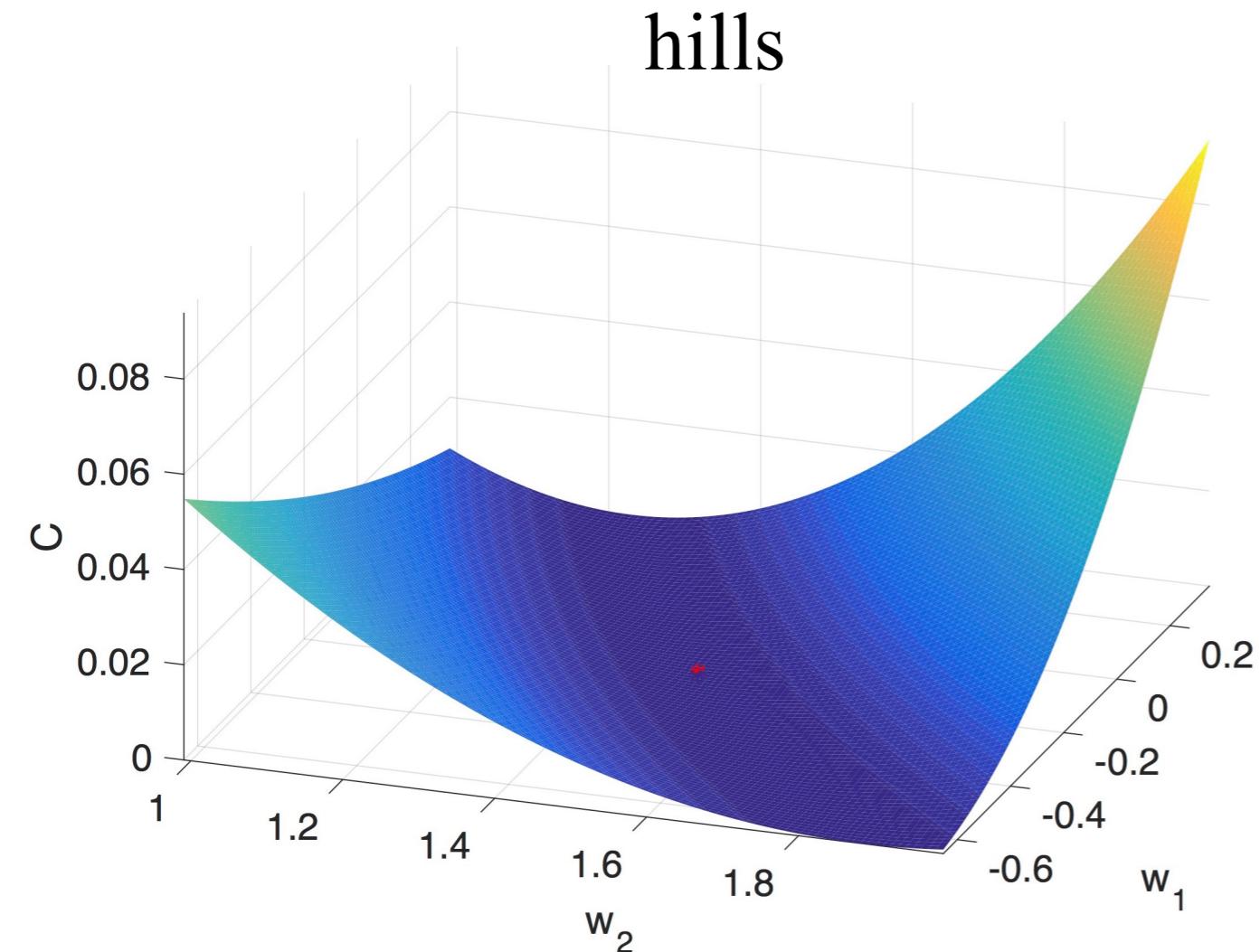


Create a cost function

Contour map of cost function:

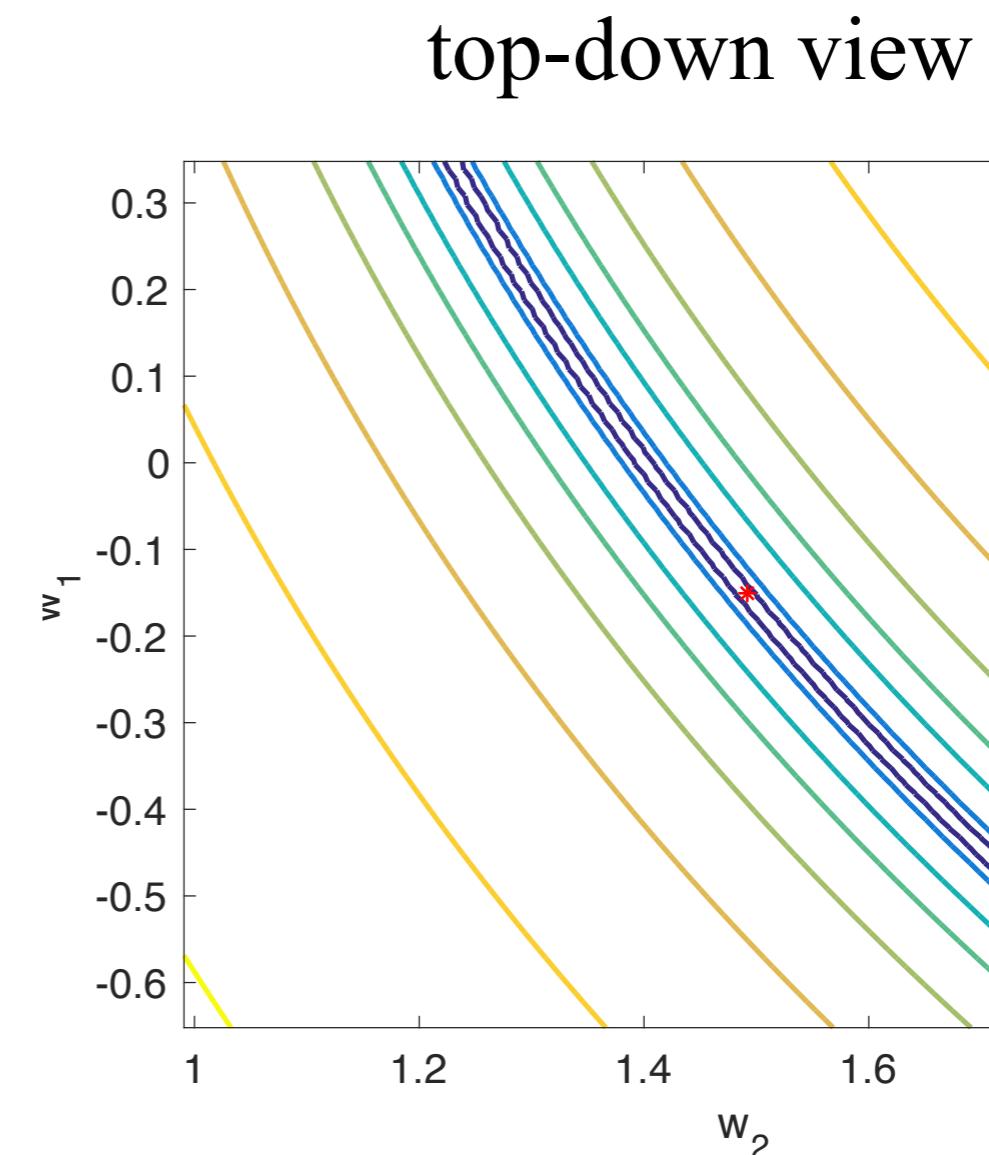
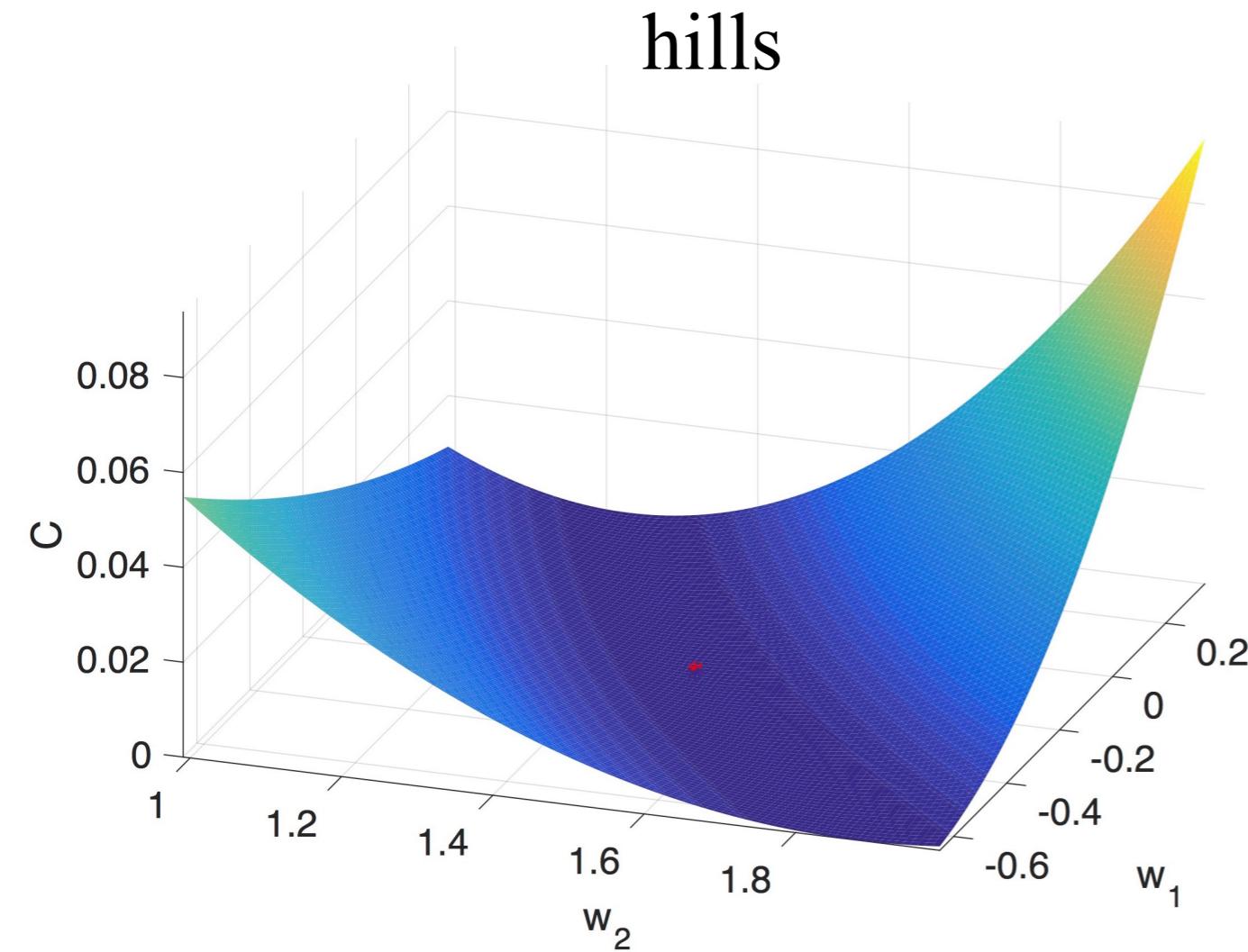
Create a cost function

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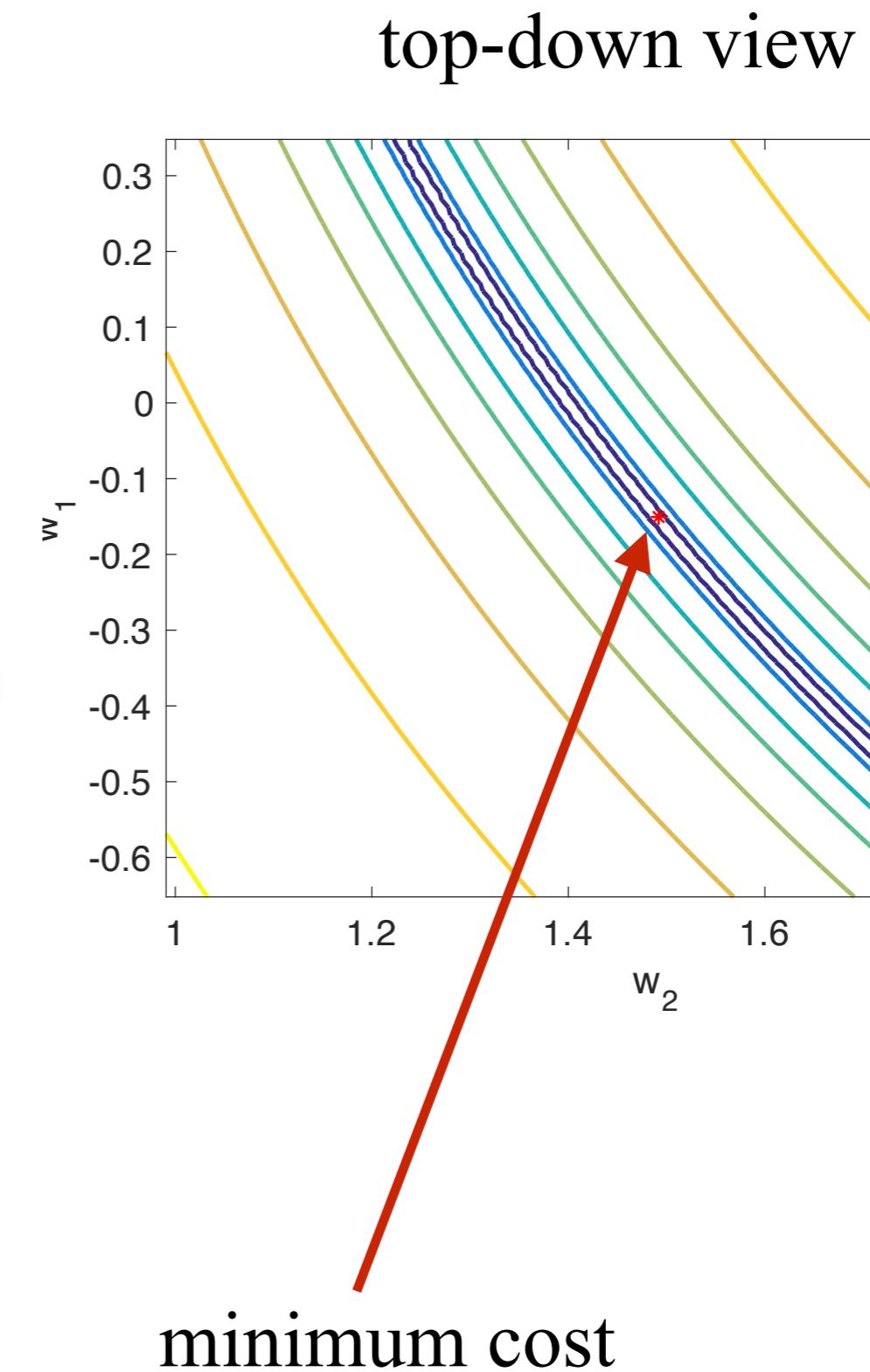
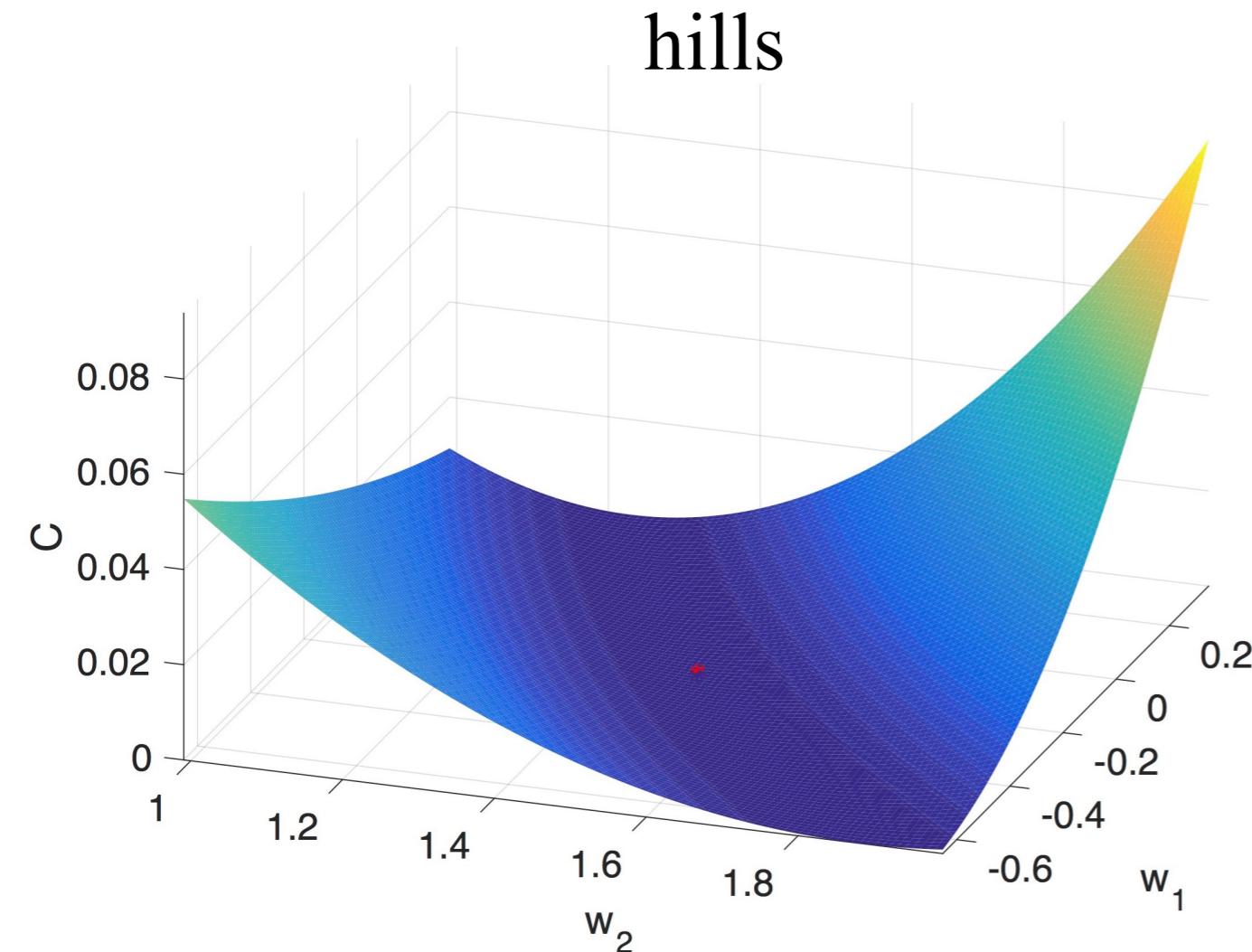
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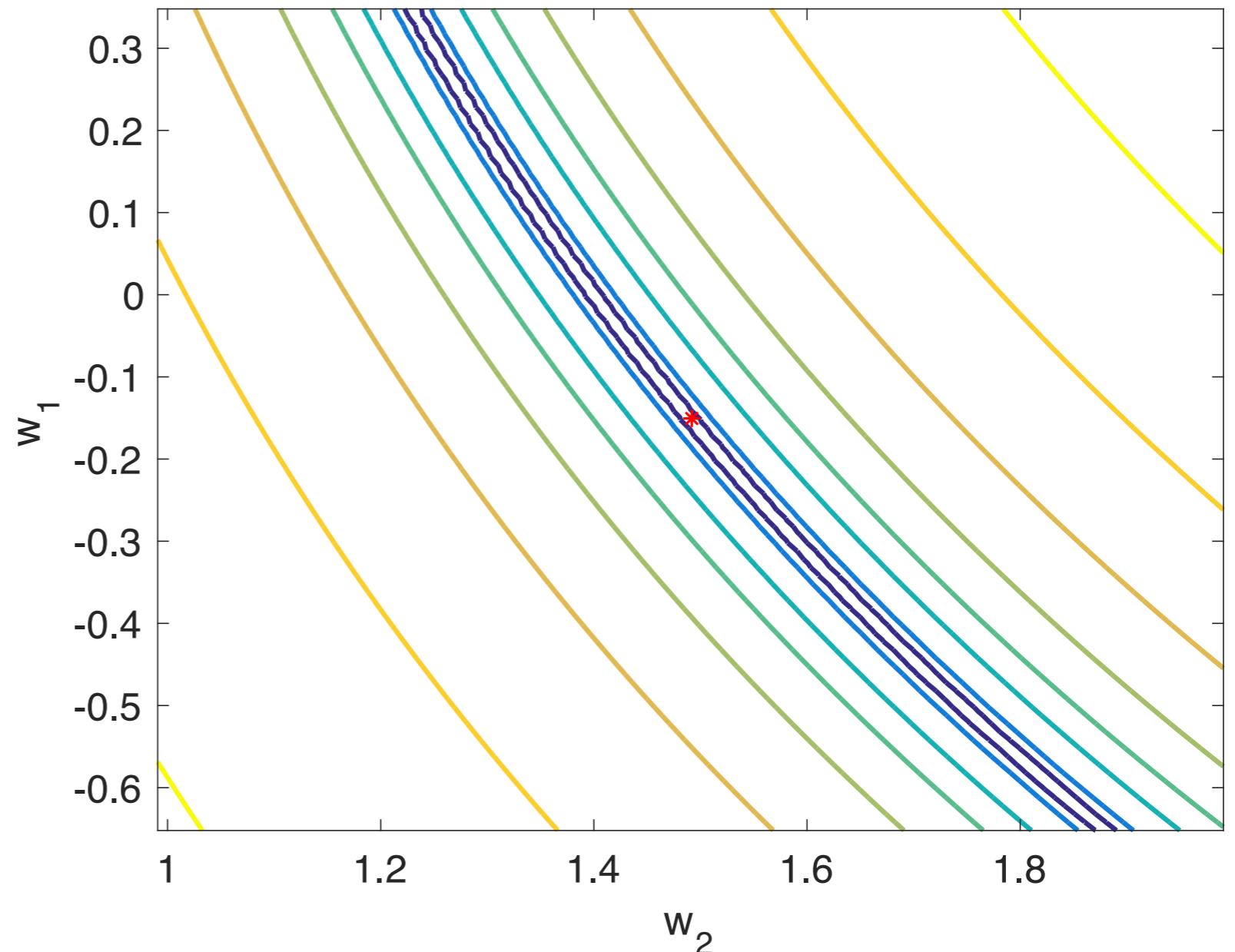


