Chap4 线性方程组的迭代解法 实验报告

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运行代码方法

环境要求：rust，gnuplot

运行代码

```
cargo run
```

q4.2

任务

实现jacobi，gs，sor三种迭代解法，利用它们分别解一个由微分方程导出的差分方程组

解题思路

jacobi，gs，sor三种迭代解法均按照课本上描述的解法即可；精度控制由用户传入eps参数，每次更新解时检查相对变化是否超过eps，如果某轮迭代没有一个分量超过即迭代完成。

关于初值，我采用了两种方式：全部赋0和按照$y = x$赋值(主要是考虑到本题的函数的大致图像情况)。

具体在实现时，因为本题的矩阵十分稀疏，我采用了稀疏矩阵的存储方式，具体来说每行只存储对角线元素(因为本节的算法都用到了对角线元素)和其他非零元素及其对应列数。

差分方程组的b向量没有在课本上描述清楚。经过分析，矩阵A和向量b的大小都应该是$n-1$，b除了最后一个分量之外都为ah^2，最后一个分量因为缺少y_n项，值应为$ah^2 - 1.0 * (\epsilon + h)$(第一个分量也缺少$y_0$，但是$y_0 = 0$，所以没有影响)

实验结果

输出如下

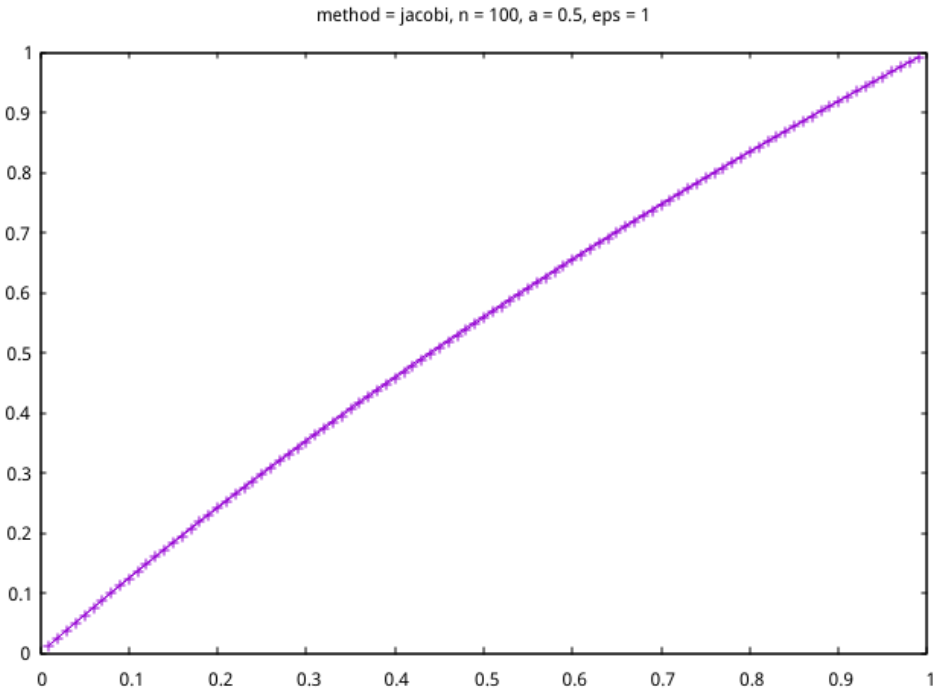
```
# init: all 0
eps = 1
jacobi: iter = 10858, inf norm dist = 0.003354437050578696
gs: iter = 5442, inf norm dist = 0.003404823354075326
sor(w = 1.1): iter = 4623, inf norm dist = 0.0028298141381259967

eps = 0.1
jacobi: iter = 5098, inf norm dist = 0.009761705106593743
gs: iter = 2578, inf norm dist = 0.009817518298027128
sor(w = 1.1): iter = 2168, inf norm dist = 0.009631073453557848

eps = 0.01
jacobi: iter = 540, inf norm dist = 0.06608628042120523
gs: iter = 320, inf norm dist = 0.0660841108742467
sor(w = 1.1): iter = 270, inf norm dist = 0.06607949954560188

eps = 0.0001
jacobi: iter = 116, inf norm dist = 0.004952583362066498
gs: iter = 108, inf norm dist = 0.004950747300598746
sor(w = 1.1): iter = 141, inf norm dist = 0.004950008892115387

# init: y = x
eps = 1
jacobi: iter = 4578, inf norm dist = 0.006589778549244585
gs: iter = 2991, inf norm dist = 0.0034038771970676396
sor(w = 1.1): iter = 2613, inf norm dist = 0.002830033772390106

eps = 0.1
jacobi: iter = 3546, inf norm dist = 0.010870718610929586
gs: iter = 2021, inf norm dist = 0.009821361136785833
sor(w = 1.1): iter = 1710, inf norm dist = 0.009632574673256844

eps = 0.01
jacobi: iter = 450, inf norm dist = 0.06611831557737685
gs: iter = 280, inf norm dist = 0.06608721442076454
```

