FDFDLH

Apellidos y nombre

AMPLIACIÓN DE MATEMÁTICAS 2019-2020
TRABAJO PRÁCTICO 3: La transformada de Fourier

Calcula la transformada de Fourier para la función $f(x) = x^2\chi_{[-1,1]}(x)$.

$$\hat{f}(\lambda) \underset{\text{def. TF}}{=} \int_{-\infty}^{\infty} f(x)\, e^{-i\lambda x}\, dx \underset{\text{def. f}}{=} \int_{-1}^{1} x^2 e^{-i\lambda x}\, dx \overset{\text{int. por partes}}{=} x^2 \left(\frac{i e^{-i\lambda x}}{\lambda}\right|_{-1}^{1} =$$

$u = x^2 \to du = 2x\,dx$

$dv = e^{-i\lambda x}dx \to v = \frac{i e^{-i\lambda x}}{\lambda} dx$

$$= -\int_{-1}^{1} 2x \cdot \frac{i e^{-i\lambda x}}{\lambda}\, dx \overset{\text{int. por partes}}{=} \frac{i}{\lambda}\Big(\underbrace{e^{-i\lambda} - e^{i\lambda}}_{-2i\,\text{sen}\lambda}\Big) - \frac{2i}{\lambda}\left(\left(x \cdot \frac{i e^{-i\lambda x}}{\lambda}\right|_{-1}^{1} - \right.$$

$u = x \longrightarrow du = dx$

$dv = e^{-i\lambda x}dx \longrightarrow v = \frac{i e^{-i\lambda x}}{\lambda}$

$$\left. - \int_{-1}^{1} \frac{i e^{-i\lambda x}}{\lambda}\, dx\right) = \frac{2\,\text{sen}\lambda}{\lambda} + \frac{2}{\lambda^2}\Big(\underbrace{e^{-i\lambda} + e^{i\lambda}}_{2\cos\lambda}\Big) - \frac{2}{\lambda^3} i \Big(\underbrace{e^{-i\lambda} - e^{i\lambda}}_{-2i\,\text{sen}\lambda}\Big) =$$

$$= 2\,\frac{(\lambda^2-2)\,\text{sen}\lambda + 2\lambda\cos\lambda}{\lambda^3} \Bigg].$$

Calcula la transformada de Fourier para la función $f(t) = \cos(\alpha t)\chi_{[-\pi,\pi]}(t)$ con $\alpha \neq 0$.

$$\hat{f}(\lambda) \underset{\text{def. TF}}{=} \int_{-\infty}^{\infty} f(t)\, e^{-i\lambda t}\, dt \underset{\text{def. f.}}{=} \int_{-\pi}^{\pi} \cos(\alpha t)\, e^{-i\lambda t}\, dt \overset{\text{int. por partes}}{=}$$

$u = \cos(\alpha t) \to du = -\alpha\,\text{sen}(\alpha t)\,dt$

$dv = e^{-i\lambda t}dt \to v = \frac{i e^{-i\lambda t}}{\lambda}$

$$= \cos(\alpha t)\left(\frac{i e^{-i\lambda t}}{\lambda}\right|_{-\pi}^{\pi} + i\frac{\alpha}{\lambda}\int_{-\pi}^{\pi} \text{sen}(\alpha t) \cdot e^{-i\lambda t}\, dt \overset{\text{int. por partes}}{=}$$