Create a cost function

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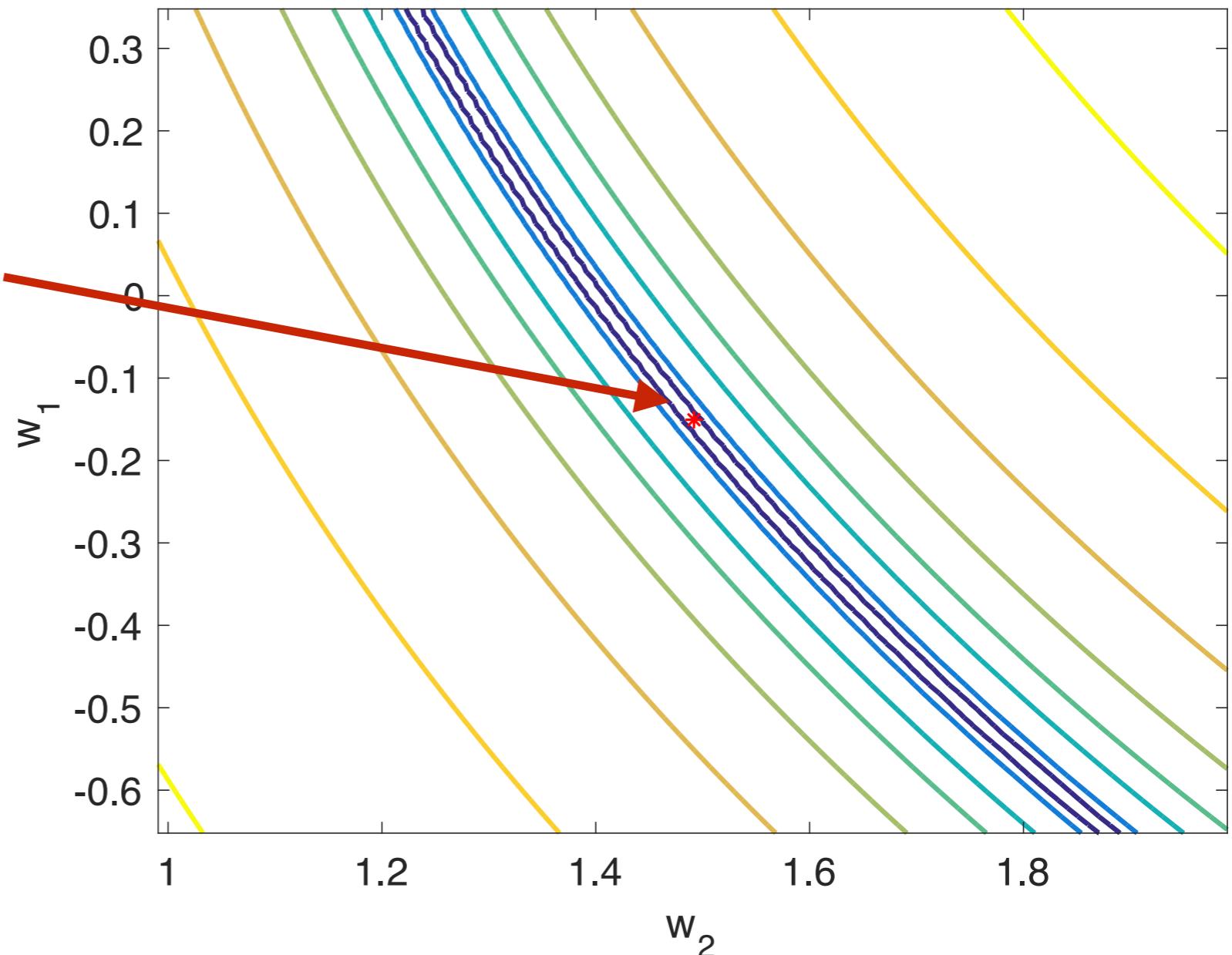


Follow the cost function



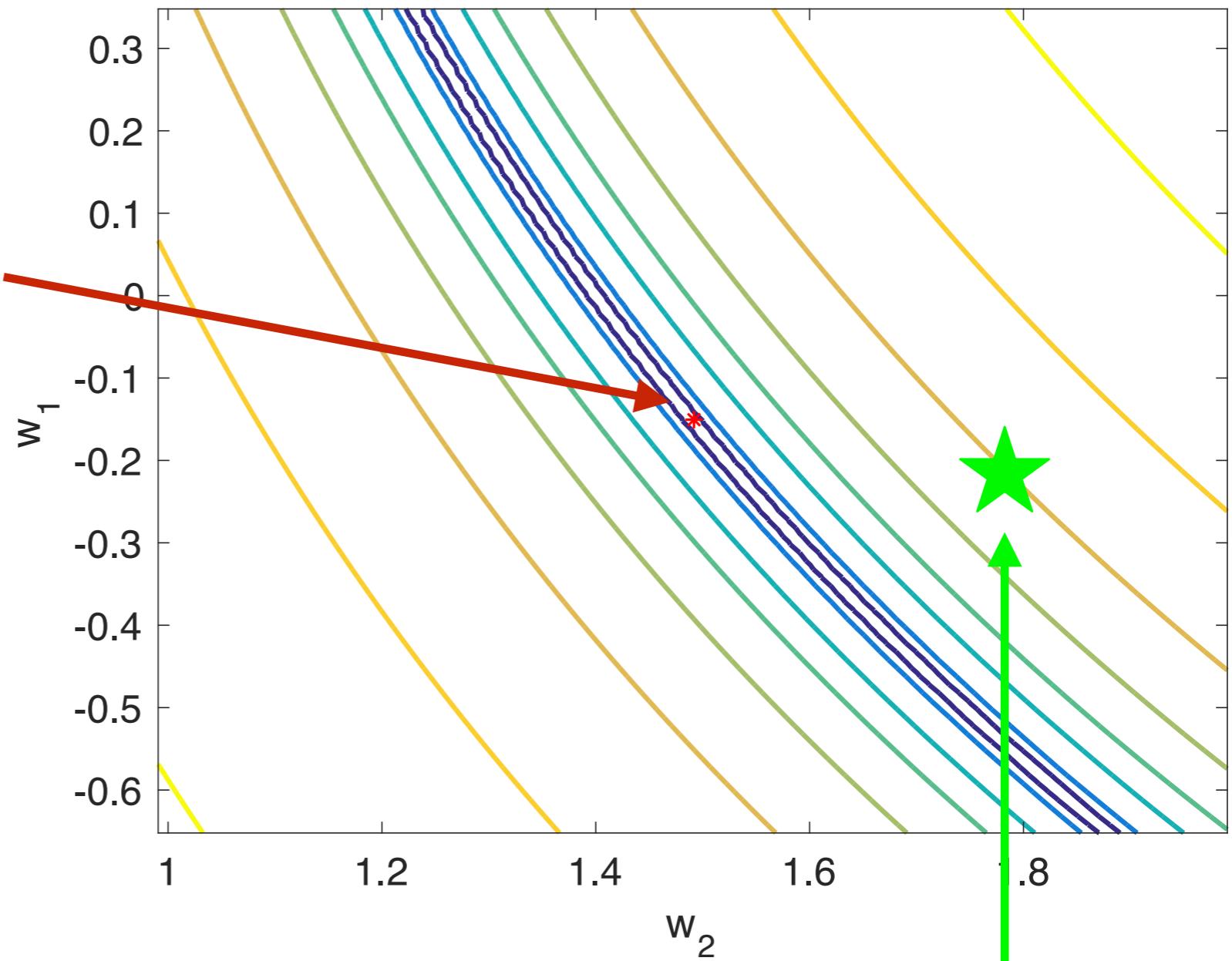
Follow the cost function

We want the weights here
at the minimum cost



Follow the cost function

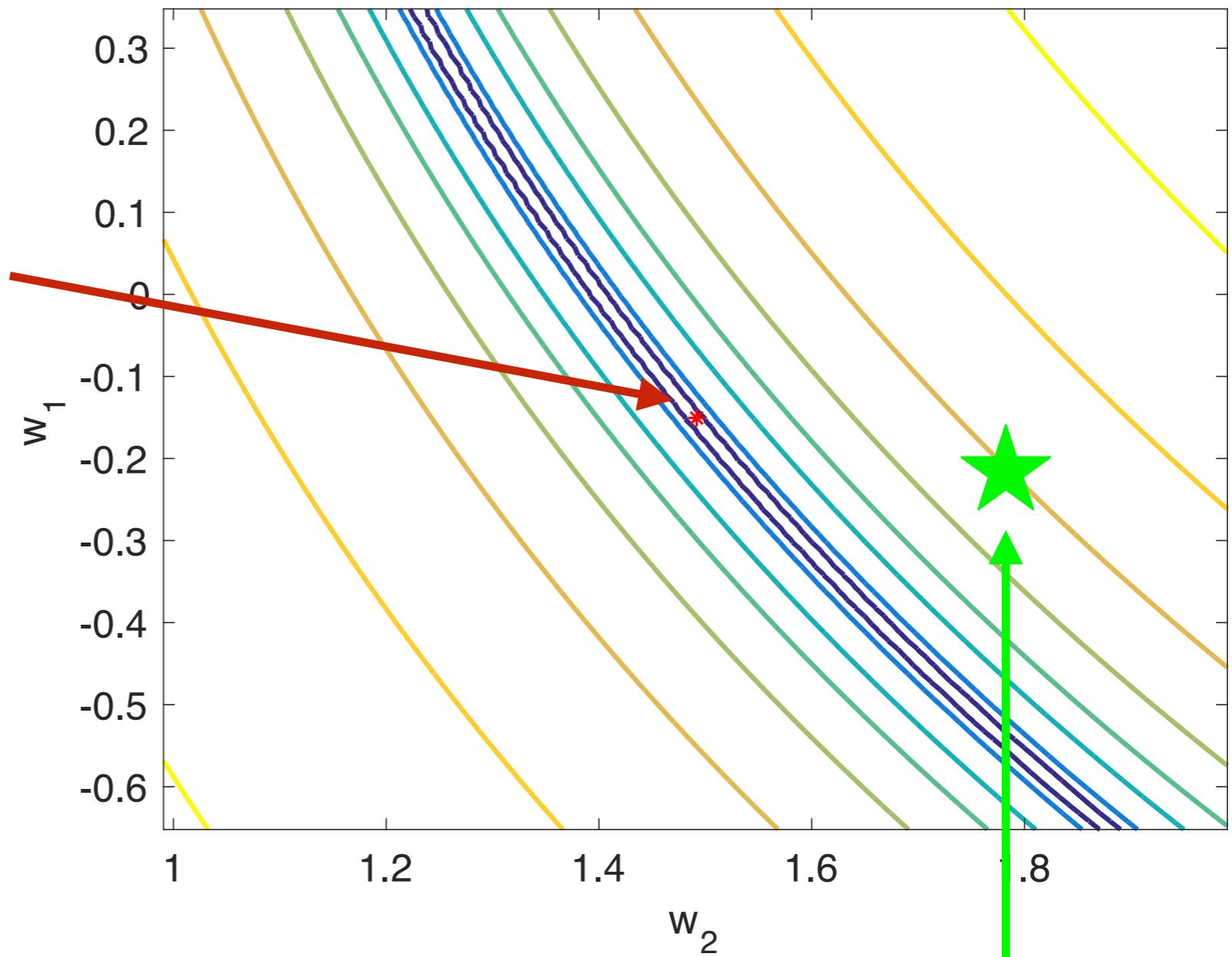
We want the weights here
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But, we have our initial,
randomly chosen weights

Follow the cost function

We want the weights here
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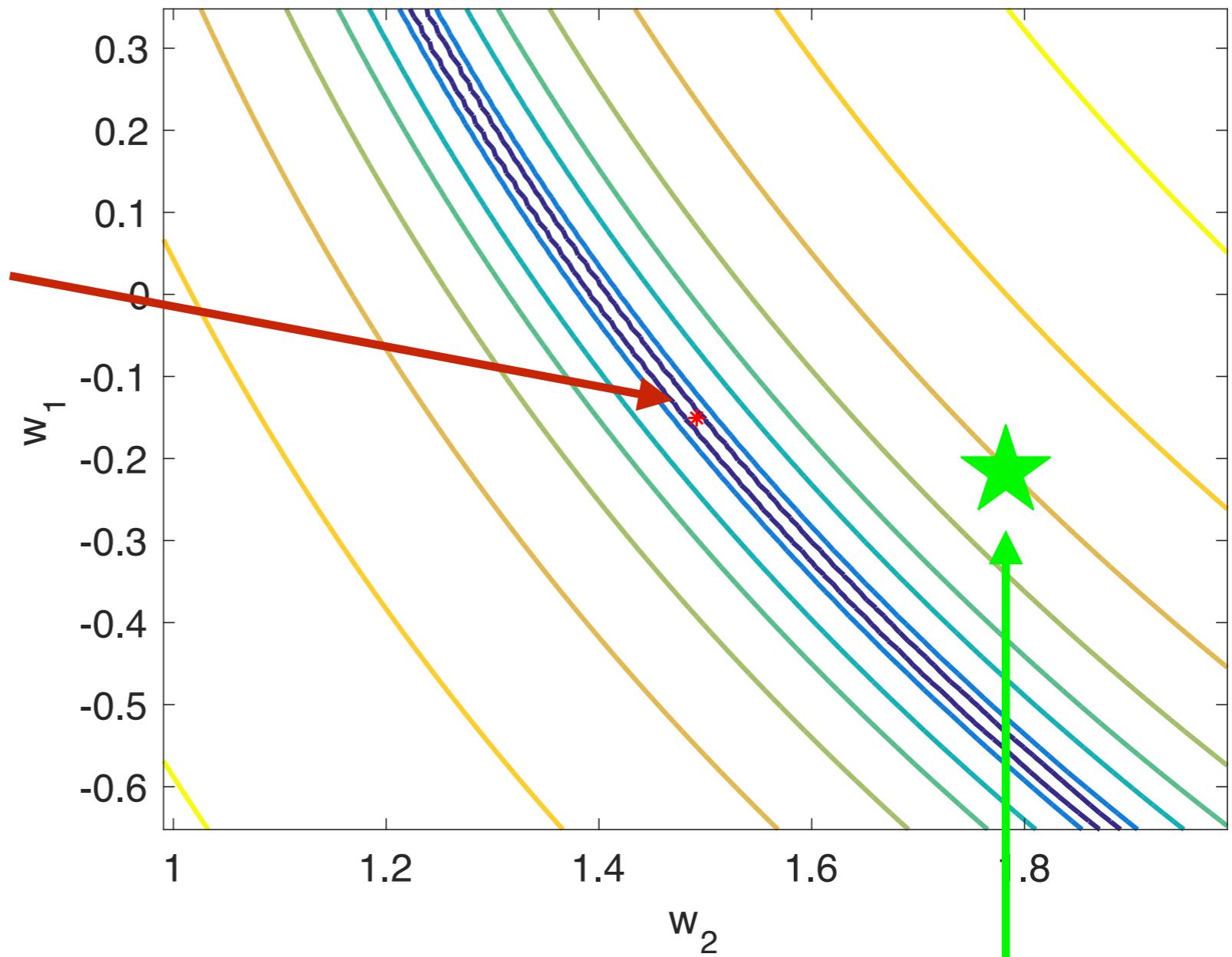


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Q: How do we adjust weights to go from ★ to minimum cost?

Follow the cost function

We want the weights here
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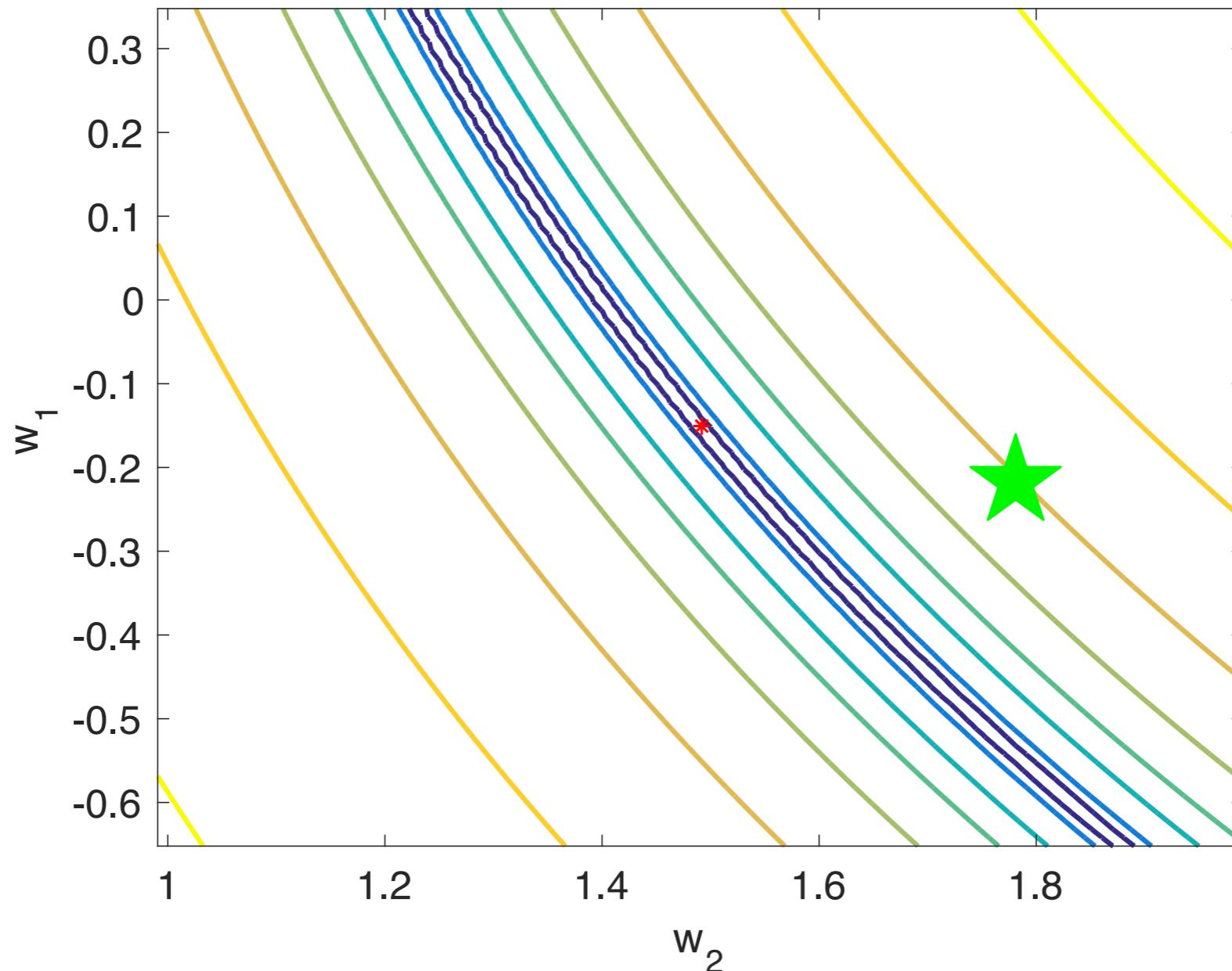


But, we have our initial,
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- Q: How do we adjust weights to go from ★ to minimum cost?
A: Move “downhill” ...

