Lecture 35: Filters III – Band Pass and Band Reject Filters

Upcoming events

1. Problem Set # 7 due Lesson 37
2. Quiz # 6 Lesson 37
3. GR # 3 Lesson 38

OBJECTIVES:

1. Understand the operation of Band Pass Filters (BPF) and Band Reject Filters (BRF)
2. Be able to design BPF and BRF using LPF and HPF as building blocks

READING

Required : Filters Handout (Available on Sharepoint), pgs 35–38

HOMEWORK

Recommended textbook problems : 3–3, 3–4, 4–1, 4–2 (Handout)

1 Introduction

Last lesson we talked about Low Pass and High Pass Filters; today we are going to look at how we can use these simple filters as building blocks to build two new types of filter: the Band Pass Filter (BPF) and the Band Reject Filter (BRF). It should be easy to figure out what these filters do from their name. A *Band Pass* Filter passes a signal when the the frequency is in a passband defined by two frequencies, ω_{c1} and ω_{c2}. A *Band Reject* Filter rejects signals whose frequency are *in band*.

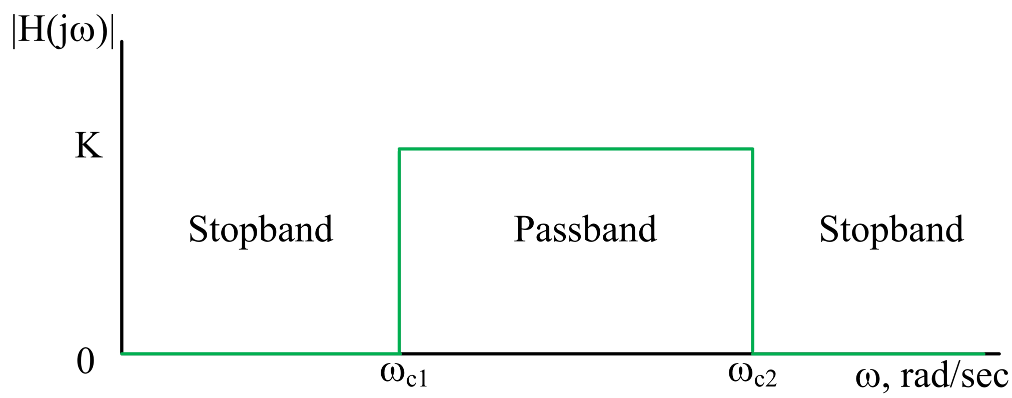
2 Block Diagrams of BPF and BRF

Before we talk about the actual design of these filters, lets step back and look at them as block diagrams using LPFs and HPFs as our building blocks.

2.1 BPF

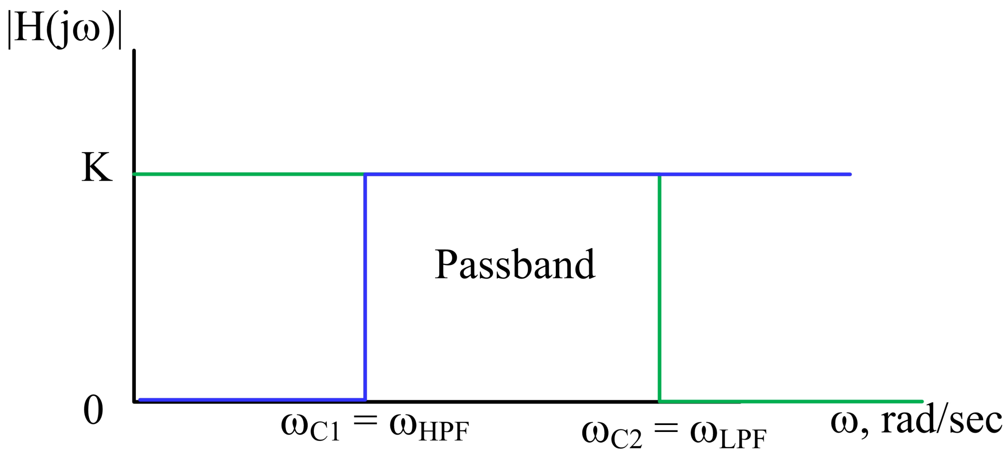
Let's start by looking at a graph of an *ideal* Band Pass Filter Transfer function.