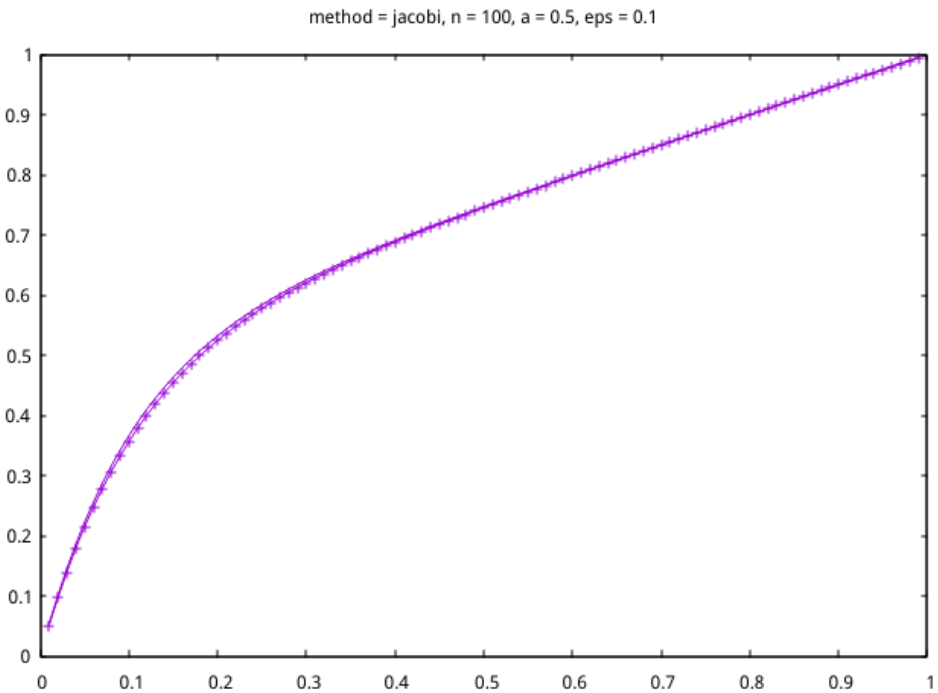
```
sor(w = 1.1): iter = 238, inf norm dist = 0.06608082591598069

eps = 0.0001
jacobi: iter = 110, inf norm dist = 0.0049552795115767045
gs: iter = 105, inf norm dist = 0.00495114521122475
sor(w = 1.1): iter = 137, inf norm dist = 0.004950070249054805
```