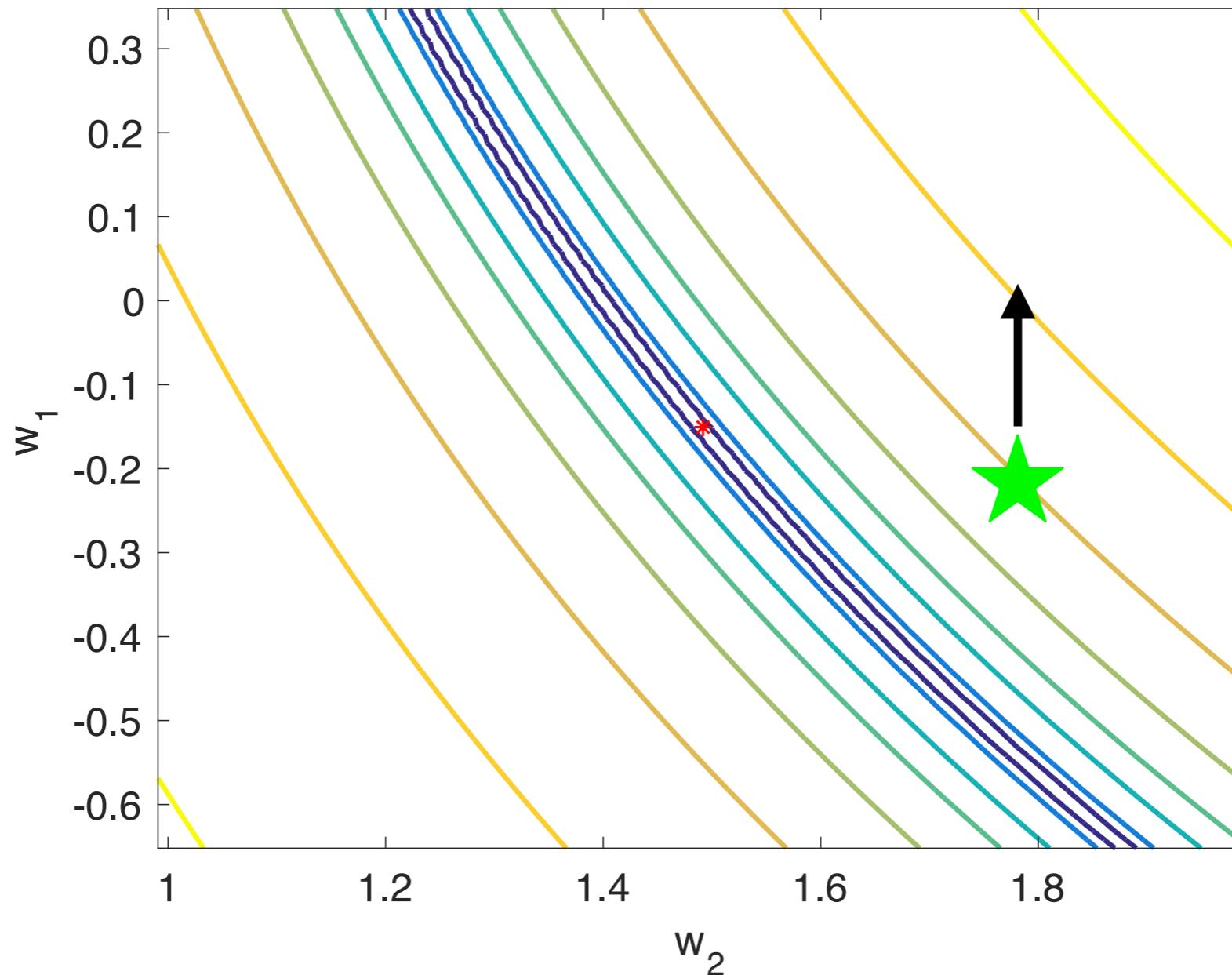
Follow the cost function

Intuition: move down the **steepest direction** of cost function



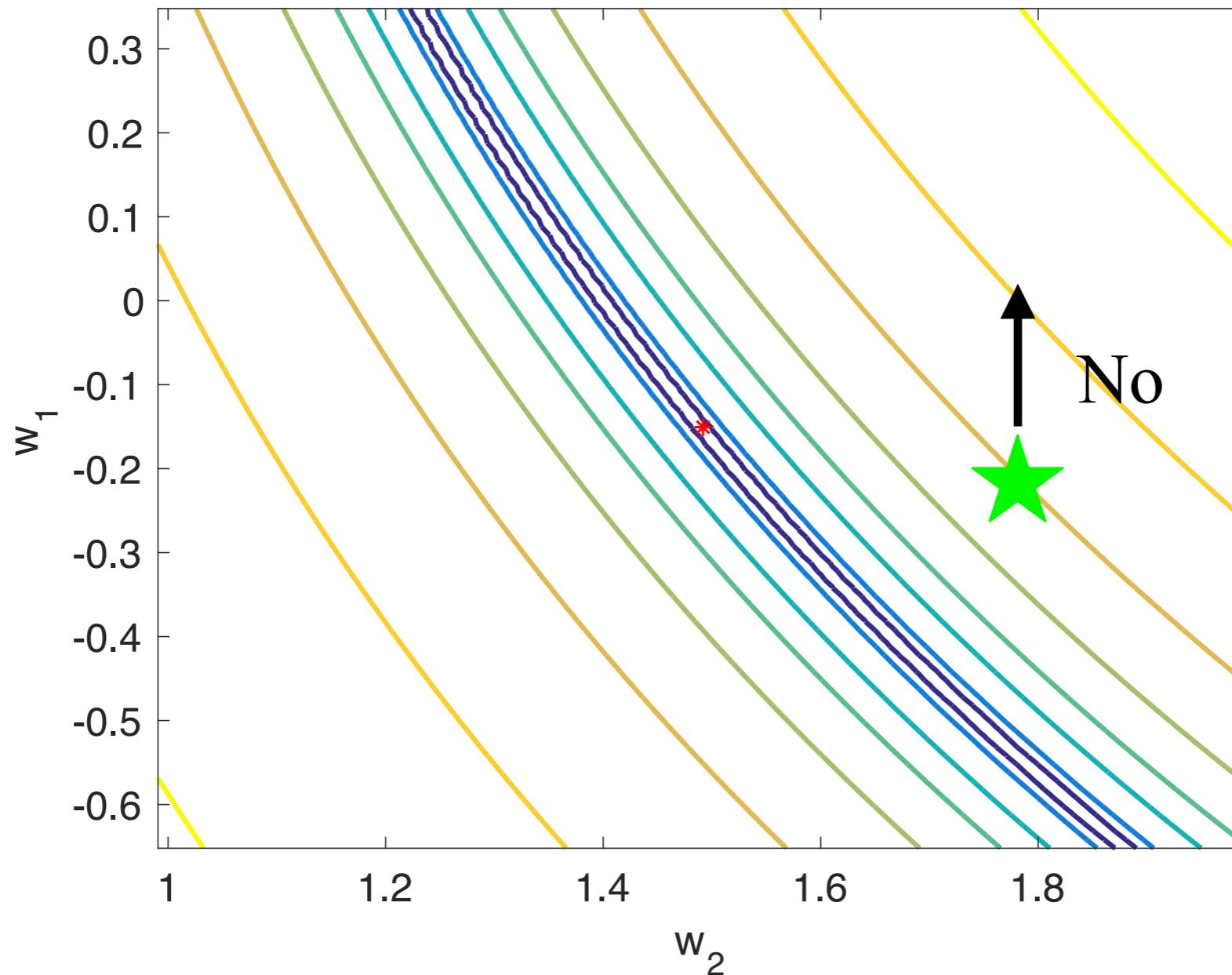
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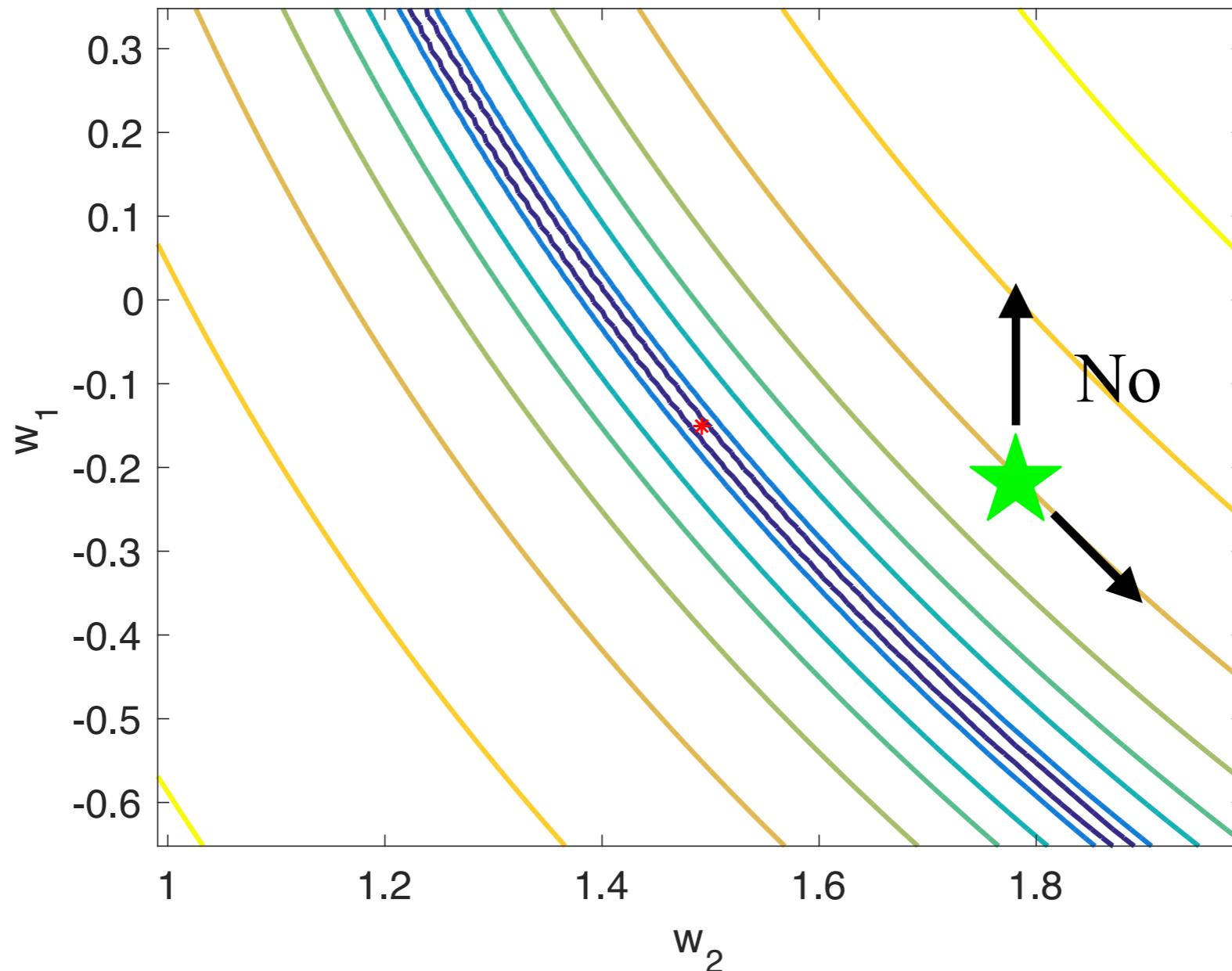
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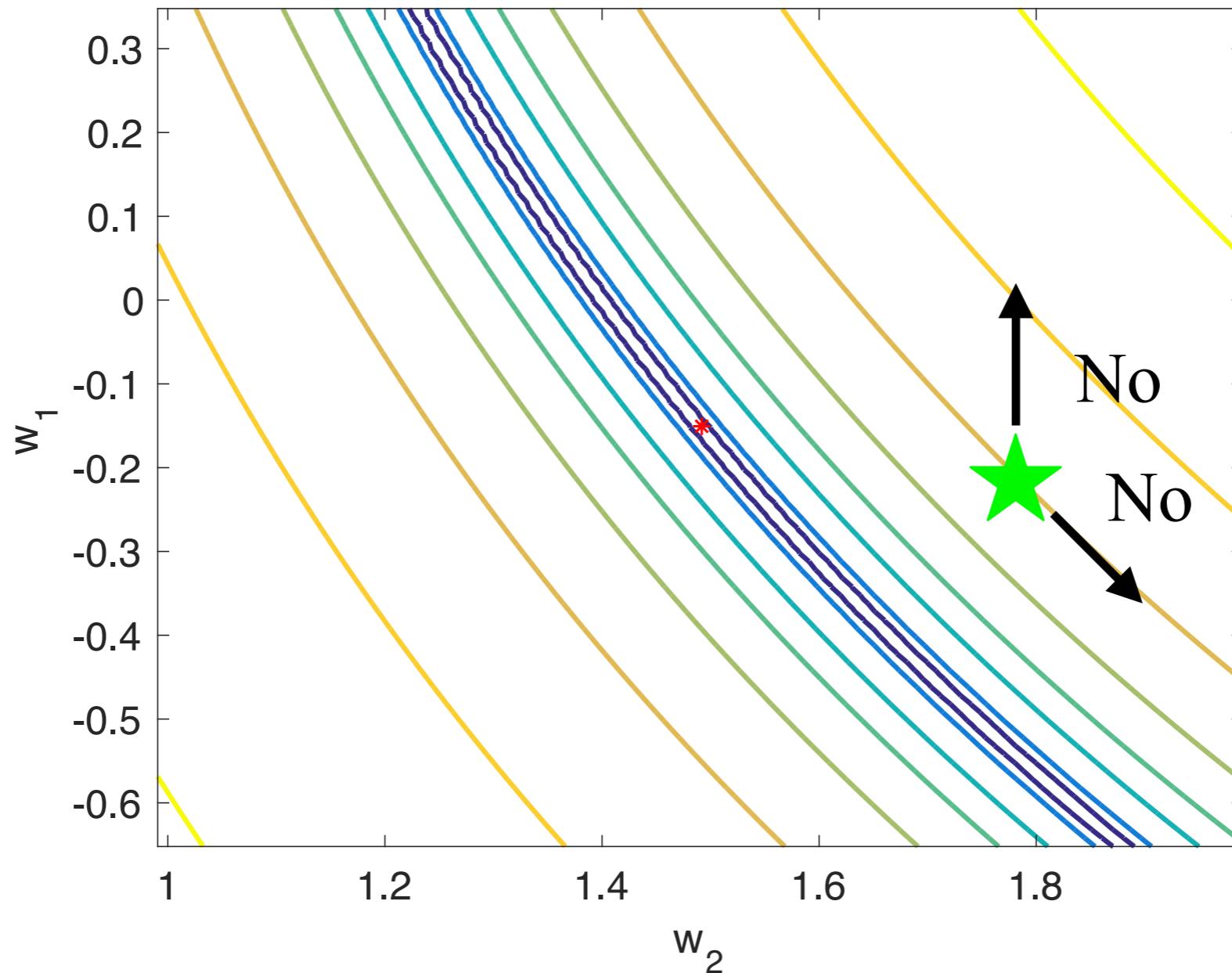
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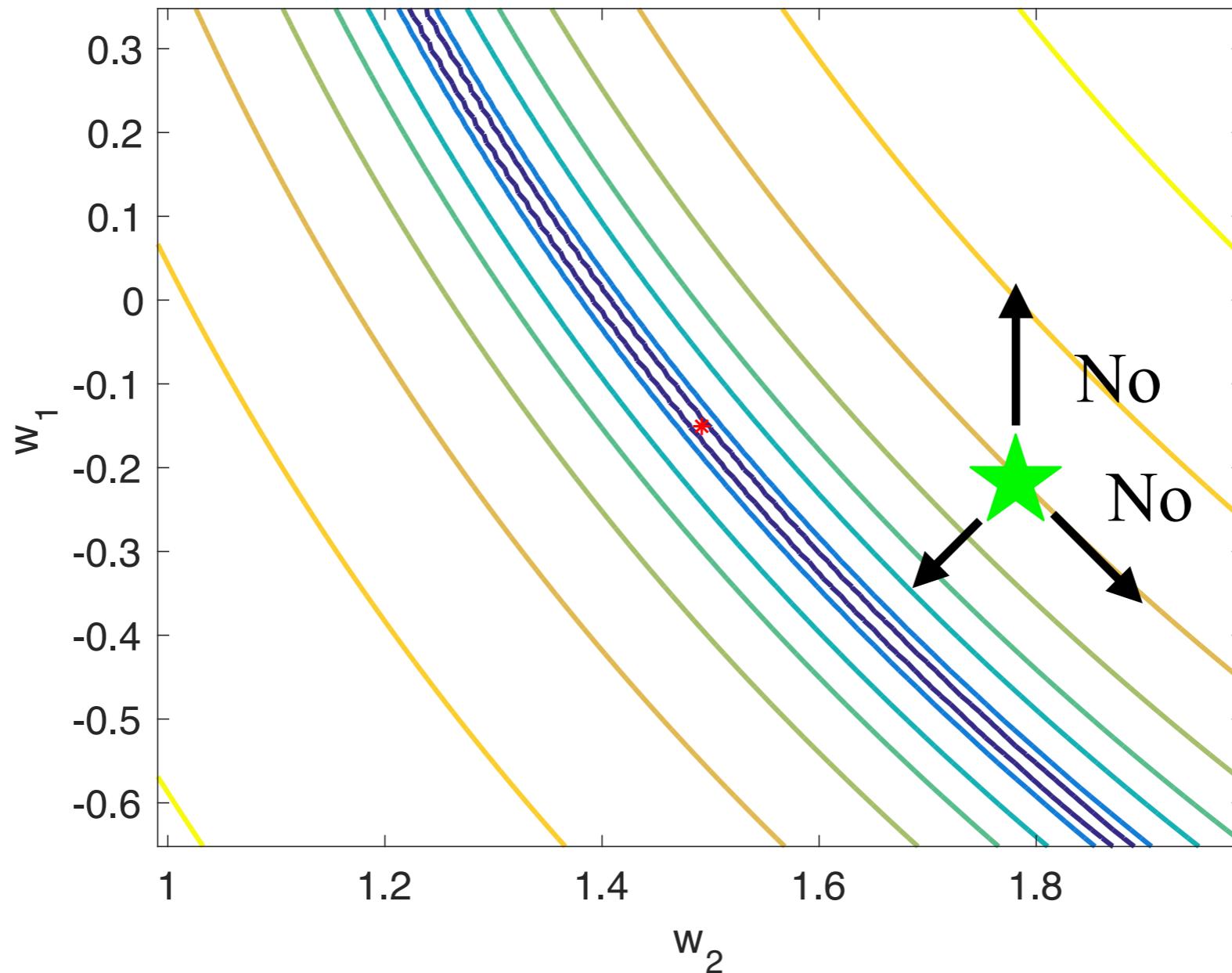
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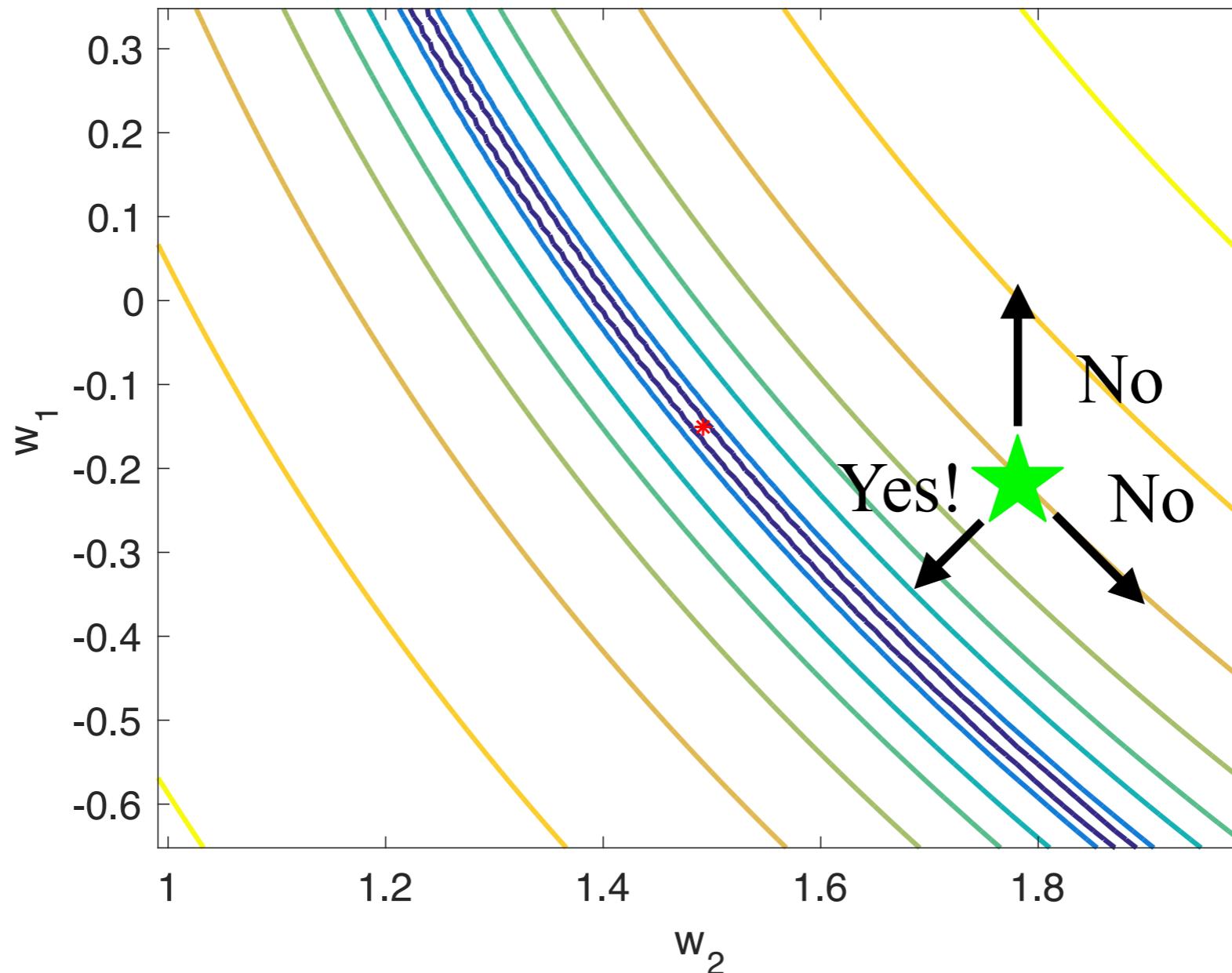
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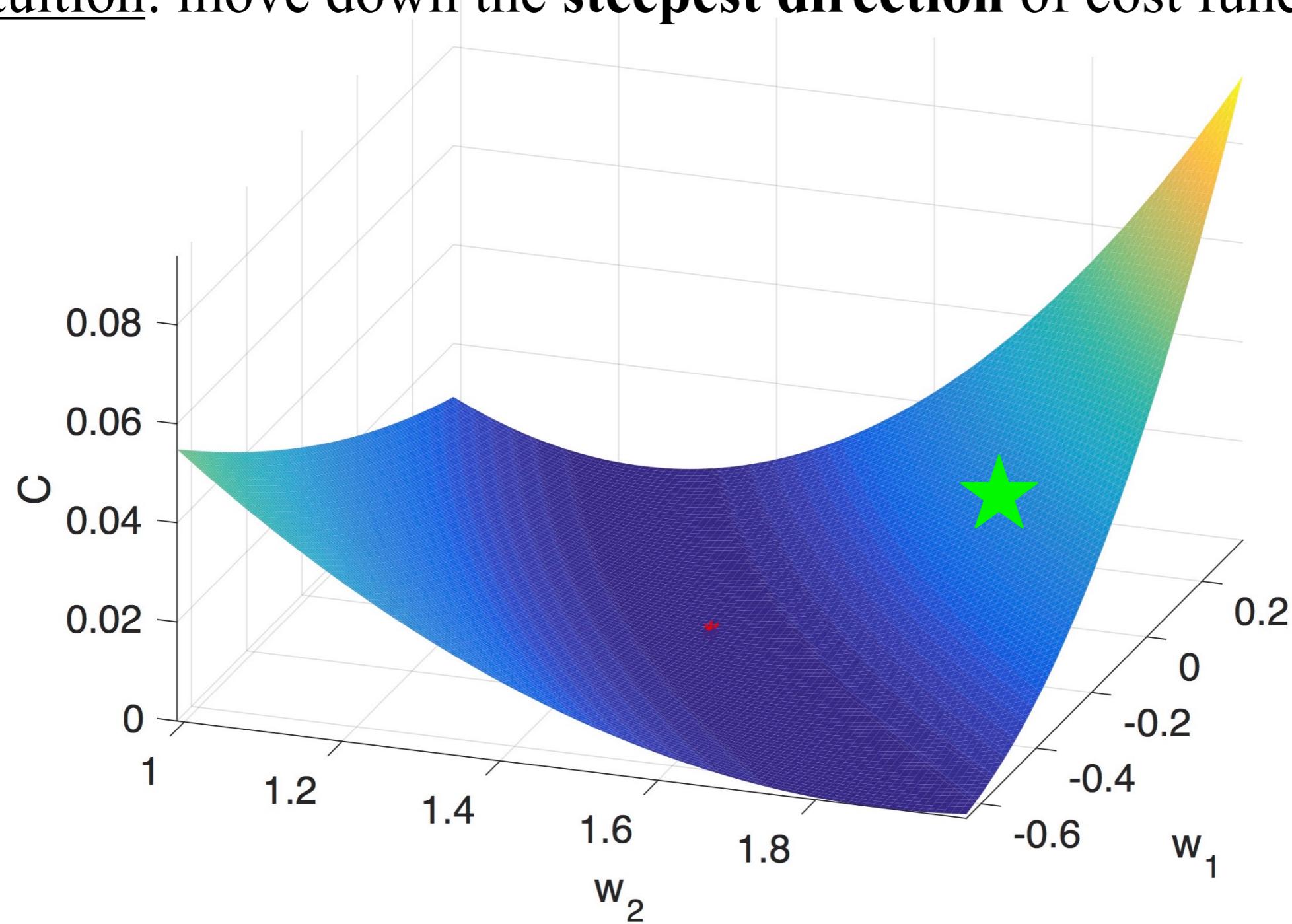
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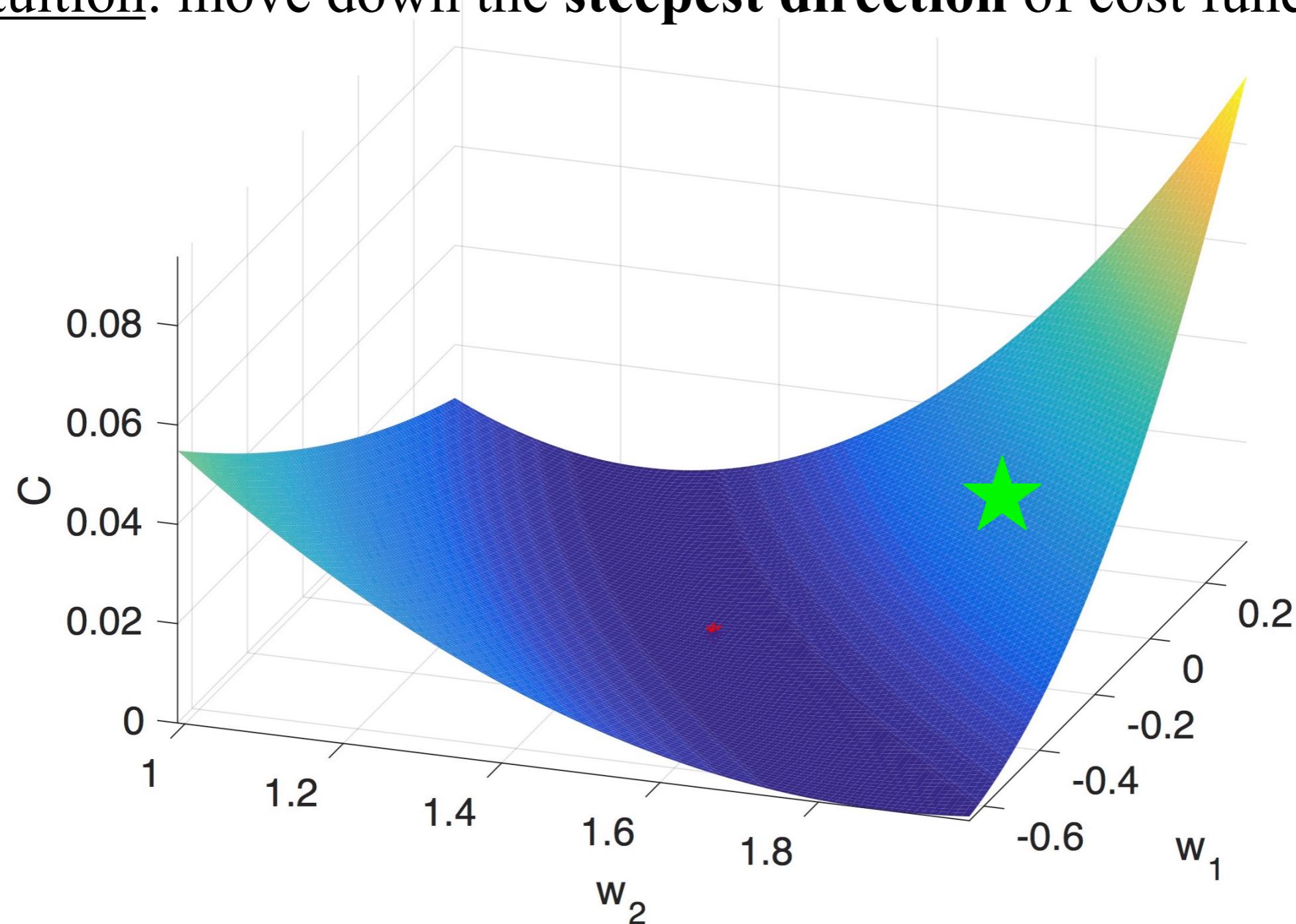
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Follow the cost function