This figure is hidden is student copy



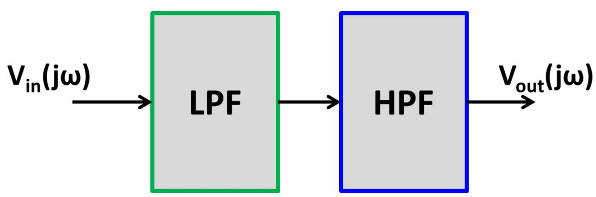
If we want to build this filter with combination of a LPF and a HPF, we can *cascade* the transfer functions to arrive at

This figure is hidden is student copy



A block diagram of this filter would look like:

This figure is hidden is student copy



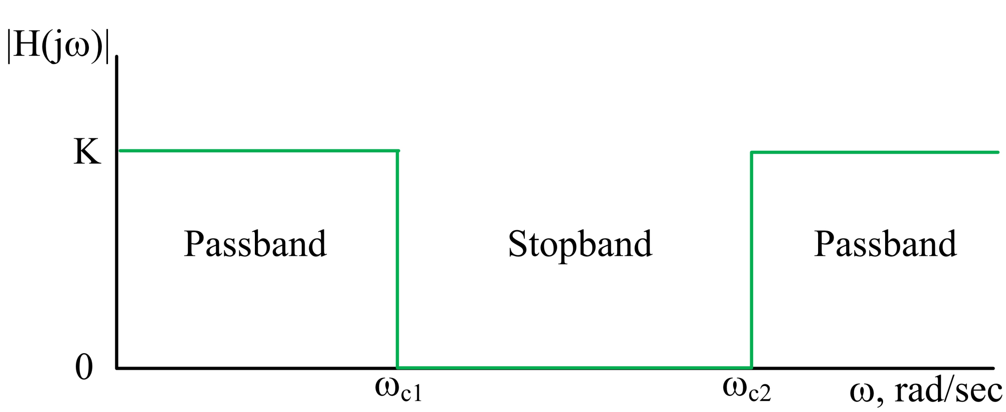
Note: The order of the filters is not important

A very important thing to remember is when designing a BPF, the cut-off for the HPF must be *smaller* than the cut-off of the LPF. **What happens if the HPF cut-off is *larger* than the LPF cut-off?** *You have designed a NO-PASS filter!*

2.2 BRF

Let's now look at a graph of an *ideal* Band Reject Filter Transfer function.

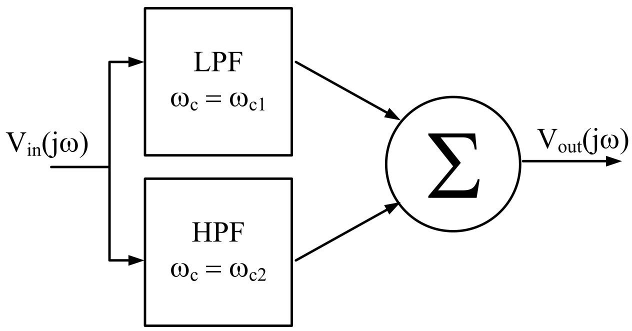
This figure is hidden is student copy



I hope at this point it is obvious we can build this by *connecting* a LPF and a HPF; what is probably not obviouse is how we connect them. **To build a BRF, you cannot cascade a LPF and a HPF!**

To build a Band Reject Filter, we will connect a LPF and a HPF using a summer:

This figure is hidden is student copy



A very important thing to remember is when designing a BRF, the cut-off for the HPF must be *larger* than the cut-off of the LPF. **What happens if the HPF cut-off is *smaller* than the LPF cut-off?** *You have designed an ALL-PASS filter!*

3 Designing Real Band Pass Filters

Let's look at the example from section 4.1 of the reading.

Example 1–Let's design a BPF with $\omega_{c1} = 500\frac{rad}{s}$ and $\omega_{c2} = 10,000\frac{rad}{s}$:

First we will design a LPF with a cut-off of $\omega_{c2} = 10,000\frac{rad}{s}$. Recalling our standard form for a LPF transfer function we can write:

$$H_{LPF}(j\omega) = \frac{10,000}{j\omega + 10,000}$$

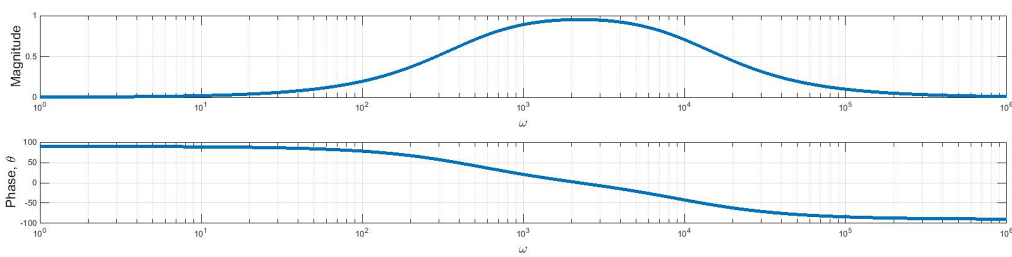
Next we deisgn a HPF with a cut-off of $\omega_{c1} = 500\frac{rad}{s}$. Recalling our standard form for a HPF transfer function we can write:

