Calculators may be used in this examination provided they are not capable of being used to store alphabetical information other than hexadecimal numbers

UNIVERSITY OF BIRMINGHAM

School of Computer Science

Advanced Robotics

Main Summer Examinations 2023

Time allowed: 2 hours

[Answer all questions]

Question 1

Consider the following matrix

$$\mathbf{R}_A^B(\phi,\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ \sin(\phi)\sin(\theta) & \cos(\phi) & -\sin(\phi)\cos(\theta) \\ -\cos(\phi)\sin(\theta) & \sin(\phi) & \cos(\phi)\cos(\theta) \end{bmatrix}$$

which denotes a rotation matrix of frame A with respect to the base frame B.

(a) Write three properties of rotation matrices and prove that $\mathbf{R}_\mathrm{A}^\mathrm{B}$ satisfies one of those properties. **[12 marks]**

(b) What is the sequence of two elementary rotations around the **fixed** frame in order to obtain frame A from frame B? **[4 marks]**

(c) What is the sequence of two elementary rotations around the **current** frame in order to obtain frame A from frame B? **[4 marks]**

Note that, if $c = cos\psi$ and $s = sin\psi$. elementary rotation matrices are

$$R_x(\psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & -s \\ 0 & s & c \end{bmatrix}, R_y(\psi) = \begin{bmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{bmatrix}, R_x(\psi) = \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 2

Recall the definition of the Denavit-Hartenberg parameters:

- d: offset along previous z-axis to the common normal
- θ : angle about previous z-axis from old x-axis to new x-axis
- a: length of common normal
- α: angle about common normal, from old z-axis to new z-axis

(a) Assign proper frames to each link of the manipulator shown on Figure 2. All frames, including the base frame must be drawn. **[8 marks]**

(b) Derive the table of Denavit-Hartenberg (DH) parameters. **[12 marks]**

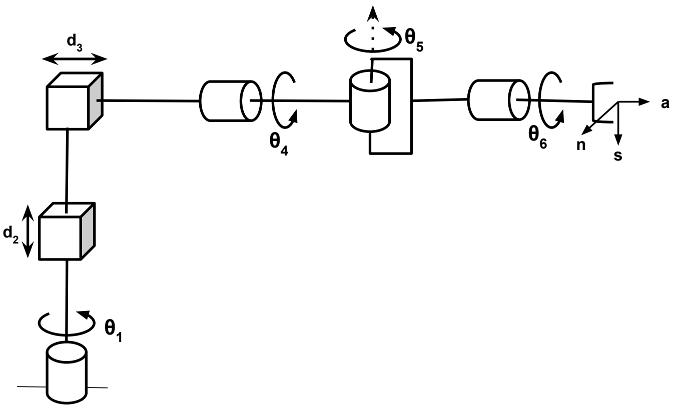

Figure 1: Robot manipulator for Question 2.

Question 3

Consider a 2DOF RR (Revolute Revolute) manipulator robot. The homogeneous transformation matrix for each link is:

$$A_1^0 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_2^1 = \begin{bmatrix} c_2 & s_2 & 0 & l_2c_2 \\ s_2 & c_2 & 0 & l_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where l_1 and l_2 are constant parameters, θ_1 and θ_2 are joint variables, $s_1 = \sin(\theta_1)$, $c_1 = \cos(\theta_1)$, $s_2 = \sin(\theta_2)$ and $c_2 = \cos(\theta_2)$. Calculate the geometric Jacobian of the robot's end effector given the formula below. Clearly explain every step of your calculations for each column of the Jacobian matrix. **[20 marks]**

$$\begin{bmatrix} \mathbf{J}_{P_i} \\ \mathbf{J}_{O_i} \end{bmatrix} = \begin{bmatrix} z_{i-1} \times (p_e - p_{i-1}) \\ z_{i-1} \end{bmatrix}$$

Question 4

Consider the 3DOF manipulator robot shown in Figure 2, with two revolute joints and one prismatic joint.The first link of the robot is a revolute joint attached to the ground; the link's length is l_1 and its center of mass (CoM) is at its center. The second link is perpendicular to the first link and slides through a fixed joint connected to the end of the first link; its CoM is at its end, i.e., at its connection with the third link. The third link, with length l_3 and CoM at the center of the link, is connected to the end of the second link through a revolute joint.

(a) Calculate the position, velocity, and acceleration of the CoM of the links in the base frame. **[8 marks]**

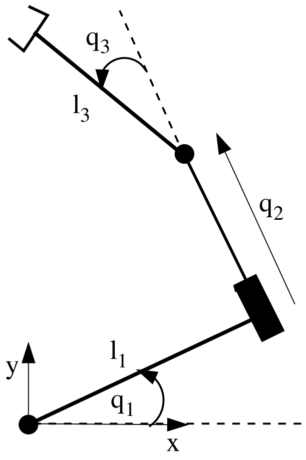

Figure 2: 3DOF manipulator robot.

(b) Assume that the moment of inertia of each link is I_z, and that the mass of the links are m_1, m_2, and m_3 respectively. Calculate the energy for each link and the Lagrangian expression for the robot. Show all intermediate steps of your computation. **[12 marks]**

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Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.