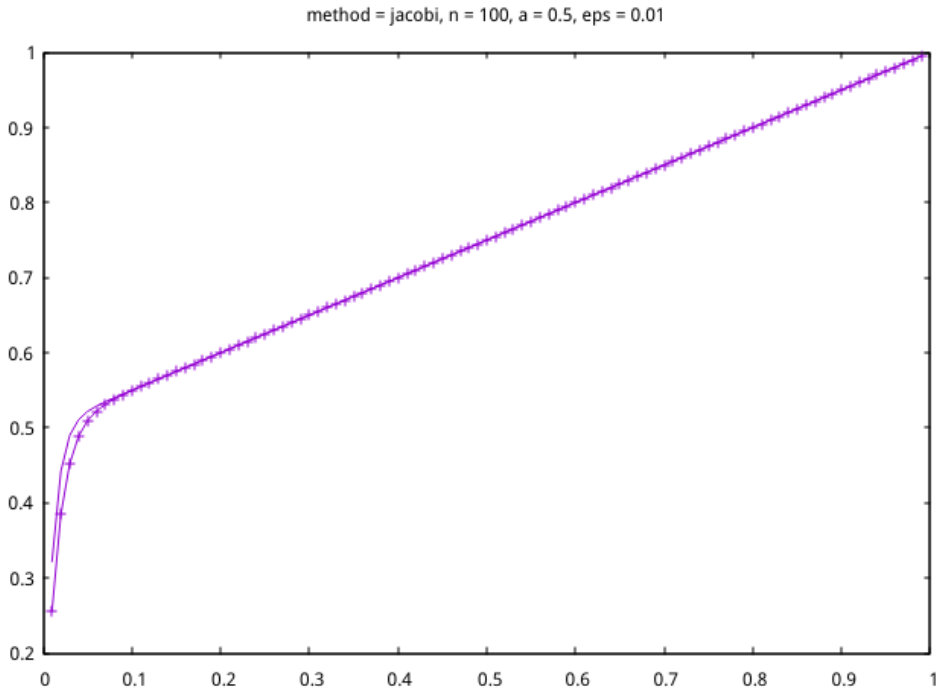
可见这个简单的初始值选择策略对收敛速度是有一定帮助的，最好情况下可以把收敛速度提高一倍多。

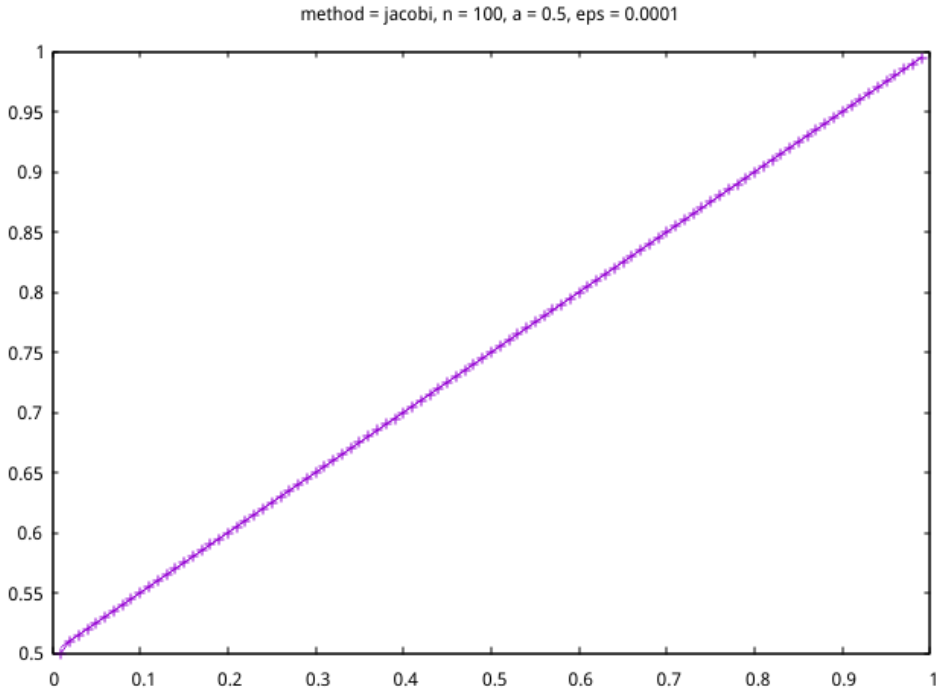
经过一些尝试我发现，对于eps = 0.0001的情况，sor解法取w尽量接近于1会收敛较快；但对于其他情况，sor解法取w稍大于1会收敛较快。最终代码里就统一取1.1了。