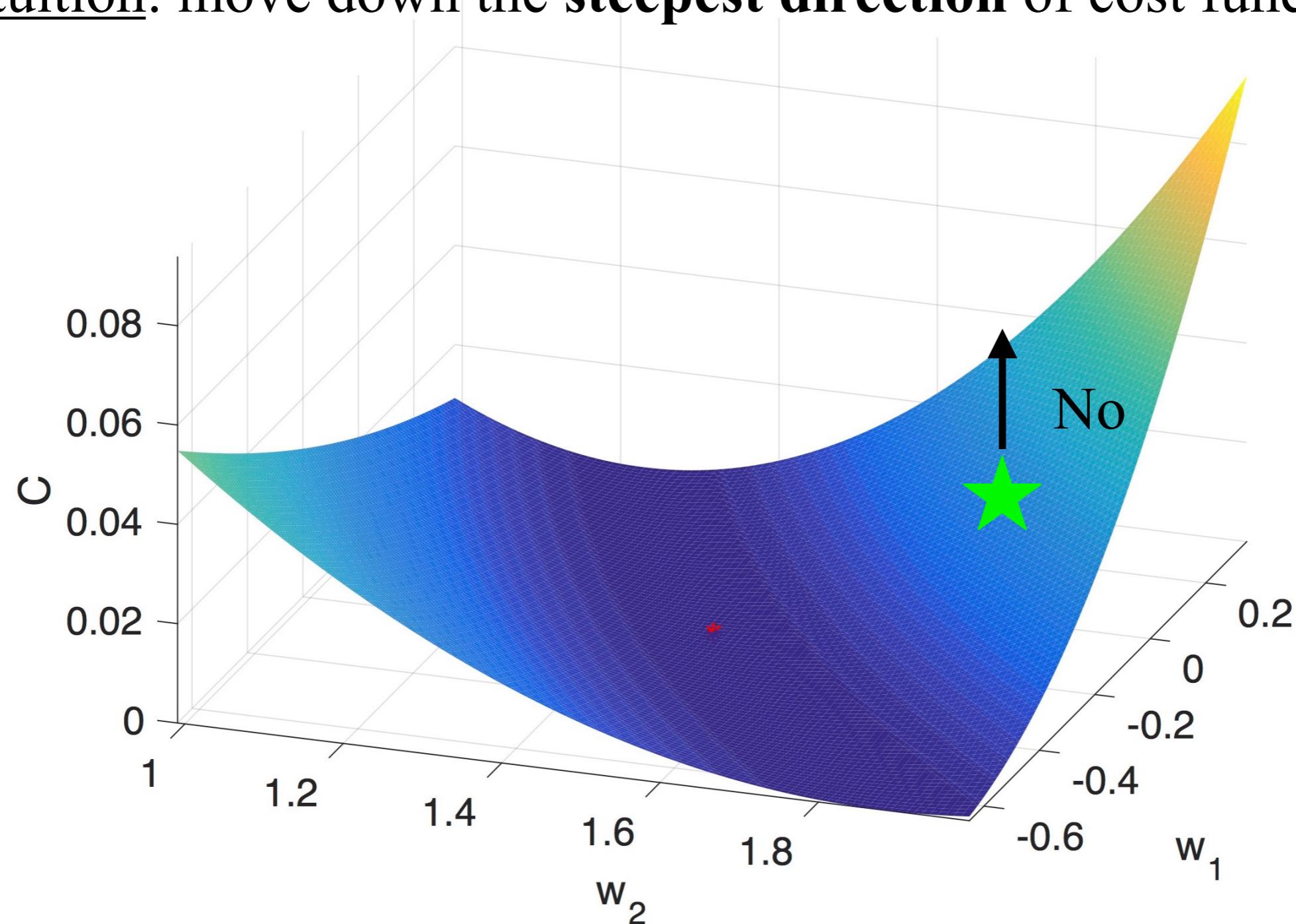
Intuition: move down the **steepest direction** of cost function



Imagine placing a marble ... where does it roll?

Follow the cost function

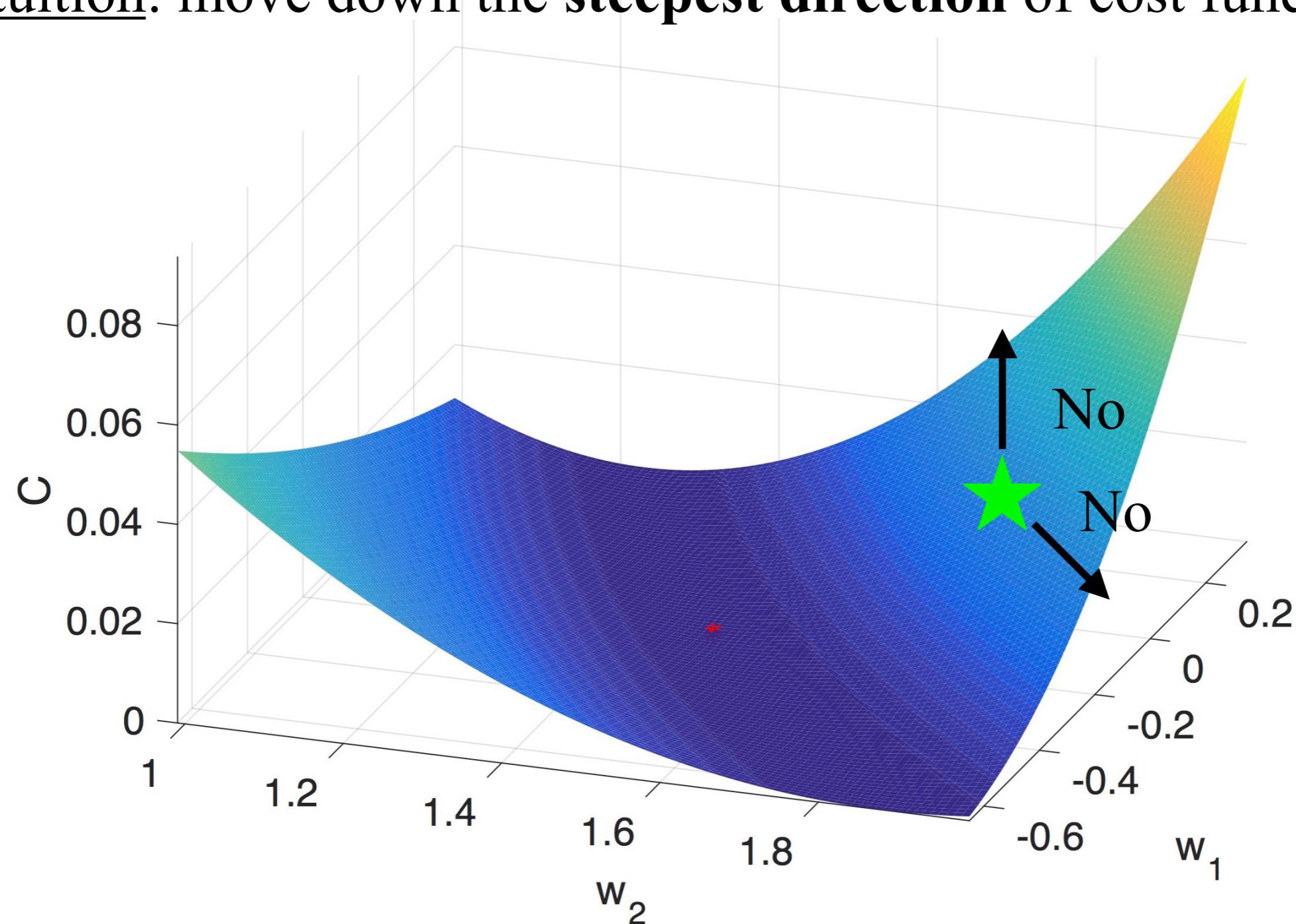
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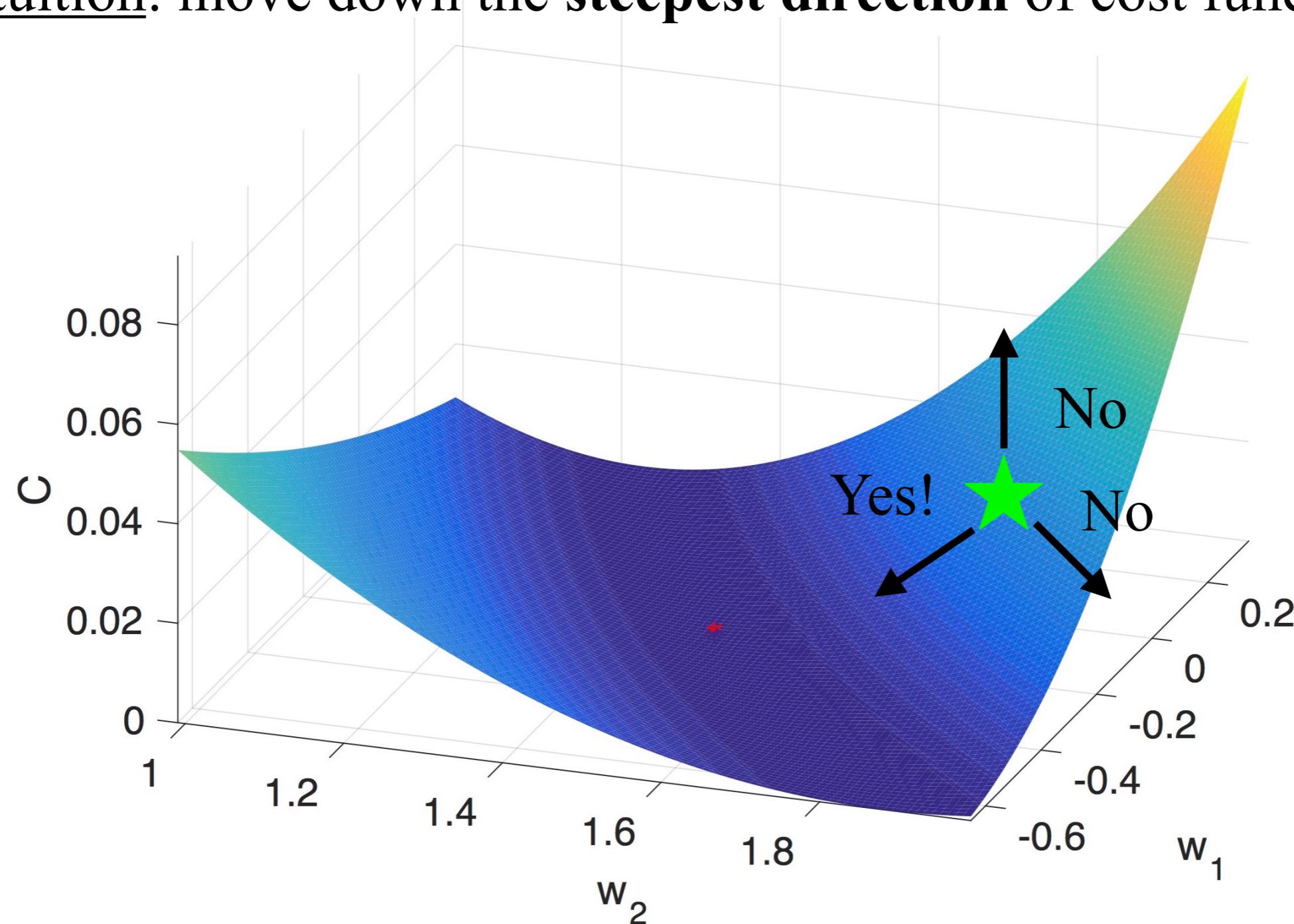
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Imagine placing a marble ... where does it roll?

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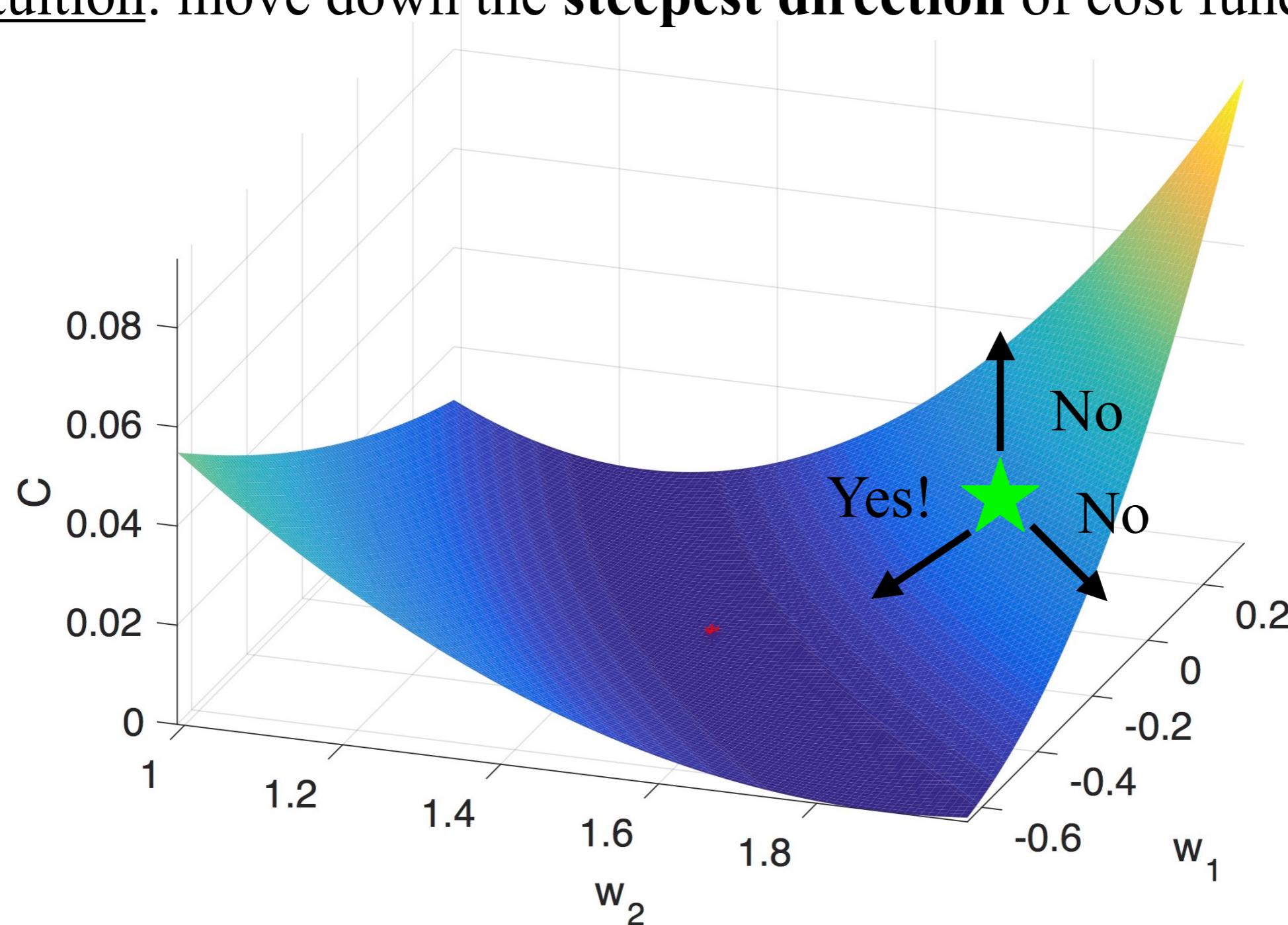
Intuition: move down the **steepest direction** of cost function



Imagine placing a marble ... where does it roll? To the minimum.

