Apellidos y nombre

AMPLIACIÓN DE MATEMÁTICAS
TRABAJO PRÁCTICO 3: La transformada de Fourier

Calcula la transformada de Fourier de la función $f(t) = (t^3 - t)\chi_{[-1,1]}(t)$

$$\hat{f}(\lambda) = \int_{-\infty}^{\infty} f(t) e^{-i\lambda t} dt = \int_{-1}^{1} (t^3 - t) e^{-i\lambda t} dt = \left[\frac{i(t^3-t)e^{-i\lambda t}}{\lambda}\right]_{-1}^{1} - \frac{i}{\lambda}\int_{-1}^{1} (3t^2-1) e^{-i\lambda t} dt =$$

$u = t^3 - t \Rightarrow du = (3t^2-1)dt$

$dv = e^{-i\lambda t} dt \Rightarrow v = \frac{e^{-i\lambda t}}{-i\lambda} = \frac{ie^{-i\lambda t}}{\lambda}$

$$= (0 - 0) - \left[\frac{i^2}{\lambda^2}(3t^2-1)e^{-i\lambda t}\right]_{-1}^{1} + \frac{6i^2}{\lambda^2}\int_{-1}^{1} t e^{-i\lambda t} dt = \frac{+2\cancel{(2)}(e^{-i\lambda} - e^{i\lambda})}{\lambda^2}$$

$u = 3t^2 - 1 \Rightarrow du = 6t\,dt$

$dv = e^{-i\lambda t} dt \Rightarrow v = \frac{ie^{-i\lambda t}}{\lambda}$

$u = t \Rightarrow du = dt$

$dv = e^{-i\lambda t} dt \Rightarrow v = \frac{ie^{-i\lambda t}}{\lambda}$

$$- \frac{6i}{\lambda^3}\left[t e^{-i\lambda t}\right]_{-1}^{1} + \frac{6i}{\lambda^3}\int_{-1}^{1} e^{-i\lambda t} = -\frac{4\,\text{sen}(\lambda)}{\lambda^2} + \frac{12i\cos(\lambda)}{\lambda^3} + \frac{12\,\text{sen}(\lambda)}{\lambda^4}$$

Calcula la transformada de Fourier de la función $f(t) = Ae^{-\alpha t}\chi_{[\pi,\infty]}$, $\alpha > 0$

$$\hat{f}(\lambda) = \int_{\pi}^{\infty} A e^{-(\alpha+i\lambda)t} dt = \left[\frac{A e^{-(\alpha+i\lambda)t}}{\alpha + i\lambda}\right]_{\pi}^{\infty} =$$

$$= \frac{A}{\alpha + i\lambda}\left(\lim_{t\to\infty} \cancel{e^{-(\alpha+i\lambda)t}} \xrightarrow{\alpha>0} 0 - e^{-(\alpha+i\lambda)\pi}\right) =$$

$$= \frac{A e^{-(\alpha+i\lambda)\pi}}{\alpha + i\lambda}$$

Utiliza el teorema de inversión para demostrar que $f(x) = \chi_{[-\frac{\pi}{2},\frac{\pi}{2}]}(x)\cos(x)$ y $g(x) = \frac{2}{\pi}\int_0^\infty \frac{\cos\left(\frac{s\pi}{2}\right)}{1-s^2}\cos(sx)ds$ son la misma función

$$\hat{f}(s) = \int_{-\pi/2}^{\pi/2} \cos(x)\, e^{-isx}\,dx = \int_{-\pi/2}^{\pi/2} \cos(x)\cos(sx)\,dx -$$

$$- i\int_{-\pi/2}^{\pi/2} \cos(x)\,\text{sen}(sx) = \frac{1}{2}\int_{-\pi/2}^{\pi/2} \left(\cos(x+sx) + \cos(x-sx)\right)dx -$$

$$- \frac{i}{2}\int \left(\text{sen}(x+sx) - \text{sen}(x-sx)\right)dx =$$

$$= \frac{1}{2}\left[+\frac{\text{sen}((1+s)x)}{1+s} + \frac{\text{sen}((1-s)x)}{1-s}\right]_{-\pi/2}^{\pi/2} -$$

$$- \frac{i}{2}\left[\frac{-\cos((1+s)x)}{1+s} + \frac{\cos((1-s)x)}{1-s}\right]_{-\pi/2}^{\pi/2} =$$

$$= \frac{1}{2}\left(\frac{\text{sen}\left((1+s)\frac{\pi}{2}\right) - \text{sen}\left((1+s)\frac{-\pi}{2}\right)}{1+s} + \frac{\text{sen}\left((1-s)\frac{\pi}{2}\right) - \text{sen}\left((1-s)\frac{-\pi}{2}\right)}{1-s}\right) -$$

$$- \frac{i}{2}\left(\frac{-\cos\left(\frac{\pi}{2}+\frac{s\pi}{2}\right) + \cos\left(\frac{-\pi}{2}-\frac{s\pi}{2}\right)}{1+s} + \frac{\cos\left(\frac{\pi}{2}-s\frac{\pi}{2}\right) - \cos\left(\frac{-\pi}{2}+\frac{s\pi}{2}\right)}{1-s}\right) =$$

$$= \frac{1}{2}\left(\frac{\cos\left(s\frac{\pi}{2}\right) + \cos\left(-s\frac{\pi}{2}\right)}{1+s} + \frac{\cos\left(s\frac{\pi}{2}\right) + \cos\left(-s\frac{\pi}{2}\right)}{1-s}\right) -$$

$$- \frac{i}{2}\left(\cancel{\frac{-\text{sen}\left(s\frac{\pi}{2}\right)(-1)\,\text{sen}\left(-s\frac{\pi}{2}\right)}{1+s}} \to 0 + \cancel{\frac{\text{sen}\left(s\frac{\pi}{2}\right) - \text{sen}\left(s\frac{\pi}{2}\right)}{1-s}}^{\,0}\right) =$$

$$= \frac{1}{2}\left(\frac{2\cos\left(s\frac{\pi}{2}\right)}{1+s} + \frac{2\cos\left(s\frac{\pi}{2}\right)}{1-s}\right) = 2\frac{\cos\left(s\frac{\pi}{2}\right)}{1-s^2}$$

$e^{i\alpha} = \cos\alpha + i\,\text{sen}\,\alpha$ y paridad

$$f(x) = F^{-1}[\hat{f}(s)](x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2\cos\left(s\frac{\pi}{2}\right)}{1-s^2}\, e^{isx}\,ds = \frac{2}{\pi}\int_0^\infty \frac{2\cos\left(s\frac{\pi}{2}\right)}{1-s^2}\cos(sx)\,ds$$