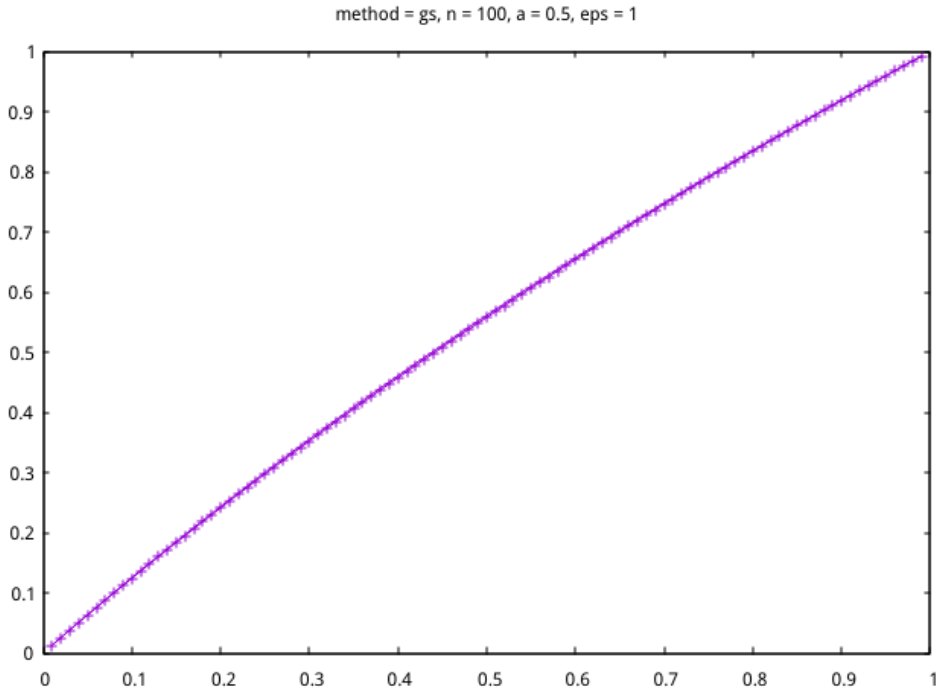
我还绘制了对应的图形，可以更加直观地看出误差情况

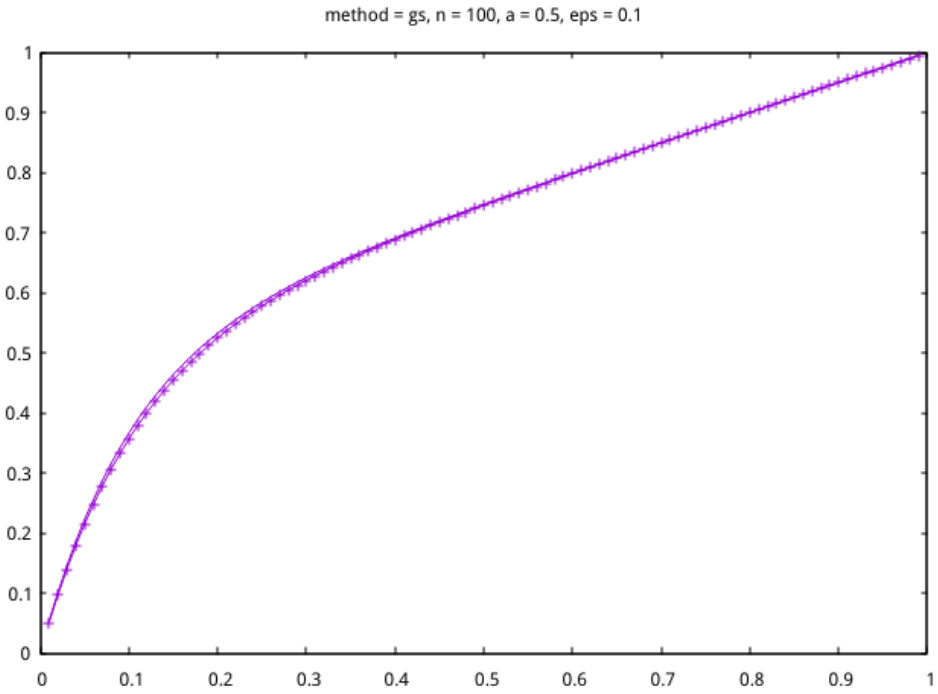


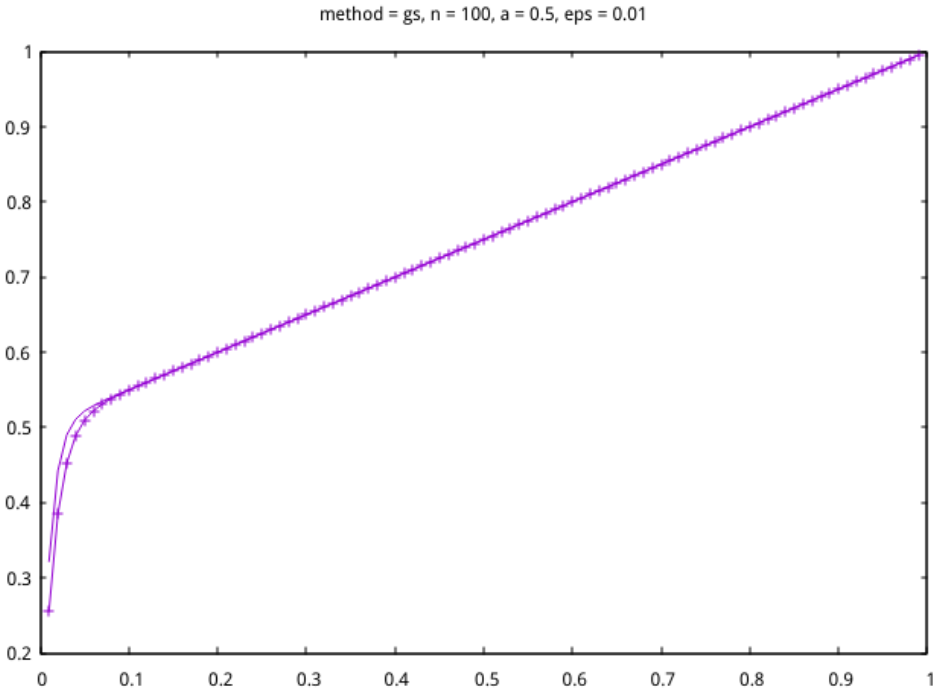


