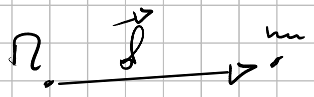
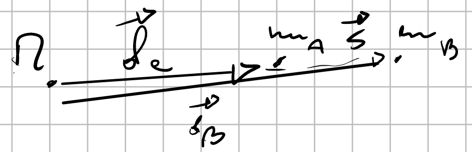
Energia Meccanica Forza Gravità

Supponiamo di avere due corpi M e m.



m si allontana



Lavoro:
$$L_{AB} = \int_A^B \vec F \cdot d\vec s = \int_A^B -\frac{GMm}{d^2}\underbrace{\hat d \cdot d\vec s}_{ds} =$$

$$= -GMm \int_A^B \frac{1}{d^2}\, ds =$$

$$= -GMm\left(-\frac{1}{d_B} + \frac{1}{d_A}\right) =$$

$$= \frac{GMm}{d_B} - \frac{GMm}{d_A}$$

$$d_B > d_A \Rightarrow L_{AB} < 0$$

È conservativa: $L_{AB} = U_A - U_B$

Assumiamo $d_B \to \infty$

$$\Rightarrow L_{AB} \to -\frac{GMm}{d_A}, \quad U_B \to U_\infty$$

$$L_{AB} = -\frac{GMm}{d_A} = U_A - U_\infty$$

$$U_A = -\frac{GMm}{d_A} + U_\infty$$

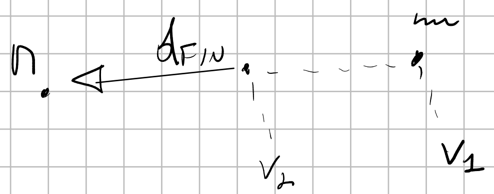
$$U_A = -\frac{GMm}{d_A}$$, a meno di costante.

Assumiamo sia zero.

$$E = K + U$$

$$E = \frac{1}{2} \cdot m \cdot v^2 - \frac{GMm}{d_A} \quad \square$$

Cambiamento Velocità



Assumiamo $d \to \infty$

$\to 0$

$$E_{IN} = \frac{1}{2} \cdot m \cdot v_1^2 - \frac{G \cdot Mm}{d}$$

$$E_{FIN} = \frac{1}{2} \cdot m \cdot v_2^2 - \frac{GMm}{d}$$

$$\frac{1}{2} \cancel{m} v_1^2 = \frac{1}{2} \cancel{m} v_2^2 - \frac{GM\cancel{m}}{d}$$

$$v_1^2 = v_2^2 - \frac{2GM}{d}$$

$$v_2^2 = v_1^2 + \underbrace{\frac{2GM}{d}}_{>0} \Rightarrow v_2^2 > v_1^2 \Rightarrow v_2 > v_1$$

$\square$

Velocità di Fuga

Troviamo la velocità che consente di poter allontanare il corpo a di distanza infinita.

$$E_{IN} = \frac{1}{2} m v_F^2 - \frac{GMm}{d}$$

$$E_{FIN} = \frac{1}{2} m \underset{0}{\underset{\shortparallel}{v_\infty}}^2 - \frac{GMm}{d_\infty} \to 0$$

$$d_\infty \to \infty$$

$$\frac{1}{2} m v_F^2 - \frac{GMm}{d} = 0$$

$$\frac{1}{2} \cancel{m} v_F^2 = \frac{GM\cancel{m}}{d}$$

$$v_F = \sqrt{\frac{2GM}{d}} \qquad \square$$

Moto Molla

Hyp:

$x(t=0) = x_0$

$v(t=0) = 0$

$$F = -k \cdot x = m \cdot a = m \frac{d^2 x}{dt^2}$$

$$m \cdot \frac{d^2 x}{dt^2} + kx = 0$$

$$\frac{d^2 x}{dt} + \frac{k}{m} x = 0$$

Eq. diff. lin. omo.: $y'' + a y = 0 \Rightarrow$ Guess: $A \cdot \cos(\omega t + \phi)$

$$x(t) = A \cdot \cos(\omega t + \phi)$$

$$x'(t) = -A \cdot \sin(\omega t + \theta) \cdot \omega$$

$$x''(t) = \underbrace{-A \cdot \cos(\omega t + \theta)}_{x(t)} \cdot \omega^2 = -\omega^2 x(t)$$