$$H_{HPF}(j\omega) = \frac{j\omega}{j\omega + 500}$$

Finally we cacade the two transfer functions to arrive at:

$$H_{BPF}(j\omega) = \frac{10,000}{j\omega + 10,000}\frac{j\omega}{j\omega + 500} = \frac{j10,000\omega}{(j\omega)^2 + j10,500\omega + 5,000,000}$$

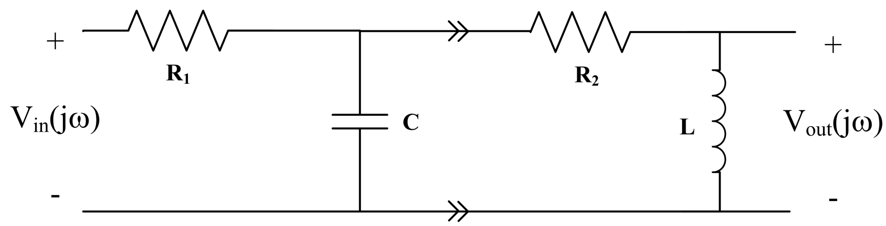
We can plot the phase and magnitude of this transfer function:



The magnitude response is what we would expect for a BPF and the phase response is zero at the center of the passband.

How would we actually build the filter from our example above?

What if we simply cascaded a LPF and a HPF as shown below?



We need to find values for the resistors, capacitor and inductor. We know:

$$\frac{1}{R_1C} = 10,000\frac{rad}{s}$$

$$\frac{R_2}{L} = 500\frac{rad}{s}$$

To satisfy the above requirements, we can use:

$$R_1 = 1\ k\Omega$$

$$C = 0.1\ \mu F$$

$$R_2 = 500\Omega$$

$$L = 1H$$

If we write Node Voltage equations for this circuit we get:

$$\frac{V_A - V_{IN}}{R_1} + V_A j\omega C + \frac{V_A - V_{OUT}}{R_2} = 0$$

$$\frac{V_{OUT} - V_A}{R_2} + \frac{V_{OUT}}{j\omega L} = 0$$

We can write the system of equations in matrix form:

$$\begin{bmatrix} \frac{1}{R_1} + j\omega C + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{j\omega L} \end{bmatrix} \begin{bmatrix} V_A \\ V_{OUT} \end{bmatrix} = \begin{bmatrix} \frac{V_{IN}}{R_1} \\ 0 \end{bmatrix}$$

You can know use Matlab's symnbolic toolbox to get the transfer function, Matlab will output (the output will not look this clean):

$$V_{out} = \frac{10,000V_{in}\omega}{j\omega^2 + 10,500\omega - j15,000,000}$$

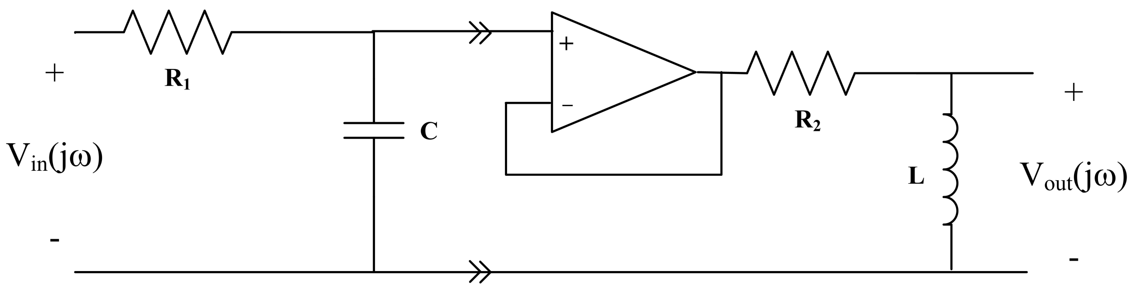
Which we can divide out the V_{in} and manipulate into standard form of a transfer function to arrive at:

$$H(j\omega) = \frac{j10,000\omega}{(j\omega)^2 + j10,500\omega + 15,000,000}$$

This is very similar to what is in the reading, however, itis not exact; it also does not match what we expect from cascading these filters. The root cause of both of these issues is stage loading!

