IMDEN Le 17/11

Homework 7

Exercice 2.1:

Let u an ideal image and v its denoised version.

We have

$$E_\Theta^{FoE}(u,v) = \frac{1}{2\sigma^2}\|u-v\|^2 + \sum_x \sum_{i=1}^{N} \phi_i(k_i * u(x)) + C.$$

Supposing the kernels k_i have size $n \times n$, we have

$$k_i * u(x) = \sum_{s=-n}^{n} \sum_{t=-n}^{n} k_i(s,t)\, u(x_1 - s, x_2 - t)$$

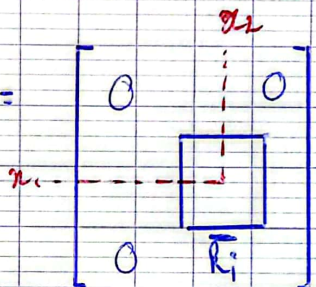
with $x = (x_1, x_2)$ the coordinate of a pixel

So we have

$$\frac{\partial}{\partial u_{s,t}} (k_i * u(x)) = k_i(x_1 - s, x_2 - t)\, \delta_{(x_1-s,\, x_2-t) \in [-n,n]^2}$$

In matrix form:

$$\nabla_u k_i * u(x) =$$



So $$\frac{\partial}{\partial u_{s,t}} \phi_i(k_i * u(x)) = k_i(x_1 - s, x_2 - t)\, \phi_i'(k_i * u(x))\, \delta_{\dots}$$

So $$\sum_{x \in \Omega} \frac{\partial}{\partial u_{s,t}} \phi_i(k_i * u(x)) = \sum_{x \in \Omega} k_i(x_1 - s, x_2 - t)\, \phi_i'(k_i * u(x))$$
$$= \bar{k}_i * \phi_i'(k_i * u).$$

So overall

$$\boxed{\nabla E_\Theta^{FoE}(u,v) = \frac{1}{\sigma^2}(u - v) + \sum_{i=1}^{N} \bar{k}_i * \phi_i'(k_i * u).}$$

If we apply this with $u = u^t$ by iteration, we find Eq 2.5.

Exercise 2.2.

We assume that the data is distributed according to $p(x)$

We have the following equivalences

$$\min_\theta \mathcal{L}_\infty(\theta) \iff \min_\theta \int \log p_\theta(x)\, p(x)\, dx$$

$$\iff \max_\theta \int \log \frac{1}{p_\theta(x)}\, p(x)\, dx + \underbrace{\int \log p(x)\, p(x)\, dx}_{\text{indep of } \theta}$$

$$\iff \max_\theta \int \log \frac{p(x)}{p_\theta(x)}\, p(x)\, dx$$

ie $\boxed{\min_\theta \mathcal{L}_\infty(\theta) \iff \max KL(p, p_\theta)}$