Deve valere:

$$x''(t)+\frac{k}{m}x(t)=0$$

$$\omega^2 x(t)+\frac{k}{m}\cdot x(t)=0$$

$$x(t)\left(\omega^2+\frac{k}{m}\right)=0$$

$$\omega=\sqrt{\frac{k}{m}}$$

$x(t=0)=x_0 \Rightarrow A\cdot\cos(\omega t+\phi)=x_0$ con $t=0$

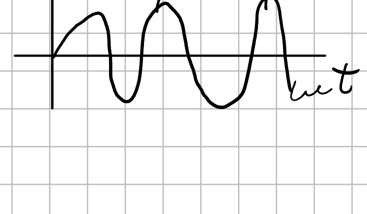
$\Rightarrow A=x_0$

$v(t=0)=0 \Rightarrow x'(t)=-\underbrace{\omega}_{=\sqrt{\frac{k}{m}}}\underbrace{A}_{=x_0}\cdot\sin(\omega \overset{=0}{t}+\phi)=0$

$\Downarrow$

$\phi=0 \Rightarrow \sin(\phi)=0$

$$x(t)=x_0\cdot\cos(\omega t) \quad \text{con} \quad \omega=\sqrt{\frac{k}{m}}$$



$$\left.\begin{array}{l}\Rightarrow \omega t=2\pi\\ t=T\end{array}\right\}$$

Periodo:

$$T=\frac{2\pi}{\omega}$$

$\square$

Energia Potenziale Forza Elastica

$$L_{AB} = \int_A^B dL = \int_A^B F \cdot dx$$

$$= \int_A^B -Kx \cdot dx =$$

$$= -K\left(\frac{x_B^2}{2} - \frac{x_A^2}{2}\right)$$

$$= -\frac{1}{2}K x_B^2 + \frac{1}{2}K x_A^2$$

$$\frac{1}{2}K x_A^2 - \frac{1}{2}K x_B^2 = U_A - U_B$$

$$U = \frac{1}{2}K x^2$$

$\left(\omega = \sqrt{\frac{K}{m}}\right)$

$x(t) = x_0 \cdot \cos(\omega t)$

$x'(t) = -x_0 \cdot \omega \cdot \sin(\omega \cdot t)$

$$v = \frac{dx}{dt}$$

(cinetica)

$$E = K + U = \frac{1}{2} m v^2 + \frac{1}{2} K x^2$$

$$= \frac{1}{2} x_0^2 \cdot \underbrace{m \cdot \omega^2}_{K} \cdot \sin^2(\omega \cdot t) + \frac{1}{2} \cdot K \cdot x_0^2 \cdot \cos^2(\omega \cdot t)$$

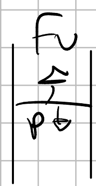
$\cos^2(x) + \sin^2(x) = 1$

$$= \frac{1}{2} x_0^2 \cdot K$$

□

Forze Viscose

$$\vec{F_v} = -\beta \vec{v}$$



$$\vec{F} = m \cdot \vec{a}$$

$$\vec{P} + \vec{F_v} = m \cdot \vec{a}$$

$$-m \cdot g - \beta v_y = m \cdot a_y \to 0$$

$$a_y = \frac{dv}{dt}$$

Per $t \to \infty \Rightarrow a_y \to 0$

$t \to \infty$: $-m \cdot g - \beta v_y = 0$

$$v_y = -\frac{m \cdot g}{\beta}$$ Per $t \to \infty$ è uguale

In generale (più preciso)

esx: $$v_y(t) = \frac{m \cdot g}{\beta}\left(e^{-\frac{\beta t}{m}} - 1\right)$$

$$a = \frac{dv}{dt} \qquad \frac{dv}{dt} = \frac{\cancel{m} \cdot g}{\cancel{\beta}} \cdot e^{-\frac{\beta t}{m}} \cdot \left(-\frac{\cancel{\beta}}{\cancel{m}}\right)$$