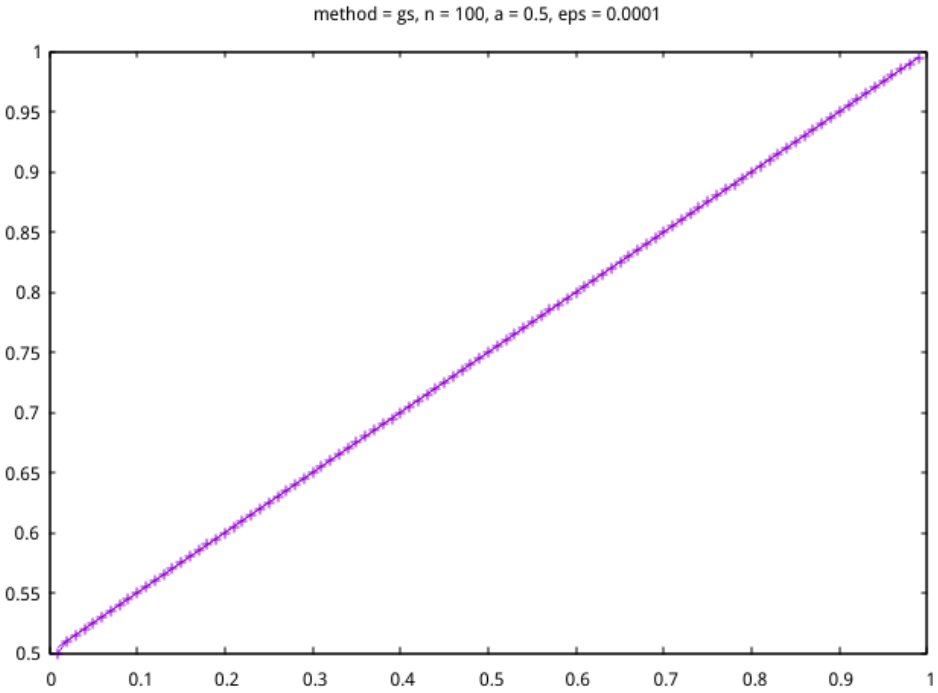

method = jacobi, n = 100, a = 0.5, eps = 0.01
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