$u = \text{sen}(\alpha t) \to du = \alpha\cos(\alpha t)\,dt$

$dv = e^{-i\lambda t}dt \to v = \frac{i e^{-i\lambda t}}{\lambda}$

$$= \frac{i}{\lambda}\cos(\alpha\pi)\Big(\underbrace{e^{-i\lambda\pi} - e^{i\lambda\pi}}_{-2i\,\text{sen}(\lambda\pi)}\Big) +$$

$$+ i\frac{\alpha}{\lambda}\left(\left(\frac{i}{\lambda}\text{sen}(\alpha t)\cdot e^{-i\lambda t}\right|_{-\pi}^{\pi} - \frac{i\alpha}{\lambda}\underbrace{\boxed{\int_{-\pi}^{\pi}\cos(\alpha t)\, e^{-i\lambda t}\, dt}}_{¡\hat{f}(\lambda)!}\right) =$$

$$= \frac{2\cos(\alpha\pi)\,\text{sen}(\lambda\pi)}{\lambda} - \frac{\alpha\,\text{sen}(\alpha\pi)}{\lambda^2}\Big(\underbrace{e^{-i\lambda\pi} + e^{i\lambda\pi}}_{2\cos(\lambda\pi)}\Big) + \frac{\alpha^2}{\lambda^2}\hat{f}(\lambda) \Leftrightarrow$$

$$\Leftrightarrow \hat{f}(\lambda) = 2\,\frac{\lambda\,\text{sen}(\lambda\pi)\cos(\alpha\pi) - \alpha\,\text{sen}(\alpha\pi)\cos(\lambda\pi)}{\lambda^2 - \alpha^2}\Bigg].$$

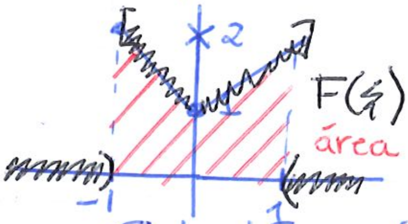
(**Examen Enero de 2019**) Sabemos que la transformada de Fourier para una determinada señal $f\colon \mathbb{R} \longrightarrow \mathbb{R}$ es

$$F(\xi) = (1 + |\xi|)\,\chi_{[-1,1]}(\xi). \equiv \hat{f}(\xi)$$

función par

Calcular el valor de $f(x)$ **enunciando correcta y completamente** el resultado que se ha empleado para dicho cálculo. Hay que utilizar la Fórmula de inversión

que nos dice que: $F^{-1}[\hat{f}](x) = \frac{1}{2\pi}\int_{\mathbb{R}} \hat{f}(\xi)\, e^{ix\xi}\, d\xi \equiv f(x)$ siempre que $F \in L^1(\mathbb{R})$



dado que $1+|\xi|$ es continua en $[-1,1]$

Sí, pues $\int_{\mathbb{R}} |F| = \int_{-1}^{1} (1+|\xi|)\, d\xi < +\infty$ ||| área 3

Calculamos la integral:

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\xi)\, e^{i\xi x}\, d\xi =$$

par x impar ≡ impar

$$= \frac{1}{2\pi}\int_{-1}^{1} F(\xi)\,(\cos \xi x + i \operatorname{sen} \xi x)\, d\xi =$$

par x par = par

$$\frac{\operatorname{sen} \xi x}{x}\Big|_0^1 = \frac{\operatorname{sen} x}{x}$$

$$= \frac{\cancel{2}}{\cancel{2}\pi}\int_0^1 (1+\xi)\cos \xi x\, d\xi = \frac{1}{\pi}\int_0^1 \cos \xi x\, dx +$$

$$+ \frac{1}{\pi}\int_0^1 \xi \cos \xi x\, dx \overset{\text{int. por partes}}{=} \xi \cdot \frac{\operatorname{sen} \xi x}{x\pi}\Big|_0^1 - \int_0^1 \frac{\operatorname{sen} \xi x}{x\pi}\, dx +$$

$u = \xi \to du = d\xi$

$dV = \cos \xi x = \frac{\operatorname{sen} \xi x}{x} dx$

$$+ \frac{\operatorname{sen} x}{x\pi} = \frac{2 \operatorname{sen} x}{x\pi} - \frac{1}{x\pi}\left(\frac{\cos \xi x}{x}\Big|_0^1 = \right.$$

$$= \frac{1}{\pi}\left(\frac{2 \operatorname{sen} x}{x} + \frac{\cos x}{x^2} - \frac{1}{x^2}\right)\Big].$$