IMDEN

le 30/10

Homework 4:

Exercice 9.1

P obeys a GMM.

$\tilde{P}$ is a noisy observation of P, of size $w \times w$, with noise σ^2

- $\mathbb{P}(\tilde{P} \mid P) = \dfrac{1}{(2\pi\sigma^2)^{\frac{w^2}{2}}} \exp\left[-\dfrac{\|\tilde{P} - P\|^2}{2\sigma^2}\right]$

- $\mathbb{P}(P) = \sum_{k=1}^{K} p_k \, \mathcal{N}(P \mid \mu_k, \Sigma_k)$

μ_k and Σ_k were obtained in the GMM training.

Then, our current problem is

$$\min_P E(P \mid \tilde{P}) \iff \min_P \frac{\|P - \tilde{P}\|^2}{2\sigma^2} - \log(\mathbb{P}(P))$$

$$\iff \max_P \log\left(\mathbb{P}(P) \times e^{-\frac{\|P-\tilde{P}\|^2}{2\sigma^2}}\right)$$

$$\iff \max_P \mathbb{P}(P) \times e^{-\frac{\|\tilde{P}-P\|^2}{2\sigma^2}}$$

$$\iff \max_P \mathbb{P}(P) \times \frac{1}{(2\pi\sigma^2)^{\frac{w^2}{2}}} e^{-\frac{\|P-\tilde{P}\|^2}{2\sigma^2}}$$

$$\iff \max_P \mathbb{P}(P) \times \mathbb{P}(\tilde{P} \mid P)$$

bayes rule

$$\iff \max_P \mathbb{P}(P \mid \tilde{P}) \times \mathbb{P}(\tilde{P})$$

$$\iff \max \mathbb{P}(P \mid \tilde{P})$$

This means that minimizing the energy is the same as solving a MAP problem.

Exercice 92:

Let $\mathcal{P}$ the set of patch projector

The likelihood of the image is the probability of each patch PU, so that they all compose the image U ie

$$\mathcal{L} = \mathbb{P}\left(\bigcap_{P \in \mathcal{P}} \{PU\}\right).$$

Thus,

$$\log(\mathcal{L}) = \log\left(\mathbb{P}\left(\bigcap_{P\in\mathcal{P}} \{PU\}\right)\right)$$

if all patches projectors are independant

ie $\log(\mathcal{L}) = \log\left(\prod_{P\in\mathcal{P}} \mathbb{P}(PU)\right)$

ie $\boxed{\log(\mathcal{L}) = \sum_{P\in\mathcal{P}} \log \mathbb{P}(PU) = EPPL}$

This is why the EPPL can be considered as the log likelihood of an image.

However, the patches are often not independant. Indeed, most of the patches are simply shifted from other patches.