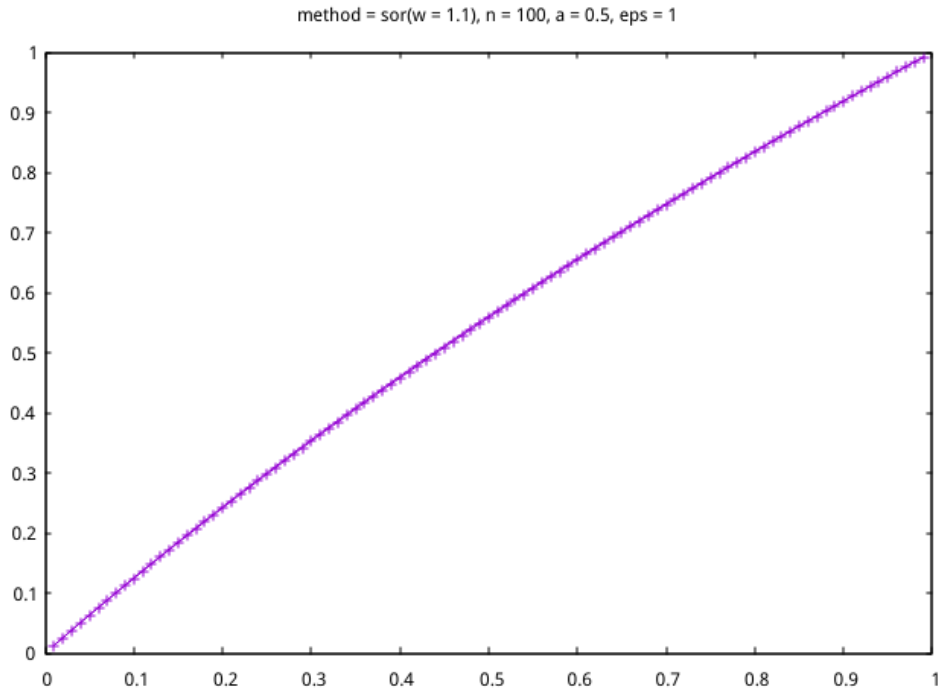

method = jacobi, n = 100, a = 0.5, eps = 0.0001
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

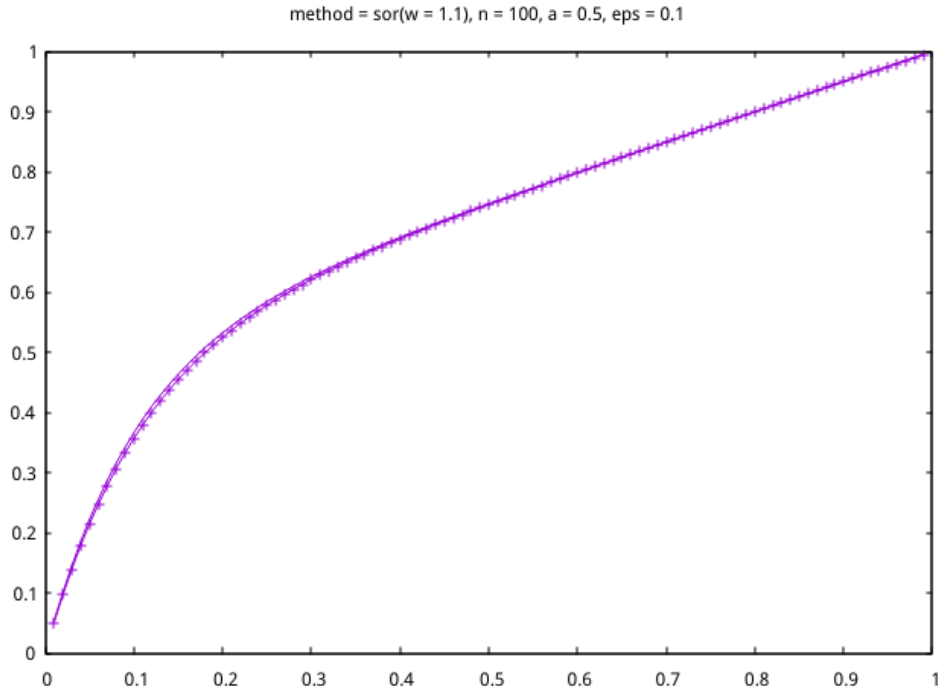

method = gs, n = 100, a = 0.5, eps = 1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