Follow the cost function

Intuition: move down the **steepest direction** of cost function

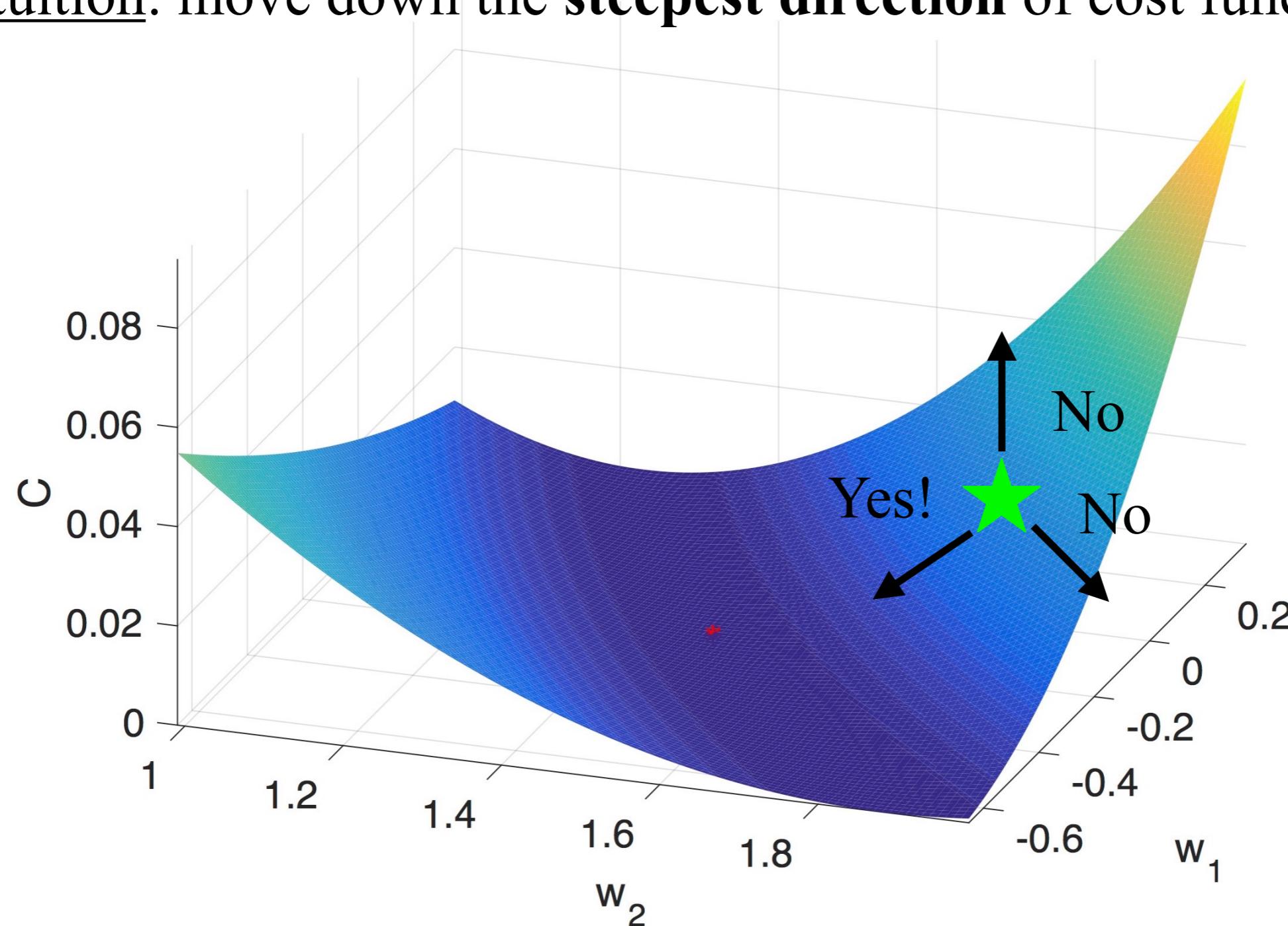


Imagine placing a marble ... where does it roll? To the minimum.

Q: How do we find the steepest direction?

Follow the cost function

Intuition: move down the **steepest direction** of cost function



Imagine placing a marble ... where does it roll? To the minimum.

Q: How do we find the steepest direction? **A:** Compute the gradient

Follow the cost function

Gradient of the cost function.

Follow the cost function

Gradient of the cost function.

- How C changes due to small changes in w_0, w_1, w_2 .

Follow the cost function

Gradient of the cost function.

- How C changes due to small changes in w_0, w_1, w_2 .

We need to compute:

Follow the cost function

Gradient of the cost function.

- How C changes due to small changes in w_0, w_1, w_2 .

We need to compute:

$$\frac{dC}{dw_0}$$

Follow the cost function

Gradient of the cost function.

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$$\frac{dC}{dw_1}$$

Follow the cost function

Gradient of the cost function.

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$$\frac{dC}{dw_1}$$

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Follow the cost function

Gradient of the cost function.

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We need to compute:

$$\frac{dC}{dw_0}$$

$$\frac{dC}{dw_1}$$

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Then, update the weights in steps proportional to the negative gradient.

Follow the cost function

Gradient of the cost function.

- How C changes due to small changes in w_0, w_1, w_2 .

We need to compute:

$$\frac{dC}{dw_0}$$

$$\frac{dC}{dw_1}$$

$$\frac{dC}{dw_2}$$

Then, update the weights in steps proportional to the negative gradient.

$$w_0 \leftarrow w_0 - \alpha \frac{dC}{dw_0}$$

$$w_1 \leftarrow w_1 - \alpha \frac{dC}{dw_1}$$

$$w_2 \leftarrow w_2 - \alpha \frac{dC}{dw_2}$$

Follow the cost function

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“new” weight

Follow the cost function

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We need to compute:

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Then, update the weights in steps proportional to the negative gradient.

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↑
“new” weight

$$w_1 \leftarrow w_1 - \alpha \frac{dC}{dw_1}$$

↑
“original” weight

$$w_2 \leftarrow w_2 - \alpha \frac{dC}{dw_2}$$

Follow the cost function

Gradient of the cost function.

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We need to compute:

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Then, update the weights in steps proportional to the negative gradient.

$$w_0 \leftarrow w_0 - \alpha \frac{dC}{dw_0}$$

“new” weight

$$w_1 \leftarrow w_1$$



$$-\alpha \frac{dC}{dw_1}$$

update

“original” weight

$$w_2 \leftarrow w_2 - \alpha \frac{dC}{dw_2}$$

Follow the cost function

Gradient of the cost function.

- How C changes due to small changes in w_0, w_1, w_2 .

We need to compute:

$$\frac{dC}{dw_0}$$

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update

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$$w_2 \leftarrow w_2 - \alpha \frac{dC}{dw_2}$$

steepest
direction of C

Follow the cost function

Gradient of the cost function.

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We need to compute:

$$\frac{dC}{dw_0}$$

$$\frac{dC}{dw_1}$$

$$\frac{dC}{dw_2}$$

Then, update the weights in steps proportional to the negative gradient.

$$w_0 \leftarrow w_0 - \alpha \frac{dC}{dw_0}$$

↑
“new” weight

$$w_1 \leftarrow w_1$$

$$-\alpha \frac{dC}{dw_1}$$

update

↑
“original” weight

$$w_2 \leftarrow w_2 - \alpha \frac{dC}{dw_2}$$

steepest
direction of C

α = learning rate

Follow the cost function

Gradient of the cost function.

- How C changes due to small changes in w_0, w_1, w_2 .

We need to compute:

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Then, update the weights in steps proportional to the negative gradient.

$$w_0 \leftarrow w_0 - \alpha \frac{dC}{dw_0}$$

↑
“new” weight

$$w_1 \leftarrow w_1 - \alpha \frac{dC}{dw_1}$$

↑
“original” weight

update

$$w_2 \leftarrow w_2 - \alpha \frac{dC}{dw_2}$$

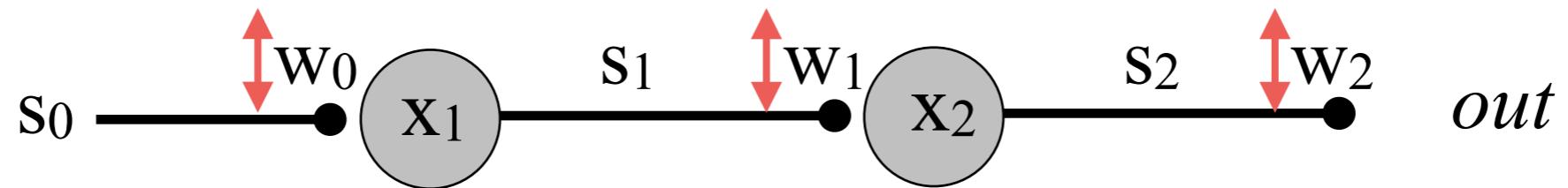
↑
steepest
direction of C

Procedure: gradient descent

α = learning rate

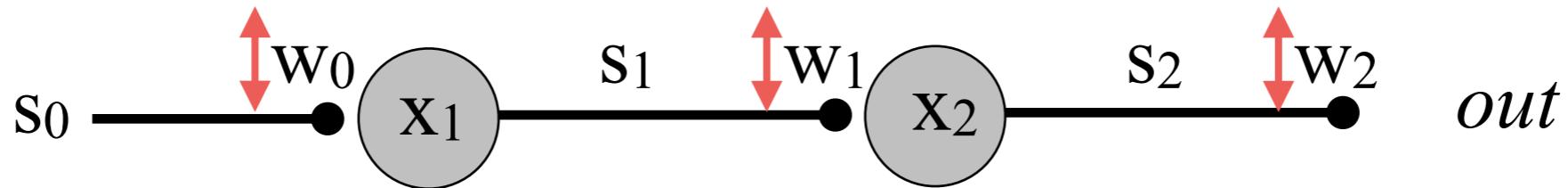
Follow the cost function

Q: How does the cost function change due to changes in weights?



Follow the cost function

Q: How does the cost function change due to changes in weights?

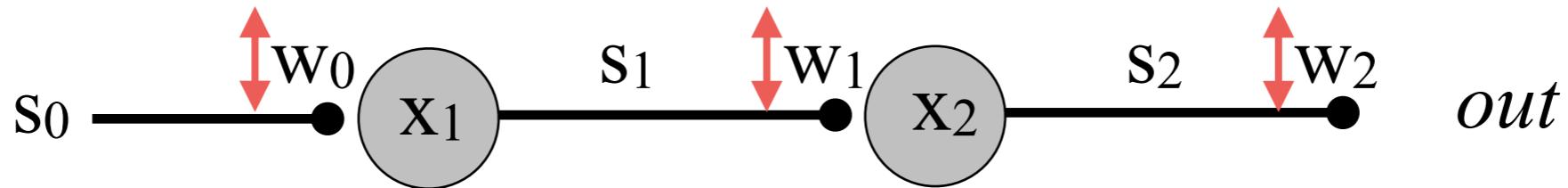


Consider:

$$\frac{dC}{dw_2}$$

Follow the cost function

Q: How does the cost function change due to changes in weights?

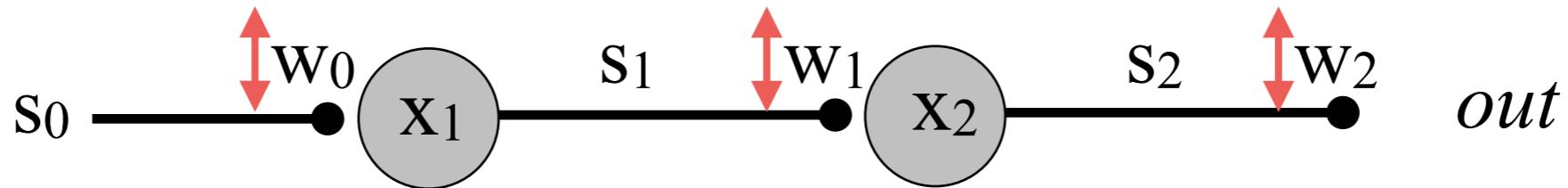


Consider:

$$\frac{dC}{dw_2} = ???$$

Follow the cost function

Q: How does the cost function change due to changes in weights?



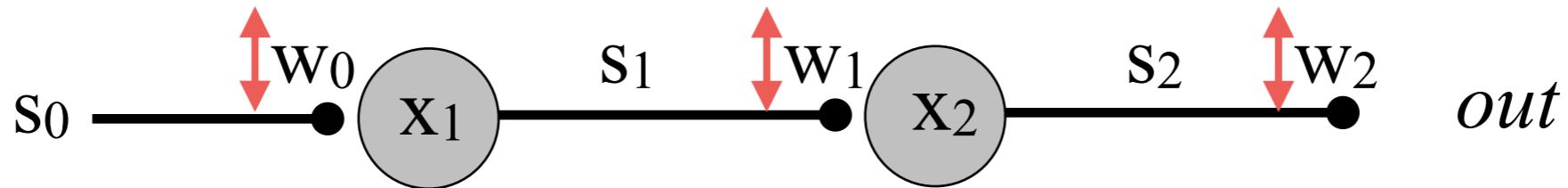
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$$\frac{dC}{dw_2} = ???$$

We know how C depends on out : $C = \frac{1}{2}(\text{target} - out)^2$

Follow the cost function

Q: How does the cost function change due to changes in weights?



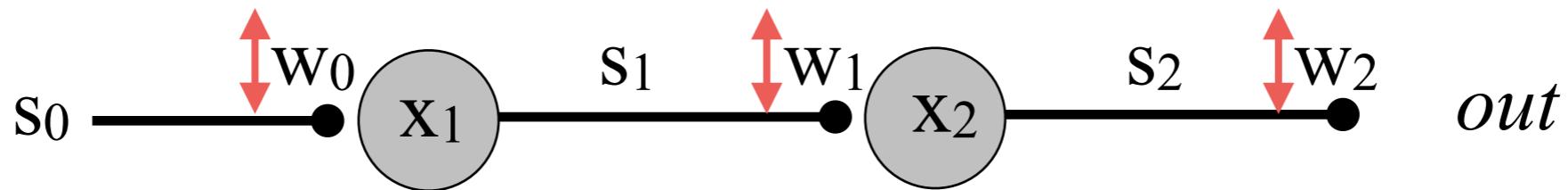
Consider:

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Consider:

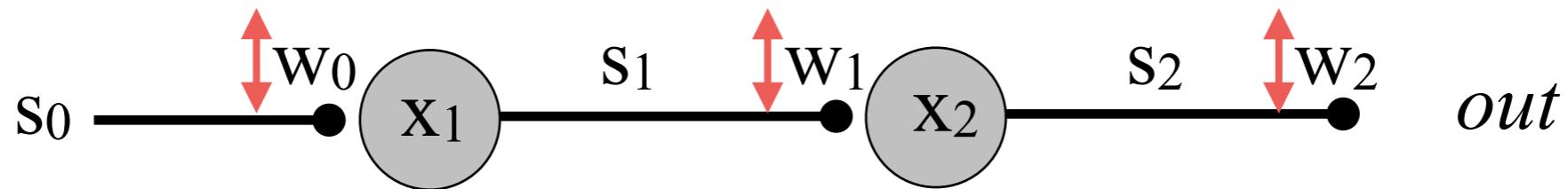
$$\frac{dC}{dw_2} = ???$$

We know how C depends on out : $C = \frac{1}{2}(\text{target} - out)^2$

And we know how out depends on w_2 :

Follow the cost function

Q: How does the cost function change due to changes in weights?



Consider:

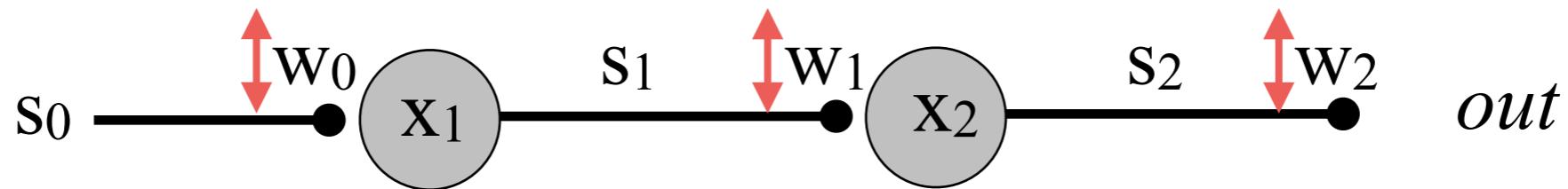
$$\frac{dC}{dw_2} = ???$$

We know how C depends on out : $C = \frac{1}{2}(\text{target} - out)^2$

And we know how out depends on w_2 : $out = s_2 w_2$

Follow the cost function

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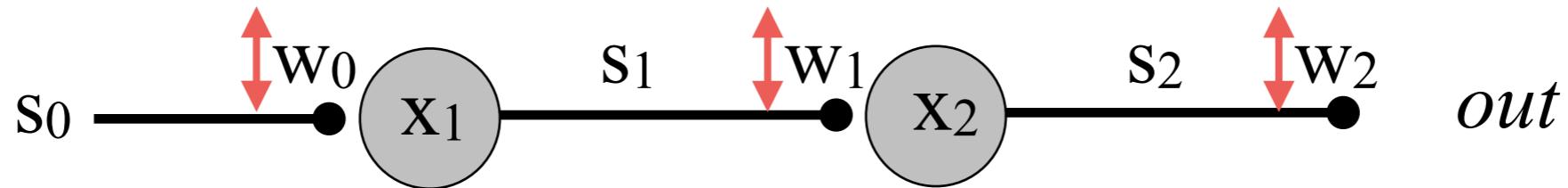
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To compute the derivative,

Follow the cost function

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To compute the derivative, use the **chain rule** ...

Follow the cost function

Our goal:

$$\frac{d C}{d w_2} = ???$$

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$$\frac{d C}{d w_2} = \frac{d C}{d out}$$

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We've already solved the first derivative!

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Slide 8

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Let's compute the next derivative ...

Follow the cost function

$$\frac{d \text{out}}{d w_2} = ???$$

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Follow the cost function

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So,

$$\frac{d \text{out}}{d w_2} = \frac{d(s_2 w_2)}{d w_2}$$

Follow the cost function

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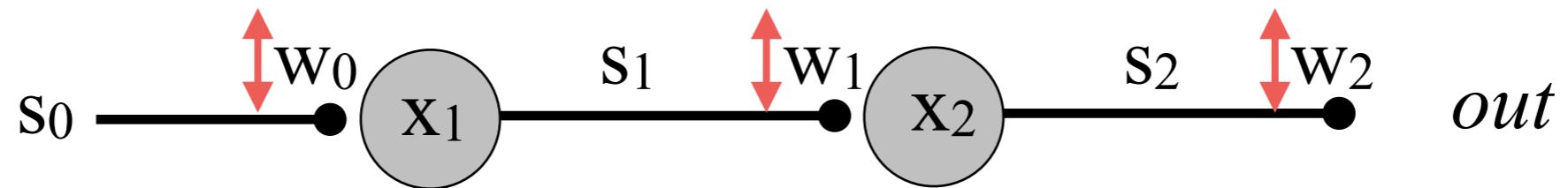
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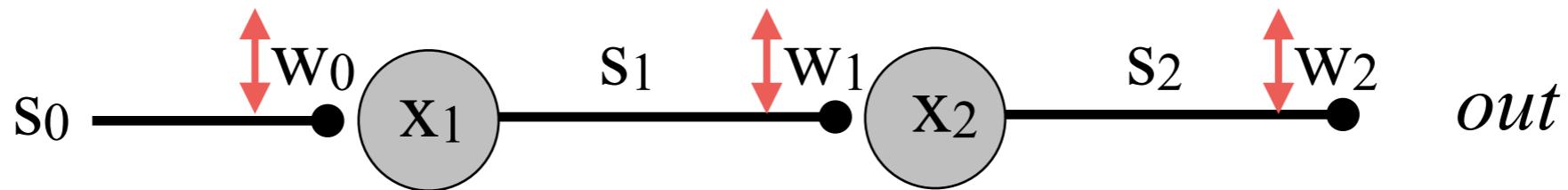
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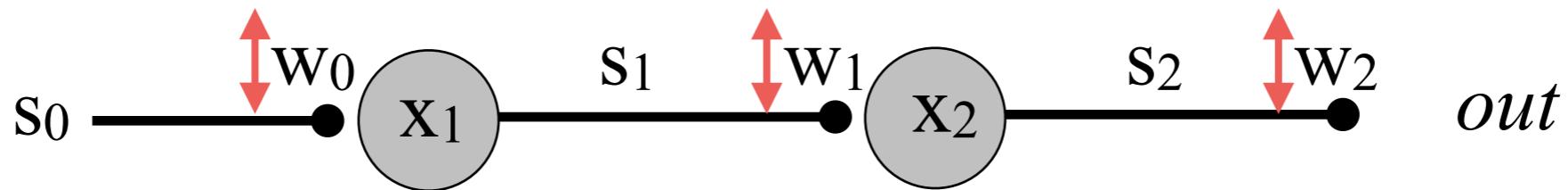


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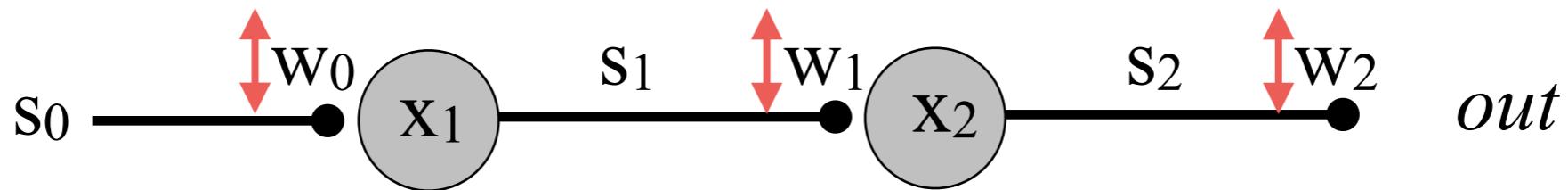
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How bad we're doing.