To avoid stage loading we insert a buffer between the two stages as shown below:

This figure is hidden is student copy

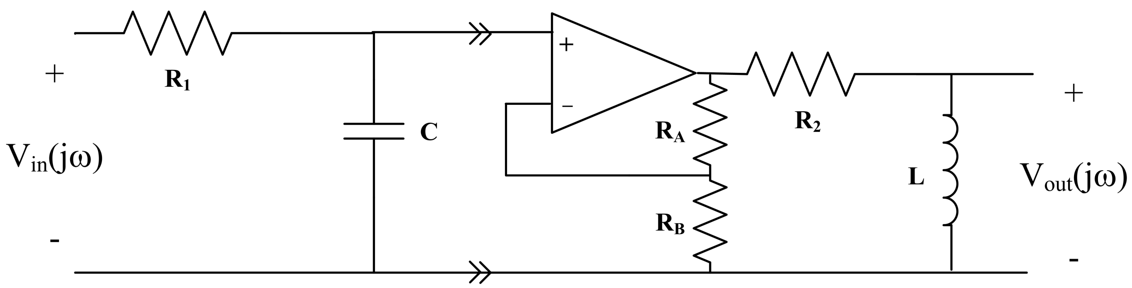


The transfer function for this circuit is:

$$H(j\omega) = \frac{j10,000\omega}{(j\omega)^2 + j10,500\omega + 5,000,000}$$

If we need an amplification stage ($K > 1$), we can replace the buffer with a *non-inverting amplifier* as shown below:

This figure is hidden is student copy The transfer function for this circuit is:



$$H(j\omega) = \left[\frac{R_A + R_B}{R_B}\right]\left[\frac{j10,000\omega}{(j\omega)^2 + j10,500\omega + 5,000,000}\right]$$

4 Designing Real Band Reject Filters

Let's look at the example from section 4.2 of the reading.

Example 1–Let's design a BRF with $\omega_{c1} = 500\frac{rad}{s}$ and $\omega_{c2} = 10,000\frac{rad}{s}$:

First we will design a LPF with a cut-off of $\omega_{c1} = 500\frac{rad}{s}$. Recalling our standard form for a LPF transfer function we can write:

$$H_{LPF}(j\omega) = \frac{500}{j\omega + 500}$$

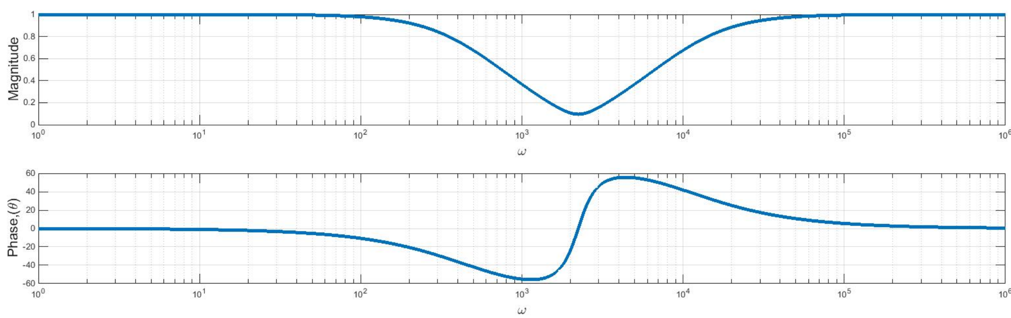
Next we deisgn a HPF with a cut-off of $\omega_{c2} = 10,000\frac{rad}{s}$. Recalling our standard form for a HPF transfer function we can write:

$$H_{HPF}(j\omega) = \frac{j\omega}{j\omega + 10,000}$$

*Finally we **SUM** the two transfer functions to arrive at:*

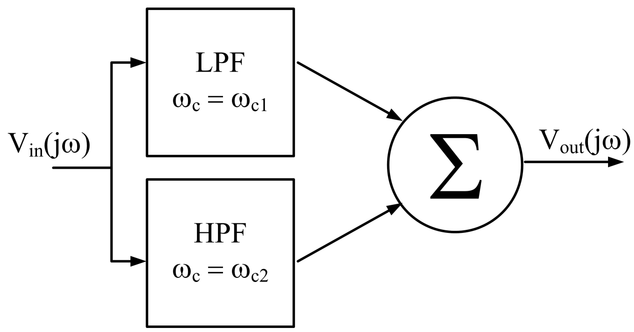
$$H_{BRF}(j\omega) = \frac{500}{j\omega + 500} + \frac{j\omega}{j\omega + 10,000}$$

We can plot the phase and magnitude of this transfer function:



The magnitude response is what we would expect for a BRF and the phase response is zero in both passbands.