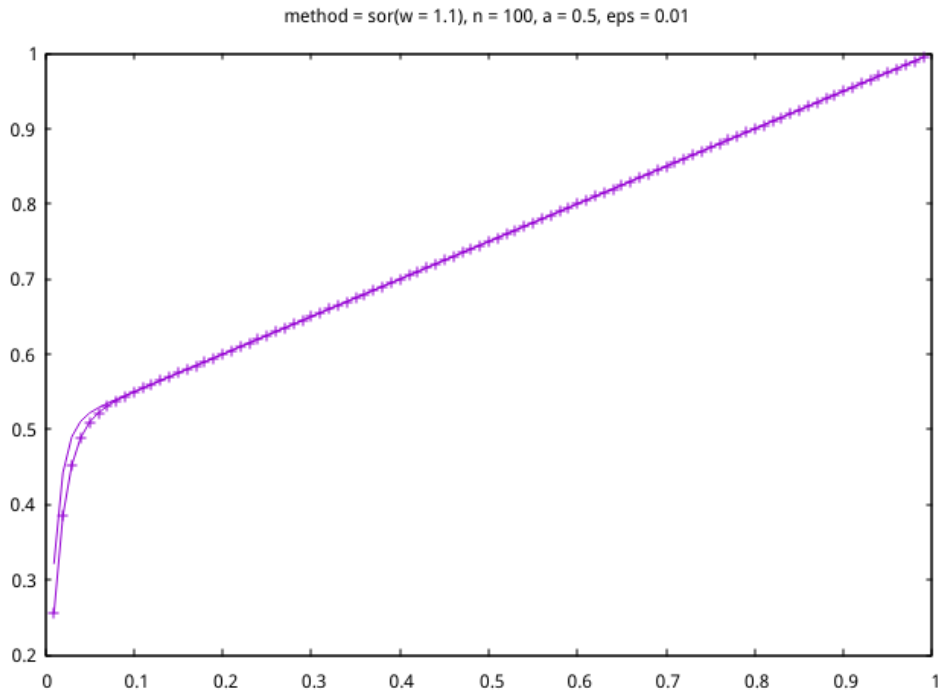

method = gs, n = 100, a = 0.5, eps = 0.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

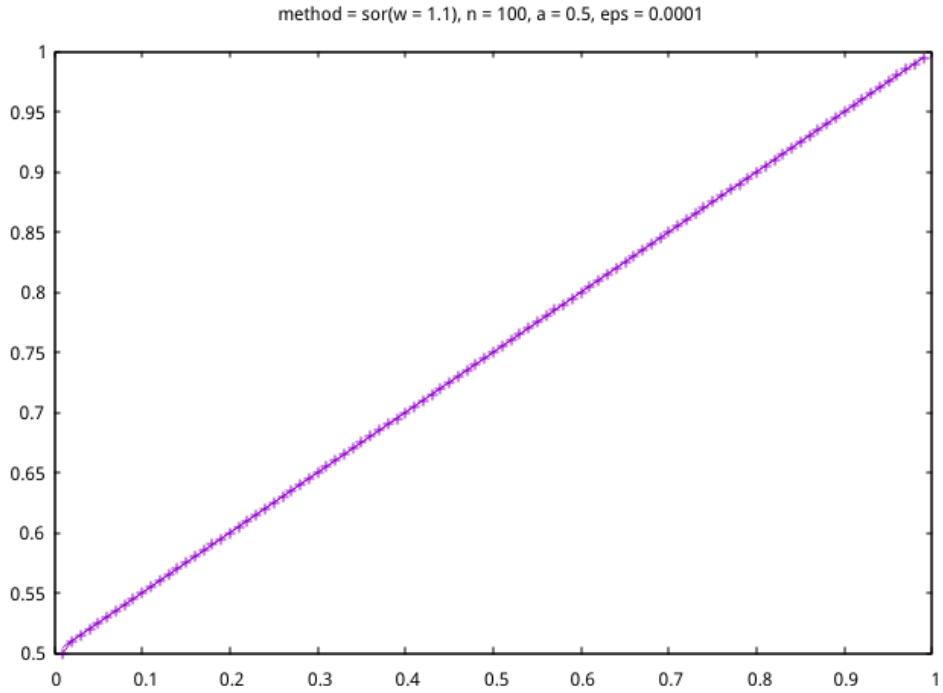

method = gs, n = 100, a = 0.5, eps = 0.01
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


method = gs, n = 100, a = 0.5, eps = 0.0001
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


method = sor(w = 1.1), n = 100, a = 0.5, eps = 1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


method = sor(w = 1.1), n = 100, a = 0.5, eps = 0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1





心得体会

其实这几个迭代方法的收敛速度都比我期望的慢一些，但是如考虑到稀疏矩阵和向量乘法的时间开销很小，那么这几种方法都还算是比较高效的。另外，一些简单的启发式的初始值选择方式可以显著加快收敛速度。