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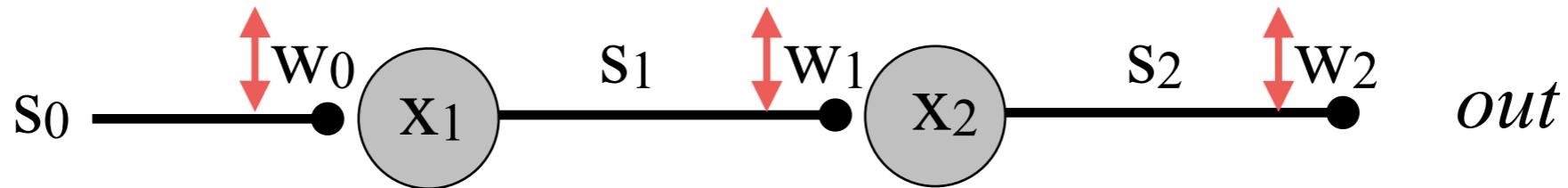
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output from x_2

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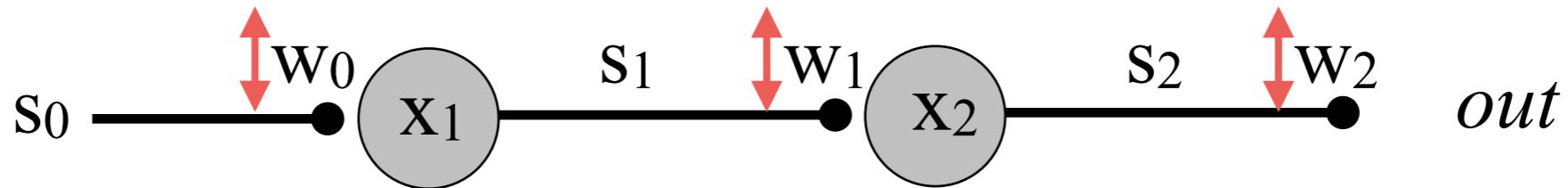
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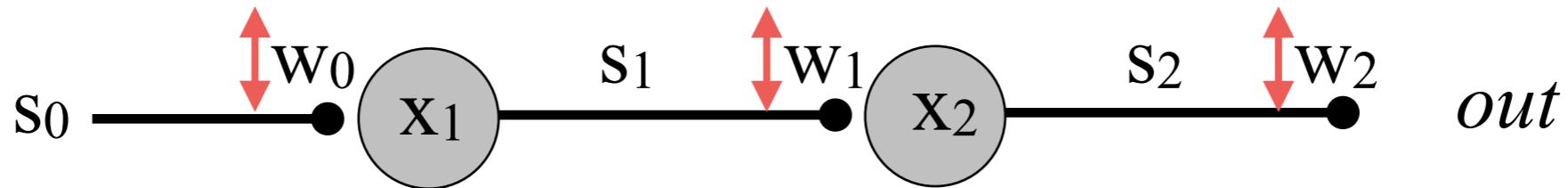
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Update the weight w_2 :

$$w_2 \leftarrow w_2 - \alpha \frac{dC}{dw_2}$$

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↑
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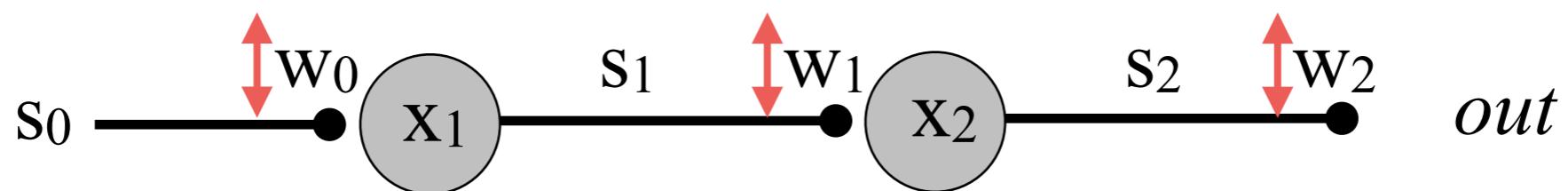
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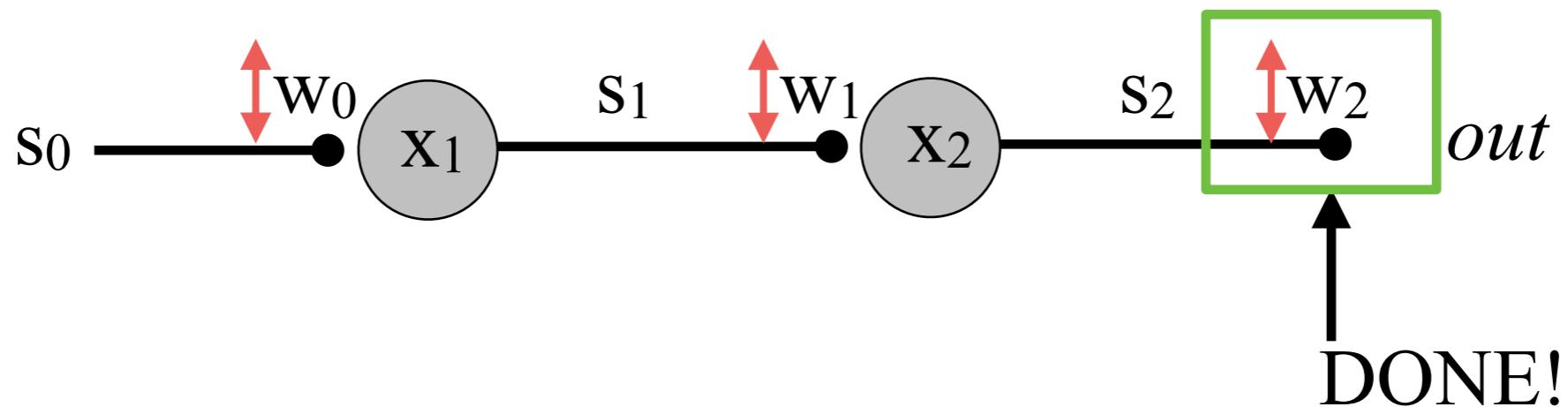
Follow the cost function

So, we've now found an equation to update one of the weights w_2 that acts to minimize the cost function.



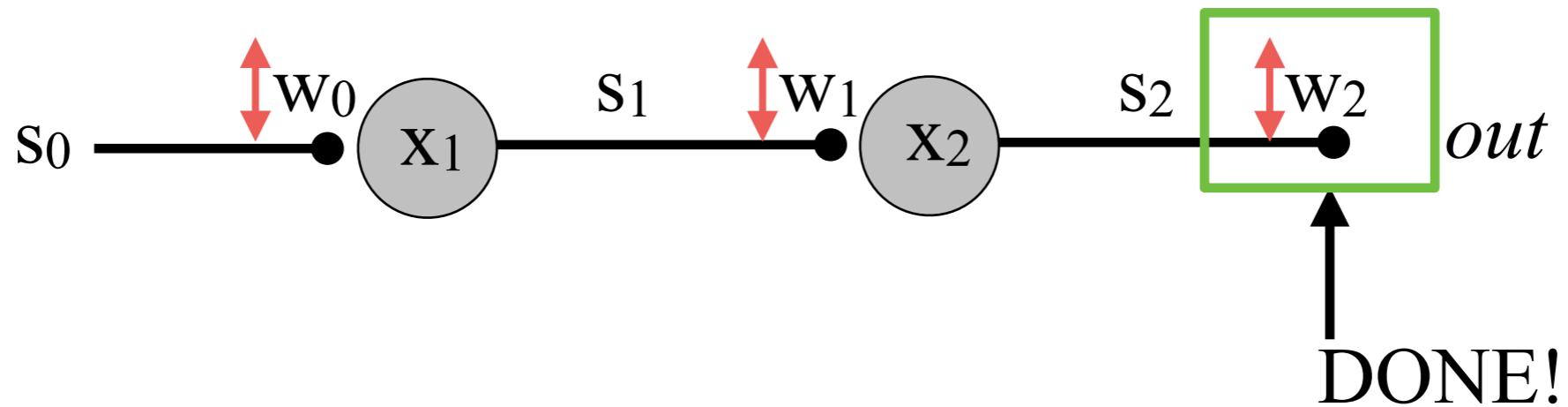
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So, we've now found an equation to update one of the weights w_2 that acts to minimize the cost function.



Follow the cost function

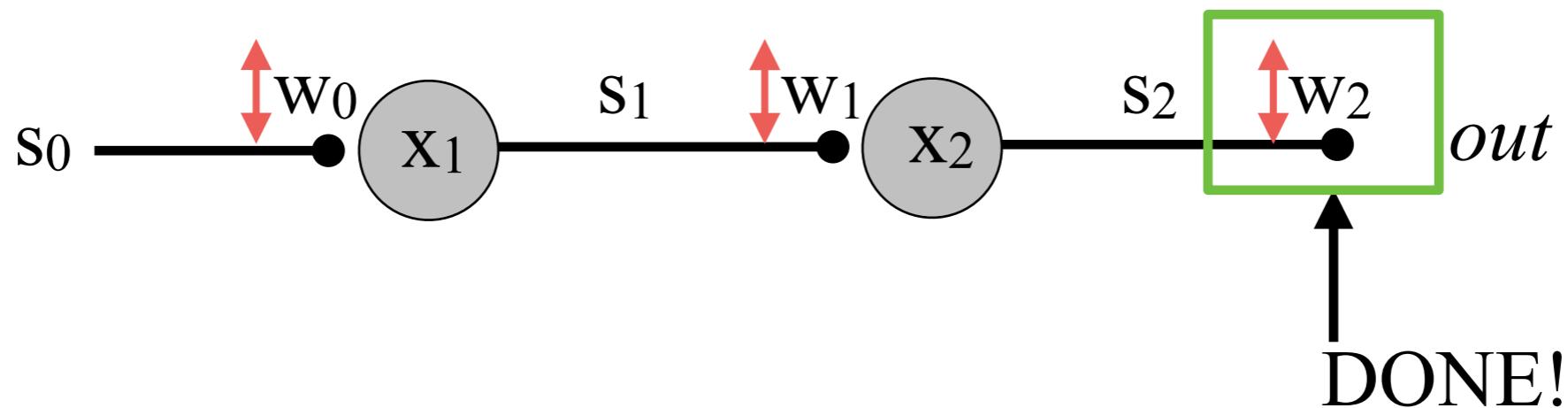
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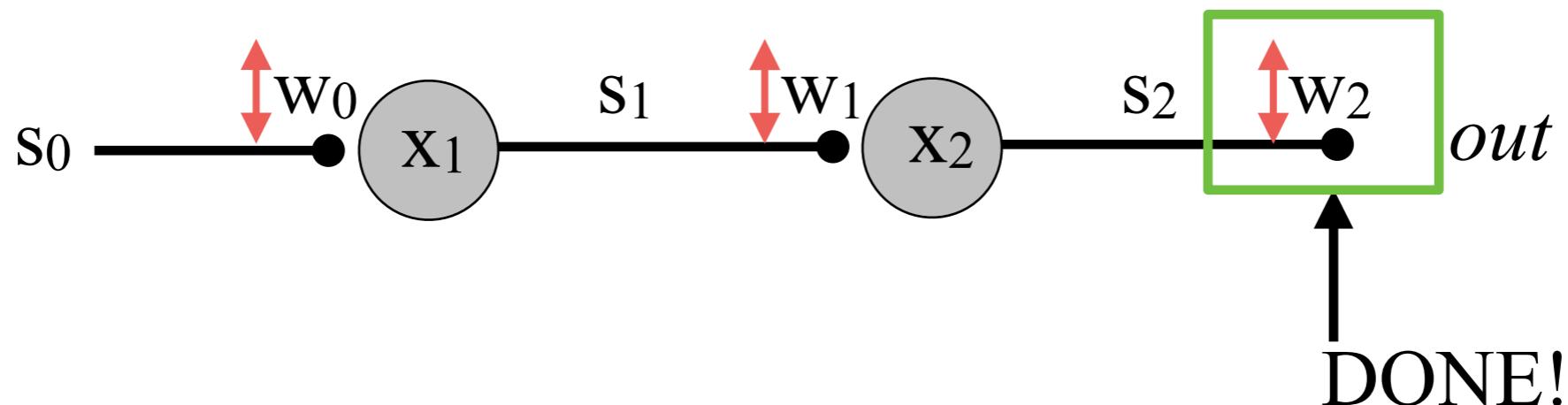


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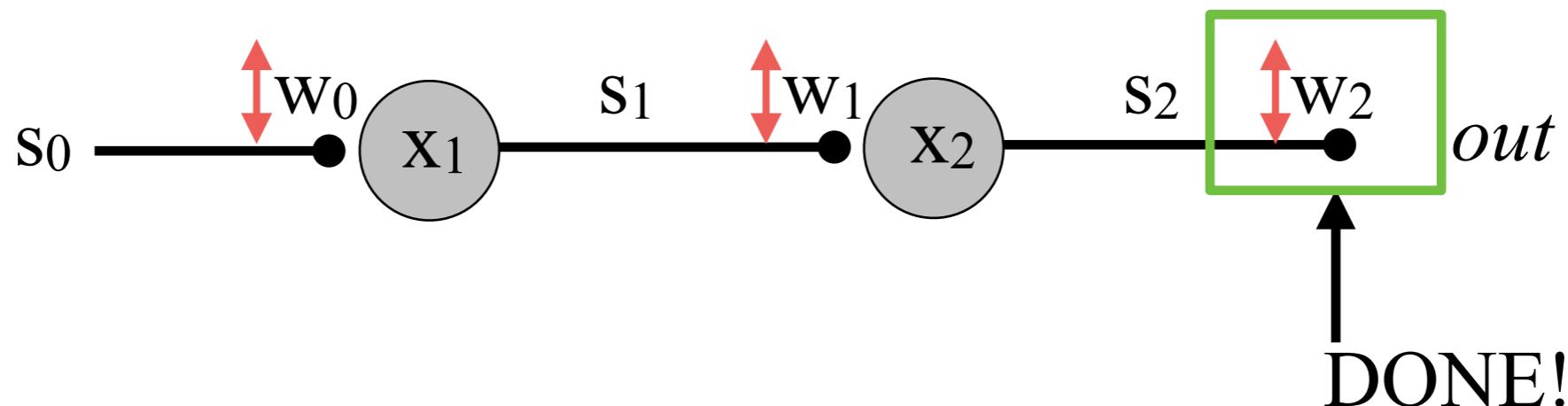
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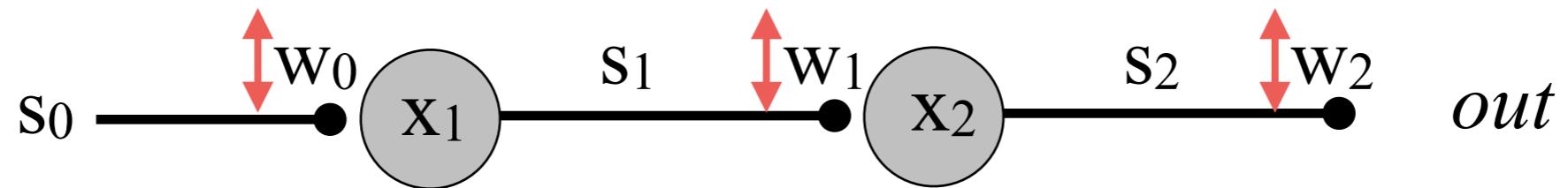
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A: Yes we can.

It'll seem difficult, but we've already done the hard work ...

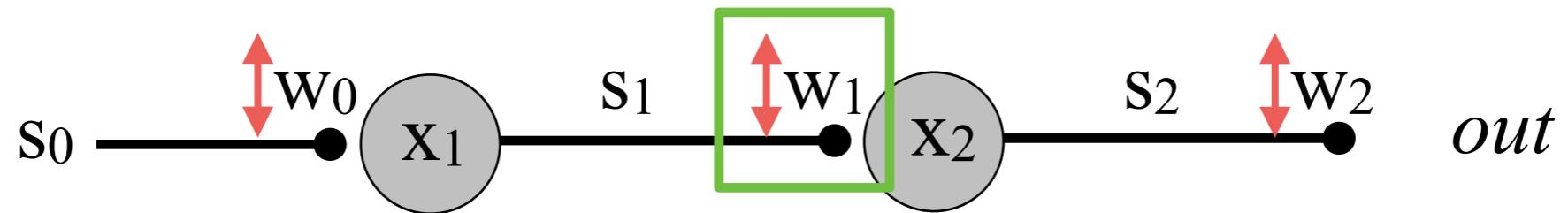
Follow the cost function

Q: How does the cost function change due to change in w_1 ?



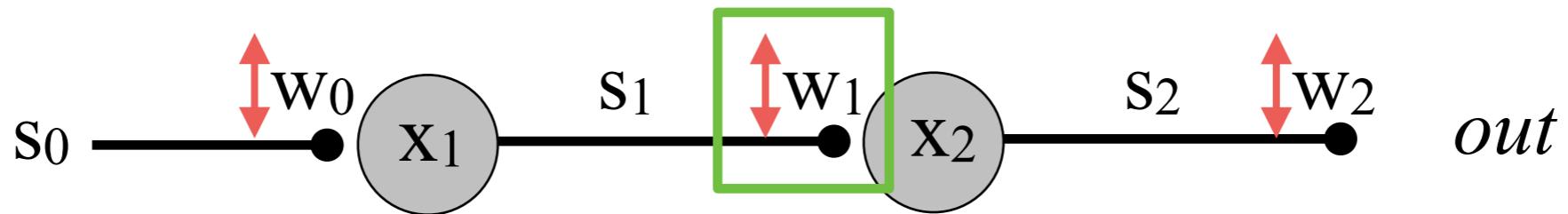
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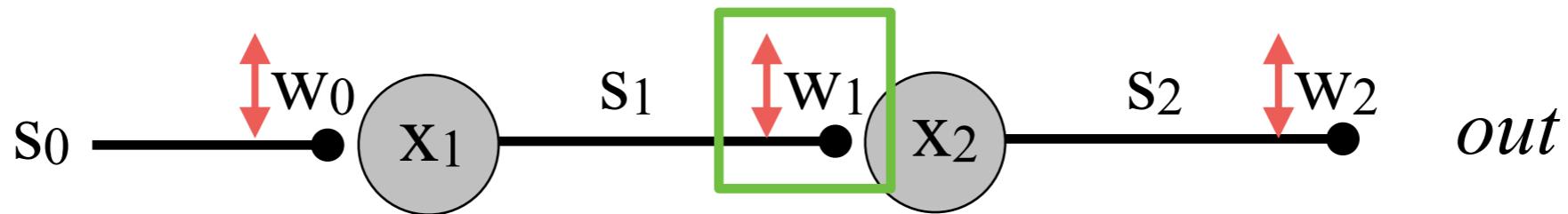
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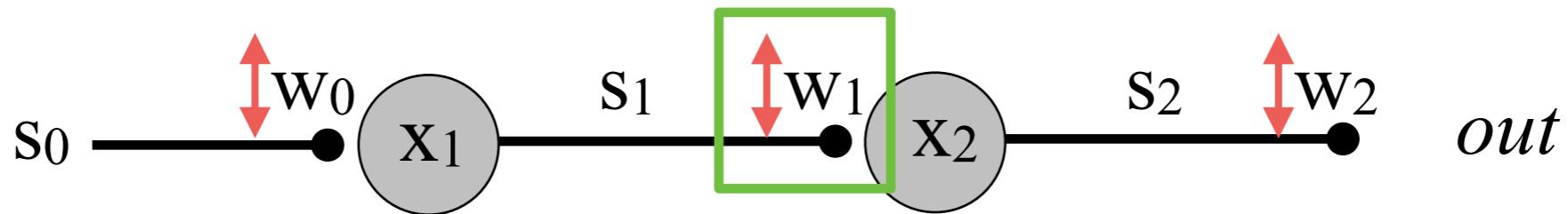


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We don't know this ...
but can write it using
things we do know.

