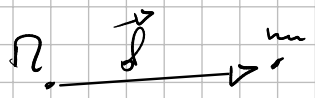
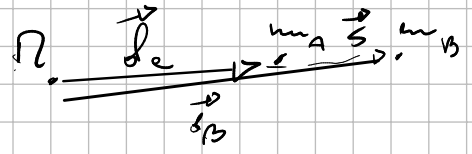
Energia Meccanica Forze Gravitazionali

Supponiamo di avere due corpi M e m.



m si allontana



Lavoro: $L_{AB} = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B -\frac{GMm}{d^2} \underbrace{\hat{d} \cdot d\vec{s}}_{ds} =$

$$= -GMm \int_A^B \frac{1}{d^2} ds =$$

$$= -GMm \left(-\frac{1}{d_B} + \frac{1}{d_A} \right) =$$

$$= \frac{GMm}{d_B} - \frac{GMm}{d_A}$$

$d_B > d_A \Rightarrow L_{AB} < 0$

È conservativa: $L_{AB} = U_A - U_B$

Assumiamo $d_B \to \infty$

$$\Rightarrow L_{AB} \to -\frac{GMm}{d_A}, \quad U_B \to U_\infty$$

$$L_{AB} = -\frac{GMm}{d_A} = U_A - U_\infty$$

$$U_A = -\frac{GMm}{d_A} + U_\infty$$

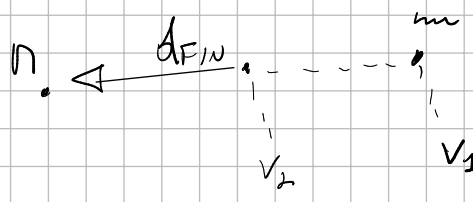
$U_A = -\frac{GM_m}{d_A}$, è nullo di costante.

Assumiamo ora zero.

$$E = K + U$$

$$E = \frac{1}{2} \cdot m \cdot v^2 - \frac{GM_m}{d_A}$$ □

Conservazione Velocità



Assumiamo $d \to \infty$

$$E_{IN} = \frac{1}{2} \cdot m \cdot V_1^2 - \frac{GM_m}{d} \to 0$$

$$E_{FIN} = \frac{1}{2} \cdot m \cdot V_2^2 - \frac{GM_m}{d}$$

$$\frac{1}{2} m V_1^2 = \frac{1}{2} m V_2^2 - \frac{GM_m}{d}$$

$$V_1^2 = V_2^2 - \frac{2GM}{d}$$

$$V_2^2 = V_1^2 + \underbrace{\frac{2GM}{d}}_{>0}$$

$$\Rightarrow V_2^2 > V_1^2 \Rightarrow V_2 > V_1$$ □

Velocità di Fuga

Troviamo la velocità che consente di fare allontanare il corpo a distanza infinita

$$E_{IN} = \frac{1}{2} m V_F^2 - \frac{G M m}{d}$$

$$E_{FIN} = \frac{1}{2} m \underbrace{V_\infty^2}_{=0} - \underbrace{\frac{G M m}{d_\infty}}_{d_\infty \to \infty} \to 0$$

$$\frac{1}{2} m V_F^2 - \frac{G M m}{d} = 0$$

$$\frac{1}{2} m V_F^2 = \frac{G M m}{d}$$

$$V_F = \sqrt{\frac{2GM}{d}} \quad \square$$

Moto Palla

Hyp:
$x(t=0) = x_0$
$v(t=0) = 0$

$$F = -k \cdot x = m \cdot a = m \frac{d^2x}{dt^2}$$

$$m \cdot \frac{d^2x}{dt^2} + kx = 0$$

$$\frac{d^2x}{dt} + \frac{k}{m} x = 0$$

Eq. diff. lin. omo.: $y'' + a y = 0 \Rightarrow$ Gruppo: $A \cdot \cos(\omega t + \phi)$

$$x(t) = A \cdot \cos(\omega t + \phi)$$

$$x'(t) = -A \cdot \sin(\omega t + \theta) \cdot \omega$$

$$x''(t) = \underbrace{-A \cdot \cos(\omega t + \theta)}_{x(t)} \cdot \omega^2 = -\omega^2 x(t)$$

Deve valere:

$$x''(t) + \frac{k}{m} x(t) = 0$$

$$\omega^2 x(t) + \frac{k}{m} \cdot x(t) = 0$$

$$x(t)\left(\omega^2 + \frac{k}{m}\right) = 0$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$x(t=0) = x_0 \quad \Rightarrow A \cdot \cos(\omega t + \phi) = x_0 \quad \text{con } t = 0$$

$$\Rightarrow A = x_0$$

$$v(t=0) = 0 \quad \Rightarrow x'(t) = -\omega A \cdot \sin(\omega t + \phi) = 0$$

