Science : the empire of certainty ?

At first glance, Science seems to be the world of necessity : nature is deterministic, and scientists make safe predictions about it. This point of view is justified by the idea that the determinism ruling the world also rules scientific method and conclusions. Moreover, this conception is supported by the omnipresence of mathematics in Science. In the deductive way of making mathematical demonstrations, conclusions derive from premisses with necessity. Although the effectiveness of mathematics in Science was called *unreasonable* by Eugene Wigner [13], the inductive approach also plays a great role in Science, and this approach appears to be uncertain by essence. The most famous example supporting that is the Sunrise : we inductively infer that the Sun will rise tomorrow because the huge number of time it did so makes the probability of having no Sunrise very low. Induction isn't logically justified, and it's founded on a probabilistic inference. Thus, certainty doesn't seem to reign on Science. We will show that the apparent correspondence between Science and certainty is incorrect and we will explore different aspects of the intervention, and even of the contribution of chance in Science.

In one of the most famous works in the history of Physics, *Philosophiae naturalis principia mathematica* (1687), Newton uses his new tool, Calculus, to study the deterministic evolution of systems in time and space, when initial conditions are given. For one set of initial conditions and forces, there is a unique possible evolution for an object. Until the XXth century, Newtonian dynamics were remarkably successful, applied to the motions of solids and fluids on Earth, of charges, or, of celestial bodies. In 1814, Laplace [7] formalized the idea of determinism :

> We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes (*A Philosophical Essay on Probabilities*).

Laplace's point of view of nature is strictly deterministic. This determinism is legislative (the existence of laws makes phenomena perfectly regular), temporal (if I know the position and the velocity of a particle submitted to a known force at a given time, I can find its position and velocity in the future and in the past), universal (the postulated homogeneity of the Universe, which is equivalent to the conservation of its momentum as Emmy Noether will show in 1915, implies that we can study movements at any place in the Universe the same way) and forbid chance (no phenomenon escapes to the strictly established laws of nature). Nevertheless, Laplace's view of knowledge isn't deterministic. The intellect he describes (today known as *Laplace's demon*) is inaccessible, we lose information. Laplace, who upgraded Newtonian laws of motion and used them in many fields, such as celestial mechanics, electrostatics or fluid dynamics, therefore developed a theory of probabilities, based on Bayes' theorem, not because of objective random behaviour of nature, but because of our limited cognitive capacities. If I flip a coin, the result (head or tail) is random because of my incapacity of prediction, but if I knew all the parameters (the force of my finger on all points of the coin, the direction of the throw, the speed of the wind...) then I could predict the result without error. This highlights the fact that even if a system is totally deterministic, its complexity leads to uncertainty.

The first example of complexity-based uncertainty was studied in Statistical Physics. The goal of Statistical Physics is to use probabilistic methods in order to deduce macroscopic properties of systems from the microscopic laws ruling elementary components. The fundamental postulate of Statistical Physics states that for an isolated system with an exactly known energy and exactly known composition, the system can be found with equal probability in any microstate consistent with that knowledge. A microstate is just the set of all the positions and velocities of the components of the system (atoms or molecules). To get back to our example of the coin, we know that the microstates (the parameters responsible for the result) are totally indeterminable, so we switch from the Newtonian attempt of computing the result to a probabilistic method : we have no *a priori* reason to privilege a result so we postulate equal probabilities of heads and tails and a great number of experiments confirms this hypothesis. So for a large number of throws, we have a good precision, without determining the behaviour of each throw. This method is well described by Henri Poincaré [11] :

> You ask me to predict the phenomena that will be produced. If I had the misfortune to know the laws of these phenomena, I could not succeed except by inextricable calculations, and I should have to give up the attempt to answer you; but since I am fortunate enough to be ignorant of them, I will give you an answer at once. And, what is more extraordinary still, my answer will be right (*Science & Method*).

