Chap3 线性方程组的直接解法 实验报告

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运行代码方法

环境要求：rust，gnuplot

运行代码

```
cargo run
```

q3.6

任务

使用Cholesky分解算法求解方程$H_nx=b$，其中H_n为n阶Hilbert矩阵，$b=H_nx$，x为n个元素全为1的向量

解题思路

按照课本上描述的算法的思路编写代码即可

Cholesky分解算法求解方程分为三步

1. 对正定矩阵A进行Cholesky分解，得到$A=LL^T$，其中L为所有对角线元素为正数的下三角阵
2. 求解方程$Ly=b$，这直接与高斯消元法的回代过程方向相反，思路类似
3. 求解方程$L^Tx=y$，这直接就是高斯消元法的回代过程

实验结果

(1) $n=10$，计算$\|r\|_\infty,\|\Delta x\|_\infty$

计算得到解x如下

```
solve = [0.9999999987384126, 1.0000001083531875, 0.9999977024253508,
1.0000208143090212, 0.9999010002715109, 1.000271511848817,
0.9995554149286598, 1.0004289087352602, 0.9997751597216618,
1.0000493815113927]
```

计算得$\|r\|_\infty,\|\Delta x\|_\infty$如下

```
inf norm of delta b = 0.00000000000000044408920985006260 // 残差
inf norm of delta x = 0.00044458507134015335 // 误差
```

(2) 在右端施加10^{-7}的扰动，观察残差和误差的变化情况

扰动后输出如下

```
inf norm of delta b = 0.00000000000000044408920985006260
inf norm of delta x = 0.700708270017177
```

可见残差没有明显变化，但是误差显著放大了

(3) 改变n为$8,12$，观察残差和误差的变化情况，这说明了什么?

完整输出如下

```
n = 10
before disturbance
solve = [0.9999999987384126, 1.0000001083531875, 0.9999977024253508,
1.0000208143090212, 0.9999010002715109, 1.000271511848817, 0.9995554149286598,
1.0004289087352602, 0.9997751597216618, 1.0000493815113927]
inf norm of delta b = 0.00000000000000044408920985006262
inf norm of delta x = 0.00044458507134015335
after disturbance
inf norm of delta b = 0.00000000000000044408920985006262
inf norm of delta x = 0.700708270017177

n = 8
before disturbance
solve = [0.9999999999709857, 1.0000000015465562, 0.9999999798871754,
1.000000108508144, 0.9999997085792339, 1.000000411543821, 0.9999997075936138,
1.0000000823906543]
inf norm of delta b = 0.00000000000000022204460492503131
inf norm of delta x = 0.000000411543821021709 3
after disturbance
inf norm of delta b = 0.00000000000000022204460492503131
inf norm of delta x = 0.02162222989802176

n = 12
before disturbance
solve = [0.9999999669702844, 1.0000041657324705, 0.9998694415563627,
1.0017744011217633, 0.9870167549994316, 1.0569612331621385, 0.8414790066453739,
1.2866651456630795, 0.6641941895670194, 1.2457673980931956, 0.8978781383305862, 1.0183901800169004]
inf norm of delta b = 0.00000000000000044408920985006262
inf norm of delta x = 0.33580581043298063
after disturbance
inf norm of delta b = 0.00000000000000055511151231257827
inf norm of delta x = 23.620154933680745
```

可见扰动之后误差的随着n的变大而显著增大，这是因为H_n的条件数随着n的增大而迅速增大导致的。

心得体会

Hilbert矩阵带来的误差非常可观，如果什么时候把它出成数据塞到oj题目里那就很有意思了