Calculators may be used in this examination provided they are not capable of being used to store alphabetical information other than hexadecimal numbers

UNIVERSITY OF BIRMINGHAM

School of Computer Science

Evolutionary Computation

Main Summer Examinations 2023

Time allowed: 2 hours

[Answer all questions]

Note

Answer ALL questions. Each question will be marked out of 20. The paper will be marked out of 60, which will be rescaled to a mark out of 100.

Question 1

The travelling salesman problem (TSP) can be described as follows: Given N cities, $1, 2, \cdots, N$ and the distances $d_{i,j}$ between each pair of them (here i and j are one of the $1, 2, \cdots, N$ cities), find a permutation $(x_1, x_2, \cdots, x_N)$ of $(1, 2, \cdots, N)$ such that the sum of distances $D = d(x_1, x_2) + d(x_1, x_2) + \cdots + d(x_{N-1}, x_N) + d(x_N, x_1)$ is minimum.

(a) What are two immediate neighbourhood solutions of a TSP? Illustrate your answer by drawing one immediate neighbourhood solution of the solution to a 6-city TSP in Figure 1.

[3 marks]

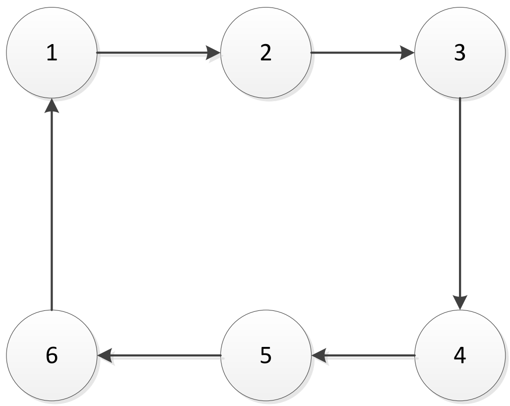

Figure 1: A solution (route) for a 6-city TSP: $1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 4 \longrightarrow 5 \longrightarrow 6$

(b) Based on your observation, derive the pseudocode of the 2-opt algorithm which searches for immediate neighbourhood solutions for TSP.

[3 marks]

(c) Discuss the pros and cons of exhaustive search, local search and stochastic local search algorithms. Comment on the suitability of for solving large-scale TSP with more than 10000 cities.

[6 marks]

(d) Read the pseudocode in Table 1, explain what is the main intuition behind Simulated Annealing. Based on the intuition, write down the function $P(e, e_{new}, T)$.

[8 marks]

```
x := x_0; e := f(x)        // x_0: initial solution; f(x): objective function
x_best := x;        // Initial "best" solution
k := 0
while (k < k_max)
        T := temperature(t_0)        // Temperature calculation.
        x_new := neighbour(x)        // Pick some neighbour.
        e_new := f(x_new)        // Compute its objective function value.
        if P(e, e_new, T) > R(0, 1) then        // Should we move to it?
                x := x_new; e := e_new        // Yes, change state.
        if e_new < e_best then        // Is this a new best?
                x_best := x_new; e_best := e_new        // Save as 'best found'.
        k := k + 1
Output x_best
```

Table 1: Pseudocode of Simulated Annealing algorithm for minimisation. Annealing schedule $temperature()$ defines how to decreased temperature from an initial temperature t_0.

Question 2

Consider the Prisoner's Dilemma (PD) payoff matrix $\begin{pmatrix} R & S \\ T & P \end{pmatrix}$, using the parameters $R = 4, S = 0, T = 6, P = 1$. We use the notation C (cooperate) for the first strategy and D (defect) for the second strategy.
Assume a population size of N. Let i be the number of agents play C at a given time, hence the relative fraction of C playing agents becomes $x := i/N$.

(a) Calculate the payoff functions $\pi_C(x), \pi_D(x)$ for an agent playing C or D, respectively. **[4 marks]**

(b) Calculate the average payoff, again assuming a fraction x of agents plays C. **[2 marks]**

(c) Derive the replicator equation for this game. **[4 marks]**

(d) Characterize the type of differential equation you obtained in (c): of which order? How many dimensions? Linear or nonlinear? **[3 marks]**

(e) Determine the fixed points and judge on their stability. **[3 marks]**

(f) Define a Nash equilibrium. Identify the Nash equilibria of this game from the payoff matrix. Discuss the relation to (e). **[4 marks]**

Question 3

We consider evolutionary optimization of connected undirected graphs of 4 nodes only, i.e., unconnected graphs are discouraged (or assigned very low fitness values).

(a) Draw all possible connected undirected graphs with 4 links (trying to avoid isomorphous graphs).

[4 marks]

(b) For each graph, identify the sequence $k_1 k_2 k_3 k_4$ of node degrees, starting from highest node degree and descending.

[3 marks]

(c) Peter and Clyde want to optimize the graphs according to two criteria. Peter wants the graph to be of intermediate link density (between sparse and fully connected). As the average of the maximal and minimal possible total number m of links in the graph is 4.5, Peter chooses to minimize the optimization function $F_1 = |m - 4.5|$. Clyde wants to have as much node degree variability as possible and chooses to maximize the optimization function $F_2 = (k_1 - k_4) + (k_2 - k_3)$.
Calculate F_1 and F_2 for all graphs and plot the results in the (F_1, F_2) plane. Identify, for each point, which graph(s) these correspond to.

[6 marks]

(d) Define a Pareto front and explain its purpose.

[2 marks]

(e) For the case discussed in (c), which graphs comprise the Pareto front?

[2 marks]

(f) Choose one of the graphs with 4 or 5 links.
For this graph, write down the adjacency matrix.
For this graph, calculate the local clustering coefficient for each node.
For this graph, calculate the average clustering coefficient.

[3 marks]

Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.