That's the opposite of what we said in the beginning of this text. Here we notice that chance is firstly compatible with knowledge, but that it also provides methods for better descriptions of the world. Of course the goal of Statistical Physics isn't to play chance games. This theory gives appropriate formalism and powerful theorems, but also strong computational methods to study large sets of objects. One of these is Monte Carlo method. Monte Carlo methods, mostly numerical methods using randomness to solve problems that might be deterministic in principle, are well known for their uses in mathematics, like the approximation of π or the computation of integrals of functions without known primitives. To compute the behaviour of N atoms, with three degrees of freedom for their position and their velocities, it's necessary to evaluate sums and integrals in $6N$ dimensions. These calculations are replaced by random sampling. Again, chance is not inevitably a problem and may be helpful. Statistical Physics predicts thermodynamic and kinetic aspects of gazes and allows us to explain magnetism and radiation of matter from its microscopic properties. An interesting effect studied by statistical physics is the Brownian motion. In 1827, the botanist

Robert Brown observed the disordered movement of tiny particles on water. It looked like that :

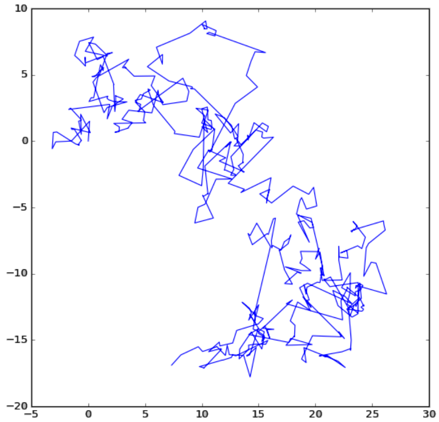

FIGURE 1 - Personal simulation (C language) of a 2D Brownian motion of 500 steps starting in (0,0).

It seems totally random[1] but this motion has a cause : multiple shocks with molecules or atoms, and this explanation, mainly theorized by Einstein and tested by Perrin, was a convincing evidence for the atomistic hypothesis. Here the atoms are so numerous and the motion is so fast that we only see a random behaviour, and Statistical Physics provides strong tools to study this motion (the number of atoms in the environment of the particle, giving an approximation of the Avogadro constant, the distance travelled by the particle...) and to make crucial ontological advance : in the beginning of the XXth century, lots of scientists didn't believe in atoms, these objects imagined by the epicureans millennia ago. Finally, the evolution of a system in Statistical Physics isn't determined by its differential equations of motion, but by a function, called entropy. Entropy characterizes the disorder, or the lack of information of a system. Of course entropy increases with the number of microstates. The theorem of maximisation of entropy states that a system evolves to an equilibrium state which corresponds to a maximum of entropy : the most probable. With the example of the coin, it is easy to see that the equal probability matches with a maximum of lack of information. Entropy defines reversibility : a system doesn't spontaneously evolve to a less probable state. This behaviour was obvious a long time before Statistical Physics gave formalism to study it rigorously, and it constituted the second principle of thermodynamics. For instance, if we let an ice cube in a water glass, it will melt and never return to its initial state. This is a certain prediction of thermodynamics. In statistical terms, the probability of the initial configuration of all molecules is very small, but this state remains possible : if we wait for an extremely long time, we can predict with a probability arbitrary close to one that the ice cube will re-form. Remember that laws ruling the molecules of the ice cube and the water are well known. Statistical Physics underlines the role of subjective chance in Science[2].

On the contrary, Quantum Physics is known as a domain of intrinsic chance. In Quantum Physics, measure has a very singular status. A quantum system is described

[1] And it can be modelled by a two dimensional random walk in the theory of probabilities.

[2] Unpredictability can also emerge from a system with only a small number of parameters, because of an important sensitivity to initial conditions. That's chaos theory.

by an algebraic object, a vector of a Hilbert space, and an observable is associated to an operator on the space. When we measure an observable, the result can only be an 'eigenvalue' (these values can be multiple) of the associated operator, and after the measure, the system is forced to be in a state associated with the measured value; this is called 'wave function collapse'. We can't predict the result of a measure in Quantum Physics (except in some precise cases) and this is a fundamental property. However, for each possible value, we can compute its *probability*, before a measurement, using Born rule. The sum of the probabilities of each value is obviously one. The fundamental difference between quantum chance and classical chance (like in a coin flip) is that quantum chance is '*true*' : our incapacity to predict a result is not in cause. Scientists experimentally showed that measures on atoms follow true intrinsic chance, not chance based on a statistical mixture of atoms existing before the measures (theoretically, it is demonstrated that random behaviour appearing during a measure and random distribution before a measure don't lead to the same types of results). To take an example, if I have electrons, I could compute that if I measure the momentum of one of them I have a probability 1/2 to get p_1 and a probability 1/2 to get p_2, not because half of the electrons have a momentum p_1 : they acquire their states during the measure. In opposition to the classical subjective chance, we will call this chance *objective*. Besides, one of the most known - and misinterpreted - notions of Quantum Physics is the 'uncertainty principle', introduced by Werner Heisenberg in 1927. Actually, physicists prefer to talk about 'Heisenberg's inequalities' because it's not a principle but a theorem and it's not about classical (subjective) uncertainty. These inequalities state that the product of standard deviations of conjugate variables, like position and momentum, is bounded below (by $\hbar/2$, where $\hbar$ is the reduced Planck constant). Heisenberg's inequalities give a fundamental algebraic property of quantum systems, not an information about the observational success of our instruments. They state that the idea of a precise value of physical observables is nonsense. This concept (like Gödel's theorems or Einstein's theories of relativity) is often used by pseudoscience and some ideologies to justify relativistic positions. It's totally absurd. Quantum Physics is not a regression : it doesn't state that we can know less than before Quantum Physics, it founds new concepts and objects of Physics. For instance, the idea of a punctual atom is replaced by a density of probability of presence of electrons in the space around a nucleus and the notion of an unique trajectory is replaced by a set of possible paths with associated probabilities.