$$a = -g \cdot e^{-\frac{\beta t}{m}}$$

$$F = m \cdot a = -m \cdot g \cdot e^{-\frac{\beta t}{m}} \qquad \square$$

Energia Interna Corpo Solido

$$Q - L = \Delta U$$

$$\left.\begin{array}{l} \delta Q - \delta L = dU \\ dL = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \delta Q = dU \\ \delta Q = m \cdot c \cdot dT \end{array}\right\} dU = m c \, dT$$

$$\int_{T_A}^{T_B} dU = \int_{T_A}^{T_B} m \, c \, dT$$

$$U(T_B) - U(T_A) = m \cdot c \, (T_B - T_A)$$

$$U(T) = m \cdot c \cdot T + \text{costante}$$

nel Gas

vol costante $\qquad Q - L = \Delta U \qquad \underbrace{\delta Q - \delta L = dU \qquad \delta L = 0}$

$$n \cdot C_V \cdot dT = \delta Q = dU$$

$$U(T) = n \cdot C_V \cdot T + \text{costante}$$

Vale sempre.

Dimostra che $C_p = C_V + R$

Pressione Costante

$$Q - L = \Delta U$$

$$\delta Q - dL = dU$$

$dL = PdV$

$$\delta Q - PdV = dU$$

$$n \cdot C_p dT = PdV + n \cdot C_V dT$$

$$PV = n \cdot R \cdot T$$

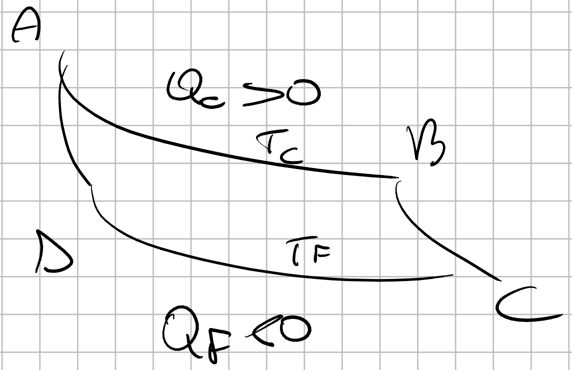
$$d(PV) = d(n \cdot R \cdot T)$$

$$PdV = n R \cdot dT$$

$$n \cdot C_p dT = n \cdot R \cdot dT + n C_V dT$$

$$C_p = C_V + R \quad \square$$

Carnot



$$L = Q = Q_C + Q_F = |Q_C| - |Q_F|$$

$$\eta = \frac{L}{|Q_c|} = \frac{|Q_c - Q_F|}{|Q_c|} = 1 - \frac{|Q_F|}{|Q_c|}$$

Nella macchina di Carnot: $\frac{|Q_F|}{|Q_C|} = \frac{T_F}{T_C}$

In generale

$$\eta = 1 - \frac{|Q_F|}{|Q_C|} \leq 1 - \frac{T_F}{T_C}$$

$$-\frac{|Q_F|}{|Q_C|} \leq -\frac{T_F}{T_C}$$

$$\frac{|Q_F|}{|Q_C|} \geq \frac{T_F}{T_C}$$

$$\frac{|Q_C|}{T_C} \leq \frac{|Q_F|}{T_F}$$

$$\frac{|Q_C|}{T_C} - \frac{|Q_F|}{T_F} \leq 0$$

$$\left.\begin{array}{l} Q_C>0 \\ Q_F<0 \end{array}\right\}$$

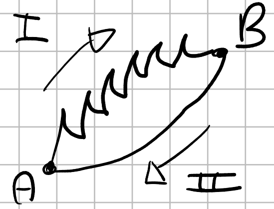
$$\frac{Q_C}{T_C}+\frac{Q_F}{T_F}\le 0$$