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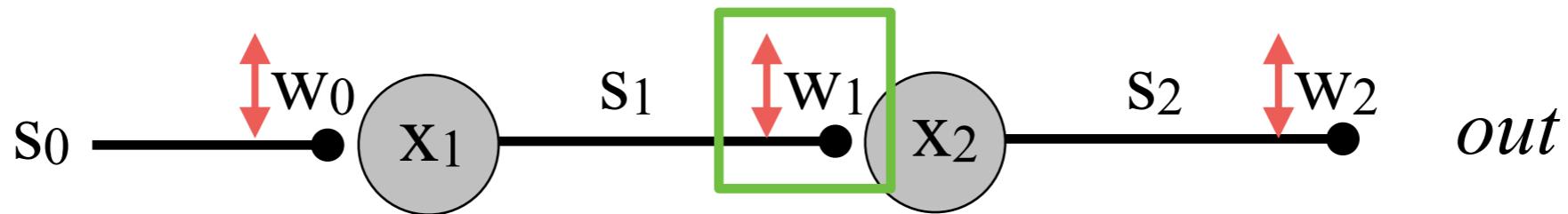
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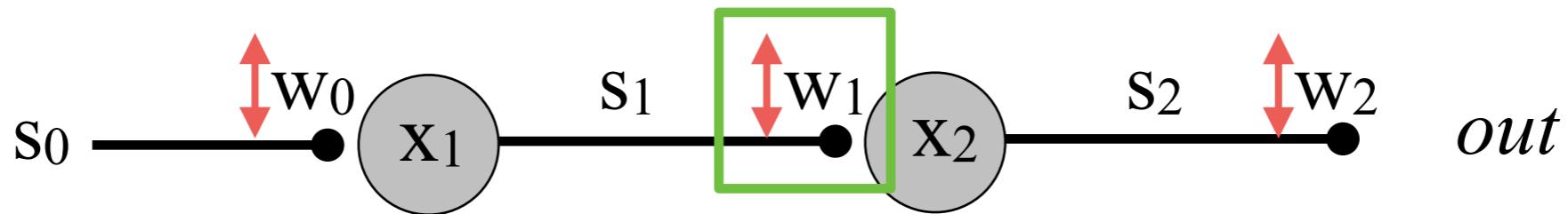
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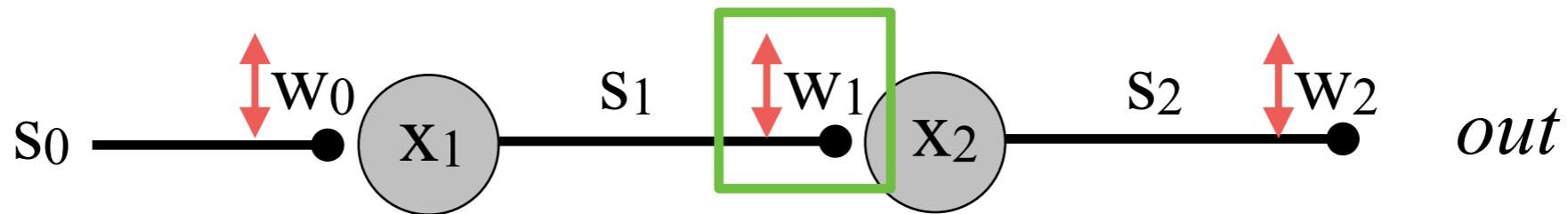
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$$\frac{dC}{dw_1} = \frac{dC}{dout} \frac{dout}{dw_1}$$

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$$\frac{dC}{dout} = out - \text{target} \quad (\text{Slide 8})$$

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$$\frac{dout}{dw_1}$$

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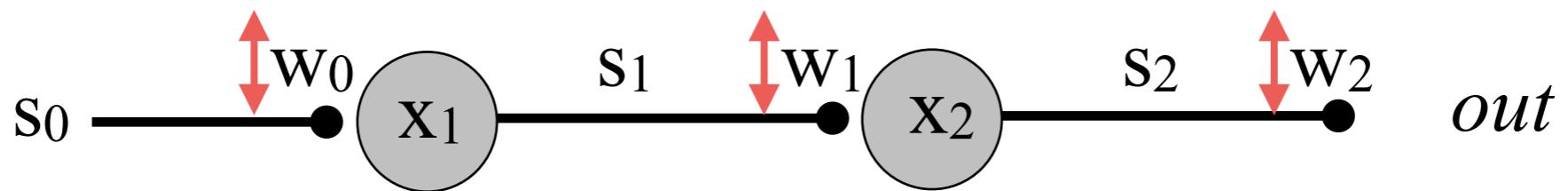
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$$\frac{dC}{dout} = out - \text{target} \quad (\text{Slide } 8)$$

$$\frac{d out}{d w_1} = w_2 s_2 (1 - s_2) s_1 \quad (\text{Slide } 23)$$

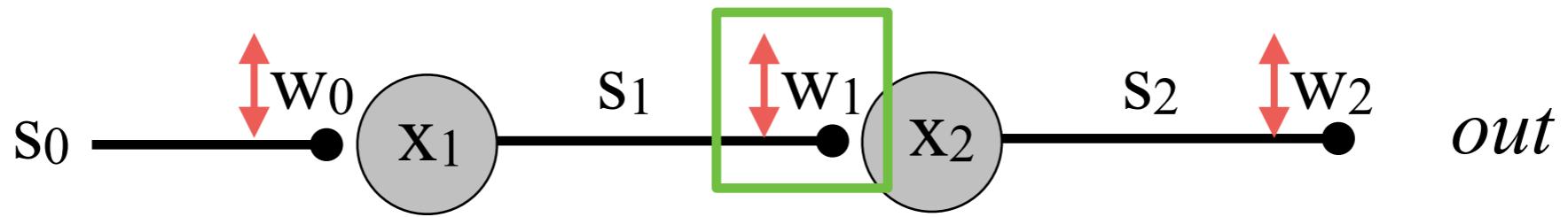
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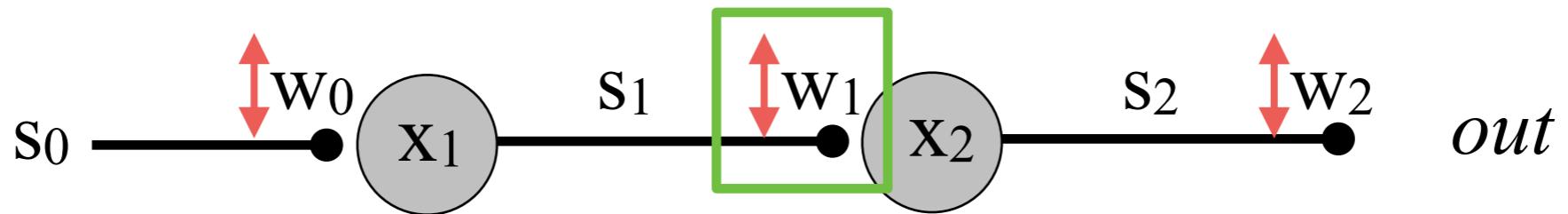
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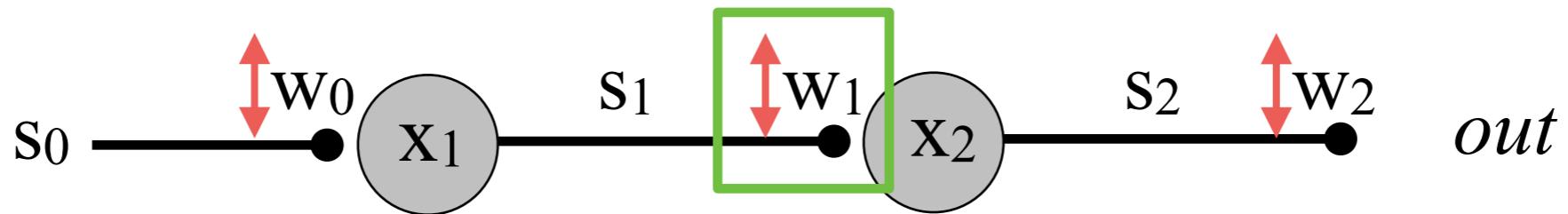
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We conclude:

Follow the cost function

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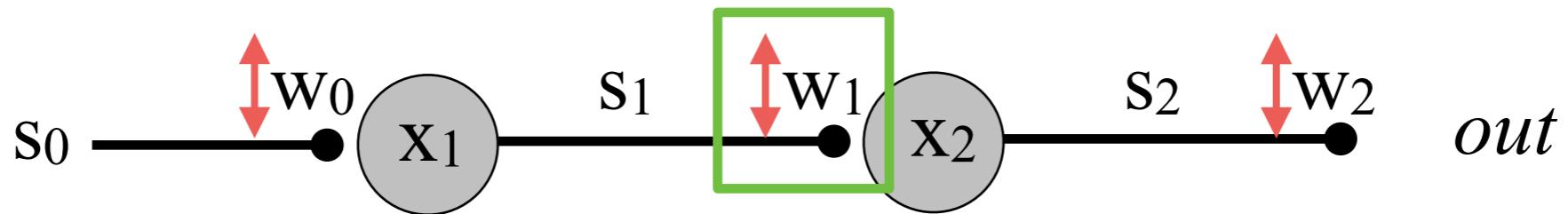


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$$\frac{dC}{dw_1} = \frac{dC}{d\text{out}} \frac{d\text{out}}{dw_1}$$

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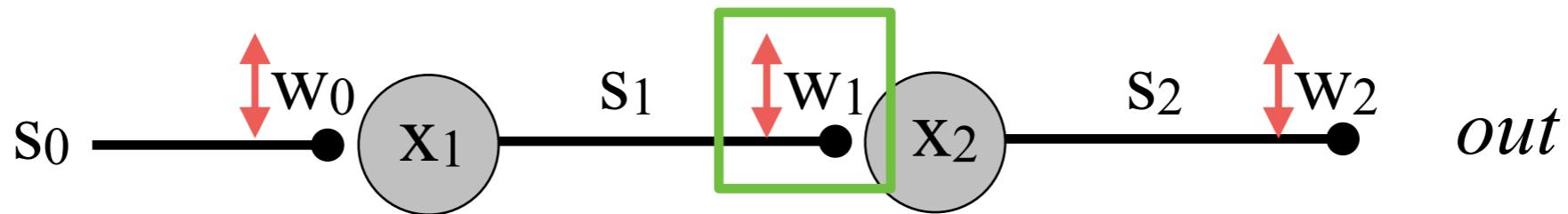
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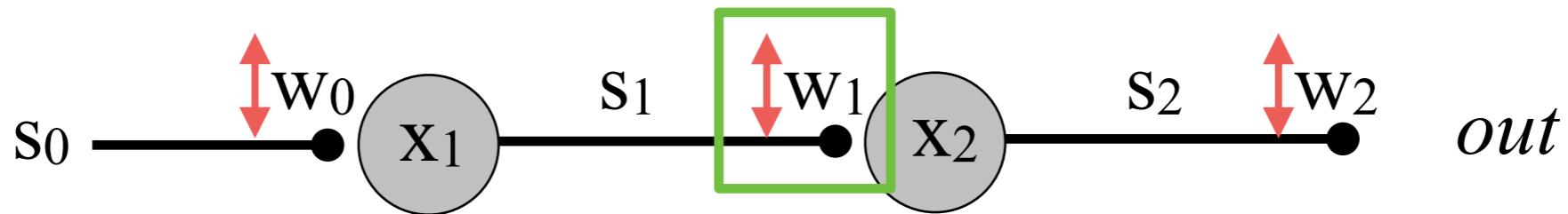
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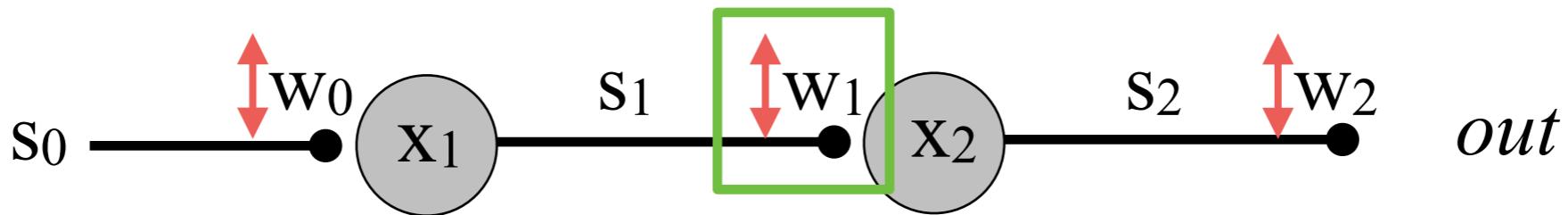
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$$\frac{dC}{dw_1} = (\text{out} - \text{target}) w_2 s_2 (1 - s_2) s_1$$

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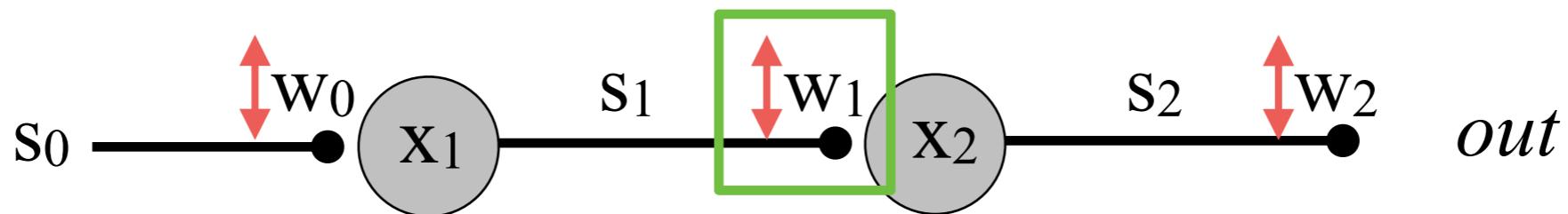
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How bad we're doing.

Follow the cost function

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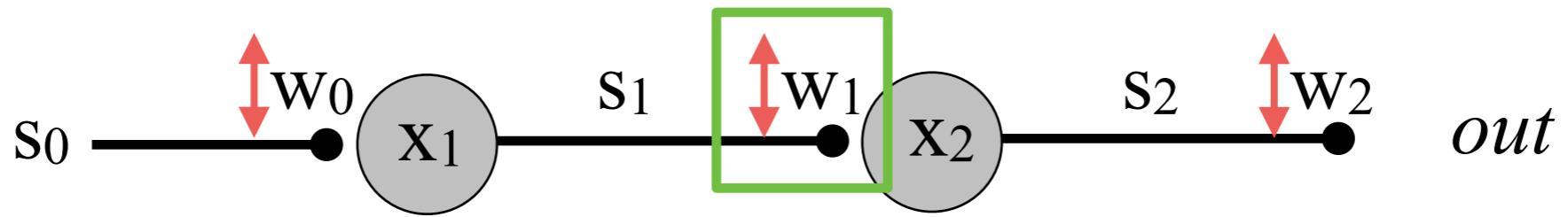
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complicated expression
of outputs and weight

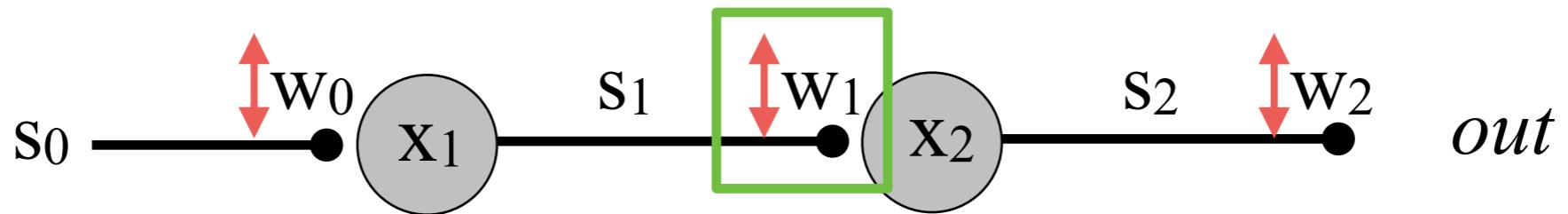
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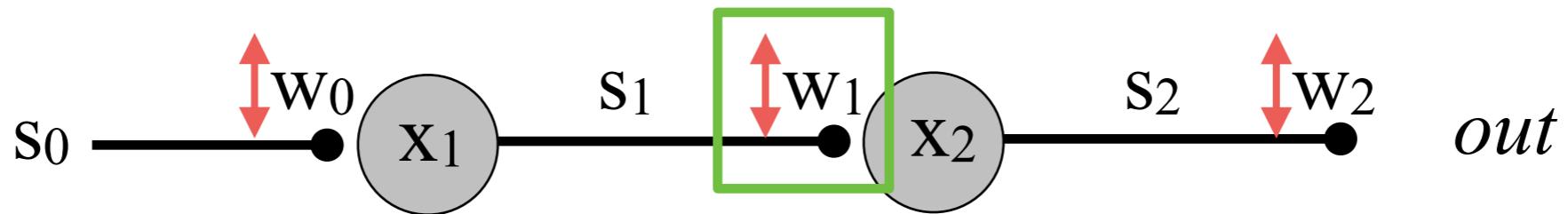
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Update the weight w_1 :

Follow the cost function

Q: How does the cost function change due to change in w_1 ?

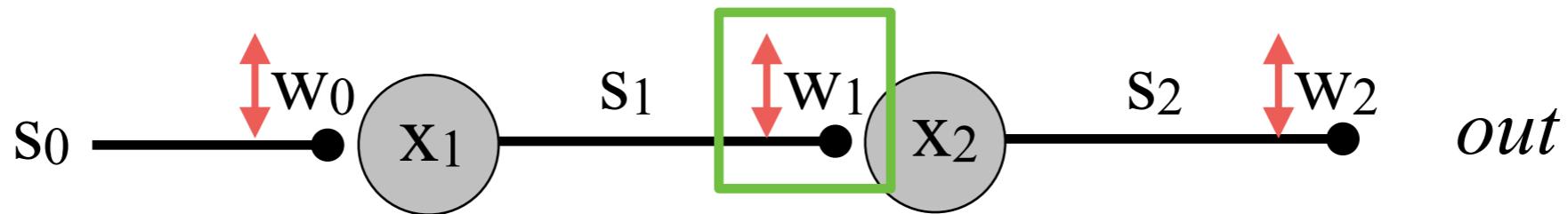


Update the weight w_1 :

$$w_1 \leftarrow w_1 - \alpha \frac{dC}{dw_1}$$

Follow the cost function

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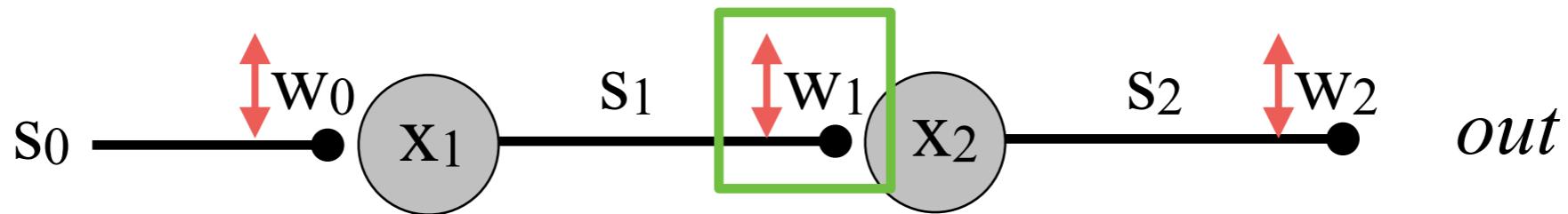
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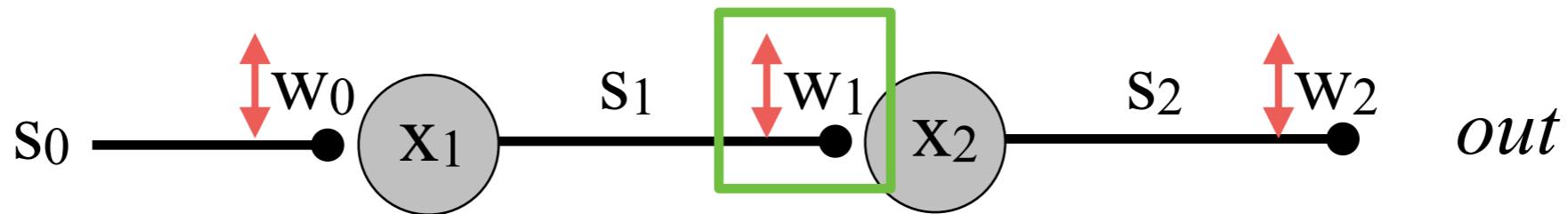
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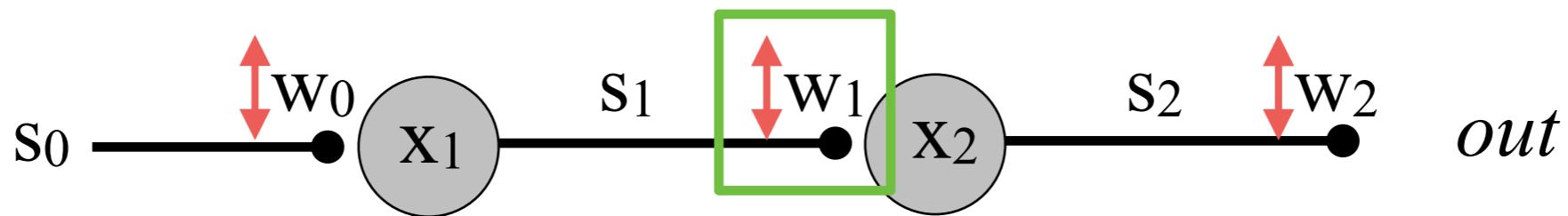
substitute in for this!