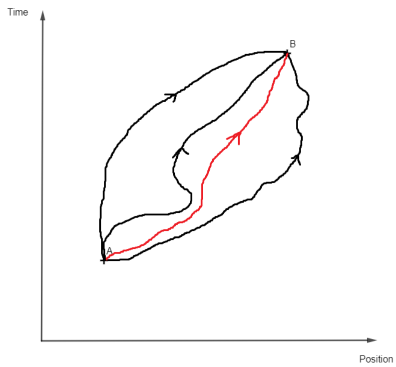

Figure 2 - Personal drawing representing multiple quantum paths, where the red path is the classical one.

It's the most powerful physical theory, for its strong explanation of phenomena, its extraordinarily precise computation of experimental measures and its fantastic prediction power. We started to associate predictability and determinism : we see now that a fundamentally indeterministic theory can predict things no other could. A last aspect (you can say that again !) of Quantum Physics, closely linked with the idea of true chance, is *entanglement* : it exists non local correlations. Two entangled particles can't be described by independent states, even if they are separated by an arbitrary large distance, and measurements of observables on them are correlated. If we measure a property (momentum, spin...) of a particle, we instantaneously have information on the other one. The famous Einstein-Podolsky-Rosen paradox was a thought experiment concluding with the incompleteness of Quantum Physics : the *spooky action at distance*[3], as Einstein called entanglement, contradicts special relativity. Now we know that Quantum Physics is not local, but we also know that we can't communicate using entanglement. Non local correlation is allowed by true chance, because true chance, indeterministic and fundamental in Quantum Physics, can manifest in several places at the same time, but it forbids communication. These concepts may have applications in everyday life. In computer science, pseudo-random numbers are used, for a program can't generate true chance, but only elements of very long pre-determined lists. Recently, quantum engineers built true random number generators. Quantum Physics teach us that nature is not deterministic and can produce true chance. Before this theory, scientists explained correlations with causal chains, even if their complexity made them impossible to follow in detail. Quantum non-locality is now a new basis for explaining correlations.

What we just said emphasizes the strong link between chance and causality : invoking chance is often admitting that there is a difficulty to establish causality. Chance is, to quote Etienne Klein [5], the *purgatory of causality*. When a deterministic causal chain is too complex, as we saw above, we resort to chance : uncertainty appears. But it's important to distinguish reduction to uncertainty and reduction to ignorance. Reduction to ignorance is the enemy of explanation, as Spinoza [12] wrote :

> If a stone falls from a roof on to someone's head, and kills him, they will demonstrate by their new method, that the stone fell in order to kill the man ; for, if it had not by God's will fallen with that object, how could so many circumstances (and there are often many concurrent circumstances) have all happened together by chance? Perhaps you will answer that the event is due to the facts that the wind was blowing, and the man was walking that way. "But why," they will insist, "was the wind blowing, and why was the man at that very time walking that way?" If you again answer, that the wind had then sprung up because the sea had begun to be agitated the day before, the weather being previously calm, and that the man had been invited by a friend, they will again insist : "But why

[3]It's interesting to notice the link with Newtonian gravitational theory. Classical Mechanics wasn't local : forces were instantaneous actions, whatever the distances of the items causing them. Electrodynamics and Relativity replaced this by the idea of field : a mass (or a charge, or other things) creates a field, which locally interacts with objects. Physics was local for only 50 years, before the concept of entanglement. Nowadays the goal of theorists is to unify the local description of General Relativity and the non locality of Quantum Physics.