$$\sum_{i=F,C}\frac{Q_i}{T_i}\le 0$$

$$\oint \frac{\delta Q}{T}\le 0$$: Disuguaglianza di Clausius

Nei cicli reversibili $\oint \frac{\delta Q_{rev}}{T}=0$

$$\underbrace{\int_A^B \frac{\delta Q_{rev}}{T}}_{S(B)-S(A)}+\int_B^A\frac{\delta Q_{rev}}{T}=0$$



$$\int_I \frac{\delta Q_{IRR}}{T}+\int_{II}\frac{\delta Q}{T}\le 0$$

$$\int_I \frac{\delta Q_{IRR}}{T}-\underbrace{\int_A^B\frac{\delta Q}{T}}_{S(B)-S(A)}\le 0$$

$$\int_I \frac{\delta Q_{IRR}}{T}\le \Delta S$$

In un sistema isolato, con trasformazione irr.

$\Rightarrow \delta Q_{IRR} = 0 \Rightarrow \Delta S \geq 0$

Abbiamo un elemento di volume con densità p_0 $\rho_0 = dm/dV$

e definito così: $dV = A_T dx$

$dm = \rho_0 dV$

L'onda passa.

- Il volume si espande o si comprime. da dx a $dx + d\xi$
- ρ_0 diventa ρ
- La pressione acquista una dipendenza da x

Gradiente di pressione

$$dP = \frac{\partial P}{\partial x} dx.$$

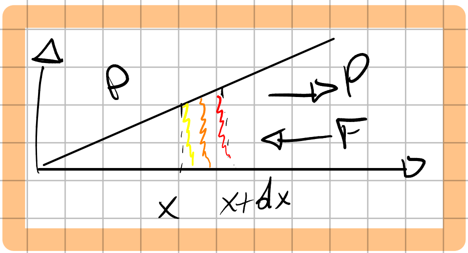
F=ma

$$dF = a\,dm = a \rho_0 dV$$

$$dF = -A_T dP,$$

$A_T \cdot dx = dV$

$$dF = -\frac{\partial P}{\partial x} dV.$$



$$a = \partial^2\xi/\partial t^2$$

$$dF = \rho_0 dV \frac{\partial^2 \xi}{\partial t^2}$$

$$\rho_0 \cancel{dV} \frac{\partial^2 \xi}{\partial t^2} = -\frac{\partial P}{\partial x} \cancel{dV}$$

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = -\frac{\partial P}{\partial x}.$$

$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial \rho}\frac{\partial \rho}{\partial x},$$

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = -\frac{\partial P}{\partial \rho}\frac{\partial \rho}{\partial x}$$

$$c_s^2 = \frac{\partial P}{\partial \rho}$$

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = -c_s^2 \frac{\partial \rho}{\partial x}$$

$$\boxed{dm = \rho_0 dV}$$

Anche quando passa l'onda, la massa è costante

$$\boxed{dV = A_T dx}$$

$$\rho_0 dV = \rho \underbrace{A_T(dx + d\xi)}_{dV_{FINALE}}$$

$$\rho_0 dx = \rho(dx + d\xi).$$

$$d\xi = (\partial\xi/\partial x) dx$$

$$\rho_0 dx = \rho\left(1 + \frac{\partial \xi}{\partial x}\right) dx$$

$$\frac{1}{1+a} \approx 1 - a,$$

$$\rho_0 = \rho\left(1 + \frac{\partial \xi}{\partial x}\right)$$

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = -c_s^2 \frac{\partial \rho}{\partial x}$$

$$\rho = \rho_0\left(1 + \frac{\partial \xi}{\partial x}\right)^{-1} \approx \rho_0\left(1 - \frac{\partial \xi}{\partial x}\right)$$

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = c_s^2 \rho_0 \frac{\partial^2 \xi}{\partial x^2}$$

$$\frac{\partial \rho}{\partial x} = -\rho_0 \frac{\partial^2 \xi}{\partial x^2}.$$

$$\boxed{\frac{\partial^2 \xi}{\partial t^2} = c_s^2 \frac{\partial^2 \xi}{\partial x^2}.}$$

$\square$