2IMW30 - FOUNDATION OF DATA MINING
Assignment 1c Report

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EXERCISE 1

In this assignment, we implement the Random Projection Algorithm for reducing the number of dimensions in the MNIST dataset.

EXERCISE 2

Since the nearest neighbor in F of a point P in R^d is the projection of P onto F, a linear mapping that maps a point to its nearest neighbor in a subspace can be simulated by a rotation followed by an orthogonal projection.

Let P_1 and P_2 be 2 points in R^d, M be the rotation matrix, and P be the projection matrix.

Let $Q_1 = M \cdot P_1$, $Q_2 = M \cdot P_2$, $P_1' = P \cdot Q_1$, and $P_2' = P \cdot Q_2$. So P_1' and P_2' are the resulting points.

We know that Rotation preserves the Euclidean distance, so $Q_1Q_2 = P_1P_2$.

Because P contains only 0 and 1, multiplying P to Q_1 and Q_2 preserves several features of Q_1 and Q_2, and set everything else to 0. Thus, $(P_1' - P_2') \leq (Q_1 - Q_2)$, which gives $(P_1' - P_2') \leq (P_1 - P_2)$.

This completes the proof.

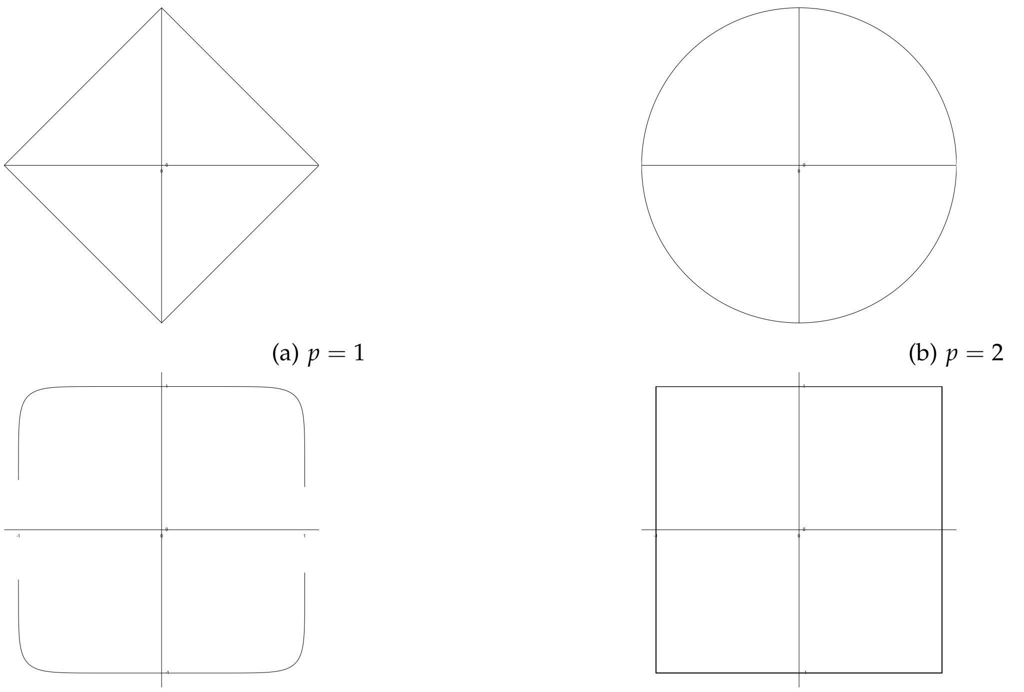
EXERCISE 3

a

Let $d(x, y)$ be the distance from a point $P(x, y)$ to the center of the circle. Then the unit circle equation is $d(x, y) = 1$.

- When $p = 1$, we have $x + y = 1$
- When $p = 2$, we have $\sqrt{x^2 + y^2} = 1$
- When $p = 10$, we have $(x^{10} + y^{10})^{\frac{1}{10}} = 1$
- When $p = \infty$, we have $max(x, y) = 1$

Plotting the graphs of those equations, we have the unit circles, shown in figure 1.



(a) $p = 1$ (b) $p = 2$ (c) $p = 10$ (d) $p = infi$

Figure 1: Unit Circles

b

We can see that the 2 unit circles (the 2 squares indeed) are quite similar. The differences are the angle ($pi/4$) and the length of the edge (2 and $\sqrt{2}$). Thus, we can preserve the distance by doing a rotation by an angle of $pi/4$ followed by a scaling by a factor of$\sqrt{2}$.

The rotation matrix is:

$$\begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

So the mapping function can be done by multiplying the following matrix to the original data:

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Now we prove that the mapping function actually preserves the distance. As we know, the difference between 2 vectors is also a vector. Let $p = (a, b)$ be a difference between 2 vectors in space. After the transformation, $p' = (a - b, a + b)$. We have:

$$||p||_1 = |a| + |b|$$
$$||p'||_\infty = max(|a - b|, |a + b|) = |a| + |b|$$

So $||p||_1 = ||p'||_\infty$. This completes the proof.