> was the sea agitated, and why was the man invited at that time?" So they will pursue their questions from cause to cause, till at last you take refuge in the will of God—in other words, the sanctuary of ignorance (*Ethics*, I, appendix).

Spinoza attacks the transformation made by finalists of ignorance to a hidden verity. This reference to a first cause, the will of God, dogmatically stops the goal of Science, and of the deterministic philosophy of Spinoza, to find the causes of phenomena. We can describe the causal chain above in another way : *why did the stone fall ?* Because it has a mass and it is attracted by other masses. *Why is it attracted ?* It is attracted because of the law of Newton. *What justifies the law of Newton ? Why does the stone have a mass ?* The law of Newton is an approximation of the law of Einstein. The stone has a mass because it is composed of atoms, composed of protons, composed of quarks interacting the the Higgs field. *And Science can't explain the origin of the Higgs field or of Einstein equations, so it explains nothing.* This reduction to ignorance is abusive : the point where the chain stops moves back with the progress of Science. Furthermore, Science predictions are more and more precise and their barrier isn't the sanctuary of ignorance at the beginning of the causal chain, but the complexity of this chain. That's why a theory of probabilistic causality could be better than a theory of deterministic causality. David Hugh Mellor [8] proposed a model, distinct from the idea of explanation, to describe causality in a probabilistic way. To him, B causes A if and only if :

1. A and B are facts.
2. The probability of A knowing B is higher than the probability of A knowing not-B (high probability exigency).
3. The higher the probability of B is, the higher the probability of A is (increasing of probability).

This idea that causes are indicators for their effects is relevant in many cases, but it also exists counterexamples for this model. Nevertheless, it constitutes an interesting approach of causal chains. We saw that there is always subjective randomness in causal chains, whose weakness of their links makes them obscure, so the point seems to be the evaluation of the degree of contingency in a deterministic causal chain.

Contingencies produce statistical fluctuations that are crucial in metrology. When scientists run experiments, a new kind of uncertainty appears, *measurement error*. Measurement errors are divided into two categories : systematic errors and random errors (as we saw, quantum uncertainty occurring during a measure is asides from these considerations). Systematic errors are due to external causes, typically the instrument of measure, take the same value when we are in the same conditions and can be controlled. Random errors are caused by unpredictable fluctuations and take different values between several measures realized in the same conditions.

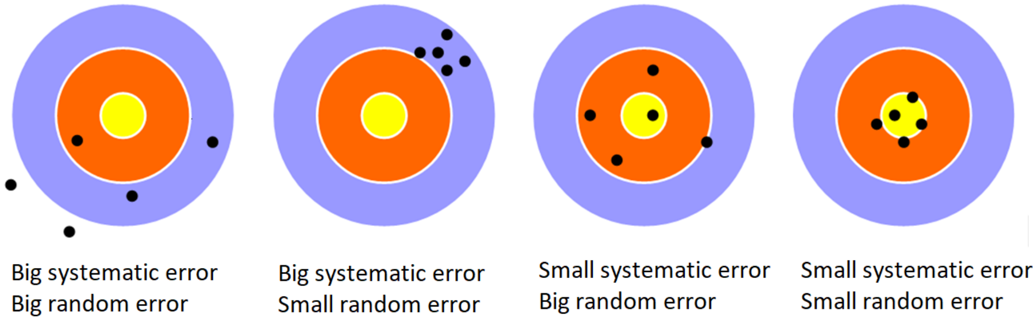

FIGURE 3 - Different classes of errors. From Oswaldo Rossi Jr, http://calibraend.blogspot.com/2013/02/voce-conhece-diferenca-entre-precisao-e.html

Random errors constitute the contingency we talked about. They are caused by many indeterminable factors independent from the observable and can be well analysed by statistics. The value of what we measure is just the mean of all the results we have and we gain precision by increasing the number of measurements. The impact of random fluctuations is given by the standard deviation of the distribution of results. The central limit theorem implies that the distribution of physical measurements is Gaußian and totally determined by its mean and standard deviation. The size of the prediction interval is then inversely proportional to the square root of the number of measurements, so each field of Science performing experiments can define his confidence level. For instance we have 32% of chance of having a difference between a result and the mean value bigger than one standard deviation. This is useful to discriminate significant results from statistical fluctuations. In particle physics experiments, like those running in the LHC at CERN, responsible for the Higgs boson discovery, scientists are very exigent. The convention is a difference of five standard deviations between a normal (fitting with the current theory) and an abnormal (indicating a new particle) result, meaning that there is one chance in 3 500 000 that what we observe is just a random fluctuation, and not *really* the particle. This general method is also used to compare therapies, to confront drugs and placebos, to compute qualities of manufactured products or to evaluate the extraordinariness of events claimed to be 'paranormal'. We can notice the apparent subjectiveness of this method : scientists chose their confidence level (one, two, three... standard deviations) and therefore the moment when a result become extraordinary. I could decide to consider a result as significant if it had less than 20% of chance to occur randomly. To avoid this and to gain in consistence, the scientific community wants a significance of at least two standard deviations, corresponding to 5% of chance of having a false positive, and changes its exigency as we saw, according to the maxim *the more a fact is extraordinary, the more it needs strong evidences*. We showed here the intervention of chance during Science experiments, and the necessity of using statistical theories to deal with it.