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$$w_1 \leftarrow w_1 - \alpha(out - \text{target})w_2s_2(1 - s_2)s_1$$

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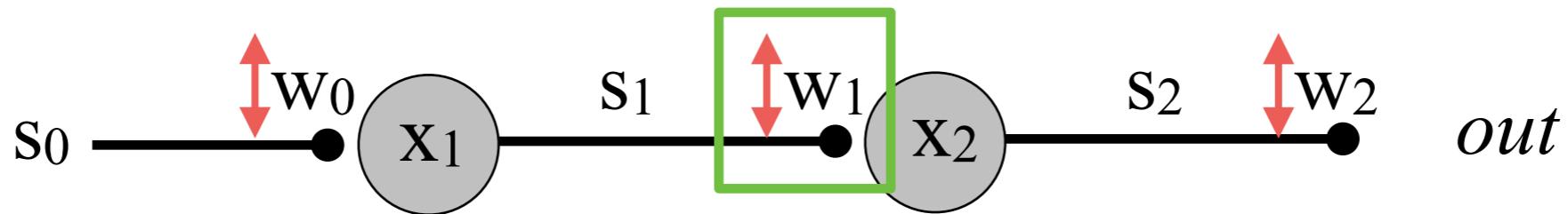
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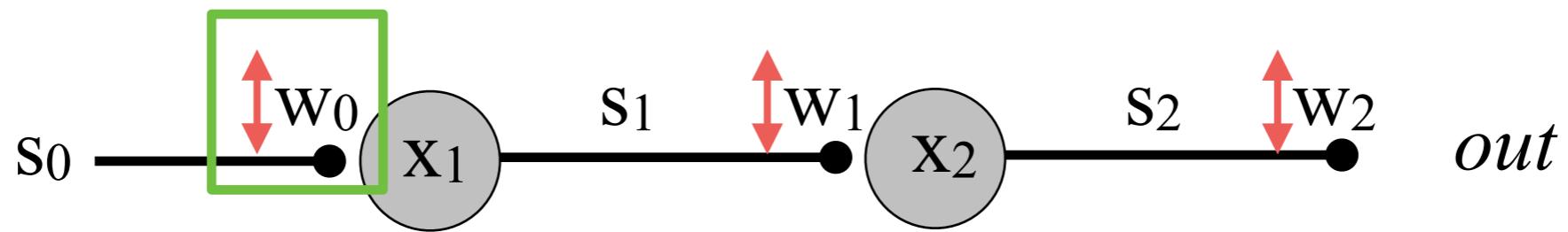
substitute in for this!

$$w_1 \leftarrow w_1 - \alpha (out - \text{target}) w_2 s_2 (1 - s_2) s_1$$

Q: What happens when $out = \text{target}$?

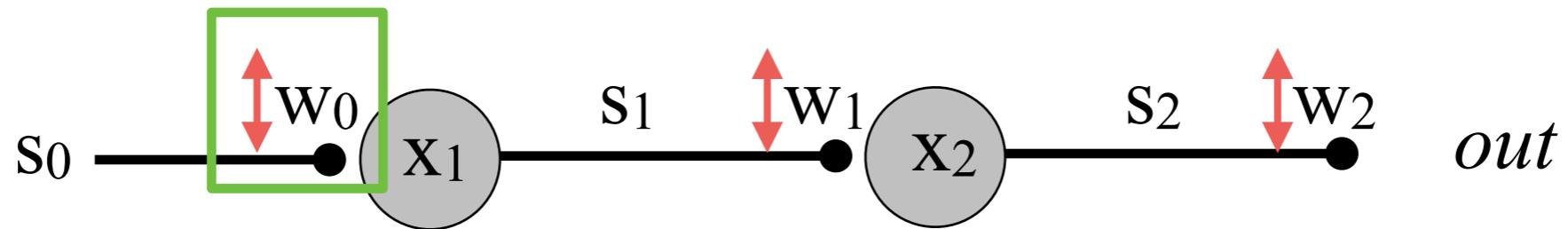
Follow the cost function

Q: How does the cost function change due to change in w_0 ?



Follow the cost function

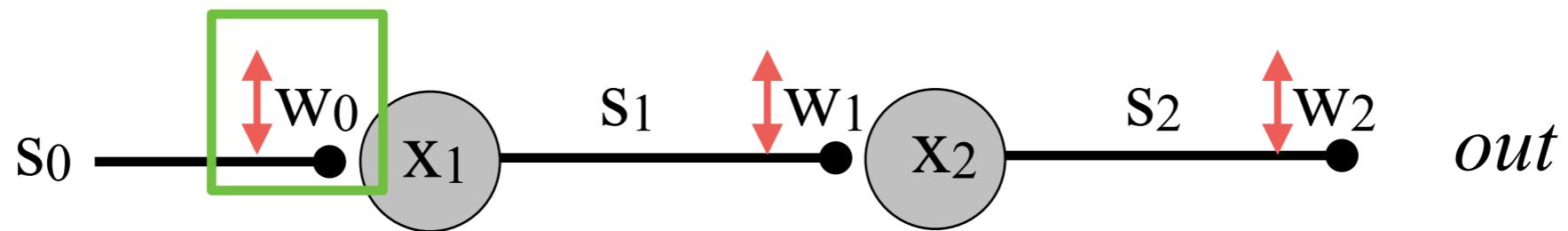
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$$\frac{dC}{dw_0} =$$

Follow the cost function

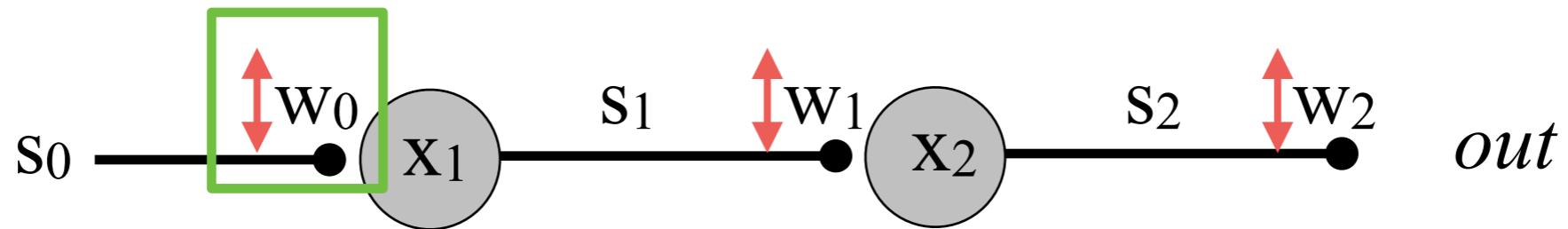
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Follow the cost function

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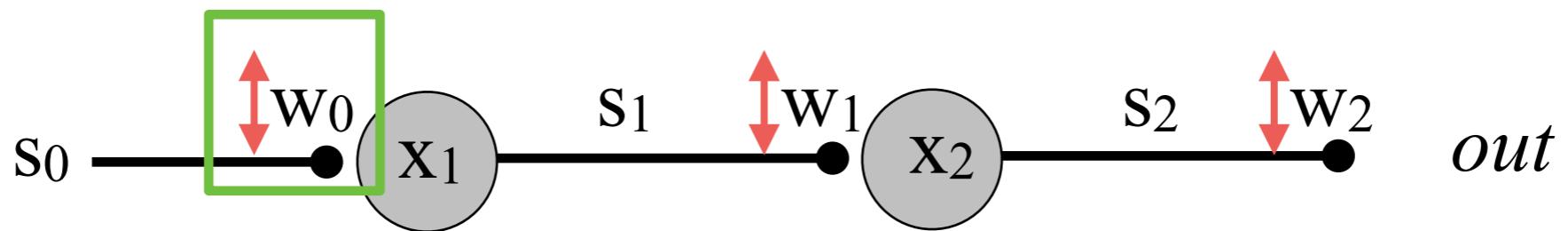


$$\frac{dC}{dw_0} = ???$$

Try it ...

Follow the cost function

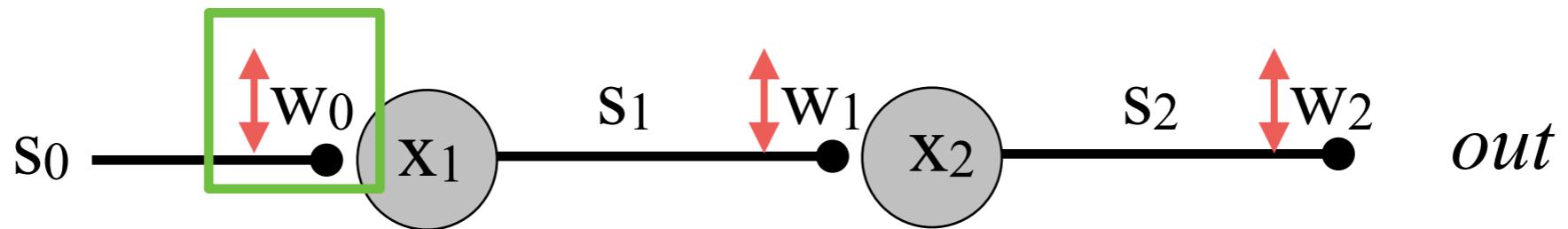
Q: How does the cost function change due to change in w_0 ?



We conclude:

Follow the cost function

Q: How does the cost function change due to change in w_0 ?

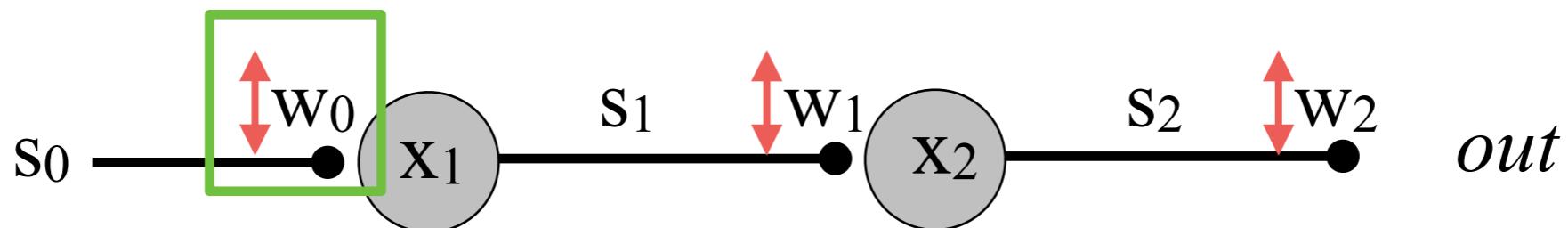


We conclude:

$$\frac{dC}{dw_0} = \text{???}$$

Follow the cost function

Q: How does the cost function change due to change in w_0 ?



We conclude:

$$\frac{dC}{dw_0} = \text{??}$$

and

$$w_0 \leftarrow w_0 - \alpha \frac{dC}{dw_0} \quad \text{where we need to replace last term ...}$$

Put it all together

Prescription to find the weights that minimize cost function
(so that out is near target).

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Put it all together

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Put it all together

Prescription (continued)

$$w_0 \leftarrow w_0 - \alpha(\text{out} - \text{target})w_2s_2(1 - s_2)w_1s_1(1 - s_1)s_0$$

Put it all together

Prescription (continued)

3. Update the weights

$$w_0 \leftarrow w_0 - \alpha(\text{out} - \text{target})w_2s_2(1 - s_2)w_1s_1(1 - s_1)s_0$$

Put it all together

Prescription (continued)

3. Update the weights

$$w_2 \leftarrow w_2 - \alpha(out - \text{target})s_2$$

$$w_0 \leftarrow w_0 - \alpha(out - \text{target})w_2s_2(1 - s_2)w_1s_1(1 - s_1)s_0$$

Put it all together

Prescription (continued)

3. Update the weights

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Note: We know all of the values required

Put it all together

Prescription (continued)

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α = learning rate, we choose this.

Put it all together

Prescription (continued)

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s_0, s_1, s_2

Put it all together

Prescription (continued)

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Note: We know all of the values required

α = learning rate, we choose this.

s_0, s_1, s_2 = calculated during forward propagation

Put it all together

Prescription (continued)

Put it all together

Prescription (continued)

4. Repeat Steps 2 & 3 until error is small enough.

Put it all together

Prescription (continued)

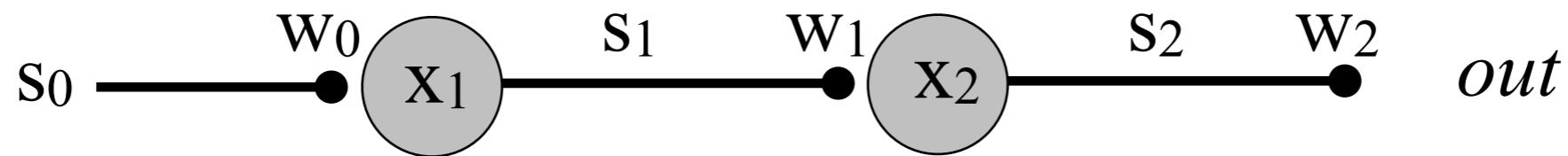
4. Repeat Steps 2 & 3 until error is small enough.
or *out* is close enough to target.

Put it all together

Prescription (continued)

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This procedure is called **backpropagation**

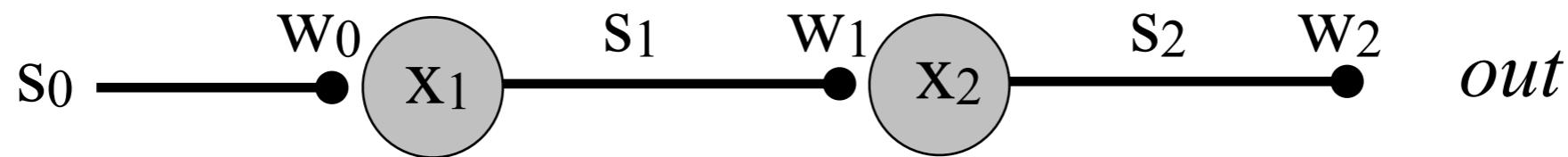


Put it all together

Prescription (continued)

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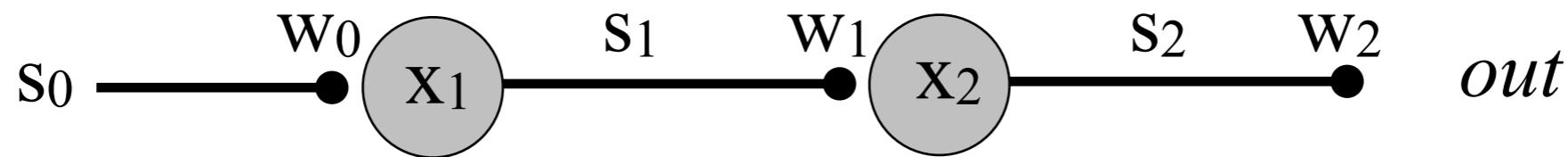
we work “backwards” through our neural network

Put it all together

Prescription (continued)

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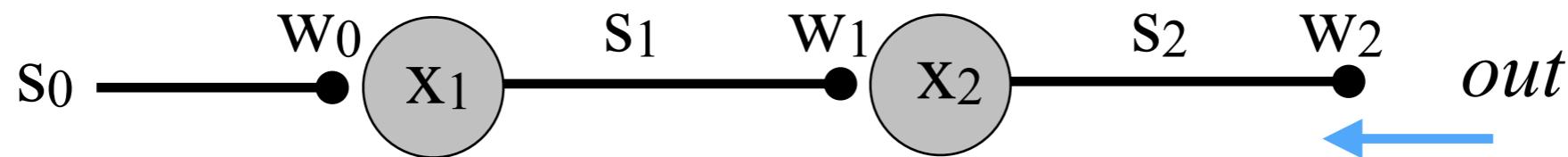
we work “backwards” through our neural network
from *out*

Put it all together

Prescription (continued)

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we work “backwards” through our neural network

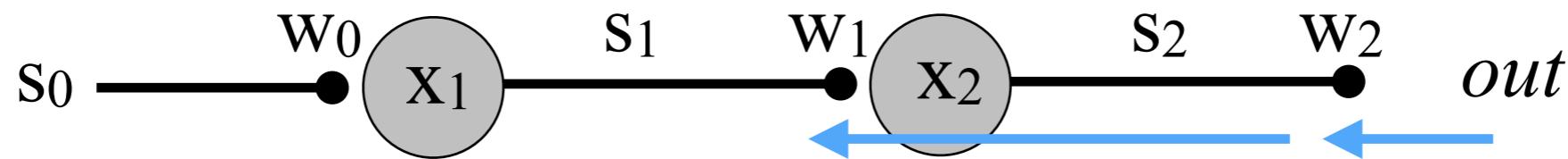
from out to changes in w_2

Put it all together

Prescription (continued)

4. Repeat Steps 2 & 3 until error is small enough.
or out is close enough to target.

This procedure is called **backpropagation**



we work “backwards” through our neural network

from out to changes in w_2 to changes in w_1

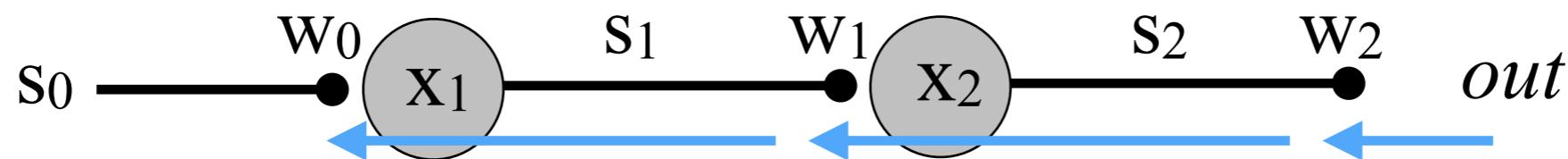
Put it all together

Prescription (continued)

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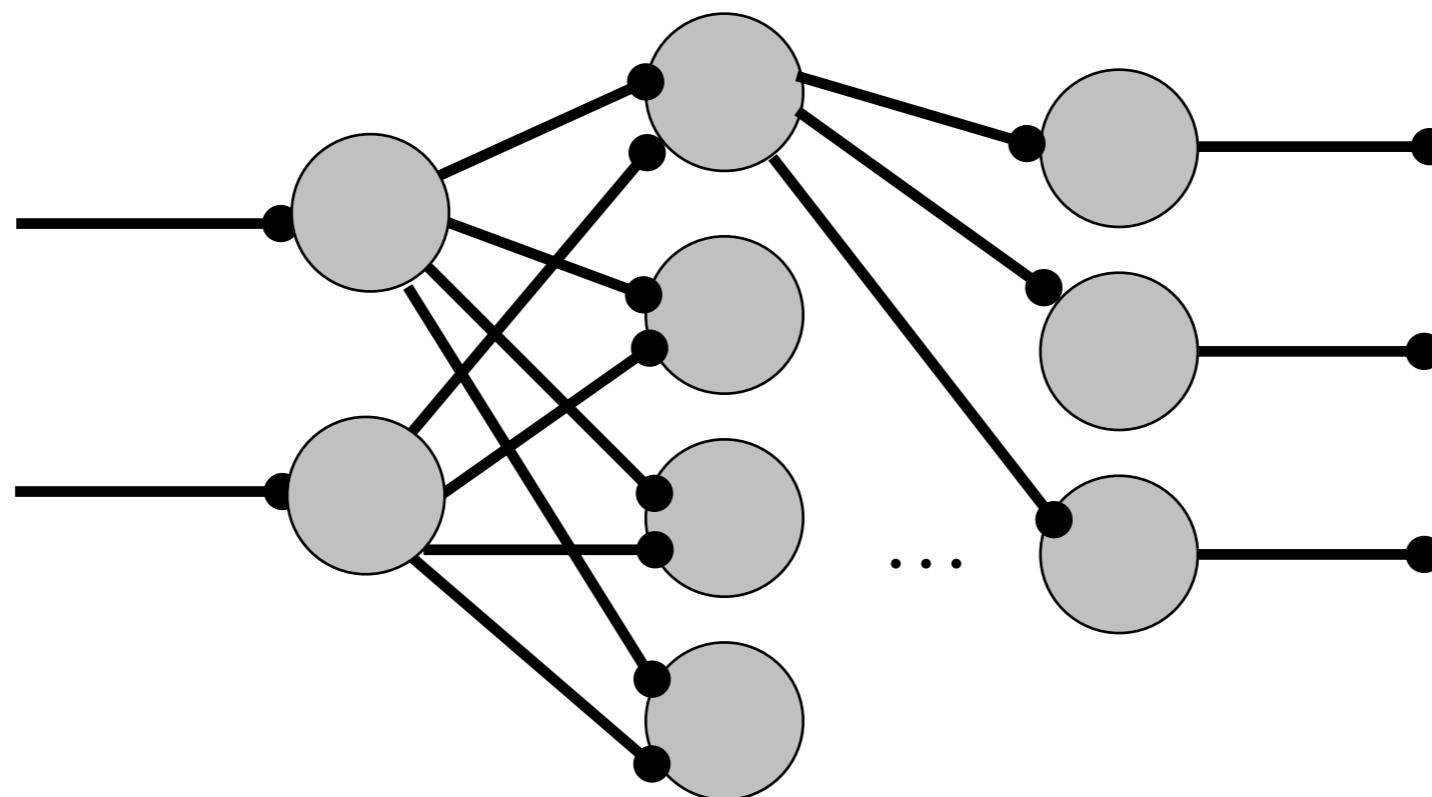


we work “backwards” through our neural network

from out to changes in w_2 to changes in w_1 to changes in w_0

Backpropagation

Can evaluate more complicated neural networks



Same ideas apply, but algebra is intense.

Cool example: playground.tensorflow.org

Next time

Implement backpropagation in Python