($\omega = \sqrt{\frac{k}{m}}$, $A = x_0$, $t = 0$)

$$\phi = 0 \Rightarrow \sin(\phi) = 0$$

$$x(t) = x_0 \cdot \cos(\omega t) \quad \text{con} \quad \omega = \sqrt{\frac{k}{m}}$$

$$\Rightarrow \omega t = 2\pi \quad t = T$$

Periodo:

$$T = \frac{2\pi}{\omega}$$

□

Energia Meccanica Forza Elastica

$$L_{AB} = \int_A^B dL = \int_A^B F \cdot dx$$

$$= \int_A^B -kx \cdot dx =$$

$$= -k \left(\frac{x_B^2}{2} - \frac{x_A^2}{2} \right)$$

$$= -\frac{1}{2} k x_B^2 + \frac{1}{2} k x_A^2$$

$$\frac{1}{2} k x_A^2 - \frac{1}{2} k x_B^2 = U_A - U_B$$

$$U = \frac{1}{2} k x^2$$

$$\left(\omega = \sqrt{\frac{k}{m}}\right)$$

$$\Rightarrow x(t) = x_0 \cdot \cos(\omega t)$$

$$\Rightarrow v = \frac{dx}{dt} \quad / \quad x'(t) = -x_0 \cdot \omega \cdot \sin(\omega t)$$

$$E = \overset{(\text{cinetica})}{K} + U = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

$$= \frac{1}{2} x_0^2 \cdot \underbrace{m \cdot \omega^2}_{k} \cdot \sin^2(\omega \cdot t) + \frac{1}{2} \cdot k \cdot x_0^2 \cdot \cos^2(\omega \cdot t)$$

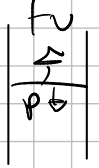
$$\cos^2(x) + \sin^2(x) = 1$$

$$= \frac{1}{2} x_0^2 \cdot k$$

□

Forze Viscose

$$\vec{F}_v = -\beta \vec{v}$$



$$\vec{F} = m \cdot \vec{a}$$

$$\vec{P} + \vec{F}_v = m \cdot \vec{a}$$

$$-m \cdot g - \beta v_y = m \cdot a_y \rightarrow \quad a_y = \frac{dv}{dt}$$

Per $t \to \infty \Rightarrow a_y \to 0$

$t \to \infty$: $\quad -m \cdot g - \beta v_y = 0$

$$v_y = -\frac{m \cdot g}{\beta}$$ ← Per $t \to \infty$ è uguale

In generale (più preciso)

$$\text{es: } v_y(t) = \frac{m \cdot g}{\beta}\left(e^{-\frac{\beta t}{m}} - 1\right)$$

$$a = \frac{dv}{dt} \qquad \frac{dv}{dt} = \frac{m \cdot g}{\beta} \cdot e^{-\frac{\beta t}{m}} \cdot \left(-\frac{\beta}{m}\right)$$

$$a = -g \cdot e^{-\frac{\beta t}{m}}$$

$$F = m \cdot a = -m \cdot g \cdot e^{-\frac{\beta t}{m}}$$ □

Energia Interna Corpo Solido

$$Q - L = \Delta U$$

$$\left.\begin{aligned} \delta Q - dL &= dU \\ dL &= 0 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} \delta Q &= dU \\ \delta Q &= m \cdot c \cdot dT \end{aligned}\right\} dU = m c\, dT$$

$$\int_{T_A}^{T_B} dU = \int_{T_A}^{T_B} m c\, dT$$

$$U(T_B) - U(T_A) = m \cdot c\,(T_B - T_A)$$

$$U(T) = m \cdot c \cdot T + \text{costante}$$

nel Gas

vol costante $\quad Q - L = \Delta U \quad \underbrace{\delta Q - dL = dU \quad dL = 0}$

$$n \cdot C_V \cdot dT = \delta Q = dU$$

$$U(T) = n \cdot C_V \cdot T + \text{costante}$$

Vale sempre.