Chance also appears when scientists discover things in a very different way. Serendipity is the fact of making an unplanned scientific discovery or invention. Some classical and questionable examples are America, radioactivity, penicillin, superconductivity, the Newtonian law of gravitation, the Cosmic Microwave Background, or the Velcro.

Serendipity regroups all the contingent discoveries. They can be made by pure chance, by error, by accident, by clumsiness or by malpractice. Let's examine the different possible links between a searched object and a discovered object : we can find what we was searching, we can not find what we was searching and we can find something we wasn't searching. The first class regroups, for instance, the discovery of Neptune in 1846, predicted by Urbain Le Verrier to explain the violation of Newton's laws by Uranus' orbit, the discovery of the neutrino in 1956, predicted by Wolfgang Pauli to conserve energy during β-decay and the discovery of the Higgs boson in 2012, predicted by Peter Higgs, Robert Brout and François Englert, to explain the origin of masses of particles and the disconnection between weak and electromagnetic interactions[4]. The third category of discoveries is often called serendipity. We gave different examples at the beginning of this paragraph. It's not totally legitimate to call random discovery any discovery that scientists wasn't searching. In every case, the discovery is based on a strong theoretical background and it's not due to the genius of a single scientist or explorer who had the prophetic intuition to perform an experiment or to explore where everybody thought there was nothing to find. Colomb knew that the Earth was round, so he had to find a continent, but this was a new continent for the Europeans. Fleming was not looking for penicillin, but he was already a renowned biochemist and microbiologist who studied staphylococci in detail. When Kamerlingh Onnes discovered that the resistance of mercury abruptly disappeared at the temperature of 4.2 Kelvin, he was putting mercury in liquid helium ! But he expected the opposite result : he thought that electrons would freeze at cryogenic temperature, forbidding electrical conduction. And of course Newton didn't discover universal gravitation by illumination when an apple fell on his head[5], he already studied dynamics of moving and falling bodies, and was *standing on the shoulders of Giants*. This examples show that chance and contingency can play a role in scientific discoveries, and even revolutions, but they are not sufficient. Discoveries never come without motivation, theoretical background is crucial.

The goal of this paper was to show that Chance and Science are not conflicting. Now we have a better view of the role of random processes in Science. Random events can belong to subjective or objective chance. In the case of subjective chance, randomness is an approximation, a model, of the deterministic behaviour of nature, but in the case of objective chance, randomness is intrinsic to nature. We should keep in mind the importance of models : what we call a random event in Quantum Physics could be a model of something better described by another theory. Since scientists developed theories of chance, like Laplace's works on Bayesian probability or the axiomatization of probabilities by Kolmogorov, they added laws of chance to laws of nature, in order to tame uncertainty and to describe and explain indeterministic events. This

[4]I didn't choose these examples randomly... They illustrate one of the two solutions to solve a contradiction between theory and experiment : change the laws or add new objects. The problem of the β-decay and the strangeness of Uranus' orbit were solved by an ontological approach, the theory was saved by new objects (on the contrary, the strangeness of Mercury's orbit couldn't be solved by the presence of a new object, and needed to change the theory, that was the first success of General Relativity). The case of the Higgs boson is both conceptual and material : it's a new object that changed the theory (before its discovery). Nowadays the problem of dark matter shows that scientists don't want to abandon their powerful theory : they invented a new object to save it.

[5]It probably didn't happen.

has applications in lots of fields, including of course Physics, but also Social Science, Economics or Medical Science. Chance also plays a role in the context of Scientific discoveries, with serendipity. We can even go further. We know that Science can't catch the truth : when our knowledge increases, our ignorance and questioning increase. Finally, Science seems to be a path to uncertainty, with chance as partner.

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