How do we build this filter? To start, let's look once again at the block diagram of the BRF:



By looking at this figure, it should be clear that we will need a HPF, a LPF and a summer (remember that thing we built with op amps); what might not be obivous yet is that we will also need a couple of buffers.

Let's jump into the design:

Let's start with our LPF design; from our standard form of a LPF transfer function, we know:

$$\frac{1}{R_{LPF}C_{LPF}} = 500\frac{rad}{s}$$

so let's use

$$R_{LPF} = 20\ k\Omega$$

$$C_{LPF} = 0.1\ \mu F$$

Next we move to the HPF design. Again from our standard form for a HPF transfer function we can know:

$$\frac{1}{R_{HPF}C_{HPF}} = 10,000\frac{rad}{s}$$

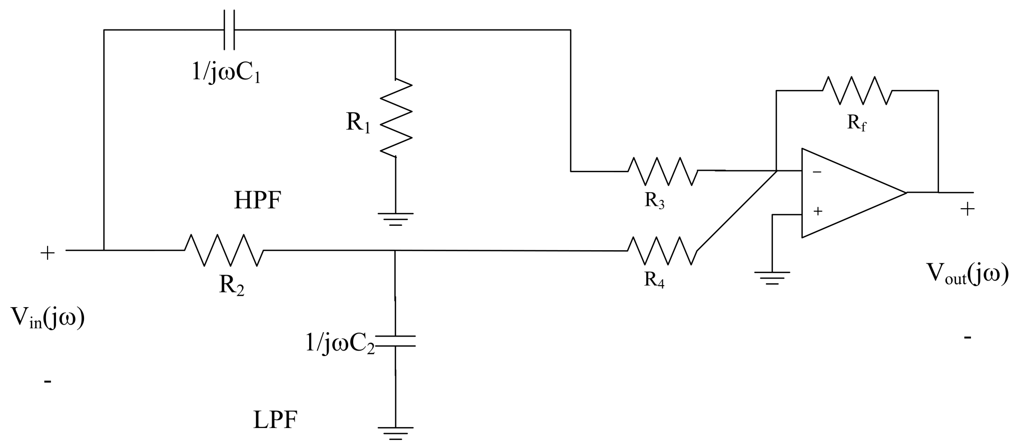
so let's use

$$R_{HPF} = 1\ k\Omega$$

$$C_{HPF} = 0.1\ \mu F$$

We now tie the filters together using a summer (**DO NOT JUST TIE THE OUTPUTS TOGETHER!**)

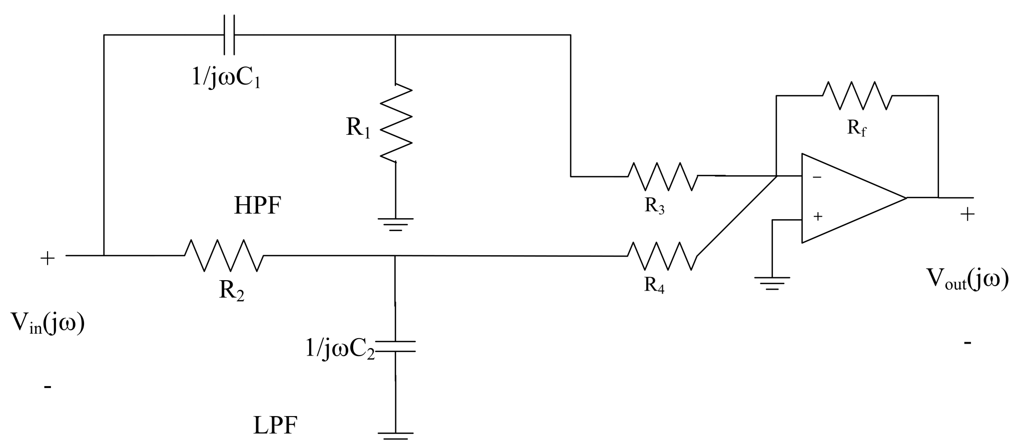
This figure is hidden is student copy



In this and the next figure $C_1 = C_{HPF}$, $R_1 = R_{HPF}$, $C_2 = C_{LPF}$, and $R_2 = R_{LPF}$

But there is a problem here.... We know an inverting summer can suffer from input loading. We need to put buffers on the summer inputs:

This figure is hidden is student copy



This filter design would give us the magnitude and phase response we expect from a BRF, with one exception. The inverting nature of the summer would cause a 180^o phase shift in the output.