Si ottiene che $C_p = C_v + R$

Pressione costante

$$Q - L = \Delta U$$

$$\delta Q - dL = dU$$

$dL = PdV$

$$\delta Q - PdV = dU$$

$$n \cdot C_p dT = PdV + n \cdot C_V dT$$

$$PV = n \cdot R \cdot T$$

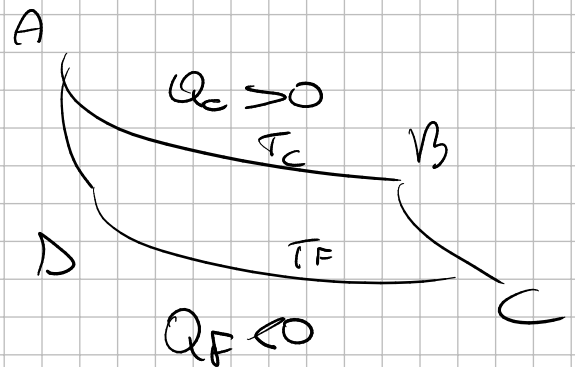
$$d(PV) = d(n \cdot R \cdot T)$$

$$PdV = nR \cdot dT$$

$$n \cdot C_p dT = n \cdot R \cdot dT + n C_V dT$$

$$C_p = C_V + R \quad \square$$

Carnot



$$L = Q = Q_C + Q_F = |Q_C| - |Q_F|$$

$$\eta = \frac{L}{|Q_C|} = \frac{|Q_C - Q_F|}{|Q_C|} = 1 - \frac{|Q_F|}{|Q_C|}$$

Nella macchina di Carnot: $\frac{|Q_F|}{|Q_C|} = \frac{T_F}{T_C}$

In generale

$$\eta = 1 - \frac{|Q_F|}{|Q_C|} \leq 1 - \frac{T_F}{T_C}$$

$$-\frac{|Q_F|}{|Q_C|} \leq -\frac{T_F}{T_C}$$

$$\frac{|Q_F|}{|Q_C|} \geq \frac{T_F}{T_C}$$

$$\frac{|Q_C|}{T_C} \leq \frac{|Q_F|}{T_F}$$

$$\frac{|Q_C|}{T_C} - \frac{|Q_F|}{T_F} \leq 0$$

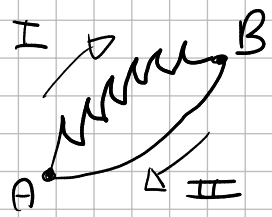
$$\left.\begin{array}{l} Q_C > 0 \\ Q_F < 0 \end{array}\right\} \qquad \frac{Q_C}{T_C} + \frac{Q_F}{T_F} \leq 0$$

$$\sum_{i \neq F,C} \frac{Q_i}{T_i} \leq 0$$

$$\oint \frac{\delta Q}{T} \leq 0 \quad : \text{Disuguaglianza di Clausius}$$

Nei cicli reversibili $\oint \frac{\delta Q_{rev}}{T} = 0$

$$\underbrace{\int_A^B \frac{\delta Q_{rev}}{T}}_{S(B)-S(A)} + \int_B^A \frac{\delta Q_{rev}}{T} = 0$$



$$\int_I \frac{\delta Q_{IRR}}{T} + \int_{II} \frac{\delta Q}{T} \leq 0$$

$$\int_I \frac{\delta Q_{IRR}}{T} - \underbrace{\int_A^B \frac{\delta Q}{T}}_{S(B)-S(A)} \leq 0$$

$$\int_I \frac{\delta Q_{IRR}}{T} \leq \Delta S$$

In un sistema isolato, con trasformazione irr.

$\Rightarrow \{Q_{IRR} = 0 \Rightarrow \Delta S \geq 0$

Abbiamo un elemento di volume con densità p_0 $\rho_0 = dm/dV$

e definito così: $dV = A_T dx$

L'onda passa.

- Il volume si espande o si comprime.

- ρ_0 diventa ρ

- La pressione acquista una dipendenza da x

da dx a $dx + d\xi$

L'equazione delle onde è un'equazione per ξ, ovvero per la perturbazione indotta dall'onda

$dm = \rho_0 dV$

Gradiente di pressione

$dP = \frac{\partial P}{\partial x} dx.$

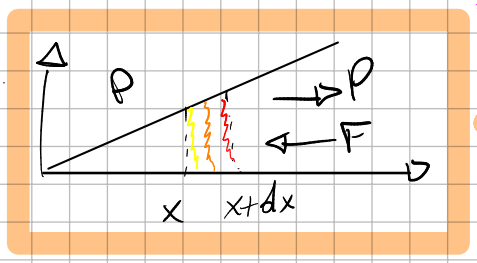
F=ma

$dF = adm = a\rho_0 dV$

$dF = -A_T dP,$

$A_T \cdot dx = dV$

$dF = -\frac{\partial P}{\partial x} dV.$



$a = \partial^2 \xi / \partial t^2$

$dF = \rho_0 dV \frac{\partial^2 \xi}{\partial t^2}$

$\rho_0 dV \frac{\partial^2 \xi}{\partial t^2} = -\frac{\partial P}{\partial x} dV$

$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = -\frac{\partial P}{\partial x}.$

$dm = \rho_0 dV$

Anche quando passa l'onda, la massa è costante

$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = -\frac{\partial P}{\partial \rho} \frac{\partial \rho}{\partial x}$ ← $\frac{\partial P}{\partial x} = \frac{\partial P}{\partial \rho} \frac{\partial \rho}{\partial x},$

$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = -c_s^2 \frac{\partial \rho}{\partial x}$ ← $c_s^2 = \frac{\partial P}{\partial \rho}$

$dV = A_T dx$

$\rho_0 dV = \rho A_T (dx + d\xi)$

dV_{FINALE}

$\rho_0 dx = \rho (dx + d\xi).$

"a tempo fissato"

$d\xi = (\partial \xi / \partial x) dx$

+ raccogli dx

$\rho_0 dx = \rho \left(1 + \frac{\partial \xi}{\partial x}\right) dx$

$$\frac{1}{1+a} \approx 1-a,$$

$$\rho_0 = \rho\left(1+\frac{\partial \xi}{\partial x}\right)$$

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = -c_s^2 \frac{\partial \rho}{\partial x}$$

$$\rho = \rho_0 \left(1+\frac{\partial \xi}{\partial x}\right)^{-1} \approx \rho_0 \left(1-\frac{\partial \xi}{\partial x}\right)$$

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = c_s^2 \rho_0 \frac{\partial^2 \xi}{\partial x^2}$$

derivi su x

$$\frac{\partial \rho}{\partial x} = -\rho_0 \frac{\partial^2 \xi}{\partial x^2}$$

$$\frac{\partial^2 \xi}{\partial t^2} = c_s^2 \frac{\partial^2 \xi}{\partial x^2}.$$

□

(1)

$dV = A \cdot dx$

$dp = \frac{\partial p}{\partial x} dx$

$dF = -A\,dp = -A \cdot \frac{\partial p}{\partial x} \cdot dx =$

$= -\frac{\partial p}{\partial x} dV$

(2)

$dx \Rightarrow dx + \partial\xi$

$a = \frac{\partial^2 \xi}{\partial t^2}$

$F = m \cdot a \Rightarrow dF = a \cdot dm$

$\Rightarrow dF = a \cdot \rho_0 dV$

$\Rightarrow dF = \frac{\partial^2 \xi}{\partial t^2} \rho_0 dV$

$\rho_0 = \frac{dm}{dV}$

$dm = \rho_0 dV$

$$-\frac{\partial p}{\partial x} dV = \frac{\partial^2 \xi}{\partial t^2} \rho_0 dV$$

(3)

$\frac{\partial^2 \xi}{\partial t^2} \rho_0 = -\frac{\partial p}{\partial x}$

$\frac{\partial^2 \xi}{\partial t^2} \rho_0 = -\frac{\partial p}{\partial \rho} \frac{\partial p}{\partial x}$ ($\frac{\partial p}{\partial \rho} = c_s^2$)

$\frac{\partial^2 \xi}{\partial t^2} \rho_0 = -c_s^2 \frac{\partial p}{\partial x}$

(4)

$m_{iv} = m_{fin}$

$\rho_0 \cdot dV = \rho \cdot dV_{fin}$

$\rho_0 A \cdot dx = \rho \cdot A (dx + \partial\xi)$

$\rho_0 dx = \rho (dx + \partial\xi)$

Fissato il Tempo: $\partial\xi = \frac{\partial \xi}{\partial x} dx$

$\rho_0 dx = \rho \left(dx + \frac{\partial \xi}{\partial x} dx\right) = \rho \left(1 + \frac{\partial \xi}{\partial x}\right) dx$

$\rho_0 = \rho \left(1 + \frac{\partial \xi}{\partial x}\right) \Rightarrow \rho \approx \rho_0 \left(1 - \frac{\partial \xi}{\partial x}\right)$

$\frac{\partial p}{\partial x} = -\rho_0 \cdot \frac{\partial^2 \xi}{\partial x^2}$

$$\frac{\partial^2 \xi}{\partial t^2} \cancel{\rho_0} = +c_s^2 \cdot \cancel{\rho_0} \cdot \frac{\partial^2 \xi}{\partial x^2}$$